

# Introduction to Complex Numbers

YouTube Workbook

Christopher C. Tisdell



Christopher C. Tisdell

# **Introduction to Complex Numbers:** *YouTube Workbook*



Introduction to Complex Numbers: *YouTube* Workbook

1<sup>st</sup> edition

© 2015 Christopher C. Tisdell & [bookboon.com](http://bookboon.com)

ISBN 978-87-403-1110-5

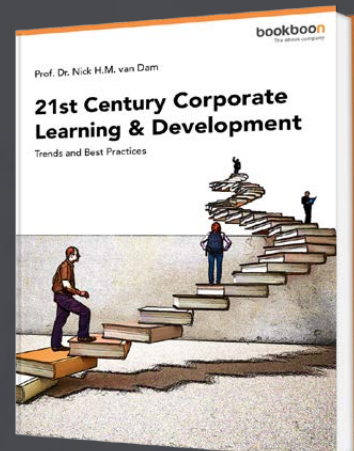
# Contents

	<b>How to use this workbook</b>	<b>8</b>
	<b>About the author</b>	<b>9</b>
	<b>Acknowledgments</b>	<b>10</b>
<b>1</b>	<b>What is a complex number?</b>	<b>11</b>
1.1	Video 1: Complex numbers are AWESOME	11
<b>2</b>	<b>Basic operations involving complex numbers</b>	<b>15</b>
2.1	Video 2: How to add/subtract two complex numbers	15
2.2	Video 3: How to multiply a real number with a complex number	16
2.3	Video 4: How to multiply complex numbers together	17
2.4	Video 5: How to divide complex numbers	19
2.5	Video 6: Complex numbers: Quadratic formula	21

## Free eBook on Learning & Development

By the Chief Learning Officer of McKinsey

[Download Now](#)



<b>3</b>	<b>What is the complex conjugate?</b>	<b>22</b>
3.1	Video 7: What is the complex conjugate?	22
3.2	Video 8: Calculations with the complex conjugate	25
3.3	Video 9: How to show a number is purely imaginary	27
3.4	Video 10: How to prove the real part of a complex number is zero	28
3.5	Video 11: Complex conjugate and linear systems	29
3.6	Video 12: When are the squares of $z$ and its conjugate equal?	30
3.7	Video 13: Conjugate of products is product of conjugates	31
3.8	Video 14: Why complex solutions appear in conjugate pairs	32
<b>4</b>	<b>How big are complex numbers?</b>	<b>33</b>
4.1	Video 15: How big are complex numbers?	33
4.2	Video 16: Modulus of a product is the product of moduli	35
4.3	Video 17: Square roots of complex numbers	36
4.4	Video 18: Quadratic equations with complex coefficients	37
4.5	Video 19: Show real part of complex number is zero	38
<b>5</b>	<b>Polar trig form</b>	<b>39</b>
5.1	Video 20: Polar trig form of complex number	39



The advertisement features a dark blue background with a large, stylized, light blue graphic of a lion's head. The text "YOU THINK. YOU CAN WORK AT RMB" is written in large, white, sans-serif capital letters across the center. In the top right corner, there is a white box containing the Rand Merchant Bank logo (a lion with a key) and the text "RAND MERCHANT BANK", "A division of FirstRand Bank Limited", and "Traditional values. Innovative ideas."

Rand Merchant Bank uses good business to create a better world, which is one of the reasons that the country's top talent chooses to work at RMB. For more information visit us at [www.rmb.co.za](http://www.rmb.co.za)

Thinking that can change your world

Rand Merchant Bank is an Authorised Financial Services Provider



Click on the ad to read more

<b>6</b>	<b>Polar exponential form</b>	<b>41</b>
6.1	Video 21: Polar exponential form of a complex number	41
6.2	Revision Video 22: Intro to complex numbers + basic operations	43
6.3	Revision Video 23: Complex numbers and calculations	44
6.4	Video 24: Powers of complex numbers via polar forms	45
<b>7</b>	<b>Powers of complex numbers</b>	<b>46</b>
7.1	Video 25: Powers of complex numbers	46
7.2	Video 26: What is the power of a complex number?	47
7.3	Video 27: Roots of complex numbers	48
7.4	Video 28: Complex numbers solutions to polynomial equations	49
7.5	Video 29: Complex numbers and $\tan(\pi/12)$	50
7.6	Video 30: Euler's formula: A cool proof	51
<b>8</b>	<b>De Moivre's formula</b>	<b>52</b>
8.1	Video 31: De Moivre's formula: A cool proof	52
8.2	Video 32: Trig identities from De Moivre's theorem	53
8.3	Video 33: Trig identities: De Moivre's formula	54



Discover the truth at [www.deloitte.ca/careers](http://www.deloitte.ca/careers)

**Deloitte.**

© Deloitte & Touche LLP and affiliated entities.



Click on the ad to read more

<b>9</b>	<b>Connecting sin, cos with <math>e</math></b>	<b>55</b>
9.1	Video 34: Trig identities and Euler's formula	55
9.2	Video 35: Trig identities from Euler's formula	57
9.3	Video 36: How to prove trig identities WITHOUT trig!	58
9.4	Revision Video 37: Complex numbers + trig identities	59
<b>10</b>	<b>Regions in the complex plane</b>	<b>60</b>
10.1	Video 38: How to determine regions in the complex plane	60
10.2	Video 39: Circular sector in the complex plane	63
10.3	Video 40: Circle in the complex plane	64
10.4	Video 41: How to sketch regions in the complex plane	65
<b>11</b>	<b>Complex polynomials</b>	<b>66</b>
11.1	Video 42: How to factor complex polynomials	66
11.2	Video 43: Factorizing complex polynomials	68
11.3	Video 44: Factor polynomials into linear parts	69
11.4	Video 45: Complex linear factors	70
	<b>Bibliography</b>	<b>71</b>

[bookboon.com](http://bookboon.com)

# Corporate eLibrary

See our Business Solutions for employee learning

[Click here](#)



Management      Time Management

Problem solving      Self-Confidence      Effectiveness

Project Management      Goal setting      Motivation      Coaching



# How to use this workbook

This workbook is designed to be used in conjunction with the author's free online video tutorials. Inside this workbook each chapter is divided into learning modules (subsections), each having its own dedicated video tutorial.

View the online video via the hyperlink located at the top of the page of each learning module, with workbook and paper or tablet at the ready. Or click on the *Introduction to Complex Numbers* playlist where all the videos for the workbook are located in chronological order:

*Introduction to Complex Numbers*

[www.youtube.com/playlist?list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP](http://www.youtube.com/playlist?list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP)

[www.tinyurl.com/ComplexNumbersYT](http://www.tinyurl.com/ComplexNumbersYT).

While watching each video, fill in the spaces provided after each example in the workbook and annotate to the associated text.

You can also access the above via the author's YouTube channel

[Dr Chris Tisdell's YouTube Channel](#)

<http://www.youtube.com/DrChrisTisdell>

There has been an explosion in books that blend text with video since the author's pioneering work *Engineering Mathematics: YouTube Workbook* [46]. The current text takes innovation in learning to a new level, with:

- the video presentations herein streamed live online, giving the classes a live, dynamic and fun feeling;
- each video featuring closed captions, providing each learner with the ability to watch, read or listen to each video presentation.



# About the author

Dr Chris Tisdell is Associate Dean (Education), Faculty of Science at UNSW Australia who has inspired millions of learners through his passion for mathematics and his innovative online approach to maths education. He is best-known for creating YouTube university-level maths videos, which have attracted millions of downloads. This has made his virtual classroom the top-ranked learning and teaching website across Australian universities on the education hub YouTube EDU.

His free online e textbook, *Engineering Mathematics: YouTube Workbook*, is one of the most popular mathematical books of its kind, with more than 1 million downloads in over 200 countries. A champion of free and flexible education, he is driven by a desire to ensure that anyone, anywhere at any time, has equal access to the mathematical skills that are critical for careers in science, engineering and technology.

Vision, leadership and management skills underpins his experience in educational change. In 2008 he dared to dream of educational experiences that featured personalized and scalable learning. His early leadership on enabling technologies such as: lecture capture; open educational resources; MOOCs; learning analytics; and gamification, has significantly influenced and positively changed L&T strategies at the institutional level.

He is a recognized leader in the online learning space at national and institutional levels, winning education awards and positively transforming learning and teaching.

As an Associate Dean (Education) at UNSW Australia he has been responsible for leading, managing and operationalising educational change at-scale, including inspiring positive transformation within 7,000 7,000 science students, 400 academic staff, 300+ courses and scores of programs within UNSW Science.

Chris has collaborated with industry and policy-makers, championed educational thought-leadership in the media and constantly draws on the feedback of key stakeholders worldwide to advance learning and teaching.

# Acknowledgments

I'm grateful to the following, who admirably transcribed audio to text for each video to create closed captions and helped me proofread drafts of the manuscript. **Thank you:**

Anubhav Ashish; Johann Blanco; Sean Cossins; Jonathan Kim Sing; Madeleine Kyng; Jeffrey Lay; Harris Phan; Anthony Tran; Koha Tran; Ines Vallely; Velushomaz; Wilson Yuan.

I would also like to express my thanks to the Bookboon team for their support.

# 1 What is a complex number?

## 1.1 Video 1: Complex numbers are AWESOME

### 1.1.1 Where are we going?

[View this lesson on YouTube](#) [1]

- We will learn about a new kind of number known as a “complex number”.
- We will discover the basic properties of complex numbers and investigate some of their mathematical applications.

Complex numbers rest on the idea of the “imaginary unit”  $i$ , which is dened via

$$i = \sqrt{-1}$$

with  $i$  satisfying the equation

$$i^2 = -1.$$

Even though the thought of  $i$  may seem crazy, we will see that is a really useful idea.

### 1.1.2 Why are complex numbers AWESOME?

There are at least two reasons why complex numbers are AWESOME:-

1. their real-world applications;
2. their ability to SIMPLIFY mathematics.

For example,  $i$  arises in the solutions

$$x(t) = e^{i\sqrt{k/m} t} \text{ and } x(t) = e^{-i\sqrt{k/m} t}.$$

to a basic spring-mass differential equation

$$m \frac{d^2 x}{dt^2} + kx = 0$$

where:  $x = x(t)$  is the position of the mass at time  $t$ ;  $m > 0$  is the mass; and  $k > 0$  is the stiffness of the spring.

Also,  $i$  appears in Fourier transform techniques, which are important for solving partial differential equations from science and engineering.

Complex numbers are AWESOME because they provide a SIMPLER framework from which we can view and do mathematics.

As a result, applying methods involving complex numbers can simplify calculations, removing a lot of the boring and tedious parts of mathematical work.

For example, complex numbers provides a quick alternative to integration by parts for something like

$$\int e^{-t} \cos t dt$$

and gives easy ways of constructing trig formulae, for example

$$\begin{aligned} \sin(x + y) &= \sin x \cos y + \cos x \sin y \\ \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \end{aligned}$$

so you might never have to remember another trig formula ever again!

### 1.1.3 What is a complex number?

Here are some examples of complex numbers:

$$\begin{array}{ll} 3 + 2i, & -7 + 3i, \\ 6 - i, & 2i, \\ -1 - 4i, & -2 - 2i. \end{array}$$

**Important idea** (What is a complex number? (Cartesian form)).

The Cartesian form of a complex number  $z$  is

$$x + yi \quad \text{or} \quad x + iy$$

where  $x$  and  $y$  are both real numbers and  $i$  is known as the imaginary unit  $i = \sqrt{-1}$  and satisfies  $i^2 = -1$ . The number  $x$  is called the “real part of  $z$ ”; while  $y$  is called the “imaginary part of  $z$ ”.



The advertisement features a central image of a smiling teacher leaning over a laptop to assist two young students, a boy and a girl. To the right, two smaller circular images show students working together and students at a computer workstation. The logo for 'e-learning for kids' is in the top left. A green oval on the right contains the following statistics:

- The number 1 MOOC for Primary Education
- Free Digital Learning for Children 5-12
- 15 Million Children Reached

**About e-Learning for Kids** Established in 2004, e-Learning for Kids is a global nonprofit foundation dedicated to fun and free learning on the Internet for children ages 5 - 12 with courses in math, science, language arts, computers, health and environmental skills. Since 2005, more than 15 million children in over 190 countries have benefitted from eLessons provided by EFK! An all-volunteer staff consists of education and e-learning experts and business professionals from around the world committed to making difference. eLearning for Kids is actively seeking funding, volunteers, sponsors and courseware developers; get involved! For more information, please visit [www.e-learningforkids.org](http://www.e-learningforkids.org).



#### 1.1.4 How to graphically represent complex numbers?

Complex numbers can be represented in the "complex plane" via what is known as an Argand diagram, which features:

- a "real" (horizontal) axis;
- an "imaginary" (vertical) axis.

## 2 Basic operations involving complex numbers

### 2.1 Video 2: How to add/subtract two complex numbers

[View this lesson on YouTube](#) [3]

To add/subtract two complex numbers just add/subtract their corresponding components.

**Example.**

If  $z = 1 + 3i$  and  $w = 2 + i$  then

$$\begin{aligned}z + w &= (1 + 3i) + (2 + i) \\ &= (1 + 2) + (3i + i) \\ &= 3 + 4i\end{aligned}$$

and

$$\begin{aligned}z - w &= (1 + 3i) - (2 + i) \\ &= (1 - 2) + (3i - i) \\ &= -1 + 2i.\end{aligned}$$

A geometric interpretation of addition is seen through a simple parallelogram or triangle law.



**Brain power**

By 2020, wind could provide one-tenth of our planet's electricity needs. Already today, SKF's innovative know-how is crucial to running a large proportion of the world's wind turbines.

Up to 25 % of the generating costs relate to maintenance. These can be reduced dramatically thanks to our systems for on-line condition monitoring and automatic lubrication. We help make it more economical to create cleaner, cheaper energy out of thin air.

By sharing our experience, expertise, and creativity, industries can boost performance beyond expectations. Therefore we need the best employees who can meet this challenge!

The Power of Knowledge Engineering

Plug into The Power of Knowledge Engineering.  
Visit us at [www.skf.com/knowledge](http://www.skf.com/knowledge)

**SKF**

## 2.2 Video 3: How to multiply a real number with a complex number

[View this lesson on YouTube](#) [3]

Multiplication of a real number with a complex number involves multiplying each component in a natural distributive fashion.

**Example.**

If  $z = 2 + 3i$  then

$$\begin{aligned}2z &= 2(2 + 3i) \\ &= (2 * 2) + (2 * 3i) \\ &= 4 + 6i\end{aligned}$$

and

$$\begin{aligned}-4z &= -4(2 + 3i) \\ &= (-4 * 2) + (-4 * 3i) \\ &= -8 - 12i.\end{aligned}$$

A geometric interpretation of (scalar) multiplication is seen through a stretching principle.



## 2.3 Video 4: How to multiply complex numbers together

[View this lesson on YouTube](#) [4]

Multiplication of two complex numbers involves natural distribution (and remembering  $i^2 = -1$ ).

**Example.**

If  $z = 2 + i$  and  $w = 1 + i$  then

$$\begin{aligned}zw &= (2 + i)(1 + i) \\ &= (2 * 1 + i * i) + (2 * i + i * 1) \\ &= (2 - 1) + 3i \\ &= 1 + 3i.\end{aligned}$$

The geometric interpretation of multiplication is seen through rotation and stretching/compression.

With us you can  
shape the future.  
Every single day.

For more information go to:  
[www.eon-career.com](http://www.eon-career.com)

Your energy shapes the future.

**e-on**



2.3.1 What is the geometric explanation of multiplication?

**Example.**

Let us consider  $z = 2i$  and  $w = 1 + i$  in the complex plane.

If we compute the distances from  $z$  and  $w$  to the origin (using Pythagoras) then we see that

$$|z| = 2, \quad |w| = \sqrt{2}.$$

Now consider the line segments joining  $z$  and  $w$  to the origin. If we compute the angles  $\theta_1, \theta_2$  to the positive real axis (using trig) with  $-\pi < \theta_k \leq \pi$  then we see

$$\theta_1 = \pi/2, \quad \theta_2 = \pi/4.$$

Now consider  $zw = -2 + 2i$ . We have

$$|zw| = 2\sqrt{2}, \quad \theta_3 = 3\pi/4.$$

We thus see that  $|zw| = |z| |w|$  and  $\theta_3 = \theta_1 + \theta_2$ .

## 2.4 Video 5: How to divide complex numbers

[View this lesson on YouTube](#) [5]

### 2.4.1 How to divide by a complex number

Division of two complex numbers involves multiplying through by a “factor of one” that turns the denominator into a real number. To do this, we use the “conjugate” of the denominator.

**Example.**

If  $z = 2 + i$  and  $w = 3 + 2i$  then

$$\begin{aligned}\frac{z}{w} &= \frac{2 + i}{3 + 2i} \\ &= \frac{2 + i}{3 + 2i} * \frac{3 - 2i}{3 - 2i} \\ &= \frac{(6 - 2i^2) + (3i - 4i)}{(9 - 4i^2) + (6i - 6i)} \\ &= \frac{8 - i}{13} = \frac{8}{13} - i\frac{1}{13}.\end{aligned}$$

Observe that the denominator is now real and we can (say) easily plot the complex number  $z/w$ .

If we interpret division as a kind of multiplication, then the geometric interpretation of division can also be seen through rotation/stretching.

2.4.2 Basic operations with complex numbers

**Example.**

If  $z = -2 + 3i$  then calculate  $z^2$ .

Consider

$$\begin{aligned} z^2 &= (-2 + 3i) * (-2 + 3i) \\ &= (4 + 9i^2) - 6i - 6i \\ &= -5 - 12i. \end{aligned}$$

Independent learning exercise: plot  $z$  and  $z^2$ . Can you see a relationship between their lengths to the origin?

© 2013 Accenture. All rights reserved.

be > your degree

Bring your talent and passion to a global organization at the forefront of business, technology and innovation. Discover how great you can be.

Visit [accenture.com/bookboon](http://accenture.com/bookboon)

**Be greater than.**  
consulting | technology | outsourcing

**accenture**  
High performance. Delivered.



## 2.5 Video 6: Complex numbers: Quadratic formula

### Applying the quadratic formula for complex solutions

[View this lesson on YouTube](#) [6]

**Example.**

Solve the quadratic equation

$$13z^2 - 6z + 1 = 0,$$

writing the solutions in the Cartesian form  $x + yi$ .

# 3 What is the complex conjugate?

## 3.1 Video 7: What is the complex conjugate?

[View this lesson on YouTube](#) [7]

As we saw when performing division of complex numbers, an idea called the conjugate was applied to simplify the denominator. Let us look at this idea a bit further.

**Important idea** (Complex conjugate).

For a complex number  $z = x + yi$  we define and denote the “complex conjugate of  $z$ ” by

$$\bar{z} = x - yi.$$

If  $z = 3 + i$  then  $\bar{z} = 3 - i$ . If  $w = 1 - 2i$  then  $\bar{w} = 1 + 2i$ . If  $u = -1 - i$  then  $\bar{u} = -1 + i$ .

For any point  $z$  in the complex plane, we can geometrically determine  $\bar{z}$  by reflecting the position of  $z$  through the real axis.

3.1.1 What are the properties of the conjugate?

**Important idea** (Conjugate properties).

Let  $z = a + bi$  and  $w = c + di$ . Some basic properties of the conjugate are:-

$$z\bar{z} = (a + bi)(a - bi) = a^2 + b^2, \text{ real and non\{neg number;}$$

$$\bar{\bar{z}} = z;$$

$$\overline{z + w} = \bar{z} + \bar{w} = (a + c) - (b + d)i;$$

$$\overline{z - w} = \bar{z} - \bar{w} = (a - c) + (d - b)i;$$

$$\overline{z\bar{w}} = \bar{z}w;$$

$$\overline{z/w} = \bar{z}/\bar{w};$$

$$\overline{z^n} = \bar{z}^n;$$

$$\frac{z + \bar{z}}{2} = a = \Re(z);$$

$$\frac{z - \bar{z}}{2} = b = \Im(z).$$

"I studied English for 16 years but...  
...I finally learned to speak it in just six lessons"  
Jane, Chinese architect

ENGLISH OUT THERE

Click to hear me talking before and after my unique course download



3.1.2 Basic operations with the conjugate

**Example.**

If  $z = -2 + 3i$  then calculate the following: a)  $\bar{z}$ ;      b)  $z + \bar{z}$ .

By definition,

$$\bar{z} = -2 - 3i.$$

Also,

$$\begin{aligned} z + \bar{z} &= (-2 + 3i) + (-2 - 3i) \\ &= -4 + 0i \\ &= 4. \end{aligned}$$



### 3.2 Video 8: Calculations with the complex conjugate

[View this lesson on YouTube](#) [8]

**Example.**

If  $z = 4 - 3i$  and  $w = 1 + 4i$  then calculate the following in Cartesian form  $x + yi$ :

- a)  $25/z$ ;      b)  $iw(\bar{z} - 4)$

3.2.1 Simplifying complex numbers with the conjugate

**Example.**

Simplify

$$\frac{2 - 7i}{3 - i}$$

into the Cartesian form  $x + yi$ .

We multiply by a factor of one that involves the conjugate of the denominator, namely

$$\begin{aligned}\frac{2 - 7i}{3 - i} &= \frac{2 - 7i}{3 - i} * \frac{3 + i}{3 + i} \\ &= \frac{(6 - 7i^2) + 2i - 21i}{(9 - i^2) + 3i - 3i} \\ &= 13/10 - 19i/10.\end{aligned}$$

**DUKE**  
THE FUQUA  
SCHOOL  
OF BUSINESS

BUSINESS HAPPENS  
**HERE.**

[www.fuqua.duke.edu/globalmba](http://www.fuqua.duke.edu/globalmba)

Learn More >



### 3.3 Video 9: How to show a number is purely imaginary

#### 3.3.1 Using the conjugate to show a number is purely imaginary

[View this lesson on YouTube](#) [9]

**Example.**

Let

$$\Im\left(\frac{z+i}{z-i}\right) = 0$$

with  $z \neq i$ . Show  $\Re(z) = 0$ .

### 3.4 Video 10: How to prove the real part of a complex number is zero

[View this lesson on YouTube](#) [10]

**Example.**

Let  $z \in \mathbb{C}$  with  $|z| = 1$ . Show

$$\Re\left(\frac{z-1}{z+1}\right) = 0.$$

## Join American online LIGS University!

Interactive Online programs  
BBA, MBA, MSc, DBA and PhD

### Special Christmas offer:

- ▶ enroll **by December 18th, 2014**
- ▶ **start studying and paying only in 2015**
- ▶ **save up to \$ 1,200** on the tuition!
- ▶ Interactive Online education
- ▶ visit [ligsuniversity.com](http://ligsuniversity.com) to find out more!

Note: LIGS University is not accredited by any nationally recognized accrediting agency listed by the US Secretary of Education. More info [here](#).



### 3.5 Video 11: Complex conjugate and linear systems

#### 3.5.1 Solving systems of equations with the conjugate

[View this lesson on YouTube](#) [11]

**Example.**

Solve the following system for complex numbers  $z$  and  $w$ :

$$2z + 3w = 1 + 5i,$$

$$3\bar{z} - \bar{w} = 4 + 3i.$$

### 3.6 Video 12: When are the squares of $z$ and its conjugate equal?

#### 3.6.1 Showing real or imag parts are zero via the conjugate

[View this lesson on YouTube](#) [12]

**Example.**

Prove the following: For all  $z \in \mathbb{C}$  we have

$$z^2 = \bar{z}^2$$

if and only if

$$\Re(z) = 0 \text{ or } \Im(z) = 0.$$

### 3.7 Video 13: Conjugate of products is product of conjugates

[View this lesson on YouTube](#) [13]

**Example.**

Prove, for all complex numbers  $z$  and  $w$ :

$$\overline{zw} = \bar{z} \bar{w}.$$

### 3.8 Video 14: Why complex solutions appear in conjugate pairs

[View this lesson on YouTube](#) [14]

**Example.**

Let  $z = \alpha + \beta i$  satisfy

$$ax^2 + bx + c = 0.$$

Show that  $\bar{z}$  is also a solution.



# 4 How big are complex numbers?

## 4.1 Video 15: How big are complex numbers?

[View this lesson on YouTube](#) [15]

To measure how “big” certain complex numbers are, we introduce a way of measuring their size, known as the modulus or the magnitude.

**Important idea** (Modulus/magnitude of a complex number).

For a complex number  $z = x + yi$  we define the modulus or magnitude of  $z$  by

$$|z| := \sqrt{x^2 + y^2}.$$

Geometrically,  $|z|$  represents the length  $r$  of the line segment connecting  $z$  to the origin.

The advertisement features a background image of a man in a green jacket looking out over a city street. In the top left corner is the IE Business School logo. In the top right corner is a badge that reads "#1 EUROPEAN BUSINESS SCHOOL FINANCIAL TIMES 2013". In the center right, there is a white speech bubble containing the hashtag "#gobeyond". Below this, the text "MASTER IN MANAGEMENT" is displayed in a white box. The main body of the ad contains the following text: "Because achieving your dreams is your greatest challenge. IE Business School's Master in Management taught in English, Spanish or bilingually, trains young high performance professionals at the beginning of their career through an innovative and stimulating program that will help them reach their full potential." This is followed by a bulleted list: "Choose your area of specialization.", "Customize your master through the different options offered.", and "Global Immersion Weeks in locations such as Rio de Janeiro, Shanghai or San Francisco." Below the list is the slogan "Because you change, we change with you." At the bottom, there is a dashed line, followed by the website "www.ie.edu/master-management", an email address "mim.admissions@ie.edu", and social media icons for Facebook, Twitter, LinkedIn, YouTube, and Instagram.

#### 4.1.1 Properties of the modulus/magnitude

**Important idea.**

Let  $z = a + bi$  and  $w = c + di$ . Some basic properties of the modulus are:-

$$|z| = \sqrt{a^2 + b^2} \geq 0;$$

$$|z| = 0 \text{ iff } z = 0;$$

$$|z^2| = |z|^2;$$

$$|z + w| \leq |z| + |w|;$$

$$|\alpha z| = |\alpha||z| \text{ where } \alpha \text{ is a real number};$$

$$|zw| = |z||w|;$$

$$z\bar{z} = |z|^2.$$

**Example.**

If  $z = 7 + i$  and  $w = 3 - i$  then calculate:

$$|z + iw|.$$

**Example.**

If  $w = 1 + 4i$  then calculate the following in Cartesian form  $x + yi$ :

$$|w + 2|.$$

We have

$$\begin{aligned} |w + 2| &= |3 + 4i| \\ &= \sqrt{3^2 + 4^2} \\ &= 5. \end{aligned}$$

## 4.2 Video 16: Modulus of a product is the product of moduli

[View this lesson on YouTube](#) [16]

**Example.**

Prove, for all complex numbers  $z$  and  $w$ :

$$|zw| = |z| |w|.$$

**SMS from your computer**  
...Sync'd with your Android phone & number

**FREE**  
30 days trial!

Go to  
[BrowserTexting.com](http://BrowserTexting.com)

and start texting from your computer!

**BrowserTexting**

The advertisement features a laptop on the left displaying a web browser interface with a list of contacts and a text message conversation. A blue double-headed arrow points from the laptop to an HTC smartphone on the right, which also displays a text message conversation. The background is a light blue gradient with a dark blue diagonal banner in the top right corner.



### 4.3 Video 17: Square roots of complex numbers

[View this lesson on YouTube](#) [17]

**Example.**

Solve

$$z^2 = (x + yi)^2 = -24 - 10i$$

for  $z \in \mathbb{C}$  by computing the real numbers  $x$  and  $y$ . Hence write down the square roots of  $-24 - 10i$ .

#### 4.4 Video 18: Quadratic equations with complex coefficients

##### 4.4.1 Square roots of complex numbers

[View this lesson on YouTube](#) [18]

**Example.**

i) Solve

$$z^2 = (x + yi)^2 = 15 + 8i$$

for  $z \in \mathbb{C}$  by computing  $x$  and  $y$  which are assumed to be integers.

Hence write down the square roots of  $15 + 8i$ .

ii) Hence solve, in  $x + yi$  form,

$$z^2 - (2 + 3i)z - 5 + i = 0.$$

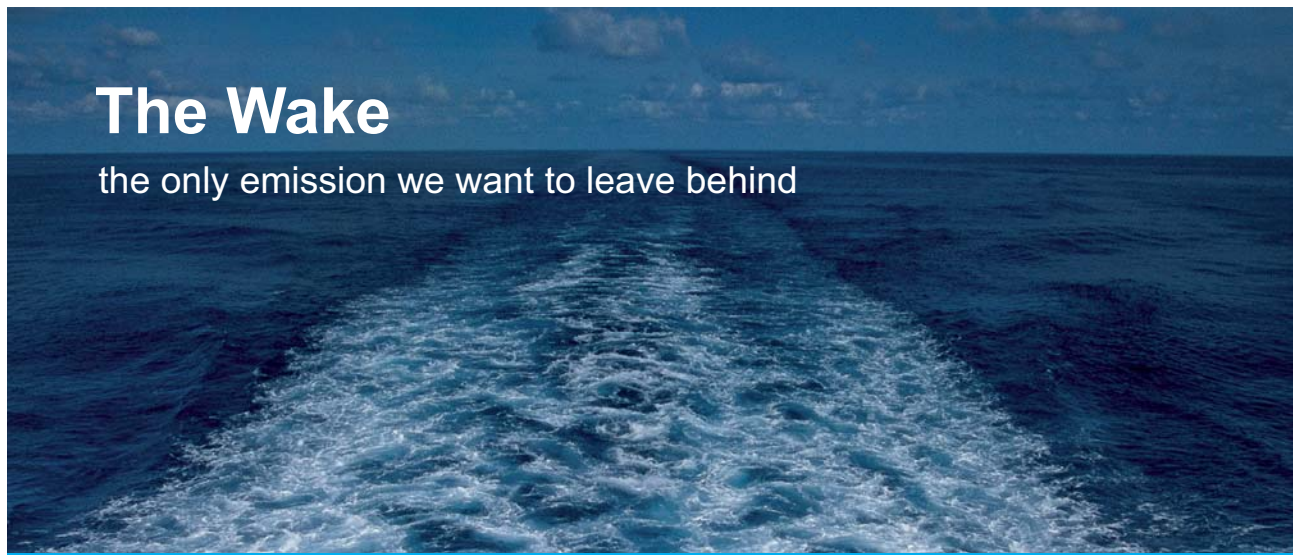
#### 4.5 Video 19: Show real part of complex number is zero

[View this lesson on YouTube](#) [19]

**Example.**

Let  $z \in \mathbb{C}$  with  $z \neq i$ . If  $|z| = 1$  then show

$$\Re\left(\frac{z+i}{z-i}\right) = 0.$$




## The Wake

the only emission we want to leave behind

Low-speed Engines Medium-speed Engines Turbochargers Propellers Propulsion Packages PrimeServ

The design of eco-friendly marine power and propulsion solutions is crucial for MAN Diesel & Turbo. Power competencies are offered with the world's largest engine programme – having outputs spanning from 450 to 87,220 kW per engine. Get up front!  
Find out more at [www.mandieselturbo.com](http://www.mandieselturbo.com)

Engineering the Future – since 1758.  
**MAN Diesel & Turbo**



 [Click on the ad to read more](#)

# 5 Polar trig form

## 5.1 Video 20: Polar trig form of complex number

[View this lesson on YouTube](#) [20]

Instead of the Cartesian  $x + yi$  form, sometimes it is convenient to express complex numbers in other equivalent forms.

Using trigonometry in the complex plane we see that we can express any (non-zero) complex number  $z$  in the form

$$z = r(\cos \theta + i \sin \theta)$$

where  $r$  is the distance to the origin and  $\theta$  is the angle to the pos. real axis.

**Important idea** (Formulae for polar trig form).

For  $z = x + yi$  a polar trig form is  $z = r(\cos \theta + i \sin \theta)$  where:

$$r = \sqrt{x^2 + y^2} = |z|;$$

$$x = r \cos \theta, \quad y = r \sin \theta, \quad \tan \theta = y/x.$$

We denote the angle  $\theta$  by  $\arg(z)$  and call  $\arg(z)$  “an argument of  $z$ ”.

Because  $\cos \theta = \cos(\theta + 2k\pi)$  and  $\sin \theta = \sin(\theta + 2k\pi)$  for all integers  $k$ , the angle  $\theta$  associated with a complex number is not unique.

For example, if  $z = 1 + i$  then we may represent  $z$  in polar trig form via

$$z = \sqrt{2}(\cos(\pi/4) + i \sin(\pi/4))$$

and

$$z = \sqrt{2}(\cos(9\pi/4) + i \sin(9\pi/4)).$$

Thus,  $\theta = \arg(z)$  is not uniquely determined by  $z$ .

To provide some definiteness, we define what is known as the principal argument of  $z$ .

**Important idea** ( $\arg(z)$  versus  $\text{Arg}(z)$ ).

For any complex number  $z = x + yi$  with  $\theta = \arg(z)$  we can always choose an integer  $k$  such that  $-\pi < \arg(z) - 2k\pi \leq \pi$ . We denote this special angle by  $\text{Arg}(z)$  and call  $\text{Arg}(z)$  “the principal argument of  $z$ ”.

## TURN TO THE EXPERTS FOR **SUBSCRIBE** CONSULTANCY

Subscribe is one of the leading companies in Europe when it comes to innovation and business development within subscription businesses.

We innovate new subscription business models or improve existing ones. We do business reviews of existing subscription businesses and we develop acquisition and retention strategies.

Learn more at [linkedin.com/company/subscribe](https://www.linkedin.com/company/subscribe) or contact  
Managing Director Morten Suhr Hansen at [mha@subscribe.dk](mailto:mha@subscribe.dk)

**SUBSCRIBE** - to the future





# 6 Polar exponential form

## 6.1 Video 21: Polar exponential form of a complex number

[View this lesson on YouTube](#) [21]

Instead of the Cartesian form  $z = x + yi$  or the polar trig form  $z = r(\cos \theta + i \sin \theta)$  sometimes it is convenient for multiplication and solving polynomials to express complex numbers in yet another equivalent form

$$z = re^{i\theta}.$$

**Important idea** (Formula for polar exponential form  $z = re^{i\theta}$ ).

For  $z = x + yi$  a polar exponential form is  $z = re^{i\theta}$  where:

$$r = \sqrt{x^2 + y^2} \text{ and } \tan \theta = y/x.$$

If we combine the polar exponential form with the polar trig form then we obtain a special identity called “Euler’s formula”

$$e^{i\theta} = \cos \theta + i \sin \theta$$

and if  $\theta = \pi$  then we obtain the famous formula

$$e^{\pi i} = -1.$$

Because  $\cos \theta = \cos(\theta + 2k\pi)$  and  $\sin \theta = \sin(\theta + 2k\pi)$  for all integers  $k$ , the angle  $\theta$  associated with a complex number is not unique.

For example, if  $z = 1 + i$  then we may represent  $z$  in polar trig and polar exp. form via

$$z = \sqrt{2}(\cos(\pi/4) + i \sin(\pi/4)) = \sqrt{2}e^{i\pi/4}$$

and

$$z = \sqrt{2}(\cos(9\pi/4) + i \sin(9\pi/4)) = \sqrt{2}e^{i9\pi/4}.$$

Thus,  $\theta = \arg(z)$  is not uniquely determined by  $z$ .

To provide some definiteness, we define what is known as the principal argument of  $z$ .

**Important idea** ( $\arg(z)$  versus  $\text{Arg}(z)$ ).

For any complex number  $z = x + yi$  with  $\theta = \arg(z)$  we can always choose an integer  $k$  such that  $-\pi < \arg(z) - 2k\pi \leq \pi$ . We denote this special angle by  $\text{Arg}(z)$  and call it “the principal argument of  $z$ ”.

## 6.2 Revision Video 22: Intro to complex numbers + basic operations

[View this lesson on YouTube](#) [22]

### Example.

Let  $z := 2e^{i\pi/6}$ . Calculate:  $z^3$ ;  $z^{-1}$ ; and  $-3z$ . In addition, plot your calculated complex numbers on the same Argand diagram.

**wethrive.net**

**How to retain your top staff**

**FIND OUT NOW FOR FREE**

**DO YOU WANT TO KNOW:**

- What your staff really want?
- The top issues troubling them?
- How to make staff assessments work for you & them, painlessly?

**Get your free trial**

Because happy staff get more done

### 6.3 Revision Video 23: Complex numbers and calculations

[View this lesson on YouTube](#) [23]

**Example.**

Define the complex numbers  $z$  and  $w$  by  $z := 2 - 5i$  and  $w = 1 + 2i$ . Calculate:

$$\frac{1 + 7i}{w}; \quad 4\bar{z}w; \quad \text{Arg}(w - 3i).$$

## 6.4 Video 24: Powers of complex numbers via polar forms

### 6.4.1 Calculations with the polar exponential form

[View this lesson on YouTube](#) [24]

**Example.**

If  $z = 2e^{5\pi i/6}$  then compute  $z^2$ ,  $1/z$  and  $\Im(z)$ . Plot  $z$ ,  $z^2$  and  $1/z$  in the same complex plane.

# 7 Powers of complex numbers

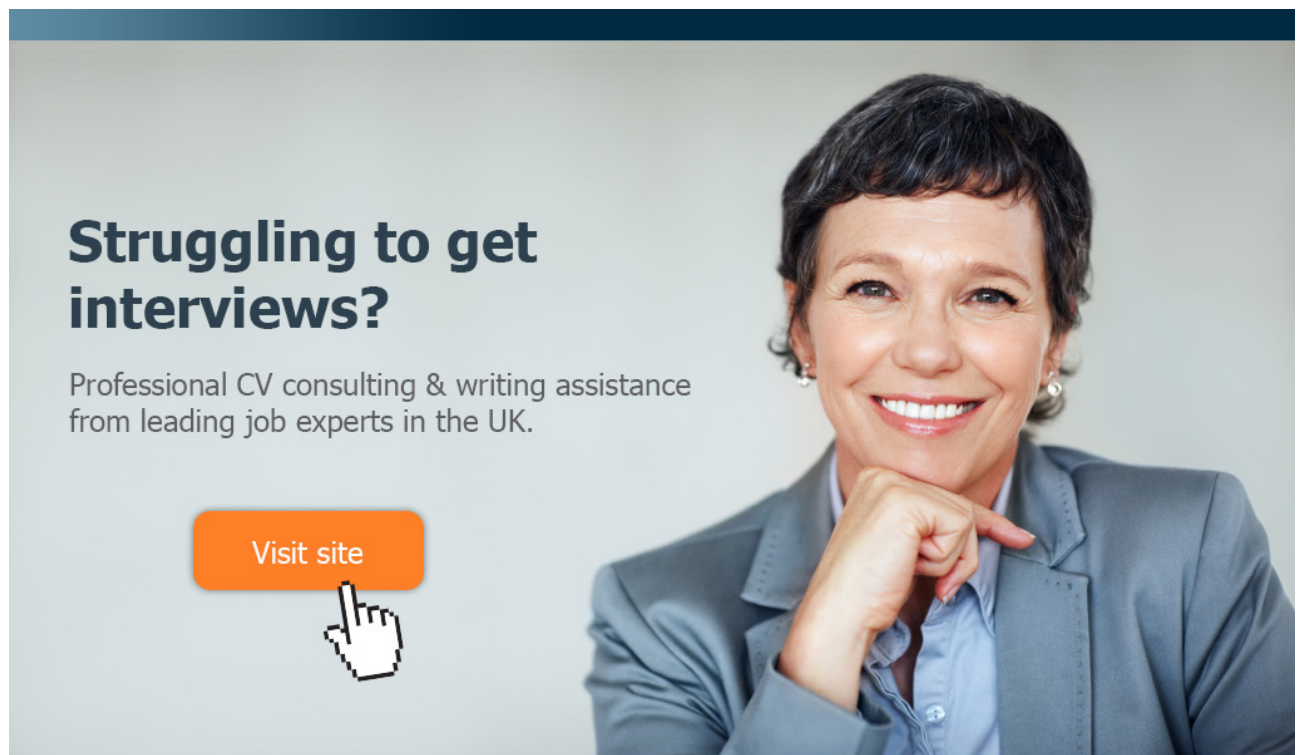
## 7.1 Video 25: Powers of complex numbers

[View this lesson on YouTube](#) [25]

**Example.**

Powers of complex numbers If  $z = -1 + i\sqrt{3}$  then:

- Calculate a polar exponential form of  $z$ ;
- Hence determine  $\text{Arg}(z^{23})$  and write  $z^{23}$  in Cartesian form.



**Struggling to get interviews?**

Professional CV consulting & writing assistance from leading job experts in the UK.

[Visit site](#)



Take a short-cut to your next job!  
Improve your interview success rate by 70%.



**TheCVagency**  
Visit [theagency.co.uk](http://theagency.co.uk) for more info.



**Click on the ad to read more**

## 7.2 Video 26: What is the power of a complex number?

[View this lesson on YouTube](#) [26]

**Example.**

Suppose  $z = 1 + i$ ,  $w = 1 - i\sqrt{3}$ . If

$$q := z^6/w^5$$

then:

- a) Calculate  $|q|$ ;
- b) Determine  $\text{Arg}(q)$ .

### 7.3 Video 27: Roots of complex numbers

[View this lesson on YouTube](#) [27]

**Example.**

Solve

$$z^5 = 16(1 - i\sqrt{3})$$

leaving your answers in simplified polar exponential form.



## 7.4 Video 28: Complex numbers solutions to polynomial equations

[View this lesson on YouTube](#) [28]

**Example.**

Determine all of the (complex) fourth roots of  $8(-1 + \sqrt{3}i)$ . You may leave your answer in polar form.

 **gaieteye**<sup>®</sup>  
*Challenge the way we run*

**EXPERIENCE THE POWER OF  
FULL ENGAGEMENT...**

.....

**RUN FASTER.  
RUN LONGER..  
RUN EASIER...**

**READ MORE & PRE-ORDER TODAY  
WWW.GAITEYE.COM**

## 7.5 Video 29: Complex numbers and $\tan(\pi/12)$

[View this lesson on YouTube](#) [29]

**Example.**

If  $z = -2 + 2i$  and  $w = -1 - i\sqrt{3}$  then:

- a) Compute  $zw$  in Cartesian form;
- b) Rewrite  $z$  and  $w$  in polar exponential form and thus calculate  $zw$  in polar exponential form;
- c) Hence determine a precise value for  $\tan(\pi/12)$ .

## 7.6 Video 30: Euler's formula: A cool proof

[View this lesson on YouTube](#) [30]

**Important idea** (Euler's formula).

We prove

$$e^{i\theta} = \cos \theta + i \sin \theta.$$

Let  $f(\theta) := \cos \theta + i \sin \theta$ . Thus,  $f(0) = 1$ . Differentiating  $f$  we obtain

$$\begin{aligned} f'(\theta) &= -\sin \theta + i \cos \theta \\ &= i^2 \sin \theta + i \cos \theta \\ &= i(\cos \theta + i \sin \theta) \\ &= if(\theta). \end{aligned}$$

We have formed a differential equation/initial value problem. Note that  $g(\theta) := e^{i\theta}$  also satisfies the IVP. By uniqueness of solutions,  $f \equiv g$ , that is,

$$e^{i\theta} = \cos \theta + i \sin \theta.$$

This also means that the polar exponential form  $re^{i\theta}$  is an accurate representation of any complex number  $z$ .

# 8 De Moivre's formula

## 8.1 Video 31: De Moivre's formula: A cool proof

[View this lesson on YouTube](#) [31]

De Moivre's formula is useful for simplifying computations involving powers of complex numbers.

**Important idea** (De Moivre's formula).

For each integer  $n$  and all real  $\theta$  we have

$$(\cos \theta + i \sin \theta)^n = (\cos n\theta + i \sin n\theta).$$

The proof utilizes Euler's formula

$$e^{i\theta} = \cos \theta + i \sin \theta.$$

We have,

$$\begin{aligned}(\cos \theta + i \sin \theta)^n &= (e^{i\theta})^n \\ &= e^{in\theta} \\ &= (\cos n\theta + i \sin n\theta)\end{aligned}$$

and thus we have proven the result.

## 8.2 Video 32: Trig identities from De Moivre's theorem

[View this lesson on YouTube](#) [32]

**Example.**

Write  $\cos 5\theta$  in terms of  $\cos \theta$  by applying De Moivre's theorem.

This e-book  
*is made with*  
**SetaPDF**



**SETASIGN**

PDF components for PHP developers

[www.setasign.com](http://www.setasign.com)



### 8.3 Video 33: Trig identities: De Moivre's formula

[View this lesson on YouTube](#) [33]

**Example.**

Write  $\sin 4\theta$  in terms of  $\cos \theta$  and  $\sin 4\theta$  by applying De Moivre's theorem. Hence, write  $\sin 4\theta \cos \theta$  as a function of  $\sin 4\theta$ .

# 9 Connecting sin, cos with e

## 9.1 Video 34: Trig identities and Euler's formula

[View this lesson on YouTube](#) [34]

### 9.1.1 More connections between $\sin \theta$ , $\cos \theta$ , $e^{i\theta}$

Euler's formula

$$e^{i\theta} = \cos \theta + i \sin \theta$$

can be manipulated to obtain the following identities

**Important idea** (Trig functions in terms of exponentials).

$$\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}.$$

For example, consider

$$e^{-i\theta} = \cos(-\theta) + i \sin(-\theta) = \cos \theta - i \sin \theta$$

and so  $e^{i\theta} + e^{-i\theta} = 2 \cos \theta$ , which rearranges to the first identity.

9.1.2 Trig identities from Euler's formula

**Example.**

Apply the identity

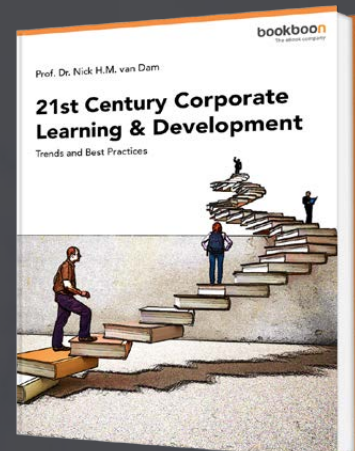
$$\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

to express  $\sin^4 \theta$  in terms of  $\cos \theta, \cos 2\theta, \dots$ .

# Free eBook on Learning & Development

By the Chief Learning Officer of McKinsey

[Download Now](#)



[Click on the ad to read more](#)



## 9.2 Video 35: Trig identities from Euler's formula

[View this lesson on YouTube](#) [35]

**Example.**

Apply the identity

$$\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

to express  $\sin^5 \theta$  in terms of  $\sin \theta, \sin 2\theta, \dots$ .

### 9.3 Video 36: How to prove trig identities WITHOUT trig!

[View this lesson on YouTube](#) [36]

**Example.**

Prove

$$\sin(x + y) = \sin x \cos y + \cos x \sin y.$$

## 9.4 Revision Video 37: Complex numbers + trig identities

[View this lesson on YouTube](#) [37]

The problem for this video is similar to Video 35.

A large blue graphic with a stylized lion in the background. The text "YOU THINK. YOU CAN WORK AT RMB" is written in large white letters. In the top right corner, there is a white box containing the Rand Merchant Bank logo (a lion with a key) and the text "RAND MERCHANT BANK", "A division of FirstRand Bank Limited", and "Traditional values. Innovative ideas." data-bbox="139 512 904 808"/>

**YOU THINK.  
YOU CAN WORK  
AT RMB**

**RAND MERCHANT BANK**  
A division of FirstRand Bank Limited  
Traditional values. Innovative ideas.

Rand Merchant Bank uses good business to create a better world, which is one of the reasons that the country's top talent chooses to work at RMB. For more information visit us at [www.rmb.co.za](http://www.rmb.co.za)

Thinking that can change your world

Rand Merchant Bank is an Authorised Financial Services Provider



Click on the ad to read more

# 10 Regions in the complex plane

## 10.1 Video 38: How to determine regions in the complex plane

[View this lesson on YouTube](#) [38]

### 10.1.1 Regions in the complex plane

We can use equations or inequalities to represent regions within two-dimensional space.

With a bit of care, we can also represent regions in the complex plane via similar techniques.

We know that the modulus  $|z|$  of any complex number  $z$  is the length of the line segment joining  $z$  to the origin. Thus, the set

$$\{z \in \mathbb{C} : |z| < 3\}$$

is the set of all complex numbers, whose distance to the origin is less than three units. This is an open disc, centred at the origin, with radius three.

Similarly, the set

$$\{z \in \mathbb{C} : |z - (2 + i)| < 3\}$$

is the set of all complex numbers, whose distance to  $2 + i$  is less than three units. This is an open disc, centred at the  $2 + i$ , with radius three.

Similarly, the set

$$\{z \in \mathbb{C} : |z - i| = 3\}$$

is the set of all complex numbers, whose distance to  $i$  is exactly three units. This is a circle, centered at the  $i$ , with radius three.

The set

$$\{z \in \mathbb{C} : |z - 2| = |z - 4|\}$$

is the set of all complex numbers, whose distance to 2 and 4 are equal. This is a vertical line, passing through 3.

Also

$$\{z \in \mathbb{C} : 0 \leq \text{Arg}(z) \leq \pi/2\}$$

is the set of all complex numbers, whose principal argument is between zero and  $\pi/2$ . This is all those points that lie in the first quadrant, covered by a quarter-turn in the anticlockwise direction about the origin.

10.1.2 Regions in the complex plane

**Example.**

Determine and sketch the set of points satisfying

$$\{z \in \mathbb{C} : |z + 4| = 2|z - i|\}.$$



Discover the truth at [www.deloitte.ca/careers](http://www.deloitte.ca/careers)

**Deloitte.**

© Deloitte & Touche LLP and affiliated entities.



Click on the ad to read more

## 10.2 Video 39: Circular sector in the complex plane

### 10.2.1 Regions in the complex plane

[View this lesson on YouTube](#) [39]

**Example.**

Determine and sketch the set of points satisfying

$$|z - 1 - i| < 3, \quad 0 < \text{Arg}(z) < \pi/4.$$

### 10.3 Video 40: Circle in the complex plane

#### 10.3.1 Regions in the complex plane

[View this lesson on YouTube](#) [40]

**Example.**

Determine and sketch the set of points satisfying

$$|z + 3| = 2|z - 6i|.$$



## 10.4 Video 41: How to sketch regions in the complex plane

[View this lesson on YouTube](#) [41]

**Example.**

Sketch the region in the complex plane dened by all those complex numbers  $z$  such that

$$|z - 2i| < 1, \quad \text{and} \quad 0 < \text{Arg}(z - 2i) \leq \frac{3\pi}{4}.$$

[bookboon.com](http://bookboon.com)

# Corporate eLibrary

See our Business Solutions for employee learning

Click here



# 11 Complex polynomials

## 11.1 Video 42: How to factor complex polynomials

[View this lesson on YouTube](#) [42]

### Important idea.

The basic theory for complex polynomials of degree  $n$

$$p(z) := a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0$$

may be summarized as follows:-

- Every polynomial  $p(z)$  of degree  $n$  has at least one root over  $\mathbb{C}$ . That is, there is at least one  $\alpha$  such that  $p(\alpha) = 0$ .
- The roots of complex polynomials with **real** coefficients appear in conjugate pairs.
- If  $p(\alpha) = 0$  for some number  $\alpha$  then  $(z - \alpha)$  is a factor of  $p(z)$ .
- Every polynomial of degree  $n$  can be factored into  $n$  linear parts. That is

$$p(z) = a_n(z - \alpha_1)(z - \alpha_2) \cdots (z - \alpha_n)$$

where the  $\alpha_i$  are the roots of  $p(z)$ .

11.1.1 Complex polynomials with real coefficients

**Example.**

- a) Solve  $p(z) := z^6 + 64 = 0$ .
- b) Hence factorize  $p(z)$  into linear factors.

## 11.2 Video 43: Factorizing complex polynomials

### 11.2.1 Complex polynomials with real coefficients

[View this lesson on YouTube](#) [43]

**Example.**

If  $p(z) := 2z^4 - 5z^3 + 5z^2 - 20z - 12$  then:

- Show  $p(2i) = 0$ ;
- Illustrate that  $z^2 + 4$  is a factor of  $p(z)$  (without division) and also find the other quadratic factor;
- Thus, factorize  $p(z)$  into quadratic factors.



**e-learning for kids**

- The number 1 MOOC for Primary Education
- Free Digital Learning for Children 5-12
- 15 Million Children Reached

**About e-Learning for Kids** Established in 2004, e-Learning for Kids is a global nonprofit foundation dedicated to fun and free learning on the Internet for children ages 5 - 12 with courses in math, science, language arts, computers, health and environmental skills. Since 2005, more than 15 million children in over 190 countries have benefitted from eLessons provided by EFK! An all-volunteer staff consists of education and e-learning experts and business professionals from around the world committed to making difference. eLearning for Kids is actively seeking funding, volunteers, sponsors and courseware developers; get involved! For more information, please visit [www.e-learningforkids.org](http://www.e-learningforkids.org).



### 11.3 Video 44: Factor polynomials into linear parts

#### 11.3.1 Complex polynomials with real coefficients

[View this lesson on YouTube](#) [44]

**Example.**

- a) Solve  $p(z) := z^7 + 3^7 = 0$ .
- b) Hence factorize  $p(z)$  into linear factors.

## 11.4 Video 45: Complex linear factors

### 11.4.1 Complex polynomials with real coefficients

[View this lesson on YouTube](#) [45]

**Example.**

If  $p(z) := z^5 + 4z^3 - 8z^2 - 32$  then:

- a) Show  $p(2i) = 0$ ;
- b) Illustrate that  $z^2 + 4$  is a factor of  $p(z)$  (without division) and also find the other quadratic factor;
- c) Thus, factorize  $p(z)$  into complex linear factors.

# Bibliography

1. Tisdell, Chris. Complex numbers are AWESOME. Streamed live on 02/04/2014 and accessed on 14/08/2014. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=YdBALaKYCO4&index=1&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
2. Tisdell, Chris. How to add and subtract complex numbers. Streamed live on 03/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=nj3qJY4QO6U&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=2>
3. Tisdell, Chris. Scalar multiply a complex number. Streamed live on 03/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=MNQPU6BQ9Ok&index=3&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
4. Tisdell, Chris. How to multiply complex numbers. Streamed live on 03/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=Kt11OMjXC6I&index=4&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
5. Tisdell, Chris. How to divide complex numbers. Streamed live on 03/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, [https://www.youtube.com/watch?v=fa7DVp\\_oNFE&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=5](https://www.youtube.com/watch?v=fa7DVp_oNFE&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=5)



**Brain power**

By 2020, wind could provide one-tenth of our planet's electricity needs. Already today, SKF's innovative know-how is crucial to running a large proportion of the world's wind turbines.

Up to 25 % of the generating costs relate to maintenance. These can be reduced dramatically thanks to our systems for on-line condition monitoring and automatic lubrication. We help make it more economical to create cleaner, cheaper energy out of thin air.

By sharing our experience, expertise, and creativity, industries can boost performance beyond expectations. Therefore we need the best employees who can meet this challenge!

The Power of Knowledge Engineering

Plug into The Power of Knowledge Engineering.  
Visit us at [www.skf.com/knowledge](http://www.skf.com/knowledge)

**SKF**

6. Tisdell, Chris. Complex numbers: Quadratic formula. Streamed live on 19/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=iNzVgErnf5w&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=6>
7. Tisdell, Chris. What is the complex conjugate? Streamed live on 19/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=C8LzaBikty8&index=7&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
8. Tisdell, Chris. Calculations with the complex conjugate. Streamed live on 19/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=WlqTBPp7sRM&index=8&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
9. Tisdell, Chris. How to show a number is purely imaginary. Streamed live on 19/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, [https://www.youtube.com/watch?v=75D\\_\\_m6q5JM&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=9](https://www.youtube.com/watch?v=75D__m6q5JM&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=9)
10. Tisdell, Chris. Complex numbers: example of how to prove the real part of a complex number is zero. Streamed live on 25/11/2008 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=QWbLhUZ6bag&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=10>
11. Tisdell, Chris. Complex conjugates and linear systems. Streamed live on 19/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=0s8XntqBrkc&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=11>
12. Tisdell, Chris. When are the squares of  $z$  and its conjugate equal? Streamed live on 19/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=U7d0NgvctMk&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=12>
13. Tisdell, Chris. Conjugate of products is product of conjugates. Streamed live on 20/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, [https://www.youtube.com/watch?v=hKe4s\\_6B0Qs&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=13](https://www.youtube.com/watch?v=hKe4s_6B0Qs&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=13)
14. Tisdell, Chris. Why complex solutions appear in conjugate pairs. Uploaded on 16/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=XkWz76dxkkI&index=14&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
15. Tisdell, Chris. How big are complex numbers? Streamed live on 20/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=NyPGV066MCM&index=15&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>



16. Tisdell, Chris. Modulus of a product is the product of moduli. Streamed live on 20/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=siePZ8yJFJU&index=16&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
17. Tisdell, Chris. Square roots of complex numbers. Streamed live on 20/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=HQ3lqtRSo-k&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=17>
18. Tisdell, Chris. Quadratic equations with complex coecients. Streamed live on 20/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=PQi-LrSWoUM&index=18&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
19. Tisdell, Chris. Show real part of a complex number is zero. Streamed live on 21/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=i8z5fDHm0JY&index=19&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
20. Tisdell, Chris. Polar trig form of a complex number. Streamed live on 21/04/2014 and accessed on 14/08/2014. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=7jT9AHJrDo&index=20&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
21. Tisdell, Chris. Polar exponential form of a complex number. Streamed live on 21/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=2ryt4n5WDnU&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=21>
22. Tisdell, Chris. Intro to complex numbers + basic operations. Uploaded on 08/09/2010 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=QeMSqlrgQYg&index=22&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
23. Tisdell, Chris. Complex numbers and calculations. Uploaded on 06/09/2010 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=0JYIh8Goblg&index=23&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
24. Tisdell, Chris. Powers of complex numbers via polar forms. Streamed live on 22/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=FtXPMSHBKgc&index=24&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
25. Tisdell, Chris. Powers of complex numbers. Streamed live on 22/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, [https://www.youtube.com/watch?v=P\\_sFeTtnQPs&index=25&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP](https://www.youtube.com/watch?v=P_sFeTtnQPs&index=25&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP)

26. Tisdell, Chris. What is the power of a complex number? Streamed live on 22/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=nZPn74GC3KM&index=26&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
27. Tisdell, Chris. Roots of complex numbers. Streamed live on 22/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=RmUazwwRqso&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=27>
28. Tisdell, Chris. Complex number solutions to polynomial equations. Uploaded on 08/09/2010 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=Y4btmS-uHWI&index=28&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
29. Tisdell, Chris. Complex numbers and  $\tan(\pi/12)$  Streamed live on 22/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=N5gRg2whooM&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=29>
30. Tisdell, Chris. Euler's formula: a cool proof. Streamed live on 02/12/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=stOZL05NvjK&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=30>
31. Tisdell, Chris. De Moivre's formula: a COOL proof. Streamed live on 23/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, [https://www.youtube.com/watch?v=NjYZS\\_XYIEQ&index=31&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP](https://www.youtube.com/watch?v=NjYZS_XYIEQ&index=31&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP)

With us you can  
shape the future.  
Every single day.

For more information go to:  
[www.eon-career.com](http://www.eon-career.com)

Your energy shapes the future.

**e.on**



32. Tisdell, Chris. Application of De Moivre's theorem. Streamed live on 23/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=JjECulRsKr8&index=32&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
33. Tisdell, Chris. Trig identities: De Moivre's formula. Streamed live on 23/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=uAj1zb1p0gg&index=33&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
34. Tisdell, Chris. Trig identities and Euler's formula. Streamed live on 23/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=Bd22Y6NvKZk&index=34&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP>
35. Tisdell, Chris. Euler's formula and trig identities. Streamed live on 23/04/2015 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=NSYYWhUpeqs&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=35>
36. Tisdell, Chris. How to prove trig identities WITHOUT trig. Streamed live on 11/12/2013 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=RGnvGjFfjBs&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=36>
37. Tisdell, Chris. Complex numbers + trig identities. Uploaded on 08/09/2010 and accessed on 14/08/2014. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=CNmK48GOCuc&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=37>
38. Tisdell, Chris. How to determine regions in the complex plane. Streamed live on 26/04/2015 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, [https://www.youtube.com/watch?v=0vjsF\\_n-DBs&index=38&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP](https://www.youtube.com/watch?v=0vjsF_n-DBs&index=38&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP)
39. Tisdell, Chris. Circular sector in the complex plane. Streamed live on 26/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, [https://www.youtube.com/watch?v=\\_2Z3qbhfa8c&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=39](https://www.youtube.com/watch?v=_2Z3qbhfa8c&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=39)
40. Tisdell, Chris. Circle in the complex plane. Streamed live on 26/04/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=sLkdqTg1-1c&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=40>
41. Tisdell, Chris. How to sketch regions in the complex plane. Uploaded on 08/09/2010 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=8gtnZ5xSLuE&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=41>
42. Tisdell, Chris. How to factor complex polynomials. Streamed live on 01/05/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=UG3TtIPTVZE&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=42>

43. Tisdell, Chris. Factorizing complex polynomials. Streamed live on 01/05/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, [https://www.youtube.com/watch?v=r\\_h\\_10ovGU0&index=43&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP](https://www.youtube.com/watch?v=r_h_10ovGU0&index=43&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP)
44. Tisdell, Chris. Factor polynomials into linear parts. Streamed live on 02/05/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=ebrLfGRLfBc&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=44>
45. Tisdell, Chris. Complex linear factors of polynomials. Streamed live on 02/05/2014 and accessed on 14/08/2015. Available on Dr Chris Tisdell's YouTube channel, <https://www.youtube.com/watch?v=9r1MSXG4ENw&list=PLGCj8f6sgswm6oVMzqBbNXooFT43yqViP&index=45>
46. Tisdell, Chris. Engineering mathematics YouTube workbook playlist <http://www.youtube.com/playlist?list=PL13760D87FA88691D>, accessed on 1/11/2011 at DrChrisTisdell's YouTube Channel <http://www.youtube.com/DrChrisTisdell>.

© 2013 Accenture. All rights reserved.

be > your degree

Bring your talent and passion to a global organization at the forefront of business, technology and innovation. Discover how great you can be.

Visit [accenture.com/bookboon](http://accenture.com/bookboon)

Be greater than.  
consulting | technology | outsourcing

accenture  
High performance. Delivered.

