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EFFECT OF TENSOR RANGE IN NUCLEAR TWO BODY PROBLEMS

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ABSTRACT

The interaction between neutron and proton in the triplet state has been investigated. The central and tensor potentials have been assumed to be of the "meson" type but of possibly different ranges. The variational-iterational method has been utilized to determine V_0, the strength of the central potential, and γV_0, the strength of the tensor potential. A wide variation in the values of both central and tensor ranges were included. Calculations of this type have proved difficult in the past principally because of the sensitivity of the results to the quadrupole moment Q of the deuteron for which accurate wave functions are necessary. It would be

*Professor Schwinger has not yet had the opportunity of reading this manuscript.

desirable to have a more accurate value of Q deduced from experiment. The percent D state in the deuteron and the effective triplet range have also been computed. The results are applied to the discussion of the magnetic moment of the deuteron, the photoelectric disintegration of the deuteron, and neutron-proton scattering. It is concluded that experimental information on neutron-proton interaction at their present accuracy does not determine the range of the tensor forces except within rather broad limits. A decrease in the uncertainty in the triplet range by an order of magnitude would be required. Finally it is concluded that charge independence of nuclear forces can be maintained with the meson well.

EFFECT OF TENSOR RANGE IN NUCLEAR TWO BODY PROBLEMS

Introduction, Determination of the Force Constants, Results, Applications
by
Herman Feshbach, Massachusetts Institute of Technology
Julian Schwinger, Harvard University

Computational Details
by
John A. Harr, Harvard University

1. Introduction.

The discovery of the quadrupole moment of the deuteron required the introduction of non-central ("tensor") forces into the phenomenological description of the neutron-proton interaction. It then became necessary to compare the experimental data, particularly for the two, three, and four-body systems, with the predictions following from the inclusion of such forces. For several years, the necessary constants for both the central and tensor constituents of the neutron-proton potential energy have been available for only one well shape, the "square" well employed by Rarita and Schwinger[1]. The analysis of experimental data indicates, however, that this well leads to predictions which are in contradiction with experiment[2],[3],[4]. We are thus led to investigate the dependence of phenomenological theory upon well shape.

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1. W. Rarita and J. Schwinger, P.R. 59, pp. 436 and 556 (1941)
2. D. Bohm and C. Richman, P.R. 71, 567 (1947)
3. J. M. Blatt and J.D. Jackson, P.R. 76, 18 (1949)
   H. A. Bethe, P.R. 76, 38 (1949)
4. E. Gerjuoy and J. Schwinger, P.R. 61, 138 (1941)
   H. Feshbach and W. Rarita, P.R. 75, 1384 (1949)
   R. E. Clapp, P.R. 76, 873 (1949)

The general form of the interaction potential is:

$$V(r) = -V_o \left\{ \left[(1-g/2) + (g/2)\, \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right] J(r) + \gamma\, S_{12}\, K(r) \right\} \quad . \qquad (1)$$

Here r is the interparticle distance, J(r) the potential well for the central force part of the potential, K(r) the potential well for the tensor part; V_o, γ, g are constants and

$$S_{12} = \frac{3\left(\vec{\sigma}_1 \cdot \vec{r}\right)\left(\vec{\sigma}_2 \cdot \vec{r}\right)}{r^2} - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \quad . \qquad (2)$$

Since our interest lies only in S and D states of the two-body systems we have not specified the exchange properties of V(r). It should be emphasized that this particular choice of non-central force has been made under the assumption that the forces are not velocity dependent; there is of course no a priori justification for this procedure.

Rarita and Schwinger employed $J = K$; $J = 0$ for $r > r_o$, $J = 1$ for $r < r_o$. In recent years other wells have been discussed. Guindon[5] employs square wells for both J and K but permits slightly different values of r_o for J and K. Biedenharn[6] and Padfield[7] have made more extensive changes which are more consistent with present day evaluation of neutron-proton scattering experiments. Hu and Massey[8] place $J = K$ and then calculate the constants for a number of wells including the square well, the exponential well $J = e^{-\beta r}$, and the Gaussian $J(r) = e^{-\beta r^2}$. Chew and

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5. W. G. Guindon, P.R. 74, 145 (1948)
6. L. C. Biedenharn, Jr., Thesis, M.I.T. (1949)
7. D. Padfield, Nature 163, 22, (1949)
8. Hu and Massey, Referred to in E.H.S. Burhop, H. N. Yadav, Nature 162, p. 738, (1948)

Goldberger[9] place J = K and employ $J = \frac{e^{-br}}{r}$. Jauch and Hu[10], and Wu and Foley[10] employed the J and K predicted by the Schwinger mixture theory. Rarita[11] placed J = K and employed the exponential well. The calculations of Hu and Massey are limited in scope by J = K; those of Biedenharn and Padfield are limited to a square well which is a rather unrealistic potential well shape and gives rise to the discomfiting necessity of introducing several new constants since the charge independence of nuclear forces fails for the square well potential. Thus it becomes necessary to have different V_o, J and ranges for the neutron-proton interaction in the triplet and singlet states; these in turn differ from those determined for the proton-proton interaction.

In the present paper the meson well will be employed. The assumption J = K is dropped so that

$$J = \frac{e^{-br}}{br} \qquad K = \frac{e^{-tr}}{tr} \tag{3}$$

where b and t are constants. The use of this well is suggested by the fact that preliminary calculations[12] showed that this combination of wells will successfully predict the experimental results for (1) neutron-proton scattering at low and moderately high neutron energies, (2) neutron scattering by ortho- and para-hydrogen, (3) deuteron binding energy, (4) quadrupole moment of the deuteron, (5) binding energy of H^3, (6) photoelectric and photomagnetic disintegration of the deuteron and, finally (7) will preserve approximately the charge independence of nuclear forces.

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9. G. F. Chew and M. L. Goldberger, P.R. 73, 1408 (1948)
10. J. M. Jauch and N. Hu, P.R. 65, 289 (1944)
    Wu and Foley, P.R. 75, 1305 (1949)
11. W. Rarita, P.R. 74, 1799 (1949)
12. H. Feshbach, P.R. 76, 185A (1949)

Potential (1) contains four constants V_o, γ, t, and b. For charge independence of nuclear forces to hold, b must be taken from proton-proton scattering data[13]. The remaining three constants are determined by

ϵ = binding energy of the deuteron[14]	2.23 MeV
Q = Quadrupole moment of the deuteron[15]	$2.73 \times 10^{-27} cm^2$
σ_o = Cross-section for the scattering of thermal neutrons by protons[16]	$20.4 \times 10^{-24} cm^2$

Calculations have been made for a sufficiently wide range in the values of t, b, and γ so as to include the effects of possible changes in the experimental data.

2. Determination of the Force Constants.

As has been just indicated, the determination of the constants in (1) is made by comparison with the known properties of the neutron-proton system in the singlet and triplet states. The potentials in each of these states are from (1):

$$\text{singlet } V = -V_o (1-2g) J(r) \tag{4}$$

$$\text{triplet } V = -V_o \left[J(r) + \gamma S_{12} K(r)\right] \tag{5}$$

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13. Jackson and Blatt - in press
H. A. Bethe, P.R. 76, 38 (1949)
G. Breit and W. G. Bouricius, P.R. 75, 1029 (1949)
14. R. E. Bell and L. G. Elliot, P.R. 74, 1552L (1948)
A. V. Tollestrup, F. A. Jenkins, W. A. Fowler, and C. C. Lauritsen, Bulletin American Phys. Soc. 24, T2 (April 28, 1949)
15. A. Nordsieck, P.R. 58, 310 (1940)
J. M. B. Kellogg, I. I. Rabi, N. F. Ramsey, and J. R. Zacharias, P.R. 57, 677 (1940)
16. R. Melkonian, L. J. Rainwater, and W. W. Havens, Jr., Bulletin American Phys. Soc. 24, G1 (Jan. 26, 1949)

For a given value of b, the constant $V_o(1-2g)$ is determined by the neutron-proton scattering cross-section at thermal energies. The values of this parameter have been tabulated by Jackson and Blatt[3] for the meson well. For given values of b and t, V_o and γ are determined by the properties of the deuteron, ϵ and Q. The calculations which constitute the main body of this paper were carried out on the Schroedinger equation for the deuteron for four values of b, five values of t for each b, and three values of γ for each pair of values of t and b, making sixty values altogether.

We employed the variational-iterational method[17]. This method is particularly valuable in the present problem because it provides (1) a method for estimating the errors in the eigenvalue by giving upper and lower bounds to them, (2) a method for systematically improving the wave functions by iteration, (3) a method for extrapolation. Inasmuch as the quadrupole moment is a determining quantity, and inasmuch as it depends rather critically upon the wave-functions and in quite a different manner than the deuteron binding energy, it was essential to have the safeguards just mentioned in order to be secure in the reliability of the final numerical values obtained.

The deuteron wave function Ψ for a given Z component of the angular momentum M_J is a linear combination of an S and D state:

$$\Psi = \frac{1}{r}\left[u(r) + \frac{1}{\sqrt{8}} S_{12}\, w(r)\right] \chi_M \qquad (6)$$

where χ_M is a neutron-proton spin function with Z component of angular momentum M_J. The differential equation determining u and w are given in

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3. J. M. Blatt and J. D. Jackson, P.R. 76, 18 (1949)
   H. A. Bethe, P.R. 76, 38 (1949)
17. H. Feshbach and J. Schwinger, in preparation

terms of the independent variable:

$$x = r/r_o \qquad r_o = 2.76 \times 10^{-13}\text{cm}^2 \,. \tag{7}$$

Then

$$\eta^2 = \frac{M\epsilon\, r_o^2}{\hbar^2} \,, \quad \Lambda = \frac{MV_o r_o^2}{\hbar^2} \,, \quad \beta = br_o, \quad \tau = r_o \,. \tag{8}$$

For the r_o given in (7) $\eta = .64$, $\frac{V_o}{\epsilon} = \frac{\Lambda}{\eta^2}$. We shall later employ the symbol $\lambda = \Lambda/\eta$ so that $\frac{V_o}{\epsilon} = \frac{\lambda}{\eta}$.

The following equations are then obtained:

$$\frac{d^2u}{dx^2} + \left[-\eta^2 + \Lambda J\right] u = -2^{3/2}\gamma\Lambda Kw \tag{9a}$$

$$\frac{d^2w}{dx^2} - \frac{6w}{x^2} + \left[-\eta^2 + \Lambda(J - 2\gamma K)\right] w = -2^{3/2}\gamma\Lambda Ku \,. \tag{9b}$$

An interesting integral connection exists between the behavior of u and w at large and small distances. From differential equations (9a and b) we have

$$u\frac{dw}{dx} - w\frac{du}{dx} = \int_x^\infty dx\left\{2^{3/2}\gamma\Lambda K(u^2 - w^2) - wu\left(\frac{6}{x^2} + 2\gamma K\Lambda\right)\right\} \,.$$

Upon placing $x = o$

$$\int_o^\infty \frac{6wu}{x^2}\, dx = 2\gamma\Lambda \int_o^\infty \left\{2^{1/2}(u^2 - w^2) - uw\right\} Kdx$$

or

$$\gamma\Lambda = \frac{3\int_o^\infty \frac{wu}{x^2}\, dx}{\int_o^\infty \left[2^{1/2}(u^2 - w^2) - uw\right] Kdx} \,.$$

The quadrupole moment is

$$Q = \frac{\sqrt{2}}{10} r_o^2 \quad \frac{\int_0^\infty x^2 \left[uw - \frac{1}{\sqrt{8}} w^2\right] dx}{\int_0^\infty (u^2 + w^2)\, dx} . \tag{10}$$

To obtain a proper form for the iteration it is necessary to replace equations (9) by equivalent integral equations:

$$u = \Lambda \int_0^\infty G_\eta^{(0)}(x;x') \left[u(x') J(x') + 2^{3/2} \gamma K(x') w(x')\right] dx' \tag{11a}$$

$$w = \Lambda \int_0^\infty G_\eta^{(2)}(x,x') \left[2^{3/2}\gamma K(x') u(x') + \left(J(x') - 2\gamma K(x')\right) \cdot w(x')\right] dx' \tag{11b}$$

where

$$\frac{d^2 G_\eta^{(\ell)}}{dx^2} - \frac{(\ell)(\ell+1)}{x^2} G_\eta^{(\ell)} - \eta^2 G_\eta^{(\ell)} = -\delta(x - x') .$$

Therefore

$$G_\eta^{(2)} = \frac{1}{\eta} \begin{cases} \left[1 + \frac{3}{\eta x} + \frac{3}{(\eta x)^2}\right] e^{-\eta x} \left[\left(1 + \frac{3}{(\eta x')^2}\right) \cdot \right. \\ \qquad \left. \sinh \eta x' - \frac{3}{\eta x'} \cosh \eta x'\right] & x' \leq x \\ \left[1 + \frac{3}{\eta x'} + \frac{3}{(\eta x')^2}\right] e^{-\eta x'} \left[\left(1 + \frac{3}{(\eta x)^2}\right) \cdot \right. \\ \qquad \left. \sinh \eta x - \frac{3}{\eta x} \cosh \eta x\right] & x' \geq x . \end{cases} \tag{12}$$

At this point it becomes possible to describe the iteration part of the calculation. Starting from an initial pair of trial functions u_o and w_o whose choice will be discussed below, a first iterate u_1 and w_1 may be

obtained by introducing u_0 and w_0 in the right hand side of equation (11). The factor λ is a common proportionality factor so that it may be dropped in computing u_1 and w_1. To obtain the second iterate introduce u_1 and w_1 in the right hand side of (11) and so on for the higher iterates.

The wave functions developed in this manner are then employed as trial functions in a variational principle for λ. Equations (11) are a pair of integral equations with eigenvalue λ; i.e.,the depth of the potential V_0 required to yield the experimental binding energy as included in the parameter is now the solution of an eigenvalue problem. This characteristic feature of the iterational scheme corresponds rather nicely to the present state of knowledge in the theory of the deuteron for the binding energy of the deuteron is known from experiment and one is looking for a corresponding value for the potential.

The necessary variational principles may be obtained from the original Schroedinger equation. However, it is somewhat simpler to work directly from (9) and (10). Let us define a matrix

$$\underset{\sim}{\Phi} = \begin{pmatrix} u \\ w \end{pmatrix} \tag{13}$$

and matrix operators

$$\underset{\sim}{A} = \begin{pmatrix} -\frac{d^2}{dx^2} + \eta^2 & 0 \\ 0 & -\frac{d^2}{dx^2} + \frac{6}{x^2} + \eta^2 \end{pmatrix} \tag{14}$$

$$\underset{\sim}{B} = \eta \begin{pmatrix} J & 2^{3/2}\gamma K \\ 2^{3/2}\gamma K & J - 2\gamma K \end{pmatrix} \tag{15}$$

Then equations (9) and (10) may be written

$$\underset{\sim}{A}\,\underset{\sim}{\Phi} = \lambda\, \underset{\sim}{B}\,\underset{\sim}{\Phi} \tag{16}$$

$$\underset{\sim}{\phi} = \lambda \underset{\sim}{A}^{-1} \underset{\sim}{B} \underset{\sim}{\phi}. \tag{17}$$

From (16) and (17) two variational principles may be obtained. Let λ_o be the eigenvalue of least absolute value. We adjust the operators so that it is positive. Then

$$\lambda_o = \text{extr} \frac{(\underset{\sim}{\phi}, \underset{\sim}{A}\, \underset{\sim}{\phi})}{(\underset{\sim}{\phi}, \underset{\sim}{B}\, \underset{\sim}{\phi})} \tag{18a}$$

and

$$\lambda_o = \text{extr} \frac{(\underset{\sim}{\phi}, \underset{\sim}{B}\, \underset{\sim}{\phi})}{(\underset{\sim}{\phi}, \underset{\sim}{B}\underset{\sim}{A}^{-1}\underset{\sim}{B}\underset{\sim}{\phi})} \tag{18b}$$

where the symbol $(\underset{\sim}{\phi}, \underset{\sim}{\psi})$ signifies the scalar product of the matrices $\underset{\sim}{\phi}$ and $\underset{\sim}{\psi}$ integrated over x from o to infinity. Upon introducing the successive iterates $\underset{\sim}{\phi}_m = \begin{pmatrix} u_n \\ w_n \end{pmatrix}$ into (18) a set of values $\lambda_o^{(n)}$ which are approximate values of λ_o is obtained:

$$\begin{aligned} \lambda_o^{(n-1/2)} &= \frac{(n-1,\ n-1)}{(n,\ n-1)} \\ \lambda_o^{(n)} &= \frac{(n,\ n-1)}{(n,\ n)} \\ \lambda_o^{(n+1/2)} &= \frac{(n,\ n)}{(n,\ n+1)} \qquad \text{etc.} \end{aligned} \tag{19}$$

where

$$(n,\ m) = (\underset{\sim}{\phi}_n,\ \underset{\sim}{B}\underset{\sim}{\phi}_m)$$

or in terms of u_n and w_n:

$$(n,\ m) = \int_o^\infty u_n \left(J u_m + 2^{3/2} \gamma K w_m \right) dx + \int_o^\infty w_n \left(2^{3/2} \gamma K u_m + (J - 2\gamma K)\, w_m \right) dx. \tag{20}$$

It may be shown[17] that the sequences $\lambda_0^{(n)}$, $\lambda_0^{(n+1/2)}$ converge to λ_0. The manner in which this occurs is complicated in this case by the non-positive definite character of $\underset{\sim}{B}$; or in equation (11) one may say that the kernel of the integral equation (11b) is not positive definite. This feature is of importance for the larger values of γ. The non-positive behavior of $\underset{\sim}{B}$ has the consequence that the eigenvalues λ are both positive and negative extending to $+\infty$ and $-\infty$. The positive sequence of eigenvalues corresponds to those states for which the S state is the principal component; i.e., it gives the values of the potential V_0 for which a state mostly S in character has the binding energy 2.23 MeV. The smallest positive λ, λ_0, is that potential for which 2.23 MeV is the binding energy of the ground state. The negative sequence of eigenvalues correspond to those states for which the D state is the principal component.

Two quantitative consequences of the above are of importance in the calculation. The inequalities satisfied by $\lambda_0^{(n)}$ and $\lambda_0^{(n+1/2)}$ as $n \to \infty$ are considerably less specific when $\underset{\sim}{B}$ is not positive definite compared to when it is. Thus for the non-positive definite case

$$\lambda_0^{(n)} \geq \lambda_0$$

$$\lambda_0^{(n)} \geq \lambda_0^{(n+1/2)} \tag{21}$$

$$\lambda_0^{(n)} \lambda_0^{(n+1/2)} \lambda_0^{(n+1)} \lambda_0^{(n+3/2)} \geq \ldots \lambda_0^{2} .$$

If λ_1 is the next eigenvalue whose absolute value is greater than λ_0 then if n is so large that

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17. H. Feshbach and J. Schwinger, in preparation

$$\lambda_1^2 \geq \lambda_o^{(n+1/2)} \quad \lambda_o^{(n+1)}$$

then

$$\lambda_o^{(n)} \geq \lambda_o^{(n+1)} \geq \ldots \geq \lambda_o. \tag{22}$$

Lower bounds as well as upper bounds to λ_o may be obtained. We give several of these:

$$\lambda_o \geq \lambda_o^{(n+1/2)} \left[1 - \frac{\lambda_o^{(n)} - \lambda_o^{(n+1/2)}}{|\lambda_1| - \lambda_o^{(n+1/2)}} \right] \tag{22a}$$

$$\lambda_o^2 \geq \lambda_o^{(n+1/2)} \lambda_o^{(n+1)} \left[1 - \frac{\lambda_o^{(n)} - \lambda_o^{(n+1)}}{|\lambda_1| - \lambda_o^{(n+1)}} \right]. \tag{22b}$$

If λ_1 is known to be negative then

$$\lambda_o \geq \lambda_o^{(n+1/2)} \left[1 - \frac{\lambda_o^{(n)} - \lambda_o^{(n+1/2)}}{|\lambda_2| - \lambda_o^{(n+1/2)}} \right] \tag{22c}$$

a considerable improvement over (22b). Because of the possibility that $\lambda_o^{(n)}$ may be <u>less</u> than λ_o another upper bound which is useful may be obtained:

$$\lambda_o \leq \lambda_o^{(n+1/2)} \left[1 + \frac{\lambda_o^{(n)} - \lambda_o^{(n+1/2)}}{|\lambda_1| + \lambda_o^{(n+1/2)}} \right]. \tag{22d}$$

To apply these inequalities some method for obtaining a lower bound for λ_1 is necessary. Here we have employed the relation:

$$\lambda_1^2 \geq \left[\text{Spur} \left(A^{-1} B \right)^2 - \frac{1}{\lambda_o^{(n)2}} \right] \tag{23}$$

where

$$\text{Spur } (A^{-1}B)^2 = \eta^2 \int dx' \int dx'' \left\{ J(x'') G^{(o)}_{\eta} (x', x'') \cdot \right.$$

$$G^{(o)}_{\eta} (x'', x') J(x') + 16\gamma^2 K(x') G^{(o)}_{\eta} (x', x'') \cdot$$

$$G^{(2)}_{\eta} (x'', x') K(x'') + \left[J(x'') - 2\gamma K(x'') \right] \cdot$$

$$\left. G^{(2)}_{\eta} (x', x'') G^{(2)}_{\eta} (x'', x') \left[J(x'') - 2\gamma K(x'') \right] \right\}. \quad (24)$$

A similar method[17] is available for determination of λ_2 in (22c).

Another remark should be made with regard to the convergence of the product sequence in (21) or the sequence in (22). The range of convergence is best when the ratio $|\lambda_o / \lambda_1|$ is very small compared to 1. For some values of the parameters this ratio is very close to 1 corresponding to an approximate degeneracy in the eigenvalues of the iterated operator in (17), $(A^{-1}B)^2$, which has eigenvalues, λ_o^2, λ_1^2..... . One may then employ the usual procedures of degenerate perturbation theory; or equivalent to this, after a sufficient number of iterations so that u and w are fixed, the amplitude of the D state is introduced as a variational parameter. Further iterations produced practically no change. In practice, it was found possible to employ a simple extrapolation technique which, it may be shown, is directly connected with the method taken from degenerate perturbation theory. This was particularly necessary for the determination of Q for the near degeneracy produced sizable fluctuations in Q with successive iterations so that it was no longer clear what to take for the final value for Q without further iterations. This was avoided by use of the extrapolation method.

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17. H. Feshbach and J. Schwinger, in preparation

Let $F^{(o)}$, $F^{(1)}$, $F^{(2)}$ ... be successive values of a quantity F obtained by successive iterations. The quantity F may be λ, or Q or the value of u or w at a given point. If the iterations at stage $F^{(o)}$ have proceeded far enough it may be assumed that

$$F^{(o)} = F + f$$
$$F^{(1)} = F + \epsilon f$$
$$F^{(2)} = F + \epsilon^2 f$$

where f is the error at stage $F^{(o)}$ while ϵ is the parameter which gives the rate of convergence. We have essentially assumed that all the error comes from the admixture of the eigenfunction of λ_1 to that of λ_o. From these three equations we may determine F:

$$F = F^{(o)} - \frac{\left[F^{(o)} - F^{(1)}\right]^2}{F^{(o)} - 2F^{(1)} + F^{(2)}} . \qquad (25)$$

If two sets of numbers F and G depend on ϵ in the same way (e.g. λ and Q) then it is possible to obtain ϵ from one sequence and use it in the other so that

$$F = F^{(o)} - (F^{(o)} - F^{(1)}) \frac{G^{(o)} - G^{(1)}}{G^{(o)} - 2G^{(1)} + G^{(2)}} . \qquad (26)$$

The extrapolation method was checked by comparison with successive iterations in some of the worst cases which occurred and proved to be very accurate.

Details of the calculation.

The first step in the calculation involves the determination of a proper initial trial function $\underset{\sim}{\phi}_o$. In the present calculation this was available from some initial trial runs by hand computers

for one set of parameters (β, τ) and various γ's. These and their iterates were employed for other sets of (β, τ) and similar γ's. However, it is more usual not to have any trial functions given a-priori. Some tests have been on the use of the Ritz Rayleigh method which showed that simple one parameter (one for u and one for w) having proper x dependence at $x = o$ and $x = \infty$, gave good initial wave functions. The values of λ_o and therefore in V_o obtained from them were in error by a few percent. However this corresponds to a considerable error in the binding energy of the deuteron of the order of more than ten percent. The error in Q was considerably larger and it must be concluded that discussion based on the Ritz method employing this simple type of function is not reliable. However after one or two iterations, the results were generally satisfactory. In the calculation reported on here two iterations were used. This provided a good internal check and permitted the employment of extrapolation formulas (25) and (26).

The iterations were performed numerically employing Simpson's rule over most of the range in x. The mesh was chosen so as to provide an overall accuracy for the calculated ratios for λ_o (18) of 1 part in 10^4. There was some difficulty in the region for $x \sim o$ for integration (11b) involves a rapidly varying function of x behaving as $1/x$ as $x \rightarrow o$. Under such conditions Simpson's rule fails. A special integration formula for this region was devised in which the known properties of u were utilized. In the region of the origin, $u = x \cdot (A_o + A_1 x + A_2 x^2 + \ldots)$. The coefficients A_i may be expressed in terms of the tabulated values of u. The integration for each power of x may be then performed and the final integral expressed in terms of tabulated values of u multiplied by known numerical coefficients.

3. Results.

A total of sixty cases were computed, a λ and Q being obtained for each. The parameter β, the inverse of the central force range was given four values. It has been customary to relate these to an equivalent meson mass by the relation $b = \frac{\mu c}{K}$, μ = meson mass. The four values and the corresponding masses are given in Table III.

Table III

Values of β and corresponding Meson masses

β	μ/m	
2.86	400	
2.50	350	
2.33	326	
2.04_4	286	m = electron mass

The value of $\beta = 2.33$ corresponds then to the range suggested by Breit et al[18], from proton-proton scattering. More recent analyses[19,20,21] suggest that a somewhat shorter range (larger values of β) may fit the data somewhat more closely. The parameter τ was given five values for each value of β; these are given in Table IV, together with their corresponding meson mass. The values of τ chosen were $\leq \beta$; this choice was indicated by the theory of

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18. Hoisington, Share and Breit, P.R. 56, 884, (1939)
19. J. D. Jackson and J. M. Blatt, in press
20. H. A. Bethe, P.R. 76, 38 (1949)
21. G. Breit and W. G. Bouricius, P.R. 75, 1029 (1949)

Table IV

Values of τ and corresponding Meson Mass

τ	μ/m
2.33	326
2.00	280
1.8	252
1.3	182
1.0	140

H^3 [4]. Finally, three values of γ were employed for each β, τ pair. The values of γ were chosen so that the ensuing values of Q should cover the possible values of Q which may deviate as much as two and one-half percent from the quoted value of 2.73×10^{-27} cm^2. Three values of Q also permit a fairly precise interpolation on γ to obtain any desired value of Q. The values of and Q for each value of τ, β, and γ are given in Table I.* The values of λ are certainly accurate to one part in 10^4, while the third figure in Q is reliable.

We now discuss the qualitative features which are apparent from Table I. The eigenvalue λ is a function of β, τ and Q or γ. For a given β and τ, it naturally decreases with increasing γ; for a given β and Q it is a decreasing function of τ, the sharpest change occurring between $\tau = 2.00$ and $\tau = 2.33$. For a given τ and Q, λ decreases as β decreases, while γ increases. These qualitative descriptions may be summarized by a set of rules which were found very valuable in predicting new values of γ for a new τ or β. They, of course, hold only very roughly. Relations (27a) and (27b) hold rather well while (27d) may be off from constant by as much as ten percent.

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4. E. Gerjuoy and J. Schwinger, P.R. 61, 138 (1941)
H. Feshbach and W. Rarita, P.R. 75, 1384 (1949)
R. E. Clapp, P.R. 76, 873 (1949)

*Tables I and II are at the end of the report.

For a given β and τ $\quad (1 + \frac{\gamma\beta}{\tau})\lambda \sim$ constant (27a)

$\frac{\gamma\lambda}{Q} \sim$ constant (27b)

For a given β and Q $\quad \frac{\gamma\lambda}{\tau^3} \sim$ constant (27c)

$(1 + \frac{\gamma\beta}{\tau})\lambda \sim$ constant (27d)

For a given τ and γ $\quad \lambda/\beta \sim$ constant (27e)

For a given τ and Q $\quad \gamma\beta \sim$ constant (27f)

We shall conclude this section by discussing the sensitivity of the force constants γ and λ to changes in the experimental data. The most serious changes may occur for Q since it has been determined with an accuracy of two and one-half percent, $2.73 \times 10^{-27} cm^2 \pm$ two and one-half percent covers the region $2.78 \times 10^{-27} cm^2$ to $2.68 \times 10^{-27} cm^2$. For small τ, $\tau = 1.0, 1.3$, a change in Q of five percent would result in changes in λ and γ of a similar order. For larger τ, $\tau = 2.33$, such a change in Q produces a similar change in $\gamma\lambda$ (see 27b), but an enormous change in both γ and λ. An error of a few percent in λ (or $\gamma\lambda$) is of the order of 1 MeV in ϵ. Errors of this magnitude have fairly large sized repercussions in the calculation of the properties of other systems of nucleons. This emphasizes two things. One, that it would be important to obtain a more accurately determined value of Q. This is not an experimental matter but rather involves improving the evaluation of the experimental results (already one percent accurate). Secondly, the method for obtaining λ and γ must give Q with sufficient accuracy or again the values of λ and γ will be falsified.

A change in the proton-proton scattering would correspond to a change in β. The consequent changes in λ and γ are given qualitatively by (27e) and (27f). A change in the binding energy of the deuteron will require a

change in r_o, the unit of length, in such a way that $\eta = \left(\frac{M \epsilon r_o^2}{\hbar^2}\right)^{1/2}$ will remain at .64. To keep the same b and Q, it would be necessary to change β and the tabulated Q's would have to be decreased by one percent. Thus the consequences of a change in ϵ may be readily obtained from Table I* and its qualitative effect from formulas (27); the answer will depend upon the initial γ, β, τ, for in this case

$$\frac{\Delta\lambda}{\lambda} \sim \frac{\Delta Q}{Q}\left(\frac{\beta\gamma}{\tau}\right) - \frac{\Delta\beta}{\beta}\frac{1}{1+\frac{\beta\gamma}{\tau}}$$

$$\frac{\Delta\gamma}{\gamma} \sim \frac{\Delta Q}{Q}\left(1+\frac{\beta\gamma}{\tau}\right) - \frac{\Delta\beta}{\beta}$$

4. Applications.

a. Magnetic Moment of the Deuteron.

The admixture of the D state gives a contribution to the magnetic moment μ_D of the deuteron so that

$$\mu_D = \mu_n + \mu_p - \frac{3}{2}(\mu_n + \mu_p - \tfrac{1}{2})\, p_D \qquad (28)$$

where the magnetic moments are expressed in terms of nuclear magnetons and P_D is the percentage of D state defined as

$$P_D = \frac{\int_0^\infty w^2 dx}{\int_0^\infty (u^2 + w^2)dx} \qquad (29)$$

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*Tables I and II are found at the end of the report.

However, because of other contributions to the magnetic moment of the deuteron arising from relativistic effects[22] equation (28) is not correct and may only be employed to obtain an upper and lower bound on p_D. In the absence of relativistic effects $p_D = .04$; relativistic effects are of the order of .015, so that $.04 \pm .015$ seems to be reasonable upper and lower bound on p_D. In Table V* we give the values of p_D corresponding to each value of γ and τ. It is seen that almost all the values from $\tau = 1.3$ to $\tau = 2.33$ give p_D which fall within these limits. It seems clear however that the regions $\tau << 1$ and $\tau >> 2$ are excluded.

b. Neutron-Proton Scattering.

Recent analyses[23][24][25][26] have shown that it is possible to express the theoretical predictions on neutron-proton scattering in terms of singlet range r_s and a triplet range r_t. We shall be primarily interested in the latter. Jackson and Blatt[24] have analyzed the data and have concluded that

$$r_t = (1.56 \pm .13) \times 10^{-13} \text{ cm} \qquad (28)$$

The parameter r_t is determined by comparison with the zero energy solution so that if k is the wave number in the center-of-mass system of the relative neutron-proton motion and δ_t is the phase shift

$$k \cot \delta_t = -\alpha_t + k^2 \left(\frac{1}{2} r_t\right) - P r_t^3 k^4 \ldots$$

where α_t is the inverse of triplet scattering length ; P is a parameter. In this paper we employ the bound state (deuteron) wave functions, and thus must

22. H. Primakoff, P.R. 72, 118 (1947), G. Breit and I. Bloch, P.R. 72, 135 (1947)
23. J. Schwinger, Notes on a course in Nuclear Theory (1947)
24. J. D. Jackson and J. M. Blatt, P.R. 76, 18 (1949)
25. H. A. Bethe and C. Longmire, in press
26. G. F. Chew and M. L. Goldberger, P.R. 75, 1637 (1949)

*Table V is at the end of the report.

replace r_t by its analogue ρ_t. Then

$$k \cot \delta_t = - \eta + (k^2 + \eta^2)\left(\tfrac{1}{2}\rho_t\right) + \ldots \quad .$$

Comparison of these two expressions shows that

$$\rho_t \sim r_t \left[1 + 4\rho {r_t}^2 \eta^2\right]. \tag{29}$$

In the case of the meson well employed here, and in the absence of tensor forces ρ_t is several percent higher so that (28) becomes

$$\rho_t = (1.66 \pm .14) \times 10^{-13} \text{ cm}$$

$$(\rho_t / r_o) = .60 \pm .05$$

We shall assume that the value of P and hence of ρ_t does not change appreciably when tensor forces are included[28]. It may be expected that the shape parameter P may be somewhat larger when τ is very much smaller than β.

The definition of ρ_t for the triplet state including tensor forces has been given by Schwinger[27], Biedenharn[28] and Christian[29]. It is

$$\rho_t = 2\mu_o \int_o^\infty \left[e^{-2\eta x} - \frac{u^2 + w^2}{1 + \zeta^2}\right] dx \tag{30}$$

where

$$\zeta = \lim_{x \to \infty} \frac{w(x)}{u(x)} \quad . \text{ Generally } \zeta << 1.$$

27. J. Schwinger, Notes on a course in Nuclear Theory (1947)
28. L. C. Biedenharn, Jr., M.I.T. Thesis
29. R. S. Christian, P.R. 75, 1675 (1949)

In Table V we have tabulated ρ_t/r_o. For a given τ, ρ_t decreases as β increases; for a given β, ρ_t decreases as τ increases: ρ_t is relatively independent of γ. It is seen that a much sharper delimitation of ρ_t experimentally is required before it would be possible to choose a best τ for a given β. However, in principle, if the uncertainty in ρ_t were diminished by a factor of 10, the value of ρ_t would be determined, provided, of course, the particular well shapes discussed here are employed. We also note that ρ_t is, for a given β and τ, independent of γ so that its determination is independent of the precise value of Q.

c. Photo Disintegration of the Deuteron.

The cross-section for the photoelectric disintegration of the deuteron may be expressed in terms of the cross-section for zero range nuclear forces, $\sigma^{(o)}$ multiplied by a "range correction"(30). In the absence of tensor forces:

$$\sigma_{p\cdot e} = \frac{\sigma^{(o)}}{1-\eta\rho_t}\left[1 + O\left(k^2\eta^2\rho_t^4\right)\right] \qquad (31)$$

where k is the relative momentum of the final neutron-proton system and

$$\sigma^{(o)} = \frac{8\pi}{3}\frac{e^2}{\hbar c}\frac{\epsilon^{1/2}\,E^{3/2}}{(E+\epsilon)^{3/2}}$$

where E is the final energy in the center-of-mass system of neutron and proton When tensor forces are included, formula (31) must be multiplied by $1/(1+\zeta^2)$ This is, however, a small correction. From Table V containing ρ_t, it is now possible to deduce the range correction $1/(1-\eta\rho_t)$. This correction is

30. H. A. Bethe and C. Longmire, in press

appreciable over the entire range of ρ_t given. However, since these ρ_t's are generally smaller than those obtained for the Rarita-Schwinger square well, the cross-section is reduced below that given by their calculation but is considerably above that obtained for zero range.

The cross-section for photoelectric cross-section may be utilized to determine ρ_t. At present, the experimental evidence is contradictory, so that as yet no reliable value of ρ_t may be deduced from it.

Computational Details

For Mark I computation, equations (11a and b) were written as follows:

$$u(x) = L(x) \int_0^x M(x') \left\{ J(x')\, u(x') + 2^{3/2}\, \gamma K(x')\, w(x') \right\} dx'$$
$$+ M(x) \int_x^\infty L(x') \left\{ J(x')\, u(x') + 2^{3/2}\, \gamma K(x')\, w(x') \right\} dx' \qquad (32a)$$

and

$$w(x) = P(x) \int_0^x R(x') \left\{ 2^{3/2}\, \gamma K(x')\, u(x') + \left[J(x') - 2\gamma K(x') \right] w(x') \right\} dx'$$
$$+ R(x) \int_x^\infty P(x') \left\{ 2^{3/2}\, \gamma K(x')\, u(x') + \left[J(x') - 2\gamma K(x') \right] w(x') \right\} dx' \qquad (32b)$$

where

$$L(x) = e^{-\eta x}, \qquad (33a)$$

$$M(x) = \sinh \eta x, \qquad (33b)$$

$$P(x) = e^{-\eta x} \left(1 + \frac{3}{\eta x} + \frac{3}{(\eta x)^2} \right), \qquad (33c)$$

$$R(x) = \left(1 + \frac{3}{(\eta x)^2}\right) \sinh \eta x - \frac{3}{\eta x} \cosh \eta x \tag{33d}$$

$$J(x) = \frac{e^{-\beta x}}{\beta x} \tag{34a}$$

and

$$K(x) = \frac{e^{-\tau x}}{\tau x} \quad . \tag{34b}$$

It was desired to obtain λ to an accuracy of one part in 10^4. Analysis of several trial runs made by hand computers showed that it would be sufficient to compute the wave functions u(x) and w(x) in the interval $0 \leq x \leq 5.00$ with a mesh of $\Delta x = .036$ near the origin and then doubled at $x = 0.936$, 1.512 and 2.664.

The first step was the computation and recording on value tape of the values of L(x), M(x), P(x) and R(x) at the mesh points stated above. This computation was performed only once since these functions are independent of the parameters β, τ, and γ.

The functions J(x) and K(x) were computed and recorded on cards for each of the β and τ combinations shown in Table I.

Trial functions $u_0(x)$ and $w_0(x)$ were introduced, along with the previously computed values of L(x), M(x), P(x), R(x), J(x) and K(x), into the right hand side of equations (32a and b) and the indicated integrations were performed using Simpson's one-third rule. Then the first iterates $u_1(x)$ and $w_1(x)$ were introduced into equations (32a and b) and second iterates obtained. Two iterations were generally sufficient to obtain λ to the required accuracy. The second integral in equation (32b) involves a function of x which behaves as $\frac{1}{x}$ as $x \longrightarrow 0$. Hence Simpson's rule fails near the origin. A special quadrature formula based on the known behavior of the integrand was employed in this region.

After two iterations approximate values of λ were obtained from equation (19) which may be written in the following form:

$$\lambda^{(n-1/2)} = \frac{(n-1,\ n-1)}{(n,\ n-1)} \quad , \tag{35a}$$

$$\lambda^{(n)} = \frac{(n,\ n-1)}{(n,\ n)} \quad , \tag{35b}$$

and

$$\lambda^{(n+1/2)} = \frac{(n,\ n)}{(n+1,\ n)} \quad , \tag{35c}$$

where

$$(n-1,\ n-1) = \int_0^\infty u_{n-1}\left\{Ju_{n-1} + 2^{3/2}\gamma K W_{n-1}\right\} dx$$

$$+ \int_0^\infty W_{n-1}\left\{2^{3/2}\gamma K u_{n-1} + (J-2\gamma K)\, W_{n-1}\right\} dx \ .$$

The integrals in equations (35a,b and c) were obtained by use of Simpson's rule. It was not necessary to compute the functions shown within the braces in equations (35a, b and c) since they were previously computed as parts of the integrands in equations (32a and b). The final values of λ, shown in Table I, were computed by the extrapolation formula:

$$\lambda = \lambda^{(n-1/2)} - \frac{\left(\lambda^{(n-1/2)} - \lambda^{(n)}\right)^2}{\lambda^{(n-1/2)} - 2\lambda^{(n)} + \lambda^{(n+1/2)}} \tag{36}$$

This extrapolation method was checked by computing the third and fourth iterates of u(x) and w(x) for several of the more slowly converging cases and found to be accurate to one part in 10^4.

The value of Q at the end of each iteration was computed by use of the equation:

$$Q = \frac{\sqrt{2}}{10}\,\gamma_o^2\,\frac{\int_0^\infty x^2\left[u(x)\,w(x) - 2^{-3/2}\,w^2(x)\right]\,dx}{\int_0^\infty \left[u^2(x) + w^2(x)\right]\,dx} \quad . \tag{37}$$

The numerical integrations were performed in the interval $o \leq x \leq 4.968$ by Simpson's rule and then correction terms added for the value of the integrals from $x = 4.968$ to ∞. These correction terms were computed from the formulas:

$$\int_{x_o}^{\infty} x^2 u(x)w(x)dx = \frac{x_o^2 u(x_o)w(x_o)}{2\eta S(x_o)}\left(1 + \frac{4}{\eta x_o} + \frac{5}{(\eta x_o)^2}\right), \tag{38a}$$

$$\int_{x_o}^{\infty} x^2 w^2(x)dx = \frac{x_o^2 w^2(x_o)\left(1 + \frac{7}{\eta x_o} + \frac{37}{2(\eta x_o)^2} + \frac{18}{(\eta x_o)^3}\right)}{2\eta S^2(x_o)}, \tag{38b}$$

$$\int_{x_o}^{\infty} u^2(x)dx = \frac{u^2(x_o)}{2\eta}, \tag{38c}$$

and

$$\int_{x_o}^{\infty} w^2(x)dx = \frac{w^2(x_o)\left[1 + \frac{6}{\eta x_o}\left(1 + \frac{1}{\eta x_o}\right)^2\right]}{2\eta S^2(x_o)}, \tag{38d}$$

where

$$S(x_o) = \left(1 + \frac{3}{\eta x_o} + \frac{3}{(\eta x_o)^2}\right)$$

and $x_o = 4.968$.

Equations (38a,b,c and d) are based on the known asymptotic behavior of u(x) and w(x). The following extrapolation formula was employed to compute the tabulated values of Q shown in Table I:

$$Q = Q^{(o)} - \left(Q^{(o)} - Q^{(1)}\right) \frac{\left(\lambda^{1/2} - \lambda^{1}\right)}{\lambda^{1/2} - 2\lambda^{1} + \lambda^{3/2}} \qquad (39)$$

where $\lambda^{1/2}$, λ^{1} and $\lambda^{3/2}$ correspond to $G^{(o)}$, $G^{(1)}$ and $G^{(2)}$ in equation (26) respectively.

The values of λ and Q tabulated in Table I are believed to be accurate to one unit in the last place printed. The wave functions u(x) and w(x) tabulated in Table II were obtained from the last iteration. These functions are estimated to be accurate to one part in 10^3. At each stage of the computation the functions were checked by use of simple identities obtained from the associative, distributive and commutative laws of multiplication and addition. An overall check was made by comparing successive values of λ.

For each pair of values of the parameters β and τ, λ and Q were computed for three γ values. The values of γ used in going from one case to the next were estimated by use of equations (27a and b). They were selected so that the three values of Q would cover approximately the range $2.73 \times 10^{-27}\,cm^2$ plus and minus two and one-half percent. The last iterates u(x) and w(x) for the previously computed case with closest γ value were used as the first approximation to the wave functions. The guess value of γ to be used in computing λ and Q for a new β and τ combination was obtained from equations (27a,b and f). The last iterates for a near β, τ and γ set were used as the trial wave functions.

The value of λ obtained after the first iteration was usually within 0.1 percent of its final value. However, Q was quite sensitive to the wave functions and after two iterations often had not completely settled down. In the few cases where two iterations and the use of equations (36) and (39) were insufficient to yield the desired accuracy in λ and Q, wave functions $u_3(x)$ and $w_3(x)$ were computed by use of the following extrapolation formulas obtained from equation (26):

$$u_3(x) = u_2(x) + \frac{\lambda^{3/2} - \lambda^1}{\lambda^{3/2}\left(\lambda^{1/2} - \lambda^1\right)}\, u_1(x) \tag{40a}$$

and

$$w_3(x) = w_2(x) + \frac{\lambda^{3/2} - \lambda^1}{\lambda^{3/2}\left(\lambda^{1/2} - \lambda^1\right)}\, w_1(x). \tag{40b}$$

Then $u_3(x)$ and $w_3(x)$ were introduced into equations (32a and b) and the iterates $u_4(x)$ and $w_4(x)$ obtained.

A case in which this process was employed is shown in Table VI.

Table VI

$\beta = 2.33 \quad \tau = 2.33 \quad \gamma = 4.73$

Iteration	λ	Q
	4.52261	
1		2.5870
	4.52118	
2		2.6647
	4.52257	
	4.52186	
4		2.6246
	4.52186	
5		2.6258
	4.52186	

The extrapolated value of λ, after two iterations, is 4.52188 and the extrapolated value of Q is 2.6264. In this case the extrapolated values of λ and Q were within the desired accuracy and therefore served as a check on the extrapolation process. A typical case in which two iterations were sufficient is shown in Table VII.

Table VII

$\beta = 2.33 \quad \tau = 2.0 \quad \gamma = 1.3392$

Iteration	λ	Q
	9.57282	
1		2.7078
	9.57259	
2		2.6797
	9.57270	

The extrapolated value of λ is 9.57266 and the extrapolated value of Q is 2.6888. In this particular case third iteration values of λ(9.57265) and Q(2.6950) were computed. The agreement between the extrapolated and and third iteration values of λ and Q in this case along with several other test cases served as a check on this method of computation.

The values of λ and Q were computed for sixty β, τ and γ combinations. The total time required was approximately six weeks.

TABLE I

β = 2.044			
τ	γ	λ	Q
1.00	0.1140	14.048	2.52
	0.1300	13.719	2.78
	0.1350	13.614	2.86
1.30	0.2400	13.140	2.53
	0.2700	12.727	2.74
	0.3000	12.329	2.89
1.80	0.8550	10.072	2.62
	0.9300	9.651	2.72
	1.0000	9.283	2.79
2.00	1.5600	7.982	2.67
	1.7000	7.556	2.74
	1.8400	7.170	2.78
2.33	6.3400	3.404	2.65
	9.2000	2.453	2.72
	13.0000	1.787	2.77
β = 2.33			
1.00	0.1000	17.388	2.64
	0.1040	17.260	2.71
	0.1080	17.131	2.78
1.30	0.2239	15.842	2.72
	0.2400	15.504	2.83
	0.3000	14.302	3.18
1.80	0.7107	12.411	2.63
	0.7660	11.935	2.71
	0.7820	11.804	2.72
2.00	1.3392	9.573	2.69
	1.4280	9.177	2.73
	1.5600	8.644	2.79
2.33	4.7300	4.522	2.63
	5.8000	3.792	2.67
	9.2000	2.504	2.75

TABLE I

τ	γ	λ	Q
	β = 2.5		
1.00	0.0920	19.562	2.67
	0.0978	19.325	2.80
	0.1140	18.652	3.08
1.30	0.1900	18.180	2.64
	0.2100	17.641	2.79
	0.2400	16.855	3.00
1.80	0.7300	12.985	2.76
	0.8100	12.231	2.85
	0.8550	11.841	2.89
2.00	1.2000	10.751	2.69
	1.3392	9.992	2.76
	1.5600	8.977	2.85
2.33	6.2400	3.605	2.70
	10.5000	2.234	2.77
	13.0000	1.827	2.79
	β = 2.859		
1.00	0.0740	24.814	2.62
	0.0770	24.623	2.71
	0.0850	24.102	2.97
1.30	0.1550	22.940	2.61
	0.1700	22.301	2.77
	0.1900	21.467	2.94
1.80	0.5800	16.474	2.72
	0.6240	15.799	2.81
	0.7107	14.603	2.91
2.00	0.9500	13.649	2.60
	1.1000	12.364	2.75
	1.2000	11.629	2.81
2.33	4.7300	4.742	2.69
	5.8000	3.943	2.72
	10.5000	2.265	2.78

TABLE II

$\beta = 2.044$			$\tau = 1.000$			
	$\gamma = 0.1140$		$\gamma = 0.1300$		$\gamma = 0.1350$	
x	U(x)	W(x)	U(x)	W(x)	U(x)	W(x)
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.036	3.3171	0.0583	1.7269	0.0337	0.9246	0.0185
0.072	6.1406	0.2333	3.2023	0.1348	1.7158	0.0742
0.108	8.5319	0.4403	4.4565	0.2544	2.3895	0.1402
0.144	10.5526	0.6667	5.5201	0.3853	2.9616	0.2125
0.180	12.2492	0.8920	6.4164	0.5157	3.4444	0.2846
0.216	13.6684	1.1152	7.1690	0.6449	3.8504	0.3561
0.252	14.8450	1.3214	7.7955	0.7643	4.1889	0.4223
0.288	15.8140	1.5134	8.3136	0.8756	4.4693	0.4840
0.324	16.6014	1.6852	8.7368	0.9752	4.6987	0.5393
0.360	17.2333	1.8394	9.0783	1.0646	4.8843	0.5890
0.396	17.7293	1.9738	9.3484	1.1426	5.0314	0.6324
0.432	18.1090	2.0910	9.5570	1.2106	5.1453	0.6703
0.468	18.3872	2.1903	9.7118	1.2682	5.2303	0.7025
0.504	18.5785	2.2741	9.8205	1.3168	5.2904	0.7297
0.540	18.6941	2.3425	9.8889	1.3566	5.3287	0.7519
0.576	18.7452	2.3977	9.9228	1.3886	5.3483	0.7699
0.612	18.7401	2.4402	9.9267	1.4133	5.3516	0.7838
0.648	18.6872	2.4716	9.9048	1.4315	5.3409	0.7941
0.684	18.5930	2.4926	9.8606	1.4437	5.3182	0.8011
0.720	18.4638	2.5048	9.7976	1.4508	5.2852	0.8052
0.756	18.3044	2.5086	9.7181	1.4530	5.2433	0.8066
0.792	18.1198	2.5053	9.6249	1.4511	5.1938	0.8057
0.828	17.9137	2.4954	9.5198	1.4454	5.1379	0.8027
0.864	17.6896	2.4800	9.4049	1.4365	5.0767	0.7979
0.900	17.4505	2.4596	9.2817	1.4246	5.0109	0.7915
0.936	17.1992	2.4349	9.1517	1.4103	4.9413	0.7837
1.008	16.6678	2.3742	8.8754	1.3751	4.7933	0.7643
1.080	16.1121	2.3035	8.5852	1.3340	4.6375	0.7417
1.152	15.5418	2.2240	8.2862	1.2879	4.4768	0.7163
1.224	14.9670	2.1398	7.9839	1.2391	4.3142	0.6893
1.296	14.3932	2.0519	7.6815	1.1881	4.1513	0.6611
1.368	13.8261	1.9628	7.3820	1.1365	3.9900	0.6325
1.440	13.2687	1.8731	7.0871	1.0845	3.8310	0.6037
1.512	12.7242	1.7844	6.7986	1.0331	3.6754	0.5752
1.656	11.6791	1.6104	6.2439	0.9323	3.3761	0.5192
1.800	10.7020	1.4487	5.7242	0.8387	3.0955	0.4672
1.944	9.7932	1.2973	5.2401	0.7510	2.8340	0.4184
2.088	8.9540	1.1599	4.7926	0.6714	2.5922	0.3742
2.232	8.1811	1.0344	4.3800	0.5988	2.3692	0.3337
2.376	7.4715	0.9219	4.0009	0.5337	2.1643	0.2975
2.520	6.8209	0.8206	3.6531	0.4751	1.9762	0.2648
2.664	6.2255	0.7304	3.3347	0.4229	1.8040	0.2357
2.952	5.1830	0.5771	2.7768	0.3341	1.5023	0.1863
3.240	4.3136	0.4576	2.3113	0.2650	1.2505	0.1478
3.528	3.5888	0.3625	1.9231	0.2099	1.0405	0.1171
3.816	2.9855	0.2881	1.5999	0.1668	0.8656	0.0930
4.104	2.4833	0.2292	1.3308	0.1327	0.7200	0.0740
4.392	2.0655	0.1828	1.1069	0.1058	0.5989	0.0590
4.680	1.7178	0.1459	0.9206	0.0845	0.4981	0.0471
4.968	1.4287	0.1166	0.7657	0.0675	0.4143	0.0376

TABLE II

$\beta = 2.044$			$\tau = 1.300$			
	$\gamma = 0.2400$		$\gamma = 0.2700$		$\gamma = 0.3000$	
x	U(x)	W(x)	U(x)	W(x)	U(x)	W(x)
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.036	1.7426	0.0451	0.6907	0.0193	1.1020	0.0333
0.072	3.2388	0.1765	1.2867	0.0758	2.0557	0.1297
0.108	4.5157	0.3289	1.7978	0.1414	2.8756	0.2415
0.144	5.6010	0.4920	2.2342	0.2117	3.5770	0.3609
0.180	6.5174	0.6511	2.6042	0.2804	4.1731	0.4774
0.216	7.2873	0.8059	2.9165	0.3475	4.6772	0.5908
0.252	7.9285	0.9457	3.1777	0.4082	5.0997	0.6932
0.288	8.4584	1.0732	3.3946	0.4636	5.4513	0.7866
0.324	8.8907	1.1844	3.5724	0.5120	5.7403	0.8681
0.360	9.2388	1.2818	3.7164	0.5545	5.9750	0.9395
0.396	9.5132	1.3640	3.8308	0.5905	6.1619	0.9997
0.432	9.7242	1.4332	3.9194	0.6208	6.3074	1.0506
0.468	9.8796	1.4893	3.9856	0.6455	6.4166	1.0919
0.504	9.9875	1.5343	4.0324	0.6654	6.4945	1.1250
0.540	10.0538	1.5684	4.0622	0.6805	6.5450	1.1502
0.576	10.0846	1.5934	4.0774	0.6917	6.5718	1.1687
0.612	10.0845	1.6097	4.0800	0.6991	6.5781	1.1808
0.648	10.0580	1.6187	4.0716	0.7033	6.5667	1.1876
0.684	10.0087	1.6209	4.0539	0.7045	6.5399	1.1894
0.720	9.9401	1.6175	4.0282	0.7033	6.5001	1.1871
0.756	9.8549	1.6088	3.9955	0.6997	6.4490	1.1809
0.792	9.7557	1.5959	3.9570	0.6943	6.3883	1.1716
0.828	9.6446	1.5790	3.9135	0.6872	6.3195	1.1594
0.864	9.5236	1.5591	3.8659	0.6787	6.2438	1.1449
0.900	9.3943	1.5363	3.8147	0.6690	6.1624	1.1284
0.936	9.2582	1.5112	3.7607	0.6582	6.0762	1.1101
1.008	8.9704	1.4553	3.6458	0.6342	5.8926	1.0694
1.080	8.6691	1.3951	3.5252	0.6082	5.6992	1.0254
1.152	8.3600	1.3313	3.4009	0.5806	5.4997	0.9788
1.224	8.0484	1.2666	3.2754	0.5525	5.2979	0.9315
1.296	7.7375	1.2013	3.1499	0.5242	5.0959	0.8837
1.368	7.4304	1.1371	3.0257	0.4963	4.8958	0.8367
1.440	7.1287	1.0740	2.9036	0.4689	4.6989	0.7905
1.512	6.8341	1.0131	2.7842	0.4424	4.5063	0.7458
1.656	6.2693	0.8972	2.5550	0.3920	4.1362	0.6608
1.800	5.7418	0.7931	2.3406	0.3466	3.7899	0.5844
1.944	5.2519	0.6988	2.1413	0.3055	3.4677	0.5150
2.088	4.8000	0.6154	1.9574	0.2691	3.1701	0.4537
2.232	4.3841	0.5413	1.7880	0.2367	2.8960	0.3992
2.376	4.0027	0.4763	1.6326	0.2083	2.6445	0.3513
2.520	3.6532	0.4190	1.4902	0.1833	2.4139	0.3092
2.664	3.3336	0.3690	1.3599	0.1615	2.2029	0.2723
2.952	2.7744	0.2862	1.1318	0.1253	1.8336	0.2113
3.240	2.3085	0.2235	0.9418	0.0978	1.5258	0.1650
3.528	1.9203	0.1750	0.7834	0.0766	1.2693	0.1292
3.816	1.5973	0.1377	0.6517	0.0603	1.0558	0.1017
4.104	1.3285	0.1087	0.5420	0.0476	0.8782	0.0803
4.392	1.1049	0.0862	0.4508	0.0377	0.7304	0.0637
4.680	0.9189	0.0686	0.3749	0.0300	0.6074	0.0507
4.968	0.7642	0.0548	0.3118	0.0240	0.5052	0.0404

TABLE II

$\beta = 2.044$			$\tau = 1.800$			
	$\gamma = 0.8550$		$\gamma = 0.9300$		$\gamma = 1.0000$	
x	U(x)	W(x)	U(x)	W(x)	U(x)	W(x)
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.036	3.1931	0.1540	3.3983	0.1710	3.8932	0.2016
0.072	6.0035	0.5756	6.3989	0.6391	7.3421	0.7533
0.108	8.4486	1.0481	9.0161	1.1643	10.3589	1.3724
0.144	10.5509	1.5337	11.2704	1.7045	12.9627	2.0093
0.180	12.3430	1.9908	13.1954	2.2135	15.1904	2.6096
0.216	13.8574	2.4214	14.8242	2.6935	17.0784	3.1758
0.252	15.1239	2.7941	16.1882	3.1092	18.6619	3.6665
0.288	16.1723	3.1211	17.3188	3.4745	19.9764	4.0978
0.324	17.0281	3.3924	18.2428	3.7778	21.0523	4.4560
0.360	17.7162	3.6182	18.9868	4.0306	21.9200	4.7548
0.396	18.2571	3.7961	19.5725	4.2300	22.6043	4.9908
0.432	18.6708	3.9348	20.0214	4.3860	23.1300	5.1754
0.468	18.9733	4.0351	20.3505	4.4989	23.5166	5.3093
0.504	19.1804	4.1039	20.5768	4.5768	23.7838	5.4019
0.540	19.3044	4.1430	20.7135	4.6215	23.9469	5.4553
0.576	19.3575	4.1580	20.7738	4.6393	24.0209	5.4769
0.612	19.3492	4.1508	20.7677	4.6322	24.0177	5.4692
0.648	19.2888	4.1259	20.7053	4.6053	23.9489	5.4380
0.684	19.1834	4.0848	20.5944	4.5603	23.8236	5.3856
0.720	19.0404	4.0312	20.4427	4.5013	23.6507	5.3164
0.756	18.8650	3.9664	20.2560	4.4296	23.4370	5.2323
0.792	18.6628	3.8931	20.0403	4.3484	23.1894	5.1368
0.828	18.4378	3.8123	19.8000	4.2588	22.9129	5.0314
0.864	18.1942	3.7261	19.5394	4.1631	22.6129	4.9188
0.900	17.9350	3.6353	19.2621	4.0622	22.2932	4.8001
0.936	17.6635	3.5415	18.9713	3.9579	21.9578	4.6772
1.008	17.0921	3.3468	18.3588	3.7412	21.2507	4.4219
1.080	16.4978	3.1512	17.7215	3.5232	20.5143	4.1649
1.152	15.8911	2.9556	17.0706	3.3052	19.7618	3.9077
1.224	15.2826	2.7660	16.4174	3.0936	19.0064	3.6581
1.296	14.6779	2.5824	15.7682	2.8887	18.2554	3.4162
1.368	14.0828	2.4078	15.1292	2.6938	17.5160	3.1860
1.440	13.5001	2.2419	14.5035	2.5085	16.7918	2.9672
1.512	12.9330	2.0860	13.8943	2.3342	16.0868	2.7613
1.656	11.8495	1.8008	12.7305	2.0156	14.7396	2.3847
1.800	10.8420	1.5560	11.6482	1.7418	13.4867	2.0611
1.944	9.9092	1.3429	10.6461	1.5034	12.3265	1.7792
2.088	9.0510	1.1611	9.7241	1.3001	11.2590	1.5387
2.232	8.2629	1.0047	8.8774	1.1250	10.2787	1.3317
2.376	7.5412	0.8714	8.1020	0.9758	9.3809	1.1551
2.520	6.8808	0.7569	7.3926	0.8477	8.5595	1.0035
2.664	6.2774	0.6591	6.7443	0.7382	7.8089	0.8739
2.952	5.2229	0.5023	5.6113	0.5626	6.4971	0.6661
3.240	4.3448	0.3868	4.6680	0.4333	5.4048	0.5130
3.528	3.6138	0.2999	3.8826	0.3359	4.4955	0.3977
3.816	3.0057	0.2343	3.2293	0.2624	3.7390	0.3107
4.104	2.4998	0.1841	2.6857	0.2063	3.1097	0.2442
4.392	2.0790	0.1456	2.2337	0.1631	2.5863	0.1931
4.680	1.7291	0.1156	1.8577	0.1295	2.1509	0.1533
4.968	1.4380	0.0922	1.5450	0.1033	1.7889	0.1223

TABLE II

$\beta = 2.044$			$\tau = 2.000$			
	$\gamma = 1.5600$		$\gamma = 1.7000$		$\gamma = 1.8400$	
x	U(x)	W(x)	U(x)	W(x)	U(x)	W(x)
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.036	1.6823	0.1054	2.9239	0.1851	0.5633	0.0363
0.072	3.1810	0.3881	5.5398	0.6796	1.0690	0.1331
0.108	4.4944	0.7010	7.8403	1.2269	1.5149	0.2403
0.144	5.6256	1.0176	9.8264	1.7801	1.9006	0.3486
0.180	6.5900	1.3113	11.5235	2.2934	2.2306	0.4491
0.216	7.4031	1.5842	12.9568	2.7706	2.5097	0.5426
0.252	8.0805	1.8160	14.1531	3.1759	2.7428	0.6219
0.288	8.6384	2.0160	15.1398	3.5258	2.9353	0.6904
0.324	9.0905	2.1779	15.9408	3.8092	3.0917	0.7460
0.360	9.4510	2.3094	16.5804	4.0395	3.2167	0.7911
0.396	9.7311	2.4092	17.0784	4.2146	3.3141	0.8254
0.432	9.9421	2.4836	17.4545	4.3453	3.3878	0.8510
0.468	10.0929	2.5332	17.7243	4.4328	3.4407	0.8682
0.504	10.1924	2.5632	17.9035	4.4858	3.4759	0.8787
0.540	10.2475	2.5745	18.0039	4.5064	3.4958	0.8827
0.576	10.2650	2.5713	18.0378	4.5014	3.5027	0.8818
0.612	10.2500	2.5546	18.0141	4.4729	3.4984	0.8763
0.648	10.2078	2.5275	17.9421	4.4261	3.4846	0.8672
0.684	10.1422	2.4911	17.8287	4.3630	3.4627	0.8549
0.720	10.0572	2.4477	17.6808	4.2876	3.4341	0.8402
0.756	9.9556	2.3981	17.5035	4.2013	3.3998	0.8233
0.792	9.8405	2.3439	17.3022	4.1071	3.3607	0.8049
0.828	9.7140	2.2860	17.0806	4.0062	3.3178	0.7852
0.864	9.5783	2.2256	16.8427	3.9007	3.2716	0.7646
0.900	9.4350	2.1631	16.5913	3.7917	3.2228	0.7433
0.936	9.2858	2.0994	16.3294	3.6806	3.1719	0.7215
1.008	8.9742	1.9698	15.7820	3.4543	3.0655	0.6773
1.080	8.6526	1.8422	15.2169	3.2312	2.9557	0.6336
1.152	8.3265	1.7169	14.6434	3.0120	2.8443	0.5907
1.224	8.0009	1.5971	14.0709	2.8023	2.7330	0.5496
1.296	7.6788	1.4826	13.5043	2.6020	2.6229	0.5104
1.368	7.3628	1.3750	12.9485	2.4135	2.5149	0.4735
1.440	7.0544	1.2738	12.4060	2.2363	2.4095	0.4387
1.512	6.7548	1.1796	11.8791	2.0712	2.3072	0.4064
1.656	6.1844	1.0097	10.8756	1.7733	2.1122	0.3480
1.800	5.6554	0.8659	9.9452	1.5211	1.9314	0.2985
1.944	5.1668	0.7425	9.0858	1.3045	1.7645	0.2560
2.088	4.7179	0.6385	8.2963	1.1218	1.6112	0.2202
2.232	4.3062	0.5498	7.5722	0.9663	1.4705	0.1897
2.376	3.9294	0.4750	6.9097	0.8347	1.3418	0.1638
2.520	3.5849	0.4112	6.3039	0.7227	1.2242	0.1419
2.664	3.2703	0.3571	5.7505	0.6276	1.1167	0.1232
2.952	2.7206	0.2710	4.7840	0.4764	0.9290	0.0935
3.240	2.2631	0.2081	3.9795	0.3658	0.7728	0.0718
3.528	1.8823	0.1610	3.3098	0.2830	0.6427	0.0555
3.816	1.5655	0.1256	2.7528	0.2209	0.5346	0.0433
4.104	1.3020	0.0986	2.2895	0.1735	0.4446	0.0340
4.392	1.0828	0.0779	1.9041	0.1370	0.3697	0.0269
4.680	0.9006	0.0619	1.5836	0.1088	0.3075	0.0213
4.968	0.7490	0.0494	1.3170	0.0868	0.2557	0.0170

TABLE II

$\beta = 2.044$			$\tau = 2.330$			
	$\gamma = 6.3400$		$\gamma = 9.2000$		$\gamma = 13.0000$	
x	U(x)	W(x)	U(x)	W(x)	U(x)	W(x)
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.036	0.5543	0.0495	0.9137	0.0826	2.8489	0.2592
0.072	1.0599	0.1760	1.7522	0.2928	5.4756	0.9162
0.108	1.5079	0.3124	2.4985	0.5196	7.8201	1.6246
0.144	1.8927	0.4455	3.1407	0.7407	9.8407	2.3139
0.180	2.2188	0.5652	3.6858	0.9396	11.5580	2.9338
0.216	2.4904	0.6733	4.1404	1.1191	12.9906	3.4932
0.252	2.7133	0.7614	4.5137	1.2655	14.1674	3.9488
0.288	2.8934	0.8346	4.8152	1.3871	15.1178	4.3274
0.324	3.0359	0.8907	5.0538	1.4803	15.8697	4.6174
0.360	3.1461	0.9336	5.2384	1.5517	16.4510	4.8396
0.396	3.2285	0.9632	5.3762	1.6010	16.8843	4.9928
0.432	3.2872	0.9826	5.4744	1.6332	17.1927	5.0930
0.468	3.3257	0.9922	5.5386	1.6491	17.3938	5.1425
0.504	3.3473	0.9942	5.5744	1.6526	17.5053	5.1534
0.540	3.3545	0.9894	5.5860	1.6447	17.5404	5.1285
0.576	3.3496	0.9793	5.5776	1.6281	17.5124	5.0769
0.612	3.3347	0.9646	5.5522	1.6038	17.4310	5.0012
0.648	3.3114	0.9466	5.5129	1.5739	17.3058	4.9081
0.684	3.2812	0.9255	5.4620	1.5390	17.1441	4.7996
0.720	3.2454	0.9024	5.4017	1.5008	16.9529	4.6805
0.756	3.2048	0.8776	5.3335	1.4596	16.7373	4.5524
0.792	3.1606	0.8517	5.2593	1.4167	16.5024	4.4187
0.828	3.1133	0.8250	5.1800	1.3723	16.2519	4.2806
0.864	3.0637	0.7978	5.0968	1.3273	15.9894	4.1405
0.900	3.0123	0.7705	5.0107	1.2819	15.7175	3.9991
0.936	2.9595	0.7432	4.9223	1.2367	15.4390	3.8582
1.008	2.8513	0.6892	4.7414	1.1470	14.8686	3.5789
1.080	2.7418	0.6376	4.5584	1.0613	14.2925	3.3118
1.152	2.6325	0.5882	4.3757	0.9792	13.7177	3.0561
1.224	2.5246	0.5420	4.1957	0.9025	13.1518	2.8170
1.296	2.4189	0.4987	4.0195	0.8306	12.5979	2.5929
1.368	2.3161	0.4588	3.8481	0.7642	12.0597	2.3858
1.440	2.2164	0.4218	3.6821	0.7027	11.5383	2.1942
1.512	2.1201	0.3879	3.5218	0.6463	11.0352	2.0182
1.656	1.9380	0.3280	3.2187	0.5466	10.0845	1.7070
1.800	1.7702	0.2784	2.9397	0.4639	9.2093	1.4491
1.944	1.6160	0.2366	2.6833	0.3944	8.4056	1.2321
2.088	1.4747	0.2020	2.4486	0.3367	7.6700	1.0519
2.232	1.3455	0.1729	2.2339	0.2883	6.9972	0.9007
2.376	1.2274	0.1486	2.0377	0.2478	6.3827	0.7743
2.520	1.1195	0.1282	1.8587	0.2137	5.8217	0.6678
2.664	1.0211	0.1109	1.6952	0.1850	5.3097	0.5780
2.952	0.8493	0.0838	1.4100	0.1398	4.4164	0.4368
3.240	0.7064	0.0641	1.1728	0.1070	3.6732	0.3343
3.528	0.5875	0.0495	0.9754	0.0826	3.0550	0.2583
3.816	0.4886	0.0386	0.8112	0.0644	2.5407	0.2013
4.104	0.4064	0.0303	0.6746	0.0505	2.1131	0.1580
4.392	0.3380	0.0239	0.5611	0.0399	1.7574	0.1248
4.680	0.2811	0.0190	0.4666	0.0317	1.4615	0.0991
4.968	0.2337	0.0151	0.3881	0.0253	1.2155	0.0790

TABLE II

$\beta = 2.330$			$\tau = 1.000$			
	$\gamma = 0.1000$		$\gamma = 0.1040$		$\gamma = 0.1080$	
x	U(x)	W(x)	U(x)	W(x)	U(x)	W(x)
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.036	0.6115	0.0115	3.9580	0.0772	2.3267	0.0468
0.072	1.1259	0.0458	7.2902	0.3065	4.2878	0.1856
0.108	1.5564	0.0860	10.0803	0.5751	5.9317	0.3483
0.144	1.9159	0.1296	12.4117	0.8660	7.3069	0.5246
0.180	2.2140	0.1726	14.3461	1.1531	8.4492	0.6986
0.216	2.4602	0.2149	15.9450	1.4354	9.3946	0.8697
0.252	2.6614	0.2536	17.2533	1.6938	10.1691	1.0264
0.288	2.8247	0.2894	18.3159	1.9325	10.7990	1.1712
0.324	2.9552	0.3211	19.1657	2.1442	11.3036	1.2997
0.360	3.0579	0.3494	19.8352	2.3328	11.7019	1.4141
0.396	3.1364	0.3737	20.3488	2.4955	12.0082	1.5130
0.432	3.1947	0.3948	20.7302	2.6360	12.2365	1.5984
0.468	3.2353	0.4124	20.9973	2.7538	12.3972	1.6700
0.504	3.2611	0.4271	21.1678	2.8520	12.5007	1.7298
0.540	3.2740	0.4389	21.2547	2.9310	12.5547	1.7779
0.576	3.2760	0.4482	21.2710	2.9936	12.5670	1.8161
0.612	3.2687	0.4552	21.2266	3.0404	12.5431	1.8447
0.648	3.2536	0.4601	21.1311	3.0737	12.4889	1.8651
0.684	3.2318	0.4632	20.9917	3.0944	12.4087	1.8778
0.720	3.2043	0.4646	20.8157	3.1043	12.3066	1.8841
0.756	3.1721	0.4646	20.6084	3.1043	12.1859	1.8842
0.792	3.1358	0.4633	20.3753	3.0958	12.0497	1.8792
0.828	3.0963	0.4608	20.1204	3.0795	11.9006	1.8695
0.864	3.0541	0.4573	19.8476	3.0568	11.7407	1.8559
0.900	3.0096	0.4530	19.5602	3.0281	11.5721	1.8386
0.936	2.9633	0.4479	19.2610	2.9945	11.3963	1.8184
1.008	2.8667	0.4359	18.6360	2.9142	11.0288	1.7699
1.080	2.7670	0.4221	17.9901	2.8227	10.6485	1.7146
1.152	2.6656	0.4069	17.3333	2.7214	10.2614	1.6533
1.224	2.5642	0.3909	16.6758	2.6152	9.8736	1.5890
1.296	2.4636	0.3744	16.0233	2.5051	9.4885	1.5223
1.368	2.3647	0.3578	15.3812	2.3942	9.1094	1.4550
1.440	2.2679	0.3411	14.7526	2.2830	8.7379	1.3876
1.512	2.1736	0.3247	14.1402	2.1735	8.3760	1.3211
1.656	1.9934	0.2927	12.9692	1.9595	7.6836	1.1913
1.800	1.8255	0.2630	11.8780	1.7615	7.0380	1.0711
1.944	1.6697	0.2354	10.8656	1.5766	6.4388	0.9587
2.088	1.5262	0.2103	9.9323	1.4090	5.8862	0.8570
2.232	1.3942	0.1875	9.0736	1.2563	5.3776	0.7642
2.376	1.2731	0.1671	8.2859	1.1196	4.9110	0.6810
2.520	1.1622	0.1487	7.5640	0.9965	4.4834	0.6062
2.664	1.0607	0.1323	6.9036	0.8869	4.0920	0.5395
2.952	0.8830	0.1045	5.7475	0.7007	3.4069	0.4263
3.240	0.7349	0.0829	4.7835	0.5557	2.8356	0.3381
3.528	0.6114	0.0656	3.9799	0.4402	2.3593	0.2679
3.816	0.5086	0.0522	3.3109	0.3499	1.9627	0.2129
4.104	0.4231	0.0415	2.7540	0.2784	1.6326	0.1694
4.392	0.3519	0.0331	2.2907	0.2220	1.3580	0.1351
4.680	0.2926	0.0264	1.9052	0.1772	1.1294	0.1078
4.968	0.2434	0.0211	1.5845	0.1416	0.9393	0.0862

TABLE II

$\beta = 2.330$			$\tau = 1.300$			
	$\gamma = 0.2239$		$\gamma = 0.2400$		$\gamma = 0.3000$	
x	U(x)	W(x)	U(x)	W(x)	U(x)	W(x)
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.036	1.1398	0.0331	3.0101	0.0913	3.6440	0.1267
0.072	2.1095	0.1279	5.5810	0.3533	6.7869	0.4874
0.108	2.9291	0.2367	7.7619	0.6547	9.4782	0.8952
0.144	3.6192	0.3518	9.6039	0.9742	11.7708	1.3637
0.180	4.1960	0.4630	11.1482	1.2836	13.7090	1.7992
0.216	4.6758	0.5705	12.4365	1.5826	15.3384	2.2213
0.252	5.0710	0.6667	13.5007	1.8505	16.6955	2.6007
0.288	5.3939	0.7537	14.3727	2.0932	17.8169	2.9456
0.324	5.6538	0.8289	15.0769	2.3031	18.7312	3.2451
0.360	5.8599	0.8942	15.6375	2.4857	19.4670	3.5064
0.396	6.0192	0.9488	16.0730	2.6383	20.0461	3.7261
0.432	6.1387	0.9943	16.4016	2.7659	20.4907	3.9106
0.468	6.2236	1.0308	16.6372	2.8680	20.8176	4.0593
0.504	6.2790	1.0595	16.7937	2.9489	21.0437	4.1780
0.540	6.3089	1.0808	16.8813	3.0090	21.1817	4.2675
0.576	6.3173	1.0959	16.9107	3.0519	21.2446	4.3323
0.612	6.3069	1.1052	16.8895	3.0784	21.2421	4.3738
0.648	6.2808	1.1096	16.8257	3.0913	21.1840	4.3959
0.684	6.2412	1.1094	16.7251	3.0915	21.0779	4.3998
0.720	6.1903	1.1055	16.5936	3.0812	20.9311	4.3886
0.756	6.1297	1.0982	16.4358	3.0614	20.7494	4.3636
0.792	6.0611	1.0880	16.2560	3.0337	20.5384	4.3273
0.828	5.9857	1.0754	16.0576	2.9990	20.3024	4.2807
0.864	5.9047	1.0607	15.8441	2.9585	20.0458	4.2257
0.900	5.8192	1.0442	15.6178	2.9130	19.7719	4.1633
0.936	5.7300	1.0263	15.3815	2.8635	19.4839	4.0950
1.008	5.5433	0.9869	14.8855	2.7541	18.8753	3.9430
1.080	5.3501	0.9448	14.3709	2.6374	18.2392	3.7798
1.152	5.1534	0.9006	13.8462	2.5147	17.5870	3.6072
1.224	4.9564	0.8560	13.3200	2.3907	16.9301	3.4323
1.296	4.7609	0.8113	12.7972	2.2662	16.2751	3.2561
1.368	4.5686	0.7674	12.2823	2.1440	15.6284	3.0827
1.440	4.3804	0.7245	11.7780	2.0244	14.9934	2.9126
1.512	4.1971	0.6831	11.2868	1.9090	14.3736	2.7481
1.656	3.8470	0.6045	10.3474	1.6899	13.1856	2.4353
1.800	3.5212	0.5342	9.4725	1.4936	12.0766	2.1543
1.944	3.2193	0.4705	8.6614	1.3158	11.0465	1.8992
2.088	2.9413	0.4143	7.9142	1.1588	10.0965	1.6737
2.232	2.6858	0.3643	7.2274	1.0193	9.2222	1.4729
2.376	2.4517	0.3206	6.5978	0.8970	8.4203	1.2967
2.520	2.2374	0.2820	6.0213	0.7893	7.6856	1.1414
2.664	2.0415	0.2484	5.4943	0.6951	7.0135	1.0055
2.952	1.6989	0.1927	4.5724	0.5393	5.8375	0.7805
3.240	1.4135	0.1505	3.8044	0.4212	4.8574	0.6098
3.528	1.1758	0.1178	3.1647	0.3298	4.0408	0.4775
3.816	0.9780	0.0927	2.6323	0.2596	3.3612	0.3760
4.104	0.8134	0.0732	2.1894	0.2050	2.7956	0.2970
4.392	0.6765	0.0581	1.8209	0.1626	2.3252	0.2356
4.680	0.5626	0.0462	1.5144	0.1294	1.9338	0.1875
4.968	0.4679	0.0369	1.2595	0.1033	1.6083	0.1496

TABLE II

$\beta = 2.330$			$\tau = 1.800$			
	$\gamma = 0.7107$		$\gamma = 0.7660$		$\gamma = 0.7820$	
x	U(x)	W(x)	U(x)	W(x)	U(x)	W(x)
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.036	1.4336	0.0707	0.7467	0.0380	4.5946	0.2362
0.072	2.6843	0.2632	1.4006	0.1416	8.6221	0.8798
0.108	3.7624	0.4778	1.9662	0.2573	12.1085	1.5982
0.144	4.6809	0.6969	2.4492	0.3755	15.0879	2.3330
0.180	5.4562	0.9018	2.8579	0.4862	17.6109	3.0212
0.216	6.1050	1.0938	3.2006	0.5899	19.7274	3.6665
0.252	6.6417	1.2587	3.4847	0.6791	21.4828	4.2214
0.288	7.0809	1.4024	3.7177	0.7570	22.9230	4.7059
0.324	7.4345	1.5206	3.9057	0.8211	24.0859	5.1049
0.360	7.7146	1.6182	4.0549	0.8740	25.0095	5.4347
0.396	7.9303	1.6942	4.1702	0.9153	25.7238	5.6920
0.432	8.0912	1.7527	4.2565	0.9472	26.2589	5.8906
0.468	8.2043	1.7940	4.3176	0.9698	26.6381	6.0314
0.504	8.2769	1.8214	4.3572	0.9848	26.8846	6.1257
0.540	8.3144	1.8358	4.3782	0.9928	27.0162	6.1758
0.576	8.3224	1.8397	4.3834	0.9951	27.0505	6.1906
0.612	8.3048	1.8339	4.3752	0.9922	27.0011	6.1727
0.648	8.2659	1.8205	4.3555	0.9851	26.8814	6.1291
0.684	8.2087	1.8002	4.3262	0.9743	26.7014	6.0621
0.720	8.1363	1.7745	4.2887	0.9606	26.4713	5.9770
0.756	8.0510	1.7441	4.2444	0.9443	26.1986	5.8759
0.792	7.9551	1.7102	4.1944	0.9260	25.8910	5.7626
0.828	7.8504	1.6731	4.1397	0.9061	25.5540	5.6388
0.864	7.7386	1.6339	4.0811	0.8850	25.1934	5.5076
0.900	7.6210	1.5928	4.0195	0.8628	24.8133	5.3700
0.936	7.4988	1.5505	3.9554	0.8400	24.4181	5.2284
1.008	7.2443	1.4633	3.8217	0.7930	23.5938	4.9358
1.080	6.9824	1.3762	3.6840	0.7459	22.7445	4.6431
1.152	6.7174	1.2895	3.5445	0.6990	21.8838	4.3516
1.224	6.4532	1.2057	3.4054	0.6537	21.0254	4.0698
1.296	6.1922	1.1248	3.2678	0.6100	20.1765	3.7975
1.368	5.9363	1.0481	3.1330	0.5684	19.3442	3.5392
1.440	5.6868	0.9753	3.0014	0.5290	18.5321	3.2939
1.512	5.4446	0.9070	2.8737	0.4920	17.7437	3.0638
1.656	4.9837	0.7824	2.6306	0.4245	16.2428	2.6436
1.800	4.5566	0.6756	2.4053	0.3667	14.8517	2.2833
1.944	4.1624	0.5828	2.1972	0.3163	13.5672	1.9701
2.088	3.8004	0.5038	2.0062	0.2735	12.3876	1.7031
2.232	3.4685	0.4358	1.8310	0.2366	11.3058	1.4735
2.376	3.1648	0.3779	1.6707	0.2052	10.3162	1.2779
2.520	2.8872	0.3282	1.5242	0.1782	9.4115	1.1100
2.664	2.6337	0.2858	1.3903	0.1552	8.5852	0.9666
2.952	2.1910	0.2178	1.1566	0.1182	7.1420	0.7366
3.240	1.8225	0.1677	0.9621	0.0910	5.9408	0.5672
3.528	1.5158	0.1300	0.8002	0.0706	4.9411	0.4397
3.816	1.2607	0.1015	0.6655	0.0551	4.1095	0.3436
4.104	1.0485	0.0798	0.5535	0.0433	3.4178	0.2700
4.392	0.8720	0.0631	0.4603	0.0342	2.8425	0.2135
4.680	0.7252	0.0501	0.3828	0.0272	2.3640	0.1695
4.968	0.6031	0.0400	0.3184	0.0217	1.9661	0.1352

TABLE II

$\beta = 2.330$			$\tau = 2.000$			
	$\gamma = 1.3392$		$\gamma = 1.4280$		$\gamma = 1.5600$	
x	U(x)	W(x)	U(x)	W(x)	U(x)	W(x)
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.036	0.7048	0.0445	0.6156	0.0390	1.0627	0.0709
0.072	1.3302	0.1626	1.1640	0.1421	2.0111	0.2593
0.108	1.8757	0.2927	1.6438	0.2555	2.8422	0.4669
0.144	2.3430	0.4234	2.0558	0.3693	3.5564	0.6757
0.180	2.7392	0.5439	2.4057	0.4741	4.1636	0.8685
0.216	3.0711	0.6553	2.6994	0.5710	4.6734	1.0470
0.252	3.3457	0.7493	2.9428	0.6527	5.0962	1.1978
0.288	3.5700	0.8300	3.1420	0.7228	5.4424	1.3272
0.324	3.7502	0.8948	3.3022	0.7791	5.7210	1.4314
0.360	3.8923	0.9470	3.4287	0.8245	5.9412	1.5154
0.396	4.0011	0.9862	3.5259	0.8585	6.1103	1.5787
0.432	4.0815	1.0151	3.5978	0.8836	6.2358	1.6253
0.468	4.1373	1.0338	3.6480	0.8999	6.3234	1.6558
0.504	4.1722	1.0446	3.6797	0.9093	6.3788	1.6736
0.540	4.1892	1.0480	3.6954	0.9122	6.4065	1.6793
0.576	4.1910	1.0454	3.6976	0.9101	6.4109	1.6756
0.612	4.1799	1.0375	3.6884	0.9033	6.3952	1.6634
0.648	4.1580	1.0255	3.6695	0.8929	6.3629	1.6445
0.684	4.1269	1.0099	3.6425	0.8793	6.3162	1.6196
0.720	4.0882	0.9914	3.6087	0.8633	6.2579	1.5903
0.756	4.0431	0.9706	3.5692	0.8452	6.1896	1.5572
0.792	3.9928	0.9480	3.5250	0.8256	6.1132	1.5212
0.828	3.9381	0.9240	3.4770	0.8047	6.0301	1.4829
0.864	3.8801	0.8990	3.4259	0.7830	5.9417	1.4430
0.900	3.8192	0.8732	3.3724	0.7607	5.8489	1.4019
0.936	3.7562	0.8471	3.3169	0.7379	5.7528	1.3601
1.008	3.6256	0.7941	3.2018	0.6919	5.5533	1.2753
1.080	3.4919	0.7421	3.0838	0.6466	5.3489	1.1921
1.152	3.3570	0.6911	2.9648	0.6023	5.1426	1.1104
1.224	3.2231	0.6425	2.8466	0.5601	4.9376	1.0326
1.296	3.0910	0.5962	2.7301	0.5197	4.7355	0.9583
1.368	2.9620	0.5527	2.6161	0.4819	4.5379	0.8885
1.440	2.8363	0.5118	2.5051	0.4463	4.3455	0.8229
1.512	2.7146	0.4738	2.3976	0.4132	4.1590	0.7620
1.656	2.4834	0.4054	2.1935	0.3536	3.8049	0.6521
1.800	2.2697	0.3476	2.0047	0.3032	3.4775	0.5591
1.944	2.0727	0.2980	1.8307	0.2600	3.1757	0.4794
2.088	1.8920	0.2562	1.6711	0.2235	2.8989	0.4122
2.232	1.7265	0.2206	1.5249	0.1925	2.6453	0.3550
2.376	1.5751	0.1905	1.3913	0.1663	2.4134	0.3066
2.520	1.4369	0.1649	1.2691	0.1439	2.2015	0.2655
2.664	1.3106	0.1432	1.1576	0.1250	2.0081	0.2305
2.952	1.0902	0.1087	0.9629	0.0949	1.6704	0.1749
3.240	0.9068	0.0834	0.8009	0.0728	1.3894	0.1343
3.528	0.7542	0.0645	0.6661	0.0563	1.1556	0.1039
3.816	0.6272	0.0504	0.5540	0.0439	0.9611	0.0811
4.104	0.5216	0.0395	0.4608	0.0345	0.7993	0.0637
4.392	0.4338	0.0312	0.3832	0.0273	0.6647	0.0503
4.680	0.3608	0.0248	0.3187	0.0216	0.5528	0.0399
4.968	0.3001	0.0198	0.2650	0.0172	0.4598	0.0318

TABLE II

$\beta = 2.330$			$\tau = 2.330$			
	$\gamma = 4.7300$		$\gamma = 5.8000$		$\gamma = 9.2000$	
x	U(x)	W(x)	U(x)	W(x)	U(x)	W(x)
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.036	1.4328	0.1240	1.3332	0.1180	0.6953	0.0643
0.072	2.7343	0.4389	2.5495	0.4179	1.3342	0.2276
0.108	3.8835	0.7775	3.6266	0.7408	1.9028	0.4035
0.144	4.8674	1.1068	4.5505	1.0549	2.3919	0.5745
0.180	5.6985	1.4020	5.3321	1.3370	2.8065	0.7281
0.216	6.3888	1.6680	5.9818	1.5912	3.1517	0.8665
0.252	6.9532	1.8840	6.5137	1.7979	3.4347	0.9789
0.288	7.4074	2.0630	6.9419	1.9693	3.6626	1.0722
0.324	7.7651	2.1995	7.2793	2.1003	3.8424	1.1435
0.360	8.0403	2.3037	7.5392	2.2003	3.9810	1.1979
0.396	8.2443	2.3749	7.7319	2.2689	4.0838	1.2352
0.432	8.3882	2.4209	7.8680	2.3133	4.1564	1.2594
0.468	8.4806	2.4429	7.9556	2.3347	4.2033	1.2711
0.504	8.5302	2.4465	8.0027	2.3386	4.2285	1.2732
0.540	8.5432	2.4332	8.0154	2.3263	4.2355	1.2666
0.576	8.5262	2.4073	7.9997	2.3019	4.2273	1.2533
0.612	8.4836	2.3701	7.9600	2.2667	4.2064	1.2342
0.648	8.4203	2.3247	7.9007	2.2236	4.1750	1.2108
0.684	8.3396	2.2721	7.8250	2.1736	4.1349	1.1836
0.720	8.2448	2.2146	7.7361	2.1188	4.0878	1.1539
0.756	8.1385	2.1530	7.6363	2.0601	4.0349	1.1219
0.792	8.0231	2.0888	7.5278	1.9989	3.9774	1.0887
0.828	7.9002	2.0226	7.4125	1.9358	3.9163	1.0544
0.864	7.7717	1.9556	7.2918	1.8718	3.8523	1.0196
0.900	7.6389	1.8881	7.1670	1.8073	3.7862	0.9846
0.936	7.5029	1.8208	7.0392	1.7431	3.7185	0.9496
1.008	7.2247	1.6878	6.7780	1.6160	3.5802	0.8805
1.080	6.9441	1.5608	6.5144	1.4947	3.4406	0.8145
1.152	6.6643	1.4395	6.2517	1.3786	3.3016	0.7514
1.224	6.3890	1.3261	5.9932	1.2702	3.1648	0.6924
1.296	6.1196	1.2201	5.7403	1.1688	3.0311	0.6371
1.368	5.8579	1.1221	5.4947	1.0751	2.9012	0.5861
1.440	5.6045	1.0316	5.2568	0.9884	2.7754	0.5389
1.512	5.3600	0.9486	5.0273	0.9089	2.6541	0.4956
1.656	4.8980	0.8018	4.5938	0.7684	2.4251	0.4191
1.800	4.4728	0.6804	4.1949	0.6521	2.2144	0.3557
1.944	4.0823	0.5783	3.8286	0.5543	2.0209	0.3023
2.088	3.7249	0.4936	3.4934	0.4731	1.8439	0.2581
2.232	3.3981	0.4225	3.1869	0.4050	1.6821	0.2210
2.376	3.0997	0.3632	2.9070	0.3481	1.5343	0.1899
2.520	2.8272	0.3132	2.6514	0.3002	1.3994	0.1638
2.664	2.5785	0.2710	2.4182	0.2598	1.2763	0.1418
2.952	2.1447	0.2048	2.0113	0.1963	1.0616	0.1071
3.240	1.7838	0.1567	1.6728	0.1503	0.8829	0.0820
3.528	1.4835	0.1211	1.3913	0.1161	0.7343	0.0633
3.816	1.2338	0.0944	1.1571	0.0905	0.6107	0.0493
4.104	1.0261	0.0741	0.9623	0.0710	0.5079	0.0387
4.392	0.8534	0.0585	0.8003	0.0561	0.4224	0.0306
4.680	0.7097	0.0464	0.6656	0.0445	0.3513	0.0243
4.968	0.5902	0.0370	0.5535	0.0355	0.2921	0.0193

TABLE II

$\beta = 2.500$			$\tau = 1.000$			
	$\gamma = 0.0920$		$\gamma = 0.0978$		$\gamma = 0.1140$	
x	U(x)	W(x)	U(x)	W(x)	U(x)	W(x)
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.036	4.8153	0.0940	1.8214	0.0372	0.6635	0.0152
0.072	8.8337	0.3709	3.3454	0.1470	1.2214	0.0602
0.108	12.1676	0.6934	4.6131	0.2752	1.6878	0.1126
0.144	14.9292	1.0407	5.6658	0.4134	2.0770	0.1690
0.180	17.1989	1.3814	6.5333	0.5493	2.3994	0.2244
0.216	19.0575	1.7149	7.2456	0.6824	2.6655	0.2787
0.252	20.5626	2.0184	7.8241	0.8038	2.8830	0.3281
0.288	21.7717	2.2976	8.2903	0.9157	3.0595	0.3737
0.324	22.7260	2.5439	8.6597	1.0145	3.2004	0.4139
0.360	23.4667	2.7622	8.9477	1.1022	3.3114	0.4496
0.396	24.0236	2.9495	9.1655	1.1777	3.3964	0.4803
0.432	24.4263	3.1105	9.3244	1.2426	3.4595	0.5067
0.468	24.6964	3.2446	9.4324	1.2968	3.5036	0.5287
0.504	24.8553	3.3556	9.4977	1.3419	3.5318	0.5470
0.540	24.9187	3.4441	9.5263	1.3779	3.5461	0.5616
0.576	24.9022	3.5135	9.5240	1.4063	3.5488	0.5731
0.612	24.8173	3.5646	9.4953	1.4273	3.5414	0.5817
0.648	24.6753	3.6001	9.4445	1.4421	3.5256	0.5876
0.684	24.4848	3.6210	9.3748	1.4510	3.5026	0.5912
0.720	24.2541	3.6296	9.2895	1.4550	3.4735	0.5928
0.756	23.9893	3.6267	9.1909	1.4543	3.4392	0.5925
0.792	23.6967	3.6143	9.0815	1.4497	3.4007	0.5906
0.828	23.3809	3.5929	8.9628	1.4416	3.3586	0.5873
0.864	23.0464	3.5643	8.8369	1.4305	3.3135	0.5827
0.900	22.6966	3.5289	8.7048	1.4167	3.2660	0.5771
0.936	22.3350	3.4881	8.5680	1.4007	3.2165	0.5706
1.008	21.5853	3.3916	8.2838	1.3626	3.1132	0.5550
1.080	20.8167	3.2827	7.9917	1.3194	3.0063	0.5374
1.152	20.0401	3.1629	7.6960	1.2717	2.8975	0.5180
1.224	19.2663	3.0380	7.4009	1.2219	2.7886	0.4977
1.296	18.5013	2.9089	7.1088	1.1704	2.6804	0.4767
1.368	17.7509	2.7791	6.8220	1.1185	2.5738	0.4556
1.440	17.0181	2.6493	6.5416	1.0665	2.4694	0.4345
1.512	16.3057	2.5216	6.2689	1.0153	2.3676	0.4136
1.656	14.9469	2.2726	5.7482	0.9154	2.1728	0.3730
1.800	13.6837	2.0424	5.2636	0.8230	1.9910	0.3354
1.944	12.5138	1.8277	4.8145	0.7367	1.8221	0.3002
2.088	11.4365	1.6334	4.4007	0.6585	1.6662	0.2684
2.232	10.4462	1.4563	4.0201	0.5873	1.5226	0.2394
2.376	9.5383	1.2977	3.6710	0.5234	1.3908	0.2134
2.520	8.7067	1.1550	3.3512	0.4659	1.2699	0.1900
2.664	7.9461	1.0280	3.0586	0.4147	1.1593	0.1691
2.952	6.6151	0.8122	2.5465	0.3277	0.9654	0.1337
3.240	5.5053	0.6441	2.1194	0.2600	0.8036	0.1060
3.528	4.5804	0.5103	1.7634	0.2060	0.6687	0.0840
3.816	3.8105	0.4056	1.4670	0.1637	0.5564	0.0668
4.104	3.1696	0.3227	1.2203	0.1302	0.4628	0.0531
4.392	2.6363	0.2573	1.0150	0.1039	0.3850	0.0424
4.680	2.1926	0.2054	0.8442	0.0829	0.3202	0.0338
4.968	1.8236	0.1642	0.7021	0.0663	0.2663	0.0270

TABLE II

$\beta = 2.500$			$\tau = 1.300$			
	$\gamma = 0.1900$		$\gamma = 0.2100$		$\gamma = 0.2400$	
x	U(x)	W(x)	U(x)	W(x)	U(x)	W(x)
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.036	3.3348	0.0934	3.4794	0.1046	1.0545	0.0345
0.072	6.1500	0.3610	6.4279	0.4037	1.9535	0.1329
0.108	8.5106	0.6676	8.9095	0.7463	2.7147	0.2456
0.144	10.4828	0.9916	10.9896	1.1082	3.3560	0.3646
0.180	12.1175	1.3039	12.7198	1.4571	3.8924	0.4792
0.216	13.4659	1.6048	14.1518	1.7934	4.3388	0.5897
0.252	14.5660	1.8730	15.3245	2.0932	4.7064	0.6882
0.288	15.4558	2.1150	16.2767	2.3639	5.0068	0.7772
0.324	16.1635	2.3234	17.0376	2.5971	5.2486	0.8538
0.360	16.7173	2.5037	17.6361	2.7989	5.4403	0.9202
0.396	17.1377	2.6536	18.0938	2.9669	5.5885	0.9754
0.432	17.4457	2.7781	18.4322	3.1066	5.6997	1.0214
0.468	17.6564	2.8770	18.6673	3.2177	5.7787	1.0580
0.504	17.7853	2.9546	18.8152	3.3049	5.8303	1.0868
0.540	17.8435	3.0115	18.8877	3.3691	5.8583	1.1080
0.576	17.8424	3.0511	18.8967	3.4140	5.8662	1.1228
0.612	17.7904	3.0746	18.8511	3.4407	5.8567	1.1318
0.648	17.6958	3.0846	18.7595	3.4525	5.8327	1.1357
0.684	17.5648	3.0821	18.6287	3.4502	5.7961	1.1351
0.720	17.4036	3.0694	18.4652	3.4365	5.7490	1.1307
0.756	17.2168	3.0473	18.2739	3.4123	5.6929	1.1229
0.792	17.0091	3.0176	18.0597	3.3796	5.6294	1.1122
0.828	16.7838	2.9811	17.8263	3.3391	5.5596	1.0990
0.864	16.5443	2.9391	17.5773	3.2925	5.4847	1.0838
0.900	16.2933	2.8922	17.3155	3.2404	5.4055	1.0668
0.936	16.0332	2.8415	17.0436	3.1841	5.3229	1.0483
1.008	15.4929	2.7304	16.4772	3.0603	5.1500	1.0078
1.080	14.9380	2.6124	15.8936	2.9288	4.9709	0.9647
1.152	14.3767	2.4890	15.3019	2.7910	4.7887	0.9195
1.224	13.8173	2.3648	14.7111	2.6523	4.6062	0.8739
1.296	13.2641	2.2404	14.1260	2.5132	4.4249	0.8283
1.368	12.7216	2.1185	13.5515	2.3770	4.2466	0.7835
1.440	12.1920	1.9995	12.9901	2.2438	4.0721	0.7397
1.512	11.6775	1.8847	12.4441	2.1154	3.9021	0.6975
1.656	10.6970	1.6674	11.4026	1.8720	3.5772	0.6175
1.800	9.7866	1.4730	10.4345	1.6542	3.2747	0.5458
1.944	8.9447	1.2971	9.5385	1.4570	2.9944	0.4808
2.088	8.1704	1.1421	8.7140	1.2830	2.7361	0.4235
2.232	7.4596	1.0043	7.9567	1.1285	2.4987	0.3725
2.376	6.8086	0.8836	7.2629	0.9930	2.2812	0.3279
2.520	6.2129	0.7774	6.6278	0.8737	2.0819	0.2885
2.664	5.6685	0.6845	6.0474	0.7694	1.8997	0.2541
2.952	4.7169	0.5310	5.0324	0.5970	1.5810	0.1972
3.240	3.9243	0.4147	4.1870	0.4663	1.3155	0.1540
3.528	3.2643	0.3246	3.4829	0.3650	1.0943	0.1206
3.816	2.7151	0.2555	2.8970	0.2873	0.9102	0.0949
4.104	2.2582	0.2018	2.4095	0.2269	0.7571	0.0750
4.392	1.8782	0.1601	2.0040	0.1800	0.6297	0.0595
4.680	1.5620	0.1274	1.6667	0.1432	0.5237	0.0473
4.968	1.2991	0.1017	1.3861	0.1143	0.4355	0.0378

TABLE II

$\beta = 2.500$			$\tau = 1.800$			
	$\gamma = 0.7300$		$\gamma = 0.8100$		$\gamma = 0.8550$	
x	U(x)	W(x)	U(x)	W(x)	U(x)	W(x)
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.036	1.3040	0.0693	2.1588	0.1190	3.2117	0.1784
0.072	2.4427	0.2570	4.0544	0.4407	6.0412	0.6589
0.108	3.4244	0.4657	5.6965	0.7985	8.4992	1.1928
0.144	4.2599	0.6783	7.0994	1.1628	10.6034	1.7357
0.180	4.9644	0.8766	8.2864	1.5029	12.3876	2.2423
0.216	5.5528	1.0622	9.2809	1.8211	13.8851	2.7161
0.252	6.0384	1.2211	10.1045	2.0938	15.1275	3.1221
0.288	6.4348	1.3595	10.7788	2.3314	16.1465	3.4757
0.324	6.7529	1.4729	11.3218	2.5263	16.9688	3.7658
0.360	7.0038	1.5664	11.7517	2.6870	17.6213	4.0049
0.396	7.1961	1.6389	12.0828	2.8118	18.1252	4.1908
0.432	7.3384	1.6945	12.3295	2.9077	18.5020	4.3336
0.468	7.4373	1.7335	12.5027	2.9753	18.7681	4.4341
0.504	7.4995	1.7592	12.6136	3.0199	18.9402	4.5007
0.540	7.5300	1.7723	12.6706	3.0430	19.0308	4.5351
0.576	7.5338	1.7754	12.6821	3.0488	19.0527	4.5438
0.612	7.5147	1.7692	12.6545	3.0387	19.0153	4.5289
0.648	7.4764	1.7558	12.5941	3.0160	18.9282	4.4953
0.684	7.4218	1.7357	12.5057	2.9820	18.7987	4.4449
0.720	7.3536	1.7105	12.3941	2.9393	18.6339	4.3813
0.756	7.2740	1.6808	12.2628	2.8888	18.4392	4.3062
0.792	7.1850	1.6478	12.1154	2.8324	18.2200	4.2224
0.828	7.0882	1.6118	11.9546	2.7710	17.9802	4.1311
0.864	6.9852	1.5737	11.7829	2.7060	17.7239	4.0344
0.900	6.8771	1.5340	11.6024	2.6379	17.4542	3.9332
0.936	6.7651	1.4931	11.4151	2.5680	17.1739	3.8292
1.008	6.5324	1.4088	11.0253	2.4237	16.5900	3.6145
1.080	6.2937	1.3247	10.6247	2.2796	15.9892	3.4000
1.152	6.0526	1.2412	10.2195	2.1363	15.3810	3.1865
1.224	5.8128	1.1604	9.8160	1.9978	14.7751	2.9802
1.296	5.5762	1.0825	9.4174	1.8640	14.1762	2.7810
1.368	5.3445	1.0087	9.0271	1.7372	13.5896	2.5920
1.440	5.1189	0.9386	8.6467	1.6168	13.0174	2.4127
1.512	4.9000	0.8729	8.2776	1.5039	12.4623	2.2443
1.656	4.4840	0.7531	7.5756	1.2977	11.4062	1.9370
1.800	4.0990	0.6503	6.9256	1.1209	10.4281	1.6734
1.944	3.7438	0.5611	6.3259	0.9673	9.5253	1.4441
2.088	3.4178	0.4850	5.7753	0.8363	8.6965	1.2487
2.232	3.1191	0.4196	5.2707	0.7236	7.9367	1.0805
2.376	2.8459	0.3639	4.8091	0.6275	7.2418	0.9372
2.520	2.5962	0.3161	4.3872	0.5451	6.6065	0.8141
2.664	2.3682	0.2752	4.0019	0.4747	6.0263	0.7090
2.952	1.9700	0.2097	3.3291	0.3618	5.0132	0.5404
3.240	1.6386	0.1615	2.7691	0.2786	4.1700	0.4162
3.528	1.3629	0.1252	2.3031	0.2160	3.4683	0.3227
3.816	1.1335	0.0978	1.9155	0.1688	2.8846	0.2521
4.104	0.9427	0.0769	1.5931	0.1326	2.3990	0.1981
4.392	0.7840	0.0608	1.3249	0.1048	1.9952	0.1566
4.680	0.6520	0.0482	1.1019	0.0833	1.6594	0.1244
4.968	0.5423	0.0385	0.9164	0.0664	1.3800	0.0992

TABLE II

$\beta = 2.500$			$\tau = 2.000$			
	$\gamma = 1.2000$		$\gamma = 1.3392$		$\gamma = 1.5600$	
x	U(x)	W(x)	U(x)	W(x)	U(x)	W(x)
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.036	1.1813	0.0757	1.3413	0.0887	1.3153	0.0895
0.072	2.2247	0.2763	2.5322	0.3237	2.4914	0.3258
0.108	3.1309	0.4966	3.5710	0.5820	3.5229	0.5857
0.144	3.9039	0.7174	4.4599	0.8410	4.4091	0.8463
0.180	4.5563	0.9205	5.2125	1.0795	5.1622	1.0864
0.216	5.1006	1.1078	5.8419	1.2996	5.7938	1.3082
0.252	5.5487	1.2654	6.3614	1.4849	6.3168	1.4950
0.288	5.9129	1.4002	6.7848	1.6436	6.7442	1.6552
0.324	6.2037	1.5081	7.1236	1.7707	7.0874	1.7837
0.360	6.4313	1.5947	7.3897	1.8729	7.3579	1.8871
0.396	6.6040	1.6593	7.5923	1.9493	7.5648	1.9646
0.432	6.7301	1.7065	7.7410	2.0053	7.7174	2.0216
0.468	6.8158	1.7368	7.8429	2.0414	7.8231	2.0585
0.504	6.8676	1.7537	7.9053	2.0617	7.8889	2.0796
0.540	6.8900	1.7581	7.9337	2.0674	7.9204	2.0859
0.576	6.8880	1.7528	7.9337	2.0616	7.9231	2.0805
0.612	6.8651	1.7386	7.9092	2.0454	7.9012	2.0646
0.648	6.8248	1.7176	7.8645	2.0210	7.8587	2.0406
0.684	6.7697	1.6905	7.8026	1.9895	7.7988	2.0092
0.720	6.7026	1.6589	7.7266	1.9527	7.7244	1.9725
0.756	6.6253	1.6233	7.6387	1.9111	7.6380	1.9309
0.792	6.5398	1.5850	7.5412	1.8663	7.5418	1.8860
0.828	6.4476	1.5442	7.4357	1.8186	7.4374	1.8382
0.864	6.3500	1.5020	7.3239	1.7691	7.3266	1.7885
0.900	6.2481	1.4585	7.2071	1.7181	7.2106	1.7373
0.936	6.1429	1.4145	7.0864	1.6665	7.0906	1.6854
1.008	5.9257	1.3252	6.8368	1.5618	6.8421	1.5800
1.080	5.7042	1.2379	6.5819	1.4591	6.5880	1.4767
1.152	5.4814	1.1524	6.3255	1.3587	6.3320	1.3754
1.224	5.2607	1.0710	6.0712	1.2630	6.0781	1.2789
1.296	5.0436	0.9935	5.8210	1.1718	5.8280	1.1868
1.368	4.8316	0.9208	5.5766	1.0862	5.5837	1.1004
1.440	4.6256	0.8526	5.3390	1.0059	5.3460	1.0192
1.512	4.4262	0.7892	5.1090	0.9312	5.1159	0.9437
1.656	4.0480	0.6750	4.6727	0.7966	4.6793	0.8076
1.800	3.6988	0.5786	4.2697	0.6830	4.2759	0.6925
1.944	3.3772	0.4959	3.8986	0.5855	3.9043	0.5938
2.088	3.0824	0.4263	3.5583	0.5034	3.5637	0.5106
2.232	2.8125	0.3671	3.2468	0.4335	3.2517	0.4398
2.376	2.5658	0.3170	2.9621	0.3744	2.9665	0.3799
2.520	2.3405	0.2744	2.7019	0.3241	2.7060	0.3289
2.664	2.1348	0.2383	2.4645	0.2814	2.4682	0.2856
2.952	1.7757	0.1808	2.0500	0.2136	2.0531	0.2168
3.240	1.4770	0.1388	1.7051	0.1640	1.7077	0.1664
3.528	1.2284	0.1074	1.4181	0.1269	1.4203	0.1288
3.816	1.0216	0.0838	1.1794	0.0990	1.1812	0.1005
4.104	0.8496	0.0658	0.9809	0.0777	0.9824	0.0789
4.392	0.7066	0.0520	0.8158	0.0614	0.8170	0.0623
4.680	0.5877	0.0413	0.6784	0.0488	0.6795	0.0495
4.968	0.4887	0.0329	0.5642	0.0389	0.5651	0.0395

TABLE II

$\beta = 2.500$			$\tau = 2.330$			
	$\gamma = 6.2400$		$\gamma = 10.5000$		$\gamma = 13.0000$	
x	U(x)	W(x)	U(x)	W(x)	U(x)	W(x)
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.036	0.4955	0.0449	0.9843	0.0898	2.9533	0.2677
0.072	0.9483	0.1587	1.8906	0.3165	5.6800	0.9420
0.108	1.3497	0.2811	2.6979	0.5604	8.1145	1.6669
0.144	1.6939	0.4001	3.3921	0.7971	10.2109	2.3700
0.180	1.9849	0.5067	3.9806	1.0095	11.9904	3.0005
0.216	2.2266	0.6026	4.4703	1.2008	13.4723	3.5682
0.252	2.4243	0.6805	4.8712	1.3561	14.6870	4.0292
0.288	2.5831	0.7450	5.1939	1.4848	15.6654	4.4113
0.324	2.7081	0.7942	5.4480	1.5830	16.4368	4.7031
0.360	2.8041	0.8316	5.6435	1.6580	17.0304	4.9258
0.396	2.8750	0.8572	5.7882	1.7093	17.4704	5.0784
0.432	2.9248	0.8737	5.8901	1.7425	17.7808	5.1774
0.468	2.9566	0.8815	5.9554	1.7585	17.9801	5.2251
0.504	2.9733	0.8827	5.9901	1.7613	18.0869	5.2337
0.540	2.9773	0.8778	5.9989	1.7519	18.1149	5.2064
0.576	2.9707	0.8684	5.9863	1.7335	18.0780	5.1522
0.612	2.9553	0.8549	5.9556	1.7069	17.9864	5.0737
0.648	2.9326	0.8384	5.9102	1.6745	17.8501	4.9778
0.684	2.9039	0.8194	5.8525	1.6369	17.6766	4.8665
0.720	2.8703	0.7987	5.7850	1.5958	17.4731	4.7447
0.756	2.8327	0.7764	5.7093	1.5516	17.2449	4.6139
0.792	2.7919	0.7533	5.6273	1.5056	16.9974	4.4776
0.828	2.7487	0.7294	5.5401	1.4582	16.7342	4.3370
0.864	2.7035	0.7052	5.4490	1.4102	16.4592	4.1945
0.900	2.6568	0.6808	5.3548	1.3617	16.1751	4.0508
0.936	2.6091	0.6566	5.2586	1.3135	15.8844	3.9075
1.008	2.5116	0.6086	5.0620	1.2179	15.2906	3.6240
1.080	2.4134	0.5628	4.8639	1.1267	14.6922	3.3530
1.152	2.3156	0.5191	4.6667	1.0395	14.0965	3.0939
1.224	2.2194	0.4782	4.4728	0.9579	13.5108	2.8515
1.296	2.1255	0.4400	4.2833	0.8815	12.9384	2.6245
1.368	2.0343	0.4047	4.0994	0.8110	12.3828	2.4148
1.440	1.9460	0.3721	3.9214	0.7457	11.8452	2.2207
1.512	1.8609	0.3421	3.7498	0.6859	11.3268	2.0426
1.656	1.7002	0.2892	3.4259	0.5800	10.3482	1.7275
1.800	1.5524	0.2454	3.1280	0.4923	9.4483	1.4665
1.944	1.4167	0.2086	2.8546	0.4185	8.6225	1.2469
2.088	1.2926	0.1781	2.6045	0.3573	7.8670	1.0646
2.232	1.1791	0.1524	2.3758	0.3059	7.1764	0.9115
2.376	1.0755	0.1310	2.1671	0.2630	6.5458	0.7836
2.520	0.9810	0.1130	1.9765	0.2268	5.9702	0.6758
2.664	0.8947	0.0978	1.8026	0.1963	5.4450	0.5849
2.952	0.7441	0.0739	1.4993	0.1483	4.5288	0.4421
3.240	0.6189	0.0565	1.2470	0.1135	3.7666	0.3384
3.528	0.5147	0.0437	1.0371	0.0877	3.1326	0.2614
3.816	0.4281	0.0340	0.8625	0.0683	2.6053	0.2037
4.104	0.3560	0.0267	0.7173	0.0536	2.1667	0.1599
4.392	0.2961	0.0211	0.5966	0.0424	1.8020	0.1263
4.680	0.2462	0.0167	0.4961	0.0336	1.4987	0.1003
4.968	0.2048	0.0133	0.4126	0.0268	1.2464	0.0800

TABLE II

$\beta = 2.859$			$\tau = 1.000$			
	$\gamma = 0.0740$		$\gamma = 0.0770$		$\gamma = 0.0850$	
x	U(x)	W(x)	U(x)	W(x)	U(x)	W(x)
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.036	2.3217	0.0461	1.4182	0.0290	0.8441	0.0186
0.072	4.2229	0.1798	2.5820	0.1132	1.5398	0.0726
0.108	5.7696	0.3332	3.5307	0.2100	2.1097	0.1348
0.144	7.0271	0.4961	4.3036	0.3130	2.5760	0.2009
0.180	8.0400	0.6538	4.9275	0.4128	2.9544	0.2652
0.216	8.8532	0.8066	5.4294	0.5096	3.2604	0.3275
0.252	9.4972	0.9439	5.8279	0.5967	3.5048	0.3836
0.288	10.0024	1.0688	6.1414	0.6761	3.6985	0.4348
0.324	10.3899	1.1777	6.3827	0.7454	3.8487	0.4795
0.360	10.6807	1.2733	6.5645	0.8063	3.9632	0.5188
0.396	10.8892	1.3542	6.6958	0.8580	4.0471	0.5521
0.432	11.0301	1.4230	6.7853	0.9019	4.1057	0.5805
0.468	11.1136	1.4794	6.8393	0.9380	4.1426	0.6038
0.504	11.1498	1.5254	6.8641	0.9676	4.1617	0.6229
0.540	11.1459	1.5614	6.8641	0.9908	4.1654	0.6378
0.576	11.1090	1.5889	6.8435	1.0085	4.1566	0.6493
0.612	11.0443	1.6083	6.8057	1.0211	4.1369	0.6574
0.648	10.9568	1.6209	6.7536	1.0295	4.1085	0.6628
0.684	10.8501	1.6272	6.6896	1.0338	4.0725	0.6656
0.720	10.7280	1.6283	6.6159	1.0347	4.0304	0.6662
0.756	10.5929	1.6244	6.5341	1.0325	3.9831	0.6647
0.792	10.4475	1.6165	6.4459	1.0277	3.9317	0.6616
0.828	10.2938	1.6049	6.3522	1.0205	3.8769	0.6570
0.864	10.1334	1.5902	6.2545	1.0114	3.8193	0.6511
0.900	9.9679	1.5727	6.1534	1.0004	3.7595	0.6441
0.936	9.7986	1.5529	6.0499	0.9881	3.6980	0.6361
1.008	9.4520	1.5074	5.8377	0.9594	3.5715	0.6176
1.080	9.1013	1.4569	5.6226	0.9275	3.4426	0.5971
1.152	8.7505	1.4021	5.4071	0.8929	3.3130	0.5748
1.224	8.4037	1.3454	5.1939	0.8570	3.1844	0.5516
1.296	8.0630	1.2872	4.9842	0.8201	3.0575	0.5279
1.368	7.7303	1.2289	4.7793	0.7831	2.9333	0.5041
1.440	7.4067	1.1709	4.5799	0.7463	2.8122	0.4804
1.512	7.0932	1.1140	4.3866	0.7101	2.6946	0.4571
1.656	6.4974	1.0033	4.0190	0.6397	2.4704	0.4118
1.800	5.9452	0.9013	3.6781	0.5748	2.2621	0.3700
1.944	5.4351	0.8063	3.3630	0.5143	2.0691	0.3311
2.088	4.9661	0.7204	3.0731	0.4596	1.8914	0.2959
2.232	4.5354	0.6422	2.8068	0.4098	1.7280	0.2639
2.376	4.1409	0.5722	2.5628	0.3652	1.5781	0.2351
2.520	3.7796	0.5093	2.3394	0.3250	1.4408	0.2093
2.664	3.4493	0.4533	2.1350	0.2893	1.3151	0.1863
2.952	2.8714	0.3581	1.7774	0.2286	1.0950	0.1472
3.240	2.3897	0.2840	1.4793	0.1813	0.9115	0.1168
3.528	1.9882	0.2250	1.2308	0.1437	0.7584	0.0925
3.816	1.6540	0.1789	1.0239	0.1142	0.6310	0.0736
4.104	1.3758	0.1423	0.8517	0.0908	0.5249	0.0585
4.392	1.1443	0.1135	0.7084	0.0724	0.4366	0.0467
4.680	0.9517	0.0906	0.5892	0.0578	0.3631	0.0372
4.968	0.7915	0.0724	0.4900	0.0462	0.3020	0.0298

TABLE II

$\beta = 2.859$			$\tau = 1.300$			
	$\gamma = 0.1550$		$\gamma = 0.1700$		$\gamma = 0.1900$	
x	U(x)	W(x)	U(x)	W(x)	U(x)	W(x)
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.036	2.8767	0.0830	1.4837	0.0455	0.7207	0.0237
0.072	5.2639	0.3164	2.7210	0.1737	1.3254	0.0902
0.108	7.2308	0.5800	3.7452	0.3188	1.8292	0.1654
0.144	8.8469	0.8547	4.5905	0.4702	2.2473	0.2438
0.180	10.1628	1.1159	5.2820	0.6144	2.5915	0.3185
0.216	11.2296	1.3648	5.8452	0.7521	2.8736	0.3898
0.252	12.0832	1.5838	6.2982	0.8734	3.1021	0.4526
0.288	12.7599	1.7794	6.6592	0.9820	3.2856	0.5088
0.324	13.2853	1.9455	6.9415	1.0743	3.4303	0.5566
0.360	13.6849	2.0876	7.1579	1.1535	3.5425	0.5976
0.396	13.9769	2.2040	7.3179	1.2185	3.6268	0.6312
0.432	14.1798	2.2992	7.4308	1.2719	3.6875	0.6588
0.468	14.3063	2.3734	7.5035	1.3136	3.7280	0.6803
0.504	14.3697	2.4303	7.5426	1.3457	3.7516	0.6969
0.540	14.3791	2.4704	7.5530	1.3685	3.7607	0.7087
0.576	14.3439	2.4969	7.5395	1.3838	3.7576	0.7166
0.612	14.2708	2.5104	7.5057	1.3918	3.7442	0.7207
0.648	14.1665	2.5135	7.4552	1.3940	3.7221	0.7218
0.684	14.0360	2.5068	7.3904	1.3908	3.6926	0.7201
0.720	13.8840	2.4922	7.3140	1.3832	3.6571	0.7162
0.756	13.7141	2.4705	7.2278	1.3716	3.6165	0.7101
0.792	13.5298	2.4430	7.1337	1.3567	3.5716	0.7024
0.828	13.3336	2.4103	7.0331	1.3389	3.5233	0.6932
0.864	13.1281	2.3735	6.9272	1.3188	3.4722	0.6828
0.900	12.9153	2.3332	6.8172	1.2967	3.4188	0.6714
0.936	12.6968	2.2901	6.7041	1.2731	3.3637	0.6592
1.008	12.2484	2.1967	6.4710	1.2217	3.2495	0.6326
1.080	11.7933	2.0989	6.2336	1.1678	3.1326	0.6047
1.152	11.3370	1.9974	5.9949	1.1117	3.0146	0.5757
1.224	10.8854	1.8958	5.7583	1.0555	2.8972	0.5466
1.296	10.4413	1.7946	5.5251	0.9995	2.7813	0.5176
1.368	10.0077	1.6959	5.2971	0.9447	2.6676	0.4893
1.440	9.5859	1.5997	5.0750	0.8913	2.5567	0.4617
1.512	9.1773	1.5072	4.8597	0.8400	2.4490	0.4352
1.656	8.4011	1.3325	4.4502	0.7429	2.2438	0.3849
1.800	7.6826	1.1766	4.0706	0.6562	2.0532	0.3401
1.944	7.0195	1.0358	3.7200	0.5778	1.8769	0.2995
2.088	6.4106	0.9118	3.3978	0.5087	1.7147	0.2637
2.232	5.8520	0.8017	3.1021	0.4474	1.5657	0.2320
2.376	5.3408	0.7053	2.8313	0.3937	1.4293	0.2041
2.520	4.8733	0.6204	2.5836	0.3464	1.3044	0.1796
2.664	4.4461	0.5463	2.3572	0.3050	1.1902	0.1582
2.952	3.6996	0.4238	1.9616	0.2366	0.9905	0.1227
3.240	3.0778	0.3310	1.6320	0.1848	0.8241	0.0959
3.528	2.5602	0.2591	1.3575	0.1447	0.6855	0.0751
3.816	2.1295	0.2039	1.1292	0.1139	0.5702	0.0591
4.104	1.7711	0.1610	0.9391	0.0899	0.4743	0.0467
4.392	1.4730	0.1277	0.7811	0.0713	0.3944	0.0370
4.680	1.2251	0.1016	0.6496	0.0568	0.3280	0.0294
4.968	1.0189	0.0811	0.5403	0.0453	0.2728	0.0235

TABLE II

$\beta = 2.859$			$\tau = 1.800$			
	$\gamma = 0.5800$		$\gamma = 0.6240$		$\gamma = 0.7107$	
x	U(x)	W(x)	U(x)	W(x)	U(x)	W(x)
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.036	1.0081	0.0538	2.6940	0.1475	0.7352	0.0423
0.072	1.8778	0.1980	5.0299	0.5423	1.3772	0.1557
0.108	2.6188	0.3568	7.0290	0.9778	1.9298	0.2810
0.144	3.2424	0.5171	8.7178	1.4176	2.3987	0.4079
0.180	3.7621	0.6653	10.1305	1.8245	2.7928	0.5257
0.216	4.1912	0.8029	11.3010	2.2026	3.1206	0.6354
0.252	4.5409	0.9197	12.2587	2.5237	3.3899	0.7288
0.288	4.8227	1.0205	13.0331	2.8012	3.6085	0.8098
0.324	5.0453	1.1023	13.6478	3.0266	3.7829	0.8759
0.360	5.2179	1.1690	14.1265	3.2107	3.9194	0.9300
0.396	5.3471	1.2201	14.4873	3.3516	4.0231	0.9717
0.432	5.4397	1.2586	14.7483	3.4582	4.0988	1.0034
0.468	5.5009	1.2849	14.9234	3.5312	4.1503	1.0254
0.504	5.5357	1.3014	15.0261	3.5775	4.1815	1.0396
0.540	5.5479	1.3088	15.0668	3.5986	4.1952	1.0464
0.576	5.5412	1.3090	15.0556	3.5997	4.1943	1.0474
0.612	5.5185	1.3025	15.0000	3.5826	4.1807	1.0431
0.648	5.4825	1.2909	14.9077	3.5511	4.1567	1.0345
0.684	5.4353	1.2746	14.7843	3.5068	4.1239	1.0221
0.720	5.3789	1.2547	14.6353	3.4527	4.0837	1.0068
0.756	5.3148	1.2317	14.4649	3.3898	4.0374	0.9890
0.792	5.2445	1.2063	14.2771	3.3205	3.9861	0.9692
0.828	5.1691	1.1790	14.0750	3.2457	3.9307	0.9477
0.864	5.0896	1.1502	13.8615	3.1671	3.8720	0.9251
0.900	5.0070	1.1204	13.6390	3.0852	3.8106	0.9015
0.936	4.9219	1.0898	13.4096	3.0014	3.7472	0.8773
1.008	4.7468	1.0271	12.9362	2.8295	3.6162	0.8275
1.080	4.5686	0.9649	12.4536	2.6587	3.4822	0.7780
1.152	4.3898	0.9033	11.9687	2.4894	3.3474	0.7288
1.224	4.2128	0.8440	11.4881	2.3264	3.2136	0.6813
1.296	4.0389	0.7869	11.0152	2.1694	3.0818	0.6356
1.368	3.8692	0.7329	10.5536	2.0208	2.9530	0.5922
1.440	3.7042	0.6817	10.1047	1.8800	2.8277	0.5511
1.512	3.5446	0.6338	9.6701	1.7481	2.7064	0.5126
1.656	3.2420	0.5465	8.8456	1.5077	2.4759	0.4423
1.800	2.9625	0.4718	8.0837	1.3018	2.2629	0.3820
1.944	2.7051	0.4070	7.3819	1.1231	2.0666	0.3296
2.088	2.4692	0.3517	6.7382	0.9708	1.8865	0.2850
2.232	2.2531	0.3043	6.1487	0.8399	1.7215	0.2466
2.376	2.0556	0.2638	5.6098	0.7283	1.5707	0.2139
2.520	1.8751	0.2292	5.1174	0.6327	1.4328	0.1858
2.664	1.7103	0.1995	4.6678	0.5509	1.3069	0.1618
2.952	1.4227	0.1520	3.8829	0.4198	1.0872	0.1233
3.240	1.1834	0.1171	3.2297	0.3233	0.9043	0.0949
3.528	0.9842	0.0907	2.6862	0.2506	0.7521	0.0736
3.816	0.8186	0.0709	2.2341	0.1958	0.6255	0.0575
4.104	0.6808	0.0557	1.8580	0.1539	0.5202	0.0452
4.392	0.5662	0.0440	1.5453	0.1217	0.4327	0.0357
4.680	0.4709	0.0350	1.2851	0.0966	0.3598	0.0284
4.968	0.3916	0.0279	1.0688	0.0771	0.2992	0.0226

TABLE II

$\beta = 2.859$			$\tau = 2.000$			
	$\gamma = 0.9500$		$\gamma = 1.1000$		$\gamma = 1.2000$	
x	U(x)	W(x)	U(x)	W(x)	U(x)	W(x)
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.036	3.1481	0.2022	0.4636	0.0309	0.8681	0.0583
0.072	5.9016	0.7329	0.8723	0.1119	1.6375	0.2107
0.108	8.2703	1.3117	1.2261	0.2004	2.3066	0.3771
0.144	10.2728	1.8875	1.5265	0.2885	2.8769	0.5427
0.180	11.9472	2.4132	1.7789	0.3690	3.3576	0.6943
0.216	13.3311	2.8954	1.9882	0.4430	3.7577	0.8335
0.252	14.4591	3.2976	2.1595	0.5048	4.0862	0.9499
0.288	15.3663	3.6393	2.2978	0.5574	4.3523	1.0490
0.324	16.0813	3.9102	2.4073	0.5991	4.5638	1.1279
0.360	16.6330	4.1257	2.4921	0.6325	4.7285	1.1908
0.396	17.0434	4.2843	2.5557	0.6571	4.8526	1.2374
0.432	17.3348	4.3981	2.6013	0.6748	4.9422	1.2710
0.468	17.5241	4.4685	2.6313	0.6859	5.0021	1.2922
0.504	17.6280	4.5052	2.6484	0.6918	5.0371	1.3036
0.540	17.6588	4.5102	2.6544	0.6929	5.0507	1.3058
0.576	17.6290	4.4909	2.6511	0.6901	5.0465	1.3009
0.612	17.5478	4.4493	2.6399	0.6840	5.0270	1.2895
0.648	17.4243	4.3908	2.6222	0.6752	4.9950	1.2732
0.684	17.2651	4.3173	2.5991	0.6641	4.9524	1.2525
0.720	17.0770	4.2327	2.5715	0.6513	4.9010	1.2286
0.756	16.8648	4.1386	2.5402	0.6370	4.8425	1.2018
0.792	16.6333	4.0377	2.5059	0.6216	4.7781	1.1730
0.828	16.3862	3.9312	2.4692	0.6054	4.7090	1.1425
0.864	16.1268	3.8212	2.4305	0.5886	4.6361	1.1110
0.900	15.8579	3.7085	2.3904	0.5714	4.5602	1.0786
0.936	15.5818	3.5946	2.3491	0.5539	4.4821	1.0459
1.008	15.0154	3.3648	2.2643	0.5187	4.3213	0.9797
1.080	14.4415	3.1406	2.1782	0.4843	4.1577	0.9149
1.152	13.8675	2.9219	2.0920	0.4508	3.9938	0.8517
1.224	13.3011	2.7142	2.0068	0.4188	3.8317	0.7915
1.296	12.7456	2.5166	1.9232	0.3885	3.6724	0.7343
1.368	12.2049	2.3316	1.8418	0.3600	3.5173	0.6806
1.440	11.6803	2.1582	1.7628	0.3333	3.3666	0.6302
1.512	11.1735	1.9971	1.6864	0.3085	3.2209	0.5833
1.656	10.2143	1.7076	1.5417	0.2638	2.9449	0.4990
1.800	9.3302	1.4632	1.4084	0.2261	2.6903	0.4278
1.944	8.5172	1.2540	1.2857	0.1938	2.4561	0.3668
2.088	7.7727	1.0778	1.1734	0.1666	2.2415	0.3153
2.232	7.0914	0.9280	1.0705	0.1435	2.0451	0.2716
2.376	6.4690	0.8014	0.9766	0.1239	1.8657	0.2346
2.520	5.9005	0.6938	0.8908	0.1073	1.7018	0.2031
2.664	5.3818	0.6024	0.8125	0.0931	1.5521	0.1763
2.952	4.4764	0.4571	0.6758	0.0707	1.2911	0.1338
3.240	3.7232	0.3510	0.5621	0.0542	1.0738	0.1027
3.528	3.0965	0.2716	0.4675	0.0420	0.8931	0.0795
3.816	2.5753	0.2119	0.3888	0.0327	0.7428	0.0620
4.104	2.1418	0.1664	0.3233	0.0257	0.6177	0.0487
4.392	1.7813	0.1315	0.2689	0.0203	0.5137	0.0385
4.680	1.4814	0.1044	0.2236	0.0161	0.4272	0.0305
4.968	1.2321	0.0833	0.1860	0.0128	0.3553	0.0244

TABLE II

$\beta = 2.859$			$\tau = 2.330$			
	$\gamma = 4.7300$		$\gamma = 5.8000$		$\gamma = 10.5000$	
x	U(x)	W(x)	U(x)	W(x)	U(x)	W(x)
0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.036	1.4313	0.1295	3.1954	0.2907	0.5783	0.0550
0.072	2.7344	0.4568	6.1165	1.0252	1.1110	0.1943
0.108	3.8857	0.8080	8.7049	1.8138	1.5856	0.3442
0.144	4.8699	1.1482	10.9221	2.5781	1.9934	0.4896
0.180	5.6997	1.4524	12.7949	3.2621	2.3389	0.6201
0.216	6.3868	1.7255	14.3480	3.8768	2.6261	0.7375
0.252	6.9466	1.9465	15.6156	4.3747	2.8610	0.8328
0.288	7.3951	2.1290	16.6324	4.7863	3.0498	0.9117
0.324	7.7464	2.2676	17.4303	5.0992	3.1984	0.9718
0.360	8.0148	2.3727	18.0410	5.3369	3.3123	1.0176
0.396	8.2118	2.4439	18.4903	5.4985	3.3965	1.0488
0.432	8.3487	2.4892	18.8040	5.6019	3.4555	1.0690
0.468	8.4344	2.5099	19.0016	5.6497	3.4930	1.0785
0.504	8.4776	2.5120	19.1028	5.6555	3.5126	1.0800
0.540	8.4846	2.4968	19.1221	5.6225	3.5171	1.0741
0.576	8.4620	2.4689	19.0741	5.5607	3.5090	1.0626
0.612	8.4144	2.4295	18.9695	5.4730	3.4904	1.0461
0.648	8.3465	2.3819	18.8187	5.3667	3.4633	1.0260
0.684	8.2618	2.3271	18.6296	5.2440	3.4289	1.0028
0.720	8.1635	2.2673	18.4097	5.1102	3.3889	0.9774
0.756	8.0541	2.2035	18.1645	4.9671	3.3441	0.9503
0.792	7.9361	2.1372	17.8996	4.8183	3.2956	0.9220
0.828	7.8111	2.0689	17.6188	4.6651	3.2441	0.8928
0.864	7.6809	1.9999	17.3259	4.5100	3.1904	0.8633
0.900	7.5466	1.9304	17.0239	4.3538	3.1350	0.8335
0.936	7.4096	1.8613	16.7154	4.1984	3.0784	0.8039
1.008	7.1303	1.7248	16.0864	3.8913	2.9628	0.7453
1.080	6.8494	1.5946	15.4536	3.5983	2.8464	0.6893
1.152	6.5703	1.4703	14.8245	3.3184	2.7307	0.6359
1.224	6.2963	1.3543	14.2066	3.0571	2.6169	0.5859
1.296	6.0287	1.2459	13.6032	2.8126	2.5059	0.5391
1.368	5.7692	1.1458	13.0179	2.5870	2.3981	0.4959
1.440	5.5182	1.0532	12.4517	2.3783	2.2938	0.4560
1.512	5.2763	0.9684	11.9060	2.1870	2.1933	0.4194
1.656	4.8199	0.8185	10.8764	1.8488	2.0037	0.3546
1.800	4.4004	0.6945	9.9299	1.5690	1.8294	0.3009
1.944	4.0156	0.5903	9.0616	1.3337	1.6694	0.2558
2.088	3.6637	0.5039	8.2674	1.1384	1.5231	0.2184
2.232	3.3420	0.4313	7.5415	0.9746	1.3894	0.1870
2.376	3.0483	0.3707	6.8787	0.8377	1.2673	0.1607
2.520	2.7802	0.3197	6.2738	0.7224	1.1558	0.1386
2.664	2.5356	0.2767	5.7218	0.6253	1.0541	0.1199
2.952	2.1089	0.2091	4.7590	0.4725	0.8767	0.0906
3.240	1.7540	0.1600	3.9580	0.3617	0.7292	0.0694
3.528	1.4587	0.1236	3.2918	0.2794	0.6064	0.0536
3.816	1.2132	0.0963	2.7377	0.2178	0.5043	0.0417
4.104	1.0090	0.0756	2.2769	0.1709	0.4194	0.0328
4.392	0.8391	0.0597	1.8936	0.1350	0.3488	0.0259
4.680	0.6979	0.0474	1.5748	0.1072	0.2901	0.0205
4.968	0.5804	0.0378	1.3097	0.0855	0.2413	0.0164

Table V*

Percent D States and Triplet Scattering Length

β	τ	γ	P_D	ρ_t/r_0
2.044	1.0	.1140	.016	.69$_5$
		.1300	.019	.71
		.1350	.020	.71
	1.3	.2400	.021	.68
		.2700	.024	.70
		.3000	.026	.70
	1.8	.8550	.032	.65
		.9300	.035	.65
		1.0000	.036	.65
	2.0	1.560	.042	.62
		1.700	.042	.62
		1.840	.042	.62
	2.33	6.340	.054	.55
		9.200	.054	.55
		13.000	.054	.55
2.33	1.0	.1000	.018	.67
		.1040	.019$_5$	.67
		.1080	.020$_5$	.67
	1.3	.2239	.025	.65
		.2400	.027	.66
		.3000	.034	.67
	1.8	.7107	.034	.62
		.7660	.036	.62
		.7820	.037	.62
	2.0	1.3392	.042	.60
		1.4280	.041	.60
		1.5600	.046	.60
	2.33	4.7300	.051	.55
		5.8000	.053	.55
		9.2000	.056	.54

β	τ	γ	P_D	ρ_t/r_0
2.50	1.0	.0920	.020	.65
		.0978	.021	.65
		.1140	.025	.665
	1.3	.1900	.024$_5$	.63
		.2100	.027	.64
		.2400	.030	.65
	1.8	.7300	.039	.61
		.8100	.040	.61
		.8550	.040	.62
	2.0	1.2000	.043	.59
		1.3392	.045	.59
		1.5600	.046	.59
	2.33	6.24	.055	.54
		10.50	.054	.54
		13.00	.052	.54
2.859	1.0	.0740	.020	.61
		.0770	.021	.61
		.0850	.023$_5$	.63
	1.3	.1550	.025	.60
		.1700	.028	.61
		.1900	.030	.625
	1.8	.5800	.039	.58
		.6240	.040	.59
		.7107	.044	.59
	2.0	.95	.044	.57
		1.10	.046	.57
		1.20	.045	.58
	2.33	4.73	.055	.53
		5.80	.0545	.54
		10.50	.059	.53

*Table V was hand-computed at M.I.T.