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PATH DIFFERENCES WITHIN WHICH SPECTRUM INTERFERENCES ARE OBSERVABLE

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1. *Reflecting systems.*—Certain earlier results (Carnegie Publications, No. 249, 1916, §56) made it seem plausible that the path differences within which interferences are obtained (i.e., the *apparent* lengths of uniform wave trains) increase as the dispersion, to which the incident collimated white light (L figure 1) is subjected, is made continually greater. With this quest in view the aim is to produce the interferences by one and the same method, but with a successive variation of the dispersion of the spectra. The method (figure 1) was first selected for this purpose (M, N, P', mirrors; T, telescope; s,s', screens), inasmuch as the use of prisms or gratings of different dispersive power at P meets the requirements, while spectra of the first and second order are equally available.

In work of this kind the spectra must be bright; otherwise the fine lines will escape detection. Deficient values will thus be obtained if the spectra are too dark. Moreover the results can not furnish data of precision, since the exact instant at which fringes, continually decreasing in size, have actually vanished, can not be fixed; and it is the fine fringes which furnish a considerable amount of the displacement. The differences, however, are so large, that orders of values are apparent, more than sufficient to substantiate the argument.

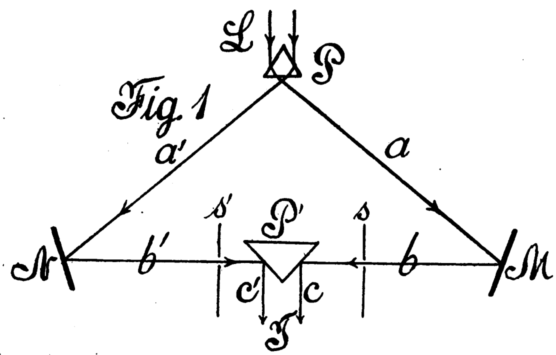
It is possible that the method (figure 1) gives the half ranges only, since the efficient pencils of light, C C', can not cross each other when M is displaced. The methods applied will nevertheless be trustworthy, since they are identical, the same telescope and other appurtenances being used throughout. Later, the grating method is to be suitably modified for corroborative experiments.

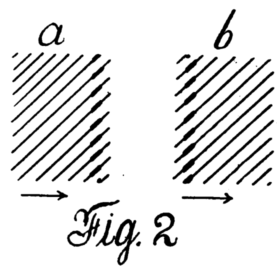
The first series of measurements was obtained with a 60° prism at P, the dispersive power $d\theta/d\lambda$ being computed (approximately) from Cauchy's equation, so that in wave length λ

$$d\delta/d\lambda = 4B \sin \tfrac{1}{2}\, \varphi/\lambda^3 \cos \tfrac{1}{2}\, (\varphi + \delta),$$

φ being the prism angle (60°) and δ the angle of minimum deviation. The dispersion constant B was estimated to be 4.6×10^{-11}.

In the remaining series with a grating at P, $d\theta/d\lambda = 1/D \cos \theta$, the usual expression, θ being the angle of diffraction and D the grating space. The dispersive power thus increases from about 800 to 17,000, over twenty times. Throughout this whole enormous range good fringes were obtained.





The values, e, show the normal displacements of the opaque mirror M, during the presence of fringes, and of the opaque mirror N, as specified. Of these, e_M is systematically larger than e_N for reasons due to residual curvature in the mirrors and surfaces, whereby fringes on the left (N) vanish sooner than those on the right (M). The datum, y, is the displacement of the right angled reflecting prism P', parallel to the component rays bb'. This value is necessarily smaller than e, as will be shown elsewhere. All measurements were frequently repeated and the means finally taken for comparison with $d\theta/d\lambda$.

In the experiments with a ruled grating at P and a concave grating reflecting at P', the phenomenon of figure 2 was observed. A wide field of faint fringes was visible, enormously accentuated and clear in the narrow strip of the linear phenomenon. As the micrometer mirror at M moves forward, these faint fringes shift bodily across the stationary bright linear strip, beginning therefore with the pattern *a* and ending with b. The faint fringes follow the rules of displacement interferometry.

TABLE 1

RANGE OF DISPLACEMENT, e, y, VARYING WITH DISPERSION

METHOD	$e_M \times 10^3$	$e_N \times 10^3$	$y \times 10^3$	θ	$d\theta/d\lambda$
	cm.	*cm.*	*cm.*		
60° prism at P	28	24	21	49°45′	760
Grating ($D = 352 \times 10^{-6}$) at P	161	136	108	9°39′	2,880
Same. 2d order	250	230	155	19°34′	6,030
Film grating ($D = 167 \times 10^{-6}$) at P	302	236	190	20°40′	6,400
Same. 2d order	470	420	440	44°56′	16,900

In addition to the data of the table, a large number of miscellaneous tests were made with the reflecting prism in different positions. Unless brought too far to the rear, when the beams are lost at the edge and e too small, the results for fine and coarser fringes were of the same order.

The data for e are not sufficiently regular in the dispersive powers above 1000 for graphic treatment (it is probable that at 16,900 the sliding along the prism surface is interfered with) but the data for the path difference, $2y$, are available. All the data, in consideration of their limitations, bear out the inference that the range of displacement within which fringes are seen, increases in marked degree with the dispersion, the average initial ratio $2y/(d\theta/d\lambda)$ is about 60×10^{-6} cm.

A very surprising result in these experiments is the efficiency of the film grating in series IV and V, not only in the first but in the second order of spectra.

After these experiments an attempt was made to obtain similar results with the more comprehensive method of two gratings G and G' (transmitting G at P and reflecting G' at P') with an appropriate change of the angle δ. But here the choice of gratings with satisfactory constants was limited and with high double dispersion the fields were apt to be too dark. Good results were obtained with the 60° prism and concave grating and with the ruled grating together with the latter. The data again showed marked increase of displacement with the dispersion $d\theta/d\lambda$.

2. *Diffraction at M, N, replacing reflection.*—The present method of observing interferences in the zeroth, first, second, third and even fourth order, successively, without essential change of the parts of the apparatus, is noteworthy. In figure 3, the incident light L from the collimator is separated into two component beams a and a' by the 60° prism, P. This is essential here, as an abundance of light is needed

(sunlight should be focused by a large lens of long focus—5 feet—on the slit). The rays a, a', are then either reflected or diffracted in any order, by the identical plane reflecting gratings G, G', into the collinear rays b, b'. These are reflected by the silvered right angled prism P' and observed in a telescope at T. G and G' and also P' should be on micrometers, so that the corresponding displacements, e, e', normal to G and G' and y in the direction $b\,b'$, may be registered.

The adjustments, if symmetry were demanded, would be cumbersome; for in addition to precise modification of the position and orienta-

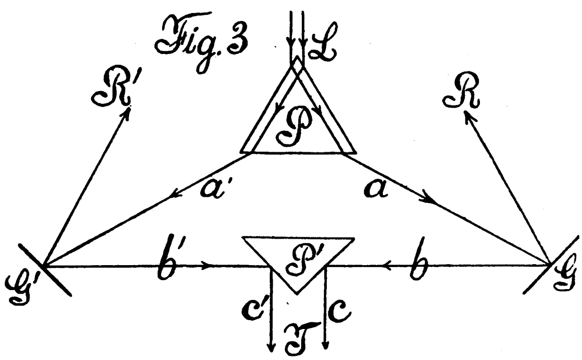


Fig. 3

tion of the prisms P, P', the grating requires fine adjustment and a means of securing parallelism of the rulings. But an approximate adjustment does very well and no pains were taken to secure symmetry. The spectra were brilliant in the low order work; but even in the fourth order the light was adequate. One may note that the gratings enhance the dispersion of the prism P, which is relatively small. Table 2 is an example of results.

TABLE 2

RANGES OF DISPLACEMENT, e, VARYING WITH DISPERSION. PAIRED GRATINGS (SPACE $D = 200 \times 10^{-6}$ cm) AND 60° PRISM. $\theta = 46°, \delta = 44°$. PATH DIFFERENCE $x = 2e \cos \delta/2$

ORDER	OBSERVED		
	$e \times 10^3$	$x \times 10^3$	$d\theta/d\lambda$
	cm.	cm.	
0	38	70	760
1	190	350	3,490
2	420	780	6,440
3[1]	520	962	9,930
4[1]	580	1,070	14,800

[1] Fringes too faint.

The fringes in the zeroth order were good and strong not inferior to any of the others, but unfortunately too shortlived. In the fourth order the fringes are weak (although the enormous sodium doublets stand out clearly), doubtless from excess of extraneous light. Here also it is difficult to prevent the beam from vanishing at the edge of the prism P'. Hence the anomalously small displacement, a discrepancy already quite manifest in the third order.

The present experiments furnish a striking example of the uniform breadth of the strip of spectrum carrying the fringes, quite apart from the dispersion of the spectra. In the prism spectrum, where the sodium doublets are indicated by a hair line just visible, to the fourth order spectra, where they stand apart like ropes, the linear interference phenomenon has the same width.

The computation of the dispersive power in these cases is peculiar. It will be seen from figure 3, that the angle ($\delta = 44°$) between the incident ray a and the diffracted ray b is constant and is $\delta = \theta + i$ in the first and second and $\delta = \theta - i$ in the third and fourth order. Hence in succession (i changing sign after the 2d order)

$$\sin(\delta - i) - \sin i = \lambda/D,$$
$$\sin(\delta - i) - \sin i = 2\lambda/D, \text{ etc.,}$$

from which equations the angle i may be computed. I did this with sufficient accuracy graphically.

Since $d\,\theta = di$, apart from sign, it follows that the dispersing power is

$$-d\theta/d\lambda = n/D\,(\cos i + \cos(\delta - i))$$

where n is the order of the spectrum. With the values of i given, the data for $d\theta/d\lambda$ were finally computed. The dispersive power of the prism was computed as above and is to be added to all the succeeding dispersive powers. The path difference, $x = 2e \cos \delta/2$ here corresponds to $2y$ above. Through the second order the rate $x/(d\theta/d\lambda)$ is about 120×10^{-6}. This value is larger than the former owing to incidental conditions. Similarly the proportionate effect of dispersion breaks down in the third and fourth orders, as already stated. The spectra themselves were still adequately bright, but the fringes were faint for some reason and I failed to make them stronger.