THE EFFECT OF COMMUTATION OF IMPEDANCES ON THE ACOUSTIC PRESSURE PRODUCED BY PAIRED TELEPHONIC SYSTEMS*

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I. *Introductory.*—The general method to which this paper contributes, is briefly described in these Proceedings of May and July. While the acoustic pressure observed in the middle of a tube, at the ends of which the telephones operate, integrates the currents, it does not obliterate the phase differences.

The paired telephones are joined by a straight tube, the nodal intensity at the middle of which is recorded by pin-hole probes communicating with the interferometer U-tube. The acoustic pressure in question, is proportional to the fringe displacement, s; or Δs, if its differential character is to be accentuated.

2. *Apparatus. Data.*—The complicated relations encountered in the preceding papers made it desirable to devise means for exchanging inductances. The adjustments are indicated in the insert of figure 1, where T and T' are the paired telephones, E the cell with periodic break, B, and S, S', the switches for reversing the current in T or T'. The inductances L and L' can be inserted either into the circuits T and T', respectively, as in the figure; or reversed so that L is inserted into the T' and L' into the T circuit. This is done by the four-fold commutator K in which the brass strips 1, 2, 3, 4, when pointing toward the left join the corresponding 4 contacts (1 to 4) below; or when pointing to the right (swivel) join the contacts 3 to 6, below. Contacts 1 and 5, 2 and 6 are metallically joined; and 1 and 2 or 5 and 6, contain the inductance L, 3 and 4 the inductance L'. The position of the strips, as in figure, will be denoted by I, the other position by II; so that for I, L is in the T and L' in the T' circuit.

In the experiments following, L is the continuously variable millihenry standard, L' the coil L_1 with or without iron cores, solid or fasciculated wire, as stated (see scheme, Fig. 1). The fringe displacements s obtained are given in figure 1 for loads L_1 of 10, 20, 30 millihenries as abscissas, when the counter-load is the coil L_1 only. In figure 2 the solid iron core is thrust into L_1 and in figure 3 the core is of wire. The three cases represent a succession of increasing inductance with the resistances constant.

The phase graphs usually present no marked peculiarity. Their mean s-values (I and II) decrease with the load L_1, while at the same time, the

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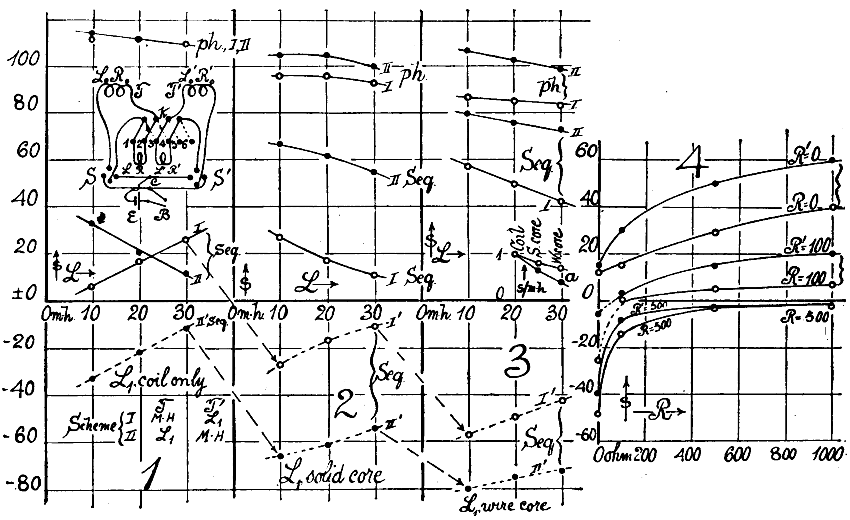
graphs nearly coincident in figure 1 (uncored) separate widely in figure 3. The *II* graph in 3 is in part even above the *II* graph in figure 2 for a lower inductance.

Very characteristic differences appear in the sequence graphs. In figure 1 the observed graphs cross. This indicates deficient singing in the position *I* of the millihenry circuit, since the graph rises with the increasing *m-h* inductance, while the L_1 (coil) circuit carries the sound. The *II* graph has therefore been reversed into the dotted line *II'*, figure 1, for the opposed conditions prevail. The two graphs, *I*, *II'*, are nearly parallel and the rate is about 1.1*s*/*m-h* for each.

In figure 2 the sound is carried by the standard *m-h* circuit, in both positions *I* and *II*. The impedence L_1 is now excessive. The graphs have



therefore both been reversed as shown in *I'* and *II'*. They are here farthest apart. The mean rates are respectively 8*s*/*m-h* for Case *I'* and 0.6*s*/*m-h* for Case *II'*.

In figure 3 the relations are of the same nature but reach a higher degree of displacement. The reversed curves, *I'* and *II'*, are lower than before, but unexpectedly nearer together. The rates have decreased further to 0.7*s*/*m-h* for Case *I'* and 0.4*s*/*m-h* for Case *II'*. This gradual fall of rate is summarized in the insert, *a*, in figure 3.

The drop below the corresponding graphs of figure 1 has been on the averages $\Delta s = 37$ in figure 2 and $\Delta s = 60$ in figure 3.

The data for Δs values of phase-sequence displacement at the same abscissas also make a coherent system, but cannot be given here.

To throw additional light on these phenomena, resistances *R* in two identical rheostats were directly compared with results such as detailed in the example, figure 4. Keeping the resistance of one circuit constant (R = 0, 100, 500 ohms), the other was increased in steps. The graphs show at once that one telephone is more efficient than the other. At $R = R'$ = 100 ohms (constant), the anterior parts of the curves pass through minima ($s = 0$). These parts have therefore been reversed in the dotted lines.

3. *Quantitative Considerations.*—The results contained in the graphs, implying that the two telephones are unequally sensitive, is a little puzzling: their resistances and inductances are the same (R_0 = 84 ohms, L_0 = 0.06 henry) and the resistances of the standard (R = 9.7 ohms) and of the L_1 coil (R = 9.9 ohms), L = 0.32 henry are about the same. Everything should therefore depend on the external resistance or inductances, and if these are the same there should be no change of fringe displacement, s, on commutation.

The difference in the paired and similar telephones may be due to the set of the plates; but as it occurs very uniformly so far as I have observed, it probably results from an induction impulse chiefly in one direction. In case of the sequence graph, the plates of the telephone would therefore be attracted and released, respectively, at any given time. These forces are liable to be unequally strong, the attraction probably being in excess.

Hence if we call the amplitudes or displacement vectors of the plate T' and T, where $T' > T$, the postulates $T \cos \theta < T$ slightly, and $T' > T \cos \theta$ in marked degree would account for most of the discrepancies, if θ is the lag due to the inductance L, initially. Subsequently T, or else T' are reduced by the successively increasing inductance of the standard. In the Case I the graph rises; in Case II the observed graph falls and $s = 0$ is reached much later.

Since $\tan \theta = L_1\omega/R$ and L_0 = 0.38 (cored coil 0.32, telephone 0.06) while R_1 = 94 (coil 10, tel. 84) and ω = 2765, we find θ = 84.9°, $\sqrt{R^2 + L_1^2\omega^2} = 1054$, $L_1\omega = 1050$.

On the side of the standard R = 94 ohms also, while the L changes from 0.010 to 0.030 henry. Thus

L =	0.010	0.020	0.030	henry
θ =	16.4°	30.5°	41.4°	henry
$\sqrt{R^2 + L^2\omega^2}$ =	98	111	125	ohms
$L\omega$ =	27.6	55.3	82.9	ohms

When L_1 and the standard are exchanged, the specifications (except L) remain about the same.

If we take the case of the sequence graph, since the currents i and i' in the branches T, T' (Fig. 1, inset) are in parallel, $E = \Delta i/\Delta \sqrt{R^2 + L^2\omega^2}$ follows as usual. But as the equality of fringe displacements s, s', due to telephones T, T', respectively, is primarily in question, we may postulate for small currents, $i = s/c$ and $i' = s'/c'$ in view of the difference of telephones in question. Hence the last equation reduces to $E = \Delta s/\{c/\sqrt{R^2 + L^2\omega^2} - c'/\sqrt{R'^2 + L'^2\omega^2}$. If $\Delta s = 0$ in the sequence graph, the denominator must also be zero. Thus $R^2 + L^2\omega^2 = (c/c')^2\ (R'^2 + L'^2\omega^2)$. Here the values of R, L, etc., are the total resistances and inductances. If we replace R by $R + R_0$, L by $L + L_0$, etc., where R_1, L are the external and R_0, L_0 the internal data, the modified equation is $(R + R_0)^2 + (L + L_0)^2\omega^2 = (c/c')^2\ \{(R' + R_0')^2 + (L + L_0')^2\omega^2\}$. This equation on commutation becomes $(R_1 + R_0)^2 + (L_1 + L_0)^2\omega^2 = (c/c')^2$-$\{(R_1' + R_0')^2 + (L_1' + L_0')^2\omega^2\}$ where L_1 and L' are read off on the standard.

Now in the above circuit the resistances R are the same, i.e., $R = R' = R_1 = R_1'$; $R_0 = R_0'$; also $L_0 = L_0'$ and $L = L_1'$ is the constant exchanged inductance. Hence if we subtract the two equations, a new equation, determining L if c/c' is known, follows, since

$$\left(\frac{c}{c'}\right)^2 = \frac{(L + L_0)^2 - (L_1 + L_0)^2}{(L' + L_0)^2 - (L + L_0)^2}$$

where L_1 and L' are given.

The difficulty of applying this equation to graphs, figure 1, et seq., is that $\Delta s = 0$, or the intersection of the graphs with the abscissa would have to be extrapolated and this is only feasible in figure 1, where, unfortunately, the L_0 of this coil (without core) was not directly determined; but equation states nevertheless if the data given be inserted

$$(c/c')^2 = ((L + 60)^2 - (65)^2)/((100)^2 - (L + 60)^2)$$

so that L must lie between 40 and 5 millihenries; or since $c/c' > 1$, between 25 and 5 *m-h*. In general, however, $i = s/c$ is not adequate, the more approximate equation being of the form $s_0 = se^{i_0/i}$ which though here inconvenient, interprets the curves as a whole, more nearly.

When the sequence graphs cross as in figure 1, the unknown inductance is determinable at once. If we re-write the first of the above equations as $\Delta s/E$, and remember that here $R = R_1'$ etc., that L (constant) is exchanged, $\Delta s/E$ remaining constant on commutation,

$$c + c' = c\,\frac{\sqrt{R^2 + L^2\omega^2}}{\sqrt{R^2 + L_1'^2\omega^2}} + c'\,\frac{\sqrt{R^2 + L^2\omega^2}}{\sqrt{R^2 + L'^2\omega^2}}$$

But at the point of intersection $L_1 = L_1'$ whence $L = L_1$. In figure 1 therefore the inductance of the coil is 23 millihenries.

From this and the numerical equation for $(c/c')^2$ the result is $c/c' = \sqrt{4421}/\sqrt{3111} = 1.19$ indicating the degree of inequality of the two telephones in relation to the sequence graphs and resulting from the assymetry of vibration of plates.

Such cases as figures 2, 3, etc., need a much larger standard of comparison, L; but these and other results are improved by the use of a small inductor (say 0.3 hen. in the secondary) and radio telephones. Omitting these, there is room here for Graph 4, obtained with the mere exchange of resistances, which merits some further attention. If R is the fixed resistance commutated, R' and R'' the counter values corresponding to positions I and II, since internally $L_0 = L_0'$, and $R_0 = R_0'$ the equations reduce to

$$c + c' = c\frac{\sqrt{(R + R_0)^2 + L_0^2\omega^2}}{\sqrt{(R'' + R_0)^2 + L_0^2\omega^2}} + c'\frac{\sqrt{(R + R_0)^2 + L_0^2\omega^2}}{\sqrt{(R' + R_0)^2 + L_0^2\omega^2}}.$$

A solution of this equation is $R' = R'' = R$, so that the paired curves of figure 4 intersect near $R = 0$, $R = 100$, $R = 500$ ohms (the case $R = 100$ left without reversal).

The case of $\Delta s' = 0$ and $\Delta s'' = 0$ for the two positions I and II, is available for $R = 100$ ohms. The other cases ($R = 0$ and $R = 500$) do not reach the abscissa. We thus have again

$$\frac{c}{c'} = \frac{\sqrt{(R + R_0)^2 + L_0^2\omega^2}}{\sqrt{(R' + R_0)^2 + L_0^2\omega^2}} = \frac{\sqrt{(R'' + R_0)^2 + L^2\omega^2}}{\sqrt{(R + R_0)^2 + L^2\omega^2}}.$$

If we insert the values $R = 100$, $R' = 50$, $R'' = 150$ ohms, as given by the graphs and the constants $L_0 = 0.06$ and $R_0 = 84$, we obtain $c/c' = 1.16$, in both cases, which agrees very well with $c/c' = 1.19$ deduced from inductances, in the preceding section.

A STATISTICAL QUANTUM THEORY OF REGULAR REFLECTION AND REFRACTION[1]

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The success of the quantum theory and more particularly of that view of the quantum theory which regards the energy of the quantum as highly localized has made it seem important to various authors to attempt an explanation on the basis of that view of some of the phenomena of optics which have been regarded as typical wave phenomena. In the following discussion, only a statistical treatment will be attempted; i.e., only large aggregates of quanta will be considered and the media traversed by these quanta will be regarded as continuous. The first of these restrictions is rendered advisable by the nature of the phenomena; the terms appropriate to the discussion of reflection, refraction and radiation pressure are scarcely