by Cayeux have the optical properties of microcline. Termier found authigenic albite crystals in the Flysch limestone of Briançonnais. Compare Rosenbusch, H., *Elemente der Gesteinslehre,* 3te Aufl., Stuttgart, 1910, (520). "Authigenic" means "formed *in situ.*"

[8] Grandjean, F., *Paris, C.-R., Acad. Sci.* **148**, 1908, (723).

[9] Daly, R. A., *Geol. Surv. Can., Memoir* No. 38, 1912, (50).

[10] Doelter, C., *Handbuch der Mineralchemie,* Dresden, Bd. 2, 2te Hälfte, 1915, (556).

[11] Chrustschoff, K., *C.-R. Acad. Sci.,* **104**, 1887, (602).

[12] Friedel, C., and Sarasin, E., *Ibid.,* **92**, 1881, (1374).

[13] Cf. Zschimmer, E., in C. Doelter's *Handbuch der Mineralchemie,* **1**, 1912, (910).

THE INTERFEROMETRY OF SMALL ANGLES, ETC. METHODS BY DIRECT AND REVERSED SUPERPOSED SPECTRA

By Carl Barus

DEPARTMENT OF PHYSICS, BROWN UNIVERSITY*

Communicated September 21, 1917

1. *Introductory.*—It occurred to me that a number of the methods treated in my papers on direct and reversed spectrum interferometry might be used directly for the measurement of small angles and possibly of the distance of the source of light. Such a procedure would have an apparent advantage, at least theoretically, of not calling for the preliminary superposition of the images of distant objects, as the superposition is inherent in the method itself. But there are large constants involved which make the result very problematical, unless these constants can be removed by a compensator. It is very questionable, moreover, whether, appreciable interferences can occur, and another difficulty which hampers the method is the decrease in the size of objects as their distances increase. A progressive investigation with the object of ascertaining to what degree the experiment is feasible is nevertheless worth while. It will be convenient therefore first to develop the methods without reference to the ulterior conditions which limit the interferences and this method has been pursued.

2. *Method with Prism.*—Figure 1 is a sketch of one of the methods in which S is the distant source of light, from which rays d and d' strike the mirrors m and n, are thence reflected to the silvered sides of the right angled prism P. After leaving it the rays enter the spectro-telescope at T in parallel. If the proper angles are selected the prism P may be replaced by one of any angle or by a reflecting grating.

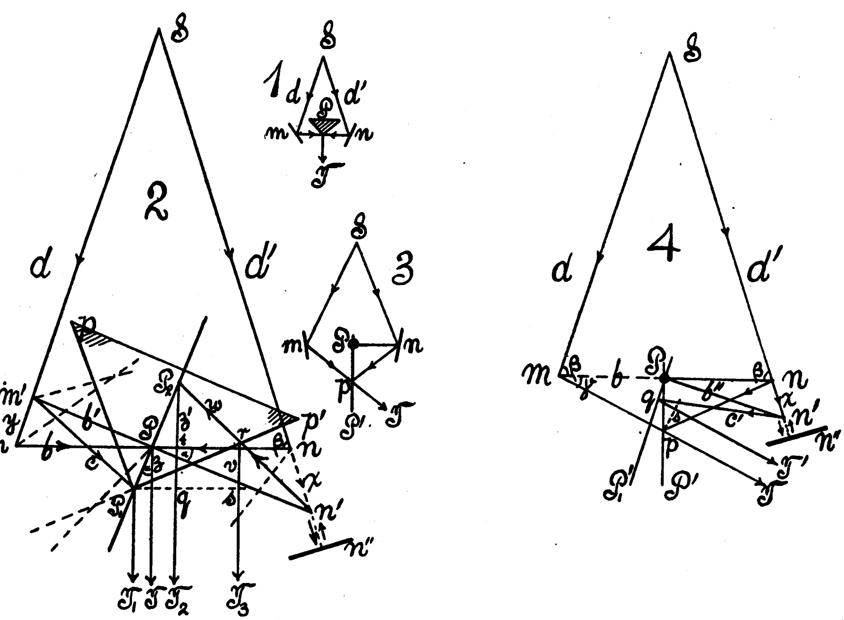
Suppose now the system mPn is securely attached to a rigid metallic beam or rail capable of rotating around a vertical axis at its center (P). This is indicated in figure 2 where the direction of rays and the normals

* Advance note from a Report to the Carnegie Institution of Washington, D. C.

of mirrors have been drawn and where the angle of rotation α has placed mPn into the position $m'Pn'$. The result is that a part y of the ray d is cut off on the left side and a part x added to the ray d' on the right side, so that the path difference which may be assumed to have been zero originally is now appreciably incremented, but not symmetrically for both sides.

It may be shown however that the rays $n'P_2T_2$ and $m'P_1T_1$ still enter the telescope in parallel and that therefore the conditions of interference have not been disturbed. This is the interesting feature of the method.

It will contribute to a more adaptable design of the apparatus for



general interferometry, if the ray Sn'' may also be reversed by reflection parallel to itself from a normal mirror n'', allowing a small lateral offset, similar on both sides for clearance of the mirrors. Again half silvers may be used at m and n for transmission and reflection, which method is probably best. These details will here be disregarded. If small angles are to be measured the direct ray method is enormously more sensitive.

3. Equations.—To derive the equations certain intercepts of the rays figure 2 in addition to x and y, may be defined. P_1 P_2 is the trace of the vertical plane of symmetry of the right angled prism, if rotated at an angle α to the right. In this case the reflected ray $n'P_2q$ on the right corresponds to the reflected ray $m'P_1$ on the left both terminating in the common wave front P_1qs before entering the telescope.

$$\begin{aligned} \text{Let } n'P_2 &= c' = b \sin\beta / \cos\alpha \sin(\beta - \alpha) \\ m'P_1 &= c \ = b \sin\beta / \cos\alpha \sin(\beta + \alpha) \\ P_2t &= z' = b \sin\alpha \sin\beta / \sin(\beta - \alpha) \\ tq &= z \ = b \sin\alpha \sin\beta / \sin(\beta + \alpha) \\ \text{and } nn' &= x \ = b \sin\alpha / \sin(\beta - \alpha) \\ mm' &= y \ = b \sin\alpha / \sin(\beta + \alpha) \end{aligned}$$

since the orginal angles at the ends of the base are β and the rotation α. The angles between incident and reflected rays are respectively $\beta - 2\alpha$ at n', $\beta + 2\alpha$ at m', $90° - 2\alpha$ at P_2 and $90° + 2\alpha$ at P_1. Finally b has changed to b' on the left and b'' on the right.

The rays however do not reach the plane of symmetry but are reflected by the faces of the right angled prism and this may be sketched in apart from size, in the rotated position (angle α) at P_1pp'. The path of the reflected rays from n' is now $n'rs$ and from m', $m'P_1$, before they meet in the common wave front P_1qs. Hence the intercepts

$$\begin{aligned} rP_2 &= w = (z + z') / \cos\alpha (\sin\alpha + \cos\alpha) \\ rs &= v = (z + z')(\cos\alpha - \sin\alpha) / (\sin\alpha + \cos\alpha) \end{aligned}$$

will enter in treating the path differences. On the left the rays have not been disturbed.

If we take the direct case first the orignal path difference SnP and SmP may be regarded zero or n and m in the same phase. On rotation therefore (angle α) the path difference is equivalent to the equation

$$n\lambda = c' - c - (w - v) + x + y$$

If the above equivalents are inserted, this equation though very complicated, may to the second order of small quantities, be reduced to a form which, since α and $90° - \beta$ are small angles for practical purposes may be abbreviated to (n, order of interference).

$$n\lambda = 2b\alpha - 2b\alpha^2 + 2b^2\alpha / d$$

The three terms correspond to the xy, wv and cc' effect.

In the case of reversed ray (fig. 2) we may consider the points m' and n' in the same phase. Hence the original path difference ($\alpha = 0$) is $x - y$. The path difference after rotation $c' - w + v - c$. The total change of path difference due to rotation is thus given by

$$n\lambda = c' - c - (w - v) - x + y$$

This differs from the preceding by the deduction of $2x$. The rays again terminate in the common wave front P_1qs to enter the telescope. Here the long rigorous equation reduces practically to

$$n \lambda = 2 b \alpha^2 - 2 b^2 \alpha / d$$

The wv effect predominates, the cc' effect is intermediate and the xy effect very small if d is large, as already instanced. If $\alpha = 1°$, $b = 1$ meter, $d = 1$ kilom., $\lambda = 6 \times 10^{-5}$ cm., the three terms of the first equation give respectively 6×10^4, 10^3, and 60 fringes.

To the first order of small quantities the equations may be written, $n \lambda = 2 b \alpha (\mathrm{cosec}\, \beta + \cot \beta)$ and $n \lambda = b\, 2 \alpha \cot \beta$. They may also be obtained geometrically by letting fall a normal from n' to the near prism face, prolonging T_3 backwards and using the isosceles triangle obtained. In this case $x + 2 (c' - w) \cos^2 (45° - \alpha) - (z' - z) + y - c$ is the path difference and reduces to the above equation. So long as the nose of the prism is near the axis of rotation, the equation of first order quantities need not be modified.

A very essential correction however is still needed. In the practical apparatus the mirrors m and n rotate on a rigid radius, b, whereas in the diagram (if $\delta = \beta - \alpha$ and $\sigma = \beta + \alpha$), b elongates on the right and contracts on the left to $b'' = b \sin \beta / \sin \delta$, and $b' = b \sin \beta / \sin \sigma$. Hence the mirrors on the right and left are displaced normally by $(b'' - b) \cos \beta / 2$ and $(b - b') \cos \beta / 2$. The path difference introduced is thus $(b'' - b) (\cos \alpha + \cos \delta) + (b - b') (\cos \alpha + \cos \sigma)$ which to the first order of small quantities may be written $2 b \alpha (1 / \sin \beta - \sin \beta + \cot \beta)$. If this quantity is deducted from the right of the above equation for path difference and direct rays there remains simply $\eta \lambda = 2 b \alpha \sin \beta$. This therefore is the equation to be used in interpreting the observations so that generally $2 \Delta N \cos i / \Delta \alpha = 2 b \sin \beta$ where $i = \beta / 2$ for the micrometer at n.

In the case of reversed rays the conditions on the left remain the same as before; whereas on the right the mirror is set at an angle $\beta / 2$ to the rail. Hence the normal displacement is $(b'' - b) \sin \beta / 2$ and the angle of incidence $90° - (\beta / 2 - \alpha)$. Thus the full path difference here to be deducted is $(b'' - b) (\cos \alpha - \cos \delta) + 2 (b - b') (\cos \alpha + \cos \sigma)$ which to the first order of small quantities reduces to $2 b \alpha \cot \beta$. Hence the equation for reversed rays is simply $2 \Delta N / \Delta \alpha = 0$ and we have an interesting appearance of terms of the second order only, which I will here omit. In general glass paths may be compensated at pleasure.

The *observations* made with the present apparatus, though quite interesting, are beyond the purpose of the present note. The two corresponding rays d and d' were obtained by cleaving the white beam from a collimator symmetrically, by a knife edged prism with silvered faces.

4. *Interferences from Rough Surfaces.*—The question now at issue is whether these interferences can be retained when the collimator is removed and the light comes directly from a ground glass surface or a Nernst filament. The spectrum fringes go at once when the slit is widened. Not so the achromatic sets. After obtaining this result with sunlight and ground glass, I replaced both by the light from a Nernst filament, under the impression that ground glass might, to a small degree, behave like plate glass. Having produced the achromatics as usual with the collimator, its objective was first removed and it was then seen that the two washed slit images were no longer superposed. Bringing the images together (by rotation of the mirror n around a vertical axis), a position was soon found in which the achromatic fringes appeared brilliantly in a white field, quite out of focus. The slit could now be widened or removed altogether, but the fringes persisted though with less brilliancy. It is thus possible to obtain these fringes directly from a Nernst filament or a narrow vertical strip of sunlight. They are so mobile with changes of Δ N and Δ α, that to find them it is necessary first to produce the spectrum fringes with collimator and spectro-telescope, then to find the achromatics on removing the spectroscope, next to remove the objective of the collimator and adjust for superposed images and finally to remove the slit. They practically cover the whole width of the washed slit image with streamers extending laterally into the glare some five times further. Slit images may even be slightly separated while each alone retains the achromatic fringes, a rather puzzling phenomenon. One may note that the slit images here are not reversed.

Experiments made as to the nature of these achromatic fringes showed that they are probably Fresnellian interferences. To prove this the objective of the collimator was removed and strong fringes obtained by passing the two vague images of the slit gradually over each other, horizontally. The fringes in this motion passed from horizontal maxima of size gradually to vertical hair lines as the images slid from contact of their nearer edges to contact of their further edges. The coarse fringes were even strongly present in the narrow dark gap between slit images prior to contact. The telescope was now focussed on the slit so that sharp linear images appeared. The fringe then vanished, but it appeared that the coarse fringes corresponded to coincident sharp slit images when observed out of focus, and the fine hair lines to sharp slit images far apart also seen out of focus. The whole phenomenon thus depends on the distance apart of two sharp lines of light and the interferences are observable before or behind their focal plane.

5. *Reversed Rays.*—If the necessary excess of path on the right is compensated by a glass column on the left (usually 10 or 15 cm. long), the spectro-telescope on adjustment shows a field of concentric *half* ellipses, all terminating in a vertical axis. These interesting phenomena are unfortunately not available for measurement as the terminator is not sharp enough and as their motion is necessarily sluggish in view of the large excess of glass path on one side. The achromatic fringes are not producible. For practical work the glass path must be replaced by an air path obtained by aid of an *offset* consisting of two pairs of parallel plates at right angles to each other. These not merely compensate without changing the direction of rays, but on rotating the plates as a whole around a horizontal and a vertical axis, they additionally serve for producing ellipses of any size and of centering them. Achromatie fringes are now brilliantly producible. The variety of observations made will however have to be given elsewhere.

6. *Plate Method.*—In view of certain difficulties encountered in the use of reflecting prisms, in particular the loss of rays at the edge, the method of figure 3 enlarged in figure 4 was devised. In this the prism is replaced by a half silver plate PP'. Hence the rays issuing at S and reflected by the opaque mirrors at m and n, are thereafter respectively transmitted and reflected by the half silvered plate PP' and then reach the spectro-telescope at T together. When the path differences are sufficiently equal, elliptic interference fringes will be seen in the spectrum. When first found they are usually very fine straight lines; but they may be rectified by plate compensators in the beams d and d' or mp and np, though the operation is not easy. Leaving these details for further consideration elsewhere, the procedure for angular measurement may advantageously be treated here. For this purpose the half silver P and one opaque mirror, n for instance are mounted on a rigid bar with an axis at P. The other mirror m is to remain fixed. If the bar is now rotated over a small angle α (fig. 4), the mirror at n is displaced to n' and the ray Sn prolonged (intercept x) is now reflected from n' to q and thence along T' into the spectro-telescope, parallel to its original direction or to the other ray mp. Hence the interferences remain intact but many fringes pass during the transfer. The *persistance* of parallelism is easily seen to be the essential feature of the method.

To control the fringes either the mirror at n (or at m) may be displaced on a micrometer screw normal to itself, or the half silver plate at P may be displaced parallel to itself. If the angle of incidence at n is i and the normal displacement of n is e the path difference introduced will be $2e \cos i$. Similarly if the normal displacement of the plate P is e' and the angle of incidence i', the path difference will be $2e' \cos i'$.

As in the preceding experiment the mirror at n may be a half silver, so that the ray, d, passes through it and may then be returned in its own path by a mirror at n'', on a fixed standard but provided with a micrometer. The displacement of this mirror parallel to itself over a distance e, introduces the path difference $2e$, while during this motion the rays np or $n'q$ are now stationary. Beams of light do not pass through each other and the interferences are kept at full intensity throughout. The glass path at n compensates the glass path PP' The air path excess $2nn''$ on the right must be specially compensated by an offset in d, as explained above.

7. *Equations.*—The rigorous equations for this case are cumbersome. If in figure 4, m and n are in the same phase and Pp is symmetrical, there will be no path difference at p. When Pn is rotated over an angle α into Pn', the path on the right becomes $nn' + n'q + qs$ while (ps, wave front) the path on the left remains mp as before. The path difference is thus the difference of these quantities to which however the increased glass path at PP' would have to be deducted. If the angle SnP is β and Pnp γ, the values of the branch paths may be found to be (since $nP = mP = b$) is $\beta - \alpha = \delta$ and $\gamma - \alpha = \tau$

$$mp = np = b / \cos \gamma$$
$$nn' = b \sin \alpha / \sin \delta$$
$$n'q = b \sin \beta / \sin \delta \sin \tau$$

Hence qs and the path difference are complicated expressions which need not be inserted here.

If α is small, so that differential expressions may be introduced, the rigorous equation (to an approximation of the second order in α) is finally equivalent to $n\lambda = b\alpha (1 + \cos(\beta - \gamma)) / \sin \beta$. If $\beta = 90°$, $n\lambda = b\alpha + p\alpha \cos \gamma$, where p is the distance Pp. The same expressions may be obtained geometrically by prolonging $n'P$ and $T'q$ and treating the isosceles triangle produced.

For the case where the ray Sn prolonged returns on itself as from n'', in figure 4, n being a half silver plate, the quantity $nn' = 2x = 2b\alpha / \sin \delta$ must be deducted. Hence $n\lambda = b\alpha (1 - \cos(\beta - \gamma)) / \sin \beta$. When $\gamma = 0$, this equation coincides with the case of the prism method apart from the factor 2.

It is finally necessary to apply the correction for the occurrence of a constant radius of rotation, whereby the mirror n is both rotated and displaced. If the distances $Pn = b$ and $Pn' = b''$, the normal displacement is $e\ (b'' - b) \cos (\beta - \gamma) / 2$. The angle of incidence being

$(\beta + \gamma) / 2 - \alpha$, the corresponding partial path difference works out as $(b'' - b)$ $(\cos(\beta - \alpha) + \cos(\gamma - \alpha))$; or finally in full as $b\,\alpha\,(\cos^2\beta + \cos\beta\cos\gamma) / \sin\beta$. Subtracting this from the first equivalent of $n\,\lambda$ above, the equation to be used for non-reversed rays becomes $n\,\lambda = b\,\alpha\,(\sin\beta + \sin\gamma)$ when α is small. As the angle of incidence at the mirror at n is $(\beta + \gamma) / 2$, and if ΔN is the displacement of this micrometer, the practical equation is thus $2\,\Delta N \cos(\beta + \gamma) / 2 = b\,(\sin\beta + \sin\gamma)\,\Delta\alpha$.

For reversed rays the normal displacement is $(b'' - b)\sin(\beta - \gamma) / 2$ and the path difference for small α therefore $b\,\alpha\cot\beta\,(\cos(\gamma - \alpha) - \cos(\beta - \alpha))$. Subtracting this from the corresponding equivalent of $n\,\lambda$, the equation for reversed rays is thus $n\,\lambda = b\,\alpha\,(\sin\beta - \sin\gamma)$ or in the practical form as the incidence is now zero, $2\,\Delta N = b\,(\sin\beta - \sin\gamma)\,\Delta\alpha$.

All equations contain the distance of the remote object at S, in $\sin\beta$, so that d occurs as a second order quantity.

Observations (also to be omitted here) were made in great variety with this apparatus, a knife edge prism at S cleaving the white beam from a collimator. To obtain strong full spectrum ellipses, the rays T and T' must not only be parallel but interpenetrate so far as possible. If p and q are a few millimeters apart all fringes vanish. For this adjustment the compensator is again convenient and for reversed rays it must be of the offset type described. One may notice that the slit images are here mirror images of each other. Nevertheless brilliant achromatics may be obtained even from rough surfaces, if narrow, by the succession of operations given above. Both these and the spectrum fringes are available for measurement. The achromatics are most serviceable if tranverse to the slit image as they then rise and fall with the play of the micrometer. To obtain them, the center of ellipses must be placed in the vertical through the telescopic field; but above or below it. In other words the spectrum fringes are to be horizontal. Even when (as in the preceding section in view of the half ellipses) horizontal spectrum fringes are precluded, the achromatic fringes will be found by adjusting as if the former were to appear. The best achromatics consist of but one or two fringes, sharply in black and white, with three or four much fainter fringes on either side. These occur frequently, but how to differentiate them systematically from the other groups of 10 or 20, more nearly uniform and therefore less serviceable fringes, I have yet to learn. All must be treated with caution, however, for they move through the field of the telescope so rapidly that if lost it is usually expeditious to seek for them again through the spectrum fringes.