A Unitary Principle of Optics, Catoptrics, and Dioptrics[1]

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The first hypothesis common to these sciences – from which the direction of any ray of light is determined geometrically – can be put in this way: *Light radiating from a point reaches an illuminated point by the easiest path, which is to be determined first with respect to planar surfaces, but is accommodated to concave and convex surfaces by considering their tangent planes.* Still, I do not have here an account of certain irregularities, perhaps having a role in the generation of colors and in other extraordinary phenomena, which are not attended to in the consideration of optics.

Hence *in simple optics, the direct ray from the radiating point C to the illuminated point E arrives* by the shortest direct path – in the same medium of course – that is *by the straight line CE*.

In Catoptrics the angle of incidence CEA and of reflection DEB are equal. For example, let C be the radiating point, D the illuminated point, and AB a plane mirror: the point E on the mirror is sought at which the ray is reflected to D. I say that it is such that as a result the whole path CE + ED becomes the least of all, or less than CF + FD if, of course, any other point F on the mirror had been taken. This will be obtained if E is taken to be such that as a result the angles CEA and DEB are equal, as is evident from geometry. Ptolemy and other ancients used this demonstration and it is found elsewhere and in particular in Heliodorus of Larissa.

In Dioptrics, the complementing sines, EH and EL of the angles of incidence CEA and of refraction GEB always preserve the same ratio, which is reciprocal to the resistance of the mediums. Let IE be air, EK water, or glass, or some other medium denser than air, C a radiating point in the air, G an illuminated point beneath the water: it is asked by which path does the light shine from the former to the latter, or what is the point E on the surface of the water AB such that the ray emitted from C is to be refracted sending it to G? This [point] E should be taken such that the path is the easiest of all. Now in different media, the difficulties of the path are in the ratio composed of the length of the paths and the resistances of the mediums. If the lines m and n represent resistance with respect to light – the former of air, the latter of water – the difficulty of the path from C to E will be as the rectangle formed by CE and m; from E to G as the rectangle formed by EG and n. Therefore so that the difficulty of the path CEG is the least of all, the sum of the rectangles CE by m and EG by n should be the least possible, or less than CF by m and FG by n – where F is taken to be any point whatsoever except E. E is sought. Since the points C and G and also the straight line AB are given by supposition, the straight lines perpendicular to the plane – we will call CH, “c”; and GL, “g”; and HL itself “h” – are therefore given as well. Also what has been sought, EH, we will call “y”, EL will be h-y, and CE will be $\sqrt{(c^2 + y^2)}$ which we will call “p”, and EG will be $\sqrt{(g^2 + y^2 - 2hy + h^2)}$, which we will call “q”. Therefore, $m \sqrt{(c^2 + y^2)} + n \sqrt{(g^2 + y^2 - 2by + b^2)}$ - or mp + nq - should be the least of all possible quantities similarly expressed and y is sought so that it will be the least. From my method of maximum and minimum, which above all notations thus far shortens miraculously the calculation, right away at first glance – almost without any calculation – it is clear that mq times y will be equal to np times h-y, or [*seu*] that np will be to mq as y will be to h-y, or [*seu*] that the rectangle CE by n will be to the rectangle EG by m, as EH will be to EL. Therefore having postulated that CE and EG are equal, the resistance of the water with respect to light will be to the resistance of the air with respect to light as EH (the sine of the complementing angle of incidence CEA in the air) is to EL (the sine of the complementing angle of refraction GEB in water), or [*seu*] the complementing sines will be in a reciprocal ratio to the resistance of the medium – as was claimed. And so if EL in an example or experiment should be discovered to be 2/3 of EH, it will be [2/3 of EH] in all other cases wherever C and G are taken to be in air and glass respectively. If E is in air and G beneath water, EL will be approximately 3/4 of EH.

We have therefore reduced all the laws of rays confirmed by experience to pure geometry and calculation by applying one principle, taken from final causes if you consider the matter correctly: Indeed a ray setting out from C neither considers how it could most easily reach point E or D or G, nor is it directed through itself to these, but the Creator of things created light so that from its nature that most beautiful event would arise. And so those who reject final causes in physics with Descartes err greatly – not to speak more harshly - since even besides the admiration of divine wisdom, they would also supply to us the most beautiful principle for discovering some properties of those things whose interior nature is still not so clearly known to us that we would be able to use proximate efficient causes and explain the machines which the Creator employed in order to produce those effects and in order to obtain his ends. Hence we also understand that the meditations of the ancients on these matters as well are therefore not to be so looked down upon as they seem to be by some people today. For it seems to me very likely that those greatest geometers Snell and Fermat – most well versed in the geometry of the ancients – extended the method that they had used[2] in Catoptrics to Dioptrics. Indeed, I suspect that Snell's theorem – which from his three unpublished books on optics is cited by that most distinguished man Issac Vossius – was discovered by almost the same method (although I believe not by such a felicitous calculation as we have used here). Indeed from our method it follows immediately as is to be demonstrated. Let a circle CBG be described with the center E and radius EC or EG, the extended tangent to this [circle] at B meets CE at V and EG at T. If an eye were at C, and an object which is seen beneath the water were at T, the point T will appear to be at V because it seems to us that we see along the straight line CEV, but since we nevertheless should really see by the broken line CET, it is clear that EV is the secant of the angle of incidence CEA[3] or of VEB which is equal to it, and ET is the secant of the angle of refraction GEB. From a known proposition of trigonometry, moreover, secants are reciprocal to their complementing sines, and therefore it follows directly that EV is to ET as EL is to EH, or [*seu*] (by our theorem) as m is to n. Therefore, given that an eye is present at C in a different medium than the object T, the apparent ray EV in the medium of the object (water) will be to the true ray ET in the medium of the object (water) as the resistance m of the medium of the eye (air) to the resistance n of the medium of the object (water). Since that ratio would always be the same – keeping the same mediums –the ratio therefore between the true ray ET and the apparent [ray] EV, will always be the same, which was Snell's theorem.

In the same way, the ratio of the sines of the complements of the angles of refraction and incidence, EL and EH, which for us is reciprocal to the resistance of the mediums, will always be the same: that is indeed the theorem of the Cartesians allowing for the different resistances of the mediums for us, which were understood by Descartes inversely[4]. Therefore not without reason, the most distinguished man Spleissius, also most well-versed in these studies, having noticed this agreement of conclusions, wondered if Descartes, when he was in Holland, had not seen Snell's Theorem; indeed he notes that [Descartes] himself had regularly omitted the names of authors and gives the example of world-vortices to which Jordanus Bruno and Johannes Kepler reached to within a finger's breadth, so that it would seem that the only thing missing to them was the name. He adds that because Descartes wanted to demonstrate this theorem using his own tools, he falls into great difficulties: indeed, since he saw that the ray CE having entered into the water from the air, and being refracted there into EG and so towards the perpendicular EK, is therefore rendered more similar to that ray whose action [*actio*] is stronger, namely, the perpendicular, he suspected that it met with less resistance in water or glass than in air. Nonetheless, in supposing the contrary[5] which is consistent with much better reasoning, the same conclusion is reached applying our principle of the easiest way. From which Fermat rightly gathered that Descartes had not given the true reason of his own theorem. Furthermore, the analogy by which he tries to illustrate his own explanation is not very apt. In fig. 2, let there be a little globe A on a polished table BC advancing in the place 1A, having arrived at the middle, it would run into the part of the table DE covered with a woolen cloth whereby it would move more slowly in 2A.

In the same way therefore he thinks that glass or another solid body will delay rays of light less than air does which is more fibrous [*villosus*]. But (as I would pass over [the fact that] the parts of water are quite soft according to Descartes himself) it is sufficient to consider that the little globe when in 2A – having been on the cloth DE – continues in turn to a polished part of the table in 3A where it would not recover the prior speed that it had at 1A before it encountered the cloth. Nevertheless when a ray of light [traveling] from a medium of greater resistance into a medium of less resistance entering again into a medium similar to the earlier medium would recover its earlier state, and having arranged the surfaces of the two similar mediums, the first and the last – the former of the emitting medium, the later of the receiving medium – to be parallel planes, the ray would recover a direction parallel to that which it had in the first medium through the later refraction.

Nevertheless, it seems that the way – worthy of the genius of him - in which Descartes explains the reflection as well as the refraction of light by an imitation of the motion of other bodies is not to be rejected but only emended. With regards to reflection, he should have first explained why some little globe, I, following along the perpendicular IE, and striking the plane AB, is thereby reflected. Indeed, we see that some bodies while [*ut*] soft are not equally reflected. The true cause of this reflection is the elasticity of the globule, or the plane, or both. For an elastic plane will yield to some degree - as we see happen when a small stone strikes a stretched membrane or an inflated bladder. And indeed, the harder it is struck the more it will yield, and in restoring itself with a commensurably greater force, it throws back what struck it with the speed and direction [*via*] from which it came. Indeed, although, as is clear from his letters, Descartes did not want to accept this explanation of reflection already brought forward by certain people in his own day, nonetheless, today it has been put beyond doubt by arguments and experiments. Since therefore the little globe would come from C to E along the straight line CE in fig. 1, and therefore by the motion composed from two - one horizontal such as CI or HE, by which it comes from CH to IE, and one perpendicular such as CH or IE, by which it comes from CI to HE - both setting out from C and terminating at E. And the surface AB would not be opposed – but rather would be parallel – to the horizontal conatus of this motion arriving at E from CH toward IE along the straight line CI or HE. Therefore it will retain the same undiminished speed and direction of the horizontal motion, and – assuming that the interval between CH and IE is equal to the interval between EI and RD – it will take as much time to come from EI to RD as it did to come from CH to IE. However, with the undiminished speed of the perpendicular motion by which it comes from CI to HE, it will be reflected in the opposite direction so that it will take the same amount of time to return from ER to ID. Therefore, since ER would be equal to EH and RD to CH, the triangles CHE and DRE will be similar and equal and therefore the angles DEB and CEA will be equal. All this will be more clear if we imagine that the segment CID parallel to the surface AB, touches AB along HR having remained parallel through CH, DR, while the globe on the segment CI is carried from C to I so that really the whole composite motion of the globe will be along the diagonal CE. But then the segment CID having reflected from the solid surface AB, will return with the same speed and direction [*via*] by which it had come, and with equal time it will arrive again at CID. Meanwhile, the globe with continuous motion moves along the segment with the same speed, and therefore in the same amount of time travels from I to D, [or] through ID which is equal to CI – for equal spaces are run through keeping the same speeds for equal times. And so, the globe is carried from E to D along the straight line ED by the composite motion of its own [motion] on the straight line along the segment ID, and the motion of the segment itself along EI, that is, its return from HR to CD.

Before refraction is to be explained, it should be noted that a medium more resistant to light (yet still not opaque) seems to be that which impedes the diffusion of light more, or (seu) its distribution through more parts of the medium, and one can say that it is less illuminable for indeed it is the nature of light to try to diffuse itself. Conversely, the more that light will affect equally the parts of the medium it illuminates, or where it will communicate its own force to more insensible parts of the illuminated place, the medium will be more illuminable and less resistant to light. Hence where the affected particles of the illuminated medium are solid and small, or less interspersed with some other heterogeneous material not affected by light, to that extent the medium will be said to be more illuminated. Indeed, it is known from principles of Mechanics that the same blow impressed at the same time on many bodies will impart less force to the individual ones than if it had been inflicted on one of them; therefore, it will happen that the more resistant a medium is to the diffusion of light, or the fewer parts that are affected, that much more strongly the single parts will be affected; in a more illuminable medium, more parts are affected, but less strongly, and the impressed impetus is weaker. Now assuming that the motion of the little globe takes place along a radius, and that the little globe is supposed to come from G to E, whereupon entering into the [new] medium, its speed or impetus would be retarded in the proportion of, say, 3/2. Therefore, if the ray came from G to E in the first medium KE in one unit of time, the same ray in the new medium EI will come from E to C in 3/2 units of time, assuming that EC and GE are equal, wherever C ultimately is[6]. But since, before the entry of the little globe into the new medium EI or at the point E, the surface AB separating the mediums will not obstruct the exercised horizontal speed on GK, LE and parallel lines (indeed this horizontal motion LEH only grazes it) however in the first moment of entering [the medium], or at the point E, (considering the little globe to be indefinitely small like a point, just as rays are customarily viewed as being without breadth) the inclination of the line CE must be determined at once: therefore such an inclination is to be assumed at the beginning so that with respect to the horizontal motion the velocity remains the same, and in the new medium it remains the same as when it first entered. Therefore, the little globe, which while it moved in the prior medium from G to E, had completed in one unit of time an interval GK or LE (between GL and KE) in the horizontal direction, now has one and half units of time during which it must go from E to C, so that it will cover the interval EH or IC (between EI and HC) in the same horizontal direction which must be one and a half times the prior interval LE because in keeping the same horizontal velocity (which the moment of refraction does not change) the distances are as times. Therefore EH is to EL in direct proportion of the times, or in reciprocal proportion to the speeds, or the resistances. Indeed, we have shown that in the case of light due to the resistance of the medium impeding diffusion, the velocity or impetus increases in proportion to the resistance, and languishes in proportion to the ease with which it diffuses itself through single particles. Conversely, the ray recovers its force and also direction when it returns again to the medium where it is diffused less, and where more rays are spent on driving fewer parts. Descartes was unable to explain that recovery – as we have mentioned above - by his own comparison with a woolen cloth or some other fibrous material.

Notes

1. The Latin text of this essay was first published by Leibniz in the *Acta Eruditorum*, June 1682. I would like to thank Paul Hoffman, Benjamin King, and Mike Stannard for many helpful suggestions that have greatly improved the present translation. I would also like to thank Paul Teeter for his technical assistance in preparing this document for the web. Readers with suggestions for further improving the present translation or notes are encouraged to contact me at

2. Taking “usu” as a slip for “usi.”

3. I.e. the secant of the angle of incidence CEA=CE/HE=EV/EB=EV/1=EV.

4. This would appear to be Leibniz’s understanding of the situation: Descartes had maintained that

the ratio of the sine of the angle of incidence to the sine of the angle of refraction is *inversely* proportional to the ratio of the velocity of the ray of incidence to the velocity of the ray of refraction, *directly* proportional to the ratio of the resistance of the medium of incidence to the resistance of the medium of refraction, and constant. In algebraic terms:

$$\frac{\sin_i}{\sin_r} = \frac{v_r}{v_i} = \frac{m}{n} = C$$

Snell, and later Fermat, had maintained that the ratio of the sine of the angle of incidence to the sine of the angle of refraction is *directly* proportional to ratio of the velocity of the ray of incidence to the velocity of the ray of refraction, *inversely* proportional to ratio of the resistance of the mediums of incidence to the resistance of the medium of refraction, and constant. In algebraic terms:

$$\frac{\sin_i}{\sin_r} = \frac{v_i}{v_r} = \frac{n}{m} = C$$

Leibniz, who goes on to argue that light travels faster in more resistant (non-opaque) mediums, maintains that the ratio of the sine of the angle of incidence to the sine of the angle of refraction is *inversely* proportional to the ratio of the velocity of the ray of incidence to the velocity of the ray of refraction, *inversely* proportional to the ratio of the resistance of the mediums of incidence to the resistance of the medium of refraction, and constant. In algebraic terms:

$$\frac{\sin_i}{\sin_r} = \frac{v_r}{v_i} = \frac{n}{m} = C$$

He is thus able to maintain that Snell was *correct* with respect to the ratio of the velocity of the incident ray to the refractive ray, but *incorrect* with respect to the ratio of the resistance of the incident medium to the refractive medium, and conversely, that Descartes was *correct* with respect to the resistance of the incident medium to the refractive medium, but *incorrect* with respect to the ratio of the velocity of the incident ray to the refractive ray. Contemporary optics agrees with Snell (and Fermat) on both counts.

5. I.e. in supposing that the ray meets with *more* resistance in water or glass than air.

6. Note Leibniz is now imagining a case where the ray's source is in water, and its sink is in air (in effect the inverse of the case he was considering earlier).