GENERAL EQUILIBRIUM

Theory and Evidence

W. D. A. Bryant

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PREFACE

General equilibrium theory is one of the major research programs in modern economics. It aims to describe and understand the functioning and properties of market economies. Developments of and reactions to this program constitute a major part of economics. In order to achieve its ends, general equilibrium theory focuses on four broad questions. First, under what conditions do equilibrium states exist? Second, under what conditions are equilibrium states optimal? Third, are equilibrium states relatively small in number, stable relative to adjustment processes and do they behave predictably in the face of shocks to the economy? Fourth, are equilibrium states congruent with actual economic data?

In this book, results concerning the theoretical and empirical properties of equilibrium states are presented and discussed. The principal aim of the work is to achieve an understanding of what general equilibrium theory has to say about the circumstances in which deregulated market economies function well, along with circumstances where this is not the case. As this is being written, economic policy makers in market economics around the world — particularly in the United States and in Europe — are intervening heavily in those economies in an effort to stabilise markets. This is at contradiction with what is often thought of as 'free market orthodoxy', a position which some think is supported by the general equilibrium theory. In fact, a close study of general equilibrium theory tends to suggest that the circumstances under which market economies can function well are potentially quite special. The need for policy intervention, particularly in establishing appropriate parameters in which the economy can operate, may therefore be more widespread than is commonly thought.

ACKNOWLEDGEMENTS

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Notwithstanding the contributions of all these people, none can surpass that of my wife Lynne who has done the hard yards with a difficult, but ultimately harmless, academic scribbler. To you this work is, with love, dedicated.

CONTENTS

Chapter 1

GENERAL EQUILIBRIUM THEORY: AN OVERVIEW

> "*Pure economics is, in essence, the theory of the determination of prices under a hypothetical regime of free competition*".
>
> L. Walras
>
> "*The most intellectually exciting question in our subject remains: is it true that the pursuit of private interest produces not chaos but coherence, and if so how is it done?*"
>
> F. H. Hahn
>
> "*There is order — not chaos.*"
>
> D. W. Katzner

1.1. Introduction

General equilibrium theory pictures the economy as a collection of economic agents who make supply and demand decisions over commodities, labour types and assets, in order to further their own interests. The general equilibrium research program then studies the equilibrium properties of the economy, so conceived. The importance of this research program to economics is noted by many writers. Fisher (1987) for instance argues that: "Economic theory is pre-eminently a matter of equilibrium analysis. In particular, the centrepiece of the subject — general equilibrium theory — deals with the existence and efficiency properties of competitive equilibrium. Nor is this only an abstract matter. The principal policy insight of economics ... rests on the intimate connections between competitive equilibrium and Pareto efficiency." Fisher (1987; p. 26). In a similar vein, Scarf remarks that: "One of the major themes of economic theory is that the behaviour of a complex economic system can be viewed as an equilibrium arising from the interaction of a number of economic units with different motivations." Scarf (1973; p. 1). Hahn (1970) has gone so far as to argue that the 'general equilibrium question' is the most intellectually exciting question in economics when he writes: "The most intellectually exciting question in our subject remains: is it true that the pursuit of private interest produces not chaos but coherence, and if so how is it done?" Hahn (1970; p. 1). Wrestling with this two part question is a constant stimulus for general equilibrium theory.

As Fisher's remark indicates, general equilibrium theory is not meant to be merely an academic exercise. Instead it is meant to provide a credible explanation of observed economic phenomena and a guide to economic policy making — which in connection it is often interpreted as providing a justification for a non-interventionist approach. As Katzner (1988) puts it: "One of the principal aims of the Walrasian theory of market behaviour, then, is to *explain* particular observed facts, that is, to impart an understanding of the economy ... Why is it that certain commodities are produced in certain quantities? What determines the specific distribution of income and final goods actually realised in the economy? How is it that the economy seems to function smoothly (in the sense that it achieves an allocation of resources) when millions of decision-making units operate independently and in their own self interest? These are a few of the general questions to which the Walrasian theory of market behaviour addresses itself." Katzner (1988; pp. 6–7; emphasis added).

In what follows, we attempt to contribute to an understanding of the equilibrium properties of 'market economies' by examining and discussing the major theorems of general equilibrium theory. Particular attention is given to those theorems that assert the existence, uniqueness, stability, optimality and predictability of equilibrium states. Consideration is also given to the congruence of equilibrium states with actual economic data as well as the policy interventions which general equilibrium theory suggests.

1.2. A brief outline of the general equilibrium research program

The desire to explain observed economic facts and to justify a largely *laissez faire* approach to policy making, appears to have motivated Walras, and many of his followers. The basic Walrasian position might be summarised as follows:

Basic Walrasian Conjecture. *The laissez faire operation of the price mechanism, in a environment of deregulated competitive markets where agents are motivated by self-interest, will produce not chaos but a coherence, in the sense of market clearing, optimal outcomes.*

The thrust of this conjecture is that a private ownership economy, in which economic agents make supply and demand decisions based on individual self-interest guided only by prices, will generally be coherent rather than chaotic.

At various points in the literature one can find narrative support for this conjecture. For instance, Katzner (1989) remarks: "On the surface, at least, modern capitalist economies seem to be chaotic ... and yet somehow the thing works. *There is order — not chaos.*" Katzner (1989; p. 1).[1] In fact Katzner is quite explicit about the likely outcome of letting the market

[1]Katzner's book is called *The Walrasian Vision of the Microeconomy: An Elementary Exposition of the Structure of Modern General Equilibrium Theory*, it seems reasonable to regard the view that order rather than chaos prevails in a typical capitalist economy as a basic Walrasian conjecture.

decide when he remarks that: “People, though not necessarily receiving an equitable share, generally do not starve. They have clothing, shelter and more.” Katzner (1989; p. 1). In a similar vein, Debreu (1998) notes: “The agents of an economy are counted in millions, if not billions. The number of commodities is similarly large. The self-interests of the independent decision makers are sometimes in agreement, sometimes in conflict. Why does one not observe for every commodity a large excess of demand over supply, evidenced, for instance, by lengthy waiting times for orders to be filled, or large excess supply over demand, evidenced, for instance, by massive inventories? . . . , i.e. why is high disorder not the result?” Debreu (1998; p. 10).

Walras and a host of subsequent workers were aware that the plausibility, in fact the logical possibility of this position depended on the satisfactory resolution of a number of specific theoretical questions. As McKenzie (1987; pp. 510–511) notes, Walras suggested the following research program. Firstly, conditions for the existence of Walrasian equilibrium were to be investigated. Secondly, consideration was to be given to the optimality properties of Walrasian equilibrium. Thirdly, an examination of the conditions under which Walrasian equilibrium is stable was to be undertaken. Fourthly, an inquiry was to be made into conditions under which Walrasian equilibrium is unique. Finally, a study of the comparative static properties of Walrasian equilibrium was to be made. As Katzner (1988) puts it: “. . . the questions of existence, uniqueness and stability of equilibria are significant in arriving at a complete understanding of the model and hence of the phenomena the model is supposed to represent.” Katzner (1988; p. 17). We now motivate each of these parts of the general equilibrium research program in a little detail.

General equilibrium theory throws light on the Walrasian conjecture, by investigating circumstances in which ‘economic chaos’ is avoided and instead ‘coherence’ is possible. The starting point of this work is an investigation of the existence question for general equilibrium. This existence question needs to be investigated because a minimal condition for a state of the economy to be regarded as coherent is that in which the plans of individual agents fit together. As Arrow and Hahn (1971) put it: "... the desired actions of economic agents are all mutually compatible and can all be carried out simultaneously." Arrow and Hahn (1971; p. 16). For that to be possible, there must exist a vector of prices that induces agents to make mutually compatible decisions. An economic system may reasonably be regarded as chaotic if economic agents frequently encounter shortages of the commodities that they wish to acquire (evidence of systematic excess demand), or are left with unwanted stocks of commodities including labour services, that they wish to dispose of (evidence of systematic excess supply). Hence general equilibrium theory begins with:

Question 1 (Existence). *Do equilibrium states exist*?

As Debreu (1998) observes proving existence is necessary in order to: "... [i]nsure against the elaboration of grand theories about the empty set." Debreu (1998; p. 21). Indeed without a positive answer to this question, a large part of general equilibrium theory would collapse. Nor is it enough in answering this question to show that an equilibrium exists for a restrictive specification of the parameters which define the economy. Indeed, for the answer to this question to be satisfactory, it is necessary that the conditions under which equilibrium exists be 'reasonable.' Debreu has made this point as follows: "If the model that has been specified requires strong assumptions to guarantee the existence of an equilibrium price vector, the explanatory power of the model will be low. In order to evaluate that model, a basic question must, therefore, be answered in the form of axioms that make it possible to prove an existence theorem." Debreu (1998; p. 21).

Debreu's remark motivates our work on the existence of equilibrium where a detailed study of the axioms which make it possible to prove existence theorems is undertaken. In view of the importance of the existence question and because of the richness of the economic insights that such a study provides, Chapters 2–6 will be devoted to exploring this part of general equilibrium theory.

The second question addressed by general equilibrium theory relates to the number of equilibrium states in a typical economy. In particular:

Question 2 (Uniqueness). *Are equilibrium states unique*?

On the face of it this might seem to be a rather esoteric question, of interest only to a limited number of people. However, as Magill and Quinzii (1996) observe, this question matters because even though the importance of uniqueness: "... is often underestimated, it provides a measure of the ability of the model to predict the outcome of economic activity and is the *sine qua non* for comparative statics". Magill and Quinzii (1996; pp. 5–6). In Chapter 7, we therefore consider the uniqueness question.

The third question considered by general equilibrium theory concerns the stability of equilibrium relative to adjustment processes at work in the economy. In particular, it is of interest to know whether or not there are processes at work in the economy that can be relied on to take the economy to equilibrium, supposing that equilibrium states exist. Therefore, general equilibrium theory explores the question:

Question 3 (Stability). *Are equilibrium states stable*?

Motivation for this question may be found, for instance in the following remark by Rader (1972): "Equilibrium, optimal but unattainable would be a 'will-o'-the wisp and a stability argument is needed to conclude a major part of research in economic theory, namely to show the viability of general market equilibrium." Rader (1972; p. 118). In commenting on the stability issue, Saari (1995) has argued that: "A lesson learned from modern dynamics is that natural systems can be surprisingly complex ... This seeming randomness, however, sharply contrasts with what we have been conditioned to expect from economics. On the evening news and talk shows, in the newspapers, and during political debate, we hear about the powerful moderating force of the market which, if just left alone, would steadily drive prices toward an equilibrium ... The way this story is invoked to influence government and even health policies highlights its important, critical role. But, is it true? I have no idea ... but then no one else does either. This is because, even though this story is used to influence national policy, no mathematical theory exists to justify it." Saari (1995). Chapter 8 is therefore devoted to the stability of equilibrium.

As noted earlier, a minimal requirement for regarding a state of the economy as coherent, is that it be an equilibrium state in which optimising actions of agents are mutually compatible. This is not all that can be asked of a 'coherent' state, however. It is reasonable to also require the allocation of commodities achieved in the economy be such that it cannot be replaced by another allocation which is 'welfare superior' to the equilibrium allocation. This idea leads to the fourth question investigated by general equilibrium theory:

Question 4 (Optimality). *Are equilibrium states optimal?*

Motivation for this question is traditionally laid at the door of Adam Smith's conjecture about the beneficence of the invisible hand. It is clearly a question of central importance to economics, a subject which sets itself the task of discovering good solutions to the economic problem of marrying 'unlimited wants' with 'limited resources.' In Chapter 9, we study the optimality of equilibrium states.

The fifth question investigated by general equilibrium theory concerns the possibility of making definitive predictions about how equilibrium states react to shocks and variations in the parameters that define the economy. Hence consideration is given to:

Question 5 (Comparative statics). *How do equilibrium states respond to variations in the parameters that define the economy*?

As well as being an inherently interesting question, the making of comparative static predictions is central to much applied economics. It was also an issue about which Walras was particularly concerned. As Arrow and Hahn (1971) note: "... Walras had a still higher aim for general equilibrium analysis: to study what is now called *comparative statics*, in other words, the laws by which equilibrium prices and quantities vary with the underlying data [of the economy] resources, production conditions, or utility functions." Arrow and Hahn (1971; p. 5). Being able to make definitive comparative static predictions also has a potentially wider methodological significance. Samuelson (1947) argued that any theory which is unable to produce 'meaningful theorems' (generally comparative static predictions) should be regarded as empty, even if it is able to rationalise the phenomena it has chosen to model. In Chapter 10, we consider the sorts of comparative static predictions that can be made in general equilibrium systems.

The final question to be considered here concerns the empirical adequacy of general equilibrium theory. In particular, it is of interest to know whether or not general equilibrium models of the economy are able to account for actual economic data.

Question 6 (Empirical congruence). *Do general equilibrium models give a satisfactorily account of actual economic data*?

This is an important question because even if general equilibrium theory were to pass various tests for 'internal consistency', it may still fail to give an accurate account of actual economic data. Katzner (1988) makes this point when he argues that: "In general, any inconsistency between actual observation and those which according to the theory, ought to obtain, casts doubt on the understanding that the theory imparts to the workings of the economy." Katzner (1988; p. 8). In light of this observation, it seems reasonable to extend the original general equilibrium research program as outlined by McKenzie (1987), to include empirically testing general equilibrium theory. Chapter 11 is devoted to this task.

Research stimulated by the questions above has lead to the rich body of results that constitutes general equilibrium theory, as well as to the execution of numerous empirical studies designed to test that theory. In grappling with each of these questions, much has been learned about the way economies operate and about the economic policies that are appropriate to their optimal functioning. To conclude this introduction, we highlight some examples of the influence of general equilibrium thinking on economic analysis and economic policy making.

1.3. Some applications of general equilibrium theory

If general equilibrium theory were aiming for nothing more than a positive account of the behaviour of certain economic phenomena, it would probably receive the sort of scrutiny that positive theories in economics routinely receive. One of the things which makes general equilibrium theory particularly interesting, and the subject of even more intense scrutiny, is the fact that the Walrasian conjecture is often a maintained hypothesis in various branches of economics and in a variety of individual pieces of applied economic analysis. As Aranjo and Monteiro (1992) observe: "... general equilibrium is nowadays perhaps the most well established framework in which to study several aspects of economic phenomena. Such diverse topics as growth, money, finance and international trade use the framework of general equilibrium." Araujo and Monteiro (1992; p. 17).

Given the widespread adoption of the general equilibrium theory in economics, it is of some interest to know whether or not the Walrasian hypothesis at its heart is true, or even plausible. Indicative of the impact of general equilibrium theory at the applied and policy level are sometimes the startling implications which flow from the theory, with regard to phenomena such as mass unemployment, international trade and tariffs, the possibility of macroeconomic policy and the appropriate management of 'economies in transition.'

1.3.1. *Equilibrium unemployment*

The capacity of general equilibrium theory to produce startling policy implications, is perhaps nowhere better illustrated than in its treatment of unemployment, as the following remark by Silvestre (1993) demonstrates: "The predominant theory of markets, namely the Walrasian or Arrow-Debreu model of general competitive equilibrium, *implies that unemployment never appears* and that *economic policy never has universally good effects*. First, it postulates that the supply and demand by price taking agents equilibrates in the market for any commodity, including labor. Hence, no unemployment occurs. Second, Walrasian equilibria are efficient as anticipated by Adam Smith's 'invisible hand' ... Thus, either economic policy has no effects or it hurts one group of citizens ... [this] is a powerful result, because it indicates that, except for distributional concerns, the public interest is well served without the need for cooperation, coordination, or public intervention." Silvestre

(1993; p. 105; emphasis added). The claim that unemployment cannot happen,[2] coupled with the assertion that collective action cannot have universally good effects, is a proposition that for many people is both startling and deeply counter-intuitive. That there are nevertheless circumstances in which the proposition is true is testimony to the ability of Walrasian general equilibrium theory to uncover potentially surprising features of economic reality. However, the power and elegance of the theory should not obscure the fact that what is being proposed by the Walrasian economics is a *theory* about how certain economic phenomena might be interpreted. Indeed, far from having arrived at the status of absolute truth, the Walrasian view about unemployment is an actively debated parts of economics, as the following remark by Malinvaud (1991) makes clear: "Today one of the most debated issues is about our fundamental ideas concerning the extent of market clearing ... Do markets clear? Should economic theory assume that markets clear? To these questions [some] economists answer 'Always'. Others think that non–market-clearing occurs and plays a significant role. Most of them then consider that it should, sometimes at least, be taken into account in the treatment of economic phenomena, hence also that some theories that assume it may have a role to play." Malinvaud (1991; p. 179). As Malinvaud notes there are views on both sides of the debate over the accuracy of the Walrasian conjecture of universal market clearing and absence of involuntary unemployment. For example Heckman and MaCurdy (1988) regard the Walrasian conjecture of market clearing as always true when they argue that: "The ambiguity inherent in proposed tests of labor-market equilibrium suggests that the choice between equilibrium and disequilibrium paradigms must

[2]Greenwald and Stiglitz (1995) have similarly noted: "Persistent unemployment, like that plaguing Europe since the early 1980's, has been a persistent problem for economic theory. Competitive equilibrium theory *assumes* that all markets clear, including the labour market. All theories of unemployment thus must reflect significant departures from that paradigm." Greenwald and Stiglitz (1995; p. 219, emphasis in original).

be made on the basis of criteria that are not strictly empirical. Intellectual aesthetics favour the equilibrium point of view. Equilibrium theory suggests a variety of market mechanisms by which workers and firms are sorted, although there is as yet little empirical evidence on such ... The fact that equilibrium theory suggests such mechanisms and motivates empirical work on them is a powerful argument in its favour as a productive research paradigm. Disequilibrium theory is necessarily incomplete and less a stimulant to empirical research because it does not articulate the mechanisms or institutional structures that prevent agents from completing mutually advantageous trades in the labor market nor does it explain how such mechanisms come into existence." Heckman and MaCurdy (1988; pp. 250–251). It is clear from these remarks that the status of the Walrasian conjecture, particularly as it relates to market clearing and unemployment, is by no means settled in the literature. Given the importance of this debate, at both a positive and normative level, the work we undertake on the existence of equilibrium suggests one resolution to the debate outlined by Malinvaud (1991). To partly anticipate our conclusion, we will show that the Walrasian view that markets always clear (or would clear if there were not 'impediments' to them doing so) does not appear to have the sound foundations which some participants in the debate believe it to have.

1.3.2. *The gains from international trade*

As Chipman (1987) observes: "... A good definition of international trade theory as it has evolved would therefore be 'general-equilibrium theory with structure'." Chipman (1987; p. 923). In introducing one of the central results in international trade theory, namely the gains from international trade proposition, Kemp (1987) makes its general equilibrium foundations clear when he states it as follows: "If a country which is initially in autarky and in a state of Walrasian equilibrium, is exposed to free commodity trade with one or more other countries, either in the whole set of producible goods or in some subset, and if preferences, technologies and endowments are restricted in the manner of Arrow and Debreu (1954) and if markets are complete, then there is a Walrasian world trading equilibrium (possibly with lump sum transfers within the country), such that no individual is worse off in the trade equilibrium than he or she was in the autarky equilibrium." Kemp (1987; p. 453). As Kemp points out: "It has been noted that [the proposition] rests on [general equilibrium] assumptions of Arrow-Debreu (1954) type. In particular, the number of goods is required to be finite and the set of markets complete. Without both those assumptions, there is no assurance that free trade is gainful to all participating countries ... That we do not observe a free trading world or even an unmistakable drift to free trade, can be traced to [among other things] the unrealism of some of the Arrow-Debreu assumptions ..." Kemp (1987; pp. 453–454).

We might add to Kemp's observation that the gains from international trade proposition also depends in a non-trivial way on satisfactory answers to the questions posed above. In particular, the proposition requires the existence of equilibrium in both autarky and under free trade. As well, there is the implicit assumption that the free trade equilibrium is stable, in the supposition that the world will arrive at the new Walrasian equilibrium, so Question 3 is assumed to be satifactorily answered. Also the final allocation of goods is assumed to be Pareto optimal, so that Question 4 is assumed to be answered in the affirmative.

It is also interesting to note that the gains from trade proposition is not necessarily robust to the failure of aspects of the Walrasian hypothesis. As for instance Benassy (1984) argues, if the country is not in equilibrium to begin with, then the GFT is by no means guaranteed. In particular: "But if one starts from a situation of less than full employment, which is of course not a Pareto Optimum, it may be that an adequate combination of commercial policies and other measures [i.e. deviations from free trade] would allow an increase in employment and production." Benassy (1984; p. 261). This remark is supported by proving a proposition which says that if in a two country world, country 1 is in a Keynesian equilibrium then an increase in the tariff rate and a variation in the home money supply sufficient to maintain the balance of payments will produce a Pareto improvement relative to free trade. Benassy is quick to point out that this result and others like it, do not allow one to take a clear position on the question of 'trade versus protectionism.' What Benassy's argument does do is highlight the potential sensitivity of the gains from international trade proposition to a failure of key parts of the Walrasian conjecture. In similar vein, Itoh and Negishi (1987; p. 22) prove that for a 'minimum wage economy', there is a possibility that autarky is superior to free trade and on the basis of this and other results which they prove when the Walrasian conjecture is not invoked, they observe that, "... In a fix-price (minimum wage) economy model, one can derive many results that do not hold in the standard model with flexible [i.e. Walrasian] prices." Itoh and Negishi (1987; p. 21).

1.3.3. *New-classical economics and the possibility of policy*

Another reason for our interest in the Walrasian conjecture (and its alternatives), stems from the debate between 'Keynes and the Classics.' As is well known, the major tenet of the classical school is the proposition that a capitalist system, operating through a network of interlocking markets will, through self-generated pressures, fully employ all its job seeking labour unless prevented from doing so by organised labour, government intervention or some other form of restraint on the operation of free markets (which is just another way of stating the Walrasian conjecture). Keynes on the other hand argued that even the most fluid of market systems would not in general reach a full employment equilibrium without explicit government intervention. The debate between the (New) Classical and (New) Keynesian Schools has often been conducted around the issue of price flexibility. As Srivastava and Rao (1990) observe: "... if we interpret the downward sloping demand and upward sloping supply schedules as depiction's of the conflicting interests of two groups of transactors, then the equilibrium models imply that such conflicts are resolved through price adjustments by the free market system. The well known monetarist, neo-classical and new classical economists use the equilibrium method of analysis. The alternative Keynesian approach, however, raised serious doubts on the ability of the free market system to attain equilibrium continuously through price flexibility ... The Keynesian alternative was seen for a long time to be a theoretically *ad hoc* approach [although] [s]ubsequent developments have shown that the Keynesian approach can be given sound theoretical foundations if the Walrasian equilibrium framework is modified to allow for disequilibrium trading and non-market clearing price adjustment ... [T]he main weakness of the equilibrium approach seems to be in an acceptance of the price flexibility assumption without any empirical and theoretical analysis of price setting behaviour in various markets." Srivastava and Rao (1990; p. 1).

Those working in the Walrasian equilibrium framework reply to this criticism that the postulate of fixed or 'sticky' prices is theoretically arbitrary and should not be accepted. However, participants on both sides of the debate appear to assume that a Walrasian equilibrium exists and would be arrived at if only prices were 'flexible enough.' Consequently, the existence question for Walrasian equilibrium is germane to this debate, as is the question of the stability of Walrasian equilibrium. Indeed it is arguable that these Walrasian hypotheses (and their Keynesian alternatives), are even more fundamental than the question of price flexibility or price fixity, because if equilibrium can be shown not to exist, or to exist only under implausible circumstances, and/or if it can be shown not to be stable under reasonable adjustment processes, then fundamental questions would be raised about the capacity of a freely operating network of interlocking markets to achieve full employment, through the operation of flexible markets. The work which we undertake in subsequent chapters aims to contribute to the New-Classical — New-Keynesian debate from a novel angle, i.e., from a point of view which does not rely on the usual fix-price or flexible price distinction, but relies instead on a careful analysis of conditions for the existence and stability of equilibrium.

1.3.4. *Economies in transition*

An important part of the theory and practice of economics is aimed at what might be called 'system design.' The nature of this activity is nicely indicated in the following remark by Blad and Keiding (1990): "… economic engineering [is] the design of institutions aimed at assuring that economic behaviour within the limits set by the institutions leads to some prescribed family of allocations." Blad and Keiding (1990; p. 277). There are numerous examples of this sort of activity in economics. The design of common currency unions, the design by central banks of prudential operating rules for financial institutions and the design of tax-subsidy schemes designed to correct externalities, are a few instances. On a grander scale, an example of system design has been underway since the early 1990's in a number of Eastern European and former Soviet Bloc countries. The exercise may roughly be characterised as the replacement of a set of central planning institutions by the institution of the market.

As Scarf (1991) puts it: "One of the major goals of the proposed economic reform in the Soviet Union is the introduction of competitive markets to replace the procedures of centralised economic decision making." Scarf (1991, p. 1). As Scarf goes on to observe, the success of such an experiment depends critically on a collection of apparently abstract issues set out for study by Walras, particularly the existence, stability and optimality questions for equilibrium. Unless market equilibrium exists, is optimal and achievable by processes at work in the economy, then the goals proposed by the economic reformers may not be achieved. The reformers' claims that markets can coherently coordinate demand and supply may therefore come to be viewed as unreliable.[3] Thus the issues raised by the Walrasian conjecture are not merely matters of technical interest. Nor are they even a matter of internal interest to economists alone. They are matters which go to the heart of some of the most important debates in the world, debates which address the appropriate design and operation of fundamental social and economic institutions.

1.4. Conclusion

There are two broad strands to the philosophy and practice of economics. One strand is concerned to answer questions of the form: 'Why is it so?,' in relation to the observed behaviour of economic variables. This strand is concerned to analyse various applied economic questions, taking a particular theoretical framework as the workhorse for the analysis. A second strand is involved in the design of social institutions which aim to produce optimal outcomes. In both strands of economics, some concept of equilibrium, often that suggested by Walras is used as an organising principle and idea.

[3]In this context our motivation for evaluating the Walrasian hypotheses has to do with the fairly often encountered suggestion that the market is able to act as a complete social system, see for example, Rader (1972b; p. 150). See also Murell (1995).

In view of the widespread use of Walrasian general equilibrium theory in economics, and the role which the Walrasian conjecture plays in numerous pieces of applied economic analysis, and considering the frequently encountered suggestion that the market is a complete social system, this monograph presents a detailed study of the six questions which underpin the Walrasian conjecture.

Chapter 2

EXISTENCE OF EQUILIBRIUM: SUFFICIENT CONDITIONS

"*If the model that has been specified requires strong assumptions to guarantee the existence of an equilibrium price vector, the explanatory power of the model will be low. In order to evaluate the model, a basic question must, therefore, be answered in the form of axioms that make it possible to prove an existence theorem.*"

G. Debreu

2.1. Introduction

The most basic question addressed in general equilibrium theory is the existence question for market equilibrium. This question is fundamental because unless circumstances can be identified in which equilibrium states exist, general equilibrium theory would collapse. As Debreu (1982) notes: "Walras himself perceived that the theory that he proposed would be vacuous without a mathematical argument in support of the existence of at least one equilibrium state." Debreu (1982; p. 697) while Mas-Colell *et al.* (1995) observe that: "Although an existence theorem can hardly be the end of the story, it is, in a sense, the door that opens into the house of analysis." Mas-Colell *et al.* (1995; p. 584). As we will see in what follows, the question is also interesting because of the economics involved in guaranteeing existence.

Notwithstanding, the importance of the existence question and the long history of work devoted to it, views about the generality of circumstances under which equilibrium exists are varied. For instance, Varian (1992), Kreps (1990), Ellickson (1993), Mas-Colell *et al.* (1995), claim that equilibrium exist under weak and general conditions. Alternatively, Debreu (1987), Duffie (1990), and Chichilnisky (1995) caution that the conditions under which equilibrium exists may be restrictive.

Motivated by the importance of the existence question, this chapter undertakes a detailed study of some sets of conditions which are sufficient for the existence of equilibrium. Section 2.2 introduces some basic definitions and presents a survey of some views on the likely existence of equilibrium. Section 2.3 introduces and studies a variety of conditions that are known to be sufficient for the existence of equilibrium. Particular attention is focused on the sorts of relationships that need to hold between the primitives that define the economy in order for equilibrium to exist. In this context, we study conditions such as: 'interior endowments', 'desirability and productivity', 'boundary endowments and production', 'resource relatedness', 'consumer productivity', 'irreducibility', 'super self-sufficiency', 'no oligarchy', 'generalised interdependence', 'normality' and 'indecomposability'. A common feature in all these conditions is that they require certain potentially restrictive relationships to hold between the primitives that define the economy. In considering each of these conditions we will be particularly interested in highlighting the economics associated with each of them. Section 2.4 presents some conclusions.

2.2. Basic ideas, definitions and views about existence

The primary object of interest in economics is the economy. An *economy* is a collection of economic agents who make supply and demand decisions over commodities, labour and various assets in order to further their own interests. In the economies considered here, the number of agents and the number of commodities will generally be finite. A *commodity* is a dated, located bundle of (possibly state contingent) characteristics, the total number of which is ℓ and the set of which is L. Although it is not necessary for the analysis, agents are traditionally divided into two groups called 'consumers' and 'producers'. There are typically n consumers with an individual consumer being denoted by i and the set of consumers being denoted by I. Consumers are characterised by their *consumption sets* $X_i \subset \Re^\ell$, their *preference orderings* $\preceq_i$ and their *initial endowment* $\omega_i \in \Re^\ell$. There are typically m firms with an individual firm being denoted by j and the set of all firms being denoted by J. Firms are characterised by their *production sets* $Y_j \subset \Re^\ell$. The economy has a *total endowment* of goods, ω. In terms of these *primitives* an economy can be written as $\boldsymbol{E} = \{X_i,\ \preceq_i,\ \omega,\ Y_j,\ \ell\}_{i=1}^{n}{}_{j=1}^{m}$. It is worth observing how intuitive and flexible this definition of an economy is. Intuitive because it captures important parts of basic economic reality and flexible because it easily admits extension. One of many possible extensions is to the case of a *private ownership economy* which is an economy where i owns a *share firm* j, θ_{ij} such that $0 \leq \theta_{ij} \leq 1$ with $\Sigma_i \theta_{ij} = 1$ for each j and the *total endowment* of the economy is held by consumers so that $\omega = \Sigma_i \omega_i$. In terms of its primitives, a finite private ownership economy may be written down as $\boldsymbol{E}_{po} = \{X_i, \preceq_i, \omega_i, Y_j, \theta_{ij}, \ell\}_{i=1}^{n}{}_{j=1}^{m}$.

Most of the primitives which define an economy are well understood and will not be discussed further here. However, it is worth saying a few words about X_i the consumption possibility set for consumer i. The standard interpretation of X_i is that it represents the set of consumptions and labour supplies that are feasible for i and are compatible with i's survival.[1] An illustration of X_i is given in Fig. 2.1.

[1] See the interesting discussion of the interpretation of X_i in Arrow and Debreu (1954, p. 269), Newman (1987) and also Mas-Colell *et al.* (1995; p. 18).

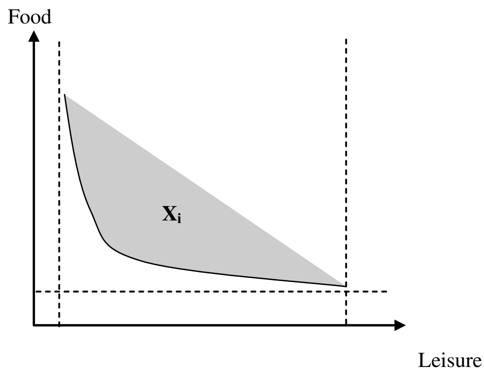

Fig. 2.1. A possible consumption set for consumer i.

The equilibrium properties of $\boldsymbol{E}$, or its variants, are the primary concern of general equilibrium theory. In specifying the characteristics of equilibrium states, it is usual to take into account: (i) what is happening with consumers; (ii) what is happening with firms; (iii) what is happening in each market of the economy. One way of doing this yields the following notion of equilibrium, named in honour of Walras.

Definition 2.1 (Walrasian equilibrium).[2] A consumption allocation (x_i^*), a production allocation (y_j^*) and a price vector $p^* \in \Re^\ell$ is a *Walrasian equilibrium* for $\boldsymbol{E}$ if: (i) for each i, x_i^* is a maximal element in the set $\{x_i \in X_i\colon p^*x_i \leq p^*\omega_i + \Sigma_j\theta_{ij}p^*y_j^*\}$ relative to $\preceq_i$; (ii) for each j, y_j^* maximises profit so $p^*y_j \leq p^*y_j^*$ for all $y_j \in Y_j$; (iii) all markets clear so $\sum_i x_i^* = \omega + \sum y_j^*$.

While this is one of the most popular ways to specify equilibrium states, there are many alternatives. For instance, a weaker market clearing requirement that is imposed by (iii) in

[2]Alternative names for this notion include: 'competitive equilibrium', 'general equilibrium' and 'market equilibrium', see for instance Mas-Colell *et al.* (1995; p. 314 and pp. 547–548).

Definition 2.1 is sometimes encountered in the literature. It leads to the idea of a 'free-disposal' equilibrium.

Definition 2.2 (Walrasian free-disposal equilibrium). A *Walrasian free-disposal equilibrium* is a Walrasian equilibrium in which (iii) is replaced by the requirement that all markets weakly clear, i.e. (iii′): $\sum_i x_i^* \leq \omega + \sum y_j^*$. Further, goods for which this inequality holds strictly have a zero equilibrium price.[3]

Remark 2.1. Condition (i) ensures that consumers submit utility maximising vectors to the market. Condition (ii) ensures that firms maximise profit subject to technical feasibility constraints. Condition (iii) requires that the optimising decisions of all the agents are compatible so that all markets simultaneously clear. The significance of this definition is emphasized by Mas-Colell *et al.* (1995) who note that: "…Walrasian equilibrium [is taken] as a positive prediction for the outcome of a system of markets in which consumers and firms are price takers and the wealth of consumers derives from their initial endowments and profit shares." Mas-Colell *et al.* (1995; p. 579). The likely accuracy of this prediction can be assessed both theoretically and empirically. Theoretical assessments typically focus on the generality of the circumstances under which equilibrium states exists (and are stable). Empirical assessments involves looking for equilibrium states in actual economic data. Both approaches will be pursued in this book and to begin, we consider some views that various authors have expressed about the likely existence of Walrasian equilibrium.

2.2.1. *Is equilibrium likely?*

There are a variety of views at the theoretical level about the likely existence of Walrasian equilibrium. One view is that such

[3]There is an interesting discussion in Arrow and Hahn (1971; p. 20) why $p = 0$ is not generally a candidate for Walrasian equilibrium.

states exist under weak and general conditions and in a wide variety of circumstances. A typical expression of such a view is that of Varian (1992) who argues: "It is worth emphasising the *very general nature* of the [existence] theorem. All that is needed is that the excess demand function be continuous and satisfy Walras' Law... The hypothesis of continuity is more restrictive [than Walras' Law] but not unreasonably so. We have seen earlier that if consumers have strictly convex preferences then their demand functions will be well defined and continuous. The aggregate demand function will therefore be continuous." Varian (1992; p. 322, emphasis added). In this statement no mention is made of the conditions which need to hold in order to guarantee excess demand continuity, although they are acknowledged to be 'more restrictive than those necessary for Walras' Law'. Certainly no attempt is made to evaluate the reasonableness or otherwise of such conditions. Nevertheless, the explicit claim is made that the existence result for Walrasian equilibrium is general, in circumstances where it is arguable that when the full range of hypotheses for the theorem is considered, the assertion of generality becomes more problematic.

In a similar fashion Kreps (1990; p. 193) asks: "...why not believe in Walrasian equilibrium?" and after listing a few considerations such as limitations on the extent of consumer knowledge, the size of the transactions which consumers might wish to undertake, the existence of rations and quotas and complications introduced by production, there then follows an optimistic view on the likely existence of Walrasian equilibrium: "If you consult [the] references, you will find the analysis is hard and the conclusions require consumers who are extraordinarily sophisticated. You might worry, in consequence, that the results derived in support of the concept of Walrasian equilibrium are not very credible. To assuage these worries, you should consult the literature on experimental economics... The results obtained [there] are usually striking in their support of

Walrasian equilibrium... [and while] these experiments do not quite get to the level of generality of a Walrasian equilibrium for a general equilibrium [model] with many interdependent markets... the repertoire of experiments is growing quickly and, except for a few special cases, those experiments that have been run are consistent with the notion of Walrasian equilibrium. *All in all, they make Walrasian equilibrium look quite good.*" Kreps (1990; p. 198, emphasis added). In this treatment of the existence result for Walrasian equilibrium, the suggestion seems to be that the 'hard analysis' of the theoretical literature can be replaced by various experimental designs, the details of which are not discussed or critiqued. Furthermore, the significance and limitations of the assumptions in the hard analysis can be glossed over in favour of the presented optimistic conclusion that Walrasian equilibria generally exist.

Similarly Jehle (1991), comments as follows at the conclusion of his treatment of an existence result for Walrasian equilibrium: "Note *how little structure* was required of excess demand to ensure the existence of market clearing prices. All we need is that excess demand be continuous and that it satisfy Walras' Law. In Theorem 7.2.1, we considered properties of agents preferences sufficient to guarantee both Walras' Law and the continuity of excess demand." Jehle (1991; p. 316, emphasis added). Theorem 7.2.1 in Jehle (1991) asserts: if preferences are complete, reflexive, transitive, continuous, monotonic and strictly convex, then the excess demand function for an (exchange) economy will be continuous for all strictly positive prices, satisfy Walras' Law, be homogeneous of degree zero and will be such that if a good is in excess supply in equilibrium, it will have a zero price. Nowhere in that theorem, or in the subsequent discussion offered by Jehle (1991), is there any discussion of the other assumptions which are needed in order to ensure the continuity of the economy's excess demand function.

Two treatments of the existence problem for Walrasian equilibrium which come to a similarly optimistic conclusion about the existence of Walrasian equilibrium but which, in the process, give some insight into reasons why it might be reasonable to be cautious about the existence of Walrasian equilibrium, are provided by Ellickson (1993) and Mas-Colell *et al.* (1995). In his treatment, Ellickson devotes considerable attention to the issue of the existence and continuity of a 'best response' (demand) function for a consumer. Towards the end of the argument (p. 225), he states a theorem which gives conditions under which the budget correspondence is continuous, something which is necessary for the continuity of the consumer's demand response. One of the conditions of this theorem is that the consumer's income at p, denoted by $w(p)$, is greater than the value of the cheapest point in the consumption set, denoted by $\inf p.X$. In a discussion of the sorts of conditions which will give rise to a situation where $w(p) > \inf p.X$ for all p, Ellickson (1993) writes: "For example, in the pure exchange case with $w(p) = p\omega_i$ a (very strong) sufficient condition guaranteeing $p\omega_i > \inf p.\ X_i$ for any $p > 0$ is to require that $X = \Re^{\mathrm{m}}_{+}$ and $\omega \gg 0$. *Clearly this condition is not very realistic, but more realistic conditions tend to be fairly complicated.* The general idea is to find some condition which guarantees that each consumer has sufficient wealth so that her budget set has nonempty interior." Ellickson (1993; p. 226; emphasis added). At this point Ellickson provides a footnote which invites the reader to see, for example, the concept of indirect resource relatedness in Arrow and Hahn (1971) 'for a much more general condition.' Ellickson's treatment of the existence problem for Walrasian equilibrium is interesting because it explicitly acknowledges the need to handle the problem of consumer demand discontinuity. In addition he comments directly, and unfavourably, on one of the most widely used conditions aimed at guaranteeing this, namely the joint assumption that the individuals' consumption possibility set is the positive orthant and that the consumer owns an initial endowment of goods in the interior of that set.[4] Having dismissed this condition as being overly strong, Ellickson does not give any insight into the nature of the other conditions which guarantee nonempty budget sets, except to say that such conditions tend to be 'fairly complicated' and 'model specific'. In spite of the reservations which he expresses, the overall message is that: "We have learned that Walrasian equilibrium exists in *a wide variety of circumstances*." Ellickson (1993; p. 331, emphasis added).

In their treatment of the existence question, Mas-Colell *et al.* (1995) proceed by first defining the notion of a Walrasian quasi-equilibrium and then by showing that there are circumstances in which such an equilibrium becomes a Walrasian equilibrium. In particular their Proposition 17.BB.2 shows that there are conditions on the preferences, technology and endowments which guarantee the existence of a Walrasian quasi-equilibrium. One of these conditions is the assumption that all consumers have an endowment which permits them to survive without entering the markets of the economy (Condition i.3 of their Proposition 17.BB.2). In order to go from a quasi-equilibrium to a true Walrasian equilibrium, Mas-Colell *et al.* (1995) note that if at a quasi-equilibrium consumption level there is a 'cheaper point' for each consumer, then the quasi-equilibrium is a full Walrasian equilibrium (see their Proposition 17.BB.1). The challenge then is to find reasonable conditions which guarantee the existence of a cheaper point for each consumer. In a footnote on p. 633, Mas-Colell *et al.* (1995) canvass various options for achieving that end and highlight two widely used assumptions. The first is an endowment and price condition similar to those considered in Ellickson (1993) and involves $p \geq 0$, $p \neq 0$ and $\omega_i \gg x_i$ for some $x_i \in X_i$. The second requires $p \gg 0$ and allows

[4]This condition along with the others alluded to by Ellickson (1993) will be discussed in detail below.

$\omega_i = x_i$ for some $x_i \in X_i$. As Mas-Colell *et al.* (1995) observe, however: "... [a]lthough convenient, neither condition can be regarded as extremely weak. It would be unfortunate if the validity of the theory were restricted to them." Mas-Colell *et al.* (1995; p. 633). The authors then go on to mention that fortunately for the theory there is another approach to guaranteeing the existence of a cheaper point, namely the assumption due to McKenzie (1959) that the economy is 'irreducible', (or as they term it 'indecomposable'). In this they also parallel Ellickson (1993) who, as we saw, made reference to the Arrow and Hahn condition of indirect resource relatedness as a way to guarantee the existence of a cheaper point. Their approach also parallels that of Ellickson in that having mentioned the McKenzie condition, Mas-Colell *et al.* (1995) are content to assert, without further argument, that this is a much weaker condition than is the interior endowment condition (which is probably true) and therefore that the existence result for Walrasian equilibrium rests on general foundations after all, something which does not necessarily follow. Even if it is true that irreducibility is a *weaker* condition than interior endowments, that does not by itself necessarily mean that it is a weak and generally applicable condition. This issue is discussed in more detail later in the chapter when the notions of interior endowments and irreducibility have been fully defined. Here, it is appropriate to simply note the overall message which Mas-Colell *et al.* (1995) give at the conclusion of their discussion which is: "The existence of a Walrasian equilibrium *can be established in considerable generality.*" Mas-Colell *et al.* (1995; p. 584, emphasis added).

In contrast to the views above, there are authors who are much more reserved in their assessment of the generality of the circumstances under which Walrasian equilibrium exists. Some are even prepared to argue that the non-existence of Walrasian equilibrium is pervasive. Debreu (1987) for instance has highlighted the restrictive nature of two conditions which typically underpin existence theorems for Walrasian equilibrium as he writes: "Some of the assumptions on which the theorems of Arrow-Debreu (1954) are based are weak technical conditions... Other assumptions were later shown to be superfluous for economies with a finite set of agents... There remain, however, *two overly strong assumptions*... They are the hypothesis that for every i, the endowment [for i] yields a possible consumption for the ith consumer (after the disposal of a suitable commodity-vector if need be), and the assumption of convexity on the total production-set... which implies non-increasing returns to scale in the aggregate." Debreu (1987; p. 217, emphasis added). In similar vein, Duffie (1990) has remarked that: "The principal contribution of general equilibrium theory has been its axiomatic validation of our benchmark model of price taking, individual rationality, and market clearing. Equilibria exist, *albeit under strong conditions*." Duffie (1990; p. 86, emphasis added). More pointedly, Chichilnisky (1995) has argued that: "The conditions known for existence [of Walrasian equilibrium] are however restrictive... [and] [t]he problem of *non-existence of a competitive equilibrium is pervasive*. Despite the fact that market allocations are regarded as a practical solution to the resource allocation problem, *many standard economies do not have a competitive equilibrium*." Chichilnisky (1995; p. 80, emphasis added). Considering these divergent views at the theoretical level over the likely existence of Walrasian equilibrium, we now examine the nature of the conditions under which equilibrium is known to exist.

2.3. Sufficient conditions for the existence of equilibrium

As noted above, an economy can be characterised by its primitives. In order for an economy to have an equilibrium, those primitives will have to have certain properties. Consequently, existence theorems for equilibrium states impose conditions on the primitives that define the economy and on the relationships which hold between the primitives. It is the interplay between the conditions which yield existence results and in that sense, no condition is more important than another in ensuring that an economy has an equilibrium state. In what follows, however, we will be particularly interested in the role played by conditions which ensure the economic viability of the agents who make up the economy.

2.3.1. *Interior endowments and 'survival without trade'*

Theorem 1 in Arrow and Debreu (1954) provides what is generally regarded as the first satisfactory set of sufficient conditions for the existence of a Walrasian equilibrium.[5] The conditions which it imposes on the economy may be written as follows.

Theorem 2.1 (Arrow and Debreu (1954; Theorem 1)). *If* $\boldsymbol{E}_{po}$ *is such that* $(ad.1)$ $\forall j$, Y_j *is closed, convex and* $0 \in Y_j$; $(ad.2)$ $Y \cap \Re^{\ell}_{\alpha +} = \{0\}$, *where* $Y = \sum_j Y_j$; $(ad.3)$ $Y \cap (-Y) = \{0\}$; $(ad.4)$ $\forall i$, X_i *is closed, convex and* $\exists \xi_i$ *such that* $\xi_i \leq x_i$ *for all* $x_i \in X_i$; $(ad.5)$ *every* $\preceq_i$ *admits a continuous utility function* $u_i : X_i \rightarrow \Re$; $(ad.6)$ $\forall i$ *and for any* $x_i \in X_i$, $\exists x'_i \in X_i$ *such that* $u_i(x'_i) > u_i(x_i)$; $(ad.7)$ $\forall i$ *if* $u_i(x'_i) > u_i(x_i)$ *and* $0 < t < 1$ *then* $u_i[tx'_i + (1-t)x_i] > u_i(x_i)$; $(ad.8)$ $\forall i$, $\omega_i \in \Re^{\ell}$ *and for some* $x_i \in X_i$, $x_i \ll \omega_i$; $(ad.9)$ $\forall i$, j, $0 \leq \theta_{ij} \leq 1$ *and* $\Sigma_i \theta_{ij} = 1$ *then there is a Walrasian free-disposal equilibrium for* $\boldsymbol{E}_{po}$ *in which all prices are non-negative and not all prices are zero.*

Proof. Arrow and Debreu (1954; pp. 274–279). □

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[5] There were many earlier attempts ranging from Walras' 'counting equations and unknowns' to the approach taken by Wald and discussed in John (1999).

Remark 2.2. Theoretical justifications, (of varying degrees of plausibility), can be given for assumptions (ad.1)–(ad.7). For example, non-increasing returns to scale might justify (ad.1), the need for inputs to produce outputs informs (ad.2), the irreversibility of production yields (ad.3) a condition which can also be justified as a consequence of the Arrow-Debreu definition of a commodity; the impossibility of an individual supplying an unbounded amount of labour can be used to justify (ad.4), while the various requirements imposed on the preference ordering by (ad.5) in order to permit representation by a utility function, might be justified by an appeal to individual rationality. Non-satiation and the desire for diversity in consumption could be invoked to justify (ad.6) and (ad.7), while (ad.9) is simply a definition of profit shares. Assumption (ad.8), the 'interior endowments assumption', is however of a quite different character to the other conditions in the theorem and will be discussed in some detail.

Definition 2.3 (Interior endowments). Individual i has an *interior endowment* if the commodities which he or she initially owns allows survival even after the disposal of a positive amount of each of the ℓ goods in the economy. The *interior endowments assumption holds for the economy* if for all i, $\exists\, x_i \in X_i$ such that $x_i \ll \omega_i$. The assumption may also be called 'survival without trade'.

Remark 2.3. As Arrow and Debreu (1954) and Moore (2005) note, this assumption effectively means that everyone in the economy has available for trade a non-zero amount of each of the ℓ goods in the economy. Numerous authors have expressed views about this condition, including Arrow and Debreu (1954; p. 274) who regard it as 'clearly unrealistic', as do Arrow and Hahn (1971; p. 80). Debreu (1987; p. 217) characterises the assumption as 'overly strong', Moore (1975; p. 287) remarks that the assumption 'seems inconsistent with a developed economy and the reality of specialisation and the necessity for exchange', McKenzie (1988; p. 109) regards it as 'very desirable to replace the assumption', while Geanakoplos and Polemarchakis (1990; p. 156) note that the assumption is 'strong, but standard'. Ellickson (1993; p. 226) regards the assumption as 'very strong', while in a telling remark Mas-Colell *et al.* (1995; p. 633) observe that 'It would be unfortunate if the validity of the theory was restricted to it', while Florenzano (2003) remarks that, "[f]rom and empirical point of view, such an assumption is of questionable plausibility." Florenzano (2003; p. 50).

Apart from objections to the interior endowments assumption that are based on plausible empirical considerations, there is also the following general theoretical objection to the assumption. The interior endowments conditions requires a particular, theoretically unexplained relationship to hold between two distinct primitive objects in the economy. The objects are ω_i and X_i. As noted earlier, X_i is the consumption set for i. X_i is determined in part by the physical and psychological characteristics and capacities of i. On the other hand, ω_i is a list of the commodities to which i has legal title before the commencement of trade. This list is determined in a potentially complicated way by prior economic activity and historical accidents. It is therefore hard to advance theoretical reasons why one of these objects, ω_i, should be located in the interior of the other, X_i, for any one consumer. It certainly stretches credulity to suppose that the needed relationship holds for all consumers, as condition (ad.8) in the theorem does. Thus, unlike the other conditions in Theorem 2.1, conditions which may be empirically inadequate but which are at least to some extent theoretically supportable, (ad.8) is both empirically inadequate and theoretically arbitrary.

2.3.2. *Why the need for such an unreasonable condition?*

The reason why condition (ad.8) appears in Theorem 2.1 is spelled out by Debreu (1962) in the following terms: "In the study of the existence of an equilibrium for a private ownership economy, one meets with the basic mathematical difficulty that the demand correspondence of a consumer may not be upper semi-continuous when his wealth equals the minimum compatible with his consumption set..." Debreu (1962; p. 257, emphasis added). The problem with this is that if individual demands behave discontinuously then the aggregate demand response will inherit this discontinuity. If aggregate demands behave discontinuously in the 'wrong' part of the price space, then Walrasian equilibrium may not exist because excess demand 'jumps' at what would otherwise be an equilibrium price. The interior endowments assumption rules out the sort of situation as can be seen by studying Fig. 2.2.

At prices such as p in Fig. 2.2, the budget set for i is not empty and a preference maximising demand vector, $x_i(p) = (x^1(p), x^2(p))$ exists. If however prices change to p' then the budget set for this consumer becomes empty and the consumer 'disappears' from the economy. In the process, individual demand changes discontinuously. This discontinuity is inherited by the excess demand response of the entire economy and that may prevent the existence of equilibrium.[6]

It might be argued that such discontinuity is of marginal significance in a 'large' economy and that various smoothing arguments could be invoked in order to get equilibrium even in the face of such discontinuity. A study of the definition of an economy reveals that this will not do. Since $\boldsymbol{E}_{po} = \{X_i, \preceq_i, \omega_i, Y_j, \theta_{ij}, \ell\}_{i=1}^{n}{}_{j=1}^{m}$ one of the things which parameterizes the economy

[6]See also the very interesting discussion of this issue in Duffie and Sonnenschein (1989) and Rizvi (1991).

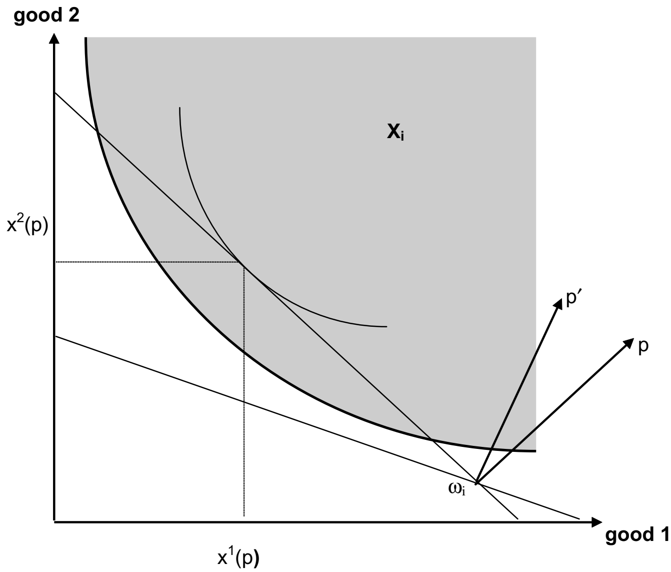

Fig. 2.2. Demand discontinuity when $\omega_i \in X_i$.

is the number of its consumers, n. If in an 'equilibrium' where only k consumers survive, with $k < n$, then this is not an equilibrium *for the original economy*. At best the existence problem has been solved for a sub-economy made up of k consumers, with the remaining $(n - k)$ individuals being eliminated. However, that was not the original problem which was to prove that the given economy had an equilibrium state. So for both the 'technical' reasons identified by Debreu (1962) and for 'fundamental' reasons to do with the definition of the economy, agent survival in equilibrium is needed in any successful existence argument. The interior endowments condition is one, unsatisfactory way to guarantee this outcome.

2.3.3. *The quasi-equilibrium approach*

A natural response to the undesirably strong nature of the interior endowments assumption is to look for weaker conditions that can be substituted for it and under which equilibrium exists. A possible first step in that direction might be to allow consumers to have endowments in the boundary of their consumption possibility sets. That there may be non-trivial difficulties associated with doing this is illustrated in the following example.

Consider a Robinson Crusoe economy of the sort illustrated in Fig. 2.3 in which i's consumption possibility set is equal to the positive orthant, $\Re^2_+$, the initial endowment of i is on the boundary of the consumption possibility set and i owns no shares in any firm. Suppose i has strictly monotonic preferences, then for prices such as p, there is positive excess demand for good 2 and negative excess demand for good 1. This disequilibrium will remain for all prices other than p'. However, at p' the budget set is unbounded and given that i's preferences are strictly monotonic, the demand for good 1 diverges to infinity. This is greater than the total endowment of good 1 and so this economy does not have a Walrasian equilibrium.

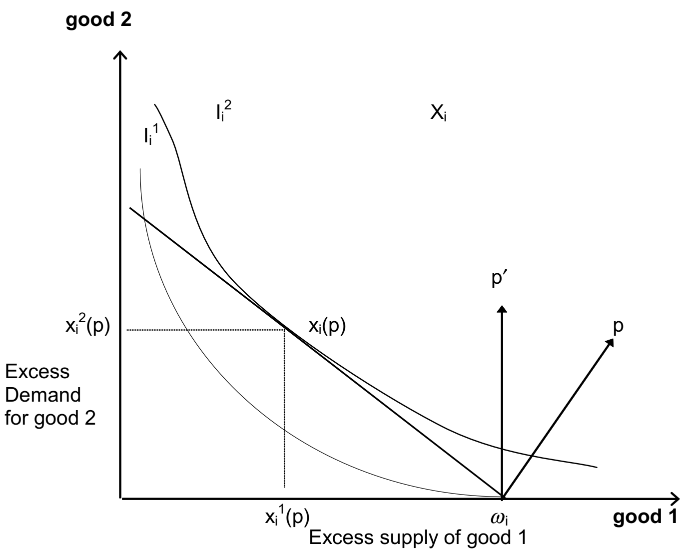

Fig. 2.3. Boundary endowments and demand discontinuity.

One approach to handling this sort of problem is to look for a form of consumer behaviour that has better continuity properties than does preference maximisation, such as 'expenditure minimisation'. This involves weakening condition (i) in the definition of Walrasian equilibrium to the requirement that x_i^* minimises expenditure at the price vector p^* on $\{x_i \in X_i : x_i^* \preceq_i x_i\}$ to yield the idea of a *quasi-equilibrium*.

Definition 2.4 (Walrasian quasi-equilibrium). A consumption allocation (x_i^*), a production allocation (y_j^*) and a price vector p^* is a *Walrasian quasi-equilibrium* for $\boldsymbol{E}$ if: (i') for each i, $p^*x_i^* \leq p^*\omega_i + \Sigma_j\theta_{ij}p^*y_j^*$ and if $x_i^* \prec_i x_i$ then $p^*x_i^* \geq p^*\omega_i + \Sigma_j\theta_{ij}p^*y_j^*$; (ii) for each firm j, y_j^* maximises profit for j so that $p^*y_j \leq p^*y_j^*$ for all $y_j \in Y_j$; (iii) $\sum_i x_i^* = \omega + \sum y_j^*$ so all markets clear.[7]

Remark 2.4. The problem is then to specify conditions under which such an equilibrium exists. Mas-Colell *et al.* (1995; p. 634) and Florenzano (2003; p. 50) provide examples of such conditions.

While the quasi equilibrium approach is an interesting way to avoid the sorts of discontinuous change in demand illustrated in Fig. 2.2, the approach is only partially successful in the overall effort to establish the existence of a Walrasian equilibrium. This is so because quasi-equilibrium allocations are not guaranteed to be utility maximising and hence are not necessarily rational for consumers. This possibility is illustrated in Fig. 2.4.

In this example, $X_i = \{x_i \in \Re_+^2 : x_{i1} + x_{i2} \geq 2\}$, endowment is $\omega_i = (1,1)$ and preferences are represented by indifference curves. The quasi-equilibrium is $(x_{i1}, x_{i2}) = (1,1)$ but this preference is inferior to the attainable point $(x_{i1}, x_{i2}) = (0,2)$. Consequently, a Walrasian quasi-equilibrium is not an ultimately satisfactory notion of equilibrium because such states need not

[7] This form of the definition follows Mas-Colell *et al.* (1995; p. 632).

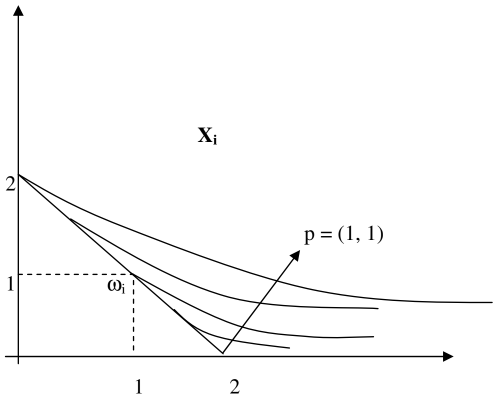

Fig. 2.4. Equilibrium compared to quasi-equilibrium.

be individually rational. An interesting question is then under what conditions will a Walrasian quasi-equilibria become a full Walrasian equilibria? The answer to this question leads to the idea of a consumers' 'cheaper point'.

Definition 2.5 (Cheaper point condition). If (x^*, y^*, p^*) is a Walrasian quasi-equilibrium, and for i if there is $x_i \in X_i$ such that $p^*x_i < p^*\omega_i + \sum_j \theta_{ij}p^*y_j^*$, then i has access to a cheaper point. If this is true for all $i \in I$, then $\boldsymbol{E}$ satisfies the *cheaper point condition.*

Lemma 2.1 (Mas-Colell *et al.* (1995)). *If (x^*, y^*, p^*) is a Walrasian quasi-equilibrium in which all X_i are convex and $\preceq_i$ are continuous, then any consumer who satisfies the cheaper consumption condition must be preference maximising in his or her budget set. If the cheaper point condition is satisfied by all i, then (x^*, y^*, p^*) is a Walrasian equilibrium.*

Proof. Mas-Colell *et al.* (1995; p. 633). □

Remark 2.5. This proposition indicates a way to pass from a quasi-equilibrium to a full equilibrium. The challenge then is to find conditions, other than the interior endowments condition, under which a cheaper point exists for each consumer.

2.3.3.1. *Desirable commodities and productive labour*

In light of their observation that the interior endowments assumption: "…is clearly unrealistic", Arrow and Debreu (1954) present a second existence theorem. This theorem works off the idea of 'desirable commodities' and 'productive labour'.

Definition 2.6 (Desirable commodities). A commodity is *desirable* if an arbitrarily small amount of it added to any consumption bundle raises utility. Let D be the set of *always desirable commodities*, so that for any i, $x_i \in X_i$ and $d \in D$ implies $\exists \lambda > 0$ such that $x_i + \lambda \delta^d \in X_i$ and $u_i(x_i + \lambda \delta^d) > u_i(x_i)$, where δ^d is the positive unit vector of the dth axis of $\Re^\ell$ and u_i is the utility function of i.

Definition 2.7 (Productive labour). Labour is *productive* if it can produce a desirable good. Let P be the set of types of labour which are *productive*, meaning that if $y \in Y$, then $d \in P$ then $y_d \leq 0$ and for some $y' \in Y$ and all $d' \neq d$, $y'_d \geq y_{d'}$ and for at least one $d'' \in D$, $y'_{d''} > y_{d''}$.

Theorem 2.2 (Arrow and Debreu (1954; Theorem 2)). *If $\boldsymbol{E}$ satisfies* $(ad.1)$–$(ad.7)$, $(ad.9)$ *of Theorem* 2.1 *and if* $(ad.8)'$ $\forall i$, $\omega_i \in \Re^\ell$, $\exists x_i \in X_i$ *with* $x_i \leq \omega_i$ *and* $x_{id} < \omega_{id}$ *for at least one* $d \in P$; $(ad.10)$ $\exists x \in X$, $y \in Y$ *and* $\xi > 0$ *such that* $x + \xi < y$; $(ad.11)$ *the set* D *is not empty*; $(ad.12)$ *the set* P *is not empty*; *then there exists a free disposal Walrasian equilibrium for* $\boldsymbol{E}$.

Proof. Arrow and Debreu (1954; pp. 282–287). □

Remark 2.6. This theorem makes do without the interior endowments assumption by introducing instead (ad.8)′, (ad.10), (ad.11) and (ad.12). This last assumption, (ad.12), asserts

(i) that no labour service, or at least none of those in P, can be produced by firms and (ii) that if no restriction is imposed on the amount used of productive labour, then it is possible to increase the output of at least one always desired commodity without reducing the output of any other good or increasing the input of any commodity other than the type of productive labour under consideration (see Arrow and Debreu (1954; pp. 280–281)). This is combined with (ad.8)$'$, which requires the endowment of every individual to be in X_i, but not necessarily in the interior of X_i. In this approach, the interior endowments assumption, (ad.8) has been replaced by the assumption that everyone in the economy is in possession of a feasible consumption and is the owner of at least one type of productive labour, i.e. labour which can be used in the production of an always desirable good.

As with the interior endowment assumption, there is no particular reason why the needed relationship between the labour endowments of each individual, the technology of the economy and the preferences of everyone else, should naturally arise in a private ownership economy so that (ad.8)$'$–(ad.12) will be satisfied. Consequently, one may reasonably have reservations about the generality of the resulting existence theorem for equilibrium. In particular, as far as the desirable good assumption is concerned, McKenzie (1959) has argued that: "An always desired good appears particularly implausible. It requires that every consumer be insatiable in this good within the supplies attainable by the whole market." McKenzie (1959; p. 58).

2.3.3.2. *Augmented aggregate production*

In an attempt to go further than Theorem 2.2, Debreu (1962) presents a set of conditions which require ω_i to be in X_i but allows the possibly that ω_i is in the boundary of X_i. The minimum wealth case is therefore allowed to occur, but there are a number of auxiliary assumptions made in order to ensure the existence of equilibrium. In particular, it is assumed that the *augmented total production set* Y which is a subset of the space which contains Y and is such that $(\{\omega\}+\underline{Y})\cap X = (\{\omega\}+Y)\cap X$, meaning that Y and $\underline{Y}$ give rise to the same set of attainable consumptions. Also let the *attainable consumption set* for i, X_i' be the set of consumptions available to i, if i had complete control of the economy, taking into account only resource limitations. If $A(K)$ denotes the asymptotic cone of a set K and D is the smallest cone of vertex 0 owning points of the form $\Sigma_i(x_i - \omega_i)$ with $x_i' \preceq_i x_i$ for $i \in \boldsymbol{E}$, we have the following result.

Theorem 2.3 (Debreu (1962)). *If* $\boldsymbol{E}$ *is such that* $(d.1)$ $AX \cap (-AX) = \{0\}$; $(d.2)$ $\forall i$, X_i *is closed and convex*; $(d.3)$ $\forall x_i' \in X_i$, $\exists x_i \in X_i$ *such that* $x_i' \preceq_i x_i$; $(d.4)$ $\forall x_i' \in X_i$, *the sets* $\{x_i \in X_i : x_i \preceq_i x_i'\}$ *and* $\{x_i \in X_i : x_i' \preceq_i x_i\}$ *are closed*; $(d.5)$ $\forall x_i' \in X_i$, *the set* $\{x_i \in X_i : x_i' \preceq_i x_i\}$ *is convex*; $(d.6)$ $\exists$ *a closed, convex augmented total production set* $\underline{Y}$ *such that for every* $i(\{\omega_i\} + A\underline{Y} - D) \cap X_i \neq \varnothing$ *and* $ri(\{\omega\} + \underline{Y}) \cap ri(X) \neq \varnothing$; $(d.7)$ $\forall j$, $0 \in Y_j$; $(d.8)$ $AX \cap AY = \{0\}$; $(d.9)$ *if in a quasi-equilibrium the condition* '$\exists i : p^*x_i^* = \min p^*X_i$' *holds at all then it holds* $\forall i \in \boldsymbol{E}$ *then a Walrasian equilibrium exists for* $\boldsymbol{E}$.

Proof. Debreu (1962; pp. 270–271). □

Remark 2.7. This existence result ingeniously avoids invoking the interior endowments assumption. However, various other hypotheses of the theorem are worthy of comment. First, notice that it is still required that $\omega_i \in X_i$ so the potentially realistic situation where $\omega \notin X_i$ is excluded. At various points in the theorem, special relationships are required to hold between other, theoretically distinct primitives which define the economy. In particular, the first part of (d.6) requires a non-empty intersection between consumer i's consumption set and a set made up by summing individual i's endowment, the asymptotic cone of the augmented total production set and D — a set which itself depends in a complicated way on i's endowments, preferences, consumption set and the resources available in the entire economy. The second part of (d.6) also requires the total endowment plus the augmented production set to have a relative interior point in common with the relative interior of the aggregate consumption set. There is no reason why these distinct objects should stand in the required relationships and therefore as Debreu notes, letting $\underline{Y}$ take the place of Y in (d.6) means: "...the requirements for the theorem are strengthened considerably". Debreu (1962; p. 268). Moreover (d.6) and the requirement that $\omega_i \in X_i$ are not the only theoretically arbitrary 'relationship' conditions in this theorem. Note that (d.8) also requires a theoretically unjustified relationship to hold between the asymptotic cone of the aggregate consumption set and the asymptotic cone of Y. As Khan (1993; p. 26) points out, condition (d.9) is used by Debreu to finesse, rather than directly address, the problem of guaranteeing that each consumer has at least minimum income by directly imposing the condition that if in a quasi-equilibrium, $px_i = \min pX_i$ for some consumer i then it occurs for all consumers i. No reason is given why this should be the case.

2.3.4. *Irreducibility and group survival with trade*

The interior endowments condition assumes individual survival without trade. A significant weakening of this condition was achieved by McKenzie (1959) who replaced the idea of individual survival without trade with the idea of group survival with trade. He called the condition which achieved this 'irreducibility' and refinements of McKenzie's basic idea has lead to a rich array of irreducibility like conditions.

The following example due to Gale (1957) provides an insight into the situation which irreducibility is designed to rule out. Consider a two-by-two exchange economy where A and B have consumption sets $X_1 = \Re^2_+ = X_2$, utility functions $u_A(x_1, x_2) = x_2, u_B(x_1, x_2) = x_1 + x_2$ and endowments $\omega_A = (1, 1), \omega_B = (1, 0)$. If $p_1 > 0$, individual A wants to sell x_1 and buy p_1/p_2 units of x_2, which cannot be supplied because B does not have any good 2, so $p_1 > 0$ cannot be an equilibrium. If $p_1 = 0$ then given their preferences, B will demand an unbounded quantity of x_1 so that $p_1 = 0$ cannot be an equilibrium either. Consequently, this economy fails to have an equilibrium state. One way to look at what is going on here is to notice that B does not have anything (i.e. good 2) that is of interest to A. To guarantee the existence of equilibrium, Gale therefore imposed the condition that the economy contained no two subgroups such that group one has commodities that group two likes, but group two has no commodities that group one likes. The idea then is that the economy cannot be decomposed or reduced into two groups of agents who have nothing interesting to trade with each other. A convenient way to express this idea is to say that the economy is 'irreducible'.

2.3.4.1. *Resource relatedness*

There are a number of formulations of the basic idea that the economy is irreducible or 'tied together' in an appropriate way. The condition we begin with is that due to Arrow and Hahn (1971) which they call 'resource relatedness'.

Definition 2.8 (Resource related and indirectly resource related). Individual i' is *resource related* to i'' if, for any increase in the quantity of those goods originally held by i' in positive quantities, there exists a reallocation of the entire economy so that no one is worse off in the new situation and i'' is strictly better off. Individual i' is *indirectly resource related* to i'' if there is a sequence of individuals i_k for $k = 0$ to n with $i_0 = i'$ and $i_n = i''$ such that i_k is resource related to i_{k+1} for $k = 0$ to $n - 1$.

If these relationships are present in the economy, then the following result is available.

Theorem 2.4 (Arrow and Hahn (1971; Theorem 5)). *If the following are satisfied in* $\boldsymbol{E}$, (ah.1) $\forall j$, $0 \in Y_j$ *and* Y_j *is closed and convex*; (ah.2) *if* $y \in Y = \times_j Y_j$ *and* $\Sigma_j y_j \geq 0$, *then* $y = 0$; (ah.3) $\exists y' \in Y$ *such that* $\omega + y' \gg 0$; (ah.4) $\forall i$, X_i *is closed*, *convex and* $x_i \geq 0$ *for* $x_i \in X_i$; (ah.5) $\exists x_i \in X_i$ *such that* $x_{ik} \leq \omega_{ik}$ *for all goods* k *and* $x_{ik} < \omega_{ik}$ *if* $\omega_{ik} > 0$; (ah.6) *for any* i, *total income is* $M_i(p) = p\omega_i + \Sigma_j \theta_{ij} p y_j(p)$ *at each* p; (ah.7) $\preceq_i$ *is defined for all pairs in the set* X_i *and satisfies transitivity, connexity, continuity, semi-strict convexity and non-satiation;* (ah.8) *every individual* i *is resource related to every other individual in the economy*; *then a Walrasian equilibrium exists for* $\boldsymbol{E}$.

Proof. Arrow and Hahn (1971; p. 119). □

Remark 2.8. Various theoretical justifications can be given for (ah.1)–(ah.4) and (ah.7), such as those discussed in relation to Theorem 2.1 above, while (ah.6) is just a definition of consumer income. There are however two conditions in this theorem, namely (ah.5) and (ah.8) for which no theoretical justification can be readily given. Assumption (ah.5) is just (ad.8)′. As was seen earlier, this is a close relative of the interior endowments assumption and is therefore subject to the criticisms already made of that condition. Of equal interest however is the sort of structure which (ah.8) imposes on the economy since it too requires a particular relationship to hold between the commodities originally owned by one consumer and the preferences of everyone else in the economy. As Arrow and Hahn put it: "…[the] property of household i' that is relevant to the definition [of resource relatedness] is the list of commodities with which it is endowed in some positive amount [while] the relevant property of household i'' is its utility function." Arrow and Hahn (1971; p. 117). Again, it is hard to see why these disparate objects, (the endowments of one individual and the preferences of another), should stand in the particular required relationship.[8] It is even harder to find a reason why the relationship should hold between *every pair* of agents in the economy as the resource relatedness condition requires. Consequently, it is not clear that one is justified in regarding this condition as general, as some of the commentaries noted earlier imply or explicitly state it is (cf. Ellickson (1993) discussed earlier in the chapter).

2.3.4.2. *Productive consumers*

In an attempt to generalise the conditions underlying Arrow and Hahn's existence theorem, in particular conditions (ah.3), (ah.5) and the second and third parts of (ah.1), Moore (1975) introduced the notion of 'productive consumers'. Let $\mathbf{X}$, the *consumption allocation set*, be the Cartesian product of all the X_i, $\mathbf{Y}$ the *production allocation set* be the Cartesian product of all the Y_j and $\mathbf{V} = \mathbf{X} \times \mathbf{Y}$. Let $\mathbf{V}^f = \mathbf{V} \cap \{(x_i, y_j) : Z[(x_i), (y_j)] = 0\}$ be the *feasible allocation set*. If $C(K)$ denotes the closure of K and $H[K]$ denotes the convex hull of K, then $Y_j^* = H[C(Y_j)]$, $Y^* = \Sigma_j Y_j^*$, $\mathbf{Y}^*$ is the Cartesian product of the Y_j^*, $\mathbf{V}^* = \mathbf{X} \times \mathbf{Y}^*$, and $\mathbf{V}^{f*} = \{(x_i, y_j) \in \mathbf{V}^* : Z[(x_i), (y_j) = 0\}$. The *attainable consumption set* for i, X_i^f is the projection of $\mathbf{V}^f$ on X_i, with analogous definitions holding for $\mathbf{Y}^f$, $\mathbf{Y}^{f*}$, Y_j^f and Y_j^{f*}. If $T \subseteq \Re^\ell$, let $\mathbf{V}_T = \mathbf{X} \times T$ and $\mathbf{V}_T^f = \{[(x_i), y] \in \mathbf{V}_T : Z[(x_i), y] = 0\}$ then T is an *equivalent technology set* for $\boldsymbol{E}_{po}$ if $[Y \subseteq T \wedge \{[(x_i), y] \in \mathbf{V}_T^f\}] \Rightarrow \exists \{[(x_i'), y'] \in \mathbf{V}^f : x_i \preceq_i x_i'\}$ for all i. Considering what i is capable of producing using only its resources yields the notion of the *individual supply set* for i, defined as $S_i = \omega_i + \Sigma_j \theta_{ij} Y_j$ and the *potential trade set* of i is given by $Z_i = \omega_i + \Sigma_j \theta_{ij} Y_j - X_i$. The ith consumer is *productive* in E_{po} given T, iff for each $(x_i, y_j) \in \mathbf{V}^f \exists (x_i^*, y_j^*) \in \mathbf{V}_T, \lambda \geq 0$ and $z_k \in Z_k$ such that

[8] As the Gale economy example considered above shows it is possible to construct nonpathological examples in which such a situation can arise.

$\lambda.(\Sigma_i x_i^* - \omega - y^*) = z_k$ and $i \in I\backslash\{k\}$ and $x_i \preceq_i x_i^*$. The *net technology set* of the ith consumer is $T_i = [c(Z_i)\backslash\{0\}] \cup Z_i$, where $c(Z_i) = \{z \in \Re^\ell : \exists z' \in Z_i,\ \lambda \in \Re_+ \text{ and } z = \lambda z'\}$ is the cone generated by Z_i.

Definition 2.9 (Productive consumers). The k^{th} consumer is *productive* in $\boldsymbol{E}$ if for any attainable allocation, there exists a feasible allocation such that everyone, with the possible exception of k, is better off and the k^{th} consumer could supply a positive amount of the excess demand of the economy in the new state.

Moore (1975; pp. 272–283) provides the following series of conditions for, and implications of, consumer productivity: (i) the k^{th} consumer is productive if and only if for each $[(x_i), (y_j)] \in \mathbf{V}^f\ \exists (x_i^*) \in X : \Sigma_i x_i^* \in \{(\omega) + T + T_k\}$ and $x_i \prec_i x_i^* \forall i \in I\backslash\{k\}$; (ii) a sufficient condition for productivity is that if the k^{th} consumer is self sufficient and able to survive on his or her own resources so that $\omega_k \in X_k$ or $0 \in Z_k$; (iii) if $0 \notin Z_k$, then for the k^{th} consumer to be productive, it must be the case that for each $(x_i, y_j) \in \mathbf{V}^f\ \exists$ an allocation $(x_i^*, y^*) \in \mathbf{V}_T, z_i^* \in Z_k$ and $\lambda_k > 0$ such that $\lambda_k[\Sigma_i x_i^* - \omega - y^*] = Z_k$ and $x_i \preceq_i x_i^*$ for all $i \in I\backslash\{k\}$; (iv) if the k^{th} consumer is productive then $X \cap (\{\omega\} + T + T_k) \neq \varnothing$. All of this culminates in the following result.

Theorem 2.5 (Moore (1975)). *If* $\boldsymbol{E}$ *satisfies* $(m.1)$ $\forall i, X_i$ *is closed, convex and bounded below*; $(m.2)$ $\forall i, \preceq_i$ *is continuous, convex on* X_i *and non-satiated on* X_i^f; $(m.3)$ $\exists (x_i, y_j) \in V^f$: $u[(x_i)] \gg 0$; $(m.4)$ $\forall j$, $0 \in Y_j$; $(m.5)$ $\forall j, y_j \in Y^* : \Sigma_j y_j \geq 0$ *implies* $y_j = 0$; $(m.6)$ *there exists an equivalent technology set* T *for* $\boldsymbol{E}_{po}$ *which is closed and convex*; $(m.7)$ $\text{int}(T) \neq \varnothing$; $(m.8)$ $AT \cap \Re_+^\ell = \{0\}$; $(m.9)$ *every consumer is productive*; $(m.10)$

every consumer is indirectly resource related to every other consumer; (m.11) $ri(X) \cap ri(\omega + T) \neq \varnothing$; *then a Walrasian equilibrium exists for* $(\boldsymbol{E}, T)$.

Proof. Moore (1975; pp. 298–299). □

Remark 2.9. Burke (1988; p. 282) has argued that Moore's conditions are the 'weakest set of sufficient conditions currently known which imply the existence of market equilibrium'. Moore argues that this theorem represents an improvement over that of Arrow and Hahn (1971, Theorem 5) because (m.3) weakens their (ah.3), (m.6) weakens the last two parts of their (ah.1) and (m.9) weakens their (ah.5). Consideration of these conditions, particularly those which are necessary for consumer productivity, reveals that the condition of consumer productivity requires particular, quite delicate relationships to hold between disparate theoretical objects in the economy. In particular, notice that there are three relationship conditions in this theorem, namely (m.9)–(m.11). Assumption (m.10) is just (ah.8) and is subject to the criticisms of that assumption outlined earlier, whilst (m.11) is closely related to the second part of (d.6) and to (mk.5). According to Moore, (m.9) is actually the only clear cut gain in terms of additional insight into the sorts of conditions necessary for the operation of a price system. We focus on (m.9) to discover circumstances in which the condition fails, in particular consider Moore's Example 2.8[9] which may be sketched as follows. Let $n = \ell = 2$, $j = 1$, $X_i = \{x_i \in \Re^2 : -1 \leq x_{i1} \leq 0, x_{i2} \geq 0\}$, $Y = \{y \in \Re^2 : y_1 + y_2 \leq 0\}$, $(\omega_{11}, \omega_{12}) = (0, 0)$, $\theta_{11} = 0$, $(\omega_{21}, \omega_{22}) = (0, 2)$, $\theta_{21} = 1$. Since $X_2 \cap (\omega_2 + \theta_{21} Y) \neq \varnothing$, a sufficient condition for productivity is satisfied, so consumer 2 is productive. The situation for consumer 1 is illustrated in Fig. 2.5. Here a necessary condition for consumer productivity fails because $X \cap (\{\omega\} + T + T_1) = \varnothing$.

[9] Moore (1975; pp. 276–278).

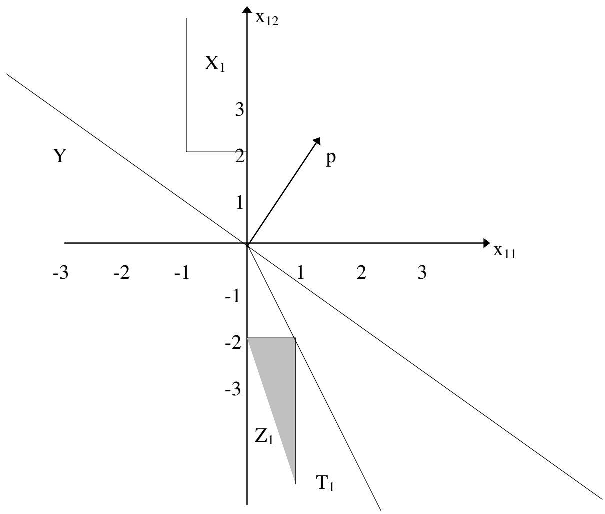

Fig. 2.5. Moore's example of a non-productive consumer.

To see this formally, note as Moore (1975; p. 278) does, that in this example $T = Y$ and if $y \in Y$ and $t \in T_1$ then $y_1 + t_1 + y_2 + t_2 = (y_1 + y_2) + (t_1 + t_2) < 0$ while if $x_i \in X_i$ for $i = 1$, 2 then $\Sigma_i(x_i - \omega_i) = (-2 - 0) + (4 - 2) = 0$ and there does not exist $t \in T_1$ such that $x_1 + x_2 = \omega_1 + \omega_2 + y + t$. Thus, although Moore's theorem is a generalisation of Theorem 2.4 due to Arrow and Hahn (1971), it shares with that theorem the need for potentially restrictive relationship conditions to hold between the primitives that define the economy.

2.3.4.3. *Irreducibility*

The McKenzie (1959) formulation of irreducibility may be stated as follows.[10]

Definition 2.10 (McKenzie-irreducible). Let Y be the aggregate production set for $\mathcal{E}$ let I^1 and I^2 be a partition of I, the set of consumers, into two non-empty sub-groups with

[10]The definition here follows McKenzie (1987).

$I^1 \cap I^2 = \varnothing$ and $I^1 \cup I^2 = I$. Also let $X^1 = \sum_i X_i$ for $i \in I^1$, let $X^2 = \sum_i X_i$ for $i \in I^2$ and let $\bar{U}^2$ be the convex hull of X^2 and the origin of $\Re^\ell$. The economy is *McKenzie-irreducible* if no matter how I^1 and I^2 are selected, if $x^1 = y - x^2$ with $x^1 \in X^1, y \in Y$ and $x^2 \in \bar{U}^2$, then there is a $\tilde{y} \in Y$ and $\omega \in \bar{U}^2$ with $\tilde{y} - x^2 - \omega = x'^1$ such that $x^1 \prec_i x'^1$ for all $i \in I^1$.

Remark 2.10. As McKenzie (1987) points out, for an economy to be irreducible, everyone in the economy must be able to supply some asset or service which is of interest to everyone else in the economy. Bergstrom (1991) notes that irreducibility is satisfied: "If each consumer could survive after having surrendered some vector of commodities that could be used to make all other consumers better off". Bergstrom (1991; p. 15). Hammond's (1993) rendition of the condition has it that an economy is irreducible if and only if every I^1 which is a proper subset of I, has available to it a Pareto improvement relative to the current allocation, provided it could have access to additional goods which duplicate those which are held as the initial endowment of the group I^2. In a private communication to Khan (1993), McKenzie made the following remarks about irreducibility: "If one considers the application of this assumption, one sees that what is really needed is that the improving change in I^1's consumption should lie in the cone from the origin spanned by $Y - y$ and $-X_2^I$. Perhaps this is the proper form of the assumption ..." Khan (1993; p. 37 footnote 67). McKenzie (1999) provides further discussion of his thinking about the irreducibility condition. He also shows that McKenzie-irreducibility is a much weaker condition than the conditions of interior endowments, desirable goods and productive labour or resource relatedness. There is no doubt that McKenzie-irreducibility represents a major advance in the search for conditions which help ensure the existence of equilibrium. However, as the example of Gale given at the beginning of this section and the example in McKenzie (1999; p. 376) show, the condition still has 'bite' in it as it requires a particular relationship to hold between the preferences of one (group of) individuals and the endowments (possibily mediated by technology) of another group. In this context, it is interesting to note the consideration of the condition given in Geanakoplos (1987) who remarks that the assumption means; "...for any two agents i and i', the endowment ω_i of agent i is positive in some commodity ℓ which (taking into account the possibilities of production) agent i' could use to make himself better off. It certainly seems reasonable that each agent's labour power could be used to make another agent better off". Geanakoplos (1987; p. 118). At first glance, this remark might seem reasonable, however a moments thought reveals that leaving aside the obvious case of people who are disabled, it is the case that Geanakoplos' remark (i) assumes that each person in the economy is capable of supplying labour services. Considering that labour generally has to be produced by food, shelter, rest, education and other inputs which are typically not free, this is not at all obvious; (ii) the qualifier 'taking into account the possibilities for production' is revealing and reinforces the point we have been making that irreducibility requires a particular relationship to hold between the endowments held by one individual and the preferences and production technology open to another, something which is not at all obvious in a given economy.

Given irreducibility, McKenzie proves what Khan (1993; p. 36) characterises as the classical theorem on the existence of Walrasian equilibrium 'in its fully developed form', incorporating as it does the Mas-Colell (1974) and Gale and Mas-Colell (1975, 1979) weakening of the conditions needed on preferences in order to get the existence of Walrasian equilibrium.

Theorem 2.6 (McKenzie (1981)). *If E satisfies* (*mk.*1) $\forall i$, *X_i is convex, closed and bounded below*; (*mk.*2) $\forall i$, $G = \{x_i' \in$

$X_i : x_i \prec_i x_i'\}$ *is open* $\forall x_i \in X_i$, $\prec_i$ *is continuous on* X_i *and* $x_i \notin H(G)$; (*mk*.3) Y *is a closed, convex cone*; (*mk*.4) $Y \cap \Re_+^\ell = \{0\}$; (*mk*.5) $ri(X) \cap ri(Y) \neq \varnothing$; (*mk*.6) E *is McKenzie-irreducible; then a Walrasian equilibrium exists for* E.

Proof. McKenzie (1981; pp. 828–834). □

Remark 2.11. Assumptions (mk.1)–(mk.4) can be justified by familiar arguments, similar to those discussed earlier in this chapter. However (mk.5) and (mk.6) are in a different category compared to the other conditions in this theorem. From the definition of irreducibility, it is clear that for an economy to be irreducible, a particular relationship must hold between the endowments of goods and labour supply capacities of each individual, the preference of all other consumers and in the case of a production economy, the production technology as well. The needed relationships may or may not hold in any particular economy. In any event, involving as it does particular relationships between theoretically distinct objects, irreducibility cannot generally be argued for on theoretical grounds, contrary to the view promoted by Mas-Colell *et al.* (1995; p. 633). As was noted earlier, these authors advance this condition over that of interior endowments on the grounds that it is a weaker condition, (which is reasonable), but in the process imply that it rescues Walrasian general equilibrium theory from the perilous position it would be in if the existence result had to rely on the interior endowments condition. This conclusion does not necessarily follow since it was observed earlier that even if irreducibility is weak*er* than interior endowments, it does not follow that irreducibility is a *weak and reasonable* condition to impose on an economy. We therefore devote some effort to exploring the nature of McKenzie-irreducibility and related conditions. To begin, notice that McKenzie-irreducibility is not the only formulation of the idea available. There have been a number of elaborations of the idea of irreducibility, including by McKenzie himself, see McKenzie (1999). Here we mention just two: the first of which is due to Bergstrom (1976) and the second is due to Florig (2001).

Definition 2.11 (Bergstrom irreducible). $\mathcal{E}$ is *Bergstrom-irreducible* if for every non-empty proper subset J of the set of consumers I and for each attainable allocation $x = (x_i)$, there exists a set of m numbers λ_i with $0 < \lambda_i < 1$ and an allocation $x' = (x'_i)$ which is preferred to (x_i) by consumers of group J and such that $\{\lambda_i x'_i + (1 - \lambda_i) b_i\}$, which is a convex combination of the survival allocation b_i and (x'_i), is attainable.

Remark 2.12. As will be seen below, this notion generalizes McKenzie-irreducibility in a number of interesting ways.

A second interesting refinement of the idea of irreducibility for an Arrow-Debreu production economy is due to Florig (2001). Florig's formulation of the condition relies on the some notation. Thus $C \subset \Re^\ell$, $posC = \{\sum_{v=1,t} \lambda_v z_v : z_v \in C,\ \lambda_v \geq 0,\ t \geq 0\}$ is the positive hull of C, and $spanC$ is the vector subspace of $\Re^\ell$ spanned by C. Also let $I^+(x, y)$ be the set of consumers in I who are not satiated at the allocation $((x), y$, 'cl' stands for closure of a set and 'co' for the convex hull). Given this we have:

Definition 2.12 (Florig-irreducibility). An m-consumer economy $\mathcal{E}$ is irreducible at an allocation $((x),\ y)$, if for every non-trivial partition of the set of consumers I, there exists an allocation $x' \in \Pi_{i \in I} X_i$ and a system of $2m$ numbers $\lambda_i \geq 0$, $i = 1, \ldots, m, \mu_i \geq 0, i = 1, \ldots, m$ such that:

(1) $i \in I^+(x, y)$ if and only if $\lambda_i + \mu_i > 0$;
(2) $x'_i \in \mathrm{cl}[\mathrm{co}\{x_i \cup$; the set of points preferred by i at $((x), y)\}]$, $\forall i \in I_1$ and $\exists i \in I_1$ such that x'_i is in the set of points preferred by i at $((x),\ y)$;
(3) $\sum_{i \in I} \lambda_i (x'_i - \omega_i - \sum_{j \in \text{Firms}} \theta_{ij} y_j) + \sum_{i \in I} \mu_i (x'_i - x_i) \in \mathrm{co}(Y - \sum_{j \in \text{Firms}} y_j)$.

Remark 2.13. As Florig (2001) notes, this condition is made up of two parts, even in the case of an exchange economy (i.e. $Y = 0$ or $Y = -\Re^{\ell}_{+}$). If $\lambda_i = 0$ for all i, then the condition requires that an 'average' of the prescribed change in consumption plans is feasible. If $\mu_i = 0$ for all i, then the condition reduces to Bergstrom-irreducibility. In general not all λ_i and μ_i are zero, so the condition involves a mixture of these two possibilities. In the case of a production economy, the condition requires that the considered changes in consumption form a *feasible* direction of change in the aggregate production plan. Florig shows that this irreducibility condition is necessary and sufficient in order to apply standard arguments for the passage from quasi-equilibrium to Walrasian equilibrium. He also shows that it is the weakest possible condition for achieving this, a point we discuss in more detail in the next chapter. Here we simply remark that Florig's condition, like the other irreducibility conditions studied, requires particular theoretically unexplained relationships to hold between the primitives that define the economy.

In trying to assess the reasonableness of irreducibility, perhaps the first thing to notice is that for an economy to be irreducible, not only has a particular relationship got to hold between the endowments and preferences of agents in the economy, but as Florig (2001; p. 138) notes, and as is clear from the definitions above, the weak survival assumption must also hold. This condition requires that each consumers endowment is at least *in* their consumption set, although unlike the interior endowments assumption, the endowment may possibly be in the boundary of the consumption set. Thus irreducibility shares with interior endowments the defect that it requires a particular relationship to hold between two distinct primitives in the economy. One can think of various informal exceptions to this condition, such as people whose labour supply capacities are limited in quantity and scope, and who are also poor in asset terms.

More formally, Moore (1970), Wilson (1981), Florenzano (1982) and Santos (1990) have all produced examples of economies which are not irreducible. Since it nicely illustrates the point that irreducibility relies on special relationships holding between the primitives of the economy, consider Florenzano's 3 person, 4 good example. Initial endowments in the economy are $\omega_1 = (\omega_{11}, \omega_{12}, 0, 0)$, $\omega_2 = (0, \omega_{22}, \omega_{23}, 0)$, $\omega_3 = (0, 0, \omega_{33}, \omega_{34})$ where each $\omega_{ii} > 0$ for $i = 1, 3$ and $i = 1, 4$. Consumption sets are $X_1 = \Re_+ \times \Re_+ \times \{0\} \times \{0\}$, $X_2 = \{0\} \times \Re_+ \times \Re_+ \times \{0\}$, $X_3 = \{0\} \times \{0\} \times \Re_+ \times \Re_+$ and $Y = \{0\}$. If $\forall i \preceq_i$ are monotone, then the economy is Arrow and Hahn resource related and both McKenzie- and Bergstrom-irreducible. If the preferences of consumers 1 and 3 are monotone and consumer 2 is indifferent between two consumptions and if neither yields $x_{22} \geq a_{22}$ and prefers with monotonic preferences all consumptions for which $x_{22} \geq a_{22}$ and if $a_{22} > 0$, then $\boldsymbol{E}$ is neither McKenzie-irreducible nor does Arrow and Hahn's resource relatedness hold. However, $\boldsymbol{E}$ is Bergstrom-irreducible. If the third commodity is not desired by consumer 2 beyond some quantity $x_{23} = a_{23} \leq \omega_{23} + \omega_{33}$, then $\boldsymbol{E}$ is not irreducible in any of the senses defined above. Florenzano's example serves to illustrate the point that the concept of irreducibility requires particular and potentially quite delicate relationships to hold between the primitives which define the economy if that economy is to be irreducible. Consequently, like all the other conditions so far encountered, the condition of irreducibility is sensitive to the details of the relationships which holds across individuals in the economy and among the primitives which define the economy. Therefore, in common with the other existence results so far encountered, those which include the hypothesis of irreducibility in its various forms, require that potentially theoretically arbitrary relationships hold among the individuals who make up the economy.

2.3.4.4. *Connected graph conditions*

Kirman (1987) makes an appeal for the use of graph theory in economics when he writes: "[a]lthough the basic concepts [of graph theory] have a simple intuitive interpretation and correspond to many features of social and economic organization, this tool has been little exploited in economics" Kirman (1987; p. 558). As Baldry and Ghosal (2005) note, graph theory has actually been used in general equilibrium analysis by Rosenblatt (1957) to provide a complete characterisation of solutions to linear input-output models, Eaves (1985) to give necessary and sufficient conditions for the existence of competitive equilibria in pure exchange economies with Cobb-Douglas preferences and by Maxfield (1997) to develop computable characterisations of irreducibility like conditions and to draw explicit attention to the survival issue. A brief account of Maxfield's work begins with the following basic ideas. The *income* of consumer i at price vector p is $\gamma_i(p) = p\omega_i + \Sigma_j \theta_{ij} p y_j$, and the income level at p, $\eta_i(p) \equiv \min\{px_i : x_i \in X_i\}$, is consumer i's *survival income.*

Definition 2.13 (Non-degenerate Walrasian equilibrium).[11] A consumption allocation (x_i^*), a production allocation (y_j^*) and a price vector p^* is a *non-degenerate Walrasian equilibrium* for $\boldsymbol{E}$ if: (i) for each i, x_i^* is a maximal element of the set $B_i = \{x_i \in X_i : p^* x_i \leq p^* \omega_i + \Sigma_j \theta_{ij} p^* y_j^*\}$ relative to $\preceq_i$; (ii) for each firm j, y_j^* maximises profit for j so that $p^* y_j \leq p^* y_j^*$ for all $y_j \in Y_j$; (iii) all markets clear so $\sum_i x_i^* = \omega + \sum y_j^*$; (iv) $p^* \neq 0$ and (v) all for all i, it is the case that $\gamma_i(p) - \eta_i(p) > 0$ so that all consumers have discretionary income.

Definition 2.14 (Degenerate Walrasian equilibrium).[12] A consumption allocation (x_i^*), a production allocation (y_j^*) and a price vector p^* is a *degenerate Walrasian equilibrium* for $\boldsymbol{E}$

[11] See Maxfield (1997; p. 45).
[12] See Maxfield (1997; p. 45).

if: (i) for each i, x_i^* is a maximal element of the set $B_i = \{x_i \in X_i : p^*x_i \leq p^*\omega_i + \Sigma_j\theta_{ij}p^*y_j^*\}$ relative to $\preceq_i$; (ii) for each firm j, y_j^* maximises profit for j so that $p^*y_j \leq p^*y_j^*$ for all $y_j \in Y_j$; (iii) all markets clear so $\sum_i x_i^* = \omega + \sum y_j^*$; (iv) $p^* \neq 0$; and (v) for at least one i, it is the case that $\gamma_i(p) - \eta_i(p) > 0$, so that at least one consumer has discretionary income.

Given these definitions, Maxfield (1997) provides another irreducibility like condition[13] using ideas from graph theory. In particular, a consumer is said to be *normal* if (A1) he or she has a feasible consumption set x_i which is closed, convex and bounded below and (A2) the preference relation $\prec_i$ which is a open valued relative to X_i, lower semi-continuous and such that $x_i \notin$ convex hull of $\{y \in X_i : x_i \prec_i y\}$. The *income* for consumer i is $\gamma_i(p) = p\omega_i + \Sigma_j\theta_{ij}py_j$. Consumer i is *non-satiable on a non-empty commodity set* A_i if for every pair (p, α) such that $\alpha > 0$, and $x_i(p, \alpha + \eta_i(p))$ defined, $p_k > 0$ is true for any good $k \in A_i$. A consumer i who is non-satiable on A_i is *demand positive on a non-empty commodity set* D_i if $\xi_i \in X_i$ and $D_i \subset A_i$ for every pair (p, α) such that $\alpha > 0$ and $x_i(p, \alpha + \eta_i(p))$ is true and that $x_{ij}(p, \alpha + \eta_i(p)) > \xi_{ij}$ for all $j \in D_i$. Denote by D the maximal such set. Let $\Omega_i = \{k : \omega_{ik} > x_{ik}^0\}$ and call Ω_i the *tradeable endowment set* for i. For every $k \in \Omega_i$, consumer i can give up a positive amount of k and still have a feasible commodity bundle left. Let $O_i = \{j : \theta_{ij} > 0\}$ be the *ownership set for consumer* i. If some firm $j \in O_i$ earns positive profit, then i gets a positive income from this source. As Maxfield notes, if a consumer is demand positive on D_i and has discretionary income, then they will demand an amount of every commodity in D_i that is strictly greater than the minimum sustainable amount. Note demand-positivity is only defined if $\xi_i \in X_i$, e.g. $X_i = \Re_+^\ell$.

[13]It is interesting to see the connections between Maxfield's subgraph connection condition and the condition of irreducibility due to McKenzie. Maxfield shows that although the notions are closely related neither condition implies the other (see Maxfield (1997; pp. 42, 43)).

Firm j with production possibility set Y_j is *normal* if $0 \in Y_j$, Y_j is closed and there are commodity sets Q_j and Z_j called *the output set* and *the factor set* such that $Q_j \cap Z_j = \varnothing$ and for every $y_j \in Y_j$ (i) $y_{jk} \geq 0$ if $k \in Q_j$ (ii) $y_{jk} \leq 0$ if $k \in Z_j$ (iii) $y_{jk} = 0$ if $k \notin Q_j \cup Z_j$. A normal firm j is *substitutable on a non-empty factor substitution set* S_j if $S_j \subset Z_j$ and for every p such that $p_{Qj} \neq 0$ and $y_j(p)$ is defined, it is true that $p_k > 0$ for every $k \in S_j$. Define S as the maximal such set. A substitutable, normal production possibility set Y_j is *factor positive on a non-empty commodity set* F_j if: (i) $F_j \subset S_j$ and (ii) for every (p, y_j) such that $y_j(p)$ is defined, $y_j \in y_j(p)$ and $p_{Qj} y_{jQj} > 0$ it is true that $y_{jk} < 0$ for all $k \in F_j$. This means that any profit maximising input-output vector which generates positive revenue has strictly negative amounts of each commodity in its factor positive set F_j. Define F as the maximal such set. A normal firm k with a production possibility set Y_j is *strictly profitable* if for every (p, y_j) that $y_j(p)$ is defined, $y_j \in y_j(p)$ and $p_{Qj} y_{jQj} > 0$ is true and $py_j > 0$. Y_j is *strictly profitless* if $py_j = 0$ for every p for which $y_j(p)$ is defined. Denote by P *the strict profitability* set, so that $P = \{j\text{: firm } j \text{ is strictly profitable}\}$. A normal firm j with a production possibility set Y_j is *robust on a non-empty commodity set* R_j if R_j is the maximal set satisfying $R_j \subset Q_j$ and for every p with $p_k > 0$ for any $k \in R_j$, there is $y_j \in Y_j$ such that $py_j > 0$. Y_j is *strictly robust* if $R_j = Q_j$. Denote by T *the strict robustness set*, i.e., $T = \{j : \text{ firm } j \text{ is robust}\}$. A firm j is the *sole source for trading for commodity* k if $k \in Q_j, \omega_{ik} = x^0_{ik} = \xi_{ik}$ for all i, and $k \notin Q_{jk}$ for any $j_f \neq j$. Define the index set $U_j = \{k : \text{ firm } j \text{ is the sole source of commodity } k\}$.

Example 2.1. Maxfield (1997; p. 31) considers a single output, multiple input production function so that $Q = \{1\}$, $b_j \geq 0$ and $Z = \{j : b_j > 0\}$. A CES production function which exhibits decreasing returns to scale, so that $y_1 \leq (\sum_{j=2,n} b_j |y_j|^{\alpha})^{\gamma/\alpha}$, $\alpha < 1$ and $0 < \gamma < 1$. This production process is: (i) substitutable on $S = Z$, (ii) factor positive on $F = Z$, (iii) strictly profitable, (iv) strictly robust, and (v) strictly normal.

If a firm is normal, then the profit associated with any input-output vector can be expressed as the inner product of the price vector and the input-output vector. If a firm is substitutable, output prices are positive and a profit maximising input-output vector produces a positive revenue, then the firm will select an infinitely large amount of some input and/or output if any factor prices are non-positive. If Y_j has a single output, then it is also strictly profitable if it is robust. If Y_k has multiple outputs, it is strictly profitable if it is strictly robust.[14] As Maxfield (1997) also notes, although the set A_i is non-empty by definition, the sets O_i, D_i and Ω_i may be empty. This possibility has interesting implications that will be explored later. Having introduced various ideas of normality for consumers and firms, Maxfield (1997; p. 34) defines a similar notion for the economy.

Definition 2.15 (Normal economy). A *normal private ownership economy* is one in which all consumers are normal, so (A1) and (A2) are satisfied and in addition (A3) if each consumer i is non-satiable on a commodity set A_i and (A4) if there exists $x_i^0 \in X_i$ such that $x_i^0 \leq \omega_i$. Firms are not necessarily normal.

Remark 2.14. In a normal economy, every consumer is normal and non-satiable and each consumers endowment allows sustainability by consuming a portion of it. We have already discussed at length how restrictive such an assumption can be. Using the index sets that arise as consequences of these definitions Maxfield (1997) defines the 'economy graphs' which show how individuals in the economy are related to each other. Each of the graphs describing the economy has $m + K$ vertices, one for each consumer and firm. Maxfield (1997; p. 35) notes the variety of graphs which are possible representations of the economy arises

[14]See Lemma 4 in Maxfield (1997) for a proof of the first half of this statement.

because there are a number of possible ways to define the arcs in the economy.

Definition 2.16 (Type m economy graph). A *type m economy graph*, denoted by $\Gamma_m(\boldsymbol{E})$, consists of $m + K$ vertices, where vertex v_i corresponds to consumer i, vertex v_{m+j} corresponds to firm j and the arcs between consumers and consumers, consumers and firms, firms and consumers and firms and firms are defined as follows: a type (a) arc exists between v_{i1} and v_{i2} if consumer i_1 $(1 \leq i_1 \leq m)$ has a tradeable endowment of at least one commodity in which $i_2(1 \leq i_2 \leq m)$ is non-satiable; a type (b) arc exists between v_i and v_{m+j} if consumer i has a tradeable endowment which is a substitution factor for firm j; a type (c) arc exists between v_{m+j1} and v_{m+j2} whenever firm j_1 has at least one output which is a substitution factor for firm j_2; a type (d) arc exists between v_{m+j} and $v_i(1 \leq i \leq m)$, if firm k has at least one output commodity for which i is not satiable and a type (e) arc exists between $v_i(1 \leq i \leq m)$, if consumer i owns a positive share in the profits of a firm which is robust in all its outputs.

Definition 2.17 (Type n economy graph). A *type n economy graph*, denoted by $\Gamma_n(\boldsymbol{E})$, consists of $m + K$ vertices, where vertex v_i corresponds to consumer i, vertex v_{m+j} corresponds to firm j and the arcs between consumers and consumers, consumers and firms, firms and consumers and firms and firms are defined as follows: a type (a) arc exists between v_{i1} and v_{i2} if consumer $i_1(1 \leq i_1 \leq m)$ has a tradeable endowment of at least one commodity in which $i_2(1 \leq i_2 \leq m)$ is non-satiable; a type (b) arc exists between v_i and v_{m+j} if consumer i has a tradeable endowment which is a substitution factor for firm j; a type (c') arc exists whenever firm j_1 is robust in at least one output commodity which is a substitution factor for j_2 or whenever j_1 is the sole source of at least one commodity for which j_2 is factor positive; a type (d') arc exists whenever an arc exists and whenever firm j is robust in at least one output commodity for which consumer i is non-satiable or whenever firm j is the sole source of at least one commodity for which consumer i is demand positive; a type (e') arc exists whenever consumer i owns a positive share of the profits of some strictly profitable firm, j.

Remark 2.15. With these definitions, what can be shown is that if either type of economy graph has a sub-graph *containing all consumer vertices*, then a Walrasian equilibrium exists for the economy. This is the content of Maxfield's main existence theorem.

Theorem 2.7 (Maxfield (1997)). *If in* E $(mx.1)$ *each consumer* i *has a feasible consumption set* X_i *which is closed, convex and bounded below*; $(mx.2)$ *each consumer* i *has a preference relation* $\prec_i$ *which is open valued relative to* X_i, *lower semi-continuous such that* $x_i \notin$ *convex hull of* $\{y \in X_i : x_i \prec_i y\}$; $(mx.3)$ *each consumer* i *is non-satiable on a commodity set* A_i; $(mx.4)$ *for each consumer there exists* $x_i^0 \in X_i$ *such that* $x_i^0 \leq \omega_i$; $(mx.5)$ $0 \in Y_j$ *for each firm* j; $(mx.6)$ *the aggregate production set* $Y \supset \Re^\ell_-$; $(mx.7)$ Y *is closed, convex and no aggregate free lunch is possible so that* $Y \cap \Re^\ell_+ = \{0\}$; $(mx.8)$ $\exists x' \in X$ *and* $y' \in Y$ *such that* $x' < y' + \omega$; $(mx.9)$ *a subgraph of* $\Gamma_m(\mathsf{E})$ *or* $\Gamma_n(\mathsf{E})$ *containing all consumer vertices, is strongly connected*; *then a Walrasian equilibrium exists for* E.

Proof. Maxfield (1997; pp. 46–48). □

Remark 2.16. It can also be shown that the assumption of free disposal, (mx.6) can under certain conditions be dropped without losing the conclusion of the theorem (see Maxfield (1997; Corollary 3)). What cannot be dropped is the requirement that the economy is 'tied together' in a way which allows a circular flow of income in which all consumers participate. Also obvious

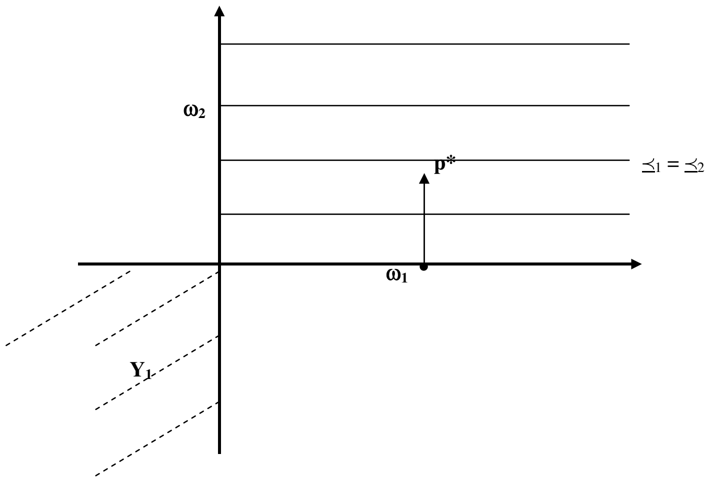

Fig. 2.6. Maxfield's example of a non-connected economy graph.

by now is that there is no reason why such a relationship condition need hold in a standard private ownership economy.

Example 2.2. Maxfield (1997; p. 47) gives an example of a strictly normal economy for which a degenerate Walrasian equilibrium exists but for which the economy graph is not connected. Consider a two consumer, two commodity one firm economy with $u_1(x_1, x_2) = x_{12}, u_1(x_1, x_2) = x_{22}, \omega_1 = (1, 0), \omega_2 = (0, 1), X_i = \Re^{\ell}_{+}, Y_1 = \Re^{\ell}_{-}$. This economy may be pictured as in Fig. 2.6.

As Maxfield notes, the economy graph is not strongly connected, but the Walrasian equilibrium $p^* = (0, 1)$, $x_1 = (0, 0)$, $x_2 = (1, 0)$ and $y = (-1, 0)$ which assigns zero income to consumer 1 exists.

2.3.4.5. *C and C′-irreducibility*

Baldry and Ghosal (2005) motivate their work by noting that Maxfield (1997) uses graph theory to provide sufficient conditions for the existence of competitive equilibria in economies with weakly monotone preferences and boundary endowments.

In particular, they observe that: "... [Maxfield] focuses attention on a particular class of private ownership economies in which utility functions and production possibility sets have specific commonly used forms, including Cobb-Douglas and CES. To each exchange economy, he associates two types of *economy graph* in which vertices represent both individuals and firms, and directed arcs link the sources of commodities and profits to users of commodities and recipients of profits. Maxfield shows that strong connectedness of either economy graph is a sufficient condition for the existence of a competitive equilibrium in the class of economies considered. Through examples, he shows that his alternative condition is "neither stronger nor weaker than either McKenzie's irreducibility or Arrow and Hahn's resource relatedness" Baldry and Ghosal (2005; p. 939). They go on to observe that there are economies which satisfy their C-irreducibility condition but "...in which Maxfield's condition does not hold (even when individuals have interior endowments). An added appeal of C-irreducibility over Maxfield's condition is its relative simplicity. As Maxfield himself points out, his notation is "somewhat awkward" — eight different index sets are used to define eight different types of arcs. C-irreducibility, on the other hand, "uses two index sets to define one single type of arc", Baldry and Ghosal (2005; p. 940). The work of Baldry and Ghosal (2005) uses graph theory to refine the notion of irreducibility in finite production economies. They do this by associating to each exchange economy at each vector of prices a 'price graph', in which each individual is represented by a vertex, and directed arcs between vertices imply a certain coincidence of the preferences and endowments of the relevant individuals. An economy will be irreducible if at each vector of prices, the price graph is strongly connected. Baldry and Ghosal (2005) provide the formal definitions as follows. Consumer i is characterised by a pair (Z_i, u_i), where Z_i is the *feasible trade set for* i equal to $X_i - \{\omega_i\}$, and u_i is i's utility function. Trades in commodities are denoted by $z = (z_1, \ldots, z_\ell)$. The *aggregate domain of trades in commodities* is $Z^0 = \sum_{i \in I} Z^i$. An *allocation* z^I is a profile of net trades $z^I = \{z^i \in Z^i : i \in I\}$ and an allocation is *feasible* if and only if $\sum_{i \in I} z^i = 0$. If z^I is a feasible allocation, then let z'^i be a trade which if added to i's allocation at z^I would make i better off, i.e. $u_i(z^i + z'^i) > u_i(z^i)$. Denote the collection of all such trades by $\Phi^i(z^I)$. Finally, denote by $Z^i_- \subset Z^i$ the set of feasible trades which are non-positive in all components and negative in some. In an exchange economy, the following graph theory formulation of irreducibility can be obtained (see Baldry and Ghosal (2005) for details).

A *directed graph* Γ consists of a set of vertices $V = \{v_i\}_{i \in I}$ and a set of arcs $A = \{a_k\}_{k \in K}$. Associated with each arc is an ordered pair of vertices so that a_k is represented by $a_k(v_i, v_j)$, meaning that the arc a_k originates at v_i and terminates at v_j. A *path* of length L is an ordered sequence of arcs $\{a_{k1}(v_{i0}, v_{i1}), \ldots, a_{kL}(v_{iL-1}, v_{iL})\}$ connecting a set of $L+1$ vertices. Such a path *connects* v_{i0} to v_{iL} via the intermediate vertices $(v_{i1}, \ldots, v_{iL-1})$. The graph Γ is *strongly connected* if for every pair of vertices (v_i, v_j), there is a path connecting v_i to v_j and v_j to v_i. A *subgraph* of Γ is a graph made of a subset of vertices of Γ and a subset of arcs of Γ both of which originate and terminate at vertices in the subset. Baldry and Ghosal (2005) then obtain:

Definition 2.18. A *type* 1 *economy graph of the exchange economy* $\boldsymbol{E}$ *at an allocation* z^I, denoted by $\Gamma^1(\boldsymbol{E}, z^I)$, is a collection of vertices, V and arcs A such that each vertex v_i corresponds to consumer i for $i = 1, 2, \ldots, I$ and an arc directed from v_i to v_j exists whenever $-Z^i \cap \Phi^j(z^I) \neq \varnothing$. A *type 2 economy graph of an exchange economy*, denoted by $\Gamma^2(\boldsymbol{E})$, is a collection of vertices, V and arcs A such that vertex v_i corresponds to consumer i, for $i = 1, 2, \ldots, I$ and an arc directed from v_i to v_j exists whenever $-Z^i_- \cap \Phi^j(z^I) \neq \varnothing$.

Definition 2.19 (Price graph). The *price graph* of the exchange economy $\boldsymbol{E}$ at prices p, denoted $\Gamma(\boldsymbol{E}(p))$, is a collection of vertices V and arcs A such that each vertex v_i corresponds to consumer i and an arc directed from v_i to v_j exists whenever i can make j better off.

Remark 2.17. Commenting on these definitions Baldry and Ghosal (2005) note, "...an arc from $v_i v_j$ exists whenever (a) individual i is a member of some subset of individuals who can supply a net trade which makes j strictly better off (at some utility maximising affordable bundle), with the additional restriction that, however we partition the aforementioned subset into two groups, the group containing i can supply a net trade which makes some individual m in the other group strictly better off (at some utility maximising affordable bundle)" Baldry and Ghosal (2005; p. 943). They then propose the following.

Definition 2.20 (C-irreducibility). Let C be the collection of price graphs of economy $\boldsymbol{E}$ at all non-zero prices. $\boldsymbol{E}$ is C-irreducible if every member of C is strongly connected.

Remark 2.18. Commenting on this definition Baldry and Ghosal (2005) note, "...the economy is C-irreducible if the economy graph $\Gamma(\boldsymbol{E}(p))$ is strongly connected for all non-zero prices. Given the way arcs are defined in the price graph, if $v_i v_j$ is an element of the arc set of $\Gamma(\boldsymbol{E}(p))$, this implies that if j has positive income at prices p... then so does individual i. If the graph $\Gamma(\boldsymbol{E}(p))$ is strongly connected, then if one individual has positive income at prices p, then so do all individuals in the economy" Baldry and Ghosal (2005; p. 945).[15] Given these ideas the following result becomes available.

Theorem 2.8 (Baldry and Ghosal (2005)). *If $\boldsymbol{E}$ satisfies* (bg.1) *for each consumer i, the set of feasible trades, Z^i, is*

[15]Note that McKenzie-irreducibility and C-irreducibility are different conditions in that neither implies the other, see Baldry and Ghosal (2005; pp. 946–949) for details.

closed, convex, bounded below and allows for free disposal; (*bg*.2) *autarchy is feasible so that for all* i, $0 \in Z^i$; (*bg*.3) *for each* i, *the utility function*, u_i, *is continuous*, *quasi-concave*, *locally non-satiated and weakly monotonically increasing*; (*bg*.4) *the aggregate production set* Y *is a closed convex cone with* $Y \cap \Re^{\ell}_{+} = \{0\}$; (*bg*.5) *for all* i, $Z^i \cap Y \neq \varnothing$; (*bg*.6) $0 \in \{Int(\sum_{i \in I} Z^i\} \cap Y$ *and is* C*-irreducible*, *then a Walrasian equilibrium exists for* E.

Proof. Baldry and Ghosal (2005; p. 945). □

Remark 2.19. As Baldry and Ghosal (2005) observe, since strong connectedness of the graph is required at all vectors of prices, C-irreducibility is a condition on preferences, consumption sets and technology. This reinforces the point we have been making that relationship conditions between, as well as structural conditions on the primitives that define the economy are needed to get existence. Also note that C-irreducibility has an advantage over McKenzie-irreducibility in that it is a condition on the relationship between pairs of agents in the economy while McKenzie-irreducibility considers 'arbitrary bipartitions of the set of agents.' Consequently the presence of C-irreducibility can be tested by assigning it to a directed graph on an adjacency matrix. They also note that since a directed graph is strongly connected if and only if the associated adjacency matrix is irreducible, standard algorithms can be used to check for irreducibility of the adjacency matrix. C-irreducibility has an advantage over irreducibility in that 'while irreducibility requires a particular relationship to hold between every pair of individuals at every feasible allocation, C-irreducibility only requires a relationship to hold between pairs of individuals for some subset of feasible allocations, which contains the set of quasi-equilibrium allocations.' Also, C-irreducibility works with quasi-concave utility functions, an added advantage over irreducibility.

As Baldry and Ghosal (2005; p. 949) note, Arrow and Hahn's notion of resource relatedness implies McKenzie-irreducibility while the converse relationship is not so obvious. However, there are conditions under which the two notions are equivalent. This answers an open question in the literature and motivates the following definition of theirs.

Definition 2.21 (Modified price graph). The modified price graph of the exchange economy E at prices p, denoted $\Gamma'(\mathsf{E}(p))$, is a collection of vertices V and arcs A such that each vertex v_i corresponds to consumer i and $A = A_1(p) \times A_2(p)$, where $A_1(p)$ are type 1 arcs and $A_2(p)$ are type 2 arcs.

Definition 2.22 (C'-irreducible). Let C' denote the collection of modified price graphs of economy E at all non-zero prices. The economy is *C'-irreducible* if every member of C' is strongly connected.

Remark 2.20. As Baldry and Ghosal (2005) note, C-irreducibility implies C'-irreducibility, but the reverse is not necessarily true. Also the existence of either a type 1 arc or a type 2 arc from i to j in the price graph $\Gamma'(\mathsf{E}(p))$ is sufficient to guarantee that if j has positive income at prices p, then so does individual i. C'-irreducibility is therefore a sufficient condition for the existence of equilibrium in economies with weakly monotone preferences and boundary endowments. This is the content of the following result.

Theorem 2.9 (Baldry and Ghosal (2005)). *If the economy E satisfies (bg.1)–(bg.6) and in addition is C'-irreducible, then a Walrasian equilibrium exists for E.*

Proof. Baldry and Ghosal (2005; p. 953). □

2.3.4.6. *Oligarchy and self-sufficiency*

Danilov and Sotskov (1990) also provide interesting extensions of the basic idea of McKenzie-irreducibility. Their approach begins with the following definition.

Definition 2.23 (Oligarchy and self-sufficiency). In an exchange economy, a *feasible allocation* $x = (x_i)$ is an assignment of a vector of goods to each i such that for all i, x_i is in X_i and $\sum_i x_i = \sum_i \omega_i$. If I^1 is a proper subset of I (the set of all consumers), then the complementary group of consumers is $I \backslash I^1$. The welfare of the members of I^1 may be *strictly improved* relative to a feasible allocation x, if there is another feasible allocation x' such that $x_i \prec_i x'_i$ for all i in I^1 and may *improve* if $x_i \preceq_i x'_i$ for all i in I^1. The group I^1 is said to be a *weak oligarchy* at allocation x if the group $I \backslash I^1$ cannot strictly improve the condition of the members of I^1 at x and is a *strong oligarchy* if $I \backslash I^1$ cannot improve the condition of the members of I^1 at the allocation x. The group I^1 is *self-sufficient* if the members of I^1 place no value on the goods held by $I \backslash I^1$ and is *super–self-sufficient* if in addition there is a consumer in I^1 who owns a good not valued by any member of I^1.

Remark 2.21. As Danilov and Sotskov show, if the set of agents is I and the set of goods is L, then a *market* (*or exchange economy*) is a finite set of agents goods, preferences and endowments $\omega_i \in \Re^\ell$ for each $i \in I$. If the preferences of each agent with respect to each good are either strictly monotonic or indifferent so that preferences can be represented by, for example, a linear utility function as in Gale (1976), then an oriented graph Γ can be assigned to a market with a set of vertices $I \cup L$. An arrow is oriented from $i \in I$ to $d \in L$ if the good d is *desired* by agent i, meaning that $\preceq_i$ is strictly monotonic with respect to x_d and from $d \in L$ to $i \in I$ if i owns d so that $\omega_{\text{id}} > 0$. A subset of the market $\Pi \subset \Gamma$ is called *self-sufficient* if there is no arrow in Γ going out from Π, and a pair of subsets $A \subset I$ and $S \subset L$ is called *super–self-sufficient* if it is self-sufficient and in the endowments of agents in A there are goods useless to all $i \in A$ but desirable for some agent in $I - A$.

Theorem 2.10 (Danilov and Sotskov (1990)). *If an exchange economy* E *satisfies*: $(ds.1)$ $\forall i, X_i = \Re^\ell_+$ *and* $\preceq_i$ *is either strictly monotonic or indifferent for each good* $d \in L$; $(ds.2)$ $\forall i$, *there is at least one desirable good*; $(ds.3)$ $\preceq_i$ *are given, convex and continuous in some neighborhood of* $\Re^\ell_+$; $(ds.4)$ *there is no super–self-sufficient pair, then a Walrasian equilibrium exists for* E.

Proof. Danilov and Sotskov (1990; p. 346). □

Remark 2.22. If I^1 is self-sufficient or oligarchic at the allocation x, then the other group $I \backslash I^1$ has nothing to offer the members of group I^1, at least nothing which given their preferences they are interested in. One group is then irrelevant to the other and the economy is in a sense reducible. Gale (1976; p. 207) shows that in an exchange economy where utility functions are linear, the presence of any super–self-sufficient group will prevent the existence of a market equilibrium. Danilov and Sotskov (1990) have also generalised this result to the production economy case: Hammond (1993) shows that self-sufficiency and oligarchy are equivalent notions in the particular economy considered by Gale (1976) but that when utilities are non-linear, oligarchy is a more general condition. Notice that as with the other conditions so far considered, these conditions require a particular relationship to hold between the endowments of one group of consumers and the preferences of another. While it *may* happen for a given economy that this is indeed the case, there are no theoretical reasons why it should be so and again, the hypothesis that there is no super–self-sufficient pair or no oligarchy in the economy cannot be so supported on theoretical grounds. Considering that the hypothesis of no super–self-sufficient pair is both necessary and sufficient for the existence of market equilibrium in the preference and technology environment which Danilov and Sotskov (1990) consider, we may reasonably question the generality of the resulting existence theorem. Hammond (1998; p. 239) further develops the idea of oligarchic and non-oligarchic allocations as follows. A proper subset H of the set of consumers I is an *oligarchy* at the feasible allocation (x, y) provided that there is no feasible alternative allocation (x', y') such that $x_i \prec_i x'_i$ for all $i \in H$. As Hammond puts it: "... when H is an oligarchy, it monopolises resources to such an extent that no redistribution of resources from outside H could possibly bring about a new allocation making all the members of H better off." Hammond (1998; p. 239). As is clear from its statement, this absence of oligarchy in the economy requires that at least to some degree, everyone in the economy is relevant to everyone else. In a related discussion of the condition in Hammond (1993), it is noted that it is related to McKenzie-irreducibility.

2.3.4.7. *Independence and interdependence*

In a further generalisation of the basic notion of irreducibility Hammond (1993) starts with the idea of oligarchy and develops that into a condition which he calls 'generalised interdependence,' which turns out to be equivalent to a desired notion of generalised irreducibility. His notions may be stated as:

Definition 2.24 (Independent and interdependent). A coalition I^1 is *independent* at x if there is no weakly Pareto superior allocation in which I^1 would benefit from access to additional resources from 'outside' the economy, where those resources are to be replicas of those held in the initial endowment of the members of the complementary group $I \backslash I^1$. The set of agents I is then said to be *interdependent* at a feasible allocation x if there is no proper subset I^1 of I that is independent at x. All agents are said to be *generalised interdependent* at a feasible allocation x if and only if for any partition I^1, I^2 of I, there exists an individual in group I^1 who could be made better off if he or she had access to a replica of the resources initially owned by the complementary coalition I^2, plus access to a replica of his or her own net trade vector or the net trade vector of any other agent in the economy.

Theorem 2.11 (Hammond (1993)). *If in* E (*h*.1) *each* X_i *is convex*; (*h*.2) *the lower contour set* $L_i(x_i) = \{y \in X_i : y \preceq_i x_i\}$ *is closed*; (*h*.3) $0 \in \Re^\ell$ *belongs to* K_i *the convex hull of the set* $X_I = \sum_{i \in I} X_i$. *Then if* (x^*, p) *is a Walrasian quasi-equilibrium in which all agents are generalised interdependent*, (x^*, p) *is a Walrasian equilibrium for* E.

Proof. Hammond (1993; p. 109). □

Remark 2.23. Hammond (1993, pp. 102–103) shows that generalised interdependence is equivalent to generalised irreducibility, and having developed these definitions, he shows that provided preferences satisfy a mild continuity condition (his Assumption 1, p. 84 which is even weaker than (h.2), then Walrasian equilibrium exists if all agents satisfy the condition of generalised interdependence. From our point of view, it is worth noting that this, along with the other existence results noted in this survey, again depends on particular, theoretically arbitrary relationships holding between distinct objects in the economy, namely endowments and preferences, relationships for which there is in general no theoretical foundation. Florig (2001) has recently proposed a definition of irreducibility which is a combination of Hammond's notion of generalised irreducibility and Bergstrom's irreducibility notion. Since the details of Florig's notion of irreducibility are similar to the notions treated already, it will not be gone into detail. However, since Florig-irreducibility is in certain circumstances necessary for existence, it will be discussed again in the next chapter and also again in Chap. 5.

2.3.4.8. *Indecomposable economies*

Reiterating a by now familiar theme, Moore (2005) observes that: "There are numerous results in the general equilibrium... literature where investigators have been looking for conditions sufficient [for] the existence of a Walrasian (competitive) equilibrium... Moreover, while authors very often state simple conditions sufficient to ensure that the quasi-competitive equilibrium obtained will actually be a Walrasian equilbrium, these simple conditions are typically patently unrealistic." Moore (2005; pp. 345–346) Motivated by this situation, he develops several conditions which allow the passage from quasi-equilibrium to equilibrium, the most general of which is 'indecomposability' a concept which is another generalization of McKenzie-irreducibility (see in particular Moore (2005; pp. 355–356)).

Definition 2.25 (Indecomposable). E is *indecomposable* at the allocation (x_i^*, y_j^*) if given any partition of consumers into two groups I^1 and I^2, there exists another allocation $(x_i', z_i') \in X_i \times Z_i$ (where Z_i is the production possibility set for consumer i), for each $i \in I$ and $\mu_i > 0$ for each $i \in I^2$ and $\hat{y} \in AY$, such that: $\sum_{i \in I1}(x_i' - \omega_i - z_i') = \Sigma_{i \in I2}(\omega_i + z_i' - x_i') + \hat{y}$ and $\forall i \in I^1 x_i^* \prec_i x_i'$, (where AY is the asymptotic cone of Y). The economy is *globally indecomposable* if it is indecomposable at each attainable allocation.

Given this idea Moore is able to prove:

Theorem 2.12 (Moore (2005)). *If* $((x_i^*), (y_j^*), p^*)$ *is a Walrasian quasi-equilibrium for* E, *and if* $(m.1)$ E *is indecomposable* *at* $((x_i^*), (y_j^*))$, $(m.2)$ $int(X) \cap [\omega + Y] \neq \varnothing$ *then* $((x_i^*), (y_j^*), p^*)$ *is a Walrasian equilibrium for* $\boldsymbol{E}$.

Proof. Moore (2005; pp. 356–357). □

Remark 2.24. Moore (2005) argues that this condition generalises the 'irreducibility' condition introduced in McKenzie. In interpreting the condition, he observes that it says that: "...given any attainable allocation in the economy, and any coalition, $I^1 \neq I$, the coalition could improve upon the given allocation for each of its members if they were allowed to choose amounts to be given up by a coalition consisting of replicas (possibly fractional) of the consumers not in I^1 and add in a production vector from AY." Moore (2005; p. 355).

All of the conditions studied here, starting with interior endowments, through various forms of irreducibility and up to indecomposability are all ingenious responses to the basic problem identified by Arrow and Debreu (1954) and elaborated by Debreu (1962, 1998). What is particularly interesting about these conditions is the *economics* that they suggest. In particular their lesson that unless the economy is 'tied together' in an appropriate way, equilibrium states — at least as they are thought of in the Walrasian tradition — generally may not exist.

2.4. Conclusion

Debreu (1998) suggests a criterion by which the reasonableness or otherwise of an equilibrium hypothesis could be judged. In particular he argued that: "If the model that has been specified requires strong assumptions to guarantee the existence of an equilibrium price vector, the explanatory power of the model will be low. In order to evaluate the model, a basic question must, therefore, be answered in the form of axioms that make it possible to prove an existence theorem." Debreu (1998; p. 21).

Motivated by this remark, a study of an important class of conditions which underpin existence theorems for Walrasian equilibrium has been undertaken here.

The major points to emerge from this work are these: (i) many of the conditions imposed by early workers, particularly conditions on consumer preferences, can be significantly weakened; (ii) some of the restrictive assumptions imposed on production can also be relaxed; and (iii) conditions imposed in order to ensure the non-emptiness of budget sets and continuity of demand responses are now apparently more general and plausibile than those imposed by earlier workers. For instance the obviously unrealistic assumption of interior endowments employed in Arrow and Debreu (1954), has been replaced by a variety of conditions which aim one way or another to ensure that the economy does not fragment and is 'irreducible'. While it is true that these conditions are weaker than the condition of interior endowments, it is also fair to say that all of these conditions are of quite a different character to the other hypotheses which typically appear in existence theorems. This importantly conditions existing arguments because there is nothing apparent in the structure or operation of an economy which ensures that such relationships between preferences, technologies and endowments will indeed hold.

Of course it may be the case that the sufficient conditions for existence which we have so far considered make unnecessarily strong demands in order to achieve their ends. It might therefore be that our critique is misplaced because these conditions can be significantly weakened without causing problems for the existence argument. In the next chapter, we examine necessary conditions for existence and show that such significant weakening of the conditions identified here is not in fact generally possible.

Chapter 3

EXISTENCE OF EQUILIBRIUM: NECESSARY CONDITIONS

"*The principal participants in the economy are consumers. The ultimate purpose of the economic organisation is to provide commodity vectors for final consumption by consumers.*"

K. Border

3.1. Introduction

A standard approach when faced with a difficult problem such as the existence problem for Walrasian equilibrium is to specify a set of conditions sufficient for its solution and to then rely on subsequent research to find ways to refine those conditions in the hope of revealing necessary and sufficient conditions for its solution. As was seen in the previous chapter, Arrow and Debreu (1954) provided a set of sufficient conditions under which Walrasian equilibrium exists. Subsequent work has relaxed a number of their original conditions — particularly those placed on preferences and on the aggregate production set. In this chapter, we follow that work in the direction of identifying necessary, as well as necessary and sufficient conditions for the existence of Walrasian equilibrium. The identification of such conditions is important because if found, they would provide a particularly sharp focus on what is needed (as opposed to what is sufficient) for the existence of Walrasian equilibrium. This could be very informative in at least the following sense. If a set of conditions 'X', is necessary for existence 'E' (so that $E \Rightarrow X$), then an application of *modus tollens* gives us that 'not X' entails 'not E'. The importance of this is that if necessary conditions for existence can be identified, a potentially sharp test of the equilibrium hypothesis can be obtained by considering the plausibility of necessary conditions for existence.

The search for necessary (as well as necessary and sufficient) conditions for existence, begins with a little-noticed remark in Arrow and Debreu (1954), which points to a necessary condition and 'important principle' for the existence of Walrasian equilibrium. Consideration of this remark culminates in the result due to Florig (2001), which establishes that in an economy where preferences do not depend on prices, Florig-irreducibility is necessary for the existence of Walrasian equilibrium. A series of conditions that have been shown to be necessary and sufficient conditions for the existence of Walrasian equilibrium are then studied. In particular, we consider the conditions which one way or another limit 'arbitrage opportunities'. As Page, Wooders and Monteiro (2000) note, conditions limiting arbitrage fall into three broad categories: (i) conditions on net trades, as in Hart (1974), Page (1987), Nielsen (1989), Page and Wooders (1993, 1996), Allouch (1999); (ii) conditions on prices, as in Green (1973), Grandmont (1977, 1982), Hammond (1983), and Werner (1987); (iii) conditions on the set of utility possibilities, particularly compactness as in Brown and Werner (1995), Dana *et al.* (1999). The details of these conditions will be studied below. One interesting feature of all these conditions is that the existence of equilibrium will depend on the right degree of 'diversity' of the economy.

3.2. A necessary condition for existence

In the course of commenting on the interior endowments assumption, Arrow and Debreu (1954) draw attention to a necessary condition for the existence of Walrasian equilibrium when they remark that: " ... [the interior endowments] assumption is clearly unrealistic. However, *the necessity of this assumption or some parallel one* for the validity of the existence theorem points up an important principle: to have equilibrium, it is *necessary* that each individual possess some asset or be capable of supplying a labour service which commands a positive price at equilibrium." Arrow and Debreu (1954; p. 270, emphasis added). It is worth studying this remark and some of its implications. Perhaps the first thing to notice is that if in order for equilibrium to exist, it is necessary 'each individual possess some asset or be capable of supplying a labour service which commands a positive price at equilibrium', then the failure of any individual to have a labour service or asset type that has a positive price at equilibrium means that equilibrium does not exist. An interesting question then arises as to whether there are processes at work in a standard private ownership economy that endogenise such circumstances? A second thing to notice is that the Arrow-Debreu restriction is, in a sense, strictly necessary for the existence of Walrasian equilibrium because it is possible to construct economically meaningful situations in which the condition is satisfied but in which a consumers budget set is empty. To see this, suppose that that $X_i = \text{int}\,\Re^2_+$, ξ_{i1} and ξ_{i2} represent the absolute minimum amounts of goods 1 and 2 needed by i contingent on large amounts of the other good being available and X_i represents the feasible combinations of goods 1 and 2 for i. Such a situation is illustrated in Fig. 3.1.

Thus while Arrow and Debreu (1954) are correct in their observation about the necessity of this sort of condition, in order for a consumer to have a non-empty budget set, not only does the equilibrium price have to be positive, but the price along with the amount of a labour service, good or asset that an individual holds has to be a high enough positive number to ensure a non-empty budget set. We now formulate the Arrow-Debreu remark

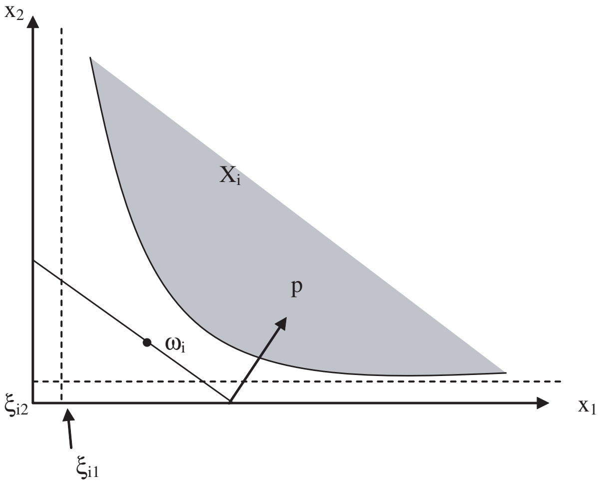

Fig. 3.1. Empty budget set with $p \gg 0$.

in the presence of general consumption sets. $B_i = \{x_i \in X_i : px_i \leq p\omega_i + \Sigma_j \theta_{ij} py_j\}$ is the *budget set for* i and it is equal to the intersection of the *purchasable set for consumer* i where $P_i = \{x_i \in \Re^\ell \colon px_i \leq p\omega_i + \Sigma_j \theta_{ij} py_j\}$ and the *consumption possibility set* X_i, i.e. $B_i = P_i \cap X_i$. For simplicity let $M_i = p\omega_i + \Sigma_j \theta_{ij} py_j$. Let the *distance* between two sets X and Y be $\inf\{|x - y|$ for $x \in X$ and $y \in Y\}$. Then we can prove the following results.

Lemma 3.1. *If* $((x_i^*), (y_j^*), p)$ *is a Walrasian equilibrium, then* $B_i \neq \varnothing$ *for all consumers.*

Proof. If $((x_i^*), (y_j^*), p)$ is a Walrasian equilibrium, then by definition, each x_i^* is a maximal element in B_i relative to $\preceq_i$. But if B_i has a maximal element for each consumer i, then $B_i \neq \varnothing$ for all consumers i. □

Lemma 3.2. B_i *is not empty if and only if the distance from* P_i *to* X_i *is zero.*

Proof.

(i) $\Rightarrow$: If $B_i \neq \varnothing$ then $P_i \cap X_i \neq \varnothing$. Therefore, $\exists z \in (P_i \cap X_i) \Leftrightarrow \exists y \in P_i$ and $x \in X_i$ and $x = y$. Therefore, $\inf\{|x-y|\} = 0$.

(ii) $\Leftarrow$: If $\text{dist}(P_i, X_i) = 0$ then $\inf\{|x-y|, x \in X_i \text{ and } y \in P_i\} = 0$. Therefore, $\exists y \in P_i$ and $x \in X_i$ such that $\inf\{|x-y|\}$, i.e. $x = y$. Therefore, $P_i \cap X_i \neq \varnothing \Leftrightarrow B_i \neq \varnothing$. □

Lemma 3.3. *If $X_i = \text{int}(\Re^\ell_+), p \in \Re^\ell_+$ and $B_i \neq \varnothing$ then $p\omega_i + \Sigma_j\theta_{ij}\ py_j > 0$. Further $p\omega_i + \Sigma_j\theta_{ij}tpy_j > 0 \Leftrightarrow p\omega_i > 0$ or $\Sigma_j\theta_{ij}py_j > 0$.*

Proof. If $x_i \in X_i$ then $x_i \gg 0$ because $X_i = \text{int}(\Re^\ell_+)$. If not all prices are zero then $px_i > 0$ for $x_i \in X_i$. Therefore, if $B_i \neq \varnothing$ then $\exists m \in \Re$ such that $px_i \leq m$, where $m = p\omega_i + \Sigma_j\theta_{ij}py_j$. The proof of the last part of the Lemma is obvious. □

Proposition 3.1. *If $\{(x_i^*), (y_j^*), p^*\}$ is a Walrasian equilibrium for an economy with $I > 0$ consumers, $J > 0$ firms and $\ell > 0$ commodities in which* (1)$X_i \subseteq (\text{int}\,\Re^\ell_+)$ *and* $X_i \neq \varnothing$; (2) $0 \in Y_j$ *for all* j; (3) *free disposal holds*; (4) *no consumer is satiated*, (c4) *not all goods needed for survival by i are free, then i must be in possession of an asset or labour type which earns a positive price in equilibrium or must have shares in a firm which earns a positive profit at equilibrium.*

Proof. In a Walrasian equilibrium at least one price is non-zero. From (3) no equilibrium prices can be negative. Therefore there exists a good $k \in [1, \ell]$ such that $p_k^* > 0$. Either $k \in X_i$ or $k \notin X_i$. (a) If $k \in X_i$ then $p_k^* x_k^* > 0$ because $X_i \subseteq (\text{int}\,\Re^\ell_+)$ and $X_i \neq \varnothing$. By (4) no consumer is satiated, the budget constraint is satisfied by every i, so $0 < p_k^* x_k^* \leq p^*\omega_i + \sum_j \theta_{ij} p^* y_j^*$. Therefore either $p^*\omega_i > 0$ or $\sum_j \theta_{ij} p^* y_j^* > 0$, or both. In the first case $(p^*\omega_{i1} + p^*\omega_{i2} + \cdots + p^*\omega_{i\lambda}) > 0$ and since $\omega_{ik} \in \Re$ for all $k = 1, \ldots, \ell$ there must be at least one commodity $k \in [0, \ell]$ such that $p_k^* > 0$. In that case i is in possession of at least one asset or one labour type which earns a positive price at equilibrium. Alternatively, suppose that $p^*\omega_i = 0$. Then for the budget constraint to be satisfied, it must be the case that $(\theta_{i1}p^*y_1^* + \theta_{i2}p^*y_2^* + \cdots + \theta_{ij}p^*y_j^*) > 0$. Since all firms maximise profit and $0 \in Y_j$ for all j, $p^*y_j^* \geq 0$ for all j. In this case it must be that i is in possession of the shares of at least one firm j for which $\theta_{ij}p^*y_j^* > 0$. If $k \notin X_i$ and if all the goods in X_i are free, then for $x_i \in X_i$, $0 = p^*x \leq p^*\omega_i + \sum_j \theta_{ij}p^*y_j^*$. In this case i is able to survive at equilibrium without owning an asset or labour type which earns a positive price at equilibrium. However, if at least one good needed for survival by i is not free then $\exists k \in X_i$ such that $p_k^* > 0$ and the earlier argument applies. □

Remark 3.1. The point being made in the original Arrow-Debreu remark and in the results above which elaborate it, is that the ownership by each agent in the economy of at least one asset or labour type which earns a positive price at equilibrium is necessary for the existence of Walrasian equilibrium, unless all the goods needed for survival are free. We have drawn attention to the Arrow-Debreu remark because it does not seem to have been widely acknowledged in the literature and because a necessary condition such as this contains important information about the economics around the existence of Walrasian equilibrium.

The importance of Arrow-Debreu's necessary condition is that it raises the following important question: Is there any natural mechanism or natural structure in a private ownership economy that guarantees the Arrow-Debreu necessary condition for the existence of Walrasian equilibrium will be satisfied? If there is no obvious mechanism at work in the economy which guarantees that all consumers have assets or labour types at all, then there is no guarantee that prices will be positive for the assets or labour types which people actually hold. However, since such a situation is identified by Arrow and Debreu (1954) as being necessary for the existence of Walrasian equilibrium, it seems unreasonable to claim, as is sometimes done (recall the discussion in Chap. 2), that theorems which assert the existence of Walrasian equilibrium are 'quite general' and 'require very little structure'. On the contrary, on the basis of the work we have done, it seems that the economy requires quite a lot of rather specific structure in order for Walrasian equilibrium to exist. Notice the argument being advanced here is not that the conditions necessary for the existence of Walrasian equilibrium *cannot* obtain in a given economy. Of course they can and nothing like a general impossibility result is being claimed here. The argument is however, that there is no mechanism operating in a private ownership economy which guarantees that each individual will be able to satisfy Arrow and Debreu's necessary condition for the existence of Walrasian equilibrium. Further, the conditions known to be sufficient for the task (interior endowments, resource relatedness, and various forms of irreducibility) are potentially fragile relationship conditions which are not guaranteed to hold in an arbitrary private ownership economy. Consequently, to claim that Walrasian equilibrium exists under very general circumstances might be a leap of faith not entirely justified by a careful analysis of what is actually involved in achieving such an outcome.

One condition on the structure of the economy which is necessary for the existence of Walrasian equilibrium, at least in the case where preferences do not depend on prices is the condition that the economy be Florig-irreducible, a notion which is discussed further in the Chap. 5. The result which establishes its necessity for existence is the following.

Theorem 3.1 (Florig (2001)). *If E is such that the aggregate production set Y and each X_i is convex, preferences are convex and defined for each (x, y) feasible for E then if E is not Florig-irreducible then E has a quasi-equilibrium (x, y, p) such that for some consumer $B_i(p) = \varnothing$.*

Proof. Florenzano (2003; p. 68). □

Remark 3.2. The importance of this result is that if a test could be devised for Florig irreducibility, perhaps in the style of Maxfield (1997), then a necessary condition for existence could be tested. A negative result of such a test would be particularly informative.

3.3. Necessary and sufficient conditions for existence

The non-emptiness of each consumers budget set is a necessary condition for the existence of Walrasian equilibrium. As we have seen, for such a condition to hold across an economy, some potentially restrictive relationship conditions must be satisfied in the economy. However, even if the non-emptiness of each consumers budget set is guaranteed, things can still 'go wrong' as far as an existence argument is concerned. Recall the two person economy studied by Gale (1976), Danilov and Sotskov (1990), and studied by Florig (2001) and discussed in the previous chapter. Another interesting example is presented by Chichilnisky (1995) and illustrated in Fig. 3.2.

In this example, $X_i = \Re^{\ell}_{+}$ for $i = 1, 2$, the preferences of person 1 are 'standard' and those of person 2 are lexicographic in good 1 and $\omega_1 \in X_1$ and $\omega_2 \in \operatorname{int} X_{3.}$ At any price vector in which the price of at least one good is positive, each consumer has a non-empty budget set. Given the way preferences and endowments are arranged in this economy, when the price of good 2 is positive, both consumers want to supply good 2 and demand good 1, at say $x_1^1(p)$ and $x_2^1(p)$ in Fig. 3.2. Therefore, $p = (p^1, p^2)$ with $p^2 > 0$ is not a Walrasian equilibrium. The only candidate for equilibrium is the price vector $p^1 > 0$ and $p^2 = 0$, (since $p = 0$ is not a candidate by definition). Such a

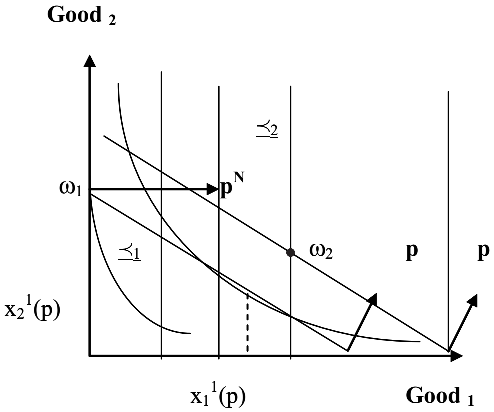

Fig. 3.2. Chichilnisky (1995) example with no Walrasian equilibrium even though budget sets are not empty.

price vector looks like p^N in Fig. 3.2. But at this price, demand for good 2 by person 1 diverges to infinity, consequently this economy does not have a Walrasian equilibrium. Thus, while the budget sets are non-empty for each consumer (a necessary condition for equilibrium), it is not sufficient. The relationship between preferences and endowments across people also has to be 'well behaved', and it is to this issue that we now turn.

3.3.1. *W-arbitrage*

In a study of necessary and sufficient conditions for the existence of Walrasian equilibrium, the best place to start is Werner (1987) who calls a commodity bundle x' *useless* for consumer i if $u_i(x+x') = u_i(x) = u_i(x-x')$, while x' is *useful* for i if $u_i(x+x') \geq u_i(x)$ and x' is not useless. Denote by W_i the set of commodity bundles that are useful for i, and by Q_i the set of commodity bundles that are useless for i. A price system $p \in \Re^\ell$ is *viable* for consumer i if their set of utility maximising demands relative to their budget set at $p \in \Re^\ell$ is not empty. The set of all viable price systems for i, (which is just the domain of the demand function for i) is denoted by D_i. A price system $p \in \Re^\ell$ is a *W-nonarbitrage price system* for i if every commodity bundle that is useful to i has a positive market value. Thus p is W-nonarbitrage if and only if $\forall x' \in W_i$, $px' > 0$. The set of W-nonarbitrage prices for i is denoted by S_i. The set of nonarbitrage prices for E is the set $S = \cap_{i \in I} S_i$ and is the set of prices that admits no W-arbitrage opportunities for any consumer in the economy. Given these notions, Werner (1987) proves the following result:

Theorem 3.2 (Werner (1987)). *If* E *is such that* $(w.1)$ $\forall i, X_i$ *is nonempty, convex and closed*; $(w.2)$ $\preceq_i$ *is a continuous and complete preference ordering on* X_i; $(w.3)$ *every utility function is concave or, more generally the 'preferred to' sets for any* $x_i \in X_i$ *are convex and have the same recession cone*; $(w.4)$ *there exists a commodity bundle that is useful for* i, *i.e.*, $W_i \neq \varnothing$; *then* (i) *every price system that admits no arbitrage opportunity for* i *is viable so* $S_i \subset D_i$; (ii) *if there is no satiation consumption in* X_i *then* $D_i \subset clS_i$ *and if the indifference curves of* u_i *do not contain any half-lines, then* $S_i = D_i$; (iii) *if* X_i *is bounded below then* $S_i \supset \Re^\ell_{++}$. $(w.5)$ *for all consumers* $i, p\omega_i > \inf pX_i$ *for all* $p \in clS\backslash\{0\}$, *then a sufficient condition for the existence of equilibrium in* E *is that* $S \neq \varnothing$, *so that there is a price system which admits no W-arbitrage opportunity for all consumers.*

Proof. Werner (1987; pp. 1412–1414). □

Remark 3.3. In (w.5) Werner assumes that the 'cheaper point' condition is somehow satisfied for every consumer (e.g. because consumers have interior endowments, or the economy is irreducible or because some other relationship conditions holds). Having gotten to that point, what is needed for existence is that *another* relationship condition holds namely that $S = \cap_{i \in I} S_i \neq \varnothing$, so that the sets of non-arbitrage prices for each consumer are in the 'right places' relative to each other, ensuring that their mutual intersection is not empty. This feature of Werner's theorem, and the other theorems of this type studied in this section strongly reinforces and support our basic contention that the existence of Walrasian equilibrium depends on some quite specific relationships holding between the primitives which define the economy.

3.3.2. *No unbounded arbitrage*

Page and Wooders (1996) provide a refinement of Werner's conditions in the case of an Arrow-Debreu exchange economy. The central notions in their work are the ideas of an 'arbitrage cone', no 'unbounded arbitrage', and a 'reconcilable economy'. The ith agents *arbitrage cone* is the closed convex cone containing the origin and given by: $CP_i = \{y \in \Re^\ell : \exists x \in X_i,$ $(x + \lambda y) \in X_i$ and $u_i(x + \lambda y)$ is non-decreasing in λ for $\lambda \geq 0\}$. The *increasing cone for* $iI_i\ (x) = \{y \in CP_i\colon \forall\lambda \geq 0\ \exists\lambda' > \lambda$ such that $u_i(x + \lambda' y) > u_i\ (x + \lambda y)\}$ and an economy satisfies the condition of *no unbounded arbitrage* if whenever $\sum_{i\in I} y_i = 0$ and $y_i \in CP_i$ for all i, then $y_i = 0$ for all i. The ith agent satisfies *extreme desirability* if for any $x \in X_i$, it is true that $I_i(x) = CP_i\backslash\{0\}$. The *economy satisfies extreme desirability* if at least all but one of the agents preferences satisfy extreme desirability. The set of *individually rational allocations* is given by the set: $A = \{(x_1, \ldots, x_n) \in X_1 \times X_2 \times \cdots \times X_n\colon \sum_{i\in I} x_i = \sum_{i\in I} \omega_i$ and $u_i(x_i) \geq u_i(\omega_i)\}$. An economy is *reconcilable* if for every consumer i, u_i is continuous, concave and is non-satiated at rational allocations. An economy is *strictly reconcilable* if it is reconcilable and satisfies extreme desirability.

Theorem 3.3 (Page and Wooders (1996)). *If* $(pw.1)$ *E is a strictly reconcilable exchange economy*; $(pw.2)$ *for every* i $\omega_i \in \operatorname{int} X_i$ *then* $(pw.3)$ E *satisfies the condition of no unbounded* *arbitrage is necessary and sufficient for the existence of Walrasian equilibrium in* $\boldsymbol{E}$ *.*

Proof. Page and Wooders (1996; p. 151). □

Remark 3.4. Commenting on their work, Page and Wooders (1996) observe that the source of non-existence of a Walrasian equilibrium, once non-emptiness of the budget set has been taken care of in (pw.2), is that the preferences of agents may be 'too dissimilar to be reconciled by price'. The notion of a reconcilable economy in (pw.1), coupled with no unbounded arbitrage works to limit the diversity of agents (see Page and Wooders (1996; p. 155)) and ensures the existence of Walrasian equilibrium. The need to *bound the diversity of agents*, in particular to impose particular relationships between the preferences and endowments of agents so that agents are not too dissimilar, is a constant and important theme in this literature. The role and nature of condition (pw.2) is by now well understood and will not be commented on further. There is however, a new relationship condition introduced by Page and Wooders in the requirement that people be 'similar enough' in terms of their preferences and endowments for markets to be able to work that is worth emphasising. It is not obvious that there are forces at work in the economy to guarantee that this additional set of relationship conditions will be satisfied in an arbitrary economy.

3.3.3. *Limited arbitrage*

Chichilnisky (1997a, 1997b, 1997c) has contributed necessary and sufficient conditions for the existence of Walrasian equilibrium, conditions which are built on the following ideas. The *space of allocations* is $X^I = \{(x_1, \ldots, x_I) \in \Re^{\ell I} : x_i \in X_i\}$; the *space of feasible allocations* is $\Gamma = \{(x_1, \ldots, x_I) \in X^I : \sum_{i \in I} x_i = \omega\}$. The *set of supports to individually rational and affordable efficient allocations* $S(E)$, is the set of prices which support those feasible allocations which all individuals prefer to their initial endowments. Thus, $S(E) = \{v \in \Re^\ell$: if $(x_1, \ldots, x_I) \in \Gamma$, with $vx_i = v\omega_i$ and $u_i(z_i) \geq u_i(x_i)$ then $v \cdot (z_i - x_i) \geq 0\, \forall z_i \in X_i$ and $\forall i = 1, \ldots, I\}$. An element v of $S(E)$ is a *support* for the allocation $(x_1, \ldots, x_I)$ and is the set of prices which supports those feasible allocations which all individuals prefer to their initial endowments. The *set of prices orthogonal to the endowments* is $N = \{v \in \Re^\ell_+ \backslash \{0\} : \exists i \in I$ s.t. $v \cdot \omega_i = 0\}$. $N = \varnothing$ if $\forall i \omega_i \gg 0$, i.e. if the interior endowment assumption holds. For agent i, define the cone of directions $A_i(\omega_i)$ along which utility increases without bound as $A_i(\omega_i) = \{x \in X_i : \forall y \in X_i, \exists \lambda > 0$ s.t. $u_i(\omega_i + \lambda x) \geq u_i(y)\}$. When augmented by the part of its boundary along which utility never ceases to increase, $A_i(\omega_i)$ defines the *global cone* $G_i(\omega_i) = \{x \in X_i$ and $\neg\exists \max_{\mu \geq 0}\ u_i(\omega_i + \mu x)\}$. In the case where $X_i = \Re^\ell$ the *market cone for agent i* is $D_i(\omega_i) = \{x \in \Re^\ell : \forall y \in G_i(\omega_i)$ the inner product $x \cdot y > 0\}$, so that $D_i(\omega_i)$ is the convex cone of prices assigning positive value to all directions in $G_i(\omega_i)$. In the case where $X_i = \Re^\ell_+$ the *market cone for agent i* is $D_i^+(\omega_i) = D_i(\omega_i) \cap S(E)$ if $S(E) \subset N$ and $D_i(\omega_i)$ otherwise. The economy has *C-limited arbitrage* if there is a common price vector at which no trader can afford an unbounded increase in utility. In the case where $X_i = \Re^\ell$, then the economy satisfies limited arbitrage when $\cap_{i \in I} D_i(\omega_i) \neq \varnothing$ so that there is a non-empty intersection among all the individual market cones in the economy. In the case when $X_i = \Re^\ell_+$, the economy satisfies limited arbitrage when $\cap_{i \in I}\ D_i^+(\omega_i) \neq \varnothing$.

It is perhaps worth looking at a couple of examples that illustrate what is going on with the condition of C-limited arbitrage. In Fig. 3.2, the arrangement of preferences and endowments there shows a situation in which limited arbitrage is not satisfied and the economy fails to have a Walrasian equilibrium. Prices like p are not in $S(E)$ because they do not support individually rational, efficient and feasible allocations. In fact the only element of $S(E)$ is p^N which is the price vector orthogonal to the endowment for agent 1. But this price vector in $S(E)$ assigns to consumer 1, zero income. Since the market cone, $D_1^+(\omega_1)$, for agent 1: "…consists of all those supporting prices at which only limited increases in utility can be afforded from initial endowments." Chichilnisky (1995; p. 87). However, as can be seen from the diagram, there are no such supporting prices since at p^N, agent 1 can afford a boundless increase in utility by demanding an unbounded amount of good 2. Therefore $D_1^+(\omega_1) = \varnothing = D_1^+(\omega_1) \cap D_2^+(\omega_2)$, limited arbitrage fails and Walrasian equilibrium does not exist for this economy. Given these definitions, Chichilnisky (1997a) establishes the following result:

Theorem 3.4 (Chichilnisky (1997a)). *If E has $i \geq 2, \ell \geq 1, X_i = \Re^\ell_+$ with $\preceq_i$ and ω_i such that (c.1) for all $i, \omega_i \in \Re^\ell_+\backslash\{0\}$, (i.e. everyone has a non-zero endowment of at least one good); (c.2) the social endowment is strictly positive in each good so that $\omega = \sum_{i\in I} \omega_i \gg 0$; (c.3) for all $i, \preceq_i$ is convex, continuous and monotonically increasing and admits representation by a continuous utility function; (c.4) for all i, if an indifference surface corresponding to a positive consumption bundle x intersects a boundary ray $r \subset \partial X_i$ in $\Re^\ell_+$ (i.e. a set which consists of all positive multiples of any vector $v \in \partial\Re^\ell_+ : r = \{w \in \Re^\ell_+/\exists\lambda > 0$ s.t. $w = \lambda v\}$), then all indifference surfaces of bundles preferred to x intersect r; then, E will have a Walrasian equilibrium if and only if $\cap_{i\in I} D_I^+(\omega_i) \neq \varnothing$.*

Proof. Chichlinsky (1997a; p. 457). □

Remark 3.5. There are a number of points worth noticing about this theorem. Firstly, it allows the consumption possibility set to be the whole of $\Re^\ell_+$, an assumption which, as was noted earlier, has no minimal survival requirement built into it. Secondly, in (c.1) everyone is assumed to own a positive amount of at least one good. This rules out the possibility that $\omega_i = 0$ for some i. Thirdly, although the restriction on preferences in (c.3) and (c.4) are standard, there are, as Chichilnisky notes, circumstances in which it is quite restrictive. The main condition of interest in this theorem is however the requirement that $\cap_{i \in I} D_I^+(\omega_i) \neq \varnothing$ and it is the main 'relationship assumption' in the theorem. The condition plays the role of imposing a restriction on the diversity of agents in the economy. In particular, as Chichilnisky observes, when C-limited arbitrage fails a situation of 'social diversity' prevails and that means $\cap_i D_i^+ = \varnothing$.

If an economy is socially diverse, this means that endowments and/or preferences are sufficiently different so that there is no common price vector at which all trades which give unbounded utility increases are unaffordable for all the agents. Such a situation is reminiscent of the example due to Gale (1976) and the thinking in Page and Wooders (1996). Making this point Chichilnisky (1995) argues that: "... the source of the problem [of the non-existence of equilibrium] is ... the diversity of the traders, which leads to discontinuous demand behaviour at the potential market clearing prices, and prevents the existence of competitive equilibrium. The value of the condition of limited arbitrage is that it ensures that the problem does not arise ... [as] *it bounds the diversity of traders* precisely as needed for a competitive equilibrium to exist." Chichilnisky (1995; p. 81, emphasis added).

3.3.4. *DLVM-limited arbitrage*

Dana *et al.* (1999) survey various notions of 'arbitrage' that have been used in the literature on the existence of Walrasian equilibrium. The key definitions and result of that paper start in a familiar place: a commodity bundle $w \in \Re^\ell$ is *W-useful for i* if $\omega_i + tw \in X_i$ and $u_i(\omega_i + tw) \geq u_i(\omega_i)$ for all $t \geq 0$. The set of all W-useful commodity bundles for i is denoted by W_i. A bundle $w \in \Re^\ell$ is *C-useful for i* if $\omega_i + tw \in X_i$ for all $t \geq 0$ and if for each $x_i \in X_i$ there exists a $t > 0$ such that $u_i(\omega_i + tw) \geq u_i(x_i)$. The set of all C-useful commodity bundles for i is C_i. The cone C_i is called the *global cone* for agent i. W_i is the asymptotic cone of the set $R(\omega_i) = \{y_i \in X_i : u_i(y_i) \geq u_i(\omega_i)\}$. A price vector $p \in \Re^\ell$ is a *DLVM-no arbitrage price* for agent i if all W-useful commodity bundles have a positive cost at p, i.e. if p is a no arbitrage price then $\forall w \in W_i \backslash \{0\}$, $pw > 0$. The set of all no arbitrage prices for i is denoted by S_i. A price vector $p \in \Re^\ell$ belongs to the *market cone* of i if all C-useful commodity bundles have a positive cost at p, i.e. $pw > 0$, $\forall w \in C_i \backslash \{0\}$. The market cone for i is denoted by K_i. A price vector $p \in \Re^\ell$ is an *arbitrage free price* for i if for all sequences $\{x_n\} \in \Re^\ell : (\omega_i + x_n) \in X_i$, $\forall n, \lim_{n\to\infty} u_i(\omega_i + x_n) = u_i^*$, where $u_i^* = \sup_{xi \in Xi}, \lim_{n\to\infty} px_n$ exists, then $\lim_{n\to\infty} px_n > 0$. A price vector $p \in \Re^\ell$ is *viable for i* if the problem: $\max u_i(x)$ such that $x \in X_i$ and $px \leq p\omega_i$ has a solution. A price vector $p \in \Re^\ell$ is a *DLVM-no arbitrage price* for the economy $\boldsymbol{E}$ if $p \in \cap_{i \in I} S_i$. There is *no unbounded arbitrage* in $\boldsymbol{E}$ if $\sum_{i \in I} w_i = 0$ and $w_i \in W_i$ for all i means that $w_i = 0$. The allocation of W-useful goods is a *DLVM-unbounded arbitrage* if $w_i \in W_i$ for all i, $\sum_{i \in I} w_i = 0$ and $w_i \neq 0$ for some i. There is *DLVM-limited arbitrage* in the economy $\boldsymbol{E}$ if $\cap_{i \in I} K_i \neq \varnothing$.

Given these ideas, Dana *et al.* (1999) are able to develop the following existence theorem.

Theorem 3.5 (Dana *et al.* (1999)). *If* $\boldsymbol{E}$ *is such that* ($dlvm.1$) $\inf pX_i < p\omega_i$ *for all* i *and* $\forall p \in \cap_i clS_i \backslash \{0\}$; ($dlvm.2$) *all utility functions* u_i *are strictly quasi-concave*; ($dlvm.3$) *the upper contour sets of* u_i *are closed, i.e.* $\forall \alpha \in \Re\{x \in X_i : u_i(x) \geq \alpha\}$ *is closed*; ($dlvm.4$) *if* $x_i \in A_i$ (*where* A_i *the projection of the* *set of individually rational and attainable allocations A onto X_i) then the set $P_i(x_i) = \{y_i \in X_i : u_i(y_i) > u_i(x_i)\}$ is not empty*; ($dlvm$.5) *the sets* $clP_i(x_i) = \{y_i \in X_i : u_i(y_i) \geq u_i(x_i)\}$ *all have the same asymptotic cone*; ($dlvm$.6) $R_i(x_i) = \{x \in X_i : u_i(x) = u_i(x_i)\}$ *does not contain a half-line for any* $x_i \in X_i$; ($dlvm$.7) *for all i and for all $x \in A$, $P_i(x_i)$ is open relative to X_i; then the existence of Walrasian equilibrium in* E *is equivalent to any of the following*: (i) $\cap_i S_i \neq \varnothing$; (ii) *no unbounded arbitrage*; (iii) *of the set of individually rational and attainable allocations in the economy A, is compact*; (iv) *the individually rational utility set is compact*; (v) *a Pareto optimum exists. If in addition the economy satisfies*: (dmv.8) $P_i(x_i) \neq \varnothing$ *for all* $x_i \in X_i$ *so i has no satiation point and* $\lim_{t\to+\infty} u_i(\omega_i + tw) = u_i^*$ *for all* $w \in W_i\backslash\{0\}$ *and all i; then the existence of Walrasian equilibrium is equivalent to DLVM-limited arbitrage.*

Proof. Dana *et al.* (1999; p. 184). □

Remark 3.6. The Dana *et al.* result emphasises the role of relationship conditions in establishing the existence of Walrasian equilibrium, particularly through (dmv.1), (dmv.7) and (dmv.8), all of which are conditions of the sort encountered and discussed by us earlier. Their result therefore supports our basic contention about the nature of this part of the foundation on which existence theorems for Walrasian equilibrium rest.

3.3.5. *Inconsequential arbitrage*

Page *et al.* (2000) introduce the idea of *inconsequential arbitrage* which is a condition on net trades that "…ensures that arbitrarily large arbitrage opportunities are inconsequential from the viewpoint of existence of equilibrium". Page *et al.* (2000; p. 441). The authors then show that under any condition which is strong enough to ensure that all equilibrium allocations are

Pareto optimal, inconsequential arbitrage is necessary and sufficient for existence of a Walrasian equilibrium. This condition advances the literature because as the authors note, in order to show that a condition limiting arbitrage is necessary for existence of equilibrium, earlier work such as Werner (1987) and Page and Wooders (1993), required that there be no half-lines in indifference surfaces, or that the number of agents with half lines in their indifference surface was at most one as in Page and Wooders (1996).

The main existence result in Page *et al.* (2000) is built on the following ideas. Consider an exchange economy $\boldsymbol{E}$ in which for each i, $X_i \subset \Re^\ell$ and preferences are representable by a utility function $u_i : X_i \rightarrow \Re$ which allows 'strong and weak upper contour sets' to be defined as $P_i(x_i) = \{x \in X_i : u_i(x) > u_i(x_i)\}$, for strict preference and $\dot{P}_i(x_i) = \{x \in X_i : u_i(x) \geq u_i(x_i)\}$ for preference and indifference. The set of *individually rational allocations* relative to an endowment allocation $\omega = (\omega_1, \ldots, \omega_n)$ are given by $A(\omega) = \{(x_1, \ldots, x_n) \in X_1 \times X_2 \times \cdots \times X_n : \sum_i x_i = \sum_i \omega_i$ and $x_i \in \dot{P}_i(x_i), \forall i\}$ and as a matter of notation, $A_i(\omega)$ is the projection of $A(\omega)$ onto X_i. The ith consumer's *arbitrage cone at endowment* $\omega_i \in X_i$ is the closed convex cone in $\Re^\ell$ denoted $R(\dot{P}_i(\omega_i))$ and equal to the set $R(\dot{P}_i(\omega_i)) = \{y_i \in \Re^\ell$: for $x_i' \in \dot{P}_i(x_i)$ and $\lambda \geq 0$, $x_i' + \lambda y_i\}$. As Page *et al.* (2000) note, this definition makes consumer i's arbitrage cone at ω_i the recession cone corresponding to the weak upper contour set $\dot{P}_i(\omega_i)$. An *arbitrage* ω is an n-tuple of net trades $y = (y_1, y_2, \ldots, y_n)$ such that y is the limit of some sequence of points $\{\lambda^k x_1^k, \ldots, \lambda^k x_n^k\}$ with $\lambda^k \downarrow 0$ and $x^k = (x_1^k, \ldots, x_n^k) \in A(\omega)$ for all k. As a matter of notation, let $\text{arb}(\omega)$ denote all the arbitrages at ω and note that this is a recession cone relative to the set of all individually rational allocations $A(\omega)$ — see Page *et al.* (2000; p. 444 for details). Also denote by $\text{arbseq}^\omega(y)$ the set of all sequences $\{x^k\}_k$ of rational allocations such that $\lambda^k x^k \rightarrow y$ for some sequence $\{\lambda^k\}_k$ of positive numbers with $\lambda^k \downarrow 0$. An arbitrage

$y = (y_1, y_2, \ldots, y_n) \in \text{arb}(\omega)$ is in the *back-up set at* ω if for each sequence $\{x^k\}_k \in \text{arbseq}^{\omega}(y)$, $\exists \varepsilon > 0$ such that for all k sufficiently large and all consumer's i, $x_i^k - \varepsilon y_i \in X_i$ and $u_i(x_i^k - \varepsilon y_i) \geq u_i(x_i^k)$. The set of such arbitrages is denoted bus(ω). With these notions, Page *et al.* (2000) make the following definition.

Definition 3.2 (Inconsequential arbitrage). An arbitrage $y \in \text{arb}(\omega)$ is *inconsequential* and is contained in the back-up set at endowment ω, if for sufficiently large allocations $x \in A(\omega)$ in the $y = (y_1, y_2, \ldots, y_n)$ directions from ω, each consumer i can reduce their consumption by a small amount in the $-y_i$ direction without reducing their utility. The economy $\boldsymbol{E}$ satisfies *inconsequential arbitrage* at ω if $\text{arb}(\omega) \subseteq \text{bus}(\omega)$.

Example 3.1. Page *et al.* (2000) give the following useful examples and pictures of situations in which the inconsequential arbitrage condition holds and fails. These examples are interesting because they show that this condition depends on certain delicate relationships between the preferences and endowments of the agents in the economy. Consider first a two person, two good economy where agent 1 has $X_1 = \{(x_{11}, x_{12}) : x_{11} \geq 1$ and $x_{12} \geq 0\}$, Leontief preferences with the kink along the curve $\ln x$ for $x \geq 1$ and endowment $\omega_1 = (2, \ln 2)$. For agent 2, $X_2 = \{(x_{21}, x_{22}) : x_{21} \leq 0$ and $x_{22} \leq 0\}$, preferences are $u_2(x_{21}, x_{22}) = |x_{22}|$ and $\omega_2 = -\omega_1$. The economy can be pictured as in Fig. 3.3.

If the economy is now changed so that for agent 2, $X_2 = \{(x_{21}, x_{22}) : x_{21} \leq 0$ and $x_{22} \geq 0\}$ and $\omega_2 = (-2, \ln 2)$, a situation pictured in Fig. 3.4, then the economy does satisfy inconsequential arbitrage.

Given the idea of inconsequential arbitrage Page *et al.* (2000) get the following result.

Theorem 3.6 (Page *et al.* (2000)). *If* $\boldsymbol{E}$ *is such that* (*pwm*.1) u_i *is upper semicontinuous and quasi-concave for all* i; (*pwm*.2)

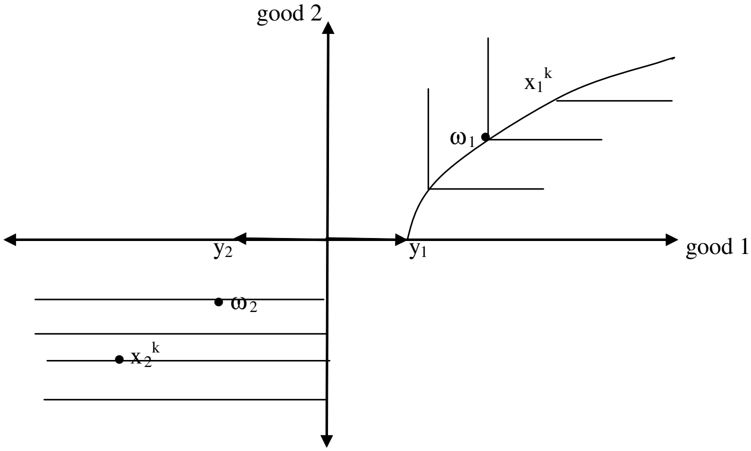

Fig. 3.3. Page *et al.* (2000) example of failure of inconsequential arbitrage.

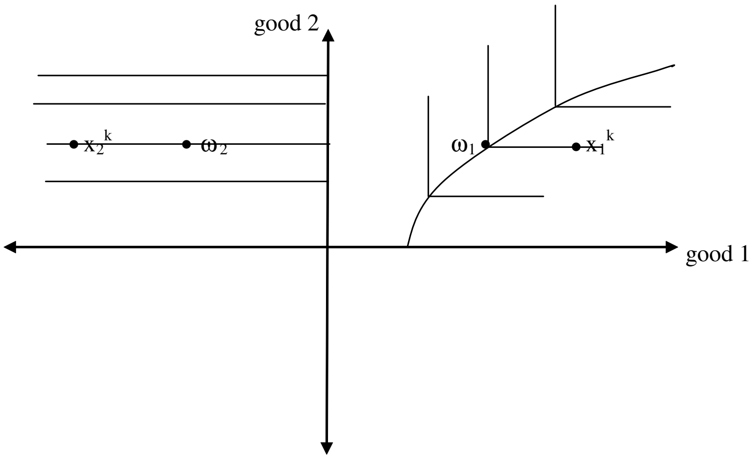

Fig. 3.4. Page *et al.* (2000) example where inconsequential arbitrage holds.

for all i, X_i *is closed and convex and* $\omega_i \in \operatorname{int} X_i$; ($pwm.3$) *all consumers are locally non-satiated at any rational allocation*; ($pwm.4$) *if* $y \in \operatorname{arb}(\omega)\backslash\operatorname{bus}(\omega)$ *then for each rational allocation there is at least one* i *such that for some* $\lambda_i > 0, x_i + \lambda_i y_i \in P_i(x_i)$; ($pwm.5$) *for all rational allocations* x *and all consumers*

$i, R(\dot{P}_i(x_i)) = R(\dot{P}_i(\omega_i))$, then a Walrasian equilibrium exists if and only if $\boldsymbol{E}$ *satisfies inconsequential arbitrage.*

Proof. Page *et al.* (2000; p. 467). □

Remark 3.7. Many of the conditions in this result are standard and have been discussed in detail already and so would not be discussed again here. It is however worth noting again that inconsequential arbitrage imposes potentially delicate relationship requirements across the primitives that define the economy — a point clearly made in the examples above.

Allouch *et al.* (2002) extend Page *et al.* (2000) and begin by noting that while there is no universally agreed upon definition of arbitrage, a good definition in the context of a finite-dimensional exchange economy might be that: "...an arbitrage opportunity is a mutually compatible set of net trades which are utility non-decreasing and, at most, costless to make". Alloch *et al.* (2002; p. 374). However, when unbounded short sales are allowed, choice sets are not bounded below and so prices at which all arbitrage opportunities can be exhausted may fail to exist along with a failure of the existence of equilibrium. The main interest in their paper from our point of view is the following existence result.

Theorem 3.7 (Allouch *et al.* (2002)). *Let* $\boldsymbol{E}$ *be a finite exchange economy in which* (*avp*.1) *for all i, X_i is closed and convex and $\omega \in X_i$*; (*avp*.2) *for all i, u_i is upper semicontinuous and quasi-concave*; (*avp*.3) *the economy is uniform in the sense that each agents arbitrage cone is invariant to the starting point of trade as long as the starting point is weakly preferred to the endowment, i.e. the arbitrage cone $= R_i, \forall x_i \in \dot{P}_i(\omega_i)$*; (*avp*.4) *all consumers are locally non-satiated in that for all $x_i \in A_i(\omega)$, $\exists$ $\{y_i^k\}_k \subset X_i$ with $\lim_{k\to\infty} y_i^k = x_i$ and $u_i(y_i^k) > u_i(x_i), \forall k$*; (*avp*.5) *for all $i, \omega_i \in \operatorname{int} X_i$ and $\forall x_i \in A_i(\omega), P_i(x_i)$ is relatively open in*

X_i; (*avp.*6) *consumers have weak no-half-lines in the sense that for all* $x_i \in \dot{P}_i(\omega_i)$ *if* $y \in \Re^\ell$ *satisfies* $u_i(x_i + \lambda y) = u_i(x_i), \forall \lambda \geq 0$ *then* $y \in L_i$, *then* $\boldsymbol{E}$ *has an equilibrium if and only if it satisfies inconsequential arbitrage.*

Proof. Allouch *et al.* (2002; p. 390). □

Remark 3.8. While there are a number of useful generalisations of conditions in this theorem relative to Theorem 3.6, inconsequential arbitrage and the restrictions that go with it for the structure of the economy are still present.

3.3.6. *Bounded arbitrage*

Page *et al.* (2000), also study the condition of 'bounded arbitrage' introduced in Allouch (1999). In developing this idea they define the idea of the *lineality space at* ω given by $L(\dot{P}_i(\omega_i)) = -R(\dot{P}_i(\omega_i)) \cap R(\dot{P}_i(\omega_i))$. As Page *et al.* (2000; p. 450) note, if $y_i \in L(\dot{P}_i(\omega_i))$, then for λ real and all $x_i \in \dot{P}_i(\omega_i)$, $x_i + \lambda y_i \in X_i$, we have that $u_i(x_i + \lambda y_i) \geq u_i(x_i)$. As they also note, if assumption (pwm.1) holds and if $y_i \in L(\dot{P}_i(\omega_i))$, then net trades in the direction of y_i or $-y_i$ and starting at the endowment ω_i are utility constant. Secondly we need the idea of 'no half-lines' an idea originally due to Werner (1987), which says that an agents utility satisfies the *no half-lines* condition if for all $x_i \in X_i$, there does not exist a non-zero vector of net trades y_i such that $u_i(x_i + \lambda y_i) = u_i(x_i)$ for all $\lambda \geq 0$. Given these ideas Allouch's notion of bounded arbitrage can be defined as follows.

Definition 3.3 (Bounded arbitrage). The economy satisfies *bounded arbitrage* if for all sequences of rational allocations $\{x^n\}_n \subset A(\omega)$, there exists (i) a subsequence $\{x^{nk}\}_k$, (ii) a rational allocation $z \in A(\omega)$ and (iii) a sequence $\{z^k\}_k \subset X_1 \times X_2 \times \cdots \times X_n$ converging to z and such that $z_i^k \in P_i'(x_i^{nk})$

and P'_i is Gale and Mas-Colell's augmented preference correspondence $P'_i(x_i) = \{x'_i \in X_i : x'_i = (1-\lambda)x_i + \lambda x'_i \text{ for } \lambda \in (0,1] x'_i \in P_i(x_i)\}$.

Given this definition, Allouch (1999) obtains the following result:

Theorem 3.8 (Allouch (1999; Proposition 5.1)). *If* E *is such that* $(pwm.1)^*$ *for all* i, u_i *is continuous and quasi-concave*; $(pwm.2)$ *for all* i, X_i *is closed and convex and* $\omega \in \operatorname{int} X_i$; $(pwm.3)$ *all consumers are locally non-satiated at any rational allocation*; $(pwm.4)^*$ *all arbitrage cones are globally uniform so that* $\forall i$, $R(\dot{P}_i(x_i)) = R(\dot{P}_i(\omega_i)) := R_i$ *for all* $x \in X_i$; $(pwm.5)$ *there are useful trades for each consumer so that* $R_i \backslash L_i \neq \varnothing$; $(pwm.6)$ *each utility function satisfies the no half-lines condition then a Walrasian equilibrium exists if and only if* E *satisfies bounded arbitrage.*

Proof. See Page *et al.* (2000; pp. 457–459) for details. □

Remark 3.9. This theorem allows quite general preference structures, however it requires quite particular relationships to hold between the primitives of the economy in order to guarantee the existence of equilibrium.

3.3.7. *Le Van-Minh no-arbitrage price*

Le Van and Minh (2007) observe that the Arrow-Debreu model imposes "…a non-satiation assumption which states that for every consumer, whatever the commodity bundle may be, there exists another consumption bundle she/he strictly prefers". Le Van and Minh (2007; p. 135). Since they want to allow for satiation, they first prove the existence of equilibrium for an economy with dividends. Dividends are thought of *a la* Aumann and Dreze (1986), as a 'cash allowance added to the budget by each trader, the function of which is to distribute among the nonsatiated agents the surplus created by the failure of the satiated agents to use their entire budget'. So agents are allowed to be satiated and if production sets satisfy the 'inaction' and 'irreversibility conditions' discussed in Chap. 2, and the utility functions satisfy a no-half line condition, then Le Van and Minh (2007) show that there exists a Walrasian equilibrium with dividends if and only if there exists a no-arbitrage price for the economy. The set up and notation is similar to that in Page *et al.* (2000) and Allouch *et al.* (2002), modified to take account of production. The key ideas and results are as follows. In Le Van and Minh's notation, W_i denotes $R(\dot{P}_i(\omega_i))$, the recession cone for i and the recession cone for Y_j is denoted by Z_j and is the set of 'useful production vectors for firm j'. If for some $\gamma_i \in Y_j$ then $\gamma_i + \lambda z_j \in Y_j$ for $\lambda \geq 0$ and $z_j \in Z_j$. Then if $p \in \Re^\ell$ then there is an *opportunity for arbitrage* associated with p if either there is a consumer i, $w_i \in W_i \backslash \{0\}$ with $p \cdot w_i \leq 0$ (so the consumer can increase without bound his or her consumption), or there is a firm j with $y_j \in Y_j$ such that $p \cdot y_j > 0$, so the firm will produce an infinite quantity they then make.

Definition 3.4 (No-arbitrage price). A price vector $p \in \Re^\ell$ is a *no-arbitrage price* for the economy if for all consumers i, $w_i \in W_i \backslash \{0\}$ implies $p \cdot w_i > 0$ and all firms j, $z_j \in Z_j$ implies that $p \cdot z_j \leq 0$.

With these ideas the following result becomes available.

Theorem 3.9 (Le Van and Minh (2007)). *If E is a production economy in which* (*vm*.1) *each X_i is nonempty, closed and convex*; (*vm*.2) *each u_i is strictly quasi-concave and upper semi-continuous*; (*vm*.3) *each Y_j is nonempty, closed and convex,*

$0 \in Y_j$, *and the aggregate production set* $Y = \sum_j Y_j$ *is closed with* $Y \cap -Y = \{0\}$; (*vm*.4) *for every consumer*, $\omega_i \in \text{int}(X_i - \sum_j \theta_{ij} Y_j)$ *and for every* $x_i \in A_i, \{x_i' \in X_i : u_i(x_i') > u_i(x_i)\}$ *is relatively open in* X_i; (*vm*.5) *for each consumer preferences satisfy a no-half line condition such that if* $w_i \in W_i \backslash \{0\}$, *then for any* $x \in \dot{P}_i(\omega_i)$, *there is a* $\lambda > 0$ *such that* $u_i(x + \lambda w_i) > u_i(x)$, *then* $\mathcal{E}$ *has a Walrasian equilibrium with dividends if and only if p is a no-arbitrage price vector.*

Proof. Le Van and Minh (2007; p. 145). □

Remark 3.10. This is an ingenious result that relaxes a number of restrictive conditions, primarily those relating to non-satiation. The requirement that preferences, endowments and technologies be so precisely arranged as to give a no-arbitrage price is still however a major restriction.

3.4. Conclusion

As Debreu (1998) makes clear, it is important to examine the restrictiveness of the axioms under which equilibrium can be shown to exist before attempting to make a judgement about the likelihood of such a situation as a representation of actual economic data. In this chapter, we have considered the nature of conditions which are necessary, as well as necessary and sufficient for the existence of Walrasian equilibrium. As a result of this work, it is clear that conditions necessary for existence, require potentially restrictive 'relationship conditions' to hold across the primitives which define the economy. This is also true of various 'no-arbitrage' conditions known to be necessary and sufficient for equilibrium. This is interesting particularly if it forces us to think seriously about the claim that 'Walrasian equilibria exist under weak and general conditions'.

What would make this work particularly interesting is if there were empirical evidence that a breakdown in the sorts of 'relationship conditions' considered here was happening in actual economies. It is to an investigation of some empirical issues concerning conditions for the existence of Walrasian equilibrium that we now turn to.

Chapter 4

EQUILIBRIUM AND IRREDUCIBILITY: SOME EMPIRICAL EVIDENCE

> *"Those without a scrap of land to work or money for the black market were starving. This would be the case for years."*
>
> A. Michaels

4.1. Introduction

As noted in the previous chapter, Arrow and Debreu's 'important principle' and necessary condition for the existence of equilibrium requires that each person be in possession of an asset or labour type that has a positive price at equilibrium. The various 'irreducibility' conditions discussed earlier give expression to this requirement by supposing that each person in the economy has something which other people are interested in purchasing, at a high enough price to guarantee survival.

Commenting specifically on McKenzie-irreducibility, Geanakoplos (1987) argues that irreducibility conditions are likely to be satisfied because: "... [i]t seems reasonable that each agent's labour power could be used to make another agent better off." Geanakoplos (1987; p. 118). In similar vein, Florig (2001; p. 184) observes that the only commodity which most people have in their endowment vector is their labour. If irreducibility is to hold, then each person's labour must be desirable to the economy at large. The principal purpose of this chapter is to consider if the Geanakoplos (1987) way of looking at things is a reasonable description of reality or is there evidence that, in actual economies, there are people whose labour is not generally interesting to others?

In order to achieve our objectives, this chapter is organised as follows: Section 2 considers what a 'reducible' economy might look like. Section 3 examines some empirical evidence from labour market studies for consistency with irreducibility like conditions. The evidence considered comes in particular from studies of unemployment, underemployment and nonemployment and also from studies of wage-employment elasticities. Section 4 considers some policy implications that follow from our theoretical and empirical arguments, particularly concerning the role that real wage reductions and retaining programs may or may not play in helping to combat unemployment. Section 5 presents some conclusions.

4.2. Reducible Economies

What would an economy in which irreducibility fails look like? To help answer that question, reconsider the model due to Gale (1957, 1976) that we discussed in Chap. 2. Here two individuals A and B, have consumption sets $X_1 = X_2 = \Re^2_+$, utility functions $u_A(x_1, x_2) = x_2$, $u_B(x_1, x_2) = x_1 + x_2$ and endowments $\omega_A = (1, 1), \omega_B = (1, 0)$. If $p_1 > 0$, individual A wants to sell x_1 and buy p_1/p_2 units of x_2, which cannot be supplied because B does not have any good 4. If $p_1 = 0$, then B will demand an unbounded quantity of x_1 so that $p_1 = 0$ cannot be an equilibrium either. In this model, which as Florig (2001) notes is an economically meaningful example in which irreducibility fails, person A is only interested in commodity 2 but person B only has commodity 1 to sell. In this economy, we see that two things are true: (i) person B would not be able to trade what they have available for sale and (ii) the demand elasticity of person A with respect to the price of good 1 (B's endowment) is zero. Supposing that commodity 1 is interpreted to be some type of labour, this means that in a 'reducible' economy, we would expect to see (i) people who were unable to trade in the labour market because what they had to trade was uninteresting to anyone else and (ii) low wage-labour demand elasticities for the types of labour that were uninteresting. Thus, it might be the case that no amount of 'flexibility' in wage-price setting would clear the markets in such an economy, because the endowments, technologies and preferences of agents in the economy are not appropriately matched. We now consider if either or both (i) or (ii) are features of actual economies.

4.3. Evidence on Irreducibility from Unemployment and Wage Elasticity Data

One way to detect a breakdown in irreducibility like conditions might be to study the behaviour of economies in which wage and price setting institutions in general, and labour markets in particular, have been made more 'flexible'. Then ask if in the face of this increased flexibility is there evidence of increased labour market clearing or not?

In his analysis of European unemployment Bean (1994) noted that the: "... huge rise in unemployment [in EC-Europe] is a major puzzle for macroeconomists, not to mention policy makers ... [and although adverse shocks have been important], there is no sequence of adverse shocks alone that seems capable of rationalising the persistence of European unemployment ... [g]iven the joint behaviour of unemployment and vacancies, the most important mechanism is in my view likely to hinge on the characteristics or behaviour of the unemployed." Bean

(1994; pp. 573, 614–615). Interestingly, a breakdown in irreducibility provides *a* way to resolve this puzzle, a way which moreover is consistent with the explanation favoured by Bean. This is because if irreducibility breaks down, then there is no fit between the preferences of one group, the technology available in the economy, and the (labour) endowments of certain others. If that happens, then vacancies (which reflect the preferences/technology of one group) and registered unemployment (which reflect the endowments of another) will behave in parallel ways, exactly as Bean (1994) notes they do for the period of history covered by his study. In the event of a breakdown in irreducibility, what one group has to offer is of no interest, from a preference and profit maximising point of view, to the other group. The behaviour of the unemployment in EC-Europe, as described by Bean (1994), appears to be consistent with a breakdown in irreducibility.

Bean (1994) presents some further evidence which is consistent with a breakdown in irreducibility. This evidence is derived from direct observation of the behaviour of unemployment in the face of increased flexibility in various economies. In presenting such evidence, Bean (1994) notes that the policy advice coming from the OECD and similar official sources starting in the mid to late 1980s is essentially Walrasian in its urging that labour markets be made more flexible, by, for example, limiting union power, reducing hiring and firing costs and 'freeing up' the wage setting process. He also notes however that such increased flexibility does not seem, in fact, to have lead to a reduction in unemployment: "The United Kingdom has probably gone the furthest in enacting such [OECD style] structural policies, although so far *with rather little beneficial effect on unemployment*." Bean (1994; p. 615, emphasis added). Likewise, Blanchard and Katz (1997) note that: " ... cross-country evidence on the relation of unemployment to rigidities is less than fully supportive. For example, although Spain and Portugal are classified by the OECD as having the most stringent legal restrictions on layoffs in Europe, Spain's unemployment rate is equal to nearly four times that of Portugal." Blanchard and Katz (1997; p. 68). Similarly, Junankar (1999) notes that unemployment is a serious problem in the OECD but that "... simple remedies like deregulation do not provide a panacea." Junankar (1999; p. 30).

In the face of this sort of evidence, it might be conceded that 'rigidities' may account for only a small fraction of unemployment without losing hope in the main thrust of the Walrasian view that most of the action on unemployment resides with the level of real wages. After an extensive review of empirical studies of European unemployment and after reflecting on the US experience, Freeman (1995) argues that: "Given the evidence reviewed in this article, it is difficult to maintain the conventional view that the way to solve Europe's unemployment problems is through wage flexibility, US-style. The sizeable reductions in pay for the less skilled in the USA have not been sufficient to maintain their employment; have impoverished them and their families; and arguably contributed to the decision of many of them to engage in crime." Freeman (1995; p. 185). Similarly, Nickell and Bell (1996) have shown that even though there has been a large fall in the relative wages of the unskilled in Britain, a fall which has not occurred in continental Europe, the "... unemployment record of the unskilled [in Britain] has been worse than in countries like Germany and the Netherlands." Nickell and Bell (1996; p. 303). This sort of appreciable variation in real wages with no appreciable changes in employment is consistent with condition (ii) above i.e. a low real wage demand elasticity because if the needed relationship structure is absent, then even very large changes in prices will have no impact on demands and amounts transacted.

Motivated by the persistent rise in unemployment in numerous industrialised countries in the 1980s, Jacobson, Vredin and Warne (1998) examined the question of whether and how real wages and unemployment are related. Using a cointegrated VAR framework (a setup which they argue is superior to the single equation error correction models often studied in the literature), and a sample of Swedish quarterly data 1965–1990, the authors find ‘only weak evidence of a short run relationship between real wages and unemployment and even less evidence for a long run relationship’. This finding is again consistent with condition (ii) above.

Mishra (1995) considers the apparent paradox of low US unemployment rates alongside high non-employment rates and argues that it is not low wages which lead US firms to hire more labour and thereby get measured US unemployment numbers down, rather it is poor and short lived social security in the United States which leads non-wealthy potential workers to drop out of the labour market entirely and therefore not show up in the unemployment statistics.[1] Consequently, a better measure than unemployment rates, and one which is able to capture exactly the sort of non-participation effect which will occur when an irreducibility like relationship condition breaks down, is ‘non-employment’.

Murphy and Topel (1997) consider a sample of 800,000 prime-aged American men drawn from Population Surveys 1968 to 1995. In their sample, non-employment has increased over the sample period from approximately 7% in 1968 to about 15% in 1995. In addition, unemployment and nonemployment has increased sharply for the less skilled *at the same time* as real wages for people in those groups has fallen significantly. In particular Murphy and Topel observe: “For workers whose skills place them at the bottom decile of the wage distribution, average wages fell by nearly 40 log points between 1969 and 1993. These are the individuals who showed the largest secular increase in

[1] This possibility is also noted by Murphy and Topel (1997) whose analysis we consider below.

nonemployment. By contrast, the average wages of individuals in the top four deciles of the wage distribution have been largely unchanged since the 1960s [and] … the employment rate for these workers has remained roughly constant over time." Juhn, Murphy and Topel (1997; p. 298). The authors conclude that explanations of unemployment which rely on real wage rigidity 'are simply not credible explanations' of the behaviour of unemployment and nonemployment in the United States. In a follow-up study, Juhn, Murphy and Topel (2002) begin by noting that in their earlier paper they: "… documented the dramatic rise between 1967 and 1989 in both unemployment and nonparticipation in the labor force among prime-aged males. Our main conclusion was that a steep and sustained decline in the demand for low-skilled workers had reduced the returns to work for this group, leading to high rates of unemployment, labor force withdrawal, and long spells of joblessness for less skilled men … [w]e concluded that *structural factors*, primarily the decline in the demand for low-skilled labor, had dramatically changed the prospects for a return to low rates of joblessness any time soon". Juhn *et al.* (2002; p. 79, emphasis added). As we have pointed out, the idea that the labour of unskilled males was not generally interesting to other agents in the economy is consistent with a breakdown in irreducibility. The motivation for the Juhn *et al.* (2002) study was the observation that since their earlier data stopped at 1989 and since the 1990s had 'the longest sustained decline in unemployment in modern US history', it was possible that the theoretical account given in the earlier study of the observed nonemployment rates would need to be modified — and perhaps jettisoned — since "… [b]y the end of that expansion, the unemployment rate had reached its lowest level since the late 1960s, falling below 4 percent for the first time since 1969". Juhn *et al.* (2002; p. 81). In particular because their account of the phenomenon "… had emphasised changes in the structure of labor demand that had made a return to low rates of joblessness unlikely, these facts presented a challenge to the earlier framework. Maybe we were just wrong — maybe the demand and supply framework of our previous work is inconsistent with rates of joblessness in the post-1990 period. If so, we would join a distinguished group of social scientists who have drawn attention to a significant empirical phenomenon only to watch that phenomenon disappear immediately thereafter. As it turns out, however, the framework that we developed for thinking about pre-1990 patterns of joblessness also does fairly well in helping to understand jobless time in the post-1990 period ... [since] [f]irst, the basic trends toward longer spells of joblessness and rising nonemployment have continued in spite of the prolonged expansion of national output and the concomitant fall in unemployment rates. Long jobless spells and labor force withdrawal were more important in the 1990s than ever before. Second, the fall in unemployment to levels close to historical lows is very misleading. Broader measures of joblessness show that the labor market of the late 1990s was more like the relatively slack labor market of the late 1980s than like the booming labor market of the late 1960s. Finally, the basic forces of supply and demand identified in our previous paper continue to have explanatory power. The theory does a reasonably good job of explaining those trends that have continued, as well as those that have changed. Recent data also provide considerable insight into what has happened in the labor market over the past decade. Over the 1990s, even as unemployment was falling, time spent out of the labor force was rising. In fact, the increase in time spent out of the labor force was so large that total joblessness, which combines the unemployed with those who have withdrawn from the labor force, was as high at the business-cycle peak in 2000 as it had been at the previous cyclical peak of 1989, even though the unemployment rate was roughly 2 percentage points lower. In terms of total joblessness, the often-praised boom of the 1990s really represented little in the way of employment progress for American males. Although the growth in the amount of time American males spend out of the labor force continues, a trend found in our earlier research [in particular], [t]rends toward longer durations of both unemployment and nonemployment continued in the 1990s, in spite of declining unemployment [and] ... over the longer term the growth in nonemployment is heavily weighted toward less skilled men. Among men at the bottom of the wage distribution, the nonemployment rate increased by 13.5 percentage points between the late 1960s and 2000 ... We conclude that long term changes in joblessness have been the result of adverse shifts in labor demand, perhaps coupled with policy-driven shifts in labor supply, among low-skilled men". Juhn *et al.* (2002; pp. 82–84).

We have quoted Juhn *et al.* (2002) at some length because it controls for obvious business cycle effects in nonemployment data and still reveals a picture of the US labour market which casts doubt on the applicability of 'irreducibility' as a description of it.

In similar vein to Murphy and Topel (1997) and Juhn *et al.* (2002), the Australian Bureau of Statistics has released data which it claimed showed 'the true state of the Australian labour market' in September 1998. Like the Murphy and Topel data, they show a big difference between officially measured unemployment rates and nonemployment. In particular the data showed 1,700,000 people who wanted work but could not find it which gave a 'real' unemployment rate of 16.3% in September 1998 compared with the official unemployment rate of 8.1%. According to Allard (1999) "... the rise in hidden unemployment puzzled economists as it traditionally falls in line with the official unemployment rate." Allard (1999; p. 3). The puzzle is however consistent with a breakdown in irreducibility. Similarly, Australian data for 2000–2006 showed that over that period between 500,000 and 1,000,000 people were jobless and willing to work but were unable to find work.[2]

Also focussing on Australian data, Gregory (1996) has presented evidence of low real wage-labour demand elasticities in Australia by observing that the large increases in female wage rates which occurred in the 1970s was not accompanied by a change in the female/male employment ratios.

Nevile (2001) surveys numerous micro and macro studies of unemployment in response to the Debelle and Vickery (1998a, 1998b) study which finds an elasticity of labour demand to real wages of −0.4, finding that "… the weight of evidence is that Debelle and Vickery's estimate of −0.4 for the elasticity of demand for labour is too high." Nevile (2001; p. 20–29). Furthermore, Nevile (2001) argues that most studies find elasticities close or equal to zero, for many types of labour, particularly unskilled labour, a finding consistent with condition (ii) above and a breakdown in irreducibility.

Based on their review of studies of Australian unemployment, Le and Miller (2000) argue for lower real wages as an important part of the policy mix for solving Australia's unemployment problem, along with high rates of economic growth. The authors however note that translating the policy recommendation concerning real wages into policy action is not easy. In fact they conclude their paper by observing that: "[i]t is disappointing that the recent debate on youth wages shows how difficult it will be to tackle the unemployment problem [via real wage reductions]." Le and Miller (2000, p. 96). Interestingly, on the basis of their survey, Le and Miller make an observation about the behaviour, over time, of the real wage elasticity of labour demand, behaviour which they describe as 'surprising'. In particular, they note the study by Debelle and Vickery (1998a) which, after reporting a labour elasticity — real wage demand

[2]See ABS publication *Underemployed Workers, Australia*, Cat. No. 6265. Data quoted here taken from September 2006 issue.

of −0.4 goes on to remark that 'the wage elasticity has been declining over time'. Commenting on this observation, Le and Miller remark: "This finding, based on rolling regressions of labour demand equations over 15-year windows, *could be categorised as surprising*." Le and Miller (2000, pp. 88, 89; emphasis added). As grounds for their surprise, Le and Miller cite the conjecture of Freebairn (1998) that with less labour market regulation and increased labour market flexibility " …one might have expected labour demand elasticities to have increased over time." Le and Miller (2000, p. 89). On the basis of our discussion in Chaps. 2 and 3 we argue that the 'surprise' finding in Le and Miller (2000) ceases to be such a surprise in an economy in which irreducibility conditions have broken down.

Lewis and MacDonald (2002) provide a new set of estimates of the elasticity of demand for labour in Australia, estimates 'that are derived using a better methodology than before'. Specifying the production side of the economy by a CES production function and using the estimation techniques of Pesaran and Shin (1999) on a sample of Australian data for the period 1959:3 to 1998:3, Lewis and MacDonald present estimates of two elasticity measures. The first is *the output constant elasticity of the demand for labour with respect to real wages*, the value of which is estimated to be −0.2. As the authors note: "… This is substantially lower than other estimates." Lewis and MacDonald (2002; p. 24). Interestingly, this lower than other estimates finding is consistent with the declining trend noticed by Le and Miller (2000). The other elasticity measure presented by Lewis and MacDonald (2002) is *the elasticity of demand for labour with respect to real wages* (allowing output to also adjust). In order to do this the authors make an *assumption* about the elasticity of demand for labour with respect to output. In particular; "Assuming an elasticity of demand for labour with respect to output of unity [and a labour to GDP share of 0.6] …" Lewis and MacDonald (2002; p. 25), they calculate the elasticity of demand with respect to real wages to be $-(0.6\times1+0.2) = -0.8$. Clearly the final elasticity estimate is sensitive to the assumed value of the elasticity of demand for labour with respect to output. Le and Miller (2000; p. 90, Table 10) present a survey of ten studies of Australian labour demand. They report that for these studies: "... output elasticity [of demand for labour] is *less than unity* ...". Some 'sensitivity analysis' on the Lewis and MacDonald estimate reveals that if the elasticity of demand for labour with respect to output is assumed to be 0.8 then elasticity of demand with respect to real wages is $-(0.6\times0.8+0.2) = -0.68$, if 0.6 then -0.56 and so on.

It is interesting to consider the evidence presented in Stegman and Stegman (2000) on the elasticity of demand for labour with respect to output. Stegman and Stegman (2000) begin by asking why, after a decade of substantial reform of Australian labour market institutions and a six year period of strong economic growth (and moderate real wage growth), have the results in terms of reductions in unemployment been so disappointing? One possibility, which Stegman and Stegman's analysis rejects, is that the reforms have not actually delivered labour market flexibility. Another possibility is that for some reason, Australia has been experiencing a 'jobless recovery'. In the limit, this would imply the appropriate value for the elasticity of labour demand with respect to output is close to 0. If that were the case, then the appropriate estimate for the *elasticity of demand for labour with respect to real wages* (allowing output to also adjust) would be closer to -0.2 (rather than to -0.8 favoured by Lewis and MacDonald (2002)). In fact, on the basis of data provided by Stegman and Stegman (2000), the elasticity of demand for labour with respect to output in Australia for the period 1993–1998 is about 0.43. This means that the *elasticity of demand for labour with respect to real wages* (allowing output to also adjust) is equal to $-(0.6\times0.43+0.2) = -0.46$. This figure is considerably different to the -0.8 reported by Lewis and MacDonald (2002), and is in line with the results reported in Le and Miller (2000) and is consistent with a relatively weak effect of real wages on employment that one might expect in an economy showing signs of a breakdown in irreducibility.

Hammermesh (1993) presents a survey of 32 studies of labour demand across a wide variety of labour types, demographics and countries. In almost all the studies reported, low or zero wage-employment elasticities were found.

In spite of the considerable body of evidence reported by Hammermesh (and others cited earlier), Nevile (1996) observes that: "... contrasts are often made between the situations in Europe, especially continental Europe, and the United States. In Europe there are high minimum wage levels and unemployment is over 10% in many countries. In the United States, both the minimum wage and the level of unemployment are much lower ... [leading to the] conventional wisdom that OECD economies have a choice between wage rates so low at the bottom end that many full time workers lived in poverty or mass unemployment." Nevile (1996; p. 209).[3] One study which attempts to address this apparent difference between Europe and the United States, due to Card, Kramarz and Lemieux (1996), compares changes in wages and employment in the 1980s in France, Canada and the United States to find that: "... the pattern of relative employment growth over the 1980s [in France and Canada] are virtually identical to those in the United States." Card *et al.* (1996; p. 29). From this Nevile (1996) concludes that "... the big fall in wages at the bottom end of the distribution appeared to have no effect in increasing employment among the unskilled in the United States." Neville (1996; p. 209). These findings are all consistent with the cause of unemployment

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[3]Blanchard and Katz (1997) for example make this sort of remark when they argue that: "What is clear is that the hypothesis works well when looking at two observations, the United States and Europe. However it is less clear that it can explain cross country differences in Europe." Blanchard and Katz (1997; p. 68).

being a breakdown in an irreducibility like relationship condition, because the findings all reveal a lack of interest in the sort of labour which the unskilled have to offer, *even when real wages have been flexible downwards to a significant degree*. Card and Krueger (1995) present a series of studies in which for the United States changes in minimum wages had mostly zero elasticities. Nevile (1996; p. 209) defines 'non-employment' as the percentage non-employed in a particular age/demographic group as: 100% (in that group employed). Taking prime age (25–54) males,[4] the non-employment rate in EC-Europe is 15%, 14% in the United States, 14% in Australia and in Britain, "… whose labour market is more like that in the United States than those in continental Europe", Nevile (1996; p. 210), and whose economic institutions have been subjected to a decade and a half of 'reforms' that have a distinctly Walrasian flavour, the rate is 18%. This is interesting empirical evidence which seems *inconsistent* with an neoclassical 'impediments' and 'wages are too high' story about unemployment. It *is* however consistent with a 'breakdown in irreducibility conditions' explanation of the sort considered here.

The natural rate of unemployment, made famous in the characterisation given it by Friedman (1968) as: 'the level which would be ground out by the Walrasian system of general equilibrium equations' may be slightly more precisely defined as: "… the rate [of unemployment] towards which the dynamic system [of the economy] is converging *for a given underlying general equilibrium stochastic structure*." Haltiwanger (1987; p. 610, emphasis added). In other words, the natural rate of unemployment is determined by the *primitives* that define the economy, i.e. preferences, endowments, technology, share ownership, and the relationships between them. As Haltiwanger (1987) notes, at the natural rate of unemployment, there will

[4]Chosen so as to avoid cultural differences in the desire for paid work by married women, differences in retirement patterns, university retention rates etc. See for details.

be some relationship between vacancies and unemployment but that it need not be one of equality and further that the natural rate depends on the heterogeneity present in the potential labour force, particularly as that heterogeneity relates to technology.

It is therefore reasonable to argue that the degree to which the economy is reducible corresponds with the level of the natural rate. If an economy is irreducible, then the natural rate is near zero, since the underlying general equilibrium structure of the economy would permit everyone to trade the labour holdings that they have. As the economy reduces into two groups (those who have a labour type that is interesting to others and those who do not), then also the natural rate will rise. These sorts of considerations find their parallel in the labour economics literature where one finds remarks such as: " labour force composition effects will alter the natural rate of unemployment [and similarly] a higher rate of structural change will yield a higher natural rate of unemployment." Haltiwanger (1987; p. 611). This point can also be made by noting that there is nothing in the operation of a competitive private ownership economy (apart from the starvation effect outlined by Coles and Hammond (1995)), which keeps the composition of the labour force and the structure of the economy aligned in a way that guarantees a low or even constant natural rate of unemployment.

As Groenewold and Hagger (2000) note, since the natural rate of unemployment is not observable, calculation of its value and remarks about its dynamic behaviour are heavily model dependent. They can also be dependent on the definition of the natural rate adopted. For instance, one approach to definition involves identifying the natural rate as the unemployment rate that would have been observed if the economy had been in constant equilibrium. An alternative is to identify the natural rate as the non–accelerating-inflation rate of unemployment. In an attempt to resolve the ambiguity at this level, Groenewold and Hagger (2000) note that the definitions of both approaches have something in common, namely, that: "... the natural rate would be observed only after shocks to aggregate demand have completely worked their way through the system." Groenewold and Hagger (2000; p. 123). This definition is an expression of the idea of Walrasian equilibrium defined in Chap. 2 and elaborated on by Coles and Hammond (1995). Variations in the natural rate, particularly upward, would be very interesting for the hypothesis being advanced here because it would indicate that the level of unemployment being ground out by the Walrasian system of general equilibrium equations is behaving in a way consistent with a breakdown in irrreducibility. Using their definition and a VAR based on the model of Blanchard and Quah (1989), Groenewold and Hagger (2000) find that for Australia, there have been significant increases in the natural rate for Australia, particularly in the late 1970s to early 1980s from about 6% to over 8%. This is in line with results by Debelle and Vickery (1998a, 1998b) and Gruen, Pagan and Thomson (1999). It is less in numerical value and somewhat different in dynamic structure to the results reported in Groenewold and Hagger (1998) which showed a persistent increase in the natural rate over the 1980s and 1990s to a rate as high as 10–11%. Nevertheless, all these studies are consistent with our hypothesis that when aggregate demand shocks are set aside, there is still significant unemployment to be explained, unemployment which may be explained in terms of a breakdown of irreducibility in the structure of the economy.

Taking up the point that the natural rate appears to move over time, there has been a resurgence of interest in 'structuralist' theories of unemployment. As Phelps (1994) notes such theories endogenise the natural rate of unemployment. In particular he remarks that, in structuralist theories "... [t]he equilibrium path of the unemployment rate always approaches the natural rate, as before. But something has been added. The natural rate moves!" Phelps (1999; p. vii). Motivated by this observation, Papell, Murray and Ghiblawi (2000) study the unemployment behaviour of 16 OECD countries[5] and find that the behaviour of unemployment in these countries: "...seem to be most congruent with the structuralist theories of unemployment." Papell, Murray and Ghiblawi (2000; p. 315). This is so because all countries in the sample have at least one significant break in the unemployment process, which is strong evidence against a constant natural rate of unemployment. Secondly, most countries in the sample had at most two breaks in the unemployment process. As Papell *et al.* (2000) argue: "[t]he combination of these two results is more congruent with the structuralist theories (which emphasise occasional changes in the natural rate of unemployment) than with traditional theories (where the natural rate is constant) or with the unit root hysteresis theories (which taken literally, imply a permanent change every period)." Papell *et al.* (2000; pp. 313–314).

Gregory (1999) presents evidence that in the period August 1975 to May 1999, the Australian labour market became increasingly divided between households that are 'work rich' (i.e. where almost everyone in the household is selling their labour) and households which are 'work poor' where nobody in the household is selling their labour. The period over which Gregory conducted his study is one in which the unemployment rate had increased from 4.6% to 7.3% and there has been a rise in the average duration of unemployment from 14.7 weeks to 55.7 weeks (see Gregory (1999; p. 1)). There could of course be macroeconomic reasons for the behaviour of unemployment, nonemployment and the polarisation in the distribution of unemployment. Gregory however dismisses this when he observes: "Although the macro labour market deteriorated, and unemployment increased it is rather puzzling, and perhaps unexpected, that couple families

[5] Australia, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, Norway, Spain, Sweden, the United Kingdom and the United States of America.

with dependent children have been so adversely affected by joblessness. Indeed the increase in families without work is even more surprising when it is realised that the number of couple families with dependent children increased by 15,700 but the number of adult members of these families with employment increased by 467,000. There should have been enough jobs to reduce joblessness. What happened?" Gregory (1999; p. 5). On the basis of the theoretical work in Chap. 2, one suggestion is that there has been a breakdown in the irreducibility like conditions which ensure that everyone is tied into the economy. In the United Kingdom, almost identical evidence to that assembled by Gregory (1999) has been gathered in a report by the Department of Social Security. The report identifies exactly the same sort of divide between 'work rich' and 'work poor' households. In particular, there are 20% of households in the United Kingdom that have no one in employment even though 500,000 new jobs have been created since mid-1997. The essence of irreducibility like conditions is that the economy does not partition in this way so the data is consistent with a breakdown of such conditions.

In searching for an explanation of the phenomenon of work rich and work poor households, Gregory (1999) notices two further things which are consistent with our 'breakdown of irreducibility' hypothesis. The first is that unemployment tends to be inversely related to education. The second is that unemployment and employment is being concentrated into geographical areas between which there is an increasing lack of wider social interaction. Recalling that irreducibility involves a situation where everyone in the economy has something which somebody else wants, it can be seen that both of Gregory's observations about the nature of current Australian unemployment are entirely consistent with a breakdown in irreducibility like conditions.

Dawkins, Gregg and Sartella (2001) continue the work of Gregory and find that in many OECD countries, including Australia, while there has been an overall recovery in employment from the lows of the 1980s recession, "... there has also been an upward trend in the number of jobless households in the majority of these countries [and also] the burden of unemployment or more generally joblessness tends to be concentrated in certain households." Dawkins *et al.* (2001; p. 1). This pattern of aggregate increases in employment accompanied by an increase in joblessness concentrated among particular groups is entirely consistent with a breakdown in irreducibility conditions discussed earlier.

4.4. Some Policy Implications

There is an argument, found at various points in the literature, (see for instance, Lindbeck and Snower (1985), Valentine (1993, 1994, 1995, 1996) and Moore (1999) and to a lesser extent Le and Miller (2000)), that the only way to reduce unemployment is to cut real wages in order to establish a labour market equilibrium. The work in the previous chapters showed that quite particular conditions are needed if an economy is to have a Walrasian equilibrium. Since the policy approach noted above implicitly assumes the existence of a Walrasian equilibrium, it seems useful to bring to this policy debate what we have learned about conditions for the existence of such equilibrium states.

When faced with any episode of unemployment, some people argue that the cause of the unemployment can be found *inside* the labour market, typically in the rules which govern trade in that market or in the level of real wages that pertain there, or both. Analysts who view unemployment this way see increased 'labour market flexibility' and/or reduced real wages as the only solution to the problem of unemployment. Valentine (1994) has argued the position that increases in Australian real wages have led to significant increases in unemployment. He further argues that a reduction in real wages in the face of adverse shocks to the economy is the only way to achieve significant reductions in unemployment. In support of such a position, he claims that: "... since 1965 all substantial increases in unemployment [in Australia] have been associated with increases in real wages. This was the case in 1974/75, 1981/82, 1982/83, 1989/90 and 1990/91 ... A solution is available [to the unemployment problem] *only if* we are prepared to make wages more sensitive to economic conditions and to allow greater flexibility in the wage structure." Valentine (1994; pp. 174 and 177, emphasis added). In similar vein, Valentine (1993) has argued: "The counterfactual experiments performed in this paper lead to the same conclusion as the earlier ones reported in Valentine (1980) — unemployment can be substantially reduced by a reduction in real wages. Valentine (1980) also concludes that employment effects of the depression of the '1930s: "... would have been minimised by a policy which related wages to unemployment *thereby replicating a market outcome*." Valentine (1993; p. 18, emphasis added). These remarks are essentially expressing the view that a decentralised economy, operating through an interlocking network of flexible markets will, through self-generated pressures, fully employ all its job seeking labour unless prevented from doing so by organised labour, government intervention or some other form of restraint on the operation of free markets. There are of course, other views about the causes of and appropriate responses to unemployment. These views generally contend that explanations of the phenomenon of unemployment that focus exclusively on the level of real wages and the operations of labour markets are mistaken. Keynes for instance argued that even the most fluid of market systems and the most flexible of labour market institutions would not generally achieve full employment without explicit government intervention. As Nicola (1997b) notes: "... it was above all unemployment that primarily concerned Keynes, who stated that he could not accept the explanation of the current economic theory according to which it would be sufficient to reduce real wages to secure full employment for workers." Nicola (1997; p. 87). In similar vein, representative of the views of those who argue that theories and policies which look for an explanation and solution to the problem of unemployment exclusively *in* the operations of the labour market are mistaken is Hargreaves-Heap (1987) who has argued: "... there is a general point here which any general equilibrium theorist should appreciate. Namely, that in the context of a general equilibrium system, it makes no sense to locate the source of market failure in the market in which it happens to occur. In a general equilibrium system, everything depends on everything else that is happening in the economy, and consequently it need not be the agents in the labour market who are responsible for the failure to generate the Walrasian equilibrium price vector ... and it remains the case that the full Walrasian equilibrium could not be achieved by changes in the real wage alone." Hargreaves-Heap (1987; pp. 746, 747). If the sorts of 'relationship conditions' discussed in this and the previous chapters, particularly those known to be necessary for the existence of equilibrium, break down, then the Keynes–Hargreaves-Heap position is supported in an interesting way. In particular in such circumstances, Walrasian equilibrium does not exist and certainly could not be made to exist by an adjustment in real wages alone, no matter how dramatic.

One possible outcome of relying on markets to solve the problem of unemployment, alluded to earlier in this chapter, has been given by Coles and Hammond (1995), who have argued that: "[o]nly after excess labour has been *removed through starvation* can general equilibrium arise." Coles and Hammond

(1995; p. 60, emphasis added). These authors go on to argue that full employment and agent survival is not guaranteed by the operation of free markets with flexible wages and in fact: "... [starvation] is *an entirely natural phenomenon* of a neoclassical economy." Coles and Hammond (1995; pp. 60–61).[6] This way of 'solving' the problem of unemployment, i.e. by killing off the unemployed, is presumably not the sort of outcome sought by those who advocate the market mechanism as their preferred — indeed, *only* — solution to the problem of unemployment. However, unless certain quite specific, theoretically fragile and potentially empirically vulnerable conditions hold, this is precisely what may happen if market forces alone are relied on to solve the problem of unemployment. Unless such conditions hold, then no matter how 'responsive' real wages become and no matter how 'flexible' the labour market is, there will be no full employment Walrasian equilibrium in the economy. This is because the structure of the economy and the relationships which hold between the primitives which define it fail to ensure the existence of Walrasian equilibrium in the first place.

4.5. Conclusion

In earlier chapters, we established the importance of irreducibility like conditions in guaranteeing the existence of a Walrasian equilibrium. Since many agents in the economy have just one commodity to sell, namely their labour, a breakdown in irreducibility may show up as high and sustained levels of non-employment. Also, if irreducibility like conditions have broken down in actual economies, we might expect to see quite small

[6] They go on to remark: " ... the fact that competitive equilibrium do[es] not require all to survive should really be no great surprise. If the analysis seems heartless in the face of human misery, that is a true reflection of the price mechanism in a *laissez faire* economy which general equilibrium theory is intended to model." Coles and Hammond (1995; pp. 60–61).

real wage — employment elasticities. In this chapter we have studied data from labour market studies in Australia, Europe, the United Kingdom, the United States and a group of OECD countries. These data are often described as 'puzzling', particularly in the face of (i) wide ranging deregulation of labour markets in these economies; (ii) little or no real wage inflation and (iii) generally supportive macroeconomic conditions. Some of this puzzlement may be relieved by noting that the phenomena being described are consistent with a breakdown in irreducibility which in the form presented by Floring (2000) is necessary for the existence of Walrasian equilibrium. This suggested explanation of at least some periods of unemployment and nonemployment, avoids any 'institutional rigidities' type explanation and reconciles unemployment with equilibrium by postulating that irreducibility has broken down across the economy.

If this is the case, then the policy response that unemployment can only be reduced if real wages are reduced may not be entirely well-founded. A more useful policy stance might involve attempts to ensure that the structural conditions needed for the existence of Walrasian equilibrium are actually present in the economy before the price system is 'freed up' to search for such an equilibrium price vector. One practical step in that direction may involve changing the characteristics of labour market entrants, through relevant training programs and by providing generous and diversified support in the transition from unemployment to employment. This may be superior to simply relying on 'market forces' to find a Walrasian equilibrium, since from the theory and empirical evidence discussed so far, it is not guaranteed that such states will always exist.

Chapter 5

EXISTENCE OF EQUILIBRIUM UNDER ALTERNATIVE INCOME CONDITIONS

"*The most remarkable achievements of modern microeconomic theory are the proof of the existence of an equilibrium and the First and Second Theorems of Welfare Economics ... Understandably, therefore, there has been much attention devoted to various interpretations, alternative proofs, and extensions of these basic results.*"

D. Luenberger

5.1. Introduction

The significance of the existence theorem for Walrasian equilibrium is hard to overestimate. Indeed Luenberger (1994a, 1994b) ranks it, along with the two welfare theorems, as 'the most remarkable achievements of modern microeconomic theory'. He also observes that, considering this status, it is understandable that "...there has been much attention devoted to various interpretations, alternative proofs, and extensions of these basic results." Luenberger (1994b; p. 147). In that spirit this chapter explores some alternative approaches to establishing the existence of Walrasian equilibrium, with a particular focus on approaches which avoid, as far as possible, invoking strong relationship conditions such as interior endowments or various forms of irreducibility. We are motivated to do this because it is not clear that there is anything at work in the economy that actually endogenizes structural relationships such as irreducibility. We are therefore keen to explore how the needed relationships — in particular those which guarantee individual survival — might appear in the economy. The first and most obvious possibility is that some form of economic policy is used to achieve the desired end. The second avenue explored here is the possibility that altruism, and the voluntary income transfers to which it might lead, could play a role in guaranteeing consumer survival, demand continuity and the eventual existence of Walrasian equilibrium. This is a interesting possibility because if such a mechanism could generally be relied on, then we would have a genuinely endogenous process, rather than simply assumed structural conditions, working in the direction of establishing Walrasian equilibrium.

In order to achieve our objectives, this chapter is organised as follows. In Section 2, some results are presented which rely on various 'economic policies' in order to obtain equilibrium existence. Section 3 considers the role that 'altruism' might play in establishing the existence of Walrasian equilibrium. Section 4 presents some conclusions.

5.2. Some 'policy induced' existence results

5.2.1. *Endowment taxes*

It might be possible to largely avoid assumptions such as interior endowments or various forms of irreducibility by using a policy instrument such as taxes and transfers in order to get around the 'fundamental mathematical difficulty' identified by Debreu (1962) and discussed in Chap. 2. The technicalities of the argument follow Arrow and Hahn (1971; pp. 101–102). To that end let the quantity $\sum_q x_{iq}$ be the sum of the demands of consumer i. This sum is calculated in the usual way for $p \in \Delta = \{p \in \Re^{\ell}_{+} : \sum_q p_q = 1\}$ at which $x_i(p)$ is defined and equals $+\infty$ otherwise. *Continuity* of $\sum_q x_{iq}$ has the usual meaning at points where the function is finite, while for sequences $\{p^v\} \to p^0$, where $\sum_q x_{iq}(p^0) = +\infty$, the function is *extended continuous* if $\lim_{v\to 0} \sum_q x_{iq}(p^v) = +\infty$ on the subsequence (if infinite) for which $\sum_q x_{iq}(p^v) < +\infty$. As usual, the excess demand function $Z(p)$ is *homogeneous of degree zero* in p if $Z(\lambda p) = Z(p)$ for $\lambda > 0$, *bounded below* if there exists a positive finite number N such that for all $p \in \Delta$ the excess demand for good q, $Z_q(p) > -N$ and is *extended continuous* if it is defined for all $p \gg 0$ and possibly other $p \in \Delta$, and is continuous when defined. If it is not defined for $p = p^0$, then $\lim_{p\to p0} \sum_q Z_q(p) = +\infty$. If in every equilibrium there is a commodity k such that $\sum_q Z_q(p) = +\infty$ when $p_k = 0$ the economy has a *numeraire* commodity (see Arrow and Hahn (1971; p. 208) for further discussion of this condition).

Remark 5.1. As Arrow and Hahn (1971) point out: "... we are unlikely to achieve continuity in any sense if income [M_i] is at the minimum possible. It will follow [from a later result] that M_i is above the minimum possible if and only if $M_i > 0$. We will therefore assume $M_i > 0$ for all possible prices — a strong assumption, because it means in effect, that [ω_i] $\gg 0$ [i.e. the interior endowments assumption holds] ..." Arrow and Hahn (1971; p. 102). The role of the condition on income noted in Arrow and Hahn's remark has been discussed by us previously and is formalised in the following result.

Lemma 5.1 (Arrow and Hahn (1971)). *If for every consumer i, X_i is a closed convex set, $x_i \geq 0$ for all $x_i \in X_i, \preceq_i$ is complete, continuous, reflexive, transitive, strictly convex and $M_i(p) > 0$ for all $p \in \Delta$ then $x_i(p)$ is continuous at least for $p \gg 0$ and is extended continuous everywhere on Δ.*

Proof. Arrow and Hahn (1971; pp. 102–104). □

The problem is to ensure that for all i, $M_i > 0$ at all p. Consider a situation where each i's income is $M_i(p) = t_{ii}p\omega_i + \sum_{h \neq i} t_{ih}p\omega_h + \sum_j \theta_{ij}py_j(p)$, where $0 \leq t_{ih} \leq 1$ for all i and h, and $\sum_h t_{ih} = 1$. The term $t_{ii}p\omega_i$ can be thought of as the after tax value of i's endowments and t_{ih} is the claim which i has on the value of the endowment of h so that $\sum_{h \neq i} t_{ih}p\omega h$ is the total transfer of value to i from all other households in the economy. If $t_{ih} = 0$ for all $i \neq h$, so $t_{ii} = 1$, then the income function is of the standard form.

Definition 5.1 (Welfare augmented private ownership economy). If the economy has a redistributive mechanism that makes transfer arrangements which satisfy the conditions that $M_i(p) = t_{ii}p\omega_i + \sum_{h \neq i} t_{ih}p\omega_h + \sum_j \theta_{ij}py_j(p)$, where $0 \leq t_{ih} \leq 1$ all i h, and $\sum_h t_{ih} = 1$, then the economy is $\boldsymbol{E}_{wa} = \{X_i, \preceq_i, \omega_i, Y_j, \theta_{ij}, t_{ih}, \ell\}_{i=1}^{m}{}_{j=1}^{n}$ is as a *welfare augmented private ownership economy.* The economy does not have *total appropriation* if $t_{ii} > 0$ for all i.

Lemma 5.2. *If* $\forall_j$, $0 \in Y_j$, $\forall$ *households* h, $\omega_h \in \Re^{\ell}_{+}$, $\omega \gg 0$ *and* $t_{ih} > 0$ *for all* i, h, *then* $M_i(p) > 0$ *for every* $i \in \boldsymbol{E}_{wa}$ *and any* $p \in \Delta$.

Proof. Since $\omega \gg 0$, $p\omega = \sum_i p\omega_i > 0$ for $p \in \Delta$. Also $p\omega_h \geq 0$ for all h because $\omega_h \in \Re^{\ell}_{+}$ and $p \in \Delta$. Therefore, there exists a household h such that $p\omega_h > 0$. If $h = i$, then $M_i(p) > 0$ because $t_{ii} > 0$ and $py_j(p) \geq 0$ for all j. If $h \neq i$, then $M_i(p) > 0$ because $p\omega_i \geq 0$, $py_j(p) \geq 0$ and $t_{ih} > 0$ for all i, h $p\omega_h \geq 0$ for all h and $p\omega_h > 0$ for at least one h. Therefore, $M_i(p) > 0$ for every $i \in \boldsymbol{E}_{wa}$ and $p \in \Delta$. □

Remark 5.2. An alternative specification of the income function that would also work here is $M_i(p) = t_ip\omega + \sum_j \theta_{ij}py_j(p)$ with $t_i > 0$ and $\sum_i t_i = 1$.

Lemma 5.3 (Arrow and Hahn (1971)). *If Y_j is bounded, strictly convex and admits free disposal, then $y_j(p)$ is a continuous function on Δ.*

Proof. Arrow and Hahn (1971; p. 71). □

Lemma 5.4 (Arrow and Hahn (1971)). *If the excess demand relation for an economy with finitely many goods is a function on Δ, homogeneous of degree zero in prices, bounded from below, extended continuous on Δ and if there is a numeraire commodity then a Walrasian equilibrium exists.*

Proof. Arrow and Hahn (1971; pp. 31–32). □

Given these results the following existence result can be established.

Theorem 5.1. *If $\mathbf{E}_{wa}$ satisfies: $(b.1)$ $\forall_i, X_i = \Re^{\ell}_{+}$; $(b.2)$ $\forall_i, \preceq_i$ is complete, continuous, transitive, reflexive and strictly convex; $(b.3)$ there is no satiation consumption in X_i for any i; $(b.4)$ $\forall_j, 0 \in Y_j$ and Y_j is closed, strictly convex, bounded and admits free disposal; $(b.5)$ there exists $i \in [1, n]$ such that $\omega_i \in \text{int}(X_i)$; $(b.6)$ $\|\omega\| < \infty$; $(b.7)$ in every equilibrium of the economy there is a numeraire commodity, say k, for which $\sum_q Z_q(p) = +\infty$ when $p_k = 0$; then a Walrasian equilibrium exist for $\mathbf{E}_{wa}$.*

Proof. By (b.2) $\preceq_i$ is strictly convex, so by Lemma 5.2, $x_i(p)$ is a function on Δ when it is defined. By Lemma 5.3, y_j is a continuous function on Δ since by (b.4), Y_j is bounded, strictly convex and admits free disposal so that $Z(p)$ is a function on Δ, whenever it is defined. The budget set for i is $\{x_i \in X_i\colon px_i \leq t_{ii}p\omega_i + \sum_{h \neq i} t_{ih}p\omega_h + \sum_j \theta_{ij}py_j(p)\}$. Therefore, $x_i(p) = x_i(\lambda p)$, $\lambda > 0$. Also, $y_j(p) = y_j(\lambda p)$ since $y_j(p) = \max_{yj} \{py_j : y_j \in Y_j\}$. Consequently $Z(p)$ is homogeneous of degree zero with respect to p. By Lemma 5.2, $x_i(p)$ is extended continuous on Δ and from this and the continuity of $y_j(p)$ on Δ, $Z(p)$ is extended continuous on Δ. By (b.7) there is a numeraire commodity in the economy and therefore from Lemma 5.3 it follows that Walrasian equilibrium exists in this economy. □

Remark 5.3. Condition (b.5) requires the existence of just *one* individual with an endowment in the interior of his or her consumption set, in place of the interior endowments requirement that *every* consumer has an endowment in the interior of his or her consumption possibility set or that an economy wide relationship requirement such as irreducibility holds. Condition (b.5) allows almost arbitrary relationships to hold between the endowments, consumption sets and preferences of the agents in the economy, in contrast to the requirements imposed by interior endowments and irreducibility. This is so because the part previously played by those assumptions is now played by the redistributive policy instrument that ties the economy together. One weakness of the result is that assumption (b.1) requires that individuals have a consumption possibility set equal to $\Re^\ell_+$. We now present a result which relaxes that restriction.

5.2.2. *Taxes and general consumption sets*

As Boyd and McKenzie (1993) note, it is desirable, where possible, to allow consumers to be characterised by consumption sets which are general subsets of $\Re^\ell$ and not just $\Re^\ell_+$. This is so because it might be desirable to model consumers as suppliers of some commodities, for which the convention that the amount supplied is a negative number is usually adopted. In addition, it is realistic to suppose that consumers have minimum requirements of certain commodities, a supposition which cannot be captured if the assumption is made that $X_i = \Re^\ell_+$. We can relax (b.1) of Theorem 5.1, with the aid of the following result:

Lemma 5.5 (Rockafellar (1970; p. 49)). *For convex sets* $X_1, \ldots, X_n$ *in* $\Re^\ell$, $ri(X_1 + \cdots + X_n) = ri(X_1) + \cdots + ri(X_n)$. *Further, if for all* i, $\dim(X_i) = \lambda$ *then* $ri(X_i) = \text{int}(X_i)$, *where*

$ri(X)$ is the relative interior of X and $\mathrm{int}(X)$ *is the interior of X.*

Proof. Rockafellar (1970; pp. 49–50) □

Theorem 5.2. *If in $\boldsymbol{E}$ $(b.1)'$ $\forall_i X_i$ is closed, convex, has non-empty interior of dimension ℓ and is lower bounded; $(b.2)'$ V_i there is no satiation consumption in X_i; $(b.3)'$ the set $\{(x, x') \in X_i \times X_i : x \preceq_i x'\}$ is closed for all i; $(b.4)'$ if x, $x' \in X_i$ are such that $x \preceq_i x'$ and $0 < r \leq 1$ then $x \prec_i (1-r)x + rx'$; $(b.5)'$ $\omega \in \mathrm{int}(X)$, where ω is the aggregate social endowment and $X = \sum X_i$; $(b.6)'$ $\forall_j$, $0 \in Y_j$; $(b.7)'$ $Y = \sum_j Y_j$ is closed and convex; $(b.8)'$ $Y \cap (-Y) = \{0\}$; $(b.9)'$ $Y \supset (-\Re^{\ell}_{+})$; then given an endowment transfer scheme which guarantees a distribution of the social endowment $(\omega_1, \ldots, \omega_n)$ such that $\omega_i \in \mathrm{int}(X_i)$ for all i with $\omega = \sum_i \omega_i$, there exists a Walrasian equilibrium for $\boldsymbol{E}$.*

Proof. From Lemma 5.4 and conditions (b.1)′ and (b.5)′, a redistribution of the social endowment exists in which after the redistribution, everyone has an initial endowment in the interior of his or her consumption set. From a result in Mas-Colell *et al.* (1995; p. 634) the conditions of the theorem are sufficient for the existence of a Walrasian quasi-equilibrium. Under the transfer scheme, $\omega_i \in \mathrm{int}(X_i)$ for all i it follows from the 'cheaper point condition' that there is $x_i \in X_i$ such that $px_i < p\omega_i + \sum_j \theta_{ij} p y_j *$ is satisfied for each i. Consequently a Walrasian equilibrium exists for $\boldsymbol{E}$. □

Remark 5.4. Condition (b.5)′ is similar to an assumption introduced by Hart and Kuhn (1975; p. 343). However in order to prove the existence of Walrasian equilibrium, they also needed to assume that $\omega_i \in X_i$ and that the economy is McKenzie irreducible or that all consumers are resource related. The approach here is able to avoid that assumption.

Remark 5.5. The usual statement of the Second Fundamental Theorem of Welfare Economics is that if certain conditions hold on the primitives of the economy then, after an appropriate redistribution of initial endowments, if necessary, there exists a Walrasian equilibrium price system which supports an arbitrary Pareto optimum (see Chap. 9 for a discussion of this result). It is interesting to note that the existence theorem which we have just proved may be summarised in similar terms. That is, if certain conditions hold on the primitives of the economy then, possibly after an appropriate redistribution of the initial endowment, a Walrasian equilibrium exists. Although the potential need for an initial redistribution in order to achieve the conclusions of the SFTWE has long been recognised, similar recognition has not been so widely given to the possible need for an initial redistribution of endowments in order to guarantee the *existence* of equilibrium.

5.2.3. *Profit share redistributions*

Endowment tax and transfer schemes of the sort studied in the previous section may be one way to facilitate the existence of equilibrium in the absence of the sorts of strong relationship assumptions discussed in earlier chapters. Another approach might involve a tax-transfer scheme based not on endowments but on profit shares. It is to a consideration of a set of circumstances in which this yields the existence of Walrasian equilibrium result that we now turn.

To begin we note an interesting argument that comes up in the treatment of the existence problem for Walrasian equilibrium due to Aliprantis *et al.* (1989). In particular they remark that each consumers' budget set is 'larger' in a production economy than it is in an exchange economy (see Aliprantis *et al.* (1989; p. 76)). Although this claim is not strictly correct for reasons detailed below, it is nevertheless interesting because it implicitly suggests a novel way in which the problem of consumer non-survival and demand discontinuity can be handled. Following Aliprantis *et al.* (1989; p. 75), an economy $\boldsymbol{E}_{nc}$ is a *neoclassical private ownership production economy* if for all i, j (i) $\preceq_i$ are *neoclassical*, meaning that preferences are defined everywhere on $\Re^\ell_+$ and are either strictly monotone and strictly convex everywhere or are strictly monotone and strictly convex on $\text{int}(\Re^\ell_+)$ and anything in $\text{int}(\Re^\ell_+)$ is preferred to anything on the boundary of $\Re^\ell_+$; (ii) for all i, $\omega_i > 0$ and $\omega = \sum_i \omega_i \gg 0$; (iii) Y_j is closed, strictly convex, bounded above, $Y_j \cap \Re^\ell_+ = \{0\}$ and the profit maximising supply of firm j at p is $y_j(p)$; (iv) there are real numbers θ_{ij} with $0 \leq \theta_{ij} \leq 1$ and $\sum_i \theta_{ij} = 1$, which represent consumer i's share of producer j's profit. The income of i in $\boldsymbol{E}_{nc}$ is $M_i(p) = p\omega_i + \sum_j \theta_{ij} p y_j(p)$ and ith consumer's budget set is given by $\{x_i \in \Re^\ell_+ \colon px_i \leq M_i(p)\}$.

Aliprantis *et al.* (1989) establish that $M_i(p) > 0$ for all i in $\boldsymbol{E}_{nc}$ because strict monotonicity of preferences means $p \gg 0$ and by assumption $\omega_i > 0$. Also $py_j(p) \geq 0$ for all firms j because $0 \in Y_j$. They then argue that: "...since each consumer shares part of each producers profit, each consumers budget set is 'larger' than the set $\{x_i \in \Re^\ell_+ \colon px_i \leq p\omega_i\}$ see Fig. 1.7–5, Aliprantis *et al.* (1989; p. 76). Their Fig. 1.7–5 is reproduced as Fig. 5.1.

The claim by Aliprantis *et al.* (1989; p. 76) that each consumer has a larger budget set in a production economy than he or she does in an exchange economy, is not strictly correct for two reasons. Firstly, the share of the profit of firm j due to consumer i is restricted by the following two conditions[1]: $0 \leq \theta_{ij} \leq 1$ and $\sum_i \theta_{ij} = 1$. These conditions do not guarantee that *each* consumer shares a part of *each* firm. Indeed these restrictions

[1]These restrictions are standard and follow in particular from condition 4 of Definition 1.7.9 in Aliprantis *et al.* (1989; p. 75) which states that: "The economy is private ownership. That is the consumers own the firms. The real number θ_{ij} represents consumer i's share of producer j's profit. It is assumed that $0 \leq \theta_{ij} \leq 1$ holds for all i and all j and $\sum_i \theta_{ij} = 1$ for all j."

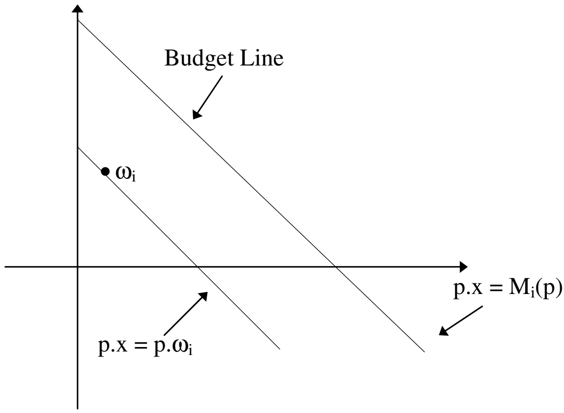

Fig. 5.1. Aliprantis *et al.* (1989) budget sets.

do not guarantee that each consumer shares any part of any producers profit, and in fact, they are consistent with just one person owning all the shares in each firm and everyone else owning nothing at all. Secondly, even if every consumer did own a positive share in each firm, the requirement that production sets are strictly convex, coupled with the other conditions which characterise production sets[2] in E_{nc}, are not strong enough to ensure that at least one producer makes a positive profit at a given price p, even if $p \gg 0$. To see why this is so consider the following production set which satisfies all the conditions imposed by Aliprantis *et al.* (1989; p. 69) $Y = \{(y_1, y_2) : y_1 < 1$ and $y_2 < y_1/(y_1 - 1)\}$. As Aliprantis *et al.* (1989; p. 80) demonstrate, the supply function of a firm operating with this technology is $y(p) = (1 - t, 1 - 1/t)$, where $t = (p_1/p_2)^{1/5}$. At the price $p = (1, 1)$, supply is $y(1, 1) = (1 - 1, 1 - 1/1) = 0$. Thus the profit of this firm at $p = (1, 1)$ is $\pi(1, 1) = (1, 1).(0, 0)^T = 0$. A picture of the situation is in Fig. 5.2.

[2]The conditions on production sets in Aliprantis *et al.* (1989; p. 69) are that Y is a non-empty bounded from above subset of a finite-dimensional $\Re^\ell$ which is closed, convex and $\Re^\ell \cap Y = \{0\}$.

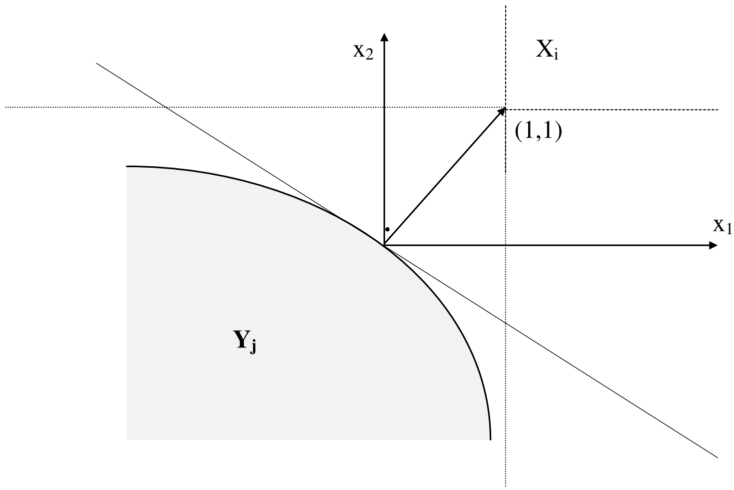

Fig. 5.2. An Aliprantis *et al.* (1989) production economy.

For both these reasons then, it is not generally correct to claim that each has a budget set which is larger in a production economy than it is in the corresponding exchange economy. However, their remark is instructive because it implicitly suggests a way to handle the consumer non-survival/non-participation and demand discontinuity problem. To see how, we follow the Aliprantis *et al.* approach to proving the existence of Walrasian equilibrium and introduce the modification that their remark suggests at the appropriate point in the argument. Following Aliprantis *et al.* (1989), assume that i's consumption set is $\Re^\ell_+$ and for all i, $\omega_i \in \Re^\ell_+ \backslash \{0\}$. While this assumption on endowments is weaker than that of interior endowments, because it does not require every individual to be endowed with a positive amount of every good, the assumption still rules out the case where an individual has no initial endowment of any good. Their argument also depends on all prices remaining positive and the consumption possibility set being $\Re^\ell_+$. The approach which their idea inspires allows a weakening of the endowment assumption to the case where for every i, $\omega_i \in \Re^\ell_+$ an assumption which allows for the empirically relevant possibility that at least some individuals do not own an initial endowment of any goods or labour types.[3]

Definition 5.2 (Profit distributed economy). An economy $\mathbfit{E}_{pd}$ is a *profit distributed neoclassical private ownership production economy* if for all i, j, the following conditions hold: (i) $\preceq_i$ are neoclassical, (ii′) $\omega_i \geq 0$, (iii′) Y_j are closed, strictly convex, bounded above, $0 \in Y_j$, and ($y \in Y_j$ and $z \leq y$) implies $z \in Y_j$, (iv′) profit shares are such that $0 < \theta_{ij} < 1$ for all i, j subject to $\sum_i \theta_{ij} = 1$, (v) the social endowment ω is $\omega = \sum_i \omega_i \gg 0$. An input-output vector $y \in Y_j$ is called *efficient* whenever $(y + \Re^{\ell}_{++}) \cap Y_j = \emptyset$.

Lemma 5.6 (Balasko (1988)). *If in $\mathbfit{E}_{pd}$, the relative interior of the set of efficient productions is open smooth and in $\Re^{\ell}$ and has non-zero Gaussian curvature everywhere, then the supply function $y_j(p)$ exists, is a smooth map $y_j : \Re^{\ell-1}_{++} \to \Re^{\ell}$ and profit, $py_j(p) > 0$ if $p \gg 0$, $\forall_j$.*

Proof. Balasko (1988; p. 202). □

Corollary 5.1. *In $\mathbfit{E}_{pd}$ $M_i(p) > 0$ for all i, whenever $p \gg 0$ even if $\omega_i = 0$.*

Proof. Since $p \gg 0$ and $\omega_i \geq 0$, $p\omega_i \geq 0$. By condition (iv′) in the definition of $\mathbfit{E}_{pd}$ $0 < \theta_{ij} < 1$ for all i, j. Then by Lemma 5.5 it is true that $\sum_j \theta_{ij} py_j(p) > 0$ for all i and by definition of $M_i(p)$ in $\mathbfit{E}_{pd}$ the result follows. □

[3] Allowing $\omega_i > 0$ instead of $\omega_i \gg 0$ also relies on all prices being positive, otherwise the income of consumer i would go to zero in the case where he or she is holding an initial endowment consisting of valueless goods. Such behaviour on the part of prices is ensured by the assumptions maintained on preferences and production sets by Aliprantis *et al.* (1989). These assumptions will also be maintained in this section in order to illustrate our central point, however it should be borne in mind that these strong assumptions on preferences and technology can also be weakened where the redistribution scheme which we here describe is operational. Such a step is taken in the next section of this chapter.

Lemma 5.7 (Following Aliprantis *et al.* (1989; p. 75)). *In any* $\boldsymbol{E}_{pd}$, *the income function* $M_i : \Re_{++}^{\ell-1} \to (0, \infty)$ *is continuous.*

Proof. From Lemma 5.5, $y_j(p)$ is smooth and therefore continuous. The conclusion follows from the joint continuity of the dot product. □

Lemma 5.8 (Following Aliprantis *et al.* (1989; p. 76–79)). *If* $x_i(p)$ *is the demand function for consumer* i *in* $\boldsymbol{E}_{pd}$ *then* $x_i(p)$ *is homogeneous of degree zero in* p, *continuous for* $p \in \text{int}(\Re_+^\ell)$ *and any sequence* $\{p_n\} \subseteq \Re_{++}^\ell$ *such that* $p_n \to (p_1, \ldots, p_\ell)$ *with* $p_r > 0$ *implies that the sequence of demands* $\{x_i^r(p)\}$ *is bounded. Further, if* $\{p_n\}$ *is a sequence of strictly positive prices satisfying* $p_n \to p \in \partial\Re_+^\ell \backslash \{0\}$ *then there exists at least one* $1 \leq r \leq \ell$ *such that either:* $\limsup n \to \infty\ x_i^r(p) = \infty$ *or* $\limsup n \to \infty\ y_j^r(p) = -\infty$, *holds for some consumer* i *or some producer* j.

Proof. Since in $\boldsymbol{E}_{pd}\omega \gg 0$, $M_i(p) > 0$ for all i and $p \gg 0$ the proof proceeds as in Aliprantis *et al.* (1989; p. 78). □

Lemma 5.9 (Following Aliprantis *et al.* (1989; p. 79)). *The excess demand function* $Z(p)$ *in* $\boldsymbol{E}_{pd}$ *is continuous, bounded below, satisfies Walras' law and is homogeneous of degree zero. If a sequence* $\{p_n\}$ *of strictly positive prices satisfies* $p_n \to p = (p_1, \ldots, p_\ell)$ *and* $p_k > 0$ *for some* k, *then the sequence* $\{Z_k(p_n)\}$ *of the kth component of* $Z(p_n)$ *is bounded and if* $p_n \gg 0$ *for each* n *and* $p_n \to p \in \partial\Re_+^\ell \backslash \{0\}$ *then* $\lim n \to \infty \|Z(p_n)\|_1 = \infty$.

Proof. The proof proceeds as in Aliprantis *et al.* (1989; p. 79). □

Theorem 5.3. *Every profit distributed neoclassical private ownership production economy* $\boldsymbol{E}_{pd}$ *has a Walrasian equilibrium price system* p^*. *That is there exists a price vector* $p^* \gg 0$ *such that* $Z(p^*) = 0$ *and all agents originally present in the economy are able to survive.*

Proof. By Lemma 5.8, the excess demand function in $\boldsymbol{E}_{pd}$ satisfies the hypotheses of Aliprantis *et al.* (1989, Theorem 1.4.8). Consequently, there exists a price $p^* \gg 0$ such that $Z(p^*) = 0$. Further, everyone originally present in the economy is able to survive because $X_i = \Re^\ell_+$ and the assumption that $0 < \theta_{ij} < 1$ for all i, j coupled with the assumptions made about Y_j ensures that $M_i(p) > 0$ for all $p \gg 0$. □

Remark 5.6. Condition (ii′) in Definition 5.2 is a weakening of condition (ii) in Aliprantis *et al.* (1989), since it allows for the possibility that consumers do not own a positive initial endowment of any good. Condition (iii′) is a slight strengthening of condition (iii) which is needed to ensure that each firm makes positive profits when $p \gg 0$, while condition (iv′) is inspired by the idea in Aliprantis *et al.* (1989) about the relative size of a consumer's budget set in an exchange versus a production economy. This approach to proving an existence result again illustrates the idea that it might be possible to use economic policy in such a way as to help ensure the existence of Walrasian equilibrium when standard conditions such as positive initial endowments and various forms of irreducibility are not invoked.

Remark 5.7. The existence theorem just proved would also hold under the weaker assumptions that either everyone has a positive share in at least one firm, or alternatively, that there is just one firm in the economy which makes a positive profit and that everyone has a non-zero share in that firm. Theorem 5.3, along with the existence theorem proposed by Aliprantis *et al.* (1989) does, however, still depend on special structure of preferences, conditions that we attempt to relax in the next section.

5.2.4. *The Gale–Mas-Colell's theorem with an explicit income function*

In a paper that Debreu (1982; p. 714) described 'remarkable' and which Khan (1993; p. 36) characterised it as being 'at the heart of the classical theorem on the existence of Walrasian equilibrium in its fully developed form', Mas-Colell (1974) presented an existence theorem which did not impose on consumer preferences the previously standard assumptions of completeness and transitivity. In a subsequent paper, Gale and Mas-Colell (1975) gave a simpler proof of Mas-Colell's original result, while in Gale and Mas-Colell (1979), the authors made two corrections to the argument presented in their earlier work. The first correction had to do with the bound used to truncate consumption sets, while the second modified the augmented preference map so that it had the needed 'open graph' property. The starting point of our work is the observation that there is another innovation in Gale and Mas-Colell (1975) which is of interest from our point of view. In particular, while the dramatic weakening of the classical assumptions on preferences achieved in Mas-Colell (1974) and Gale and Mas-Colell (1975) naturally captured most attention, there are according to the authors, two other aspects of their work which also involves a significant generalisation of the standard Walrasian model. They are (i) the specification of an individual's income function as any continuous function of prices and (ii) their weak assumption on the production technology which requires only that it is not possible to obtain infinite output with zero input (see Gale and Mas-Colell (1975; pp. 9–10) for further discussion). It is the treatment of individual incomes by Gale and Mas-Colell (1975) that will be the focus here. Of particular interest is their remark that: "... [our] condition (10) guarantees that no trader will be allowed to starve no matter what the prices are. The need for this sort of condition is familiar. In pure exchange models for example, it is achieved by the customary, and unpleasant, assumption that all traders have a strictly positive initial endowment. In our present, more general way of looking at equilibrium the assumption (of non-starvation) becomes more palatable. Not many economies in the present day are so extremely *laissez-faire* as to permit people to starve." Gale and Mas-Colell (1975, p. 12).

The spirit of this remark is not being contested since, as has been argued at length in earlier chapters, it is highly desirable to avoid strong assumptions, such as interior endowments, in order to ensure consumer survival, *en route* to making an existence proof for Walrasian equilibrium. Indeed the fact that strong 'relationship assumptions' like interior endowments cannot be avoided when attempting to prove the existence of Walrasian equilibrium in standard private ownership economies is at the heart of our argument that the existence of Walrasian equilibrium, in such economies, is more problematic than is sometimes thought. However, there are two problems with the Gale and Mas-Colell (1975) argument which were not addressed in Gale and Mas-Colell (1979). Firstly, it is not the case that the assumptions made by Gale and Mas-Colell about the consumption and production sets in their economy are in fact strong enough to ensure survival income for everyone in the economy. We show that their assumption that individual incomes will always exceed the sustenance level no matter what prices prevail in the market, is not guaranteed by the structure imposed by them on their economy. Secondly, having pointed out that survival income is essential for the existence of Walrasian equilibrium and having dismissed as implausible the standard assumption for achieving that end, Gale and Mas-Colell (1975) say nothing about the mechanism by which survival income is guaranteed in their model. Newman (1987) draws attention to this when he remarks that: "[s]ome models … attempt to justify Slater-like conditions directly on the grounds that 'Not many economies in the present day are so extremely *laissez faire* as to permit people to starve' … This justification clearly fails as long as the behaviour of the public agency whose actions allegedly prevent such starvation is not modelled *explicitly*, like that of private agents." Newman (1987; p. 617, emphasis in original). In what follows we address this concern and make explicit an income mechanism which achieves what Gale and Mas-Colell assume as far as individual incomes are concerned.

We begin by describing an aspect of the economy specified by Gale and Mas-Colell (1975). In particular we are interested in their Condition (10) and their Eqs. (2) and (3). Using their notation their Eqs. (2) and (3) may be written as follows:

(A2): For any p in Δ, define the *profit function* $\Pi(p)$ by the rule $\Pi(p) = \sup pY$ and define $\Delta' \subset \Delta$ as the set of p in Δ for which this supremum is finite.

(A3): Postulate the existence of m real valued functions α_i on Δ' (to be called *income functions*) satisfying $\sum_i \alpha_i(p) = \Pi(p)$ for all p in Δ'.

These two conditions yield (M): $\sum_i \alpha_i(p) = \sup pY$ for $p \in \Delta'$. Equation (10) in Gale and Mas-Colell (1975) requires that (A10): $\alpha_i(p) > \inf pX_i$ for $p \in \Delta'$ holds. (A2), (A3) and (A10) together imply that in the Gale and Mas-Colell economy: (S): $\sup pY = \sum_i \alpha_i(p) > \sum_i \inf pX_i$ for $p \in \Delta'$. The assumptions made about X_i and Y in Gale and Mas-Colell (1975) are: (A8): $X_i \subset R^n$, is not empty, closed, convex and bounded below and (A9): $Y \subset R^n$, is closed, convex, contains the negative orthant and has bounded intersection with the positive orthant. We now show that it can happen that the economy satisfies conditions (A8) and (A9) but where it is not the case that $\sup pY > \sum_i \inf pX_i$ for all $p \in \Delta'$ as required by (A2), (A3) and (S). Let the ith consumer's consumption set is $X_i = \{(x_{i1}, x_{i2}) \in R^2 : x_{i1} \geq 1,\ x_{i2} \geq 1\}$ and the economy's production set is $Y = \{(y_1, y_2) \in R^2 : y_1 \leq 1,\ y_2 = y_1/(y_1 - 1) + 1/2\}$. Clearly for this economy, $\sum_i \inf pX_i > \sup pY$ for $p = (1/2, 1/2) \in \Delta'$ so that the conditions imposed on X_i and Y via conditions (A8) and (A9) are not sufficient to guarantee that (A10) also holds for $p \in \Delta'$.

We now present a series of conditions on the economy which are strong enough to ensure that everyone has at least a survival income, no matter what market prices prevail. We also provide an explicit mechanism by which survival income can be ensured, namely an 'economic policy', appropriately arranged, and in the process meet the objection raised by Newman (1987) to the argument of Gale and Mas-Colell (1975).

Theorem 5.4. *If in* E (*a*.1) *for all* j, Y_j *is closed, strictly convex, bounded above,* $0 \in Y_j$ *and* $y \in Y_j$ *and* $z < y$ *implies* $z \in Y_j$; (*a*.2) $\omega \in int(X)$ *and there is a redistributive mechanism in the economy which guarantees* $\omega \in int(X_i)$ *for all* i; (*a*.3) *for all consumers* i, $X_i \subset R^n$; (*a*.4) *for all* i *the strict preference maps* $\prec_i$ *are irreflexive have open graphs in* $X_i \times X_i$ *and their values are non-empty convex sets*; (*a*.5) *the* i*th consumers income function is given by* $\alpha_i(p) = p\omega_i + \sum_j \theta_{ij} p y_j$; *then Walrasian equilibrium exists for* E.

Proof. We need to show that if (a.5) governs the way income is determined in the economy, then these income functions are continuous for all $p \in \Delta'$ and satisfy the condition $\alpha_i(p) > \inf p.X_i$. From Balasko (1988; p. 202), we know that if (a.1) describes the production technology, then $y_j(p)$ exists and is a smooth map. Since $\omega_i \in \mathrm{int}(X_i)$ for all i and $p \in \Delta$, individual income functions are all continuous by virtue of the smoothness of $y_j(p)$ and the joint continuity of the dot product and the condition $\alpha_i(p) > \inf pX_i$ is satisfied for all i. The rest of the proof then proceeds as in Gale and Mas-Colell (1975, 1979). □

Remark 5.8. Condition (a.1) involves a strengthening of the requirements on Y relative to Gale and Mas-Colell (1975, 1979). Assumption (a.2) introduces an explicit redistribution mechanism plus an assumption that the social endowment is in the interior of the aggregate consumption possibility set. The payoff from this is that when income is defined as in (a.5), and everyone is guaranteed a positive share in the profit of every firm then, condition (M) will be satisfied by the economy. By this device, we have avoided the difficulty noted in Gale and Mas-Colell's original argument. We have also addressed the concern of Newman (1987) by specifying a mechanism by which survival income is to be guaranteed.

5.2.5. *Tax-transfers and 'patching' irreducibility*

As was noted in Chap. 2, research aimed at finding a more plausible condition than 'interior endowments' has produced a rich list of alternatives, many of which are related to the idea of 'irreducibility' introduced by McKenzie (1959) and refined by McKenzie (1981) and others. As we have also seen, while these conditions differ in detail, they share in common the feature that they require particular relationships to hold among the primitives that define the economy. As there is nothing in the operation of an economy that guarantees these relationships will hold, it is of interest to ask if public policy, in particular a tax-transfer scheme, may be able to 'patch' a breakdown in irreducibility if that were to occur. In this section, we show that a breakdown of irreducibility can be repaired by a tax and transfer scheme, at least as far as the existence of equilibrium is concerned. It is perhaps worth observing that while there are a number of papers which establish the existence of equilibrium with taxes and/or transfers (e.g Mantel (1975), Shafer and Sonnenschein (1976), Dieker and Haller (1990)) in these papers taxes and transfers are regarded as a complicating institutional features to be incorporated into an existence argument. The orientation here is different in that we show how the preconditions for equilibrium can be engineered by an appropriate tax-transfer scheme. Following the notation and nomenclature in Florenzano (2003), let I be a finite set of consumers, J be a finite set of producers, X_i be the consumption possibility set for consumer

i and Y_j be the production possibility set for firm j. The set S of normalized prices is $\{p \in \Re^\ell : \|p\| = 1\}$. The preference correspondence for i is a map $P_i : \prod_{i\in I} X_i \times \prod_{j\in j} Yj \times S \to X_i$ with $P_i(x, y, p) = \{x'_i \in X_i : x'_i \succ_i x_i\}$ and $x_i \notin P_i(x, y, p)$. Also let $\hat{P}_i(x, y, p) = \{x'_i \in X_i : x'_i = x_i + \lambda(x''_i - x_i), 0 < \lambda \leq 1, x''_i \in P_i(x, y, p)\}$ be the 'augmented preference correspondence' for i. θ_{ij} is the share of firm j's profit going to consumer i and consumer i's endowment is $\omega_i \in \Re^\ell$. The total wealth of consumer i at p is $w_i = p.\omega_i + \sum_{j\in J} \theta_{ij} p.y_j$. The *disposal cone* Z, is a convex cone with vertex $0 \in Z$ and contained in $\Re^\ell_+$. $Z^0 = \{p \in \Re^\ell : p.z \leq 0 \; \forall z \in Z\}$ is the polar cone of Z. The set of attainable allocations for $\boldsymbol{E}$is $A(\boldsymbol{E}) = \{(x, y) \in \prod_{i\in I} X_i \times \prod_{j\in J} Y_j : \sum_{i\in I} x_i - \sum_{j\in J} y_j - \omega \in Z\}$. $\hat{X} = \{x \in \prod_{i\in I} X_i : \exists y \in Y$ and $\sum_{i\in I} x_i - y - \omega \in Z\}$ are the attainable consumption allocations and $\hat{Y} = \{y \in Y : \exists x \in \prod_{i\in I} X_i$ and $\sum_{i\in I} x_i - y - \omega \in Z\}$ is the attainable total production set. An allocation $(x, y) \in \prod_{i\in I} X_i \times \prod_{j\in J} Y_j$ is called *attainable* if $\sum_{i\in I} x_i - \sum_{j\in J} y_j - \omega \in Z$.

Definition 5.3 (McKenzie-Debreu irreducible). Let $T_y(Y) = \text{cl}\{z \in \Re^\ell : z = \lambda(y' - y), \lambda > 0,\; y' \in Y, y = \sum_{j\in J} y_j\}$ then an economy $\boldsymbol{E}$ is *McKenzie-Debreu irreducible* if, for any partition of I into two non-empty sub-sets $\{I_1, I_2\}$ and for each $(x, y, p) \in A(\boldsymbol{E}) \times (S \cap Z^0)$, there exists $x' \in \prod_{i\in I} X_i$ such that:

(i) $x' \in \text{cl}(\hat{P}_i(x, y, p))$ for each $i \in I_1$ with for some $i_1 \in I_1, x'_{i1} \in \hat{P}_{i1}(x, y, p)$;
(ii) $\sum_{i\in I1}(x'_i - x_i) + \sum_{i\in I2}(x'_i - \omega_i) = y \in (T_y(Y) + Z)$.

Remark 5.9. Inspection of (ii) in this definition shows that the existence of a feasible trade from the group of consumers I_2 (a set which may contain just one element), depends on the location of the ω_i's relative to the x'_i's. Note that Part (ii) of the definition can be rewritten as: $\sum_{i\in I2} \omega_i = \sum_{i\in I} x'_i - y - \sum_{i\in I1} x_i$ for $y \in (T_y(Y) + Z)$. It is clear from this expression that for McKenzie-Debreu irreducibility to hold, a relationship needs to hold between the endowments of the consumers in group I_2 and the preferences of the consumers in group I_1. Suppose that the needed conditions for McKenzie-Debreu irreducibility do not obtain, are there circumstances in which some form of public policy induce it?

Definition 5.4 (Tax induced McKenzie-Debreu irreduciblity). A *tax-transfer scheme that induces McKenzie-Debreu irreducibility* is a scheme that allocates to consumer i the share $t_i\omega$ of the total endowment ω, such that the following holds. For any partition of I into sub-sets $\{I_1, I_2\}$ such that $I_1 \cap I_2 = \emptyset$ and $I_1 \cup I_2 = I$ and for each $(x, y, p) \in A(\boldsymbol{E}) \times (S \cap Z^0)$, there exists $x' \in \prod_{i \in I} X_i$ such that:

(i) $\text{cl}(\hat{P}_i(x, y, p))$ for each $i \in I_1$ with for some $i_1 \in I_1$, $x'_{i1} \in \hat{P}_{i1}(x, y, p)$;
(ii) $\sum_{i \in I1}(x'_i - x_i) + \sum_{i \in I2}(x'_i - t_i\omega) = y \in (T_y(Y) + Z)$.

Similarly, using the definition of Bergstrom-Florig irreducibility.

Definition 5.5 (Bergstrom-Florig irreducibility). An economy $\boldsymbol{E}$ is *Bergstrom-Florig irreducible* if for any partition of I into two non-empty sub-sets $\{I_1, I_2\}$ and for each $(x, y, p) \in A(\boldsymbol{E}) \times (S \cap Z^0)$, there exist real numbers $\theta_i > 0$, $i \in I$ and $x' \in \prod_{i \in I} X_i$ such that:

(i) $x' \in \text{cl}(\hat{P}_i(x, y, p))$ for each $i \in I_1$ with for some $i_1 \in I_1, x'_{i1} \in \hat{P}_{i1}(x, y, p)$;
(ii) $\sum_{i \in I} \theta_i(x'_i - \omega_i - \sum_{j \in J} \theta_{ij} y_j) \in (T_y(Y) + Z)$.

Remark 5.10. Inspection of the condition for Bergstrom-Florig irreducibility indicates that similar idea to that contained in Definition 5.4 will work in this case also.

To complete the existence argument in this case we use the formulation of quasi-equilibrium and equilibrium due to Florenzano (2003):

Definition 5.6 (Quasi-equilibrium and equilibrium). A tuple (x^*, y^*, p^*) consisting of an attainable allocation (x^*, y^*) and a non-zero price vector p^* is called a *quasi-equilibrium* if: (i) for every $i \in I$, $p^*x_i^* \leq w_i^*$ and $x_i \in P_i(x^*, y^*, p^*)$ implies $p^*x_i \geq p^*x_i^*$; (ii) for each $j \in J$ and $y_j \in Y_j$, y_j^* is profit maximising at p^* so $\forall y_j \in Y_j \; p^*y_j \leq p^*y_j^*$; (iii) $p^* \in Z^0$ and $p^* \cdot \sum_{i\in I} x_i^* = p^* \cdot \sum_{j\in J} y_j^* + p^* \cdot \sum_{i\in I} \omega_i$. A quasi-equilibrium is *non-trivial* if $\exists i \in I$ and $x_i \in X_i$ such that $p^*x_i < p^*x_i^*$. Let $\delta_i(p^*) = \{x_i \in X_i : p^*x_i < p^*x_i^* = p^*\omega_i + \sum_{j\in J} \theta_{ij}p^*y_j\}$ be the set of 'cheaper points' relative to the allocation that consumer i receives in the quasi-equilibrium. A tuple (x^*, y^*, p^*) consisting of an attainable allocation (x^*, y^*) and a non-zero price vector p^* is called an *equilibrium* if: (i) for every $i \in I$, $p^*x_i^* \leq w_i^*$ and $x_i \in P_i(x^*, y^*, p^*)$ implies $p^*x_i > p^*x_i^*$; (ii) for each $j \in J$ and $y_j \in Y_j$, y_j^* is profit maximising at p^* so $\forall y_j \in Y_j p^*y_j \leq p^*y_j^*$; (iii) the cost of the disposal needed to achieve equilibrium is zero so that $p^* \in Z^0$ and $p^* \cdot \sum_{i\in I} x_i^* = p^* \cdot \sum_{j\in J} y_j^* + p^* \cdot \sum_{i\in I} \omega_i$.

Proposition 5.1. *If* (x^*, y^*, p^*) *is a non-trivial quasi-equilibrium for* $\boldsymbol{E}$, *then it is also an equilibrium for* $\boldsymbol{E}$ *if* $(bt.1)$ *each* X_i *and* Y_j *are convex*; $(bt.2)$ *for all* $i \in I$ *and* $(x, y, p) \in A(\boldsymbol{E}) \times (S \cap Z^0)$ *if* $z_i \in P_i(x, y, p)$ *and* $v_i \in X_i$ *then there exists* $0 < \lambda \leq 1$ *such that* $(\lambda v_i + (1-\lambda)z_i) \in P_i(x, y, p)$; $(bt.3)$ $\boldsymbol{E}$ *is tax-induced McKenzie-Debreu irreducible or tax-induced Bergstrom-Florig irreducible.*

Proof. The proposition follows from Proposition 2.3.3 in Florenzano (2003; p. 65). □

5.3. Voluntary transfers, altruism and the existence of Walrasian equilibrium

The Walrasian vision of an harmonious, efficient, market clearing economy in which every agent is able to make all budget feasible welfare improving trades, is both powerful and appealing. That the vision depends on the economy having a particular structure is by now well understood. If the needed structure, particularly with respect to agent survival, is not in place *a priori* then, as we saw in the previous section, one way to proceed might be to deploy various economic policy instruments to ensure the preconditions for the existence of Walrasian equilibrium are present. While this approach is an improvement to the 'let's just hope the conditions needed for equilibrium are present' outlook, it would be nice if some mechanism *endogenous* to the economy could be found which guaranteed that consumers always had enough income so that a cheaper point always existed in their budget sets. One such mechanism might be 'altruism' and the voluntary transfers of income that it might stimulate. In this section, we consider something of the potential for this mechanism in guaranteeing the existence of Walrasian equilibrium.

We begin by again recalling the economy studied by Gale (1957, 1976) and by showing that an altruistic redistribution could restore equilibrium to an economy where none existed before. In the Gale example, there are two individuals A and B with consumption sets $X_1 = X_2 = \Re^2_+$, utility functions $u_A(x_1, x_2) = x_2$, $u_B(x_1, x_2) = x_1 + x_2$ and endowments $\omega_A = (1, 1), \omega_B = (1, 0)$ and for reasons spelled out in Chap. 2, the economy does not have a Walrasian equilibrium. Now suppose we change the example so that A has altruistic concern for the welfare of B and transfers $1/2$ a unit of good 2 to individual B. The endowments of the two people are now $\omega'_A = (1, 1/2)$, $\omega'_B = (1, 1/2)$ and an equilibrium price vector $p^e = (1/2, 1)$ emerges. The economy may be sketched as in Fig. 5.3.

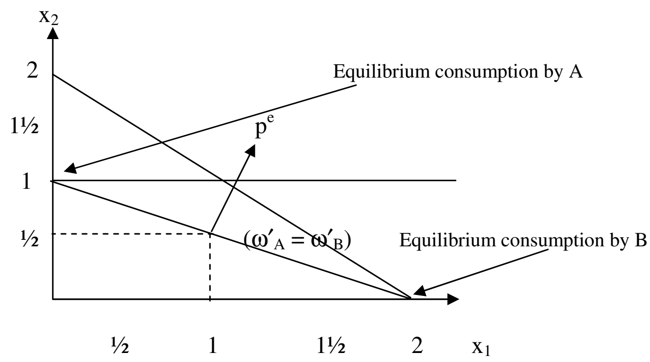

Fig. 5.3. Existence of equilibrium with voluntary redistribution.

This example demonstrates that an altruistic redistribution may restore Walrasian equilibrium in a model where equilibrium previously did not exist. It is also interesting to note that one person, (B) is strictly better off and the other person, (A) no worse off in this equilibrium relative to the situations they would have experienced had the transfer not taken place. While this example is interesting, it is necessary to ask at least two questions before concluding that altruism may provide a generally interesting mechanism for helping to establish Walrasian equilibrium. The first question that needs to be asked is, not withstanding the above example, does the presence of altruism create particular difficulties when it comes to proving the existence of Walrasian equilibrium? The second question is can altruistically motivated voluntary transfers generally be relied on, particularly in large economies?

The answer to the first question appears to be 'no', as the following result due to Kranich (1988) shows. Consider an exchange economy with n agents and ℓ commodities. Agents are indexed by i, j, k and the set of agents is I. Commodities are indexed by ℓ and the set of commodities is L. Agent i is characterised by a consumption possibility set $X_i \subseteq \Re^{\ell}_{+}$, a domain of transfers $T_i \subseteq \Re^{(n-1)\ell}_{+}$, a non-zero endowment $\omega_i \in X_i$ and continuous preferences represented by a utility function $u_i : X_i \times \Re^\ell_+ \to \Re$. *Total endowment* is ω and $\omega = \sum_{i \in I} \omega_i$, and X and T denote, respectively, the Cartesian products of all the X_i's and T_i's. It is assumed that $\omega \in \Re^\ell_{++}$. Agents are permitted to engage in commodity transfers. Let $t^{ij} \in \Re^n_+$ denote *the transfer from agent i to agent j*. Since transfers are voluntary these numbers are restricted to be non-negative. A *transfer plan* for the ith agent is a list $t^i = (t^{i1}, \ldots, t^{i(i-1)}, t^{i(i+1)}, \ldots, t^{in}) \in T_i$. The *net transfer from i to j* is $\tau^{ij} \equiv t^{ij} - t^{ji}$. In this economy preferences are specified over the pairs $(x_i, \theta) \in X_i \times \Re^n_+$, where x_i is person i's own consumption vector and θ is an n-dimensional vector which describes the distribution of income in the economy. Given this set up, Kranich (1988) proves the following existence theorem.

Theorem 5.5 (Kranich (1988)). *If the economy* $\boldsymbol{E}$ *satisfies*: $(k.1)$ $\forall_i, X_i$ *is compact, convex and* $0 \in X_i$; $(k.2)$ $\forall_i$, T_i *is compact, convex and* $0 \in T_i$; $(k.3)$ $\forall_i$, $u_i(x_i, \theta)$ *is twice continuously differentiable*; $(k.4)$ $\forall_i, u_i(x_i, \theta)$ *is quasi-concave*; $(k.5)$ $\forall_i, u_i(x_i, \theta)$ *is non-satiated so that for any* $\ell \in L, \partial u_i / \partial x_\ell > 0$ *then a Walrasian equilibrium exists for* $\boldsymbol{E}$.

Proof. Kranich (1988; p. 377). □

Remark 5.11. This theorem establishes that the presence of altruism is not in itself inimical to the existence of Walrasian equilibrium. In addition, as Kranich (1988) notes, the existence of equilibrium is not dependent on the form of the altruism, that is agents could be benevolent, malevolent or neutral toward each other without that influencing existence.[4]

[4]It is interesting to note however that another result in Kranich (1988) establishes that a Walrasian equilibrium with altruistic transfers is generally not efficient in the sense that the equilibrium is generally not Pareto optimal. This is an interesting theoretical result because even if altruism could be relied on to guarantee the existence of equilibrium then another extremely important part of the Walrasian vision, namely market efficiency, may fail. This possibility is discussed further in Chap. 9.

We now consider the second question about whether altruism can generally be relied on to effect voluntary redistributions of wealth. There is a long literature on this subject, dating at least from Hochman and Rogers (1969) who noticed that if the economy consists of just two people, one rich and one poor, and if the welfare of the poor depends solely on their income while the welfare of the rich person depends on their income and the welfare level of the poor, then depending on the details of the rich persons utility function, a transfer of income from the rich person to the poor person might occur. However, as Hochman and Rogers (1969) demonstrate, this result vanishes when there are two or more rich people in the economy. Now the welfare of the poor, if it enters the utility functions of the rich, has the characteristics of a public good. There is then a potential for each of the rich agents to attempt to free ride on any transfer that the other(s) might make. Bergstrom (1970), Nakayama (1980), Arrow (1983) and Hammond (1987) all elaborate and reinforce the basic message of the Hochman and Rogers model that like standard public goods, voluntary transfers are likely to be under-provided. As Hammond (1987) puts it: "... this [analysis makes] a *prima facie* argument for public intervention to redistribute income." Hammond (1987; p. 85), similarly Kranich remarks that: "... government transfer programs may be better suited for redistributing income than private philanthropies." (1988; p. 370). This is in the spirit of the work presented in the previous section of this chapter.

Recently however, Harrington (2001) has proposed a model which seems to challenge the Hochman and Rogers *et al.* analysis in that he finds conditions under which, even in a large economy, the probability that someone will receive 'help' (here thought of as income supplementation), is bounded below and away from zero. This is an interesting possibility and we devote some effort to studying and then modifying the model in Harrington (2001).

Harrington begins by noting the experimental and field studies reported by Latane and Nida (1981), which establish an inverse relationship between the probability that someone in need will receive help and the size of the group of potential helpers. He proposes a 'rational choice' model which attempts to account specifically for the following two phenomena that appear in these studies: (A) the likelihood that a person will help another who is in trouble declines as the size of the group to which the potential helper belongs increases; and (B) the probability that a particular person helps another person decreases as the number of potential helpers increases, but the probability that help is given is nevertheless bounded below and away from zero. Outcome B is at variance with the results in the models of Hochman and Rogers (1969), and may strengthen the case for the existence of an endogenous process, in even large economies, that may ensure survival and help guarantee the existence of Walrasian equilibrium. In Harrington's model, there are N agents (potential helpers) and U_i is the utility function of agent i. Let H_i denote the outcome 'i helps another person', let $\neg H_i$ denote the outcome 'i does not help anyone', let $H_{N/i}$ denote the outcome 'somebody other than i helps another person' and let $\neg H_N$ denote the outcome 'nobody helps another person'. The utility levels associated with these possible states are:

the utility of i when they help and nobody else helps $\equiv U_i(H_i \wedge \neg H_{N/i}) = a$;
the utility of i when they do not help and somebody else helps $\equiv U_i(\neg H_i \wedge H_{N/i}) = b$;
the utility of i when they help and somebody else also helps $\equiv U_i(H_i \wedge H_{N/i}) = c$;
the utility of i when nobody helps $\equiv U_i(\neg H_N) = d$.

Definition 5.7 (Nash equilibrium). A *Nash equilibrium in the helping game* is defined by strategies, one for each of the

N players, in which a player's strategy maximises their payoff given the strategies adopted by other players, and this condition holds for all players.

Harrington (2001; p. 391) shows there are N pure strategy Nash equilibria in this game, each involving one player choosing H and the other $N-1$ players choosing $\neg H$, provided that $a > d$ and $b > c$.[5] Since the game is symmetric, the symmetric Nash equilibria necessarily involve players using mixed strategies. As Harrington (2001; p. 391) notes, a mixed strategy involves randomisation which is represented by the probability of choosing H. Let p be the probability of agent i choosing H and $(1-p)$ be the probability of them choosing $\neg H$. Then i is indifferent between the actions H and $\neg H$ if the expected utilities of the two actions are the same.[6] Using the notation introduced earlier this occurs if:

$$(1-p)^{N-1}a+[1-(1-p)^{N-1}]c = (1-p)^{N-1}d+[1-(1-p)^{N-1}]b. \tag{5.1}$$

Expanding this and collecting terms we get:

$$\begin{aligned}
&(1-p)^{N-1}(a-c) + (1-p)^{N-1}(b-d) = (b-c)\\
&\quad\Leftrightarrow (1-p)^{N-1}[(a-c)+(b-d)] = (b-c)\\
&\quad\Rightarrow (1-p)^{N-1} = (b-c)/[(a-d)+(b-c)]\\
&\quad\Rightarrow (1-p) = \{(b-c)/[(a-d)+(b-c)]\}^{1/(N-1)}.
\end{aligned}$$

This gives the optimal randomisation probability, p^* as a function of utility values as:

$$p^* = 1 - \{(b-c)/[(a-d)+(b-c)]\}^{1/(N-1)}. \tag{5.2}$$

[5]Suppose that one player chooses H and the other $N-1$ players choose $\neg H$. Consider one of the non-helpers. If they choose $\neg H$ then they get a payoff 'b' while if they choose H they get a lower payoff 'c'. Thus choosing $\neg H$ is optimal. The player who chooses H gets 'a' which exceeds the payoff 'd' from choosing $\neg H$.

[6]$(1-p)^{N-1}U_i(H_i \wedge \neg H_{N/i}) + [1-(1-p)^{N-1}]U_i(\neg H_i \wedge H_{N/i})(1-p)^{N-1}U_i(\neg H_N) + [1-(1-p)^{N-1}]U_i(\neg H_i \wedge H_{N/i})$.

If the utility that i receives from helping when nobody else does is greater than the utility they get from the situation where nobody helps then $U_i(H_i \wedge \neg H_{N/i}) > U_i(\neg H_N)$ and $a > d$. If the utility i gets when they do not help but somebody else does help is greater than the utility they get when they help and somebody else helps $U_i(\neg H_i \wedge H_{N/i}) > U_i(H_i \wedge H_{N/i})$ and $b > c$ makes just this assumption.

Assumption H (Harrington (2001; p. 390)). *The preferences of everyone in the group of potential helpers is such that $a > d$ and $b > c$.*

If Assumption H holds, then the term $\{(b-c)/[(a-d)+(b-c)]\} > 0$. From (5.2) we see that as the size of the economy increases, i.e. as $N \to \infty, 1/(N-1) \to 0$. Therefore:

$$\therefore \lim_{N\to\infty} p^* = \lim_{N\to\infty} 1 - \{(b-c)/[(a-d)+(b-c)]\}^{1/(N-1)} = 0. \tag{5.3}$$

Thus the probability that a particular agent will help, declines as the size of the group of potential helpers grows and this feature of the model rationalises (A). What about the probability that some help is given? Denote the equilibrium probability that at least one person helps by $Q(N)$ then $Q(N) = 1-(1-p^*)^N = 1-\{(b-c)/[(a-d)+(b-c)]\}^{N/(N-1)}$. The probability that nobody helps is $1-Q(N) = \{(b-c)/[(a-d)+(b-c)]\}^{N/(N-1)}$. Then

$$\ln(1-Q(N)) = N/(N-1)\ln\{(b-c)/[(a-d)+(b-c)]\}. \tag{5.4}$$

The RHS of (5.4) yields: $[1/(N-1)-N/(N-1)^2].\ln\{(b-c)/[(a-d)+(b-c)]\}$

$$\begin{aligned} &= [(N-1)/(N-1)^2 - N/(N-1)^2] \\ &\quad \times \ln\{(b-c)/[(a-d)+(b-c)]\} \\ &= -1/(N-1)^2 \ln\{(b-c)/[(a-d)+(b-c)]\}. \end{aligned}$$

Since $\partial \ln(1-Q(N))/\partial N = \partial(\ln(1-Q(N))/\partial(1-Q(N)) \cdot \partial(1-Q(N))/\partial Q(N)) \cdot \partial Q(N)/\partial N$,

$$
\begin{aligned}
&1/[1-Q(N)] \cdot (-1) \cdot \partial Q(N)/\partial N = \partial\{(N/N-1) \\
&\quad \times \ln\{(b-c)/[(a-d)+(b-c)]\}/\partial N \\
&\therefore 1/[1-Q(N)] \cdot (-1) \cdot \partial Q(N)/\partial N = -1/(N-1)^2 \\
&\quad \times \ln\{(b-c)/[(a-d)+(b-c)]\} \\
&\Rightarrow 1/[1-Q(N)] \cdot \partial Q(N)/\partial N = 1/(N-1)^2 \\
&\quad \times \ln\{(b-c)/[(a-d)+(b-c)]\} \\
&\Rightarrow \partial Q(N)/\partial N = [1-Q(N)] \\
&\quad \times \ln\{(b-c)/[(a-d)+(b-c)]\} \Rightarrow \\
&= 1/(N-1)^2\{(b-c)/[(a-d)+(b-c)]\}^{N/N-1} \\
&\quad \times \ln\{(b-c)/[(a-d)+(b-c)]\}. \qquad (5.5)
\end{aligned}
$$

Provided $\{(b-c)/[(a-d)+(b-c)\} < 1$, (which is guaranteed by Assumption H), then $\ln\{(b-c)/[(a-d)+(b-c)]\} < 0$ and from (5.5), $\partial Q(N)/\partial N < 0$. Thus the addition of more potential helpers to the economy lowers the probability that anyone helps. However, since $\partial Q(N)/\partial N < 0$ and $\lim_{N\to\infty} Q(N) = (a-d)/[(a-d)+(b-c)]$ we have that $Q(N) > (a-d)/[(a-d)+(b-c)]$ for all N, so there is a lower bound on the probability that someone helps (see Harrington (2001) for further discussion). As noted earlier and as Hammond (1987) points out, the models of Hochman and Rogers (1969), Nakayama (1980) and Arrow (1981), 'helping' (in the form of endowment or income transfer), *might* happen in a two person economy where there is one potential helper (high income person) and one victim (low income person), but as soon as the number of helpers gets to two or more, 'helping' becomes a classic public good and the Nash equilibrium amount of help (redistribution) goes to zero with probability one. This is at variance with the predictions of Harrington (2001) where the equilibrium probability that at least one person helps is bounded below by $(a-d)/[(a-d)+(b-c)]$. Can the results of these two groups of models be reconciled?

Consideration of Assumption H reveals that the results in Harrington (2001) depend on the assumed preference structure. In particular, it is required that the utility an agent enjoys if they help and nobody else helps (‘a’) is greater than the utility they receive if nobody at all helps (‘d’). From an economic and psychological point of view, this is not the only possible preference structure. If for example, an agent helps and nobody else helps, they may feel ‘put upon’ or exploited by the rest of the group. The positive benefit derived from the altruistic act of helping may be partially or totally negated by the disutility coming from the sense of being ‘used’ and being the only one in the group of helpers who is actually making a material contribution. If this is the case, then the value ‘a’ may approach ‘d’. We now explore this consequences of this possibility by replacing Harrington’s assumption that $a > d$ with the assumption $a \geq d$.

Assumption H′. *The preferences of everyone in the group of potential helpers is such that $a \geq d$ and $b > c$.*

Under Assumption H', let $a = d + \varepsilon$ then $Q(N) = 1 - \{(b-c)/[(\varepsilon) + (b-c)]\}^{N/(N-1)}$ and as $\varepsilon \downarrow 0$ in this expression, $Q(N) \to 0$ independent of N provided only that $N \geq 2$. Also since $\partial Q(N)/\partial N = 1/(N-1)\{(b-c)/[(\varepsilon)+(b-c)]\}^{N/N-1}$. $\ln\{(b-c)/[(\varepsilon)+(b-c)]\}$ as $\varepsilon \downarrow 0$ this term is dominated by $\ln\{(b-c)/[(\varepsilon)+(b-c)]\} \to 0$. Therefore in the case where $a \approx d$, adding another person to the economy makes no difference to the probability that somebody helps.

Remark 5.12. By extending the model in Harrington (2001) to allow utilities to be such that $a \geq d$, we have allowed for the possibility that agents may resent being the only active ones in a group of potential helpers. If this resentment makes agents indifferent between being the only one to help and seeing the outcome where there is no help at all, then like the models of Hochman and Rogers (1969), Nakayama (1980) and Arrow (1981), this modified version of Harrington's model predicts zero help in an economy in which the group of potential helpers has two or more members. Under such circumstances, altruism could not generally be relied on to ensure that each agent in the economy has at least minimum wealth, something which, as we have seen is necessary for the existence of Walrasian equilibrium.

5.4. Conclusion

We have explored the possibility that implementable economic policies or endogenously occurring redistribution schemes might be used in place of structural conditions such as interior endowments or irreducibility. It is interesting to note that while the general need for redistribution of endowments in order to achieve desired welfare levels has long been recognised (see the discussion of the Second Welfare Theorem in Chap. 9), there does not seem to be the same level of recognition that redistribution may be necessary for the existence of equilibrium.

The possibility that policy induced redistributions may be unnecessary because altruistic feelings among agents in the economy may lead to voluntary redistributions is particularly interesting. If that were the case, then an important part of the proof of existence theorems would be 'endogenised'. However, this possibility runs into the general presumption in economic analysis, dating at least from Hochman and Rogers (1969), that the redistribution which altruism inspires will, by virtue of being a public good, be subject to significant under provision. Recent work by Harrington (2001) seems to challenge this presumption and suggests that, even in large economies, we might expect to see voluntary redistribution. An analysis of Harrington's model shows that his result depends on a particular preference structure holding across agents in the economy. By generalising the assumed preference structure, we were able to show that the general market failure results of Hochman and Rogers (1969) and others, would re-emerge.

The work in this and in the previous chapters, has been conducted for the case of a complete market Arrow-Debreu economy. It is possible that if we move outside that framework, then the argument we have been making will breakdown and that the sorts of 'relationship conditions' we have been studying will not be needed in order to guarantee the existence of equilibrium. In the next chapter, we show that this is not the case where we bring our study of the existence question to a close by showing that even more 'relationship conditions' are needed to guarantee the existence of Walrasian equilibrium in certain non–Arrow-Debreu environments.

Chapter 6

EXISTENCE OF WALRASIAN EQUILIBRIUM IN SOME NON–ARROW-DEBREU ENVIRONMENTS

> "*When markets are incomplete, agents' activities are not well co-ordinated in an equilibrium. Establishing this property — which was for a long time a Folk Theorem — leads us to introduce some techniques that play an important role in the analysis of the model ...*"
>
> M. Magill and M. Quinzii

6.1. Introduction

We have so far considered the sorts of conditions needed for the existence of Walrasian equilibrium largely in the context of an Arrow-Debreu specification of the economy. There are at least four features of an Arrow-Debreu specification of the economy which when relaxed, generate alternative and arguably more realistic models of the economy. The features are: (i) the assumption of market completeness meaning that there is a market for each commodity. Recalling that commodities are defined by their characteristics, their date of availability and their location (possibly also by a state of nature), this means in particular that the economy has a complete set of *futures* markets. A consequence of this is that in an Arrow-Debreu economy, agents never have to form expectations about the future. This particularly applies to future prices because if markets are complete, agents are able to observe the prices that they will *actually* obtain at all future dates; (ii) there is a single mythical 'day zero', which occurs before the start of historical time, and on which all equilibrium prices and trades are determined for the entire life of the economy; (iii) the absence of any incentives for agents to hold money — a consequence actually of (ii), (see Duffie (1990) for a fuller discussion); (iv) the absence of a past for the economy. This feature also arises because of the 'day zero' assumption, after which day the economy merely 'unfolds forward' and the contracts which were agreed on that day zero are honoured. One important consequence of this is that agents' budget sets are never impinged upon by prior commitments — possibly onerous commitments.

It is worth considering whether or not the sorts of conditions that were encountered in the Arrow-Debreu case, also play a role in guaranteeing the existence of Walrasian equilibrium in non–Arrow-Debreu environments. This question needs to be considered because it is possible (although unlikely), that the conditions we have identified and critiqued, are merely artifacts of the Arrow-Debreu set-up. We therefore consider the existence question for Walrasian equilibrium in economic environments generated by a relaxation of the above four 'institutional' features of an Arrow-Debreu economy. It will be shown that the need for the sorts of conditions identified earlier as being needed for the existence of Walrasian equilibrium in an Arrow-Debreu framework is not avoided by moving away from the Arrow-Debreu set up. If anything, the sorts of structural conditions needed for the existence of Walrasian equilibrium are more stringent in non–Arrow-Debreu environments than they are in an Arrow-Debreu framework. It will be shown that there are environments in which, even if conditions such as interior endowments or irreducibility hold, the existence of Walrasian equilibrium is not guaranteed.

The chapter is organised as follows. Section 2 considers the role of relationship conditions for the existence of Walrasian equilibrium in a temporary equilibrium framework. Section 3 considers some conditions in guaranteeing the existence of Walrasian equilibrium in a monetary economy. Section 4 considers the role of relationship conditions in guaranteeing the existence of Walrasian equilibrium in an economy which has a past (as well as a present and expected future). Section 5 presents our conclusions which are basically that in each non–Arrow-Debreu environment studied, it seems that the sorts of conditions we have previously studied certainly have no less a role (and indeed may have even more work to do), in helping to guarantee the existence of Walrasian equilibrium than they do in the Arrow-Debreu case.

6.2. Existence of equilibrium in temporary equilibrium

Instead of supposing a single ‘day zero’ on which all prices and quantities are determined for the entire life of the economy, consider a model in which there is a sequence of days (‘Mondays’), on which agents meet and where prices and trades are arranged in the spot markets and the limited futures markets that might be open. An equilibrium that might be reached on a particular Monday was called by Hicks a ‘temporary equilibrium’ (see de Vroey (2006) for an interesting discussion of the evolution of Hicks’ thinking about temporary equilibrium models). Suppose that individuals are generally uncertain about the prices which will prevail for goods on future Mondays (apart from those limited number of goods about which future contracts have been written). Suppose also that they are uncertain about the endowments they will receive in the future (the state of their health, likely inheritance etc.). In such an environment, agents will generally try form expectations about future prices and endowments. They will also condition their current demand and supply decisions, in part, on those expectations. The existence problem for a temporary equilibrium in this 'sequence economy' now involves showing that Walrasian equilibrium is possible when consumers are maximising utility, producers are maximising profit and both groups of agents exhibit behaviour which is dependent, in part, on their expectations. A typical existence result is due to Green (1973) and it illustrates that the need for the sorts of conditions discussed earlier has not been diminished in the move to a temporary equilibrium environment.

Consider a two period world where the current period is 1 and the future is 2. Let consumers be uncertain about the prices they will face and the endowments they will have in period 2. Let x_{i1} and x_{i2} be the goods to be delivered to or by i in periods 1 and 2 and let the function $\Psi_i(p_1, q_1)$ summarise the expectations function of consumer i about their second period endowment ω_{i2} and the period 2 spot prices p_2, given current spot prices p_1 and futures prices q_1 observed on the futures markets that are open in period 1.

Theorem 6.1 (Green (1973)). *If* $(g.1)$ *for all* i, $X_{i1} = \Re_+^{n1}$ *and* $X_{i2} = \Re_+^{n2}$; $(g.2)$ *for all i preferences over trades in the first period can be represented by a concave, monotone utility function of the von Neumann-Morgenstern type*; $(g.3)$ *the expectation function* $\Psi_i(p_1, q_1)$ *is continuous*; $(g.4)$ *for every* (p_1, q_1), $\Psi_i(p_1, q_1)$ *gives probability* 1 *to the set of* (ω_{i2}, p_2) *for which* p_2 *is positive*; $(g.5)$ *the support of* Ψ_i *is independent of* (p_1, q_1) *and the convex hull of the projection of the support of* Ψ_i *on the second period price space has non-empty relative interior* Π_i; $(g.6)$ *for all* i $\omega_{i1} \gg 0$; $(g.7)$ *the intersection of all the* Π_i *is non-empty then a Walrasian equilibrium exists in period* 1 *for this economy.*

Proof. Green (1973; pp. 1116–1117). □

Remark 6.1. Assumption (g.6) is the interior endowment assumption, and for the usual reasons, it appears in this existence result. (g.7) has introduced a new requirement across agents in the economy in that individual expectations must agree at least to the extent that $\cap_i \Pi_i \neq \emptyset$. As McKenzie (1987) observes, in the case of point expectations, this condition is particularly restrictive because it means that everyone has *identical* expectations. Grandmont (1982, Theorem 1) shows that (g.7) is actually necessary and sufficient for the existence of temporary equilibrium. In commenting on the result, he also observes that: "...It is clear beforehand that the properties of agents' expectations will play a central role in the analysis [of the existence] of temporary equilibrium." Grandmont (1982; p. 887).

Sondermann (1974) obtains the following extension of Green (1973) to cover the case of a production economy. Let the set of consumers be indexed by I, producers by J and goods by L. The economy is imagined to operate over two periods 1 and 2 (which may be t and $t+1$ in an infinite sequence of 'Hicksian weeks'). At the beginning of period 1, commodity and capital markets are open. All commodities traded are either consumed or used as productive inputs during the first period and the only store of value is money or shares in firms, which are the assets traded on capital markets. There is a finite number of goods ℓ whose prices are +, 0 or − depending on whether the goods are scarce, free or noxious in equilibrium so the space of prices at t is $P^t = \Re^\ell$. There are as many risky assets as there are firms in the economy. A contract on A_{jt} units of the jth firm concluded in t means a commitment to buy or sell the fraction A_j of the firm in period $t+1$. There also exists a safe asset A_0 ('money') which yields no return and which has a price $\equiv 1$. A *portfolio* is a point $A = (A_1, \ldots, A_j, A_0)$ in the asset space $M = \{A \in \Re^{J+1} : 0 \leq A_J \leq 1$ for all $j \in J\}$ and short selling is not allowed. An asset price vector r_t is a point in the space $R^t = \{r_t = (r_{1t}, \ldots, r_{Jt}) \in \Re_+^{J+1} : r_{0t} = 1\}$. Nonnegative prices mean that the shareholders have limited liability so that they may lose all that they have invested but they are not required to make good the losses of the firm. Capital markets open jointly with commodity markets and agents get a signal $s_t = (p_t, r_t) \in S_t$. Given the signal, each agent has to formulate a feasible supply and demand plan $x_{it} \in X_{it}$. The environment of the consumer, Ω, is the set of current signals, S_1 and the states of the world unknown to the consumer are period 2 prices and period 2 endowments. Let $\underline{\Omega} = S_2 \times \Re^\ell$ and let F be the σ-algebra generated by $\underline{\Omega}$. If expectations are governed as in Theorem 6.2, we have an equilibrium.

Theorem 6.2 (Sondermann (1974)). *If the economy satisfies*: $(s.1)$ *for all* i *and* t, X_{it} *is a compact, convex subset of* $\Re^\ell$; $(s.2)$ *at the beginning of period* 1, *each consumer has fixed strictly convex preferences defined over the intertemporal consumption, set* $C_i = X_{i1} \times X_{i2}$, *such that* $c_1 \neq c_2 \in C_i$ *and* $0 < \lambda < 1$ *implies* $c_1 \prec_i [\lambda c_1 + (1-\lambda)c_2]$; $(s.3)$ *for all* $c' \in C_i$ *the sets* $\{c \in C_i : c \prec_i c'\}$ *and* $\{c \in C_i : c' \prec_i c\}$ *are closed, so that preferences are continuous*; $(s.4)$ *every* i *is endowed at the beginning of period* 1 *with* $e_{i1} = (\omega_{i1}, A_{i0}) \in X_{i1} \times M_0$. *The amount of money in the portfolio,* $A_{i0} > 0$ *for all* i, *so that for any price system* $s_1 \in S_1 = P^1 \times \Re$, *it is true that* $p_{i1}X_{i1} < W_i(s_1) = p_1\omega_{i1} + r_1 A_{i0}$. *In addition, each consumer expects an endowment* $\omega_{i2} \in \mathrm{int}X_{i2}$ *for period* 2; $(s.5)$ $\Psi_i(w, .)$ *is a probability measure on* $(\underline{\Omega}, F)$ *for all* $\omega \in \Omega$; $(s.6)$ $\Psi_i(., E)$ *is a continuous function on* Ω *for all closed events* $E \in F$; $(s.7)$. *For all* $\varepsilon > 0$, *there exists a compact set* $K \subset S_2 \times X_{i2}$ *such that for all* $\omega \in \Omega, \Psi_i(\omega, K) \geq 1 - \varepsilon$, *meaning that expectations are not 'too widely spread' then a Walrasian temporary equilibrium exists for the economy.*

Proof. Sondermann (1974; pp. 256–258). □

Remark 6.2. Through (s.4), the existence of a cheaper point is assumed directly by Sondermann (1974) in a fashion reminiscent of Gale and Mas-Colell (1975). Our remarks in the previous chapter about such an approach apply here also.

Assumptions (s.5) and (s.6) guarantee that Ψ_i is a continuous Markov kernel from Ω to $\underline{\Omega}$. The interpretation of this is that $\Psi_i(\omega, E)$ is the subjective probability attached by i to E given that at present, ω is observed while assumption (s.7) implies that $\Psi_i(\omega, S_2 \times X_2) = 1$ for all $\omega \in \Omega$. As Sondermann points out, this means that: "...the consumer is almost sure that his initial endowment in period 2 will be contained in his consumption set, or expressed more dramatically, starvation is excluded from his forecasts." Sondermann (1974; p. 240). Thus in a temporary equilibrium set up considered here, both (s.4) and (s.7) impose relationship conditions on the economy and indeed add some relationship requirements, relative to the Arrow-Debreu framework, because of the need to guarantee that individual expectations are also well behaved. Consequently, it is reasonable to argue that moving from an Arrow-Debreu environment to a Hicksian temporary equilibrium set-up does not reduce the importance of the sorts of relationship conditions identified earlier. If anything, this change in environment increases the demands on such conditions and the remarks made earlier about the dependence of the existence of Walrasian equilibrium on theoretically arbitrary relationship conditions applies with at least as much force here. The points we have been making here have been illustrated by some quite early theorems in the literature. However, recent work has not changed the basic story elaborated above, as the discussion of temporary equilibrium in Grandmont (2008) demonstrates.

6.3. A general equilibrium monetary economy

One of the features of economic reality not well captured by the Arrow-Debreu framework is the presence of money. As noted earlier, this is essentially due to the 'day zero' feature of that model. Duffie (1990) developes an explicitly monetary economy in which agents are characterised in the usual way (i.e. consumption possibility set, preferences, endowment of goods), except they now also have an endowment $m_i \in [0, \infty)$ of money and a transactions technology $T_i \subset \Re^\ell_+ \times \Re^\ell_+ \times \Re^\ell_+$, where $(b, s, z) \in T_i$ means that purchasing the bundle b and selling the bundle s can be achieved at the cost of bundle $z \in \Re^\ell_+$. A choice $(b, s, z) \in T_i$ by i yields the consumption $x_i = \omega_i + b - s - z$. This is budget feasible at buying prices p^b and selling prices p^s provided $x_i \geq 0$ and $p^b.b - p^s.s \leq m_i$. A budget feasible choice (b, s, z) is *optimal* for i provided $\omega_i + b - s - z \succeq_i \omega_i + b' - s' - z'$ for any budget feasible choice (b', s', z'). A *Walrasian monetary equilibrium* for the economy $\boldsymbol{E}_T = (X_i,\ \succeq_i,\ \omega_i,\ M_i,\ T_i)$ is a collection $\{(b_1, s_1, z_1), \ldots, (b_m, s_m, z_m),\ (p^b, p^s)\} \in (\Re^{3\ell})^m \times \Re^{2\ell}$ such that given prices (p^b, p^s) (i) the choice $(b_i, s_i, z_i) \in T_i$ is optimal for each i and (ii) the allocation $\{(b_i, s_i, z_i)\}$ is feasible so that $\sum_i b_i \leq \sum_i s_i$.

Theorem 6.3 (Duffie (1990)). *If in the economy* $\boldsymbol{E}_T$ (*df.*1) *the allocation* $0 \in \Re^{3\ell m}$ *is not efficient*; (*df.*2) *for all* i, T_i *is closed and convex and* $0 \in T_i$; (*df.*3) *for all* i, *if* $(b, s, z) \in T_i$ *and* $(b', s') \in [0, b]\ x\ [0, s]$ *then* $z' \geq z \Rightarrow (b', s', z') \in T_i$; (*df.*4) *for all* i, *if* $(b, s, z) \in T_i$ *and* $(b, s) \neq 0$ *then* $z \neq 0$; (*df.*5) *for all* i, *there exists* $(b, s, z) \in T_i$ *such that* $s \gg 0$; (*df.*6) *for all* i, $\succeq_i$ *is continuous*, *strictly monotone*, *semi-strictly convex and non-satiated at all feasible choices*; (*df.*7) *for all* $i, \omega_i \gg 0$ *and* $X_i = \Re^\ell_+$ *then* $\boldsymbol{E}_T$ *has a Walrasian monetary equilibrium.*

Proof. See Duffie (1990; pp. 486–487). □

Remark 6.3. Condition (df.7) is the interior endowments assumption. In this context, it is just as unreasonable as it was in the Arrow-Debreu set up, something which Duffie (1990) recognises when he characterises the assumption as extremely restrictive. Although the interior endowments assumption can be weakened and replaced by an irreducibility like condition, as was seen earlier this does not avoid the criticism that the resulting existence theorem depends on a theoretically arbitrary relationship condition. Thus moving from an Arrow-Debreu framework to a monetary economy, has not in this instance diminished the need for strong structural assumptions in the economy.

6.4. A 'Keynesian' economy

Some readings of Keynes conclude that Keynesian phenomena can only occur if there are rigidities in markets — specifically when prices are prevented from 'moving freely'. As Srivastava and Rao (1990) note, this interpretation arises from the view that: "…the free market system, if left to itself, can clear all markets at every point in time *through price changes*. Therefore, demands and supplies always match and there can never be any involuntary unemployment of resources [unless prices are inflexible]." Srivastava and Rao (1990; p. 1, emphasis added). Mukherji (1990) similarly remarks that: "Now the Walrasian equilibrium denies the very existence of such a situation [as unemployment]; it is assumed that prices are flexible and hence that they should adjust in such a manner that demand and supply match. This was the general view of unemployment: that (real) wages must be too high to sustain full employment and that all that is required is a reduction in real wages. It is against such a view that Keynes (1936) argued." Mukherji (1990; p. 171).

An alternative reading suggests that Keynes was interested in circumstances in which Walrasian equilibria do not exist and in which as a consequence no amount of price flexibility will clear all markets. In support of this reading, Arrow and Hahn (1971) remark that: "…[the non-existence of temporary Walrasian equilibrium] is a matter of interest because Keynes has often been interpreted as claiming that, in fact, a temporary equilibrium (as we have defined it) may not exist." Arrow and Hahn (1971; p. 347). Motivated by this idea, Arrow and Hahn pose two questions: "The first, straightforward question is whether it can be argued that Keynes discovered features of an economy that... make it impossible to establish the existence of a temporary [Walrasian] equilibrium. (It should be emphasised that for our purposes temporary equilibrium implies the clearing of all markets including that for labour)... The second question is whether such an equilibrium, even if it can be shown to exist, is 'sensible'. By 'sensible', of course we can mean all sorts of things. Certainly, though, we would not be much interested in an equilibrium with zero real wage." Arrow and Hahn (1971, pp. 354–355).

A contribution to answering these questions might come from noting that the existence of Walrasian equilibrium may actually be problematic even in Arrow-Debreu environments, which are completely free of the complications introduced by the economy having a past. We have argued at length that a break down in irreducibility like conditions may be enough to prevent the existence of Walrasian equilibrium even in economies entirely free of the 'institutional rigidities' sometimes considered central to Keynesian outcomes. In fact, what we have demonstrated is what Jossa (1997) regards as a non-negotiable requirement for giving a Keynesian explanation of unemployment when he remarks: "Every explanation of unemployment that wishes to be Keynesian must, in my opinion, reconcile unemployment with equilibrium [because] every explanation that hypotheses rigidities of various kinds (without basing them on rational behaviour) challenges the letter and spirit of the *General Theory*." Jossa (1997; p. 173). We have shown that it is possible for Keynesian phenomena, in particular non-market clearing states which are insensitive to variations in prices, to occur even in 'classical' (i.e. Arrow-Debreu) environments in which there are no impediments to price flexibility, no destabilising expectations, no money, and a complete set of (futures) markets. What is required for 'Keynesian phenomena' to appear in such economies is for certain conditions (e.g. irreducibility) to break down. We have previously considered reasons why these conditions may be theoretically onerous. We have noted evidence from actual labour market studies is consistent with the break down of such relationship conditions. Furthermore, since Florig-irreducibility is known to be necessary for all individuals to have a non-empty budget set (see the argument in Chap. 3), we can see that if this condition fails then there is nothing in the operation of a private ownership economy which guarantees that the resulting price system is 'sensible', to use Arrow and Hahn's phrase, i.e. there is no guarantee that real wages will remain positive for all types of labour.

Our work also appears to throw interesting light on the following remark by Hahn: "…Walrasian equilibrium of the Arrow-Debreu variety and involuntary unemployment are incompatible. But of course if a description of the economy is best approximated by such an equilibrium…[then] the whole Keynesian opus [is] irrelevant…" (1987; p. 1). What we have been able to show is that all the *institutional* features of the Arrow-Debreu set-up can be retained (complete futures markets, no money, perfect price flexibility, no history impinging on the economy), and yet the Keynesian opus can retain it's relevance, or at least Keynesian phenomena, such as involuntary unemployment, can be observed in the economy, if irreducibility like conditions break down.

However, if we do consider one of the complications considered by Keynes to a classical Arrow-Debreu environment, then the situation as far as the existence of Walrasian equilibrium is concerned appears to be even more fragile. The feature we have in mind involves adding a 'past' to the economy. Following the formulation of the 'Keynesian model' in Arrow and

Hahn (1971), we may develop this innovation as follows. Take a two period temporary equilibrium set up (present and future) in which ω_{ib} is i's initial endowment of bonds (= the anticipated volume of receipts in period 2), ω_{im} is the stock of money held by i and θ_{ij} is the share portfolio carried by i from the previous period. The two budget constraints and transactions costs constraint faced by individual i are:

$$p^1 x_i^1 + p_b(x_{ib} - \omega_{ib}) + p_m(x_{im} - \omega_{im}) \\ \leq p^1\omega_i^1 + \sum_j \theta_{ij}(py_j) + \sum_j (\theta_{ij} - \theta ij)K_j \qquad (6.1)$$

$$p^2 x^2 \leq \omega_{ib} + p_m^2 x_{im} \qquad (6.2)$$

$$J(x_i^2, p^2, p_m^2, x_{im}) \leq T \qquad (6.3)$$

where $J(\bullet)$ represents the transactions technology, T is the available time, p_m is the price of money, p_b is the price of bonds and K_j is the capital value of firm j. Now let there be three time periods 0, 1, 3. If the agent is in period 1, then 0 is viewed as the 'past' and 2 is the 'future'. The use of a superscript "tk" will denote the expectations formed in period t of values of the variable under consideration in period k. Assume that bonds issued in period 0 by firm j are repaid in period 1 and that the firm issues a quantity of new bonds $p_j^{12}y_j^{12}$ which is equal in value to the profits it expects to make in period 2, where that expectation is formed in period 1, the present. Let ϖ_{ib}^1 be the actual value of receipts in period 1 plus the value of receipts expected then from period 2, the value of ϖ_{ib}^1 is given by:

$$p^1\omega_i^1 + p_b^1(p_i^{12}\omega_i^{12}) + \sum_j \theta_{ij}^1[p^1 y_j^1 + p_b^1(p_j^{12}y_j^{12}) - p_j^{0F}y_j^{0f}] \\ + p_b^1 \sum_j \theta_{ij}^1[\max_i(p_i^{12}y_j^{12}) - p_j^{12}y_j^{12}]. \qquad (6.4)$$

The first two terms in this expression for ϖ_{ib}^1 give the value of proceeds from current endowment sales and the present expected value from future endowment sale. The first square bracket gives for any j the payment by j to shareholders after payment of the bonds issued in period 0. The second square bracket gives the profit expected of firm j by the most optimistic individual after repayment of the bonds issued in period 1. If i is not the most optimistic, then $\theta^1_{ij} = 0$. If K^1_j denotes the capital value of firm j in period 1, then $K^1_j = p^1 y^1_j + p_b \cdot \max_i[(p^{12}_i y^{12}_j) - p^{0F}_j y^{0F}_j]$. The importance of this term is as follows: "Since the economy now has a history, the firm has past commitments that must be taken into account in the valuation placed on it in period 1. It is plain that we now cannot assume $K^1_j \geq 0$. If $K^1_j < 0$, then a household with a share θ^0_{ij} in the firm is responsible for $\theta^0_i K^1_j$ of the net debt and cannot escape this obligation." Arrow and Hahn (1971; p. 352). The potential importance of this for the existence of Walrasian equilibrium, is that the capital value of the firm not guaranteed to be positive at all prices. The consequence this may have for consumer budget sets, is anticipated by Mas-Colell *et al.* (1995) who explicitly rule out this possibility when they require that: "…to every consumer i, price vector p and production profile $y = (y_1 \ldots y_J)$ [is assigned] a *limited liability amount of wealth* $M_i = p\omega_i + \text{Max}\{0, \sum_j \theta_{ij} p y_j\}$." Mas-Colell *et al.* (1995; p. 636, emphasis added). The assumption might also be called a 'no liability' condition because the consumers wealth is given by $\max\{0, \sum_j \theta_{ij} p y_j\} \geq 0$. As will be shown below, this is not an innocuous requirement.

If it is assumed that $K^1_j < 0$ implies $\theta^1_{ij} = \theta^0_{ij}$ and that $K^{01}_j \geq 0$, the income available to the household at the beginning of period 1 is M^1_i which is given by the expression:

$$[p^1\omega^1_i + p^1_b(p^{12}_i\omega^{12}_i) - p^{0F}_i\omega^{0F}_i] + \left[\sum_j(\theta^1_{ij}K^1_j - \theta^{01}_{ij}K^{01}_j)\right] - \sum_j(\theta^0_{ij} - \theta^{01}_{ij})K^{01}_j + x^0_{ib} + p^1_m x^0_{im}. \tag{6.5}$$

Several situations can now arise which are ruled out in an Arrow-Debreu context. Firstly, note that (p^1, p_b^1) might be such that $p^1y_j^1 + p_b^1p_j^{12}y_j^{12} - p_j^{0F}y_j^{0F} < 0$, in which case (a) the firm j cannot repay the debt of the previous period, and is therefore 'technically bankrupt'. There may, however, be households more optimistic than the firm who are prepared to make good these debts provided they are not involved in a loss so that (b) even if at $(p^1, p_b^1)K_j^1 < 0$ as long as the present value of profits of j is positive, it is worthwhile to continue to operate the firm. If however at (p^1, p_b^1), K_j^1 is sufficiently negative to cause $M_i^1 < 0$, then (c) the bankruptcy point of the *individual* is reached, and the individual suffers a discontinuous change in wealth and also a discontinuous change in demand. This leads Arrow and Hahn (1971) to make the following important remark: "Clearly, the actual bankruptcy procedure is at least a matter of law, but it seems plain that the *history of the economy* may make it impossible to guarantee the continuity properties of the various [excess demand] functions and correspondences *and this is bad for existence proofs*... In addition to all this another problem arises. It will be recalled that the strategy of our existence proofs was to establish the existence of a compensated [or quasi] equilibrium and then to show that it was, in fact, a [Walrasian] equilibrium. For this last step, we needed to ensure that every household disposed of a value of resources that exceeded the value of its minimum consumption vector. It is plain that this last step may not be possible now, even if the existence of a compensated equilibrium could be demonstrated. In a compensated equilibrium, if a household is bankrupt, it disposes of no resources, and *we cannot use our assumption of resource relatedness* [or irreducibility] to reach the desired conclusion. The fact that some household has an 'effective' demand for the resources of the bankrupt household does not help the latter to any 'disposable' resources." Arrow and Hahn (1971; pp. 354–356; emphasis added).

These remarks represent an important development in our consideration of the existence question for Walrasian equilibrium, because what they claim is that *even if* the relationship condition of resource relatedness (or one of its irreducibility relatives) is assumed, the fact that agents now have a history in which they will typically have incurred financial obligations, may prevent the existence of Walrasian equilibrium. Thus, by a relatively trivial and seemingly realistic generalisation of the economic situation facing agents that allows for past obligations to impinge on current budgets, it may be the case that even if an assumption such as resource relatedness or irreducibility is made, that may no longer guarantee the existence of Walrasian equilibrium.

To see this possibility a little more clearly, it might help to note that an agents situation in a Keynesian economy is, in a sense, the 'inverse' of that considered by Aliprantis *et al.* (1989; p. 76) — and studied by us in Chap. 5. In particular, a consumer's budget set may be 'smaller' in a Keynesian production economy than it is in the corresponding Arrow-Debreu economy. Even if the very strong assumption of interior endowments is invoked, this may not be enough to guarantee the existence of Walrasian equilibrium in a Keynesian economy. A sketch of this possibility is provided in Fig. 6.1.

In this example, $X_i \subset \Re^2_+$ and that $\omega_i \in \mathrm{int} X_i$ and the value of i's past liability at current prices is L_i. Then the budget set for i is $\gamma_i\ (p, \omega_i, L_i)$. Clearly the budget set can be empty, even though $\omega_i \in \mathrm{int}(X_i)$, if the past liabilities L_i are great enough, which is the case illustrated. We can formalise this situation as follows. Consider the existence proof developed by Mas-Colell *et al.* (1995; pp. 632–640). Note in particular the importance of the No Personal Bankruptcy assumption, repeated below, in their argument.

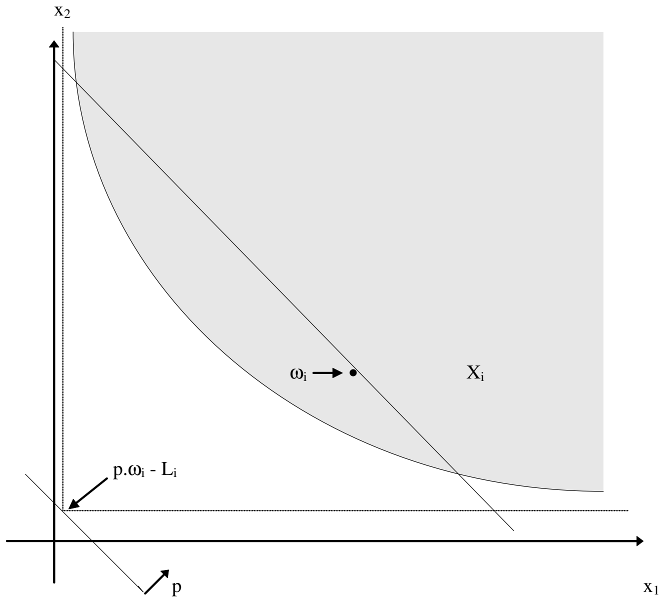

Fig. 6.1. A budget set with 'personal bankruptcy' permitted.

Assumption NPB (Mas-Colell *et al.* (1995; p. 636)). Assign to every consumer i at price vector p and production profile $y = (y_1, \ldots, y_J)$ the *limited liability amount of wealth* defined as $M_i(p, y) = p\omega_i + \text{Max}\{0, \sum_j \theta_{ij} p y_j\}$.

If, contrary to this assumption the Keynesian possibility is admitted, then it can happen that $(p \cdot \omega_i + \sum_j \theta_{ij} p.y_j + L_i) < \min p \cdot X_i$, by virtue of the liabilities that i has acquired in the past. In such an eventuality, the optimal response sets $x_i^b(x, y, p)$ will be empty for at least some i, contrary to the requirements of existence proof for a quasi-equilibrium developed by Mas-Colell *et al.* (1995; pp. 632–640). This outcome also prevents the transition from a Walrasian quasi-equilibrium (if one existed), to a Walrasian equilibrium because under the assumed conditions, consumers do not have access to a 'cheaper point'. For both these reasons then, the existence proof for Walrasian equilibrium cannot go through.

6.5. Conclusion

In this chapter, we have briefly considered what is needed for the existence of Walrasian equilibrium in certain non–Arrow-Debreu environments. From Chap. 2, it is fairly clear that any model in which demands are derived from utility maximisation will need to ensure somehow that budget sets are non-empty and that demands behave continuously in the whole price space. It is however interesting to note that even more stringent conditions have to hold in order to ensure the existence of Walrasian equilibrium in these more general and probably more realistic non–Arrow-Debreu settings in which the influence of the expected future and actual past are allowed to be felt. Particularly interesting is the case in which the past is admitted to the model and where the limited liability assumption of the Arrow-Debreu framework is replaced by the Keynesian possibility of potentially ruinous obligation. In such a situation, even if the strongest survival condition (namely, interior endowments) is assumed, this may not be enough to facilitate the existence of Walrasian equilibrium. It is also interesting that Keynesian phenomena such as non-clearing goods and labour markets and low demand-price elasticities can be rationalised in an otherwise 'classical' environment by simply supposing that irreducibility like conditions have broken down rather than having to appeal to institutional or wage rigidities to achieve the outcome.

Clearly, conditions under which Walrasian equilibrium exists could arise in actual economies. However, when the details of the nature of the conditions needed for such an outcome are closely examined, it is not at all clear that a given economy will necessarily have the needed *a priori* structure to guarantee the existence of Walrasian equilibrium. As a consequence, policy interventions of the sort considered in Chap. 5 may have added relevance in ensuring the existence of Walrasian equilibrium non–Arrow-Debreu environments.

Chapter 7

UNIQUENESS OF EQUILIBRIUM

"*The uniqueness of equilibrium can only be obtained from highly restrictive assumptions.*"

E. Dierker

"*But conditions for the uniqueness of general equilibrium are significantly more restrictive than conditions for existence and one is lead to require local uniqueness instead of global uniqueness ... However, even with this lowered requirement, one cannot expect local uniqueness to hold for every general equilibrium price vector under weak assumptions.*"

G. Debreu

"*Uniqueness proven is meaningless if it is based on unrealistic assumptions.*"

K. Sasakura

7.1. Introduction

The existence question for Walrasian equilibrium is a fundamental and heavily researched question in general equilibrium theory. It is not the only question of interest however as was pointed out in Chap. 1. Perhaps disappointingly, when compared with the existence theorems, it is generally the case that less attention has been given to the uniqueness of equilibrium. As Keenan (1982) puts it this way: "One of the drawbacks of the general equilibrium model is that, at the level of generality of the existence ... theorems, there are few results available concerning uniqueness..." Keenan (1982; p. 23). The uniqueness of equilibrium is important to general equilibrium theory for at least the following reasons. Firstly, if equilibrium prices are not unique, then Walrasian general equilibrium theory is not able to furnish a theory of *value*, although it may be able to supply a theory of *prices*. This is so because as Allingham (1987) notes: "In general equilibrium theory, equilibrium prices may be interpreted as those prices which coordinate the buying and selling plans of all the various agents in the economy; equivalently, they may be interpreted as the values of the commodities. Such values will only be well defined if there is only one system of coordinating prices, that is, if the equilibrium is unique." Allingham (1987; pp. 753–754). Secondly, if equilibrium is not unique, then some equilibria will be unstable. Thirdly, in the absence of uniqueness, it is generally not possible to obtain unambiguous comparative static predictions. Since generating comparative static predictions is one of the central purposes of most economic theories, this is a potentially serious outcome. Making comparative static predictions is also generally the motivation for the construction of applied general equilibrium models. As Kehoe (1998) notes: "Consider an economist working with an applied general equilibrium model. This economist starts by calibrating, or statistically fitting, the parameters of the model so that it has an equilibrium that replicates transactions observed in the data. He or she then changes some of the parameters to simulate a change in policy ... This is the comparative static method. If there is more than one possible equilibrium after the parameter change, the method becomes problematic." Kehoe (1998; p. 38). In a similar vein but from a methodological point of view, there is the argument due to Samuelson (1947) that any economic research program is essentially vacuous unless it can generate *meaningful theorems*. For Samuelson, a meaningful theorem is an assertion, generated by a model or theory, that could conceivably be refuted by actual data. Commonly, such assertions are the comparative static propositions which a model or theory generates. If general equilibrium models are not able to make unambiguous comparative static predictions on account of equilibrium states not being unique, and if Samuelson's methodology is adhered to, then general equilibrium theory might be in danger of being meaningless. Fourthly, as Ghiglino and Tvede (1997) note: "... economic agents have to coordinate their actions in order to make them mutually compatable and thereby obtain a feasible state. This coordination among agents can be obtained through a form of consistency of expectations ... In economies with a unique equilibrium, it is natural that agents expect the same state to prevail, namely the equilibrium, thus uniqueness of equilibrium leads to coordination. However for economies with multiple equilibria, it is less clear how and on what agents coordinate, so multiplicity of equilibria can lead to market failures due to lack of coordination". Ghiglino and Tvede (1997; p. 1). For these reasons at least, and also because the question is inherently interesting, conditions for the uniqueness of Walrasian equilibrium have been sought in a literature stretching at least from Wald (1936) to Kehoe (1998), Jerison (1999) up to and including the interesting discussion in McKenzie (2008b).

Mukherji (1997) provides the following useful overview of the main ways in which conditions for the uniqueness of equilibrium have been sought: "Two types of conditions have been used to ensure that equilibrium is unique ... one class consist of conditions which are assumed to hold at every price in the domain of definition of the excess demand functions ... the other type of conditions are those that are required to be satisfied either on the set of equilibrium prices or only on some other suitably defined set." Mukherji (1997; p. 509). Mukherji goes on to note that examples of the first type include the assumption that the weak axiom of revealed preference holds in aggregate; or that all goods are gross substitutes at all prices; or that the Jacobian of the excess supply function (with the numeraire row and column deleted), has the Gale property. Conditions of the second type include the Jacobian of excess supplies — again without the numeraire row or column — has the Gale property at equilibrium prices; or that this Jacobian has a dominant diagonal at equilibrium prices; or that it always has a positive determinant at equilibrium prices; or that the entire Jacobian of the excess demand functions now bordered by prices, has a determinant of sign $(-1)^{l-1}$ at equilibrium; or that the Jacobian of the excess demand functions is negative quasi-definite on appropriate subspaces of the price space. Later in this chapter, we provide a discussion of these various conditions along with a study of the particularly interesting approach in Mukherji (1997).

With these introductory remarks in place, this chapter proceeds as follows. Section 2 identifies what must be happening to excess demands in an economy in which equilibrium is not unique and then considers what must be happening in an economy where such an outcome to be avoided. Section 3 begins with a brief discussion of the Sonnenschein-Mantel-Debreu theorem, since that result conditions attitudes to any proposed conditions for uniqueness. We then review a number of conditions that are known to ensure the uniqueness of Walrasian equilibrium. We also consider a criterion for uniqueness based on the convergence of a price adjustment process. The central conclusion of this section is the observation that the uniqueness of Walrasian equilibrium appears to hold only under quite restrictive conditions. Section 4 begins by noting that if this is the case, then it is interesting to know if anything can be said about the size and structure of the equilibrium set. Debreu (1970) gave the seminal answer when he showed that most exchange economies had finitely many isolated equilibria, a result that has been extended by Fuchs (1974), Smale (1974), Ghiglino and Tvede (1997) and Pascoa and da Costa Werlang

(1999) among others. Using a result of Balasko (1980, 1988) and Journee (1992), we note that Debreu's 'finiteness' conclusion can be refined to yield the conclusion that the equilibrium set is likely to be a small finite set. We also use an argument from generic analysis to establish a result about the genericity of Walrasian equilibria. Section 5 offers some concluding remarks.

7.2. Conditions for the uniqueness of Walrasian equilibrium

In his study of the uniqueness problem for exchange economies, Debreu (1970) remarked that: "... The uniqueness property, however, has been obtained only under strong assumptions and economies with multiple equilibria must be allowed for." Debreu (1970; p. 387). Similarly Dierker (1983) notes: "The uniqueness of equilibrium can only be derived from highly restrictive assumptions." Dierker (1983; p. 796). For the production economy case, Kehoe (1985) suggests: "Unfortunately, the conditions required for the uniqueness of equilibrium in production economies appear to be [even] more restrictive than those in pure exchange economies ... this observation suggests that non-uniqueness of equilibrium is a less pathological situation than is sometimes thought." Kehoe (1985; pp. 120–121). This assessment is reinforced by the 'state of the art' result in Mas-Colell (1991) and by Mas-Colell *et al.* (1995) who argue: "We shall see ... that the uniqueness of equilibrium is assured only under special conditions." Mas-Colell *et al.* (1995; p. 590). Kehoe (1998) similarly contends that: "As we shall see, useful conditions that guarantee the uniqueness of equilibrium are very restrictive." Kehoe (1998; p. 38), while Debreu (1998) observes: "... conditions for the uniqueness of general equilibrium are considerably more restrictive than conditions for existence." Debreu (1998; p. 35).

Since there is not much division of opinion about the strength of the conditions needed to ensure that Walrasian equilibrium is

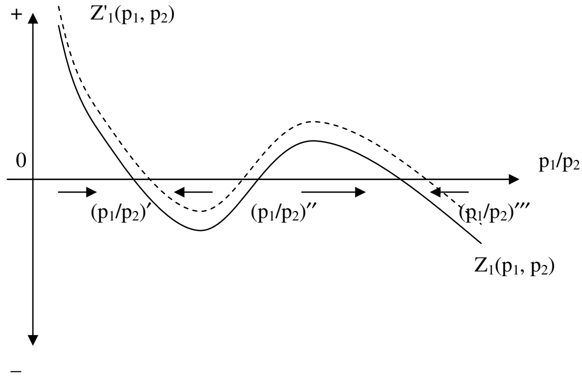

Fig. 7.1. Indeterminate comparative statics and non-uniqueness.

unique, our review of the literature on conditions for uniqueness will be less extensive than it was in the case of existence. Nevertheless, we will review the main approaches to specifying conditions for uniqueness and attempt to make what contributions we can to this difficult problem.

7.2.1. *The index theorem approach*

Consider the following two good example and in particular the excess demand function for good one $Z_1(p_1, p_2)$. An economy with multiple equilibria $(p_1/p_2)'$, $(p_1/p_2)''$ and $(p_1/p_2)'''$ is illustrated in Fig. 7.1

This example suggests that non-uniqueness of equilibrium is due to the slope of the consumer excess demand function varying in sign across equilibrium price vectors. If the excess demand function had the same, non-zero sign at each price ratio, then equilibrium would be unique. Therefore, conditions which rule out the slope of the excess demand function changing sign are candidate conditions for ensuring uniqueness of equilibrium.[1]

[1]Notice two other things about this example. Firstly, $(p_1/p_2)''$ is unstable and secondly there is the potential for strange comparative static outcomes such as an increase in excess demand for good 1 (represented by the dotted line), leading to a decrease in the relative price of good 1 if $(p_1/p_2)''$ was the original equilibrium.

The 'index theorem' introduced by Dierker (1972) and Varian (1974), allows the insight in Fig. 7.1 to be generalised to an economy with $\ell > 2$ commodities. Find a function $g(p) = (g_1(p), \ldots, g_\ell(p))$, where $g : \Delta \to \Delta$ and $p^* = g(p^*)$ iff $Z(p^*) \leq 0$, (where Δ denotes the unit simplex) so that the fixed points of $g(p)$ are Walrasian equilibria. Then use the following theorem to identify conditions under which equilibrium is unique.

Theorem 7.1 (Dierker (1972)). *If* $g : \Delta \to \Delta$ *and if* $g(p)$ *is continuously differentiable at its fixed points, all its fixed points are in* $\text{int}(\Delta)$, $[I - D_p g(p)]$ *is non-singular and the index of* $(p) \equiv \text{sgn}(\det[I - D_p g(p)])$, *then* $\sum_{p=g(p)}$ *index* $(p) = +1$.

Proof. Kehoe (1998; p. 52). □

Remark 7.1. The illustration in Fig. 7.2 is for an economy where $\ell = 2$ where there is no equilibrium at $p_1 = 0$ or $p_1 = 1$ and define the *index of* p at points where $p = g(p)$ as $+1$ for 'crossings from above' and -1 for 'crossings from below'. Then

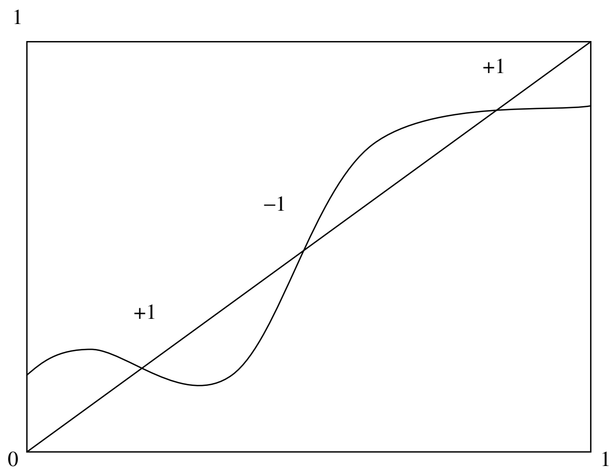

Fig. 7.2. The index theorem illustrated following Kehoe (1998).

the index theorem asserts that $\Sigma_{p=g(p)}$ index $(p) = +1$, (see Kehoe (1998; p. 51) for further details).

Remark 7.2. This index theorem suggests a sufficient condition for uniqueness of a fixed point of g(p), and hence of a Walrasian equilibrium, namely that $\det[I - D_p g(p)] > 0$ at every fixed point. It also provides a necessary condition in that if at any fixed point the $\det[I - D_p g(p)] < 0$ then there must be multiple fixed points. As Kehoe (1991, 1998) notes, in order to make an economic application of the index theorem an economically meaningful g — a function needs to be specified whose fixed points can be interpreted as Walrasian equilibria. One such function can be constructed by letting $g(p)$ be the point of Δ that is closest to $p + z(p)$, i.e. let $g(p)$ solve the problem: min $(1/2)\Sigma_k(g_k - p_k - Z_k(p))^2$ subject to $\Sigma_k\ g_k = 1$ and $g_k \geq 0$. Kehoe (1991) shows that $p^* = g(p^*) \Leftrightarrow Z(p^*) \leq 0$ and that $\text{sgn}(\det[I - D_p g(p)]) = \text{sgn}(\det[-D_p Z(p)])$, where $D_p Z(p)$ is the Jacobian of the excess demand function of the economy with the last row and column deleted. Thus in the case of economies in which the Jacobian of the excess demand function with the last row and column deleted is not singular at any equilibrium, so called 'regular economies', the restriction $\det[-D_p Z(p)] > 0$ is necessary and sufficient for the uniqueness of Walrasian equilibrium. In light of Dierker's index theorem, the literature has searched for conditions on the economy which ensure that the Jacobian of the excess demand function has the desired properties. Some of the conditions which do this are now discussed.

7.2.2. *Gross substitutes, the weak axiom, diagonal dominance and the Gale property*

We begin with some basic ideas. An economy has the *gross substitute property* if whenever p and p' are price vectors such that for some good j, $p'_j > p_j$ and $p'_i = p_i$ for all $i \neq j$

then $Z_i(p') > Z_i(p)$ for all $i \neq j$. If $Z(p)$ is differentiable, then $\partial Z_i(p)/\partial p_j > 0$ is sufficient for gross substitutability and $\partial Z_i(p)/\partial p_j \geq 0 (i \neq j)$ is necessary (see McKenzie (2008a)). If the matrix of these excess demand price effects is denoted D_pZ with typical element (z_{ij}), then the off-diagonal elements of $D_pZ(p)$ are positive (see Mas-Colell *et al.* (1995; p. 939)). An economy $Z(p)$ satisfies *the weak axiom of revealed preference* (WARP) if $p.Z(p') \leq 0$ and $Z(p) \neq Z(p')$ implies that $p' \cdot Z(p) > 0$. An economy has *diagonal dominance* if the excess demand for each good is 'more sensitive' to a change in the price of that good than it is to a change in the prices of all other goods combined (see Arrow and Hahn (1971; p. 233)). This means that for any p in the interior of the domain of $Z(p)$, there is $(h_1, h_2, \ldots, h_\ell) \gg 0$ such that $|h_i Z_{ii}| > \sum_{j \neq i} |h_j Z_{ij}|$, (see Mas-Colell *et al.* (1995; p. 939)). An excess demand map has the *Gale property* if for any small change in prices, relative to the numeraire, there is at least one commodity for which its price and excess demand move in opposite directions (see Arrow and Hahn (1971; pp. 210, 211)). A function $h : X \subset \Re^\ell \to \Re$ is *psuedomonotone* on $Y \subset X$ if for $p, p' \in Y, h(p')(p - p') \leq 0$ implies $h(p)(p - p') \leq 0$. An economy satisfies the *weak axiom of revealed preference* if it is pseudomonotone. In the case where the demand function is differentiable, it has the weak axiom of revealed preference property iff $\forall (p, \omega) \in \Re^\ell_{++} \times \Re^\ell_{++}$, the associated Slutsky matrix is *negative semi-definite*.

It has been known since Wald (1936) that if an exchange economy has the gross substitute property, there is only one Walrasian equilibrium (see Kehoe (1998; p. 43) for a neat proof of this). It is also well known that if the economy satisfies the WARP, then the set of equilibria is convex. If there are only finitely many equilibria (the 'regular economy' case), this also implies uniqueness (again see Kehoe (1998; p. 44) for a proof). Arrow and Hahn (1971; Theorem 9.12, p. 234) shows that diagonal dominance plus a desirable good will yield uniqueness.

Arrow and Hahn (1971; Theorem 9.1, p. 211) show that if excess demands are differentiable, are homogeneous of degree zero, satisfy Walras' Law, bounded below, are weakly continuous and a desirable good exists, then if the economy has the Gale property, then equilibrium is unique. Potent as these conditions are for establishing uniqueness, they all share a basic flaw in that they are not implied by the underlying microeconomics of the economy. There is nothing in the optimizing behaviour of individual agents which guarantees that these properties hold for the aggregate excess demand function. Also properties such as WARP do not generally aggregate in the sense that even if all consumers have personal excess demand functions that satisfy WARP, it is not necessarily the case that the aggregate $Z(p)$ for an exchange economy will also satisfy WARP (see Kehoe (1998; p. 45) for an example to illustrate this). Since it is known from an argument of Scarf (1973) that WARP is the weakest condition on $Z(p)$ alone that will guarantee uniqueness in an arbitrary production economy (see Kehoe (1998; p. 63)), it is of considerable interest to try to develop conditions on the primitives of the economy for such a condition to hold at the aggregate level.

One line of inquiry in this direction, initiated by Hildenbrand (1983), involves restricting the shape of the distribution of income in the economy. In pursuing this approach, Hildenbrand (1983) along with Hardle *et al.* (1991) restrict the distribution of income to be a nowhere increasing function. In the context of price independent income and identical consumers, such a restriction on income distribution implies that WARP holds in the aggregate. Relaxation of the price independent income assumption was achieved by Hildenbrand and Kirman (1988) and Hildenbrand (1989) in the case where endowments are collinear. Chiappori (1985) allowed income density functions which were sometimes increasing in the case where all consumers are identical and their Engel curves have a particular functional form. Also working in this tradition, Marhueda (1995) argues that the essential step in achieving the uniqueness of equilibrium involves controlling the behaviour of the income term in the (aggregate) Slutsky equation. In the context of an economy with a continuum of agents, price dependent incomes and endowments that are not necessarily collinear, Marheuda derives sufficient conditions for the uniqueness of Walrasian equilibrium. These conditions involve a trade-off between aggregate features of the consumption sector and the distribution of income and also seem to depend on there being a continuum of agents in the economy. The later condition appears to be an essential if the 'income distribution approach' is taken in view of the fact that Kirman and Koch (1986), working in an economy with a non-increasing income distribution function, a finite number of agents who have identical preferences, and price independent incomes, obtain nonuniqueness, while Hildenbrand (1983), using similar assumptions and a continuum of agents, derives a downward sloping aggregate demand relation.

As an alternative to considering restrictions on the distribution of income, Grandmont (1992) considers restricting the shape of the distribution of agents' characteristics. Assuming: (i) that there is a commodity that is always desired at the aggregate level no matter what its price, (ii) all agents who have the same preferences have the same income, (iii) there is a continuum of agents, he obtains conditions on the excess demand function of the economy such as gross substitutability, diagonal dominance and WARP which allow the uniqueness of Walrasian equilibrium to be established. In the spirit of Hildenbrand (1983) and Grandmont (1992), Quah (1997) identifies conditions sufficient to guarantee that aggregate demand is approximately linear in income and satisfies a restricted form of the Law of Demand, even when income is price dependent. In particular Quah (1997) set out "...to identify some plausible assumptions on the distribution of characteristics among agents that will guarantee that an exchange [or production] economy has a unique ...equilibrium..." Quah (1997; p. 1421). In the event Quah (1997) achieves his objective via an assumption that *preferences of agents in the economy are distributed independently of the endowments of agents in the economy.*

Remark 7.3. Commenting on this assumption, Quah (1997) makes the following observation: "...the independence assumption here has an unpleasant implication: it excludes the possibility that agents face a labor-leisure choice." Quah (1997; p. 1423). If this is regarded as implausible, then this work has not been able to furnish a plausible uniqueness argument, even though it is an important extension of the Hildenbrand–Grandmont approach.

Operating directly on the excess demand function of the economy, Mukherji (1997) establishes that if: (a) $Z(p)$ is continuously differentiable; (b) $Z(p)$ satisfies Walras' Law; (c) $Z(p)$ is homogeneous of degree zero; (d) $\Sigma_i \; Z_i(p) \rightarrow +\infty$ if the price sequence $\{p^s\} \rightarrow p^0 \in \partial\Delta$; (e) equilibrium is locally unique; (f) there are no boundary equilibria; (g) $\forall p \in \Delta$, $Z(p)^T D_p Z(p) = 0 \Rightarrow Z(p) = 0$, then equilibrium is unique (see Mukherji (1997; Proposition 3)). The crucial condition in all this is that $\forall p \in \Delta, Z(p)^T D_p Z(p) = 0 \Rightarrow Z(p) = 0$. This condition would be violated in an economy where (i) there is no first order change in excess demand even though relative prices have changed and (ii) there is a way to change prices such that all excess demands increase. If these possibilities are regarded as unreasonable, then Mukherji's conditions are applicable (see Mukherji (1997; pp. 514–517) for further discussion). Jerison (1999) has proposed an economically interpretable condition that guarantees that the (mean) excess demand satisfies the WARP and that Walrasian equilibrium is unique. This is an interesting contribution and some time will be spent studying it. The essential definitions in Jerison (1999) are the following.

Definition 7.1 (Increasing dispersion of excess demand). Let $Z_\lambda^\alpha(p) = x^\alpha(p, \lambda + p\omega^\alpha) - \omega^\alpha$ be the *excess demand for household type* α at price vector p and wealth $\lambda + p\omega^\alpha$. *Increasing dispersion of excess demand* (*IDED*) (*non-decreasing dispersion of excess demand* (*NDED*)) holds at p if a slight increase in λ, starting at 0, increases (does not decrease) the variance of the household excess demands in any direction orthogonal to p and the total excess demand vector $Z(p) = \int [x^\alpha(p, \lambda + p\omega^\alpha) - \omega^\alpha] d\mu$.

Definition 7.2 (NAS). The consumption sector of an economy has *non-positive average substitution* at p if the mean of the households' Slutsky matrix $\int S^\alpha(p, p\omega^\alpha) d\mu$ is negative semi-definite at p.

Proposition 7.1 (Jerison (1999)). *Suppose the consumption sector satisfies NAS and has a regular mean excess demand function Z. If the consumption sector satisfies NDED, then Z is pseudomonotone and so satisfies WARP and the set of Walrasian equilibrium price vectors is convex. If in addition, there is no production or if the consumption sector has NAS or IDED then Walrasian equilibrium is unique.*

Proof. Jerison (1999; pp. 37, 38). □

Remark 7.4. In commenting on this result and on the conditions which permit it, Jerison (1999) notes that IDED requires stringent restrictions on the Engel curves of consumers if it is to be satisfied for every distribution of collinear consumer endowments. Figure 7.3 shows a situation in which the condition fails.

There are two other difficulties with the result. Firstly, the sufficient condition given for uniqueness in the proposition only guarantees one equilibrium with strictly positive prices. In particular: "The proposition would not rule out other equilibria with some prices equal to zero." Jerison (1999; p. 27). As seen

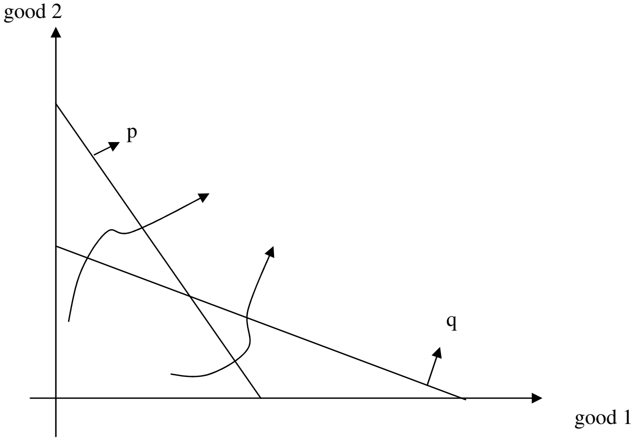

Fig. 7.3. Example of violation of NDED from Jerison (1999).

in the previous chapter, the possibility of equilibrium with some prices zero is a real one and this may weaken Jerison's result. Secondly, in the commonly encountered case of an economy in which there are fixed supplies of a primary non-produced good: "It can be shown that in these models IDED is generically violated." Jerison (1999; p. 27). A more fundamental difficulty with the proposition and the condition on which it depends is as Jerison (1999; p. 34) points out in considering the definition of IDED: "... we see that the property cannot be expected to hold for highly disaggregated commodities." The reason for this is that when commodities are highly disaggregated, inferior goods will generally exist and when they are present, the differences between excess demands decrease as households get richer (see Jerison (1999; pp. 34, 35) for further discussion). Recalling the definition of an Arrow-Debreu commodity, it is clear that disaggregation comes with the definition and is therefore a key feature of the model. Consequently, according to Jerison's argument, it is unlikely that IDED will hold in the sorts of economic environments considered in general equilibrium theory.

7.2.3. *Uniqueness with production*

Kehoe (1998) argues that the literature tends to concentrate on conditions for uniqueness in exchange economies: "In particular, there has been too little effort devoted to research on uniqueness in production economies." Kehoe (1998; p. 83). One of Kehoe's own contributions in this context involves developing an index condition for uniqueness in a production economy. One consequence of this work appears to be that conditions that guarantee uniqueness in exchange economies, restrictive as they might be, are not generally strong enough to guarantee uniqueness in a production economy context. Furthermore, it turns out that the index condition in the production economy case is almost impossible to interpret in an economically meaningful way because of the complex feedback effects from the profits generated by production into consumer demand. This later observation is the primary motivation for us to develop an alternative uniqueness criterion based on contraction mappings.

Kehoe (1998) illustrates his claim that conditions for uniqueness which work in the exchange case will not generally do so in a production context, with an example where the aggregate consumer excess demand functions satisfies gross substitutability, the production side of the economy is described by an activity matrix and the economy has three equilibria. From this example, Kehoe concludes that gross substitutability on the aggregate consumer excess demand function is generally not sufficient in a production economy, to guarantee uniqueness of Walrasian equilibrium. In the case where production can be described by an activity matrix A, the economy (z, A) is called *regular* if no column of $A = (a_{ij})$, can be expressed as a linear combination of fewer than ℓ other columns, every activity that earns zero profit in equilibrium is associated with a positive activity level and $I - D_p g(p^*)$ is non-singular at every equilibrium p^*, where $g(p)$ is a map defined by Kehoe (1985; p. 125). An economy is *input-output* if there is one non-produced good

ℓ with $z_\ell(p) > 0$ at every equilibrium and there is no joint production so that $a_{ij} > 0$ for each ij, and some non-negative vector of activity levels y exists such that $\Sigma_j a_{ij} y_j > 0$. Kehoe (1985) shows that if z is an aggregate consumer excess demand function which violates WARP in the sense that there are prices $p' \neq p''$, with at least one strictly positive, such that $p''z(p') \leq 0$ and $p'z(p'') \leq 0$, then there exists an activity matrix A such that the economy (z, A) has multiple equilibria. On the other hand, if the economy is such that the aggregate consumer excess demand function z exhibits gross substitutability and either $A = -I$, or $\ell \leq 3$, or (z, A) is a regular economy in which z satisfies WARP, or (z, A) is an input-output economy in which no columns of A can be expressed as a linear combination of fewer than ℓ other columns, then Walrasian equilibrium is unique.

The question which Kehoe (1998) then addresses is what sort of conditions will yield uniqueness of equilibrium in a general production economy where production is not necessarily describable by an activity matrix. In particular, in the case where production exhibits decreasing returns to scale, the situation becomes more complicated, primarily because of distributed profits in the income functions of households. In particular the index of an equilibrium price vector p^* is given by:

$$(-1)^\ell \text{sign det} \begin{bmatrix} 0 & \iota^T \\ \iota, & D_p z(p^*, \pi^*) + D_\pi z(p^*, \pi^*) \cdot B^T(p^*) - H(p^*) \end{bmatrix}, \tag{7.1}$$

where $D_\pi z(p, \pi)$ is the matrix of partial derivatives of the consumer excess demand function with respect to the vector of profits of the individual firms and $H(p)$ is the Jacobian matrix of the supply function of the economy. This Slutsky type expression is central to the search for conditions for the uniqueness of equilibrium. However: "Although [this index] looks much like that for a pure exchange economy, it is difficult to interpret.

The problem is that the term $D_\pi z(p^*, \pi^*) \cdot B^T(p^*)$ does not depend on consumer demand alone; it involves a complex interaction between income effects from the demand side of the economy and activities from the production side. It seems impossible to develop easily checked conditions to guarantee that $D_p z(p^*, \pi^*) + D_\pi z(p^*, \pi^*) \cdot B^T(p^*) - H(p^*)$ has the required sign pattern." Kehoe (1985; p. 144). In similar vein after 13 more years thinking about the problem Kehoe (1998) remarks: "Unfortunately, $D_\pi z(p^*, \pi^*) \cdot B^T(p^*)$ does not depend on consumers utility and endowments alone. It also involves a complex interaction of income effects in consumption and production. It *may* be possible to develop conditions that ensure it has the required sign pattern." Kehoe (1998; p. 73, emphasis added). However at this stage, no such conditions are apparent.

If attention is restricted to economies with convex, constant returns to scale aggregate production sets Y, then profits are zero and the problems alluded to by Kehoe (1985, 1991, 1998) do not arise. Let $z(p) = \Sigma_i[x_i(p, p\omega_i) - \omega_i]$ be the aggregate excess demand function for consumers, then in the economy given by $\boldsymbol{E} = [z(p), Y]$ the following condition is necessary for uniqueness.

Proposition 7.2 (Mas-Colell *et al.* (1995)). *The satisfaction of WARP by $z(p)$ is a necessary condition for the uniqueness of Walrasian equilibrium when Y is a convex, constant returns to scale aggregate technology set and the economy has finitely many equilibria.*

Proof. Mas-Colell *et al.* (1995; p. 609). □

Remark 7.5. As Takayama (1974), Hildenbrand (1989), and Jerison (1999) all point out, satisfaction of WARP is a strong condition to impose on $z(p)$. It is particularly interesting that it is necessary for uniqueness when Y is CRS.

7.2.4. *Special structure on preferences and technologies*

One approach to specifying conditions for the uniqueness of equilibrium in a production economy involves considering the sorts of special structure on production and utility functions that will guarantee uniqueness. One example of this approach is due to Mas-Colell (1991) who establishes, via an index argument, that Walrasian equilibrium is unique if every utility and production function is *super–Cobb-Douglas.*

Definition 7.3 (Super–Cobb-Douglas). A function $h : \Re^\ell_+ \to \Re$ is *super–Cobb-Douglas* if at every $x \geq 0$, there is a Cobb-Douglas function $h_x : \Re^\ell_+ \to \Re$ such that $h_x(x) = h(x)$ and $h_x(x') \leq h(x')$ for all x' in the neighbourhood of x. On the basis of this definition Mas-Colell (1991) proves:

Theorem 7.2 (Mas-Colell (1991)). *If every utility function and every production technology is super–Cobb-Douglas, then in every regular economy Walrasian equilibrium is unique.*

Proof. Mas-Colell (1991; pp. 294, 295). □

Remark 7.6. Kehoe (1998) shows that what has to be the case for production or utility functions to be super–Cobb-Douglas is that *locally* the production or utility function admits as much substitutability as a Cobb-Douglas function (see Kehoe (1998; p. 74) for an illustration of this). The examples of super–Cobb-Douglas functions given by Mas-Colell (1991, p. 292) reveal that they form a restricted class of function (e.g. the CES functions with elasticity of substitution greater than or equal to one). In his commentary on the theorem, Mas-Colell is quick to point out that the conditions of the theorem are sufficient 'but far from necessary' and that what really matters for the result is that the economy is Cobb-Douglas 'in the average' (see p. 294). The condition that the economy is on average Cobb-Douglas, nevertheless imposes restrictions on the preferences of consumers and the technology of producers which are of the sort routinely rejected in applied work. In modern empirical work, it is almost obligatory the use of flexible functional forms designed precisely to capture demand and supply behaviour generated by non-Cobb-Douglas preferences and technologies (see for example the remarks in Lau (1983), Cooper and McLaren (1993), and Nahm (1996) and Chap. 11).

Remark 7.7. John (1989) has taken a similar approach and has established conditions for the uniqueness of Walrasian equilibrium when all utility and production functions are of the CES variety.

7.2.5. *The Sonnenschein-Mantel-Debreu result*

The search for economically reasonable conditions for the uniqueness of equilibrium (meaning conditions implied by the underlying microeconomics of the economy) is not straightforward. In part this is a consequence of the Sonnenschein-Mantel-Debreu theorem. Roughly stated, this result says that, apart from continuity, homogeneity of degree zero and Walras' Law, essentially no structure is imposed on the aggregate excess demand function and its associated Jacobian matrix of price effects, by agent optimisation. More precisely, as Mas-Colell *et al.* (1995) put it: "[The SMD result] tells us that for any finite collection of price vectors $\{p^1, \ldots, p^N\}$ and matrices of price effects $\{D_pZ(p^1), \ldots, D_pZ(p^N)\}$, we can find an economy with ℓ consumers for which these price vectors are equilibrium price vectors and $\{D_pZ(p^1), \ldots, D_pZ(p^N)\}$ are the corresponding price effects at these equilibria. The result implies that to derive further restrictions on Walrasian equilibria [such as uniqueness and stability] we will need to make additional (*and* *as we shall see, strong*) assumptions." Mas-Colell *et al.* (1995; p. 604, emphasis added). Grandmont (1992) has gone so far as to describe the result in the following way: "The shattering result in the area is Sonnenschein's indeterminacy theorem... as refined by McFadden, Mantel, Mas-Colell, Richter, Debreu and others [which says]: individual optimisation ('microeconomic rationality') does not place any restriction on aggregate excess demand, on any given compact set of prices, other than homogeneity and Walras' Law. This would be true even if we required all traders to have homothetic preferences!" Grandmont (1992; p. 2). Specifically, why this result should be so significant for uniqueness is brought out by the following remark by Grandmont (1992): "Whereas such findings do not threaten the usual results about the existence and efficiency of market equilibrium, they appear to jeopardise two important ingredients of the neoclassical paradigm. There is apparently little chance indeed to obtain gross substitutability, diagonal dominance or the weak axiom of revealed preference in the aggregate or any other properties that have long been known to be needed for *uniqueness* and *stability* of competitive equilibrium. The implications of such results are unfortunately rarely discussed..." Grandmont (1992; p. 2, emphasis in original).

Given that the conditions for uniqueness are restrictive, difficult to interpret economically, or imposed in the sense that they do not follow naturally from the microeconomics of the economy it seems reasonable to conclude, along with the literature, that uniqueness of Walrasian equilibrium cannot generally be relied on. We are therefore motivated to explore an alternative possible route to establishing uniqueness. The approach we explore follows a remark by Friedman (1991) who notes that there are two basic approaches to establishing the uniqueness of equilibrium. One, inspired by index theorems, involves conditioning the Jacobian of the excess demand function. The other involves studying properties of the fixed point map other than those that depend directly on its Jacobian. As we have seen, the first approach has been studied extensively in the literature, but as Friedman (1991) points out: "In practice it is useful to have all of these uniqueness theorems available." Friedman (1991; p. 87). In the next section, we therefore consider uniqueness from the second point of view identified by Friedman (1991).

7.3. Uniqueness via adjustment processes

7.3.1. *Exploiting regularity and boundary conditions*

In line with Friedman (1991), the approach Mukherji (1997) takes starts with an observation by Arrow and Hahn (1971; p. 236) in connection with their Theorem 15 that: "... [the] method of proof is one where the dynamics of a particular adjustment process is exploited." Mukherji (1997; p. 510). This is the approach taken in the current section where we first study Mukherji's contribution and then present an argument that also exploits the dynamics of a particular adjustment process to establish a uniqueness result. Mukherji's basic set up and result is e summarised in the following proposition.

Theorem 7.3 (Mukherji (1997)). *The price set* P *is such that* $\Re^{\ell}_{++} \subset P \subset \Re^{\ell}_{+}$ *and if the economy is characterized by an excess demand function* $Z : P \rightarrow \Re^{\ell}$ *which satisfies* $(mu.1)$ $\forall p \in P$, $Z(p)$ *is a continuously differentiable function of* p, *is bounded below and has continuous second order partial derivatives*; $(mu.2)$ *satisfies Walras' Law so* $p^T Z(p) = 0$, $\forall p \in P$; $(mu.3) Z(p)$ *is homogeneous of degree zero in price so for any* $\lambda > 0$ *and* $\forall p \in P, Z(\lambda p) = Z(p)$; $(mu.4)$ *if* $S = \{p \in \Re^{\ell}_{+} : \sum_i p_i^2 = 1\}$ *and* $T = S \cap P$ *then if* p^s, $s = 1, 2, \ldots$ *is a sequence,* *with* $p^s \in T$ *for all* s, *and if* $p^s \to p^0 \notin P$ *then the boundary condition* $\Sigma_i Z_i(p^s) \to +\infty$ *holds*; $(mu.5)$ *if* $E = \{p \in T : Z(p) = 0\}$ *is the equilibrium price set and* $\partial S = \{p \in S : p_k = 0$ *for some good* $k\}$ *is the boundary of* S, *then* $E \cap \partial S = \emptyset$ *so there are no boundary equilibria*; $(mu.6)$ *if* $p^* \in E$ *then there is a neighbourhood* $N(p^*)$ *such that* $p^* = N(p^*) \cap E$ (*local uniqueness*) *and* $p^{*T} J^T\ (p) Z(p) \leq 0$ *for all* $p \in N(p^*) \cap T$ (*weak regularity*) *where* $J(p) = \partial Z_i / \partial Z_j(p) \equiv Z_{ij}$ *for* $i, j = 1, \ldots, l$ *is the Jacobian of excess demands*; $(mu.7)$ $\forall p \in T, Z(p)^T J(p) = 0 \Rightarrow p \in E$, *equilibrium is unique and* $E = \{p^*\}$.

Proof. Mukherji (1997; p. 518). □

Remark 7.8. As the argument in Mukherji (1997; pp. 517–518) makes clear, this uniqueness result exploits properties of the adjustment process $\dot{p} = -J(p)^T Z(p)$, which he characterizes as being like 'the global Newton method', under the assumed conditions of the theorem. As for those conditions, (mu.1)–(mu.3) are standard as are the boundary conditions (mu.4) and (mu.5). Condition (mu.6) is a condition weaker than the requirement that p^* is a regular equilibrium while (mu.7) is a global version of a local implication of the regularity of p^*. The theorem, its relative sympathy with the SMD result and in particular, the exploitation of properties of the adjustment process, is therefore an interesting contribution to the literature.

7.3.2. *Exploiting contraction mappings*

In this section, we use contraction mappings in an attempt to specify conditions for the uniqueness of Walrasian equilibrium. Contraction mappings have been used to establish uniqueness by Friedman (1991) in a model of Cournot oligopoly and by Cornes *et al.* (1999) in a public goods model. We begin as usual with some basic ideas and definitions. (X, ρ) a *metric space* if X is non-empty and ρ is a measure of distance that satisfies:

(i) $\forall x, y \in X, \rho(x, y) \geq 0$; (ii) $\forall x \in X, \rho(x, x) = 0$; (iii) $\forall x, y, z \in X, \rho(x, y) \leq \rho(x, z) + \rho(z, y)$. X is a *complete metric space* if (X, ρ) is a metric space and X is a *complete set* meaning that for any sequence $\{x_1, x_2, x_3, \dots\}$ in X, such that $\rho(x_m, x_n) \to 0$ as $m, n \to \infty$, then that sequence is convergent and its limit is in X.

Definition 7.4 (Contraction mapping). Let f be a mapping such that $f\colon X \to X$, then if there is a $0 < k < 1$ such that $\rho[f(x), f(x')] \leq k \cdot \rho(x, x')$ for all $x, x' \in X$, then f is a *contraction mapping* on X.

Lemma 7.1 (Banach). *If $f\colon X \to X$ is a contraction mapping on a complete metric space (X, ρ), then f has a unique fixed point, i.e. $\exists! x^* \in X$ such that $x^* = f(x)$.*

Proof. Bryant[2] (1985; pp. 59–60). □

Lemma 7.2. *The unit simplex Δ is a complete metric space.*

Proof. If (M, ρ) is a complete metric space under the metric ρ and X is a closed subset of M, then X is a complete metric space under the induced metric ρ^X. To see this let (x_n) be a Cauchy sequence in X. The (x_n) is a Cauchy sequence in M. Therefore there exists $x \in M$ such that $x_n \to x$. But then $x \in X$, because X is closed and so X is complete. Now Δ is a closed subset of $\Re^\ell$ and $(\Re^\ell, \rho)$ is a complete metric space (see Beals (1973; p. 23)) and so (Δ, ρ^Δ) is a complete metric space. □

Proposition 7.3. *If $g(p)$ maps $\Delta \to \Delta$ such that the fixed points of $g(p)$ occur only at points where $Z(p) \leq 0$ and if $g(p)$ is a contraction mapping, then the economy which generates $Z(p)$ has a unique Walrasian equilibrium.*

Proof. The result follows immediately from Lemmas 7.1 and 7.2. □

[2]No relation to the Author.

Remark 7.9. Lemma 7.1 is our parallel to the index theorem in that it provides a mathematical criterion for uniqueness which Proposition 7.2 translates into an economic context. In order to try to get economically meaningful conditions for the uniqueness of Walrasian equilibrium, we need at least a g-function whose fixed points are the equilibria of the economy. In the case of a general production economy, Kehoe (1991) shows that one way to obtain such a function is as a solution to the problem:

$$\begin{aligned} &\mathrm{Min}_{g(p)} \frac{1}{2}[g(p) - p - Z(p)]^T [g(p) - p - Z(p)] \\ &\qquad \text{subject to } g(p) \in \Delta. \end{aligned} \tag{7.2}$$

Uniqueness of Walrasian equilibrium then follows if

$$\begin{aligned} &|g(p) - g(p')| \leq k.|p - p'| \\ &\qquad \text{for all } p, p' \text{ in } \Delta \text{ and for some } k < 1. \end{aligned} \tag{7.3}$$

In general, a closed form solution to this problem cannot be obtained. However, in the case where the production side of the economy is described by an activity matrix A, Kehoe (1991; p. 2081) shows that the optimisation problem can be written as:

$$\begin{aligned} &\mathrm{Min}_{g(p)} \frac{1}{2}[g(p) - p - Z(p)]^T [g(p) - p - Z(p)] \\ &\qquad \text{subject to } A^T g \leq 0 \quad \text{and} \quad e^T g = 1. \end{aligned} \tag{7.4}$$

If B is the matrix of columns of A associated with strictly positive Lagrange multipliers λ_j in the first-order conditions, $g(p) - p - Z(p) + Ay + \lambda e = 0$, let C be the matrix $[Be]$ be the $e^{\ell+1}$ vector with $e_j^{\ell+1} = 1$ for $j = \ell + 1$ and $= 0$ otherwise. Then the following closed form solution for $g(p)$ emerges.

$$g(p) = (I - C(C^T C)^{-1} C^T)(p + Z(p)) + C(C^T C)^{-1} e^{\ell+1}. \tag{7.5}$$

Substituting this expression into (7.3) gives a criterion for uniqueness, which is an alternative to that provided by the index theorem approach. The criterion is that:

$$|(I - C(C^TC)^{-1}C^T)||p + Z(p) - p' + Z(p')|$$
$$\leq k \cdot |p - p'| p, p' \in \Delta \text{ and some } k < 1. \qquad (7.6)$$

An activity analysis structure on the production side of the economy is however restrictive. It would be interesting to extend this criterion for the general production economy case. In such a general production context it is, as Kehoe (1991, 1998) notes, generally impossible to get an economically interpretable condition for uniqueness via the index theorem route.

Some headway may be made, via the contraction mapping approach, if we consider the map $g(p)$ defined following Mas-Colell *et al.* (1995; p. 588). Let $Z^+(p)$ be defined on Δ by the function $Z^+_\ell(p) = \max\{Z_\ell(p), 0\}$ and let $\alpha(p) = \Sigma_\ell[p_\ell + Z^+_\ell(p)]$. The function $g(p) = [1/\alpha(p)](p + Z^+(p))$ has fixed points just at the equilibria of the economy (see Mas-Colell *et al.* (1995; p. 588) for a proof of this). Then an alternative condition for uniqueness of Walrasian equilibrium is that:

$$|p'/\alpha(p') + Z^+(p')/\alpha(p') - p/\alpha(p) - Z^+(p)/\alpha(p)|$$
$$\leq k \cdot |p' - p|. \qquad (7.7)$$

Multiply throughout by $\alpha(p) \cdot \alpha(p')$. Then (7.7) implies that equilibrium is unique if:

$$|\iota^T(p + Z^+(p))p' - \iota^T(p' + Z^+(p'))p$$
$$+ \iota^T(p + Z^+(p))Z^+(p') - \iota^T(p' + Z^+(p')Z^+(p)|$$
$$\leq k \cdot (\iota^T(p' + Z^+(p')) \cdot \iota^T(p + Z^+(p)) \cdot |p\prime - p|$$
$$\text{where } \iota \text{ is the unit vector.} \qquad (7.8)$$

This contraction mapping criterion for uniqueness, unlike the index theorem based criterion, does not require differentiability of the $g(p)$ function. However, in the case where $g(p)$ is differentiable, then we may use the following result to obtain a uniqueness criterion.

Lemma 7.3 (Bryant (1985) and Friedman (1991)). *Let $f : [a, b] \to [a, b]$ be differentiable. Then f is a contraction of $[a, b]$ if and only if there is a number $k < 1$ such that $|f'(x)| \leq k$ for all $x \in (a, b)$. In the case where $f : \Re^m \to \Re^n$, then if $\Sigma_{i=1}, m|\partial f_j/\partial x_i| \leq k$ for each component $f_j(x)$ of $f(x) = (f_1\ (x), \ldots, f_n(x))$, then $f(x)$ is a contraction mapping.*

Proof. Bryant (1985; p. 62). □

Remark 7.10. In the case where $g(p) = [1/\alpha(p)](p + Z^+(p))$, Lemma 7.3 yields that $g(p)$ is a contraction mapping (and hence equilibrium is unique), when $\Sigma_{i=1,n}|\partial g_j/\partial p_i| \leq k < 1$. Since $g : \Delta \subset \Re^\ell \to \Delta \subset \Re^\ell$, then $(p_1, p_2, \ldots, p_\ell) \to g_1(p_1, p_2, \ldots, p_\lambda) \ldots g_\ell(p_1, p_2, \ldots, p_\ell))$. Specifically, $g_j(p_1, \ldots, p_\ell) = (p_j + Z_j^+(p)/[\sum_\ell(p_\ell + Z_\ell^+)]$. Therefore,

$$\partial g_j/\partial p_i = -(p_j + Z_j^+(p)) \cdot \left(1 + \sum_\ell \partial Z_\ell^+/\partial p_i\right) \Big/ \left[\sum_\ell (p_\ell + Z_\ell^+(p)\right]^2, \quad j \neq i$$

$$= \left[(1 + \partial Z_i^+/\partial p_i) \cdot \left(\sum_\ell (p_\ell + Z_\ell^+(p)\right) - (p_i + Z_i^+(p))\right.$$

$$\left. \times \left(1 + \sum_\ell \partial Z_\ell^+ \Big/ \partial p_i\right)\right] \Big/ \left[\sum_\ell (p_\ell + Z_\ell^+(p)\right]^2, j = i.$$

Therefore in evaluating $\sum_{i=1,n} |\partial g_j/\partial p_i|$, for $j \neq i$, the terms going into the sum are:

$$|p_j + Z_j^+(p)| \cdot |\left(1 + \sum_\ell \partial Z_\ell^+/\partial p_i\right) \Big/ \left[\sum_\ell (p_\ell Z_\ell^+(p)\right]^2.$$

For $j = i$, the terms going into the sum are:

$$|(1 + \partial Z_i^+/\partial p_i)| \cdot \left|\sum_{\ell \neq i} (p_\ell + Z_\ell^+(p))\right| \cdot \left|1\right.$$

$$\left. + \sum_\ell \partial Z_\ell^+ \Big/ \partial p_i\right| \Big/ \left[\sum_\ell (p_\ell + Z_\ell^+(p)\right]^2,$$

therefore:

$$\sum_{i=1,n} |\partial g_j/\partial p_i| = \Bigg\{ |p_j + Z_j^+(p)| \cdot \Bigg(\sum_i \Bigg| \Bigg(1 + \sum_\ell \partial Z_\ell^+/\partial p_i \Bigg) \Bigg] \Bigg| \cdot \Bigg| \times \sum_{\ell \neq i} (p_\ell Z_\ell^+(p)) \Bigg| \cdot \sum_i \Bigg[\Bigg| (1 + \partial Z_i^+/\partial p_i) \Bigg| \cdot \Bigg| 1 + \sum_\ell \partial Z_\ell^+/\partial p_i \Bigg| \Bigg] \Bigg\} \Bigg/ \Bigg[\sum_\ell (p_\ell + Z_\ell^+(p) \Bigg]^2. \tag{7.9}$$

So if $\sum_{i=1,n} |\partial g_j/\partial p_i| \leq k < 1$ then equilibrium is unique.

Remark 7.11. A further generalisation of this result can be obtained by noting that it is not necessary for $g(p)$ itself to be a contraction mapping for it to have a unique fixed point. It will suffice if the composition of g with itself N-times, for some $N < \infty$, is a contraction mapping. If f: $X \to X$, then $f(f(x))$ also maps $X \to X$. Call $f(\ldots f(f(f(f(x)$ (N-times) the Nth-*iterate of f denoted by f^N*. Bryant (1985; p. 65) shows that if (X, ρ) is a complete metric space and f: $X \to X$ then if, for some N, the Nth-iterate of f is a contraction mapping on X, then X has a unique fixed point under f. In the case where $g(p)$ is described by (7.5) then provided the Nth iterate of $g(p)$ is a contraction mapping, then the argument above will still apply.

Remark 7.12. Our attempts to come up with conditions under which Walrasian equilibrium is unique has proceeded by showing that if a function is a contraction mapping, then it has only one fixed point and uniqueness of equilibrium is guaranteed. The strength of the conditions needed for this result reinforces the conclusion obtained by the index theorem approach that it is only under special and restrictive conditions that the economy has just one equilibrium.

7.4. The size of the equilibrium set

Given that uniqueness cannot be expected in general, the next hope is that the number of elements in the equilibrium set is not 'too big' and at least that it is a finite set. As Allingham (1987) puts it: "If [uniqueness] does not obtain, then at least the set of equilibrium price systems should not be too large, that is there should be only a finite number of equilibria." Allingham (1987; p. 754). In a celebrated paper, Debreu (1970) found a set of conditions under which almost all exchange economies have finitely many locally isolated equilibria. It turns out that this result has a number of interesting implications and refinements, some of which we explore.

7.4.1. *'Small finite' or 'large finite'?*

Even if generally there are only finitely many Walrasian equilibria, that still leaves open the possibility that the finite number may be numerically large. Indeed just such a possibility has been noted by Mas-Colell *et al.* (1995), who remark that: "...the 'finiteness of the number of equilibria' is a blunt conclusion. *It is not the same* if the 'finite' stands for three or for a few million. Unfortunately, short of going all the way to uniqueness conditions... we have no technique that allows us to refine our conclusions. We want to put on record, however, that it should not be presumed that in all generality 'finite' means 'small'." Mas-Colell *et al.* (1995; p. 597, emphasis added). As Mas-Colell *et al.* (1995) correctly observe, the fact that the equilibrium set is finite does not rule out the possibility that the number of equilibria is numerically large. What is not correct are their twin claims that: (i) without going all the way to uniqueness, we have no technique that allows us to refine the 'finiteness' conclusion and (ii) that it should not be presumed that 'finite' means 'small'. We now show that there is a technique which allows 'finiteness' to be refined, at least in probability and in the direction of 'small'. The results which justify this claim are due to Balasko (1980, 1988) and Journee (1992). Consider a compact subset K of the space of exchange economies Ψ and let $\boldsymbol{E}_n(K)$ denote the set of economies in K having at least n Walrasian equilibria. Also let $\mu(\boldsymbol{E}_n(K))$ denote the Lebesgue measure of that set.

Theorem 7.4 (Balasko (1988)). *There is a constant $c(K)$ such that the inequality $\mu(\boldsymbol{E}_n(K)) \leq c(K)/n$ is satisfied for all $n \geq 1$.*

Proof. Balasko (1988; pp. 108, 109). □

Remark 7.13. In interpreting this result, Balasko (1988; p. 109) argues that it shows that: "…the size of the set of economies with at least n equilibria tends to zero at a rate greater than $1/n$ [and] [a]lthough the probability of observing economies with *multiple* equilibria is far from being equal to zero, the probability of observing economies with a *large number of equilibria is rather small.*" Balasko (1988; p. 109, emphasis added). Another way to put this is to say that while 'finite' does not necessarily mean 'small' it probably does not mean 'large'. Thus while Mas-Colell *et al.* (1995) are correct to point out that 'finite' could still mean that there were say $10^{10,000} + 1$ equilibria in the economy, Balasko's result shows that the probability of that is very small. Consequently and contrary to the argument in Mas-Colell *et al.* (1995; p. 597), 'finite' probably does mean 'small'. Journee (1992) has extended Balasko's result to cover the case of a production economy.

Remark 7.14. Knowing that the equilibrium set is likely to be a small finite set is actually a move in the direction of the following methodology announced by Mas-Colell *et al.* (1995; p. 590) which has it that theories about social phenomena should be parsimonious with respect to inputs and able to predict unique outcomes. The Balasko-Journee result is also important in light of another remarkable result established by Balasko (1988; p. 188, Theorem 7.7.9). This theorem shows that knowing just the *number* of equilibria also gives information about the *values* taken by prices at equilibrium. As Balasko (1988) puts it, having the number of solutions to the equilibrium equations: "... is just equivalent to knowing the precise values of these solutions." Balasko (1988; p. 189). Therefore knowing something about the likely size of the equilibrium set is a first step to obtaining *quantitative* information about equilibrium prices from a purely theoretical model. There is however another implication of finiteness (small or large), which has to do with the genericity or not of equilibrium states.

7.4.2. *The measure, density and likelihood of equilibrium prices*

In this section, we consider the following question: are Walrasian equilibria relatively abundant and likely to be encountered in the price space or are they sparse and relatively rarely encountered? do they form an 'unlikely' set? The usual approach to answering questions of this type is to ask whether or not the object or condition of interest is 'typical' or 'exceptional' relative to the family of objects or events in which it resides. The following remark by Grandmont *et al.* (1974) gives an indication of the motivation for, and the nature of this 'generic' way of thinking: "A problem of major concern to economists is that of knowing whether a certain phenomenon is 'likely' or not. For example the econometrician would doubtless be happy if it were true that Cobb-Douglas production functions were very 'likely' in the sense that every production function could be closely approximated by a function of the Cobb-Douglas type. Such results would provide excellent protection against those critics who claim that such functions are only special cases ..." Grandmont *et al.* (1974;

p. 289). Generic analysis approaches the question of the likelihood of a particular object, event or property in the following way. Having defined a class of abstract objects (e.g. production functions, price vectors, economies), if the subset of objects with a given property is 'large' in an appropriate sense relative to the entire set, then the property is typical or generic. If the subset is 'small' it is unusual or rare and as Chillingworth (1976) puts it: "It is common practice in classifying any set S of mathematical objects to begin by trying to sort them into two types: *usual* (regular, non-degenerate, tame, nice…) and *unusual* (singular, degenerate, wild, pathological). One way to do this is to put a *measure* on the set S and to associate usual with *occupying a subset of large measure*. Another is via a topology on S…" Chillingworth (1976; p. 221).

An informal illustration of the generic way of thinking in an economics context is provided by the following remark due to Feldman (1987): "In spite of the multiplicity of optima in a general equilibrium model, *most states* are non-optimal. If the economy were a dart board and consumption and production decisions were made by throwing darts, the chance of hitting an optimum would be zero." Feldman (1987; p. 890, emphasis added). Chillingworth also indicates there are a number of ways to formally investigate the genericity of a particular property depending on the criterion used to measure size. The need for alternative criteria arises because if the set of possible environments can be described by a finite number of parameters, then a property is generic in a measure theoretic sense if it only fails to hold on a set of Lebesgue measure zero in the set of parameters. If the environment cannot be described by a finite number of parameters, Lebesgue and other natural measures are not available. In such situations, a topological approach is needed in which negligible sets are defined by the conditions that: (i) open subsets are non-negligible, (ii) a subset of a negligible set is negligible, (iii) the union of negligible sets is negligible. A property is then regarded as if it holds on an open dense set and is non-generic or exceptional if it only holds on a closed nowhere dense subset of the set of possible parameters. Mas-Colell *et al.* (1995) characterise the situation as follows: "...we could say that in a system defined by finitely many parameters (taking values in say, an open set), a property is *generic in the first sense* if it holds for a set of parameters of full measure (i.e. the complement of the set for which it holds has measure zero). The property is *generic in the second sense* if it holds in an open set of full measure. A full measure set is dense but it need not be open. Hence the second sense is stronger than the first... In some applications there is no finite number of parameters and no notion of measure to appeal to. In those cases, we could say that a property is *generic in the third sense* if the property holds in an open and dense set. When no measure is available, this still provides a sensible way to capture the idea that the property is typical..." Mas-Colell *et al.* (1995; p. 595).

Generic analysis has been used extensively in mathematics (see for example Chillingworth (1976) for an historical discussion). In economics, the first explicit use of generic analysis appears to be in Debreu (1970), who established that in the space of exchange economies those with finitely many isolated equilibria were generic and those with a continuum of equilibria were exceptional. Debreu (1983) summarises his 1970 result as follows: "...a given equilibrium theory associates with each economy $\boldsymbol{E}$ in Ψ the set $W(\boldsymbol{E})$ of equilibrium states of $\boldsymbol{E}$, a subset of Δ... [it is shown that] (a) the critical set C of non-regular economies is a negligible set and (b) every regular economy has a discrete set of equilibria... One can obtain considerably stronger conclusions by making suitable assumptions on the behaviour of the excess demand function near the boundary of S... In this case the set C of critical economies is closed, in addition to being of measure zero and is therefore negligible in a strong sense. Moreover (b′) every regular economy has a finite set of equilibria…" Debreu (1983; pp. 232–233, 238). Fuchs (1974) and Smale (1974) have extended Debreu (1970) to cover private ownership production economies as follows: let Λ be the set of private ownership production economies. Then under standard conditions on the parameters of these economies, there exists an open dense set Θ in Λ with the following properties: if $\boldsymbol{E} \in \Theta$ then (i) each Walrasian equilibrium for $\boldsymbol{E}$ is locally unique, (ii) the set of Walrasian equilibria is continuous at $\boldsymbol{E}$, (iii) the set of Walrasian equilibria is finite in number. Generic techniques have been used to investigate questions such as the uniqueness of equilibrium (Grandmont *et al.*, 1974), the uniqueness of maximising elements in consumer problems (Hildenbrand, 1975), manipulable equilibria (Safra, 1983), differentiability of demand, linked Walrasian allocations, smoothness of excess demand, regularity of weak optima, single valuedness of aggregate excess demand (Mas-Colell, 1985), market demand functions which satisfy the weak axiom of revealed preference (Hildenbrand, 1989), and the regularity of Walrasian equilibrium (Balasko, 1992). The application we have in mind here is to say something about the genericity or otherwise of Walrasian equilibrium prices.

Definition 7.5 (Chillingworth (1976; p. 222)). Let S be any topological space. A subset G of S is called a *residual set* if it is the intersection of a countable number of sets each of which is both open and dense in S. Since most topological spaces are Baire spaces in which residual sets are automatically dense, it is possible to identify 'occupying a residual set' with 'usual' and 'occupying a closed nowhere dense set' with 'unusual'.

Lemma 7.4 (Milton and Tsokos (1976)). *If A is a countable subset of $\Re^n$ then A is a Borel set and A has Lebesgue measure zero.*

Proof. Milton and Tsokos (1976; pp. 49, 50). □

Proposition 7.4. *In the space of possible price vectors, the set of Walrasian equilibria is typically a closed nowhere dense set of Lebesgue measure zero. The set of Walrasian equilibria is therefore a negligible or not generic set in the space of possible prices.*

Proof. Let $\Delta = \{p \in \Re^{\ell}_{+} : \Sigma_h p_h = 1\}$ be the space of possible prices and let the set $W(\boldsymbol{E}) = \{p \in S : Z_E(p) = 0\}$ be the set of Walrasian equilibria for the economy $\boldsymbol{E}$. From the Debreu–Fuchs–Smale result, we know that typically $\#W(\boldsymbol{E}) < \infty$ and the equilibria are locally isolated for all but a negligible set of economies. Thus $W(\boldsymbol{E})$ is a countable subset of Δ and by the Lemma 7.4, $W(\boldsymbol{E})$ has Lebesgue measure zero. Also, since $W(\boldsymbol{E})$ is a finite set of locally isolated points, it is the finite union of singleton sets. Singleton sets are closed, and any finite union of closed sets is closed. Therefore, $W(\boldsymbol{E})$ is a closed set. If $W(\boldsymbol{E})$ were a dense subset of Δ, then its closure would equal Δ. But since $W(\boldsymbol{E})$ is closed and finite, we know that $W(\boldsymbol{E})$ is a proper subset of Δ. Consequently, $W(\boldsymbol{E})$ is a closed nowhere dense set with null measure. □

Corollary 7.1. *The set of non-Walrasian prices is generic in the space of possible prices.*

Proof. The set of non-Walrasian prices is $NW = \Delta \backslash \{p \in S : Z_E(p) = 0\}$. We have shown that $\{p \in \Delta : Z_E(p) = 0\}$ forms a closed nowhere dense set of null measure. Since Δ is a closed subset of $\Re^{\ell}$, it follows that NW is a residual set. Therefore from Definition 7.5, the conclusion follows. □

Remark 7.15. This proposition establishes the interesting result that for a large class of economies, the set of Walrasian prices is a negligible set. In line with the usual practice in the literature (e.g. Debreu (1970), Feldman (1987), Mas-Colell *et al.*

(1995)), it could be argued on this basis that although it is possible to observe Walrasian prices, it is *a priori* unlikely since such prices form a small subset of the set of possible prices. Of course this argument, as with all generic arguments, depends on there being no bias toward Walrasian prices, or to continue Felman's analogy,[3] the Walrasian points on the (price) dart board are not 'magnetised', for example by virtue of being stable under some price adjustment process. Put another way, the Balasko-Journee result plus our argument based on generic analysis yields the interesting conclusion that the likelihood of 'randomly' encountering a Walrasian equilibrium in the space of all probable prices is small. This is of course subject to the important caveat that there is not some process which biases the selection of price vectors in the direction of Walrasian equilibrium prices. An obvious mechanism that might achieve such a bias is a price adjustment process which converged to Walrasian equilibrium prices. Price adjustment processes and their convergence properties are the subject of the next chapter.

7.5. Conclusion

The uniqueness question for Walrasian equilibrium has been extensively studied in the literature. The major conclusion to emerge in the literature is that the uniqueness of equilibrium will only be obtained under restrictive conditions.

If uniqueness is not generally available, it is of interest to know something about the size and structure of the equilibrium set. Debreu (1970) established that for almost all exchange economies the Walrasian equilibrium set has finitely many locally unique elements, a result that has been extended in

[3]This way of putting it was also pointed out to me by Russel Cooper in private communication.

various directions. This result however leaves open the possibility that the number of equilibria is nevertheless numerically large. Results due to Balasko (1980, 1988) and Journee (1992) establish however that the equilibrium set is probably small. Interestingly, even if the probability argument developed in Balasko-Journee should fail we are able to show that the set of Walrasian equilibria is, in a precise sense, negligible. This result has potentially important implications for assessing the likelihood of an economy finding a Walrasian equilibrium state. The conclusion is however subject to the important caveat that Walrasian prices are not 'privileged' by the existence in the economy of a price adjustment processes which converges to equilibrium prices. The convergence properties of price adjustment processes is a topic to which we now turn.

Chapter 8

STABILITY OF EQUILIBRIUM

"*A universally convergent iterative mechanism for finding equilibrium is the Holy Grail of general equilibrium theory in economics.*"

V. Bala and N. Kiefer

"*Equilibrium, optimal but unattainable, would be a will-o'-the-wisp.*"

T. Rader

"*The results concerning... global stability, i.e. the market's ability to attain [Walrasian] equilibrium, are unquestionably negative.*"

B. Ingrao and G. Israel

"*This [instability result] has extremely important implications. Indeed, it is not too strong to say that the entire theory of value is at stake.*"

F. Fisher

8.1. Introduction

The stability question for Walrasian equilibrium is an essential part of the Walrasian program. Indeed, Bala and Kiefer (1994) have described the discovery of a universal and globally stable adjustment process as the 'Holy Grail' of general equilibrium theory. The reason the stability question has this status is easy to see. Without an argument establishing the existence of a price adjustment process that converges to Walrasian equilibrium, Walrasian states, even if they exist and are optimal, lose both descriptive and normative relevance. In particular, if no convincing stability argument can be made then, in a sense, Walrasian states might as well not exist because nothing in the operation of the economy will lead to their realisation.

Numerous authors have discussed the importance of the stability question and many have also noted the largely negative results concerning the issue. Fisher (1987) for example has argued that: "…the very power and elegance of equilibrium analysis often obscures the fact that it rests on a very uncertain foundation. We have no similarly elegant theory of what happens out of equilibrium… As a result, *we have no rigorous basis* for believing that equilibrium can be achieved." Fisher (1987; p. 26, emphasis added). In similar vein, Hahn (1982) summarises his survey of the stability literature in the following terms: "The conclusion of the ensuing survey will be this: a great deal of skilled and sophisticated work has gone into the study of processes by which an economy could attain equilibrium. Some of the (mainly) technical work will surely remain valuable in the future. But the whole subject has a distressing *ad hoc* aspect… [and] the subject can aspire to no more than the study of a series of suggestive examples." Hahn (1982; p. 747). Similarly, McKenzie (1987) claims that: "… [all] global stability results are very special and relatively unconvincing." McKenzie (1987; p. 511), while Ingrao and Israel (1990) conclude their survey of the stability literature by arguing that: "The results concerning… global stability, i.e. the market's ability to attain equilibrium, are unquestionably negative." Ingrao and Israel (1990; p. 361). Reinforcing these views, Saari (1995) has made the following stiring remark: "On the evening news and talk shows, in the newspapers, and during political debate, we hear about the powerful moderating force of the market which, if just left alone, would steadily drive prices towards an equilibrium with the desired balance between demand and supply. The way this story is invoked to influence government and even health policies highlights its important, critical role. But is it true? I have no idea... but, then, no one else does either. This is so because, even though this story is used to influence national policy, *no mathematical theory exists to justify it*." Saari (1995; p. 284, emphasis added). Saari supports his contention that no mathematical theory exists to justify a belief in the stability of Walrasian equilibrium by showing that even very simple economies can exhibit complex dynamics and that such dynamics generally do not lead to Walrasian equilibrium. Recently Gintis has argued that while "[t]here have been notable analytical contributions to general equilibrium dynamics... Franklin Fisher's assessment (Fisher, 1983) remains valid: we have no plausible analytical model of multi-sector dynamics with heterogeneous agents." Gintis (2007; p. 1303)

The reason that such negative conclusions have been reached concerning the stability question might be summarised as follows. Informationally undemanding and economically plausible adjustment processes, such as processes where prices change in response to excess demands, are only guaranteed to converge if excess demand functions have particular structure such as satisfying conditions like the weak axiom of revealed preference, gross substitutes, or diagonal dominance. In light of the Sonnenschein-Mantel-Debreu (SMD) result (see the pervious chapter), it is known that such conditions are not generally implied by the underlying microeconomics of the economy. If excess demand maps are allowed to be relatively unstructured (up to continuity, Walras' Law and homogeneity of degree zero) then adjustment processes are only guaranteed to converge if they are of the informationally demanding 'Global-Newton' or similar type. However, such processes make considerable informational demands because the adjustment of prices requires the evaluation of the vector of excess demands plus, pretty much, an evaluation of the whole Jacobian matrix of the excess demand map, at each iteration. That this is necessary for adjustment process stability was demonstrated by Saari and Simon (1978). The Saari-Simon result, and subsequent results due to Saari (1985, 1995), have deep implications for attitudes to the stability question and will be considered in detail below.

Given the importance of the stability issue to general equilibrium theory and given the negative conclusions reached by earlier surveys of the literature, this chapter after a brief review of some classic stability results, considers work on the stability question done after the point where the Ingrao and Israel (1990) survey left it.

In order to achieve our objectives, this chapter is organised as follows: Section 2 introduces some basic ideas and briefly reviews some classic stability results. We then consider the Saari-Simon result and work inspired by it, including work on Global-Newton processes and simplical algorithms. We also consider the convergence properties of discrete tatonnements as well as the stability properties of some 'agent based' price adjustment processes. Section 3 offers some concluding remarks.

8.2. Some adjustment processes and their convergence properties

8.2.1. *Basic concepts and ideas*

A *price adjustment process*, F, is a map from the space of prices $\Re^\ell$ into the space of prices $\Re^\ell$. A price adjustment process is *globally* (*asymptotically*) *stable* if, for any starting point p^0 in the price space, the solution path $p(t)$ of F, approaches an equilibrium price vector as $t \to \infty$. An adjustment process is *locally* (*asymptotically*) *stable* if for any starting point in the neighbourhood of an equilibrium, the solution path of F approaches an equilibrium as $t \to \infty$. F is *universal* if it generates a solution path that converges to equilibrium for any type of excess demand map. A particular equilibrium is *locally* (*globally*) *stable* if starting in a neighbourhood of the equilibrium (starting anywhere in the space of prices), the adjustment process at work in the economy restores *that* equilibrium.

8.2.2. *Classical tatonnement*

Walras (1874) initiated the study of the stability of adjustment processes with his suggestion of a 'sequential-tatonnement' in which: (i) if $p(t)$ is a price vector at t then this price is changed if and only if $p(t)$ is not an equilibrium; (ii) agents are permitted to trade if and only if $p(t)$ is an equilibrium; (iii) one market at a time is considered and a price is sought to clear that market before the next market was worked on. As Joosten and Talman (1998; p. 16) demonstrate, the problem with this sort of adjustment process is that it is easy to construct economies in which this process does not converge.

Samuelson (1941, 1942) suggested a modification of Walras' approach to allow for a 'simultaneous-tatonnement' in which features (i) and (ii) of Walras' process were retained but (iii) was replaced by: (iii′) prices on all markets move at the same time in response to the excess demand experienced in each market. Such a process is given by:

$$\frac{dp_i}{dt} = H_i[Z_i(p)], \quad \text{for all goods } i = 1, \ldots, \ell$$

$$\text{with } H_i(0) = 0 \text{ and } H_i' > 0$$

$$\text{and } \frac{dp_i}{dt} = 0 \quad \text{if } p_i = 0 \text{ and } Z_i(p) < 0. \tag{8.1}$$

The process in (8.1) is economically attractive because it mirrors what seems to be an intuitive state of affairs where prices are driven up if excess demand is positive and down if excess demand is negative. It is also attractive because it requires relatively little information, just the state of excess demand in each market. However, such a process is not guaranteed to converge in all economies, and a particular structure needs to be imposed on the excess demand map in order to ensure that it does. Typical of classic stability results in this context is that due to Arrow and Hahn (1971).

Theorem 8.1 (Arrow and Hahn (1971)). *Characterise the economy by its excess demand map $Z(p)$ and suppose that this map is a continuous function which satisfies Walras' Law, is homogeneous of degree zero in prices and is bounded below. If $Z(p)$ is such that all goods are gross substitutes at all prices, then the adjustment process in* (8.1) *is globally asymptotically stable.*

Proof. Arrow and Hahn (1971; pp. 288–289). □

Remark 8.1. Similar results can be proved when the hypothesis of gross substitutes is replaced by that of the aggregate weak axiom of revealed preference, weak gross substitutes or diagonal dominance and results of this sort are presented in Arrow and Hahn (1971; pp. 263–323), (see also the remarks in McKenzie (2008a)). It is perhaps interesting to note that the basic reason why such conditions work can be traced to the following proposition of Uzawa (1960).

Lemma 8.1 (Uzawa (1960)). *If p^* is a Walrasian equilibrium and if the value of excess demand at any other p not proportional to p^* is positive when evaluated by p^* (i.e. $p^* Z(p) > 0 \, \forall \, p \neq \alpha p^*$), then prices generated as a solution to the differential equation in* (8.1) *converge to p^* from any point $p^0 \in \Delta$, the unit simplex.*

Proof. See Uzawa (1961; pp. 627–628) and Mas-Colell *et al.* (1995; pp. 623–624). □

Remark 8.2. Conditions such as gross substitutes, the weak axiom of revealed preference and diagonal dominance all work to guarantee that equilibrium is unique and that if p^* is an equilibrium, then $\forall p \neq \alpha p^*$, $p^* Z(p) > 0$ and as the discussion in Mas-Colell *et al.* (1995; pp. 623–633) show that inequality occurs in many stability arguments.

Unfortunately, conditions such as gross substitutes, the weak axiom of revealed preference and diagonal dominance, do not necessarily arise from the microeconomics of even well behaved economies. This point was made by Scarf (1960) who presented an example of an exchange economy with three agents, (a, b, c), three goods (x, y, z), utility functions $U^a(x, y, z) = \min(x, y), U^b(x, y, z) = \min(y, z), U^c(x, y, z) = \min(x, z)$ and endowments $\omega^a = (1, 0, 0), \omega^b = (0, 1, 0), \omega^c = (0, 0, 1)$. Excess demands generated by this economy are: $Z_x(p) = -p_y/(p_x + p_y) + p_z/(p_x + p_z)$, $Z_y(p) = -p_z/(p_y + p_z) + p_x/(p_x + p_y)$, $Z_z(p) = -p_x/(p_x + p_z) + p_y/(p_y + p_z)$ and the unique equilibrium is $p^* = (1/3,\ 1/3,\ 1/3)$. However a price adjustment process such as (8.1) will never approach p^* except in the trivial case where the starting point of the process is the equilibrium (see Scarf (1960) for details). In light of the SMD result, it is known that the instability instanced by Scarf (1960) can be multiplied almost indefinitely, even in economies with smooth and homothetic preferences. Consequently, approaches to stability which seek to impose minimal structure on the excess demand map have been actively sought.

8.2.3. *A stability-information tradeoff*

In light of the SMD result, one branch of the literature has attempted to find adjustment processes that are convergent for arbitrary excess demand maps. Motivating this approach, Herrmann and Kahn (1999) observe: "Following [the SMD] results, the stability question took the form: assuming that agents are heterogeneous in preferences and endowments, does there exist a mechanism that: one, computes an equilibrium from any starting point in the price simplex; and, two, is robust to specifications in the underlying characteristics of the economy?" Herrmann and Kahn (1999; p. 422). In this context, the authors also make the following interesting remark: "Of course, it is certainly desirable to find a mechanism that mimics the actual market adjustment process. But this is a far harder fundamental problem and one on which there is no consensus on an approach." Herrmann and Kahn (1999; p. 422). We will return to this issue later in the chapter. We now consider some answers to the questions posed by Herrmann and Kahn (1999; p. 422).

Partly as a mathematical exercise, Smale (1976) studied the convergence properties of a 'Global-Newton' process of the form $D_pZ(p) \cdot dp/dt = -\lambda(p)Z(p)$. Here $D_pZ(p)$ is the Jacobian matrix of the excess demand function, with the last row and column removed and $\text{sign}[\lambda(p)] = (-1)^{\ell-1}\text{sign}[\det\ D_pZ(p)]$. If $\det(D_pZ(p)) \neq 0$,[1] the Smale process may be written as:

$$dp/dt = -\lambda(p) \cdot D_pZ(p)^{-1} \cdot Z(p). \tag{8.2}$$

Smale (1976) showed that such an adjustment process converges to equilibrium for an arbitrary $Z(p)$, provided that the process is started on the boundary of the price space. Thus one half of the Holy Grail (universality) is achieved by a process of this form, but the other half (global stability) is not necessarily guaranteed. In addition, the processes described in (8.2) represents a radical departure from the tradition that informed (8.1) in terms of the amount of information needed at each step in the adjustment process. As Saari and Simon (1978) put it: "The above 'Generalised Newton Method' [(8.2)] requires knowledge of $Z(p)$ and the gradients of all but one of its component functions ... i.e. $(\ell-1)^2 + (\ell-1)$ quantities at each price, including how the jth commodity affects the rate of change of the demand for the

[1] The non-singularity of this determinant is known to hold almost always by the result of Debreu (1970) discussed in the previous chapter.

kth commodity for all j and k. For practical problems this is a staggering amount of information... " Saari and Simon (1978; pp. 1098–1099, emphasis added). Just how much information is involved is indicated by the following analogy due to Saari (1985) who points out that there have been about 7.884×10^{14} minutes since the big bang. This number is about equal to the number of bits of information that a process like (8.2) needs *at each iteration* to adjust the markets in an economy with 28 million goods and services. Considering that the population of the United States is about 300 million, there are about five times that many commodities, in potential labour services alone, in that economy. Thus, knowledge of all excess demands $Z(p)$ *along with all the gradients* of almost all the excess demands, $D_pZ(p)$, is needed at each point in the adjustment process in order for a process of type (8.2) to operate.

It is of course possible that the informational requirement in (8.2) is overly strong and that Smale's process merely provides sufficient conditions for convergence on a relatively arbitrary excess demand map. It is therefore of interest to know what sort of deviation from a Global-Newton process can be entertained before the guarantee of convergence is lost. One way to address this question is to discover how many 'ignorable co-ordinates' there are in a convergent Global-Newton style process. This is the motivation for Saari and Simon (1978) who ask: "...how much information [does] a price adjustment mechanism *need to have* in order to be effective for all standard economies?". Saari and Simon (1978; pp. 1098–1099, emphasis added). To answer the question Saari and Simon (1978) write the Global-Newton process of Smale (1976) as:

$$dp/dt = -D_pZ(p)^{-1}Z(p) = F[Z(p), D_pZ(p)]. \tag{8.3}$$

Definition 8.1 (Locally effective). An adjustment process $F(\mathbf{z}; y_{11}, \ldots, y_{\ell\ell})$ is a *locally effective price mechanism* (LEPM) if for any smooth excess demand function Z and for any p^* such that $Z(p^*) = 0, p^*$ is an attractor for the solution path to Eq. (8.3) provided it starts sufficiently close to p^*.

Remark 8.3. If a LEPM starts at any price near an equilibrium, then the mechanism will adjust prices so that they tend asymptotically to the equilibrium point. As Saari and Simon (1978) note: "Consequently the limitation of LEPM is that you must start *near* an equilibrium point." Saari and Simon (1978; p. 1103).

Definition 8.2 (Effective mechanism). F is an *effective price mechanism* (EPM) if for any smooth excess demand function Z the following are satisfied: (a) if $Z(p) = 0$ then $F[Z(p), D_pZ(p)] = 0$; (b) for almost all p in some open subset V of Δ the solution of (8.3) through p tends asymptotically to a zero of Z as $t \to \infty$; and (c) there exists some p and Z^* such that $Z^*(p) = 0$, $D_pZ^*(p)$ is non-singular and p is a non-singular zero of $dp/dt = F_{Z^*}(p)$.

Remark 8.4. For an EPM, it is only required that the mechanism finds *some* Walrasian equilibrium, but there is no guarantee which one it might be. As Saari and Simon note, a potentially interesting feature of this process is that: "...it may turn out that the adjusted prices (solutions of Eq. (8.3)) pass arbitrarily close to one zero of $Z(p)$, only to leave this neighbourhood and converge to a second zero of $Z(p)$." Saari and Simon (1978; p. 1104).

Definition 8.3 (Ignorable co-ordinates). A co-ordinate y_{ij} of the process $F(Z(p), D_pZ(p))$ is *ignorable* if $\partial F/\partial y_{ij} = 0$.

Remark 8.5. An ignorable co-ordinate of the adjustment process corresponds to a direction, or piece of information, that is not necessary for the effectiveness of the process (see Saari and Simon (1978; p. 1100)). The interesting question is how many ignorable co-ordinates are there in an adjustment process if it is EPM or LEPM?

Theorem 8.2 (Saari and Simon (1978)). *Let Z be an excess demand map for a standard exchange economy with ℓ commodities and let $F : \Re^{\ell-1} \times \Re^{(\ell-1)(\ell-1)} \to \Re^{\ell-1}$ be a price adjustment mechanism. Then if $\ell \geq 3$ and if F has some ignorable co-ordinate y_{ij} in some neighbourhood of $Z = 0$, then F cannot be a LEPM. Furthermore, if $\ell \geq 3$ and if y_{ij} and y_{hk} are ignorable co-ordinates of F for some neighbourhood of $Z = 0$, where $i \neq h$ and $j \neq k$ and if a_{ij} and a_{hk} is not identically equal to zero on $\{0\} \times \Re^{(\ell-1)2}$, then F cannot be an EPM. Also if the vector y_j and some y_{ik} for $k \neq j$ are ignorable co-ordinates for $F(Z; y_1, \ldots, y_\ell)$, then F cannot be an EPM. Finally, if $\ell = 3$ or 4 and F has two ignorable co-ordinates y_{ij} and y_{hk} with $i \neq h$ and $j \neq k$, then F cannot be an EPM. If $\ell = 5$ and F has three ignorable co-ordinates $y_{h1k1}, y_{h2k2}, y_{h3,k3}$ with the h's not all equal and the k's not all equal, then F cannot be an EPM.*

Proof. Saari and Simon (1978; pp. 1108–1112). □

Remark 8.6. This result shows that, if the SMD theorem is respected and the excess demand functions are allowed to be arbitrary (up to continuity, Walras' Law and homogeneity), then a Global-Newton style process is not only sufficient for stability but is also pretty much necessary for stability. As Hahn (1982; p. 768) notes, this is a salutary result because it shows that the informational burden of essentially a full Global-Newton process cannot be dispensed with if effectiveness (or even local effectiveness) of a price adjustment process is desired. As Saari and Simon put it: "…our results show that the informational requirements [of the Global-Newton type] cannot be relaxed by any significant amount. That is, should some price mechanism require 'a low amount of information'… then there can be found a classical exchange economy for which the mechanism is not effective." Saari and Simon (1978; p. 1099, emphasis added).

8.2.4. *Discrete time adjustment processes*

One possible response to the Saari-Simon result is to note that processes (8.2) and (8.3) run in continuous time. It might be the case that a discrete time counterpart to these processes are less informationally demanding than the continuous time versions. In fact, the informational requirements for convergence discussed above more than carry over to the case where iterative adjustment processes are considered in place of continuous processes. As Weddephol (1997) notes: "... in an exchange economy the discrete time tatonnement process need not converge to a Walrasian equilibrium, even if the economy satisfies conditions (gross substitutability for example) that guarantee a continuous time tatonnement to converge." Weddephol (1997; p. 551). An example due to Mukherji (1999) usefully illustrates this point. Take a two by two exchange economy where people A and B have Cobb-Douglas preferences over and endowments of, goods x, y such that $u_A(x, y) = x^{\alpha}y^{1-\alpha}, \omega_A = (x_A, 0)$; $u_B(x, y) = x^{\beta}y^{1-\beta}$, $\omega_B = (0, y_B)$. Let y be the numeraire (so that $p_y = 1$) then the excess demand for x is $Z_x(p) = \beta y_B/p - (1 - \alpha)x_A$ with $p \equiv p_x/p_y = p_x$, then the unique equilibrium is $p^* = \beta y_B/(1 - \alpha)x_A$. Let the discrete time tatonnement on prices be $p(t + 1) = p(t) + \gamma Z(p(t))$ with $\gamma > 0$, some constant speed of adjustment. By substitution, the tatonnement can be written in terms of the parameters of the economy as $p(t+1) = p(t) + \gamma[\beta y_B/p - (1 - \alpha)x_A]$. Define $K = \gamma[(1 - \alpha)x_A]^2/\beta y_B$. Mukherji (1999) then shows that (i) if $K < 2$ then p^* is locally stable for the adjustment process and p^* locally stable for the process implies that $K \leq 2$; (ii) if $2 < K < 2.5$, then the process generates a cycle of period 2; (iii) if $K \in (3.0, 3.6)$, then the adjustment process exhibits topological chaos; (iv) if

$K = 25/9$, then the process exhibits ergodic chaos and also there is a value in the interval (3.0, 3.6) for which the process exhibits ergodic chaos.[2] The point that this example makes is that even for a low dimension economy with Cobb-Douglas preferences and constant speed of adjustment, then depending on the exact distribution of agent characteristics very complex dynamics and non-convergent dynamics can emerge from a discrete time tatonnement.

Saari (1985) codifies observations of this sort to obtain a general result which says that if an iterative mechanism depends on only a *finite* amount of local information such as $Z(p),\ D_pZ(p), D_p^2(p), \ldots, D_p^N(p)$ for N a positive integer, then such a mechanism will generally not be effective. Even for local effectiveness, iterative processes depend on $Z(p)$ and $D_pZ(p)$ as in the differential version of the process considered by Saari and Simon (1978). In particular Saari (1985) considers the case where (8.3) is generalised to: $P_{t+1} = M[Z(p_t),\ D_pZ(p_t), \ldots, D_p^N(p_t), \ldots, Z(p_{t-k}), \ldots, D_p^N(p_{t-k})]$. In this context, he proves the following result: *for every member of a class of adjustment processes described by* (8.4) *there exists an exchange economy for which any adjustment process in the class will fail to converge to an equilibrium of the economy.* Bala and Majumdar (1992) highlight the significance of Saari (1985) as follows: "Saari's (1985) result is particularly important: he showed that for *any* price adjustment rule which the economy follows, there exists an open set of excess demand functions... where the price adjustment rule fails to converge to an equilibrium starting from an open set of initial conditions." Bala and Majumdar (1992; pp. 437, 438)

One response to the results of the Saari type is to look for a mechanism outside the class considered by him and to study its convergence properties. Bala and Kiefer (1994) do this by

[2]See Bala and Majumdar (1992; p. 441) and Mukherji (1999; p. 743) for definitions of these concepts.

considering a two good economy in which the second good is the numeraire. Here $Z : \Re \to \Re$ is the excess demand map. Let a simple Newton process be responsible for price adjustment so that successive iterates in the price space are:

$$p_{t+1} = p_t - [Z(p_t)/Z'(p)]. \tag{8.4}$$

As an example, consider the excess demand function $Z(p) = \sqrt[3]{(3/p - 4)}$, then the unique equilibrium is $(p_1^*, p_2^*) = (0.75, 1)$. However, even if the initial price is $p^0 = (0.751, 1)$, which might be thought of as close to equilibrium, (0.75, 1) the zero of $Z(p)$ is not approached by the process in (8.4) as t becomes large (see Bala and Kiefer (1994; p. 300) for a plot of the solution path of the process). How must the process be modified in order to guarantee convergence? For a given excess demand function $Z(p)$, if the process described by (8.4) is replaced by:

$$p_{t+1} = p_t + N[Z(u(p_t)), Z'(v(p_t))], \tag{8.5}$$

where u and v are complicated functions chosen so as to avoid the phenomena identified by Saari (1985). The process operates essentially by 'sampling' a new point at each iteration and by moving to that point if the value of $Z(p)$ is closer to zero at the new point than it was at the previous best point (see Bala and Kiefer (1994; p. 314) for a detailed description of the process).

An interesting thing about this process is, as Bala and Kiefer (1994) note, while the process works without adding *additional* informational requirements relative to Saari (1985), it nevertheless makes the *same* informational demands as the process in Saari (1985). For local effectiveness, this is the same amount of information as the Saari and Simon (1978) informational requirement for global effectiveness in continuous time. For global effectiveness, the Bala and Keifer process requires an infinite amount of information. Thus in terms of reducing the informational requirement for stability, the Bala and Kiefer approach does not represent an improvement relative to the approaches that preceded them.

8.2.5. *Simplical algorithms*

Given the large informational demands of Global-Newton processes, various attempts have been made to design mechanisms which reduce the burden when possible without sacrificing the global convergence of the process. The results of Saari and Simon (1978) and of Saari (1985) show that in general, this will not be possible, nevertheless it is interesting to see what can be done in particular cases.

One ingenious attempt in this direction is due to Kamiya (1990), who formulates an adjustment process which is a weighted average of a simple tatonnement and a Global-Newton process and is described as:

$$[D_p Z(p)/\|Z(p)\| - I/\|p - p^0\|]dp/dt = -\lambda(p)Z(p). \qquad (8.6)$$

As Kamiya (1990) notes, when p is approximately at the starting price p^0, the second term on the LHS of (8.6) dominates so that the process is approximately a simultaneous tatonnement. As equilibrium is approached, however the first term asserts itself and the process becomes a pure Global-Newton process of the Smale-Saari and Simon variety. As Kamiya (1990) shows: "Our process always converges to an equilibrium unless the initial price vector belongs to a set of measure zero in [the price space]". Kamiya (1990; p. 1482). Thus while Kamiya's process starts out like a simple tatonnement, in order to ensure convergence to equilibrium, it must ultimately make the same informational demands as Global-Newton processes.

Noting that Scarf (1967) and Kuhn (1968) had devised artificial algorithms which compute equilibria in applied general equilibrium models, van der Laan and Talman (1987), Herings (1997), and Joosten and Talman (1998) thought to combine the

Scarf and Kuhn processes to obtain a universal and globally stable price adjustment process. The van der Laan and Talman (1987) process may be described as follows. Define two sets $P(s)$ and $D(s)$, where $P(s)$ contains information about the location of p with respect to the starting price p^0 and $D(s)$ contains information about the excess demand at p. The 'auctioneer' running this process keeps in mind the starting price vector and also keeps in mind the reaction of agents in the market, as reflected by excess demand. The prices of all goods in positive excess demand are increased and the price of all goods in negative excess demand are decreased in such a way that the ratios of the prices of any two goods with positive or negative excess demands are kept constant. Prices are kept in these ratios and adjusted until one market gets to equilibrium. Then prices are adjusted, again respecting the above 'ratio rule', so as to maintain this equilibrium. This process is global and universal, however it is highly artificial. As van der Laan and Talman (1987) also note, it is informationally demanding since: "For the adjustment mechanism induced by this process the auctioneer needs information about [excess demands] *and the corresponding gradients.* Moreover, the auctioneer has to keep in mind the starting price vector." van der Laan and Talman (1987; p. 123, emphasis added). Van den Elzen and Kremers (2006) have labelled such processes 'simplical algorithms.'

Herings (1997) demonstrates that the adjustment process of van der Laan and Talman (1987) can be followed using the numerical algorithm developed by Doup *et al.* (1987). The information needed at every price vector for the operation of this process are the $(n+1)$ price vectors already generated, the excess demands at these prices and the initial price vector. Informationally: "This means that the amount of information needed is roughly the same as the amount indicated by Saari and Simon (1978)." Herings (1997; p. 168). Thus, although the van der Laan and Talman process is ingenious construction, it still needs information of essentially Saari-Simon magnitude in order to achieve its ends. Van den Elzen (1997) has also produced a tatonnement process in which price adjustments are determined by the current state of the market along with the starting price vector. He works with semi-algebraic economies and so starts with the definitions that a set in $\Re^l$ is *semi-algebraic* if it is the finite union of sets of the form $\{x \in \Re^l : f_1(x) = 0, \ldots, f_a(x) = 0; v_1(x) < 0, \ldots, v_d(x) < 0\}$, $f_h, h \in I_a \cup \{0\}$ and $v_h, h \in I_d \cup \{0\}$ are polynomials with real coefficients. If $f : A \to B$, where A and B are semi-algebraic sets, then f is a semi-algebraic correspondence if its graph is a semi-algebraic set. An economy is semi-algebraic if all the sets and functions which define the economy can be described by polynomial (in)equalities — see van den Elzen and Kremers (2006; p. 2). Van den Elzen (1997) demonstrates that for a class of semi-algebraic convex economies with production, the process defines at least one path connecting the starting price and an equilibrium. The need for the adjustment process to keep track of the initial price is essential to the success of Elzen's algorithm but is also a major limitation in terms of the realism of the process. Van den Elzen and Kremers (2006) extend this reasoning to cover the case of a non-convex production economy as follows. They call a *market condition* a combination of market prices and production vectors at which each consumer determines their utility maximising bundle of commodities and each producer determines if the market price is acceptable or not, so that the behaviour of the producer is modelled as a pricing rule that relates a set of 'acceptable' price vectors to every market condition. The adjustment process in this model works as follows. Commodity prices are represented by $p \in \Delta$, the unit simplex. Each producer j is characterised by a tuple (Y_j, ϕ_j) consisting of a production possibility set Y_j and a pricing rule ϕ_j. The interesting production vectors are the *weakly efficient* $F_j = (y_j \in Y_j : \hat{y}_j \gg \hat{y}_j \Rightarrow y_j \notin Y_j\}$. The pricing rule 'relates admissible price vectors for firm j to each relevant pair of prices and production vectors' and is the map[3] ϕ_j: $\Delta \times F \to \Delta$, where $F = \Pi_j F_j$. Each element $(p, y) \in \Delta \times F$ with $y = (y_1, y_2, \ldots, y_m)$ is a *market condition* and producer j is in equilibrium at a market condition (p, y) if $p \in \phi_j(p, y)$. At a *production equilibrium*, all producers find the market condition acceptable so $p \in \cap_j \phi_j(p, y)$. Let $S = \Pi_j \Delta^j \times \Delta$ then adjustment process of van den Elzen and Kremers (2006) is based on the Villar (1994) correspondence $\Gamma : S \to \Re^{\ell(m+1)}$, where $\Gamma(p, q) = \{Z(p, q), p - \phi_1(p, q), p - \phi_2(p, q), \ldots, p - \phi_m(p, q)\}$. The first component of this $(m+1) \cdot \ell$-vector map is the excess demand of the economy, while the subsequent components 'can be interpreted as the difference between the price vector proposed by the auctioneer and those prices that are acceptable to each firm'. The vector q is a transformation of y in which for any $q_j \in \Delta$ high values of $q_{j\ell}$ indicate that ℓ is an output for j while low values indicate ℓ is an input — see van den Elzen and Kremers (2006; p. 5) for details. The adjustment of prices proceeds as follows:

Definition 8.4 (Adjustment process EK). For any initial market condition $(p^0, q^0) \in \text{int}(S)$, define the set $P(p^0, q^0; \Gamma)$ as the set of tuples (p, q, z, π) consisting of market conditions $(p, q) \in S$, excess demands $z \in Z(p, q)$ and acceptable price vectors $\pi = (\pi_1, \pi_2, \ldots, \pi_m)$ with $\pi_j \in \phi_j(p, y)\ \forall\ j$, satisfying for each commodity ℓ: (i) $z_\ell < 0 \Rightarrow \min_k p_k/p_k^0 = p_\ell/p_\ell^0$; (ii) $z_\ell = 0 \Rightarrow \min_k p_k/p_k^0 \leq p_\ell/p_\ell^0 \leq \max_k p_k/p_k^0$; (iii) $z_\ell > 0 \Rightarrow p_\ell/p_\ell^0 = \max_k p_k/p_k^0$; and for each producer j; (iv) $p_\ell - \pi_{j\ell} < 0 \Rightarrow \min_k q_{jk}/q_{jk}^0 = q_{j\ell}/q_{j\ell}^0$; (v) $p_\ell - \pi_{j\ell} = 0 \Rightarrow \min_k q_{jk}/q_{jk}^0 \leq q_{j\ell}/q_{j\ell}^0 \leq \max_k q_{jk}/q_{jk}^0$; (vi) $p_\ell - \pi_{j\ell} > 0 \Rightarrow q_{j\ell}/q_{j\ell}^0 = \max_k q_{jk}/q_{jk}^0$ with $\min_k(p_k/p_k^0) = \min_k(q_{jk}/q_{jk}^0)$. Such an algorithm will be called an *EK-adjustment process*.

[3]Note to maintain notational consistency with earlier notation in the book, Δ and S have been transposed here relative to the use in van den Elzen and Kremers (2006).

Remark 8.7. As van den Elzen and Kremers (2006) note, this price adjustment process is like that introduced in van der Laan and Talman (1987) and: "[t]he adjustment of prices and quantities in $P(p^0, q^0;\ \Gamma)$ can be interpreted as a tatonnement process [where] relative market prices are kept minimal (maximal) for those commodities ℓ, which are in excess supply (excess demand). The relative market prices p_ℓ/p_ℓ^0 of the commodities ℓ whose markets are in equilibrium are allowed to vary between their lower bound $\min_k(p_k/p_k^0)$ and their upper bound $\max_k(p_k/p_k^0)\ldots$ As soon as the relative market price on a market in equilibrium reaches its upper (lower) bound, the equilibrium is disturbed and the market is brought into excess demand (excess supply)." Elzen and Kremers (2006; pp. 6, 7). Given these ideas the following stability result can be established.

Theorem 8.3 (van den Elzen and Kremers (2006; Theorem 3.1)). *Let the economy be* $\boldsymbol{E} = \{X_i, \preceq_i, \omega_i, Y_j, \phi_j, \theta_{ij}, \ell\}_{i=1}^{n}{}_{j=1}^{m}$. *If* $\boldsymbol{E}$ *is such that* $(ek.1)$ $\forall j, Y_j - \Re_+^\ell \subset Y_j, Y_j$ *is closed*, $0 \in Y_j, \phi_j$ *is a non-empty, closed, convex valued upper-hemicontinuous correspondence*; $(ek.2)$ $\forall i, \omega_i \gg 0,\ X_i = \Re_+^\ell$ *while preferences are locally non-satiated and representable by a continuous, quasi-concave utility function*; $(ek.3)$ *the set of attainable allocations in the economy is compact*; $(ek.4)$ *the wealth of consumer* i *at* $(p, y) \in \cap_j \phi_j(p, y) \times F$ *is given by* $r_i(p, y) = p\omega_i + \sum_j \theta_{ij} p y_j$ *and is such that for all* i *and for all market conditions*, $r_i(p, y) \geq \delta$ *for any* $\delta > 0; (ek.5)$ *all utility functions, production sets and pricing rules are semi-algebraic*; $(ek.6)$ *prices follow adjustment process EK then for an initial market condition* $(p^0, q^0) \in \mathrm{int}(S)$ *there exists a path connected subset in* $P(p^0, q^0; \Gamma)$ *which includes* (p^0, q^0, z^0, Π^0) *with* $z^0 \in Z^0(p^0, q^0)$ *and* $\Pi^0 \in \Pi_j \phi_j(p^0, q^0)$ *and the tuple* $(p^*, q^*, 0, (p^*, \ldots, p^*))$ *with the equilibrium market condition* $(p^*, q*) \in \mathrm{int}(S), 0 \in Z(p^*, q^*)$ *and* $\forall j, p^* \in \phi_j(p^*, q^*)$.

Proof. van den Elzen and Kremers (2006; pp. 7–8). □

Remark 8.8. As the authors note, this result represents an interesting alternative to the Kamiya (1988) result because this process can start from anywhere whereas in Kamiya (1988), the set of initial market conditions consistent with convergence is limited.

Joosten and Talman (1998) present another adjustment process that also has its roots in the algorithms of Kuhn and Scarf. This process, which may start anywhere in Δ, allows the price of the commodity with the largest excess demand to initially increase and the price of the commodity with largest excess supply to decrease. The price of all other goods is initially not changed. In a fashion similar to the process in van der Laan and Talman (1987), it converges under general conditions on the excess demand functions. Apart from its artificial and economically counterintuitive nature, this process also makes some rather unusual informational demands. Joosten and Talman (1998) show their process can be followed by what they term an '$\ell(\ell+1)$-ray variable restart algorithm'. Although this algorithm uses slightly less information than that needed to follow the process in van der Laan and Talman (1987), there is still a large informational requirement. In fact the information needed is in the order of the $(\ell-1)^2+(\ell-1)$ bits of information needed by Saari and Simon (1978). As Joosten and Talman (1998) note: "The informational requirements for the globally convergent price adjustment process consists of local information obtained from the excess demand function and global information about the location of the current price vector in relation to the starting price." Joosten and Talman (1998; p. 24). The process is also unsatisfactory from an economic point of view because it requires holding constant the prices and excess demands of all goods which are not at the maximum value.

8.2.6. *'Random' adjustment processes*

Keisler (1996) proposes a random price adjustment process in which there is a market maker who sets prices and a large number of agents who trade only with the market maker (auctioneer) and not with each other. The auctioneer holds an inventory of every good and is prepared to trade with any agent according to that agent's demand at current prices. At each discrete point in time, a randomly chosen agent trades with the auctioneer. The auctioneer then adjusts prices in the direction of the trade. If it is assumed that the initial price is in the domain of attraction of the limit price (which is a Walrasian equilibrium), and if this remains the case for the whole trading process then with probability arbitrarily close to one, the process converges to a Walrasian equilibrium. This is an interesting adjustment process because it addresses at least three shortcomings of the tatonnement type processes: (i) trade out of equilibrium is permitted; (ii) the set of trades open to an agent no longer depends on the holdings of all other agents in the economy; (iii) the auctioneer is no longer required to determine the excess demands of all agents in the economy before prices are changed.[4] As Keisler (1996) also notes, the process has some drawbacks. Firstly, it may fail to converge for certain excess demand maps and is therefore not universal. Secondly, there is no motivation for the auctioneer to behave the way it does. Thirdly, agents are not permitted to speculate on future prices. Of particular interest is Keisler's observation that: "Saari (1985) showed that a tatonnement price adjustment mechanism which always approaches a stable limit requires data equivalent to the derivatives of the average excess demand. An open problem is to show that our process escapes that requirement by proving that with probability arbitrarily close to one, the price will eventually be

[4]See Keisler (1996; p. 31) for further discussion.

captured by some stable limit." Keisler (1996; p. 31). The informational demands made by this process, even to achieve local stability, are unclear. It is also the case that the process makes the restrictive assumption that the initial price vector is in the region of attraction of an equilibrium limit point. It is therefore, at best, a locally effective mechanism.

Ermoliev *et al.* (2000) also introduce stochastic elements into a tatonnement process. Taking the SMD theorem as their starting point, the authors note that making assumptions like WARP on aggregate excess demands is too restrictive. They therefore propose to work with excess demand maps which are HDO, continuous, satisfy Walras' Law and the boundary condition: $\max_l Z_i(p) > 0$ if $p_l = 0$, where l indexes commodities. Explaining their approach Ermoliev *et al.* (2000) remark: "Our strategy will be to compensate for the lack of properties [on excess demands] through additional stochastic mechanisms that impose a 'wild' shock in case the process is not converging satisfactorily, simply to start anew at a different spot." Ermoliev *et al.* (2000; p. 179). As the authors note, this process in addition to being *ad hoc*, runs into some significant informational requirements. If the entire excess demand function is known to the adjustment process (an informational requirement in the order of that required by Saari-Simon), then equilibrium can be achieved in a finite number of steps by stochastic search, although, as the authors note, 'this may take a long time'. In the case where the excess demand map is imperfectly known then: "...in this case even if the equilibrium price is given at the outset, it still takes an infinite sampling of excess demands to verify that this price clears the market." Ermoliev *et al.* (2000; p. 179). Either way the process requires an enormous amount of information (of Saari-Simon magnitude or more), in order to guarantee global stability. For that reason, it is subject to the information requirement critique made earlier.

8.2.7. *The return of 'special structure' on excess demands*

Comparing a simple excess demand driven tatonnement processes with Global-Newton processes, Herrmann and Kahn (1999) remark: "...tatonnement has appeal over various Global-Newton methods (introduced since the mid-1970s), in that tatonnement has an economic justification as a market adjustment process: prices move in the direction of excess demand, and agents act as price takers ..." Herrmann and Kahn (1999; p. 420). By contrast, Global-Newton type processes lack immediate economic appeal and also impose significant informational requirements. Given this situation, it is perhaps not surprising that another major branch of the stability literature research has continued into 'special structure' on excess demand maps to ensure stability, the SMD result notwithstanding, Work aimed at generalising the classical conditions for convergence of processes like (8.1), includes an investigation of a generalisation of the 'Morishima condition' due to Keenan (1990) and Dohtani (1993, 1998). A *Morishima condition* holds in an economy in which substitutes of substitutes and complements of complements are substitutes and substitutes of complements and complements of substitutes are complements. As Keenan (1990) notes, this condition imposes the following structure on $D_pZ(p)$ — along with $(\partial Z_i/\partial p_i) < 0$ and indecomposability:

$$\begin{aligned} \operatorname{sgn}(\partial Z_i/\partial p_j) &= \operatorname{sgn}(\partial Z_j/\partial p_i) && \text{for } i \neq j \\ \operatorname{sgn}(\partial Z_i/\partial p_j) &= \operatorname{sgn}(\partial Z_i/\partial p_k)\cdot(\partial Z_k/\partial p_j) && \text{for } i \neq j \neq k \neq i. \end{aligned} \tag{8.7}$$

The Morishima condition generalises GS, and it is known that if it holds a process like (8.2) is globally stable. Keenan (1990) generalises the Morishima condition by allowing for the possibility that $\partial Z_i/\partial p_j = 0$ for some i, j. He then shows that under standard conditions on $Z(p)$ such as Walras' Law, HDO, a boundary condition, differentiability and boundedness below, that a process like (8.1) is almost everywhere globally stable (see Keenan (1990; p. 11)). However, as Dohtani (1998) notes: "In spite of Keenan's valiant effort to develop his theorem to a stronger result on global stability, he was not able to succeed. The 1990 paper he finally published was a second best solution." Dohtani (1998; p. 181). In his contribution Dohtani (1993, 1998) proceeds to extend the basic notion of Diagonal Dominance that was introduced in the previous chapter, a condition which as Dohtani (1993) notes imposes restrictions on the rows of the Jacobian matrix of the excess demands. He introduces the following two generalisations of Diagonal Dominance, one of which is a column analogy of the condition introduced by Arrow and Hahn (1971).

Definition 8.5 (Diagonal Dominance with constant weights). A set of excess demand functions satisfies *diagonal dominance with constant weights* at $p > 0$ if there exists $h(p) = (h_1(p), \ldots, h_\ell(p))$ such that $\partial h_j Z_j/\partial p_j + \sum_{l>j\neq i} |\partial h_i(p) Z_i/\partial p_j| < 0$, where $h(p)$ is continuously differentiable on $\Re^l_{++}, \forall i, h_i(p_i) > 0$ for $p_i > 0$ and $|\int_{pi} h_i(s)ds| \rightarrow +\infty$ as $p_i \rightarrow \pm\infty$.

Definition 8.6 (Row and Column Diagonal Dominance). An excess demand map satisfies the *row dominant diagonal condition with weights* if $\exists \beta_i > 0$ such that: $\sup_u\{\partial Z_u/\partial p_u + \sum_{i\neq u} \beta_i/\beta_u.|\partial Z_u/\partial p_i|) < 0$ for any $p \in \Delta$. A set of excess demand functions satisfies a *column dominant diagonal condition with weights* if there exists $\beta_i > 0$ such that: $\sup_u\{\partial Z_u/\partial p_u + \sum_{i\neq u} \beta_i/\beta_u.|\partial Z_i/\partial p_u|) < 0$ for any $p \in \Delta$.

Given these definitions, Dohtani (1993, 1998) proves the following results:

Theorem 8.4 (Dohtani (1993, 1998)). (1) *If the excess demand map Z satisfies the boundary condition $Z_i(p) > 0$ if $p_i = 0$, has at least one zero and is of the dominant diagonal with constant weights variety then a process like* (8.1) *is globally stable in* Δ. (2) *Furthermore, if there is a set of positive real numbers* $\{\beta_k : 1 \leq k \leq L\}$ *such that either*: (i) $\max_{1 \leq u,w \leq L \text{ and } u<w}\{\partial Z_u/\partial p_u + \sum_{i \neq u,w} \beta_u/\beta_i \partial Z_u/\partial p_i| + \partial Z_w/\partial p_w + \sum_{i \neq u,w} \beta_w/\beta_i |\partial Z_w/\partial p_i|\} < 0$; *or* (ii) $\max_{1 \leq u,w \leq L \text{ and } u<w}\{\partial Z_u/\partial p_u + \sum_{i \neq u,w} \beta_i/\beta_u/\partial Z_i/\partial p_u| + \partial Z_w/\partial p_w + \sum_{i \neq u,w} \beta_i/\beta_w/\partial Z_i/\partial p_w/\} < 0$ *then* (8.1) *is globally stable in* Δ.

Proof. Part 1, Dohtani (1993; pp. 79–81) and Part 2 Dohtani (1998; p. 170). □

Another interesting approach to structuring the Jacobian of $Z(p)$ is the contribution of Keenan and Rader (1985). Their result depends on the following definition:

Definition 8.7 (Weak law of demand). The *weak law of demand* holds if the trace of the Jacobian of the excess demand map is negative for all p, i.e. $\operatorname{tr} D_p Z(p) < 0$, $\forall\, p \in \Delta$.

Theorem 8.5 (Keenan and Rader (1985)). *If there are just two goods in the economy, then there is global stability of a process like* (8.1) *on a* $Z(p)$ *that is smooth, HD*0, *satisfies Walras' Law, satisfies the boundary condition* $Z_i(p) \to \infty$ *if* $p_i \to 0$, *if and only if the weak law of demand holds. If there are three goods in the economy and if all equilibria are isolated, then the weak law of demand implies convergence to equilibrium.*

Proof. Keenan and Rader (1985; pp. 469–470). □

Remark 8.9. This is an intriguing result but it only seems to guarantee stability in the case of an economy with at most three goods and as Keenan and Rader (1985) demonstrate, the result is already invalid if there are four goods.

In a related work, Keenan and Kim (2000) investigate various forms of the law of demand and their implications for the stability of a tatonnement process. Consider the Jacobian of the excess demand $D_pZ(p)$, which is the matrix of uncompensated price effects for the economy. Symmetrise this to form: $K(p) = [D_pZ(p) + D_pZ(p)^T]$. Rank the eigenvalues of $K(p)$ in decreasing size order as $\lambda_1(p), \lambda_2(p), \ldots, \lambda_{n+1}(p)$. The key idea in Keenan and Kim (2000) is contained in the following definition:

Definition 8.8 (Law of demand of order *i*). $Z(p)$ obeys the *law of demand of order i* if $\lambda_1(p) + \cdots + \lambda_{n+2-i}(p) < 0$.

Remark 8.10. For the law of demand of order $i = 1$, this means that $\lambda_1(p) + \cdots + \lambda_{n+1}(p) < 0$, so $2 \times \sum \text{diag} D_pZ(p) < 0$. This requires very little structure on the uncompensated price responses of the economy and is therefore called the *weak law of demand*. At the other extreme, if the law of demand holds for order n, then $\lambda_1(p) + \lambda_2(p) < 0$. Then all eigenvalues, bar one, must be negative. This is called the *strong law of demand*.

Theorem 8.6 (Keenan and Kim (2000)). *Economies obeying the strong law of demand (i.e. the law of demand of order n) have a globally stable equilibrium relative to a simple tatonnement $dp/dt = Z(p)$.*

Proof. Keenan and Kim (2000; p. 315). □

Remark 8.11. As the order of the law of demand declines from n towards 1, Keenan and Kim (2000) show, by means of concepts drawn from chaos theory, that tatonnement dynamics become more and more arbitrary and generally not convergent to a Walrasian equilibrium. Keenan and Kim's work represents an interesting summary result concerning the 'conditions on excess demands and their Jacobians' literature and it opens the question of what needs to be happening in the microeconomics of the economy to ensure that the various laws of demand hold.

Mukherji (2007) presents the following interesting extension on the 'law of demand' approach to obtaining global stability in the case of a three good economy. Excess demands $Z_i(p_1, p_2, p_3) : \Re^3_{++} \to \Re$ for $i = 1, 2, 3$ good 3 is the numeraire and for notation let $p \equiv (p_1, p_2)$. The price adjustment process is $p_i = h_i(p), i = 1, 2$ with $p_3 = 1$ and it defines a path of prices beginning at any positive price p^0. The price configuration at any point in time is $(\varphi_t(p^0, 1) = (p_1(t), p_2(t), 1)$. Interest is focussed on the limit points of the trajectory $\varphi_t(p^0)$ as $t \to \infty$, the so called ω — *limit set*, $L_\omega(p^0)$. Make the following assumptions about this economy.

(A) The excess demand functions are continuously differentiable and have continuous partial derivatives, satisfy Walras' Law, are homogeneous of degree zero in prices and satisfy the boundary condition that for any price sequence $p^s \in \Re^3_{++}$ if $p^s_i = 1 \; \forall s$ for any good i, and $\|p^s\| \to \infty$ as $s \to +\infty$ then $Z_i(p^s) \to +\infty$ for that good.
(B) The price adjustment process $h_i(p)$ is a continuously differentiable function and has the same sign as $Z_i(p, 1), (i.e. +, -, 0), \forall p \in \Re^3_{++}, i = 1, 2$.
(C) (i) the solution trajectory $\varphi_t(p^0)$ of $h_i(p)$ is always in a bounded region R of $\Re^2_{++}$; (ii) there is a differentiable function $\theta(p) : \Re^2_{++} \to \Re$ such that $\mathbf{div}(\theta(p)h_1(p), \theta(p)h_2(p))$ has the same non-zero sign everywhere on R; (iii) $\mathbf{div}(\theta(p)h_1(p), \theta(p)h_2(p)) \neq 0$ and the $\det J((\theta(p)h_1(p), \theta(p)h_2(p)) \neq 0$, on the set of equilibrium prices, where for two functions $f(x, y)$ and $g(x, y)$ the

matrix of partial derivatives $\left[\begin{smallmatrix} f_x & f_y \\ g_x & g_y \end{smallmatrix}\right]$ is the Jacobian $J(f,g)$, $\det J$ is its determinant and $\mathbf{div}(f,g) = \operatorname{tr} J = f_x + g_y$.

Theorem 8.7 (Mukherji (2007)). *If the three good economy satisfies conditions A, B and C, then all solutions to the adjustment process $\dot{p}_i = h_i(p)$ converge to an equilibrium so that for any $p^0 \in R, L_\omega(p^0) = p^*$, where p^* is an equilibrium price vector.*

Proof. Mukherji (2007; p. 588). □

Remark 8.12. As Mukherji (2007; p. 587) notes, the assumption $\mathbf{div}(Z_1, Z_2) < 0$ implies C(i) and C(ii) when $h_i = Z_i$ and $\theta(p) = 1$. Since Keenan and Rader (1985) call $\mathbf{div}(Z_1, Z_2) < 0$, the weak law of demand, it is reasonable to call condition C the *general law of demand*. This is an interesting generalisation of the conditions of Keenan and Rader (1985) to ensure stability of adjustment processes operating in the plane.

8.2.8. *Individual characteristics and the law of demand*

Imposing conditions on the price derivatives of the excess demand function is sometimes equivalent to imposing conditions on the income and substitution effects in the Slutsky equation. It is therefore interesting to see what is required in the microeconomics of the economy if income and substitution effects are to be stabilising, or at least not destabilising. This approach takes the SMD theorem 'head on' and attempts to find plausible conditions on the structure of the economy under which it does not apply. The usual way to do this is to look for conditions under which the *uncompensated law of demand* holds.

Definition 8.9 (Uncompensated law of demand). Consumer i's demands $x_i(p, m_i)$ satisfy the *uncompensated law of* *demand* if $(p' - p)[x_i(p', m_i) - x_i(p, m_i)] \leq 0$ for any p, p' and m_i with strict inequality if $x_i(p', m_i) \neq x_i(p, m_i)$. The economy's aggregate demand satisfies the uncompensated law of demand if $(p' - p)[x(p', m) - x(p, m)] \leq 0$ for any p, p' and $m = \sum_i m_i$, with strict inequality if $x(p', m) \neq x_i(p, m)$.

Milleron (1974) and Mitushin and Polterovitch (1978) identified conditions on individual preferences under which the ULD will hold for individuals (and hence for the economy since this property unlike some, e.g. WARP, aggregates). The condition is that: $-x_i^{\cdot} D^2 u_i(x_i)^{\cdot} x_i < 4x_i^{\cdot} Du_i(x_i)$ for all i. This says that the consumers indifference curve family has to 'fan' and 'curve' in the right way to prevent income effects from overpowering substitution effects. While this is an interesting result, two features of it should be noted. Firstly, it is derived for an economy in which income is independent of prices, and therefore has little relevance to the sort of economies we are considering. Secondly, it is actually only a sufficient condition for aggregate demands to have the sort of nice properties needed for simple tatonnement processes to work (this point was made by Mas-Colell *et al.* (1995; p. 113)).

Taking up both these observations in a contribution, we noted in connection with uniqueness in the previous chapter, Quah (1997) set out "...to identify some plausible assumptions on the *distribution* of characteristics among agents that will guarantee that an exchange [or production] economy has a unique and stable equilibrium ..." Quah (1997; p. 1421, emphasis added). In the event Quah (1997) achieves his objective but at the price of a particularly unrealistic 'independence' assumption. The assumption may be stated as follows:

(QH): Preferences of agents in the economy are distributed independently of the endowments of agents in the economy.

Remark 8.13. As was noted earlier, Quah (1997) has this to say about the limitations of this assumption: "...the independence assumption here has an unpleasant implication: it excludes the possibility that agents face a labor-leisure choice." Quah (1997; p. 1423). If this is regarded as implausible, then this work has not been able to furnish a stability argument.

8.2.9. *Stabilising income effects*

We now follow up the suggestion by Mukherji (1974) that the role of 'stabilising income effects' and 'strong substitution effects' in guaranteeing the stability of Walrasian equilibrium is worthy of investigation. Following Mukherji (1974), we make this definition of stabilising income effects:

Definition 8.10 (Stabilising income effects). Let $V(p(t)) = 1/2\sum_h [Z_h(p(t))]^2$ be a Lyapounov function where $Z_h(p)$ is the excess demand for good h at p. Then $V(p) = 0$ if p is an equilibrium price vector. Also if $Z_h(p)$ is differentiable, then $dV(p(t))/\mathrm{d}t$ exists. *Income effects are stabilising* provided they ensure that $dV(t)/dt < 0$.

Remark 8.14. Mukherji (1974) shows that income effects are stabilising if the condition: $Z^T[\sum_i Z^i_{k\cdot}\ \partial x^i_k/\partial M^i)]Z \geq 0$ is satisfied. Mukherji (1974) investigates circumstances under which the microeconomic structure of the economy ensures the existence of stabilising income effects. He obtains the following two sets of circumstances: (i) if every individual has the same propensity to consume each good at equilibrium then income effects are stabilising; (ii) if the income effects of net buyers are not weaker than the income effects of net sellers' and if net buyers of commodity h are likely to have small marginal propensities to consume commodity $k \neq h$, then (8.1) is locally stable. (See Mukherji (1974; p. 251.) Commenting on these conditions Mukherji (1974) remarks: "The above [conditions] perhaps indicate the difficulty of obtaining income effects that are stabilising. Stabilising income effects is too strong a condition to insist [on]... Thus it seems better to seek conditions under which the income effects are outweighed by substitution effects." Mukherji (1974; p. 251). We now follow up this suggestion using the set-up in Mas-Colell (1985) and Nachbar (1998) and in the process make an interesting discovery about how limited this sort of approach is likely to be.

The economy has $1 \leq i < \infty$ consumers, $1 \leq k < \infty$ inputs and $2 \leq \ell < \infty$ outputs. Assume that consumers are endowed only with inputs and have preferences only over outputs. Let $\omega_i \in \Re^k_{++}$ be the endowment for consumer i and let $p \in \Re^l_{++}$ be the vector of output prices with the ℓth goods price being normalised to be 1.

Assumption 8.1. The demand by consumer i is given by a continuously differentiable function $\phi_i : \Re^{\ell+1}_{++} \to \Re^{\ell}_{++}$ which depends on output prices and i's income: $\phi_i(p, m_i)$. Aggregate demand is ϕ: $\Re^{\ell+1}_{++} \to \Re^{\ell}_{++}$, where $\phi(p, m_1, \ldots, m_\ell) = \sum_i [\phi_i(p, m_i)]$.

Assumption 8.2. The ownership of endowments and firm shares is collinear across consumers. (This assumption will be dropped later.)

Lemma 8.2 (Nachbar (1998; p. 405)). *If the ownership of shares and endowments is collinear across consumers, then there exists* $\alpha_i > 0, \sum_i \alpha_i = 1$ *such that* $m_i = \alpha_i m$ *for any vector of input and output prices. Aggregate demand is then* $\phi(p, \alpha_1 m, \ldots, \alpha_\ell m)$ *which may be written* $\phi(p, m)$. □

Proof. Nachbar (1998; pp. 405–406). □

Remark 8.15. One other consequence of this lemma is, that as far as the production side of the economy is concerned, it is enough to consider aggregate production possibilities. The aggregate production technology will be represented in dual form through an aggregate revenue function $\rho : \Re^{\ell+k}_{++} \to \Re^{\lambda}_{+}$. This function depends on p and the aggregate endowment $\omega = \sum_i \omega_i$.

In particular $\rho(p, \omega)$ is a solution to the problem:

$$\text{Max } p^y \text{ such that the output } y \in \Re^l_+$$
$$\text{can be produced from inputs } \omega.$$

It will be assumed that ω is fixed throughout the analysis and so the dependence of ρ on ω will generally be suppressed except when a shock to endowments needs to be studied. Nachbar (1998) shows that this can be done by introducing a 'production shock' parameter ξ. Output $(y^1, \ldots, y^\ell)$ is feasible at $\xi = \xi^*$iff$(y^1/\xi^*, \ldots, y^\ell/\xi^*)$ is feasible. Let $\rho(p, \xi)$ denote aggregate revenue at prices p and production shock ξ then given (p, ξ) aggregate income is $m = \rho(p, \xi)$.

Assumption 8.3. ρ is twice continuously differentiable at least near an equilibrium.

Remark 8.16. It is well known that ρ is convex in prices. Therefore, from Assumption 8.4, for a given (p, ξ), $D_{pp}\rho(p, \xi)$ will be positive semi-definite. From Hotellings' lemma, $D_p\rho(p, \xi) = y$. Therefore aggregate excess demand for consumption goods can be written as $f(p, \xi) = \phi(p, \rho(p, \xi)) - D_p\rho(p, \xi)$. A Walrasian equilibrium is characterised by $f(p, \xi) = 0$ and we fix a reference equilibrium price vector at p^* for the ξ^* economy. Mukherji's suggestion that we focus on substitution effects which outweigh income effects is the same thing as seeking conditions under which the endogenous income law of demand holds.

Definition 8.11. The *endogenous income law of demand* holds at equilibrium $(p^*, 1)$ if and only if $v^T D_p f(p^*, 1)v < 0$ for any $v \in \Re^{\ell-1} \times \{0\}, v \neq 0$.

Remark 8.17. In particular if v is the ℓth unit vector, then the law of demand implies that $\partial f^\ell/\partial p^\ell < 0$ so that aggregate own price effects are negative, which means that its aggregate substitution effects have outweighed income effects. It is now of interest to see what sort of situations imply that outcome.

In the case we are dealing with here, the Slutsky equation is

$$D_p f = D_p \phi + D_m \phi D_p \rho - D_{pp} \rho. \quad (8.8)$$

Since $D_p \rho = \phi^T$, (8.8) yields,

$$D_p f = \sum_i S_i - \sum_i D_{mi} \phi_i \phi_i^T + D_m \phi \phi^T - D_{pp} \rho. \quad (8.9)$$

Again under standard conditions, each S_i is negative definite as is $-D_{pp}\rho$ and the endogenous income law of demand can only fail if $-\sum_i D_{mi}\phi_i\phi_i^T$, the aggregate income term fails to be negative semi-definite. Rewrite the aggregate income effect as follows:

$$\begin{aligned} - \sum_i D_{mi} \phi_i \phi_i^T + D_m \phi \phi^T \\ = - \sum_i (D_{mi} \phi_i - D_m \phi).(\phi_i - \alpha_i \phi)^T. \end{aligned} \quad (8.10)$$

Thus from (8.10), it can be seen that the endogenous income law of demand will only hold for good ℓ if consumers with higher than average propensities to consume good ℓ purchase their 'share' (or more than their share) of the good, where consumer i's share is $\alpha_i \phi^\ell$. While this is slightly different to the conditions derived by Mukherji (1974), the stabilising income effects approach is essentially equivalent to the substitution effect dominating in the endogenous income case (even with collinear endowments and shares). As Mukherji (1974) and numerous authors since note, this sort of condition is not very promising.

8.2.10. *Stabilising aggregate substitution effects*

Keenan (2000) considers an approach to establishing global stability that he characterises as follows: "Stability conditions, as seen in economics, are commonly expressed in terms of Jacobians, and the current method makes more direct use of such conditions than does the usual method of Lyapounov functions. Furthermore, in the case of tatonnement, the method has consequences closely related to traditional substitution and income reasoning." Keenan (2000; p. 317). The set-up of the approach is as follows. Let $p \in \Re^{\ell}_{+}$ be prices, m income, $\omega \in \Re^{\ell}_{+}$ endowment, and $x_i(p, m_i) \in \Re^{\ell}_{+}$ is person i's demand. In an exchange economy, $m_i = p\omega_i$ and the individual excess demand function $z_i(p) = x_i(p, p\omega_i) - \omega_i$ the Slutsky decomposition yields by familiar reasoning the following expression for the Slutsky substitution matrix:

$$\begin{aligned} S_i(p) &= D_p z_i(p) + D_m x_i(p, p\omega_i)(x_i^T - \omega_i) \\ &= D_p z_i(p) + D_m x_i(p, p\omega_i) z_i^T(p). \end{aligned} \tag{8.11}$$

The familiar homogeneity and adding up conditions hold so $D_p z_i(p)p = 0$, $pD_p z_i(p) + z_i^T(p) = 0$ and $pD_m x_i(p, p\omega_i) = 1$ yielding $pS_i(p) = 0^T$ and $S_i(p)p = 0$. Keenan (2000) makes the observation that: "[t]he income effect only acts on post-multiplied vectors in the direction $z_i(p)$, and so all compensated vectors v such that $vz_i(p) = 0$ are affected by $D_p z_i(p)$ only in so much as they are affected by the substitution matrix $S_i(p)$." Keenan (2000; p. 323). For the aggregate excess demand function $Z(p) = \sum_i z_i(p)$ which satisfies Walras' Law and homogeneity of degree zero, Keenan (2000) defines the idea of a *compensated price change* as being any dp such that $Z(p)\mathrm{d}p = 0$ and so the *aggregate substitution effect* concerns $S(p)|T_{Z(p)}$, where $S(p) \equiv D_p Z(p) + \alpha(p) Z^T(p)$ for any function $\alpha(p)$ homogeneous of degree -1 and $T_{Z(p)}$ is the hyperplane orthogonal to $Z(p)$. Since $S(p)$ contains the entire substitution effect, in the way $S_i(p)$ does for individuals, it may reasonably be called the *aggregate substitution matrix* — see Keenan (2000; p. 324) for further discussion.

For the price adjustment process, Keenan (2000) considers a *simple normalised tatonnement* $\dot{p} = \hat{Z}(p)$ for $p \in \Re^{\ell-1}$

(the prices of the $\ell - 1$ non-numeraire goods) and $\hat{Z}(p) = (Z_1(p,1), \ldots, Z_{\ell-1}(p,1))$ is the vector of their excess demands with $p_\ell = 1$. Diagonal dominance is defined by taking the vector norm $|x|$ as the $\max_i |\beta_i x_i|$ for $\beta_i > 0$, $i = 1, m$ then the Lozinski log norm becomes $\mu(A) = \max_i \beta_i a_{ii} + \sum_{k \neq i} \beta_k |a_{ik}|$ for any $m \times m$ matrix A, so that $\mu(A) < 0$ with negative diagonal gives the *row dominant diagonal* condition $\beta_i |a_{ii}| > \sum_{k \neq i} \beta_k |a_{ik}|$. Choosing the vector norm to be $\sum_i |\beta_i x_i|$, $\beta_i > 0$, the Lozinski norm becomes $\mu(A) = \max_k \beta_k a_{kk} + \sum_{k \neq i} \beta_i |a_{ik}|$ so that with negative diagonal, the *column dominant diagonal* condition becomes $\beta_k |a_{kk}| > \sum_{k \neq i} \beta_i |a_{ik}|$ — see Keenan (2000; p. 325) for this and further discussion. A *non-normalised tatonnement* where $p = Z(p)$. Homogeneity of degree zero in prices of $Z(p)$ means that one eigenvalue of $D_p Z(p)$ is zero so negative definiteness of this matrix can be expressed as the condition $v D_p Z(p) v < 0$ for $v \in T_p$, the hyperplane orthogonal to p. Call this condition the *strict form of monotonicity.* The compensated or net form of this condition is that $vS(p)v < 0$ for $v \in T_p$, where $S(p)$ is as defined above. This condition will be referred to as the *strict form of WARP* — see Keenan (2000; p. 326). If $S(p)$ is of the form $S(p) = D_p Z(p) - \beta(p) Z(p) Z^T(p)$, then for some large enough value of the scalar function $\beta(p)$, the strict (local) form of WARP for some $\alpha(p)$ is equivalent to $v D_p Z(p) v < 0$ for $v \in T_p \cap T_{Z(p)}$. If the aggregate substitution matrix is such that $\alpha(p)$ is homogeneous of degree -1 and is such that $p\, \alpha(p) = 1$, then we say that $\alpha(p)$ satisfies a *budget like condition.* We may then summarise Keenan's main results as follows:

Theorem 8.8 (Keenan (2000; Theorems 2 and 3 and Corollary 1)). *If the excess demand functions $Z(p)$ are smooth, bounded from below, homogeneous of degree zero, satisfy Walras' Law and a boundary condition and if either the normalised substitution matrix or the total price effects matrix* *satisfy row or column diagonal dominance for constant weights with negative diagonal, then the simple normalised tatonnement is globally stable. If either the aggregate substitution matrix* $S(p) = D_pZ(p)+\alpha(p)Z^T(p)$ *always satisfies strict WARP or has positive off diagonals and* $\alpha(p)$ *satisfies a budget like condition, or the total price effects matrix* $D_pZ(p)$ *always satisfies strict monotonicity, then a non-normalised tatonnement is globally stable.*

Proof. Keenan (2000; pp. 325 and 327–328). □

Remark 8.18. The interesting consequence of this result is that, in a sense, income effects do not matter. As Keenan (2000) puts it: "… assumptions on the total price-effect matrix [$D_pZ(p)$] influence stability only in terms of what they imply for the substitution matrix, so that for every successful stability condition on the total matrix [such as WARP, diagonal dominance, gross substitutes], there is a corresponding one on the substitution matrix which implies global stability, without regard to income effects." Keenan (2000; p. 328).

8.2.11. *Stability via 'institutions': Rader (1996), the Hahn process and modern banking*

Apart from the restrictive microeconomics which have to hold in order to guarantee that $D_pZ(p)$ has particular structure, the approach considered in the previous section is subject also to the observation made by Radner, and reported to Saari and Simon (1978), that conditions like GS, DD and WARP all represent an *exchange of information* relative to processes of a Global-Newton type. As Saari and Simon (1978) put it: "[o]f course if we have *a priori* knowledge concerning a given vector field $Z(p)$, we may be able to design simpler mechanisms. However this is merely an exchange of type of information used, and, as Radner pointed out to us, the expense of determining this type of information may be very high." Saari and Simon (1978; p. 1099). Thus, even if general conditions on the microeconomics of the economy could be found to ensure stability, the informational cost involved in actually knowing when it is safe to use a simple tatonnement, instead of a Global-Newton process, would as Radner, observes probably be prohibitive. In the next section, we therefore return to the case where the excess demand map is allowed to be general and the role of certain 'institutional' assumptions in ensuring global stability is explored.

Rader (1996) specifies an adjustment process which is universal and globally stable in an environment where the SMD result is largely respected and little special structure is imposed on $Z(p)$. Rader also tries to avoid the unrealistic informational demands of a Global-Newton process type by: (i) adopting the Hahn process as part of the mechanism by which prices are adjusted; and (ii) by arguing that if certain *institutional features* of the economy were incorporated into the analysis, then global stability of a modified Hahn process can be achieved, without having to resort to the informationally demanding Global-Newton type process. In particular, Rader (1996) argues that if one market, the money market, is always in equilibrium, as a result of the institution of 'modern banking', then the Hahn process will converge without any of the familiar additional structure, such as WARP, GS or DD being imposed. This is a potentially interesting approach and some time will be spent exploring it. We follow the notation and set-up of Fisher (1974) to formulate the Hahn process.[5] In Rader's model, there are $\ell+1$ commodities, the $(\ell+1)$th of which is the numeraire, money. The price of the kth good is p_k. There are $i = 1, \ldots, I$ households. The actual stock of the kth good held by the ith household is ω_{ik}. The desired stock of the kth good for household is x_{ik}. The *excess demand* of the ith household for good k is $z_{ik} \equiv x_{ik} - \omega_{ik}$.

[5]See also the commentary in Bryant (1996).

The household's stock of money is ω_{im} and its desired stock of money is m_i.

The household maximises a utility function $U_i(x_i, m_i)$ which is strictly quasi-concave and twice continuously differentiable. The first partials are denoted by $U_{ik} \equiv \partial U_i/\partial x_{ik}$ for $k = 1, \ldots, \ell$ and $U_{im} \equiv \partial U_i/\partial m_i$.

Assumption 8.4. For all $I, U_{im} > 0$, so that households are not satiated in money.

Let W_i denote the wealth of household i, then the household budget constraint is

$$p^T x_i + m_i = W_i. \tag{8.11}$$

As usual the wealth of the household consists of the value of the households endowment, money holdings and shares in the profits of firms (s_i minus any dividends already paid d_i). Thus:

$$W_i \equiv p^T \omega_i + \omega_{im} + (s_i - d_i). \tag{8.12}$$

There are $j = 1, \ldots, J$ firms and the *desired* commitment of the jth firm with respect to the kth commodity is v_{jk} while the *actual* commitment is $\bar{v}_{jk}$. The difference is given by $g_{jk} \equiv v_{jk} - \bar{v}_{jk}$. The jth firm's desired commitment with respect to the numeraire commodity is denoted by y_j and the actual commitment is $\bar{y}_j$. As Fisher (1974) notes, the numeraire commodity plays a dual role in the economic system. It can be produced and used as an input just like any other good, but it alone has the role of medium of exchange. The jth firm's production possibility set is $\phi_j(v_j, y_j) \leq 0$ and the efficient production surface is

$$\phi_j(v_j, y_j) = 0. \tag{8.13}$$

ϕ_j is assumed to be twice continuously differentiable with the first partial derivatives denoted by $\phi_{jk}(\equiv \partial \phi_j/\partial v_{jk}$ for $k = 1, \ldots, \ell)$ and $\phi_{jy} \equiv \partial \phi_j/\partial v_{jy}$ for the numeraire commodity. It is assumed that (8.13) is satisfied at the origin so that the firm always has the option of not producing any output or using any input. Each firm is assumed to be profit maximising subject to its production technology. Also firms have to meet past commitments and may have earned profits from past commitments, denoted by $\bar{\pi}_j$. Thus the profit of the jth firm π_j is given by:

$$\pi_j \equiv p^T(v_j - \bar{v}_j) + (y_j - \bar{y}_j) + \bar{\pi}_j, \tag{8.14}$$

where

$$\bar{\pi}_j(t) = \int_{0,t} \{p(\tau)^T d\bar{v}_j/dt + d\bar{y}_j(\tau)/dt\} d\tau + \bar{\pi}_j(0). \tag{8.15}$$

Let q_j be the total payments made to shareholders by the jth firm to date, then $q_j \leq \bar{\pi}_j$. If $\bar{r}_j$ is the total money stock held by the firm then

$$\bar{r}_j = \bar{\pi}_j - q_j - \bar{y}_j. \tag{8.16}$$

The hth households share of firm earnings, s_i, and dividends received, d_i, are defined as:

$$s_i = \sum_j \theta_{ij}\pi_j \quad \text{and} \quad d_i = \sum_j \theta_{ij} q_j. \tag{8.17}$$

There are various aggregates which Fisher (1974; p. 475) defines as sums of the corresponding individual entities as follows:

$$\begin{aligned}
&X_k \equiv \sum x_{ik}; && X \equiv \sum_i x_i; && \bar{X}_k \equiv \sum_i \omega_{ik}; && \bar{X} \equiv \sum_i \omega_i;\\
&Z_k \equiv X_k - \bar{X}_k; && Z \equiv X - \bar{X}; && M \equiv \sum_i m_i; && \bar{M} \equiv \sum_i \bar{m}_i;\\
&V_k \equiv \sum_j v_{jk}; && V \equiv \sum_j v_j; && \bar{V}_k \equiv \sum_j \bar{v}_{jk}; && \bar{V} \equiv \sum_j \bar{v}_j;\\
&G_k \equiv V_k - \bar{V}_k; && G \equiv V - \bar{V}; && Y \equiv \sum_j y_j; && \bar{\pi} = \sum_j \bar{\pi}_j;\\
&Q = \sum_j q_j; && \bar{R} = \sum_j \bar{r}_j; && \bar{Y} = \sum_j \bar{y}_j.
\end{aligned} \tag{8.18}$$

Denote by $\bar{X}(0)$ and $\bar{M}(0)$ the holdings of the household sector of commodities and money, respectively at some initial time, 0. Some of these holdings may be the result of commitments and distributions made by the firms up to that time. Denote by $\bar{V}(0)$ and $[\bar{Y}(0)+Q(0)]$ the holdings by households of goods and money that result from such commitments and distributions. Then Fisher (1974; p. 476) shows the following adding up conditions hold:

$$\bar{X} \equiv \bar{X}(0)+\bar{V}-\bar{V}(0); \quad \bar{M} \equiv \bar{M}(0)+\bar{Y}(0)+Q-Q(0). \quad (8.19)$$

Using condition (8.19), the various definitions in (8.18) and adding up the budget constraints for all the households in (8.11) we get Walras' Law for this economy:

$$p^T Z - p^T G + (M-\bar{M}) - (Y+\bar{R}) \equiv 0. \quad (8.20)$$

This is Rader's *Axiom W* — see Rader (1996; p. 116). The idea of a Walrasian equilibrium for the economy is standard, involving as it does conditions on consumers, firms and the overall state of the markets. However, following Fisher (1974; p. 476), it is worth restating the definition in the current notation:

Definition 8.12. A *Walrasian equilibrium* is a state in which: (i) for every consumer i, $m_i = m_i$ and $z_{ik} \leq 0$ for all $k = 1, \ldots, n$; (ii) for every firm $j, y_j = \bar{y}_j$ and $g_{jk} \geq 0$, for all $k = 1, \ldots, n$; and (iii) $\bar{X} = \bar{X}(0)+\bar{V}-\bar{V}(0)$ and $\bar{M} = \bar{M}(0)+\bar{Y}+Q-\bar{Y}(0)-Q(0)$.

In a Walrasian equilibrium, all profits are realised and distributed and the price of any good in excess supply is zero (see Fisher (1974; Lemma 2.1)).

Like the model constructed by Fisher (1974), the economy which Rader (1996) has in mind is a monetary production economy in which the 'Hahn process' operates to adjust prices. One way to ensure that the economy is monetary is to insist that purchases can only be made with money and to say that demands only become *effective* if there is money to back them up. This also helps avoid some problems that the Hahn process can get into otherwise (see Fisher (1974; p. 477) for further discussion). Fisher (1974) therefore distinguishes between *target* and *active* excess demands.

Definition 8.13 (Target and active excess demands). Excess demands are *effective* if there is money to back them up. *Target excess demands* are excess demands that would prevail if the institutional restriction that purchases have to be backed by money did not apply.

Denote by a_{ik} the active excess demand for good k by household i and by a_i^+ the vector of those a_{ik} that are positive, with p_k^+ the vector of corresponding prices. We now formalise the idea that negative excess demands are active while positive excess demands are only active if there is money to back them up. Furthermore, it is assumed that the available money is distributed over commodities for which there is positive excess demand so that some purchases of each commodity is attempted.

Assumption 8.5. (i) if $z_{ik} \leq 0$, $a_{ik} = z_{ik}$; (ii) if $z_{ik} > 0$ and $m_i > 0$ then $0 < a_{ik} \leq z_{ik}$ and $p_k^{+T} a_i^+ \leq m_h$; if $z_{ik} > 0$ and $m_i = 0$, then $a_{ik} = 0$.

The same restriction applies to firms and may be written as follows:

Assumption 8.6. (i) if $g_{jk} \geq 0$, $c_{ji} = g_{jk}$; (ii) if $g_{jk} < 0$ and $r_j > 0$, then $0 > c_{jk} \geq g_{jk}$ and $p_j^{-T} c_j^- \geq -r_j$; (iii) if $g_{jk} < 0$ and $r_j \leq 0$ then $c_{jk} = 0$.

Like Fisher (1974), Rader (1996) bases his price adjustment process on that of Hahn (1961). In commenting on the process he remarks: "The Hahn system is an obvious competitor to the one of Scarf, that was to substitute for tatonnement. It involves less abstract mathematics and is highly reasonable . . . " Rader (1996; p. 116). Hahn (1982) has described the process as expressing the idea of 'orderly markets'. As Fisher (1974) points out, what this means is that markets are sufficiently well organised so that there cannot be both active excess demands and active excess supplies in the *same* commodity. Thus, if there is disequilibrium in a market, only one side of the market is frustrated in its sales or acquisition plans. We may write this feature of the economy formally as follows. Add up the active excess demands and supplies across the households and firms to form $A_k \equiv \sum_i a_{ik} - \sum_j c_{jk}$. Thus A_k indicates the state of the market for good k. The essence of the Hahn process, namely that markets are 'orderly', is then captured in the following definition (for further details, see Hahn (1961, 1982), Arrow and Hahn (1971), Fisher (1974) and Rader (1996)).

Definition 8.14 (Hahn process).[6] (i) for all times, t, all households, i, and all commodities k, if $a_{ik} \neq 0$ then $a_{ik}A_k > 0$; and (ii) for all times, t, all firms, j and commodities k, if $c_{jk} \neq 0$ then $c_{jk}A_k < 0$.

Price Adjustment Rule (RP).[7] (i) If $p_k > 0$ or $p_k = 0$ and $A_k \geq 0$ then $dp_k/dt = F_k(A_k)$, where F_k is continuous, sign preserving and bounded away from zero except as A_k goes to zero; (ii) if $p_k = 0$ and $A_k < 0$ then $dp_k/dt = 0$. Also for every commodity k there exists a positive scalar k_k such that $F_k(A_k) \leq k_k A_k$.

Given these assumptions and definitions, we may establish global stability of a Hahn process in the sort of economy imagined by Rader (1996) as follows. As prices change, so does the wealth of households through: (i) capital gains or losses on the stocks of goods owned by the household; and (ii) changes in the future profits of firms in which the household owns shares. In particular household wealth evolves according to the equation:

Assumption 8.7. $p^T(d\omega_i/dt) + dm_i/dt = dd_i/dt$

[6] This definition follows Fisher (1974; p. 478).

[7] Rader (1996) actually specifies the adjustment process as $(dp_k/dt)/p_k = A_k/\sum_i a_{ik}$ but we prefer to work with the more slightly more general form in Fisher (1974).

As a consequence of profit maximisation each firm's target profits, π_j, will be non-increasing over time and will be strictly decreasing if the firm cannot achieve its objectives. Rader (1996; p. 117) assumes this directly in his Axiom Π. In Fisher (1974) it comes out as a theorem (see Fisher (1974; p. 479)). In any case we have:

Target Profits Condition: For all firms j and all time t, if $\bar{r}_j > 0$ then $d\pi_j/dt \leq 0$. Moreover, $d\pi_j/dt = 0$ iff for all commodities k $v_{jk} \geq \bar{v}_{jk}$ and $v_{jk} > \bar{v}_{jk}$ only for those i's for which $dp_k/dt = 0$.

Target Utility Condition[8]: For every household i and every time t, if at t, $\bar{m}_i > 0$ and also $\bar{r}_j > 0$ for every j such that $\theta_{ij} > 0$, then $dU_i/dt \leq 0$. Further, $dU_i/dt = 0$ iff $x_{ik} \leq \omega_{ik}$ and $v_{jk} \geq \bar{v}_{jk}$ for every j such that $\theta_{ij} > 0$, with strict inequality for those commodities k for which $p_k = 0$.

A central step in constructing the stability proof involves ensuring that prices, actual endowments of households and actual commitments of firms remain bounded. As a preliminary it is necessary to guarantee that the price of money is always positive. Rader (1996) guarantees this by the twin assumptions that (i) $p_k = 0 \Rightarrow$ positive excess demand for good k; and (ii) his condition M which ensures that the money market is always in equilibrium (see Rader (1996; pp. 116, 117)).[9] For reference we state these axioms formally as follows:

(B) : $p_k = 0 \Rightarrow$ positive excess demand for good k. (8.21)

(M) : The demand for money is always equal to the supply of money. (8.22)

[8] This also comes out as a theorem in Fisher (1974) see p. 480.

[9] Fisher (1974) achieves this end by assuming that if the price of money becomes zero, while other commodity prices are positive, then total demand for it exceeds total supply, Fisher (1974; p. 481).

Given these assumptions, the price of money always stays positive and it is possible to prove boundedness of the price path.

Lemma 8.3. *Given* (B), (M) *and Walras' Law the time path of prices is bounded under the price adjustment process* (8.21).

Proof. Following Fisher (1974; p. 481), let $N = 1/2 \sum_i p_k^2/k_k$ then $dN/dt = \sum_k p_k(dp_k/dt)/k_k = \sum_k p_k F_k(A_k)/k_k \leq \sum_k p_k A_k$. From the definition of A_k and from Assumptions (8.5) and (8.6), $\sum_k p_k A_k \leq p^T(Z - G)$. By Walras' Law, $p^T(Z - G) = \bar{M} + \bar{R} - M + Y$, but $\bar{M} + \bar{R} - M + Y$ is just the difference between money demand and money supply. By Rader's assumption M this is always 0. So if the price adjustment process starts at a price vector in which all entries are finite then no price will diverge to infinity as time goes on since $dN/dt = 0$. Thus, given (B), (M) and Walras' Law, the time path of prices is bounded under the price adjustment process (RP). □

Remark 8.19. As Fisher (1974) notes, this is the key result in establishing global stability of the Hahn process in the current context. Consequently, it is interesting to see where Rader's key assumption, that the money market is always in equilibrium, comes into play and to ensure the boundedness of prices at each stage in the adjustment process. It is also interesting to note that the assumption avoids the need to place special conditions on the production and utility functions in order to obtain boundedness of prices.

Lemma 8.4 (Fisher (1974; p. 483)). *If the time path of prices is bounded then the time paths of* v_j *and* y_j *are also bounded.*

Proof. Fisher (1974; pp. 483–484). □

The final group of assumptions needed to ensure stability of the Hahn process are familiar from the work on existence of equilibrium in Chap. 2. The first assumption involves households and firms having positive money holdings in all disequilibrium situations. As Fisher (1974; p. 484) notes, this is a very restrictive assumption but one that is indispensable in any attempt to ensure that the adjustment process is quasi-stable. The second assumption, needed to ensure global stability, is that the economy is irreducible.

Assumption 8.8. For all t for which the system is not in equilibrium, $\bar{m}_i > 0$ for each household i and $\bar{r}_j > 0$ for each firm j.

Definition 8.15 (Lyapounov function). A function $V(p)$ that is continuous in its arguments and such that $V[p(t|p(0)]$ converges mathematically for all admissible starting points $p(0)$ and is constant if and only if $p(0)$ is an equilibrium is called a *Lyapounov function.*

Definition 8.16 (Quasi-globally stable). An adjustment process, or more precisely the set of differential equations defined by it, is said to be *quasi-globally stable* if it has the property that it allows the existence of a *Lyapounov* function. Then for every starting price $p(0)$, the solution path of the process approaches arbitrarily close to the set of equilibria of the economy and every limit point of the path of prices, stocks and commitments is an equilibrium.

Theorem 8.9. *The price adjustment process as described in* (RP) *is quasi-globally stable.*

Proof. From Fisher (1974; p. 481), we know that provided money stocks are positive, the sum of household target utilities can be taken as a *Lyapounov* function provided this sum is bounded below. It follows from the boundedness of prices (see Lemma 8.2) that this is the case. We have therefore established that the price adjustment process in (RP) is quasi-globally stable. □

Remark 8.20. As Arrow and Hahn (1971; p. 274) note, what results of this sort of guarantee is that after a sufficiently long time has elapsed, prices will be arbitrarily close to some equilibrium if they are guided by a price adjustment process of type (RP) and in the sort of economy described here. This is a considerable amount of information. However, if we want to prove global stability, in the strong sense of convergence to an equilibrium then the conditions of the model need to be strengthened as follows:

Assumption 8.9 (Indecomposable). Let L be the set of commodities, including money, which have positive prices. Then at any equilibrium point and for every proper subset L' of L, there exists a pair of commodities $a \in L'$ and $b \in L, b \notin L'$, such that at least one of the following is true: (i) there exists an i with $x_{ia} > 0$ and $x_{ib} > 0$ or (ii) there exists a j with $v_{ja} \neq 0$ and $v_{jb} \neq 0$.

Remark 8.21. As Fisher (1974; p. 485) notes, Arrow and Hahn (1971; p. 345) fail to make this assumption and so that there is stability proof is not completely general. It is interesting to note here that this 'relationship condition', the nature of which was extensively discussed in Chap. 2, has made an appearance also in this stability argument.

Theorem 8.10. *Given the boundedness of prices and the quasi-stability of the adjustment process, if the economy is indecomposable then the price adjustment process RP is globally stable.*

Proof. The proof is inspired by an argument in Arrow and Hahn (1971; pp. 274–275). Suppose the economy has at least two equilibria, p^* and p^{**}, where p^* and p^{**} are limit points of the adjustment process. If these equilibria are isolated, then it is possible to find neighbourhoods $N(p^*)$ and $N(p^{**})$ of these prices such that $N(p^*) \cap N(p^{**}) = \varnothing$. For t arbitrarily large, $p(t)$ must lie in $N(p^*)$ for by definition there is a sequence of points on the path such that for t large enough, $p(t) \in N(p^*)$. Similarly for t' large enough, $p(t') \in N(p^{**})$. Then for some t^* such that $t < t^* < t', p(t^*)$ must be on the boundary of the closed neighbourhood of $N(p^*)$. Since the boundary of $N(p^*)$ is bounded, there is a sequence of points $p(t^*_\lambda)$ converging to some limit p^{***} which lies on the boundary of $N(p^*)$. But that contradicts the assumption that p^* is an isolated equilibrium and we have that if the set of limit points of a solution path contains more than one point, then these points cannot be isolated. The consequence of this is that if the equilibria of the economy are isolated and if the adjustment process RP is quasi-globally stable, then applying Arrow and Hahn (1971; p. 275, Corollary 4), RP is globally stable. □

Remark 8.22. In his evaluation of the result Rader (1996) remarks that: "The Hahn article as modified [here], permits the conclusion of a major part of research in economic theory, namely to show viability of general market equilibrium, as least as far as it concerns the convergence of an attractive version of the law of markets ... To be sure, we use B which is special, [and $d \gg 0$ and Π], but whether realistic or no the computation succeeds." Rader (1996; p. 118). Interesting as Rader's approach is, the argument above has its limitations, in particular the modification of the Hahn process on which it depends and which has been discussed at length by Kugawa and Kugo (1980) along with the assumption of one market always in equilibrium. His argument therefore probably constitutes what Hahn (1982) would regard as a 'suggestive example' rather than as a result which overturns the general negative thrust of the stability literature.

8.2.12. *Agent based price adjustment and experimental results on tatonnement*

Hahn (1982) observed that the really basic axiom in general equilibrium theory is that agents are attempting to improve their welfare. In that spirit Katzner (1999) notes that there is something methodologically odd about the 'auctioneer driven tatonnement story' for price adjustment in a general equilibrium context. The oddity is that there are no agents in the economy who are actually changing prices. Prices change in response to excess demand or supply pressures, not at the behest of any participant in the economy, but at the behest of a fictional auctioneer who is explicitly ruled out as an agent in the economy. Motivated by this situation, Katzner (1999) argues: "...it is reasonable and appropriate to ask about the possibility of introducing stories and adjustment rules that do not rely on the standard auctioneer or something similar [and instead are] 'agent price–adjustment stor[ies]." Katzner (1999; pp. 20–22). Given this motivation, Katzner (1999) provides the following analysis of agent price–adjustment rules. Let $p^i = (p_{1i}, \ldots, p_{\ell i}) > 0$ be the possible price announcements by agent i for $i = 1, \ldots, i$. The change in the announced price p^i varies directly with the difference between i's desired trades at p^i and those trades required in response to the desires of the remaining market participants at p^i. Agent i changes p^i directly with the market excess demand function $Z(p^i)$. Let $\theta_i \neq 0$ be a known constant for i. Then one collection of price adjustment rules is $dp^i/dt = \theta_i Z(p^i)$, which we will call *Katzner agent adjusted.* As Katzner (1999) points out, although this process has the same form as an auctioneer based tatonnement it "...has a different interpretation and significance." Katzner (1999; p. 28). The stability result that emerges from this work can be stated as follows:

Proposition 8.1 (Katzner (1999; Theorem 6)). *In an exchange economy in which* $(k.1)$ *all initial endowment are* *positive*; (k.2) *preferences can be represented by utility functions that are continuous, increasing, strictly quasi-concave and Cobb-Douglas*; (k.3) *the price adjustment process is of the Katzner agent adjusted form then starting at a collection of I price vectors p^i each one different from the unique equilibrium price p^*, then the individual behaviours of the I agents in the economy will eventually lead everyone to p^*. The adjustment process is therefore globally stable in this environment.*

Proof. Katzner (1999; pp. 16–18). □

Remark 8.23. As Katzner (1999; footnote 13) observes, the result continues to hold if production is allowed, provided that all production functions are also Cobb-Douglas. As far as the Cobb-Douglas restriction is concerned, he remarks that: "[o]f course, it would be better to have a story and associated price-adjustment rules... which were expressible with greater generality than the above specification of Cobb-Douglas utility functions for every agent [however these issues] have not yet been resolved." Katzner (1999; p. 29).

Gintis (2007) notes that although the general equilibrium model 'is the centrepiece of modern economic theory, progress in understanding its dynamical properties has been meagre'. In order to advance understanding, he provides an agent-based model of price adjustment in a Walrasian economy. To get an insight into how Gintis' model works, it is useful to follow his treatment and the modification of the Scarf (1960) example discussed earlier in this chapter. There are three goods x, y, z and one 'producer' of each good. The x-producer is endowed with $\omega_x = 10$ units of x, the y-producer is endowed with $\omega_y = 20$ units of y, the z-producer is endowed with $\omega_z = 400$ units of z. The parameter values are chosen, following Anderson, Plott, Shimomura and Granat (2004) to ensure equilibrium relative prices are very unequal. The x-producer consumes x and y in proportion $x/\omega_x = y/\omega_y$ and has utility function: $u_x(x, y, z) = \min(x/\omega_x, y/\omega_y)$, the y-producer consumes y and z in proportion $y/\omega_y = z/\omega_z$ and so has the utility function $u_y(x, y, z) = \min(y/\omega_y, z/\omega_z)$, the z-producer consumes x and z in the proportion $x/\omega_x = z/\omega_z$ and so has the utility function $u_z(x, y, z) = \min(x/\omega_x, z/\omega_z)$. Let z be the numeraire, then equilibrium prices are $p_x^* = \omega_z/\omega_x$, $p_y^* = \omega_z/\omega_y$ and $p_z^* = \omega_z/\omega_z$. Numerically $p^* = (40, 20, 1)$. Suppose a price tatonnement starts at disequilibrium prices $p_x = p_x^* + 3, p_y = p_y^* - 2$ and $p_z = p_z^* = 1$. In this process the auctioneer *publicly* announces the price vector, receives demands from the agents and updates the prices so that $p_x' = p_x + Z_x/100$ and $p_y' = p_y + Z_y/100$, where Z_g is the excess demand for good g. The process is repeated indefinitely and after 5200 iterations Gintis shows the picture in Fig. 8.1 emerges. This perfectly replicates the analytically derived non-convergence result in Scarf (1960).

Reflecting on this example and on the Walrasian model in general, Gintis argues that "[a] major attraction of the Walrasian economy is that the only information that an individual needs to have is his personal preferences and endowments, as well as the

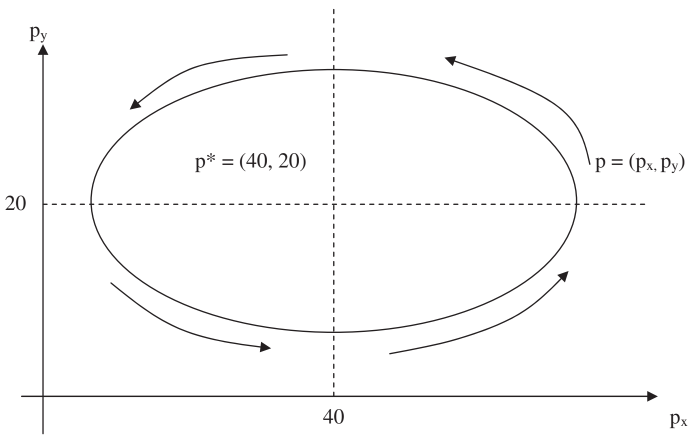

Fig. 8.1. Adjustment in initial Gintis experiment.

prices of all goods, and the only information a firm needs to have is its production function and the prices of all inputs and outputs [unfortunately] these assumptions are both too strong and too weak. They are too strong because the dynamic properties of the system are improved if we assume that the economic actors have no public information whatever but rather each agent has a private set of prices that he updates through experience. Similarly, for firms ..." Gintis (2007; pp. 1303, 1304). This observation leads him to modify the Walrasian underpinnings of the Scarf example as follows. Suppose there are now 1000 traders of each of the three types and each trader is given at the beginning of the price adjustment process a set of *private* prices randomly drawn from the uniform distribution on (0, 1). There are 2500 generations of traders and 10 periods per generation. At the start of each period, each agent's endowment is re-initialised to ω_g units of the good $g = x, y, z$ for which they are the producer and zero units of the other goods. Each agent takes turns at being a trade initiator and is randomly paired with a responder who can accept or decline the proposed trade. The responder will accept the proposal if they have some of the good desired by the initiator and if the value of what they get is greater than the value of what they give up according to their *private* prices. The initiator's selected trades and good ratio is also determined by their private prices. He shows that when the Scarf example is modified to have private prices that: "... in sharp contrast to the Scarf economy with public prices, convergence to a steady state is rapid and complete." Gintis (2007; p. 1286).

The other aspect of the Walrasian framework is that: "[t]he general equilibrium assumptions are too weak because they do not take into account that agents can learn from one another's successes and failures... agent-based models allowing traders, consumers, workers and firms to imitate successful others leads to an economy with a reasonable level of stability and efficiency"

Gintis (2007; pp. 1303–1304). Gintis takes these insights and builds an 'agent-based Walrasian economy' which incorporates the idea of private prices and imitation (see Gintis (2007; pp. 1287–1293 for a complete description). He then specifies the details of the 'agent-based algorithm' for trade, price setting and imitation (see Gintis (2007; pp. 1293–1294) for details).

In summarising the properties of his agent-based adjustment process, Gintis argues that: "... Franklin Fisher's assessment (Fisher, 1983) remains valid: we have no plausible analytical model of multi-sector dynamics with heterogeneous agents. The article presents the first general, highly decentralised, agent based model of the dynamics of general equilibrium [and finds] that a plausible dynamic exists in which prices and quantities converge to their market clearing values with a stochastic error term that exhibits moderately large excursions from zero, at irregular intervals." Gintis (2007; p. 1303).

Remark 8.24. Gintis (2007) acknowledges that there are many limitations to his particular model (no inter-industry trade, one financial asset, consumers with hybrid CES consumption functions and homogeneous labour for instance). He also argues that agent based modelling is not an alternative to analytic modelling but is rather a stimulus to formulating better analytical models, once 'an empirical investigation of complex agent-based systems has been undertaken in a controlled laboratory setting'.

8.3. Conclusion

This chapter has considered the part of general equilibrium theory that is concerned with the stability of price adjustment processes. This is an essential part of the program for without a convincing argument in favour of (global) stability general equilibrium theory is at some risk. The results considered here suggest that, in fact, it is difficult to tell a convincing story, which yields a universal, and globally stable adjustment process as the norm. This view also emerged from earlier work on the stability question (see for instance the surveys by Hahn (1982) and Ingrao and Israel (1990)). The key reason this appears to be the case is that if economically intuitive processes, such as an excess demand driven tatonnement is specified, then there is a large classes of excess demand maps for which it fails to converge to equilibrium. Alternatively, processes which are known to converge on general excess demand maps generally require an implausibly large amount of information in order to function. Such a situation is a sure stimulus for continuing research aimed at finding a palatable combination of structure on the excess demand map and informational requirements on the adjustment process which guarantee convergence to equilibrium.

Chapter 9

OPTIMALITY OF EQUILIBRIUM

"*The principal policy insight of economics... rests on the intimate connections between competitive equilibrium and Pareto efficiency.*"

F. Fisher

"*The First Fundamental Theorem of Welfare Economics provides a set of sufficient conditions for a price system to efficiently coordinate economic activity. It is a beautiful result with a surprisingly simple proof.*"

L. Makowski and J. Ostroy

"*The second welfare theorem has a central place in our understanding of the decentralisation properties of the market system ...*"

J. Campbell

"*[The second welfare theorem] says that under convexity assumptions (not required for the first welfare theorem), a planner can achieve any desired Pareto optimal allocation by appropriately redistributing wealth in a lump-sum fashion and then 'letting the market work'. Thus, the second welfare theorem provides a theoretical affirmation for the use of competitive markets in pursuing distributional objectives.*"

A. Mas-Colell, D. Whinston and J. Green

9.1. Introduction

Having found circumstances under which equilibrium states exist, are stable and finite in number, general equilibrium theory considers whether Walrasian equilibrium prices can be relied upon to decentralise optimal commodity allocations. In particular, is it generally true that equilibrium states are coherent, not only in the sense that markets clear, but also in the stronger sense that equilibrium states are 'the best possible states, according to some criterion'? The imposingly titled 'Fundamental Theorems of Welfare Economics', summarise what general equilibrium theory has to say about the optimality of Walrasian equilibrium. As a consequence, this chapter is devoted to a close examination of the two Welfare Theorems, particularly for the light they might shed on the role of markets in achieving economically and socially optimal outcomes. In particular, as well as providing a statement of the two welfare theorems under 'classical' conditions (which many books do), we will also explore extensions and generalisations of the theorems in an attempt to get a sense of their robustness. The chapter is organised as follows: Section 2 considers the First Welfare Theorem, Section 3 considers the Second Welfare Theorem, Section 4 offers some concluding remarks.

9.2. The first fundamental theorem of welfare economics

9.2.1. *Preliminaries and definitions*

Unless otherwise indicated, the terms 'allocation', feasible' and 'Pareto optimal' (or efficient) will be used in the following way. An *allocation* $(x, y) = (x_1, \ldots, x_I, y_1, \ldots, y_J)$ is a consumption vector $x_i \in X_i$ for each consumer i, and a production vector $y_j \in Y_j$ for each firm j. An allocation is *feasible* if $\sum_i x_i = \omega + \sum_j y_j$.

A feasible allocation (x^*, y^*) is *optimal in the sense of Pareto* (or *Pareto optimal*), if there is no other feasible allocation (x', y') such that for all consumers i, $x_i^* \preceq_i x_i'$ and $x_i^* \prec_i x_i'$ for at least one consumer.

9.2.2. *The First Welfare Theorem under 'classical' conditions*

The First Fundamental Theorem of Welfare Economics (FWT) provides a set of conditions under which Walrasian equilibrium prices decentralise Pareto optimal commodity allocations. The theorem is generally interpreted to mean that markets are efficient and should be permitted to operate unfettered in order to solve 'the economic problem'. Indeed, the FWT is often viewed as a formal statement of Adam Smith's invisible hand proposition that the pursuit of private interest, coordinated only by prices, will lead to a social optimum. In this vein, Geanakoplos (1987) argues that it expresses: "…the efficiency of the ideal market system" Geanakoplos (1987; p. 120), while Mas-Colell *et al.* (1995) remark: "…[since] any equilibrium allocation is a Pareto optimum, the only possible welfare justification for intervention in [a competitive] economy is the fulfilment of distributional objectives." Mas-Colell *et al.* (1995; p. 524). Note is also often made of the apparently short list of conditions in the theorem and the generality of circumstances under which it holds. Mas-Colell *et al.* (1995) for instance argue that: "A single, very weak assumption, the *local nonsatiation of preferences*…is all that is required for the result." Mas-Colell *et al.* (1995; p. 549) and consequently that it 'holds with great generality'. We are therefore motivated to get some perspective on the generality of the FWT. As a first observation in that direction, notice that in order for the theorem to be non-vacuous (as opposed to simply being true), Walrasian equilibrium states need to exist, and at least one of them needs to be stable relative to the adjustment processes at work in the economy. Unless both the existence and stability of equilibrium are guaranteed, then although the FWT might be true under appropriate conditions, the result would have much less impact as a guide to, and attitude about, economic policy. Making this point, Chichilnisky (1995) argues: "A necessary precondition for using the market solution is the existence of a competitive [Walrasian] equilibrium ... [note] that it would be possible to use lesser concepts of equilibrium, such as quasi-equilibrium and compensated equilibrium, or equilibria where there may be excess supply in the economy. These exist under quite general conditions, but fail to provide Pareto efficient allocations and are therefore less attractive from the point of view of resource allocation." Chichilnisky (1995; pp. 79–81). Similarly Ledyard (1987) in his discussion of the FWT lists as a cause of market failure 'the non-existence of Walrasian equilibrium' (see Ledyard (1987; p. 326)). As to the need for stability, we can do no better than recall the remark of Rader (1972b) who pointed out that 'equilibrium optimal but unattainable would be a will-o'-the wisp'. With these preliminary remarks in place, we now state and prove a 'classical' version of the FWT.

Theorem 9.1 (First Fundamental Theorem of Welfare Economics).[1] *Consider an economy* $\boldsymbol{E} = \{X_i, \preceq_i, \omega_i, Y_j, \theta_{ij}, \ell\}_{i=1}^{n}{}_{j=1}^{m}$ *in which* $(ft.1)$ *there is a complete set of markets*; $(ft.2)$ *all consumers and producers are price takers on all markets*; $(ft.3)$ *the preferences of all consumers are complete*, *continuous*, *reflexive*, *transitive and locally non-satiated*; $(ft.4)$ *the commodity space is* $\Re^{\ell}$, *then Walrasian equilibrium prices yield Pareto efficient commodity allocations.*

[1] This statement of the theorem is informed by the versions in Ledyard (1987; p. 326), Mas-Colell *et al.* (1995; pp. 308, 549) and Hammond (1998; p. 233).

Proof.[2] Let (x^*, y^*, p) be a Walrasian equilibrium for $\boldsymbol{E}$ then $x'_i \preceq_i x^*_i$ for $x'_i \in X_i$ such that $px'_i \leq px^*_i$, $py'_j \leq py^*_j$ for all $y'_j \in Y_j$ and $\sum_i x^*_i - \sum_j y^*_j - \sum_i \omega_i = 0$. Suppose (x^*_i, y^*_j) is not Pareto efficient then there exists a feasible allocation (x'_i, y'_j) such that $\forall i, x^*_i \preceq_i x'_i$ and $x^*_{io} \prec_{io} x'_{io}$ for some i_o. Then (i) $px^*_i \leq px'_i$ and (ii) $px^*_{io} < px'_{io}$. To see (i) suppose not, then $px'_i < px^*_i$ and $x^*_i \sim_i x'_i$ means $\exists \varepsilon > 0$ such that $p(x'_i + \varepsilon) < px^*_i$. By (ft.3) preferences are locally non-satiated. Therefore $x^*_i \prec_i (x'_i + \varepsilon)$ and x^*_i and $x'_i + \varepsilon$ are both available when x^*_i is chosen, which violates x^* being a Walrasian equilibrium consumption. To see (ii) suppose not, then $px'_{io} \leq px^*_{io}$ and $x^*_{io} \prec_{io} x'_{io}$ means x^*_{io} would not be a Walrasian consumption. Since $py^*_j \geq py'_j$, $\forall\, y'_j \in Y_j$ by profit maximisation $\sum_i px'_i > \sum_i px^*_i = \sum_j py^*_j + \sum_i p\omega_i \geq \sum_j py'_j + \sum_i p\omega_i = \sum_i px'_i$ [!]. This contradiction is arrived at by supposing that a Walrasian equilibrium does not yield with a Pareto optimum. □

Remark 9.1. It is often noted in the literature that if condition (ft.1) is dropped, so that markets are incomplete and 'externalities' of various sorts are present in the economy, or if (ft.2) is changed to allow non-competitive behaviour, then the FWT can fail. Quirk and Saposnik (1968; pp. 131–134) note that if (ft.3) is changed so that preferences are simply complete, transitive and reflexive or (ft.4) is changed to allow for indivisible commodities, then 'thick' indifference curves are possible and the theorem may again fail. Considering the centrality of the FWT in economic theory and its role in informing economic and social policy, it is important to explore directions in which the theorem can be extended relative to its classical form and also directions in which it fails.

[2]No claim of originality is made for this proof. It is provided to show that, contrary to assertions sometimes found in the literature, several features of the economy are exploited in proving the FWT.

9.2.3. *The FWT under some non-classical conditions*

9.2.3.1. *Alternative preference conditions*

Luenberger (1994a, 1994b) provides what he characterises as 'a slight extension of the FWT' by modifying (ft.3) in Theorem 9.1 as follows. If $p \in \Re^\ell$ is a price vector and m_i is an income of i, then i's preference relation $\preceq_i$ on X_i is *income regular for* (p, m_i) if for any $m_i < m_i'$ there is an $x_i' \in B_i(p, m_i')$ with $x_i \prec_i x_i'$ for all $x_i \in B_i(p, m_i)$. If all preference relations are income regular and if the Walrasian equilibrium is not at a bliss point, then the FWT continues to hold. Also operating on condition (ft.3), Gale and Mas-Colell (1977) and Fon and Otani (1979) show that something like the FWT survives, even if preferences are incomplete and intransitive. However as Weymark (1985) notes, Gale and Mas-Colell (1977) define a feasible allocation as Pareto optimal if there is no other feasible allocation which *all* consumers prefer in order to get their result. Fon and Otani (1979) characterise a feasible allocation as Pareto optimal if there is no other feasible allocation which a subset of consumers strictly prefer and which leaves the remaining consumers with the consumption bundles of the original allocation. Consequently these results are not strictly a generalisation of the FWT because of the somewhat different notion of optimality employed. Weymark (1985) achieves a generalisation of the FWT, under the standard notion of Pareto optimality. He assumes that for each i, $\preceq_i$ is a binary relation on X_i that induces a *strict preference* relation $\prec_i$ defined by $[x_i \prec_i x_i' \Leftrightarrow x_i \preceq_i x_i'$ and it is not the case that $x_i' \preceq_i x_i]$, an *indifference relation* $\sim_i$ defined by $[x_i \sim_i x_i' \Leftrightarrow x_i \preceq_i x_i'$ and $x_i' \preceq_i x_i]$ and a *noncomparability relation* $\not\preceq_i$ defined by $[x_i \not\preceq_i x_i' \Leftrightarrow$ not $(x_i \preceq_i x_i')$ and not $(x_i' \preceq_i x_i)]$. The preference relation so defined is *strongly convex* if for $x_i, x_i' \in X_i$ with $x_i \sim_i x_i'$ then $x_i \prec_i [\gamma x_i' + (1 - \gamma) x_i]$ with $0 < \ell < 1$. With these definitions the following result becomes available.

Theorem 9.2 (Weymark (1985)). *If in* $\boldsymbol{E} = \{X_i, \preceq_i, \omega_i, \ell\}_{i=1}^{n}$ (*wy*.1) *all* X_i *are convex sets*, (*wy*.2) *each* $\preceq_i$ *is reflexive and strongly convex but not necessarily complete or transitive, then a Walrasian equilibrium is Pareto optimal.*

Proof. Weymark (1985; p. 158). □

Remark 9.2. Qizilbash (2005) also obtains an extension of the FWT when preferences are not complete, but $\preceq_i$ is a reflexive and transitive *quasi-ordering* which is also *PI-transitive* so that $\forall x, y, z \in X_i$ then $(y \prec_i x \wedge y \sim_i z) \Rightarrow z \prec_i x$. Commenting on this result, he argues that "... [v]iolations of the axioms of microeconomic theory which may arise from incommensurability do not undermine the central insight of welfare economics [i.e.] [t]here remains a case for not interfering with the Walrasian equilibrium..." Qizilbash (2005; p. 672).

Another interesting variation on (ft.3) involves allowing interdependent and altruistic preferences[3] among agents in the economy. In pioneering work, Bergstrom (1971) showed that if no agent is altruistic to more than one other agent, the FWT continues to hold. Yi (1987) generalises this by showing that if agents can be ordered in such a way that each agent is altruistic only towards subsequent agents in the ordering, then the FWT continues to hold. Kranich (1988) demonstrates that something like Yi's condition cannot be dispensed with by proving that in a Walrasian model with voluntary transfers among altruistic agents, equilibrium is generally not Pareto efficient under otherwise standard conditions on the economy. Kranich (1988) takes a pure exchange economy where agents are indexed by i and j, the set and number of agents is denoted by I, number and index of commodities is ℓ and the set is L. Agent i is characterised in

[3]This generalisation may not merely be of academic interest, because according to the *Statistical Abstract of the United States* individual charitable donations in 2007 were around US$306.4 billion.

the usual way (consumption set, preferences and endowment), but in addition i has a *domain of transfers* $T_i \subseteq \Re_+^{(I-1)I}$, where $t_{ij} \in \Re^I$ is the *transfer from* i *to* j. Then $x_i \in X_i$ is i's own consumption and $\theta \in \Re_+^I$ a wealth vector across the economy. Agent i is *altruistic towards* j if $\partial u_i(x_i, \theta)/\partial\theta_j \geq 0$ for all $(x_i, \theta) \in (X_i \times \Re_+^I)$ and *self-biased* if $\forall j \in I, \theta_i = \theta_j$ then $\partial u_i(x_i, \theta)/\partial\theta_i > \partial u_i(x_i, \theta)/\partial\theta_j$.

Theorem 9.3 (Kranich (1988, Theorem 4.2)). *There is an* $\mathsf{E} = \{X_i, T_i, \omega_i, \preceq_i, \ell\}_{i\in I,\ell\in L}$ *in which* $(k.1)$ *all* $X_i \subseteq \Re_+^\ell$ *are compact, convex and* $0 \in X_i$; $(k.2)$ *all* T_i *are compact, convex and* $0 \in T_i$; $(k.3)$ *all* $\preceq_i$ *can be represented by a twice continuously differentiable utility function* u_i *which is quasi-concave, non-satiated and defined for* $(x_i, \theta) \in X_i \times \Re_+^I$; $(k.4)$ *i is altruistic towards all agents* $j \in I$ *and self-biased and in which Walrasian equilibrium is not Pareto optimal.*

Proof. Kranich (1988; pp. 379–380). □

Remark 9.3. Kranich's proof is constructive proof in that he produces an economy which has the claimed properties (see Kranich 1988; pp. 379–380). In commenting on the result, he points out that failure of the FWT is not due to some bizarre feature of the example but that: "…the first welfare theorem fails to hold in the present context. Furthermore, the Cobb-Douglas economy used in the example is extremely well behaved." Kranich (1988; p. 380). Reflecting on his results overall, Kranich (1988) interprets them to mean that: "…when agents wish to effect an equitable distribution of wealth, the Walrasian mechanism is ill suited for allocating resources". Kranich (1988; p. 369). In subsequent work, Kranich (1994) shows that in a one commodity, three agent *gift-economy* in which agents have preferences that are *anonymous* (meaning that $\forall x \in \Re_+^3$

and $\forall j \neq k \neq i$, $u^i(x) = u^i(\pi_{jk}(x))$, where $\pi_{jk}(x)$ is the allocation obtained by transposing the jth and kth coordinates of x) and *self-biased* (which in this context means $\forall x \in \Re^3_+$ and $\forall j \neq i, x_j = x_i \Rightarrow (\partial u^i/\partial x_i)(x) > (\partial u^i/\partial x_j)(x)$), Walrasian equilibrium is generally not Pareto optimal.

9.2.3.2. *Indivisible commodities*

As Quirk and Saposnik (1968) note and as condition (ft.4) in Theorem 9.1 makes clear, the classical version of the FWT assumes that all commodities are divisible. Van der Laan, Talman and Yang (2002) establish the FWT when this condition is to some extent relaxed and indivisible commodities are allowed. They consider a finite economy with n consumers denoted by i, $\ell + 1$ divisible commodities (commodity ℓ_0 being thought of as 'labour' or 'capital') and m indivisible commodities denoted by 'c' (for chunky). Each agent is endowed with one indivisible commodity, and $\omega_{i_0} > 0$ of commodity 0. Only divisible goods are producible, $A = [a^1_, a^2, \ldots, a^\ell]$ is the $(\ell + 1) \times \ell$ input-output matrix which describes the linear production technologies in the economy. A_0 is the first row of A and $\hat{A}$ is A with the first row deleted. Let $I_k = \{1, \ldots, k\}$ be the set of the first k positive integers and for agent $i \in I_k$ let $e(i)$ denote the ith k-dimensional unit vector, while 0_k and 1_k are k-dimensional zero and unit vectors, respectively. $E_k = \{0_k, e(1), e(2), \ldots, e(k)\}$ the set of k-dimensional unit vectors and the k-dimensional vector of zeros. The definitions of feasible allocations, Walrasian (or competitive) equilibrium and Pareto efficiency are standard, once allowance is made for divisible goods (see van der Laan, Talman and Yang (2002; Definitions 2.1, 4.1 and 4.2)).

Theorem 9.4 (van der Laan, Talman and Yang (2002)). *Let the economy* $\boldsymbol{E} = \{X_i, \preceq_i, \omega_i, e(i), A\}_{i=1}^n$ *be such that* (*lty*.1)

$X_i = \Re_+^{l+1} \times \boldsymbol{E}_n$; ($lty.2$) $\preceq_i$ can be represented by a utility function $u_i : \Re_+^{l+1} \times \boldsymbol{E}_n \to \Re$ under which i derives utility from at most one indivisible commodity; ($lty.3$) endowments are such that each i owns a non-zero amount of one of the divisible goods along with one indivisible commodity; ($lty.4$) A_0 is a strictly negative row vector (so labour or capital is needed in any production process), $\hat{A}$ is regular (e.g. Leontief) and $\hat{A}^{-1}$ is nonnegative; ($lty.5$) for all i, u_i is strongly monotonic in commodity 0 and weakly monotonic in the indivisible goods and if (x^*, c^*) is a Walrasian allocation and p^* the equilibrium prices of the divisible goods then (x^*, c^*) is Pareto efficient.

Proof. van der Laan, Talman and Yang (2002; p. 425). □

Remark 9.4. Extending the FWT to cover this case is an important undertaking because as van der Laan, Talman and Yang (2002) note, many commodities traded in real markets are indivisible. It should be noted however that, at this stage, the extension is only possible with the help of some reasonably artificial assumptions about preferences (and also endowments) over indivisible goods — see in particular (lty.2) in Theorem 9.4.

9.2.3.3. *Missing markets, externalities and public goods*

Consideration of the FWT when condition (ft.1) in Theorem 9.1 is relaxed is important because many economies display varying degrees of market incompleteness. As Varian (1984) notes, the situation as far as the FWT is concerned is quite different in for instance, a temporary equilibrium set-up compared where expectations play a role, compared with the situation in a complete market economy. In particular, he argues that: "Unfortunately, there is no reason to believe that temporary equilibria are efficient. The equilibria depend very much on the expectations of agents [and unless 'rational expectations are assumed] the situation will undoubtedly reveal intertemporal inefficiencies."

Varian (1984; p. 234). Looked at from another angle, Mas-Colell *et al.* (1995; pp. 358–359) note another source of market incompleteness is the presence of externalities. They argue that 'externalities are inherently tied to the absence of certain markets', an observation originally due to Meade (1952) and considerably elaborated by Arrow (1969). Since public goods are goods which generate widely felt externalities (in the limit for a 'pure' public good the entire economy), these cases are the leading cases which motivate the consideration of the robustness of the FWT when markets are incomplete. Classical results due to Arrow (1970) and numerous results from game theory (see Conley and Smith (2005) for some examples), give many circumstances in which the FWT fails and equilibria are not efficient. The FWT can be re-established however if the path pioneered by Arrow (1970) is followed and the market incompleteness is corrected by attaching to the economy *Arrovian commodities*, which is the proxy for externalities, along with competitive markets in which they are traded. Unfortunately, as Starrett (1972) showed this is not an entirely satisfactory solution because Arrovian commodities introduce 'fundamental non-convexities' in production sets and put at risk the existence of equilibrium (along with the second welfare theorem). Conley and Smith (2005) therefore construct a 'externality rights model' in which the FWT holds. Their model can be summarised as follows. There are I individuals and J firms. There are ℓ^c *private commodities* 'c', ℓ^g *public goods* 'g' and ℓ^r *public externality rights* 'r'. The number of commodities and goods is $\ell^c + \ell^g + 2\ell^r \equiv \ell$. The reason why '$2\ell^r$' appears here is that i's consumption bundle is $(x_i^c, x_i^G, x_i^r, x_i^R) \equiv x_i$, where $x_i^c \in \Re^{\ell c}$ is a bundle of private commodities, $x_i^G \in \Re^{\ell g}$ is a bundle of public goods, $x_i^r \in \Re^{\ell r}$ is a bundle of *privately used* externality rights and $x_i^R \in \Re^{\ell r}$ is a bundle of *publicly held* externality rights. As Conley and Smith (2005; p. 690) point out, the last two components of x_i 'represent two different uses of the same commodity: externality rights'. They have a nice example of this. Suppose each i derives a benefit from burning leaves in their yard (x_i^r), but are also harmed by the smoke generated by other people exercising the same right to burn leaves in their yard. If x_i^R is thought of as the net level of publicly held externality rights which are *not used*, it may be thought of as the total level of pollution abatement undertaken and is therefore a kind of public good. Each consumer has a preference relation $\preceq_i$ defined over $X_i \subset \Re^{\ell}$, is endowed with private commodities and externality rights so $\omega_i = (\omega_i^c, 0, \omega_i^r, 0)$ which corresponds with an aggregate endowment vector $\omega = \sum_i \omega_i$. Each firm j is characterised by a production set $Y_j \subset \Re^{\ell + \ell g}$ and a typical production plan $y_j = (y_j^c, y_j^g, y_j^G, y_j^r, y_j^R)$, where y_j^c is the net output bundle of private commodities for j, y_j^g is the gross output of privately produced public goods by j, y_j^G is the input bundle of public goods, y_j^r is the input bundle of privately used externality rights and y_j^R is an input bundle equal to the total level of externality rights consumed across all agents in the economy. Firms can be benefited by using externality rights y_j^r, and harmed by the total rights collectively used in the economy, y_j^R. Aggregate production possibilities are affected by firms' use of public commodities as inputs and also by the externalities produced by consumers. So the *global production set*, defined relative to $x^r = \sum_i x_i^r$, is not generally the sum of all the Y_j's but is a correspondence denoted by $Y(x^r) : \Re^{\ell + \ell g} \to \Re^{\ell + \ell g}$. An *allocation* for this economy is a list $a = (x_1, \ldots, x_I, y_1, \ldots, y_J)$ and it is a *feasible allocation* if it satisfies standard adding up conditions (modified to suit the current context — see Conley and Smith (2005; p. 692)), while *Pareto efficient* allocations are defined in the standard way. The *price space* in this economy is $\{p \in \Re^L \backslash \{0\}\}$, where $L \equiv \ell^c + (I+J)\ell^g + (I+J)\ell^r$. The idea here is that the first ℓ^c components of p are the private commodity prices and these are common to all consumers; the next $I\ell^g$ components are consumer personalised prices for public goods; the next $J\ell^g$ components are producer personalised prices for public goods; the next $I\ell^r$ components are consumer personalised prices for abatement and the last $J\ell^r$ components are producer personalised prices for abatement (see Conley and Smith (2005; p. 693 for further discussion)). Defining the profit for firm j, the profit shares θ_{ij} for consumers i from j and budget sets in the usual way (again modified to fit the current contex — see Conley and Smith (2005; p. 694)), a Walrasian equilibrium can be defined as a feasible allocation that is utility maximising and budget consistent for all consumers and profit maximising for all firms. The following extension of the FWT then becomes available.

Theorem 9.5 (Conley and Smith (2005)). *If* $\boldsymbol{E} = \{\{X_i, \preceq_i, \omega_i, Y_j, \ell\}_{i=1}^{n}{}_{j=1}^{m}$ *is an economy with private commodities, public goods and externality rights of the type described above and if* (*cs*.1) *for all* i $X_i \subset \Re^\ell$, *where* $\ell = \ell^c + \ell^g + 2\ell^r$; (*cs*.2) *for all* i $\preceq_i$ *is defined on* X_i *and is complete, continuous, transitive, weakly convex and locally non-satiated*; (*cs*.3) *for all* j Y_j *is a non-empty, closed and convex set*; (*cs*.4) *the global production set relative to* x^r *is an upper hemi-continuous correspondence then if a pair* (a^*, p^*) *with* a^* *a feasible allocation and* $p^* \in \Re^L \backslash \{0\}, L \equiv \ell^c + (I+J)\ell^g + (I+J)\ell^r$ *together constitute a Walrasian equilibrium then* a^* *is Pareto efficient.*

Proof. Conley and Smith (2005; p. 695). □

Remark 9.5. Conley and Smith (2005) note that the environment studied is interesting because public goods are allowed to enter firms production sets; firms and consumers are influenced by externalities and consumers as well as firms can produce externalities. That a version of the FWT holds in this context is particularly interesting because as the authors observe: "...these seem like natural features for an economy with externalities to possess ..." Conley and Smith (2005; p. 688). As they also point out 'there doesn't seem to be a general equilibrium treatment of such an economy elsewhere in the literature.'

Remark 9.6. One of the interesting features of the Conley and Smith (2005) model is that not all prices are the same for all agents. Hervés-Belosos *et al.* (2005) also consider a situation in which agents have a different experience of the economic environment, but in their case, the difference is due to differential information. They show however, that under standard conditions on a differential information exchange economy and, as a consequence of their Theorem 4.1, a Walrasian allocation is also Pareto efficient. Therefore the FWT holds in such a context (see Hervés-Belosos *et al.* (2005; Remark 4).

9.2.3.4. *Budget constraints that hold with equality*

Cass (2008) begins with the observation that, as a consequence of what he calls 'the competitive hypothesis', the consumer must solve a preference optimization problem subject to a budget constraint that holds with *equality.* This is different to the set-up in much of the literature (e.g. Debreu (1959; p. 62), Arrow and Hahn (1971; p. 79), Mas-Colell *et al.* (1995; p. 50) and McKenzie (2002; p. 3)), where the consumers problem is stated as $\text{Max}_{x\geq 0}\, U(x)$ subject to $p^T x \leq m$. The apparently dramatic consequence of replacing '$\leq$' with '$=$' in this problem is that the FWT may fail to hold. An example which Cass (2008) produces to make this point is the following. Consider a two-by-two exchange economy with x_i is the consumption vector for i and x_{ik} is the amount of good $k = 1, 2$ that $i = 1, 2$, consumes. Let 1's preferences be $U_1(x_1) = -x_{11}$, endowment be $\omega_1 \gg (1, 1)$ and consumption set be $X_1 = \{x_1 \in \Re^2 : x_{12} \geq 1/x_{11}$ and $x_{11} > 0\}$. For person 2, $U_2(x_2) = x_{21}, \omega_2 \gg (0, 0)$ and $X_2 = \Re^2_+$. This economy, its equilibria and optima can be pictured as in Fig. 9.1.

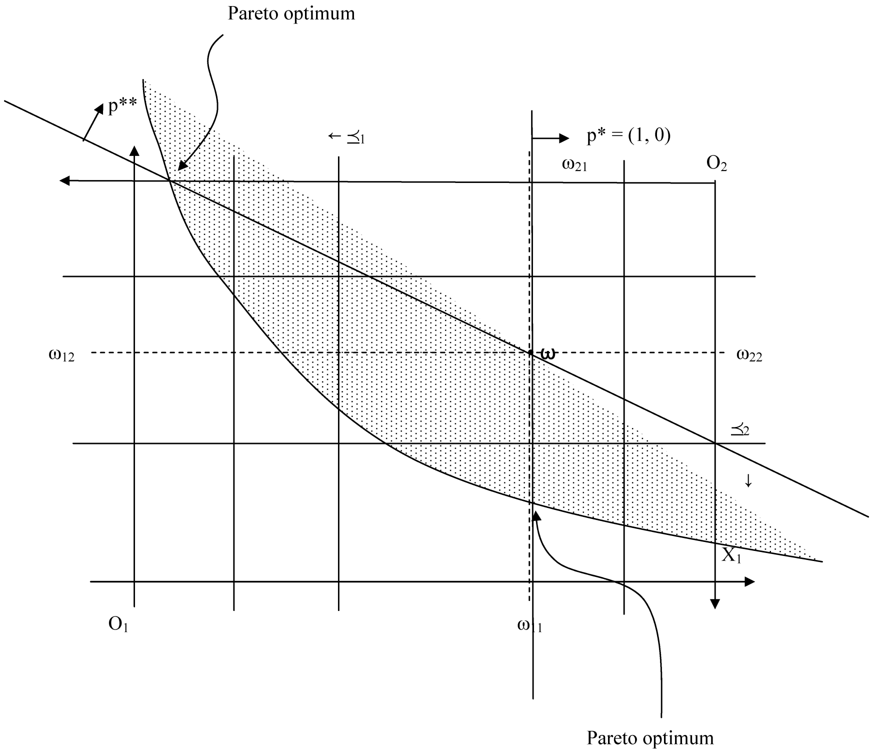

Fig. 9.1. The Cass example.

In this example, there is a continuum of allocations associated with the equilibrium $p^* = (1, 0)$, but only one of them is Pareto optimal. Since there are allocations associated with equilibrium prices (1, 0) that are not Pareto optimal, the FWT fails in this context. Note, the other Pareto optimum for this economy is the allocation in the upper left corner and this is decentralized by p^{**}.

9.2.4. *Summary on the FWT*

The FWT is a remarkable result which establishes that, under certain circumstances, Walrasian equilibrium prices decentralise

Pareto optimal allocations of goods. Because of it's importance, it is of interest to explore directions in which it can be extended and directions in which it fails. The work we have done here shows that the theorem can be extended to cover situations where preferences have weak structure, goods are indivisible, where some markets are missing and to situations where agents have differential information. The result fails to extend in certain other directions, notably to economies where there are certain forms of altruism and where budget constraints hold with equality. All of this needs to be borne in mind when one encounters claims in the literature that the result holds 'with great generality and under minimal conditions.'

Interesting as it is, the FWT has a well known limitation. It is entirely concerned with the efficiency of equilibrium allocations and has nothing to say about the distributional justice of those allocations. We therefore now consider a result that Debreu (1959; p. 95) characterises as being 'much deeper' than the FWT, namely the Second Fundamental Theorem of Welfare Economics (SWT). This theorem is motivated by the observation that even if equilibrium allocations are Pareto efficient, the resulting allocations may be ethically objectionable (see for instance Coles and Hammond (1995) and Lengwiler (1998) for circumstances in which Pareto efficient allocations involve starvation by some agents). Since, as Feldman (1987) notes, there is nothing in the FWT that ensures any reasonable distributional outcome, this has lead some to advocate the abandonment of markets and their replacement by central planning. Others have argued that in order to achieve distributional objectives, market prices should be manipulated using taxes and subsidies, price controls and the like.

An alternative approach argues that distributional concerns can be addressed without dispensing with, or even over-riding, markets. As Mas-Colell *et al.* (1995) point out, the result in economic theory which is appealed to in order to support such a position is the SWT and that: "...the second welfare theorem provides a theoretical affirmation for the use of competitive markets in pursuing distributional objectives." Mas-Colell *et al.* (1995; p. 524). It is therefore of considerable interest for us to study that result in some detail.

9.3. The second fundamental theorem of welfare economics

9.3.1. *The SWT under 'classical' conditions*

In order to obtain a statement of the SWT under 'classical' conditions, we begin with the following result in Debreu (1959).

Lemma 9.1 (Debreu (1959; p. 95)). *Let (x^*, y^*) be a Pareto optimal allocation in a complete market Arrow-Debreu economy $\boldsymbol{E} = \{X_i, \preceq_i, \omega_i, Y_j, \theta_{ij}, \ell\}_{i=1}^{n}{}_{j=1}^{m}$. If $\boldsymbol{E}$ is such that:* $(d.1)$ *$\forall i, X_i$ is convex*; $(d.2)$ *$\forall i, \preceq_i$ are complete*, *continuous*, *transitive*, *reflexive and convex*; $(d.3)$ *$\exists i$ for which $\preceq_i$ is non-satiated*; $(d.4)$ *the set $\sum_j Y_j$ is convex*, *then there is a price system $p^* \neq 0$ such that $(\alpha) x_i^*$ minimises $p^* x_i$ on consumptions $\{x_i \in X_i : x_i \succeq_i x_i^*\}$ for every i, and $(\beta) y_j^*$ maximises $p^* y_j$ on Y_j for every j, so there is a Walrasian quasi-equilibrium which supports the Pareto allocation (x^*, y^*).*

Proof. Debreu (1959; p. 96). □

Remark 9.7. This result is not the statement that optima can be supported by Walrasian equilibria since what has been shown is that optima can be supported by quasi-equilibria. However, as Debreu (1959; p. 96) points out: "[i]f the exceptional case where $p^T x^*$ is the smallest expenditure relative to p in the consumption set X_i does not occur, then (α) of [Lemma 9.1] implies $((x_i^*), (y_j^*))$ is indeed an equilibrium relative to $p^{[*]}$."

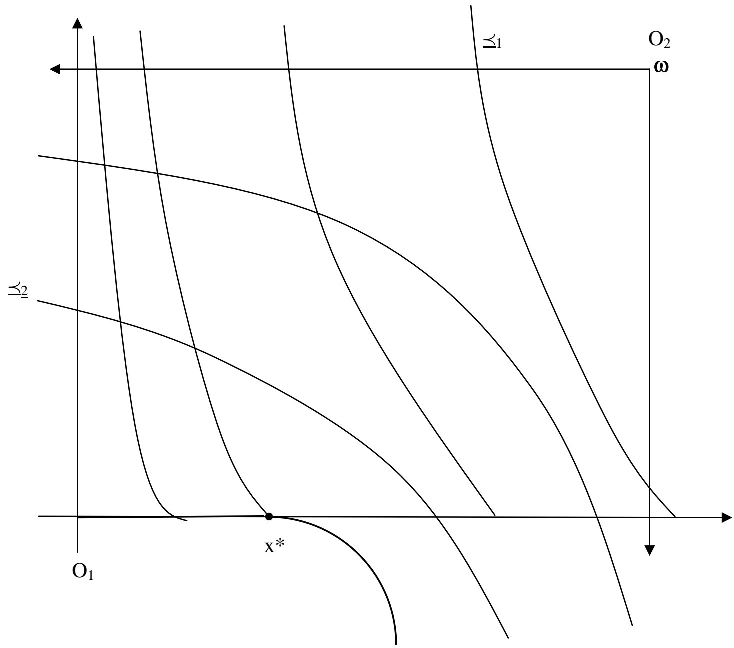

Fig. 9.2. Mas-Colell's example.

The reason why this is the case is made clear in the following example due to Mas-Colell (1985).

In Fig. 9.2, the Pareto optimum x^* can be supported by a quasi-equilibrium but not as an equilibrium (a similar example is provided by Mas-Colell *et al.* (1995; p. 555). A standard way to solve this problem is to make a 'cheaper point' assumption (recall the discussion of this condition in Chap. 2) — see for instance Mas-Colell *et al.* (1995; p. 555). A neat way to do this, and in the process to establish the SWT, is provided by Hammond (1998). Hammond's approach depends on the idea of *nonoligarchic allocations* (also discussed in Chap. 2), along with the idea of 'relevant commodities'. Let $V \equiv \sum_j Y_j - \sum_i X_i$ which can be thought of as 'the set of net export vectors which the economy could provide to the rest of the world' and assume

$0 \in \text{int}(V)$. As Hammond (1998; p. 237) points out, this interiority condition means that for each good g and the corresponding unit vector $e_g = (0, 0, \ldots, 1_g, 0, \ldots, 0)$, which has one unit of good g and zero of all other goods, there exits a small enough $\varepsilon > 0$ such that both εe_g and $-\varepsilon e_g$ are in V. As Hammond further notes, this means that 'the economy is capable of absorbing a positive net import or capable of supplying a positive net export of each good'. Hammond (1998) calls this the condition that *all goods are relevant*. He also says a feasible allocation (x^*, y^*) is *weakly Pareto efficient* if there is no feasible allocation (x, y) such that for all $i \in I, x_i^* \prec_i x_i$. With these ideas we have the following:

Theorem 9.6 (SWT, Hammond (1998, Proposition 7)). *Let (x^*, y^*) be a weakly Pareto optimal allocation in an economy $\mathsf{E} = \{X_i, \preceq_i, \omega_i, Y_j, \theta_{ij}, \ell\}_{i=1}^{n}{}_{j=1}^{m}$ in which $(h.1)$ all commodities are relevant; $(h.2)$ all $\preceq_i$ are complete, continuous, reflexive, transitive, convex and locally non-satiated; $(h.3)$ $\sum_j Y_j$ is convex; $(h.4)$ (x^*, y^*) is a nonoligarchic allocation; $(h.5)$ markets are complete, then there exists a $p^* \neq 0$ such that (p^*, x^*, y^*) is supported as a Walrasian equilibrium for E.*

Proof. Hammond (1998; p. 240). □

Remark 9.8. Hammond's approach is a very neat way to avoid the problems caused by Arrow's celebrated 'exceptional case'. Also, as Hammond (1998; p. 240) points out, the conclusion of the theorem can actually be strengthened by noting that under the conditions of the theorem (in particular non-satiation), any weakly Pareto efficient allocation is actually fully Pareto efficient.

Remark 9.9. The SWT can be viewed as an existence theorem for a price vector which supports a particular Pareto optimal allocation of commodities. It is important to notice that the

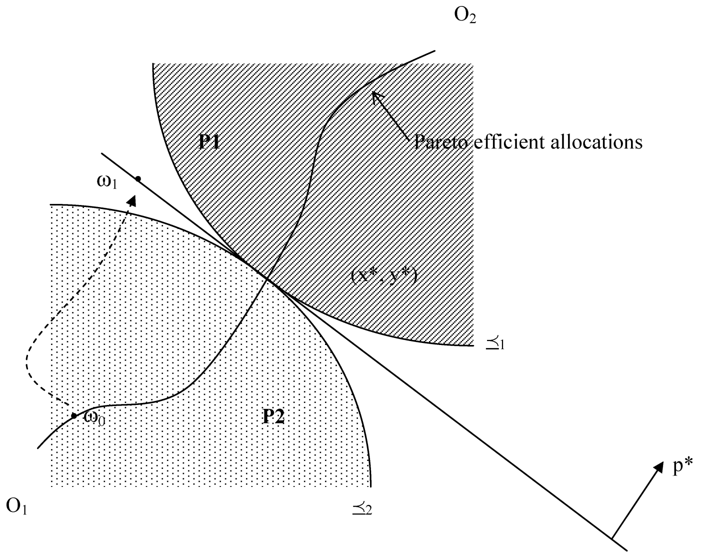

Fig. 9.3. A Pareto optimum supported by redistribution and Walrasian prices.

theorem does not claim that the supporting equilibrium price vector is necessarily an equilibrium for the current economy, and in particular, for the current initial endowments. For instance, if the current distribution of endowments is at ω_0 in Fig. 9.3, then the current economy does not have a Walrasian equilibrium which supports (x^*, y^*). However, if the wealth of individuals were those associated with the endowment ω_1 in Fig. 9.3 then there is a supporting Walrasian equilibrium for (x^*, y^*).

In light of this, the SWT is therefore sometimes written in such a way as to make explicit the possible need of an initial redistribution of wealth. Such a formulation of the theorem can be found in Mas-Colell *et al.* (1995; p. 308). The other thing which Fig. 9.3 indicates is that the SWT typically depends on the mathematics of *separating hyperplane theorems*. Consider the upper contour sets P1 and P2 for agents 1 and 2 in Fig. 9.3. Being able to find a supporting equilibrium for the Pareto allocation (x^*, y^*) comes down to being able to (weakly) separate the two sets P1 and P2 with a non-degenerate hyperplane. Sufficient conditions for being able to do that involve int(P1) $\cap$ (P2) $= \varnothing$ and P1, P2 being convex sets. A typical separating hyperplane result may be stated as follows (see Mas-Colell *et al* (1995; 947, 948) for further discussion): *Take* $\alpha \in \Re$ *and* $p \in \Re^{\ell}\backslash\{0\}$ then the hyperplane $H(p, \alpha)$ generated by p and α is $H(p, \alpha) = \{x \in \Re^{\ell} : p^T x = \alpha\}$. *If* $X, Y \subset \Re^{\ell}$ *are two convex sets that have disjoint interiors then* $\exists\ p \in \Re^{\ell}\backslash\{0\}$ *such that* $p^T x \leq \alpha$, *and* $p^T y \geq \alpha$, $\forall x \in X$ *and* $\forall y \in Y$. Similar arguments are used in production economies and help explain why convexity conditions, on preferences and production sets, appear in classical statements of the SWT but not in the FWT.

Given the significance of the SWT in economic theory and its influence in conditioning thought about appropriate economic policy, it is important to know something about the directions in which it can be generalised relative to the conditions in Lemma 9.1 and Theorem 9.6. It is also important to have an appreciation of some directions in which it fails to generalise.

9.3.2. *Some generalisations, extensions and limitations of the SWT*

9.3.2.1. *Interdependent preferences*

Winter (1969) shows that in the absence of gifts, the SWT holds in an economy with a benevolent non-participatory agent. Archibald and Donaldson (1976) replace Winter's non-malevolent preferences assumption with a much weaker non-participation assumption without losing the conclusion of the theorem (although they need to introduce a strong preference separability condition to achieve that end). Goldman (1978) showed that the SWT fails for a 'gift economy'. Rader (1980) shows that the Archibald and Donaldson approach can be relaxed and preferences can be allowed to display general interdependence without losing the SWT, provided that 'the utility of any one individual depends only on the utility of others and not on the form of others' consumption'. In the context of the model of 'general altruism' discussed in the previous section, Kranich (1988) shows that the SWT fails. A result similar to that established by Kranich (1994) shows that in Goldman's model if symmetry holds (i.e. the identity of agents is not known, so that people are anonymous) and there is a slight bias on the part of individuals towards their own consumption, then the SWT is restored in models of the Goldman (1978) type. Ythier (2000) provides a general result for models of this type. He supposes individuals have utility functions which are non-paternalistically interdependent and have three options for the use of commodities: private consumption, individual gift or exchange on competitive markets. The *consumption* of i is $x_i \in \Re^\ell$ and $x = (x_1, x_2, \ldots, x_n)$ is the economy wide vector of individual consumptions; a *gift* from k to i is denoted by $t_{ki} \in \Re^\ell$ with the *net gift* accruing to i being $\sum_{k \in I} (t_{ki} - t_{ik}) \equiv \Delta_i t$. The economy wide gift vector is $t = (t_1, t_2, \ldots, t_n)$, the *net trade* of i is $z_i \in \Re^\ell$, where $z_{i\ell}$ is the difference between purchases and sales of good ℓ by i and $z = (z_1, z_2, \ldots, z_n)$ is the economy wide vector of individual net trades. From these definitions, $x_i = z_i + \omega_i + \Delta_i t$ for each i, where ω_i is the *endowment* of i, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ and $\sum_{i \in I} \omega_i = e$. A *social state* is a vector (x, t, z) and if a pair (z_i, t_i) is called an *action* for i and denoted by a_i and $a = (a_1, a_2, \ldots, a_n)$ is the economy wide vector of actions then the *social state determined by* a is $(x(a), t(a), z(a))$. To capture preference interdependence, each i has two 'preference relations', both of which are assumed to be representable by real valued functions. The first is an *ophelimity function* which describes i's preferences over their own consumption and is $u_i : \Re^\ell \to \Re$ which generates an *ophelimity profile* across the economy $(u_1(x_1), u_2(x_2), \ldots, u_n(x_n))$ represented by a map $u : \Re^{\ell n} \to \Re^n$. The second is a *utility function* w_i which maps the set of ophelimity profiles $u(\Re^{\ell n})$ in $\Re$. The composition of w_i with u, $w_i \circ u$ describes i's preferences over economy wide allocations. A *social system* is a list $(w_1 \circ u, w_2 \circ u, \ldots, w_n \circ u)$ denoted by w while a *social system of private property* is a pair (w, ω). An *equilibrium* of (w, ω) is a price-action pair (p^*, a^*) in which all markets clear and all agents budget consistent choices are such that they are satisfied with their choices of x_i^* and t_i^* given prices p^* and the choices of others. An *optimum* of a social system w is a feasible allocation x such that there is no other feasible allocation x' such that $u(x') \geq u(x)$ and $u(x') \neq u(x)$, i.e. it is a Pareto optimum with respect to the ophelimity functions of its members and the set of optima is O. An allocation x^* is $(i,\ k)$-*maximal* if there exists an ophelimity profile $(\hat{u}_1, \hat{u}_2, \ldots, \hat{u}_n) \in u(\Re^{\ell n})$, where x^* solves $\text{Max}\{w_i(u(x) : x$ is a feasible allocation and $u_f(x_f) \geq \hat{u}_f$ for all $k \neq f\}$. The set of (i, k)-maximal allocations is M_{ik} and the set $\cap_{i \in I} M_{ik}$ is M. Given this set-up Ythier (2000) is able to prove:

Theorem 9.7 (Ythier (2000, Theorem 2)). *Consider a social system of private property (w, ω) with ℓ goods, n consumers and in which for all consumers i, $(y.1) X_i = \Re^{\ell}_+; (y.2) \omega_i > 0; (y.3) u_i$ is continuous in $\Re^{\ell}_+$, differentiable and increasing in $\Re^{\ell}_{++}$ and such that $x_i \gg 0$ whenever $u_i(x_i) > 0$ with $u_i(0) = 0; (y.4) w_i$ is continuous in $\Re^n_+$ and differentiable with respect to its kth argument in $\{\hat{u} \in \Re^n_+ : \hat{u}_k > 0\}$ for all k and is increasing in its ith argument; $(y.5) w_i \circ u$ is convex in that $w_i(u_i(\lambda x + (1 - \lambda)x')) > w_i(u_i(x'))$ for $0 \leq \lambda \leq 1$ and all $(x, x') \in \Re^{\ell n}_+ x \Re^{\ell n}_+$ while $w_i(u(x)) = 0$ if $u_i(x_i) = 0$. Then for any allocation $x \gg 0$ in M, there exists a price vector p^* and a vector ω^* of individual endowments with $\omega^* = x$ such that the price-action vector $(p^*, 0)$ is an equilibrium of (w, ω^*).*

Proof. Ythier (2000; p. 59). $\square$

Remark 9.10. As Ythier (2000) points out, this result extends the SWT to this class of model by establishing that the set $M \cap \Re^{\ell n}_{++}$ "...is the set of decentralizable allocations ... and any allocation in this set is an equilibrium allocation for properly chosen vectors of market prices and individual endowments." Ythier (2000; p. 50).

9.3.2.2. *General preference orderings*

Hildenbrand (1969) establishes the SWT without convexity on preferences in the context of an economy with an uncountable infinity of agents. Khan and Rashid (1975) produce an asymptotic version of the SWT in a large finite economy. Mas-Colell (1985) gives an approximate version of the SWT in an economy with nonconvexities while Anderson (1988) shows that for most large finite exchange economies, all appropriately bounded Pareto optima 'are close to Walrasian equilibria with appropriately chosen income transfers'. Also working in the direction of relaxing classical conditions on the preference ordering, Ryder and John (1985) show that the SWT holds in an exchange economy where preferences are locally satiated and in a production economy provided at least one consumer has non-satiated preferences. They also show that the conclusion of the theorem fails if all preferences are satiated.

Jofre and Cayupi (2006) extend the SWT to economies where preferences have quite weak properties. Among other things, they make use of the ideas of distance, subgradients and subdifferentials which they define as follows. Let Z be a subset of $\Re^\ell$ then the *distance function* to Z, denoted by $\mathrm{d}_Z(.) \equiv \inf_{z \in Z} \|x - z\|$. Consider a map $f : \Re^\ell \to \Re^* \equiv \Re \cup \{+\infty\}$ with $f \not\equiv +\infty$ and lower semicontinuous. For $x \in \Re^\ell$ such that $f(x)$ is finite, call $f^-(x; v) = \liminf [f(x + tu) - f(x)]/t \,|_{u \to v, t \to 0+}$ the *subderivative* of f at x in the direction $v \in \Re^\ell$ and call $\partial^- f(x) = \{p \in \Re^\ell : p^T v \leq f^-(x; v) \text{ for all } v \in \Re^\ell\}$ the set of *regular subgradients* of f at x. If $f(x) = +\infty$ then $\partial^- f(x) = \varnothing$. The *subgradient set* of f at the point $x \in \Re^\ell$ is defined as $\partial f(x) = \limsup \partial^- f(z) \mid_{z \to x, f(z) \to f(x)}$ and an element of $\partial f(x)$ is called a *subgradient* of f at x (see Jofre and Cayupi (2006; p. 38) for further details). Let $(x^*, y^*) \in \Re^{\ell n} \times \Re^{\ell m}$ be an allocation in an economy with ℓ commodities, n consumers and m firms. If for each i, $X_i \subseteq \Re^\ell$ and if $x^* = (x_1^*, x_2^*, \ldots, x_n^*) \in \prod X_i$ is a consumption allocation across the individuals in the economy then $P_i(x^*) \subseteq X_i$ is the set of elements that i prefers to x^* in X_i and the $\mathrm{cl}P_i(x^*) \subseteq X_i$ is the set of consumptions in X_i which are preferred or indifferent to x^* by i. Thus $P_i : \prod X_i \to X_i$ is a set-valued mapping which generalises the idea of a preference preorder (or utility function) in that it is not required that preferences be transitive. An important condition in the extension of the SWT that Jofre and Cayupi (2006) achieve, is a condition they call *asymptotically included* and if $B(z, \varepsilon)$ denotes the ball centre z radius ε it is defined by them as follows.

Definition 9.1 (Asymptotically included condition: Jofre and Cayupi (2006; p. 48)). The economy $\boldsymbol{E} = \{X_i, P_i, \omega_i, Y_j, \ell\}_{i=1}^{n}{}_{j=1}^{m}$ satisfies the *asymptotically included condition* at (x^*, y^*), if there is a consumer i_0, an $\varepsilon > 0$ and a sequence $(h_k) \to 0$ so that for large enough integer values of k

$$- h_k + \sum_i \mathrm{cl}P_i(x^*) \cap B(x_i^*, \varepsilon) - \sum_j Y_j \cap B(y_j^*, \varepsilon) \subseteq P_{i_0}(x^*)$$

$$+ \sum_{i \neq i_0} \mathrm{cl}P_i(x^*) - \sum_j Y_j.$$

In this set-up, Jofre and Cayupi (2006) obtain the following generalisation of the SWT.

Theorem 9.8 (Jofre and Cayupi (2006; Theorem 2)). *Let (x^*, y^*) be a Pareto optimal allocation for a nonconvex, nontransitive economy $\boldsymbol{E} = \{X_i, P_i, \omega, Y_j, \ell\}_{i=1}^{n}{}_{j=1}^{m}$ in which for all* *i* $(jc.1) X_i \subseteq \Re^\ell; (jc.2)$ *for all* $j, Y_j \subseteq \Re^\ell; (jc.3)\ \omega \in \Re^\ell; (jc.4)$ *the asymptotically included condition holds at* (x^*, y^*) *then there exists a non-zero price vector* p^* *such that* $p^* \in \cap_j \partial d_{Yj}(y_j^*)$ *and* $-p^* \in \cap_i \partial d_{\mathrm{cl}Pi(x^*)}(x^*)$ *so that* (x^*, y^*) *is decentralised by a Walrasian equilibrium.*

Proof. Jofre and Cayupi (2006; p. 49). □

Remark 9.11. As Jofre (2000; p. 5) notes (jc.4), the asymptotically included condition, involves 'a constraint on the aggregated production and preference sets' and it is weaker than the local non-satiation condition that appears in classical versions of the SWT (e.g. (h.2) in Theorem 9.6). He also points out that a sufficient condition for (jc.4) is that the preference ordering P_i is *epi-lipschitz* for some consumer $i \in I$. A subset $Z \subseteq X$ is epi-lipschitzian at a point $z \in Z$, if $\exists d \in X \backslash \{0\}$ and open neighbourhoods N_z and N_d of z and d respectively, and $\lambda > 0$ such that for each $z' \in Z \cap N_z$ and $0 < t < \lambda$, we have $z' + tN_d \subseteq Z$. As Jofre (2000; p. 5) and Jofre and Cayupi (2006; p. 48) note other economically interpretable conditions that imply (jc.4) include: (a) for some consumer $i_0, P_{i_0}(x^*)$ is a closed set; (b) for some $i_0 P_{i_0}(x^*)$ is a convex set with non-empty interior; (c) there exists a non-trivial sequence $(h_k) \to 0$ such that for some i_0 and sufficiently large integer values of $k, -h_k + \mathrm{cl}P_{i_0}(x^*) \subseteq P_{i_0}(x^*)$; (d) there is an i_0 such that $\forall x \in \mathrm{cl}P_{i_0}(x^*), x + \Re^\ell_{++} \subseteq \mathrm{cl}P_{i_0}(x^*)$. The last condition has the familiar economic interpretation that there is a consumer i_0 for whom if x is preferred or indifferent to x^*, then so is the allocation x plus a positive amount of all goods.

Remark 9.12. Jofre (2000; p. 8) gives the following useful interpretation of the conditions on the supporting prices, i.e. the conditions $p^* \in \cap_j \partial d_{Yj}(y_j^*)$ and $-p^* \in \cap_i \partial d_{\mathrm{cl}Pi(x*)}(x^*)$. Consider the production set Y (with the subscript j deleted for notational ease) then the subgradient to the distance function to the production set Y is given by the following *proximal normal* *formula* $\partial d_Y(y^*) = \{\lim_k p_k^* : p_k^* \in PN_Y(y_k^*), y_k^* \to_Y y^*\}$, where $PN_Y(y_k^*)$ is the set of *proximal normal vectors* to Y at y_k^* which is defined as: $p_k^* \in PN_Y(y_k^*) \Leftrightarrow$ for some $t_k > 0, p_k^{*T}(y - y_k^*) \leq (1/2t_k)\|y - y_k^*\|$ holds for each $y \in Y$. As Jofre (2000; p. 8) points out, this implies that $p^* \in \partial d_Y(y^*) \Leftrightarrow$ there is a sequence $p_k^* \to p^*$ and $y_k^* \to y^*$, where $y_k^* \in Y$ is the solution to the following optimization problem: $\max_{y \in Y}[p_k^{*T} y - (1/2t_k) \parallel y - y_k^* \parallel]$. Similar expenditure minimising behaviour holds for consumers when $-p^* \in \partial d_{\mathrm{cl}Pi}(x^*)$ is considered.

9.3.2.3. *Non-convex production sets*

Foley (1970), Gusenerie (1975), Kahn and Vohra (1987), Bonnisseau and Cornet (1988), Malcolm (1998), all work in the direction of relaxing the assumption that individual or aggregate production sets are convex. The following result due to Mas-Colell *et al.* (1995) is an interesting example of the sorts of results that are available in such an environment.

Theorem 9.9 (Mas-Colell, *et al.* (1995; Proposition 16.G.1)). *Consider an economy* $\boldsymbol{E} = \{X_i, \preceq_i, \omega_i, Y_j, \theta_{ij}, \ell\}_{i=1}^{n}{}_{j=1}^{m}$ *in which for all* $i(mwg.1) X_i = \Re_+^\ell; (mwg.2)$ *each* $\preceq_i$ *can be represented by a twice continuously differentiable strictly monotonic and quasi-concave utility function* u_i; $(mwg.3)$ *for each* j, $Y_j = \{y \in \Re^\ell : F_j(y) \leq 0\}$, *where* $F_j : \Re^\ell \to \Re$ *and is twice continuously differentiable with* $\nabla F_j(y_j) = (\partial F_j/\partial y_{1j}, \ldots, \partial F_j/\partial y_{\ell j}) \gg 0$, *but* Y_j *is not necessarily convex. Then if* (x^*, y^*) *is a Pareto optimal allocation for* $\boldsymbol{E}$, *then there is a price vector* $p \in \Re^\ell$ *and wealth levels* $(w_1, w_2, \ldots, w_I)$ *with* $\sum_{i \in I} w_i = p^T\omega + \sum_{j \in J} p^T y_j^*$ *such that* (i) *for any firm* $j, p = \gamma_j \nabla F_j(y_j^*)$ *for some* $\gamma_j > 0$; (ii) *for any* i, x_i^* *is maximal for* $\preceq_i$ *in the budget set* $\{x_i \in X_i : p^T x_i \leq w_i\}$ *and* $\sum_{i \in I} x_i^* = \omega + \sum_{j \in J} y_j^*$.

Proof. Hara *et al.* (1996; pp. 16–18). □

Remark 9.13. The work in Jofre and Cayupi (2006) discussed in the previous section, also achieves a version of the SWT without needing to impose convexity on individual (or aggregate) production sets.

9.3.2.4. *Incomplete markets*

Allard *et al.* (1989) begin by noting that the assumption of complete markets is counterfactual to much economic reality and/or supposes immense computational abilities on the part of economic agents. Working in a temporary equilibrium model with incomplete futures markets where agents have to generally forecast future prices and incomes. They consider a model where x_0 is a consumer's vector of current consumptions and x_1 is their vector of planned future consumptions. Let a typical agents price expectations be given by the function $p^e = \psi(p_0, \delta, y_0, s)$, where p_0 is the vector of current prices, δ is a discount factor ('interest rate'), y_0 is current income and s are 'other' signals the agent might get from the economy. Similarly, income expectations are given by the function $y^e = \rho(p_0, \delta, y_0, s)$ and the *expectation function* for the individual is the pair (ψ, ρ). The fate of the SWT then depends on how expectations behave and in particular the sort of regularity conditions they satisfy (or violate). Allard *et al.* (1989; p. 666) say that expectations are *weakly Roy-consistent* if (i) (ψ, ρ) is continuously differentiable; (ii) the marginal utility of current wealth is always positive; and (iii) there is no consumer in the economy who is so optimistic that they interpret an increase in p_0 as beneficial because they think such a price change will mean their future real income will increase at a higher rate, similarly for a change in the discount rate. To get the stronger consistency notion on expectations, two more conditions are needed. Suppose it is true that a currently compensated variation in current prices has no impact on the expected change in future real income; and (v) temporary indirect utility — but not necessarily expectations — are independent of s. If (i), (ii), (iv), and (v) are true, then the expectation function (ψ, ρ) is *strongly Roy-consistent*. Given these definitions, Allard *et al.* (1989) prove that a (temporary) Pareto optimum is a (temporary) competitive equilibrium so the SWT can be extended to the missing futures market environment under these conditions. However, if what they call 'the general case of weak Roy-consistency' only holds, then the SWT does not generalise.

Operating in the context of a two period real asset incomplete markets model, Pan (1995) shows that a *constrained* Pareto optimum can be supported by Walrasian prices after a suitable redistribution of first period endowments, provided a spanning condition on the substitution matrix across consumers was satisfied. Unifying these two strands of the 'market incompleteness' literature, Balasko (2003) has recently shown that in a two period exchange economy with financial assets and a temporary financial equilibrium can be interpreted as a (complete markets) Arrow-Debreu economy in which preferences are price dependent. So far, only existence, local determinateness and comparative statics have been investigated. However, the prospects of being able to decentralise Pareto efficient allocations is not clear because as Arrow and Hahn (1971) point out: "...the significance of Pareto efficiency becomes obscure, since an allocation that is dominated at one set of prices is not dominated at another." Arrow and Hahn (1971; pp. 129–130). This is an interesting area for future research.

9.3.2.5. *Some other extensions of, and limits to, the SWT*

Leuenberger (1994a, 1994b) using a dual approach to Pareto efficiency discussed in the previous section, gets the SWT under conditions similar to the classical conditions, but again using the notions of 'income regularity' and 'dual bliss points' — see Luenberger (1994a; Theorem D2) for details. In the model with indivisibilities specified by Laan, Talman and Yang (2002) and considered in the previous section, the SWT holds for the economy $\boldsymbol{E} = \{X_i, \preceq_i, \omega_i, e(i), A\}_{i=1}^n$ provided (i) A_0 is a strictly negative row vector (so labour or capital is needed in any production process), $\hat{A}$ is regular (e.g. Leontief) and $\hat{A}^{-1}$ is nonnegative; (ii) all u_i are weakly monotonic in commodity 0 and in the indivisible goods, also continuous with respect to the indivisible goods. Then if (x^*, c^*) is a Pareto efficient allocation such that for every consumer i, there exists a vector of divisible goods $x_i \in \Re_+^{l+1}$ and some $y \in \Re_+^l$ satisfying $x_i = Ay + \hat{\omega}_i$, where $\hat{\omega}_i = (p^* x_i^*, 0, \ldots, 0)$ and $\max_{k \in \text{In}} u_i(0^{\ell+1}, e(k)) < u_i(x_i, c^*)$ — see the previous section and Laan, Talman and Yang (2002; Theorem 4.9) for details. Under conditions previously discussed for their model, Herves-Beloso *et al.* (2005) obtain the SWT in the differential information context studied by them — see Herves-Beloso *et al.* (2005; Remark 4) for details. In the economy with public goods and externalities considered by Conley and Smith (2005) and described in the previous section, a SWT holds under conditions similar for the existence of equilibrium that model — see Conley and Smith (2006; p. 694). Brito *et al.* (2006) on the other hand show that in a model with externalities, Coasian bargaining among individuals in a private ownership context may limit the set of Pareto optima that can be supported. They find that "…the Second Fundamental Theorem of Welfare Economics …does not hold for Coasian bargaining." Brito *et al.* (2006; p. 885).

9.3.3. *Summary on the SWT*

The SWT is typically motivated by the observation that while the FWT identifies circumstances under which 'equilibria are optima', that result is capable of tolerating very inequitable distributions of welfare in the economy. In response to arguments which suggest that consequently market prices should be modified or that the market mechanism should be abandoned entirely in order to address distributional concerns, the SWT argues that, under certain circumstances, once a desired Pareto optimum has been identified, market equilibrium prices can be used to support it. Because the SWT is such an important result in economic theory and has been such an influential guide to policy we have been concerned to consider the 'classical' version of the result and then to explore its robustness in various directions. While no attempt has been made to be encyclopedic, the results considered in this section indicate that taking 'blanket' positions on the applicability of the SWT is not generally possible and that particular economic circumstances need to be considered before a policy maker should conclude that a particular state can be supported (or not) as a market equilibrium. Another related issue is that an unfortunate habit has grown up in interpretations of the SWT in which the word 'achieved' is substituted for 'supported'. Reasons why this is a non-trivial — and generally invalid — substitution, is the subject of the next section.

9.3.4. *'Achieving' a Pareto optimum versus 'supporting' a Pareto optimum*

The SWT is an important result in economic theory and it has also been influential in forming attitudes about the way economic policy should be conducted. For instance, Gravelle and Rees (1981) argue that: "The market economy can be thought of as an efficient 'black box' or resource allocation machine: feed in an initial wealth distribution, the mechanism churns away and out comes a Pareto Optimum… A policy maker who dislikes the welfare distribution implied by a given market mechanism can best improve things *not* by interfering with the market mechanism — the works of the black box — but rather by changing the wealth distribution directly [since] the policy of lump sum redistribution together with unfettered operations of the market mechanism is superior [to any distortion of market prices]. [Indeed] it is possible to prove the following proposition: *every Pareto Optimal resource allocation can be achieved as a competitive market equilibrium given an appropriate initial distribution of wealth* (for a proof see Takayama (1974; p. 185–201))." Gravelle and Rees (1981; p. 485). The view about how policy should be conducted expressed here can be found widely in the literature. For instance Shone (1975; p. 262) states that: "... The requirements of Theorem 10.5 [the SWT] ensures the conditions under which a Pareto Optimal state can always be *reached* by perfect competition." Russell and Wilkinson (1979; p. 356) write: "... The significance of the second fundamental optimality principle is that, given our assumptions, any Pareto-optimal allocation can be *achieved* by the decentralised decision making of consumers pursuing their own self interest in competitive markets. *Centralised planning is unnecessary.*" Allingham (1983; p. 28) argues that: "... the second theorem of welfare economics [states that] any optimal allocation may be *obtained* as an equilibrium allocation, given the appropriate endowment allocation." Cornwall (1984; pp. 382–383) claims: "... the result of this section gives a much deeper result than the previous section. It can be paraphrased: if certain assumptions including convexity hold, then all Pareto efficient outcomes can be *gotten* as outcomes of a market process very similar to competitive equilibrium." Varian (1987; p. 502) adds: "... The Second Theorem of Welfare Economics asserts that under certain conditions, every Pareto efficient allocation can be *achieved* as a competitive equilibrium. What is the meaning of this result? The Second Welfare theorem implies that problems of distribution and efficiency can be separated ... whatever your criteria for a good or just distribution of Welfare, you can use competitive markets to *achieve* it...” Feldman (1987; p. 891) contributes the claim that: “...The Second Fundamental Theorem of Welfare Economics established that the market mechanism, modified by the addition of lump sum transfers, can *achieve* virtually any desired optimal distribution.” Blad and Keiding (1990; p. 124) state that: “It turns out that Theorem 4.3 has a converse: all Pareto optimal allocations can be *obtained* through the market.” Kreps (1990; p. 200) claims that: “...In other words, if one imagines that it would be difficult for a social dictator to find an equitable and efficient allocation of the social endowment, and if the economy has well functioning markets, the dictator might choose to reallocate initial endowments in an equitable fashion and then *let the market take over*.” Silberberg (1990; p. 588) asserts: “...The [SWT] is the statement that there is an allocation under perfect competition for any overall Pareto optimum. That is, starting now with a point on the Pareto frontier, there exists a competitive solution which *achieves* that optimum.” Eaton and Eaton (1991; p. 421) argue: “Suppose that we have identified some Pareto-optimal allocation that we would like to implement. The second theorem tells us first to redistribute the initial endowment and then to rely on competitive markets to *achieve* Pareto optimality. Varian (1992; p. 346) insists that: “...the above proposition [SWT] shows that every Pareto efficient allocation can be *achieved* by a suitable reallocation of wealth.” Finally, Mas-Colell *et al.* (1995; p. 308) write: “*The Second Fundamental Welfare Theorem* [says] If household preferences and firm production sets are convex, there is a complete set of markets with publicly known prices, and every agent acts as a price taker, then any Pareto optimal outcome can be *achieved* as a competitive equilibrium if appropriate lump-sum transfers of wealth are arranged”. These authors supplement this formal statement with the following commentary: “...it [SWT] says that under convexity assumptions (not required for the first welfare theorem), a planner can *achieve* any desired Pareto optimal allocation by appropriately redistributing wealth in a lump-sum fashion and then 'letting the market work'. Thus, the second welfare theorem provides a theoretical affirmation for the use of competitive markets in pursuing distributional objectives." Mas-Colell *et al.* (1995; p. 524). Finally as Anderson (1988) points out: "...the interpretation [usually] placed on the second welfare theorem [is] that it would be better for government to redistribute income, and then allow the workings of the market to determine the allocation of commodities to individuals rather than have the government establish subsidies for certain commodities or to allocate goods through non-market mechanisms." Anderson (1988; p. 361).

We advance the argument that there are general circumstances in which these claims are not true. We do this by showing that crucial word in the SWT and one overlooked in all the interpretations of it cited above is the word 'supported' (see Theorem 9.6). It is not generally the case, for reasons detailed below, that the SWT allows the substitution 'achieved' or any of its synonyms for 'supported'.

One reason why the cited interpretations of the SWT are not generally correct was noted by Samuelson (1974), followed by him considering a situation where, for a given initial distribution of endowments, Walrasian equilibrium is not unique. In the face of non-uniqueness, and assuming for a moment the global stability of the market adjustment process, we cannot be sure which equilibrium allocation the market will select. Therefore, if a particular Pareto optimum is *desired* on equity grounds, there is no guarantee that, without intervention and guidance, the market will actually converge to the desired Pareto optimum. The difficulty which non-uniqueness causes for common interpretations of the SWT is illustrated in Fig. 9.4.[4]

[4]See also the discussion of this point in Anderson (1988), Bryant (1994) and Hammond (1998).

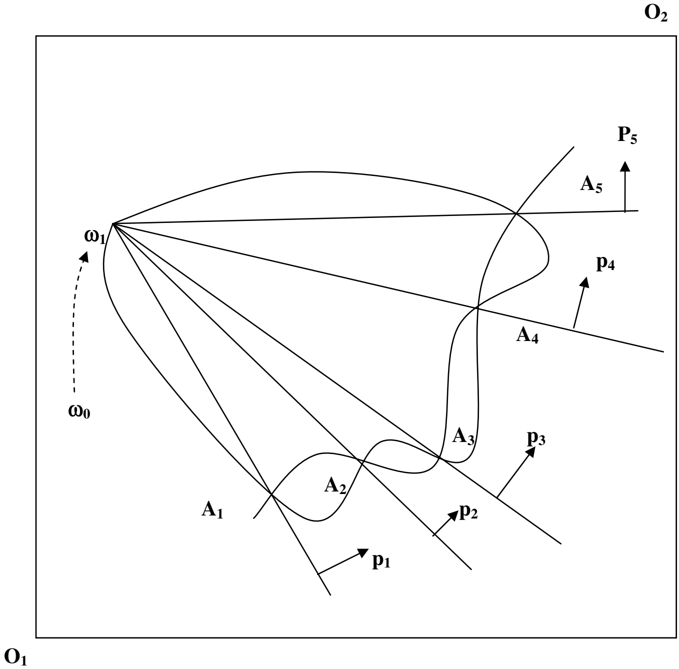

Fig. 9.4. SWT and non-uniqueness.

Walrasian equilibrium in this economy, which has an allocation of the social endowment at ω_1 (possibly achieved from the initial allocation in the economy at ω_0), occurs at price vectors denoted by $\mathbf{p^1}$, $\mathbf{p^2}$, $\mathbf{p^3}$, $\mathbf{p^4}$, $\mathbf{p^5}$. These price vectors support the Pareto optimal allocations $\mathbf{A_1}$, $\mathbf{A_2}$, $\mathbf{A_3}$, $\mathbf{A_4}$, $\mathbf{A_5}$. Suppose that the allocation $\mathbf{A_3}$ is regarded as desirable from an equity point of view. Is it true that this Pareto optimum will necessarily be *achieved* by a competitive market, as the cited interpretations of the SWT assert is the case? The answer is clearly 'no' because there is nothing to prevent the price mechanism from selecting $\mathbf{p^1}$, $\mathbf{p^2}$, $\mathbf{p^4}$ or $\mathbf{p^5}$ and hence decentralising allocations $\mathbf{A_1}$, $\mathbf{A_2}$, $\mathbf{A_4}$ or $\mathbf{A_5}$. Thus although the SWT guarantees the existence of an equilibrium price system like $\mathbf{p^3}$ to *support* $\mathbf{A_3}$, there is no guarantee that in this economy that the market mechanism will actually *achieve* $\mathbf{A_3}$ contrary to the cited interpretations of the theorem.

What is happening in Fig. 9.3 can be illustrated in an example due to Mas-Colell *et al.* (1995). Consider a two person exchange economy with the following utilities and endowment vectors: $u_1(x_{11}, x_{12}) = x_{11} - 1/8x_{12}^{-8}, \omega_1 = (2, r)$ and $u_2(x_{21}, x_{22}) = -1/8x_{21}^{-8} + x_{22}, \omega_2 = (r, 2)$. Then the offer curves for these two individuals are:

$$\mathrm{OC}_1(p_1, p_2) = [2 + r(p_2/p_1) - (p_2/p_1)^{8/9}, (p_2/p_1)^{-1/9}]$$

$$\mathrm{OC}_2(p_1, p_2) = [(p_1/p_2)^{-1/9}, 2 + r(p_1/p_2) - (p_1/p_2)^{8/9}]$$

In the case where $r = 2^{8/9} - 2^{1/9}$, the three Walrasian equilibrium price ratios for the economy are: $p_1/p_2 = 2/1, 1/1, 1/2$. Suppose that the allocation $[(1 + r, 1), (1, 1 + r)]$ is desired on distributional grounds. Then there is a Walrasian equilibrium and initial distribution that supports this allocation, namely $(p_1, p_2) = (1, 1)$ and $\omega = [(2, r), (r, 2)]$. For this example, the initial endowment allocation equals the initial distribution that supports the desired final allocation. Despite this, there is nothing in the structure of the problem to ensure that the desired price equilibrium is selected. Hence there is no guarantee that 'letting the market work' will actually result in the desired allocation being achieved.

Since the failure of the market to necessarily achieve the desired Pareto optimum is, in this case, a consequence of the non-uniqueness of equilibrium, it is reasonable to ask whether non-uniqueness may be regarded as an uninteresting pathology, which can safely be ignored, or is it something which must be allowed for as a phenomenon likely to occur in most market economies? As we saw from our work in Chap. 7, non-uniqueness of equilibrium appears to be the normal case and consequently the Walrasian vision which inspired policy recommendations and interpretations of the SWT appear to be in error on this count.

From this point of view, consider the interpretation of the SWT by Hammond (1998) which is that: "...A much more promising defense of perfectly competitive markets is based, of course, on the second efficiency theorem of welfare economics [which states] any Pareto efficient allocation of resources in which no individual is on the margin of being forced below subsistence can be reached through perfectly competitive markets, provided that the invisible hand is supplemented by a suitable method for redistributing wealth." Hammond (1990; p. 8). This interpretation survives non-uniqueness in the sense that provided a policy maker is trying to decentralise a Pareto allocation 'above subsistence' and provided all the Pareto efficient allocations in the lens formed by the two individuals indifference curves through the initial endowment are above subsistence, then non-uniqueness would not cause a problem in hitting *some* non-subsistence level.

There is however a further problem with common interpretations of SWT, even when equilibrium is unique and that occurs if the price adjustment process at work in the economy is not globally stable. If that is the case, then again it cannot be asserted that the market mechanism will necessarily *achieve* a desired Pareto optimum, precisely because the market process may fail to arrive at any equilibrium price vector at all. Given this, it is reasonable to ask what is known about the stability of market mechanisms and from the work in the previous chapter, it seems reasonably clear that global stability of informationally plausible adjustment processes cannot generally be relied on. Therefore the implicit reliance of popular interpretations of the SWT on the hypothesis of global stability is not well founded and the approach to economic policy which it engenders is not generally theoretically supported.

Even though global stability of adjustment processes seems not to be generally guaranteed, suppose for the moment that the market mechanism is assumed to be globally stable. This means that the price adjustment process at work in the economy leads to *some point* in the set of Walrasian equilibria. Recalling that the processes which seem most likely to have this feature are variants on the Global-Newton type processes, it is of interest to note a feature of such processes. While such processes might be successful in selecting an equilibrium, there is generally no relationship between the starting point and the end point for such processes. As Herrmann and Kahn (1999) point out: "...the mechanism computes *an* equilibrium (among a finite/discrete set of equilibria) that has *no* relation to the starting point of the process. The later drawback means that GNM can say nothing about which equilibrium is computed and why it was selected. That is there is no control over which equilibria is selected." Herrmann and Kahn (1999; pp. 422–423). Since there is no guarantee about which equilibrium is selected by such a mechanism, then even allowing the apparently dubious hypothesis of global stability does not generally validate the interpretations of the SWT cited earlier. For the cited interpretation of the theorem to be true, either Walrasian equilibrium has to be globally stable *and* unique or some other conditions have to be present to ensure that the desired equilibrium allocation is actually selected. In commenting on this argument, Parinello (1998) remarks that: "...Bryant (1994) convincingly criticised the usual interpretation of the theorem because it is based on an arbitrary substitution of words... in what follows we shall call the set of unpleasant properties [identified by Bryant] *equilibrium snags* ..." Parinello (1998; p. 211). To these equilibrium snags, Parinello (1998) has added a number of *informational snags* to the cited interpretations of the SWT. See Parinello (1998) for details.

What might allow 'achieved' to be used in this context? A hint comes from the following diagram.

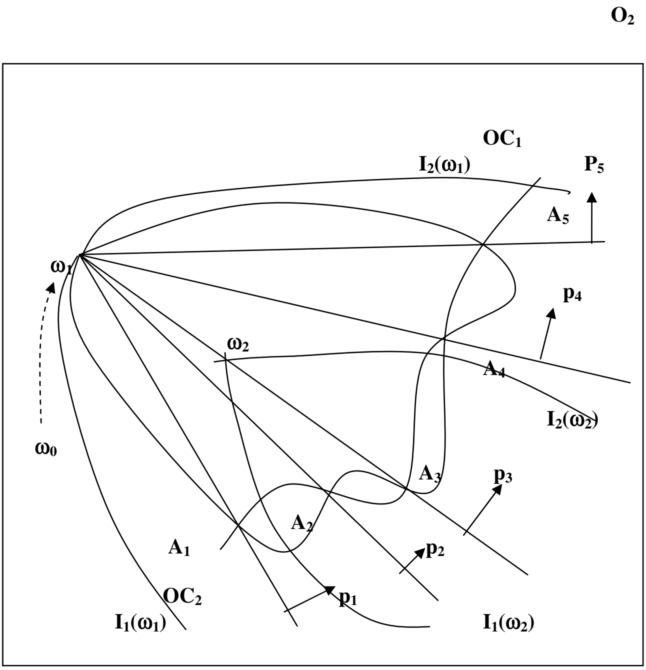

Fig. 9.5. Approximate achievement of a desired allocation.

Suppose the desired Pareto optimal allocation is $\mathbf{A}_3$ in Fig. 9.5. Then if the initial redistribution takes the economy from ω_0 to ω_1, then the operation of markets may result in the desired allocation being missed by a 'large distance' (e.g. allocations at $\mathbf{A}_1$ or $\mathbf{A}_5$ might result, instead of the desired allocation $\mathbf{A_3}$). However, by construction, the offer curve of an individual must lie in the upper contour set defined by the indifference curve for that person through the initial endowment point. If the redistribution gets closer to the final desired allocation, the set of possible equilibria cannot increase and the size of the possible error in achieving the desired allocation cannot increase either, provided the allocation stays in the lens formed by the two individual indifference curves through the endowment point (and certainly if it stays on the hyperplane supporting the desired allocation). The equilibria associated with the distribution of endowments ω_1 will support allocations $\mathbf{A_1}$, $\mathbf{A_2}$, $\mathbf{A_3}$, $\mathbf{A_4}$ or $\mathbf{A_5}$. However, if the starting point is the allocation ω_2, then the equilibria must be contained in the lens formed by the indifference curves $I_1(\omega_2)$ and $I_2(\omega_2)$. Thus the equilibria, wherever they are exactly (and they have not been drawn in here because the diagram is already busy enough), will be closer to $\mathbf{A}_3$ than either $\mathbf{A}_1$ or $\mathbf{A}_\mathrm{a}$ or $\mathbf{A}_5$. In this diagram, the closer the initial allocation comes to $\mathbf{A}_3$ the better the approximation, until it is exact, when it is at the desired Pareto optimum, as the theorem asserts.

If enough conditions are put on the economy so that the equilibrium that supports the desired Pareto optimum is locally stable, then in an economy with a multiplicity of equilibria (i.e. the normal case), 'achieved' can be validly substituted for 'supported' in the SWT if the appropriate redistribution takes the economy into the neighbourhood of the desired Pareto optimal allocation. However, this again leaves little room for 'letting the market work' in the pursuit of distributional objectives, since all the work in achieving the desired distribution of welfare has been done by the non-market act of redistribution. Consequently, we see that in all these cases, the argument of Mas-Colell *et al.* (1995), and others listed above, that the SWT guarantees that a planner can *achieve* any desired Pareto optimal allocation by appropriately redistributing wealth in a lump-sum fashion and then 'letting the market work' may be an overstatement. The SWT provides a theoretical affirmation for the use of competitive markets in pursuing distributional objectives in economies with multiple equilibria, only when the 'appropriate' redistribution involves taking the economy directly to (or at least into the neighbourhood of) the desired Pareto optimum. The point we are making is nicely put in the words of Mandler (2007): "There is a well known puzzle about the second welfare theorem: if a policymaker knows the preferences and endowments of all agents, then it might as well act like a central planner and just assign agents the Pareto optimal allocation that it wants them to consume. If on the other hand, the policy maker is uncertain about the economy's primitives, it will be unable to even identify Pareto optima, let alone design transfers that implement them. So in what sense does the second welfare theorem recommend markets as an allocation mechanism?" Mandler (2007; pp. 523–524).

9.4. Conclusion

This chapter has been concerned with the capacity of equilibrium prices to decentralise optimal commodity allocations. General equilibrium theory summarises its arguments in this regard in the First and Second Welfare Theorems.

These theorems show that there are circumstances under which equilibrium allocations are optimal and circumstances under which optimal allocations can be supported as equilibria. There are also interesting 'non-classical' environments to which the FWT can be extended, but there are also circumstances in which it fails. The same is true of the SWT. In addition it was shown that sometimes too much is claimed on behalf of the SWT on account of an implicit reliance in interpretations of the theorem on uniqueness of equilibrium and adjustment process stability. Since neither the FWT nor the SWT are condition free, no blanket claim can be made to the effect that 'markets are efficient' (or not) or that 'distributional objectives can be handled by markets' (or not). The connection between equilibria and optima therefore needs to be considered in particular economic environments.

Chapter 10

COMPARATIVE STATICS OF EQUILIBRIUM STATES

> "*... comparative static results on Walras equilibria cannot be obtained with any reasonable degree of generality. This does not mean that no set of assumptions is forceful enough to yield comparative static results ... but that such assumptions tend to be more restrictive than we are ready to accept.*"
>
> M. Blad and H. Keiding

10.1. Introduction

Having established the existence of equilibrium and having studied various qualitative properties of equilibrium states, general equilibrium theory turns to the problem of making predictions about how equilibrium prices and quantities will respond to changes in the primitives which define the economy. It can be argued that working out of the comparative static properties of equilibrium was regarded by Walras as the central purpose of his whole program. Indeed, according to Balasko (1975): "... the main aim that Walras had for general equilibrium analysis was the study of what is now called comparative statics, in other words the laws by which the equilibrium prices and quantities vary with the underlying data of the economy: resources, production conditions and utility functions." Balasko (1975; p. 95, emphasis added). In similar vein, Arrow and Hahn (1971) remark: "Finally, Walras had a still higher aim for general equilibrium analysis: to study what is now called 'comparative statics', in other words, the laws by which the equilibrium prices and quantities vary with the underlying data [of the economy]." Arrow and Hahn (1971; p. 5).

There are at least two reasons for being interested in the comparative static properties of Walrasian equilibrium. Firstly, from a methodological point of view, there is the argument due to Samuelson (1947) that any theory or research program which cannot make predictions about how the variables explained by the theory change in response to 'shocks' to the economy is flawed. According to Samuelson (1947), a theory or research program that cannot generate 'meaningful theorems' is essentially empty. For general equilibrium theory, 'meaningful theorems' in Samuelson's sense are just the comparative static propositions which the theory is able to produce. From a methodological point of view, it is therefore important to know what can be said about comparative statics in a general equilibrium context. Secondly, as was also noted in Chap. 1, numerous policy analyses depend on being able to make well defined comparative static predictions about what will happen to prices, quantities and utility levels if particular changes are made to the structure of the economy. In order for such applied work to proceed, it is necessary to have reliable and determinate comparative static results.

For at least these reasons, this chapter is devoted to a consideration of general equilibrium comparative statics. The chapter is organised as follows. Section 2 considers various approaches to obtaining comparative static predictions in general equilibrium models. Section 3 considers a particularly interesting class of 'welfare comparative static' results associated with the transfer of endowments between agents (or countries). These comparative static results have been studied as examples of 'competitive perversity' in the mathematical economics literature, and as 'transfer paradoxes' in the international trade literature. Section 4 offers some concluding remarks.

10.2. Some comparative static approaches and results

10.2.1. *The basic framework*

In an economy with numerous commodities, diverse consumers, a multitude of production techniques and many interconnected markets, it is unsurprisingly difficult to predict what will happen to equilibrium prices, quantities and utility levels when the parameters that define the economy change. Nevertheless, it would be disappointing for any theory or research program which attempts to explain economic reality, if its set of comparative static predictions were severely limited or empty. What then can be said about the comparative static predictions in general equilibrium? In an early assessment of the situation Arrow and Hahn (1971) argued: "The most notable conclusion of our investigation . . . appears to be that for very many interesting problems of comparing equilibria, the information provided by the foundations of the models [i.e.] profit and utility maximisation, are insufficient to give us definitive answers to our questions." Arrow and Hahn (1971; p. 261). More recently Geanakoplos (1987) has argued that: "There is an unfortunate side to this [Walrasian] comparative static story. One would like to show not only that comparative statics are well defined, but also that they have a definite form . . . Given the strong rationality hypothesis of the Arrow-Debreu model, one would hope for some [definitive] results. [However the assumptions of the model] do not permit any *a priori* predictions about the changes that must occur in equilibrium given exogenous changes in the economy . . . [this] is disappointing. It means that to make even qualitative predictions, the economist needs detailed data on the excess demands." Geanakoplos (1987; p. 121). Along similar lines, Hardle *et al.* (1991) have argued that: "When general equilibrium models are used to make comparative static predictions, they cease to be general ... [unfortunately] in most analyses, conclusions depend on aggregating consumers into a single representative, or by assuming restrictive forms for utility or production functions." Hardle *et al.* (1991; p. 1525). As these and other authors note, the lack of general comparative static results is a serious problem for general equilibrium theory and it motivates the search for techniques and circumstances by and under which comparative static results might be established.

To see formally why comparative static results are so hard to establish in the context of general equilibrium models, consider the following argument.[1] An equilibrium price vector for an economy $\mathcal{E}$ may be characterised as a zero of the map $Z : P \times A \to \Re^\ell$, where $P \subset \Re^\ell$ is the space of prices, $A \subset \Re^m$ is a set of parameters which define the economy and Z is the excess demand map of the economy. Suppose equilibrium exists for a vector of parameters $\alpha^\circ \in A$, then there exists a $p^\circ \in P$ such that $Z(p^\circ; \alpha^\circ) = 0$. Assume enough differentiability of Z so that the Jacobian matrix of excess demands with respect to prices, $D_p Z(p^\circ; \alpha^\circ)$, exists. If the conditions of the implicit function theorem hold at (p°, α°), then in a small neighbourhood of $p^\circ \in P, p^\circ = Z^{-1}(0, \alpha^\circ)$ is a locally unique Walrasian equilibrium. In addition, p varies continuously with the parameter α near α°. There then exists a continuous function $p(\alpha)$ such that the identity $Z(p(\alpha), \alpha) \equiv 0$ holds. Consequently, the impact on p of a small change in α can be deduced as follows. If good ℓ is 'always desired' so that if $p_\ell \to 0$, then $Z_\ell(p) \to +\infty$, then good ℓ can be made the numeraire and its price can be normalised

[1] The treatment presented here follows that in Kehoe (1987) and Mas-Colell *et al.* (1995).

so that $p_\ell = 1$. Then writing out the equilibrium conditions we have:

$$\begin{aligned} Z_1(P_1, \ldots, p_{\ell-1}, 1;\ \ \alpha_1, \ldots, \alpha_m) &= 0 \\ Z_2(p_1, \ldots, p_{\ell-1}, 1;\ \ \alpha_1, \ldots, \alpha_m) &= 0 \\ \vdots \qquad\qquad\qquad & \vdots \\ Z_{\ell-1}(p_1, \ldots, p_{\ell-1}, 1; \alpha_1, \ldots, \alpha_m) &= 0, \end{aligned} \tag{10.1}$$

which can be written more compactly as $Z(p, \alpha) = 0$. Then the (p^0, α^{o}) defined above is a solution to (10.1). Suppose that $Z(p, \alpha)$ is differentiable and that the $(\ell - 1) \times (\ell - 1)$ matrix of partial derivatives (10.2) is invertible.

$$\begin{pmatrix} \partial Z_1/\partial p_1(p^0, \alpha^{\text{o}}) \ldots \partial Z_1/\partial p_{\ell-1}(p^0, \alpha^{\text{o}}) \\ \partial Z_2/\partial p_1(p^0, \alpha^{\text{o}}) \ldots \partial Z_2/\partial p_{\ell-1}(p^0, \alpha^{\text{o}}) \\ \vdots \qquad\qquad\qquad\qquad \vdots \\ (\partial Z_{\ell-1}/\partial p_1)(p^0, \alpha^{\text{o}}) \ldots \partial Z_{\ell-1}/\partial p_{\ell-1}(p^0, \alpha^{\text{o}}) \end{pmatrix} \tag{10.2}$$

Then as noted above, in a small neighbourhood of $p^0, p^0 = Z^{-1}(0, \alpha^{\text{o}})$ is a locally unique solution to the equilibrium conditions. Since the implicit function theorem guarantees the existence of a continuous function $p(\alpha)$ such that $Z[p(\alpha), \alpha] \equiv 0$, the impact of a small change in α on p can then be calculated as:

$$\underset{(\ell-1)\times(\ell-1)}{D_p Z(p^{\text{o}}; \alpha^{\text{o}})} \cdot \underset{(\ell-1)\times m}{Dp(\alpha^{\text{o}})} + \underset{(\ell-1)\times m}{D_\alpha Z(p^{\text{o}}; \alpha^{\text{o}})} = 0, \tag{10.3}$$

which inverting $D_p Z(p^{\text{o}};\ \alpha^{\text{o}})$ yields

$$\underset{(\ell-1)\times m}{Dp(\alpha^{\text{o}})} = -\underset{(\ell-1)\times(\ell-1)}{[D_p Z(p^{\text{o}}, \alpha^{\text{o}})]^{-1}} \cdot \underset{(\ell-1)\times m}{D_\alpha Z(p^{\text{o}}, \alpha^{\text{o}})} . \tag{10.4}$$

The typical element in $Dp(\alpha^{\text{o}})$ is $\partial p_g/\partial \alpha_k$ and these are the terms that general equilibrium comparative statics tries to sign. If it is known how the variations in the α's influence excess demands, then the 'shock' matrix $D_\alpha Z(p^o; \alpha^o)$ is determined. The matrix of comparative static effects, $D_\alpha p(\alpha^o)$ would then also be known provided the structure of $[D_p Z(p^o; \alpha^o)]^{-1}$ is known. However, the Sonnenschein-Mantel-Debreu result asserts that the underlying microeconomics of the economy does not generally endow this matrix with any particular structure. A consequence of this is that 'anything goes' as far as comparative static predictions are concerned. As Mas-Colell *et al.* (1995) note that: "... the matrix of price effects $[D_p Z(p^0, \alpha^0)]$ is unrestricted [so] that without further assumptions, the 'anything goes' principle applies to the comparative statics of equilibrium ..." Mas-Colell, *et al.* (1995; p. 616). Thus, a given shock to the economy represented by the $D_\alpha Z(p^o, \alpha^o)$ matrix — which could also be very complex — interacting with the (inverse of) an excess-demand–price response matrix $[D_p Z(p^o; \alpha^o)]^{-1}$ that has weak structure means that the pattern in the price response matrix $D_\alpha p(\alpha^o)$ can be almost anything. It is therefore of considerable interest to ways in which comparative static predictions may be arrived at.

10.2.2. *Structuring the Jacobian and the shock matrix*

An obvious route to obtaining comparative static predictions is to impose some structure on the Jacobian of the excess demands of the economy (the $D_p Z(p^0, \alpha^0)$ matrix — or more precisely its inverse) and also on the types of shocks that impinge on the economy (the $D_\alpha Z(p^o; \alpha^o)$ term). An illustration of this general approach is provided in the following result due to Arrow and Hahn (1971). A shock to the parameters of the economy at a price vector p is *binary* if when $Z'(p)$ is the excess demand map after the shock and $Z(p)$ is the excess demand map before the shock, $Z'(p)-Z(p)$ has only two non-zero components. This then structures the shock term in (10.4). To complete the exercise, the Jacobian of the excess demands needs to be structured and an example of such structuring is the following. If the excess demand map is differentiable at p, then good i is a *gross substitute* for good j if $\partial Z_i/\partial Z_i(p) > 0$. If all goods are gross substitutes for each other, then all the off-diagonal entries in $D_pZ(p;\alpha)$ are positive. Furthermore, since the excess demand functions are homogeneous of degree zero in prices, application of an Euler theorem for homogeneous functions yields that the diagonal entries of $D_pZ(p;\alpha^o)$, i.e. the $\partial Z_i/\partial Z_i(p)$ terms are negative. With this set-up, the following result becomes available:

Theorem 10.1 (Arrow and Hahn (1971; Theorem 10.2)). *Let p^* be an equilibrium for the economy **E** characterised by its excess demand map Z. Let the parameter changes at p^* be binary and such that $Z_1'(p^*) > Z_1(p^*)$ and $Z_2'(p^*) < Z_2(p^*)$ for goods 1 and 2 and let all goods be gross substitutes at all prices. Then at the new equilibrium price vector p^{**}, $p_1^{**} > p_1^*$ and $p_2^{**} < p_2^*$.*

Proof. Arrow and Hahn (1971; pp. 246–247). □

Remark 10.1. It is interesting to note that as Eq. (10.4) makes clear, it is the *combination* of shocks to the economy and structure on the Jacobian of the excess demands that delivers definitive comparative static results in this case. Part of the reason why this happens is that as Mas-Colell *et al.* (1995; Proposition 17.G.3) show, if the price effects matrix $D_pZ(p^0, \alpha^0)$ has negative diagonal and positive off-diagonal entries (so that $D_pZ(p^0, \alpha^0)$ is a *proper Metzler matrix*), then $[D_pZ(p^0, \alpha^0)]^{-1}$ has all negative entries. This approach of putting conditions on the Jacobian of the excess demands and on the shock matrix will be explored in detail in what follows.

As we have noted several times, an economy as far as general equilibrium theory is concerned is made up of consumers and producers who in turn are characterised by consumption sets, preferences, endowments and production technologies. Variation in any of these characteristics, along with variations in the number of consumers or producers or commodities and the distribution of share ownership in the economy, all potentially give rise to variations in equilibrium prices and quantities. General equilibrium comparative static analysis has tended to focus on two sorts of shocks: changes in preferences (which are sometimes thought of as 'demand shocks') and changes in endowments (which are sometimes thought of as 'supply shocks') although in general whatever the source of the shock it shows up as a variation in excess demands. Under the assumption that all goods are gross substitutes at all prices, Hicks (1939) established the three *Hicksian Laws* concerning a shift in preference from the numeraire to another good which, following Hale *et al.* (1999) can be written:

Theorem 10.2 (Hale *et al.* (1999; p. 175)). *Consider an economy* E *in which there is an always desired numeraire good* ℓ *and all goods are gross substitutes for each other at all prices. Then* (i) *a shift in excess demand from good* ℓ *to good* i *increases the equilibrium price of good* i; (ii) *a shift in excess demand from good* ℓ *to good* i *increases the equilibrium price of all goods in the economy, apart from good* ℓ; (iii) *a shift in excess demand from good* ℓ *to good* i *increases the equilibrium price of good* i *proportionally more than it increases the prices of all other goods in the economy.*

Proof. Hale *et al.* (1999; p. 175). □

Remark 10.2. Hicks (1939) for a small number of goods and Morishima (1970) for the general ℓ good case, showed that if the gross substitutes assumption were replaced with the condition that substitutes of substitutes and complements of complements are substitutes and substitutes of complements and complements of substitutes are complements, then (a) a shift in excess demand from the numeraire to good i increases the equilibrium price of good i; and (b) a shift in excess demand from the numeraire to good i increases the equilibrium price of all substitutes and decreases the equilibrium price of all complements. However as Hale *et al.* (1999; p. 176) point out, these conditions do not generally imply stability of equilibrium.

An illustration of a result concerning the effect of a change in endowments on equilibrium prices is the following result due to Mas-Colell *et al.* (1995).

Proposition 10.1 (Mas-Colell *et al.* (1995; p. 619)). *If the economy* $\boldsymbol{E}$ *is such that all goods are gross substitutes at* $(p^{\mathrm{o}}, \alpha^{\mathrm{o}})$, *consumer i's demand behaviour is normal at* $(p^{\mathrm{o}}, \alpha^{\mathrm{o}})$, *then if* $\omega_{i\ell}$ *the endowment of good* ℓ *for a single consumer i falls, then the price of good* ℓ *relative to that of all other goods rises or, equivalently, the price of all other goods falls relative to the price of good* ℓ.

Proof. Mas-Colell *et al.* (1995; p. 619). □

Remark 10.3. It is striking how restrictive the conditions have to be on both the shock matrix (one consumer and an endowment decline on one good) and the Jacobian of the excess demands (local gross substitutes). It is of interest to see how close to necessary these conditions are because as it stands, the criticism of Hardle *et al.* (1991) that this approach takes the 'general' out of general equilibrium analysis, has some force.

In a series of papers, Nachbar (2002, 2004, 2008) derives 'minimal conditions' under which intuitive comparative static results, such as those above, hold. Following Nachbar (2004) his results can be expressed as follows. Consider a production economy $\boldsymbol{E} = \{X_i, \preceq_i, \omega_i, Y_j, \theta_{ij}, \ell\}_{i=1}^{n}{}_{j=1}^{m}$, where for all i, $X_i = \Re_+^\ell$, $\omega_i \in \Re_+^\ell \backslash \{0\}$ and $x_i \in X_i$ is a consumption by vector for $i, x = (x_1, \ldots, x_n)$ is a *consumption allocation*, $\overline{x} = \sum_i x_i$ is an *aggregate consumption* vector, $\omega = (\omega_1, \ldots, \omega_n)$ is an *endowment allocation* and $\overline{\omega} = \sum_i \omega_i$ is aggregate endowment. The demand function for i is a map $x_i : \Re^{\ell+1}_{++} \to \Re^{\ell}_{+}$ denoted by $x_i(p, m_i)$. Assume that budgets are exhausted so that $p^T x_i(p, m_i) = m_i$ and $X(p, \omega) = \sum_i x_i(p, \omega_i)$ is *aggregate demand.* Consider the difference between $\overline{x}$ and $X(p, \omega)$ and denote it by $\overline{\psi}^{\Delta}(p, \overline{x}, \omega)$. Central to Nachbar's results is the behaviour of wealth effects. He shows that if $\overline{x}$ is aggregate consumption at prices p, then $\overline{\psi}^{\Delta}(p, \overline{x}, \omega) = X(p, \omega) - \overline{x}$ can be interpreted as the *aggregate wealth effect* when the wealth of i is changed to $p^T \omega_i$. For unchanged prices, if $p^T \overline{\psi}^{\Delta}(p, \overline{x}, \omega) \neq 0$, then the discrete analogue of the marginal propensity to consume, namely the *aggregate incremental propensity to consume* can be defined through the expression $\overline{\mu}^{\Delta}(p, \overline{x}, \omega) = (1/p^T \overline{\psi}^{\Delta}(p, \overline{x}, \omega))$. $\overline{\psi}^{\Delta}(p, \overline{x}, \omega)$ and this captures the vector of consumption changes when prices are constant and the wealth of consumer i is changed to $p^T \omega_i$. Since the sum of incremental propensities to consume at constant prices is one, $p^{*T} \overline{\mu}^{\Delta}(p^*, \overline{x}, \omega) = 1$. So if $p'^T \overline{\mu}^{\Delta}(p^*, \overline{x}, \omega) > 0$ then the term $[p'/(p' \overline{\mu}^{\Delta}_x) - p^*]$ can be thought of as a vector of normalised price changes for the price normalisation $p^T \overline{\mu}^{\Delta}(p^*, \overline{x}, \omega)$. Finally, the aggregate demand $X(p, \omega)$ satisfies the *strong form of the weak axiom of revealed preference* at the price pair p^*, p' and the endowment profile $\omega \Leftrightarrow [p^* X(p', \omega) \leq p^* X(p^*, \omega)$ and $p' X(p^*, \omega) \leq p' X(p', \omega)]$ both hold $\Leftrightarrow X(p^*, \omega) = X(p', \omega) \Leftrightarrow p^*, p'$ collinear. Given this set-up Nachbar (2004) obtains the following result on the effect of an endowment redistribution.

Theorem 10.3 (Nachbar (2004)). *Consider* $\boldsymbol{E} = \{X_i, \preceq_i, \omega_i, \ell\}_{i=1}^n$ *an exchange economy parameterised by* ω. *Let* $\boldsymbol{E}^*$ *and* $\boldsymbol{E}'$ *be the economies associated with endowment distributions* ω^* *and* ω', *and* (p^*, x^*), (p', x') *be the equilibrium price and consumption allocation for* $\boldsymbol{E}^*$ *and* $\boldsymbol{E}'$, *respectively. If* $(n.1)$ $\overline{\omega} \gg 0$; $(n.2)$

$X(p,\omega)$ satisfies the strong form of the weak axiom of revealed preference; $(n.3)$ $\overline{\psi}^{\Delta}_{\omega} = \overline{\psi}^{\Delta}(p^*, \overline{x}^*, \omega'), p^*\overline{\psi}^{\Delta}_{\omega} \neq 0, \overline{\mu}^{\Delta}_{\omega} = \overline{\mu}^{\Delta}(p^*, \overline{x}^*, \omega')$ *with* $p'\overline{\mu}^{\Delta}_{\omega} > 0$ *then* (1) $[p'/(p'\overline{\mu}^{\Delta}_{x}) - p^*].(\overline{x}' - \overline{x}^*) \leq 0$ *with equality if and only if* p', p^* *are collinear if and only if* $(\overline{x}' - \overline{x}^*) = \overline{\psi}^{\Delta}_{\omega}$; (2) *if* $\overline{\psi}^{\Delta}_{\omega} = 0$ *then* $(p' - p^*).(\overline{x}' - \overline{x}^*) \leq 0$ *with equality if and only if* p', p^* *are collinear if and only if* $\overline{x}' = \overline{x}^*$.

Proof. Nachbar (2004; pp. 159–160). □

Remark 10.4. This result establishes that an aggregate wealth effect caused by an endowment redistribution in an economy where goods are weakly normal and where the aggregate demand function satisfies a strong form of the weak axiom, will produce aggregate demand changes that are negatively correlated with the vector of price changes. This result represents an advance relative to Proposition 10.1 for instance, because it allows a change in the entire distribution of endowments, not just a change in the endowment of one good and the aggregate normality condition allows some degree of flexibility for non-normal demand behaviour by some individuals. Unfortunately, as Nachbar (2004; p. 160) notes, the assumption that the strong form of the aggregate weak axiom holds is both necessary and sufficient for the result. Nachbar (2002; p. 2069) also notes that the aggregate excess demand satisfies the weak axiom in the neighbourhood of a *regular* equilibrium p, then the aggregate (excess) demand for the economy $X(p)$, $[Z(p,\omega)]$ satisfies *the law of demand* if $(p-p')^T(X(p)-X(p') \leq 0, [(p-p')^T Z(p) - Z(p')] \leq 0$. If that is the case and if p is an equilibrium price vector, then $v^T D_p\ X(p)v \leq 0$ for any $v \in \Re^{\ell}$ with $D_p\ X(p)v = 0 \Leftrightarrow$ either $v = 0$ or v is collinear with p (correspondingly for $Z(p)$). Given that satisfaction of the weak axiom (or the law of demand) is necessary and sufficient for Theorem 10.3, it is of interest to see what sort of conditions on the microeconomics of the economy, if any, imply these restrictions hold.

In this regard, Jerison and Quah (2008) provide the following interesting argument. Consider a consumer in a standard competitive price taking environment with ℓ goods, facing prices $p \in \Re^{\ell}_{+}\backslash\{0)$. If they are a 'type α' consumer, they express demand $x(p, m, \alpha) \in \Re^{\ell}$, where this demand satisfies the budget constraint $p^T x(p, m, \alpha) = m$ and is differentiable enough to have a Slutsky matrix. Demands also satisfy the usual adding up conditions. Suppose type α consumers have an endowment ω_α, then $x(p, m, \alpha) = x(p, p^T \omega_\alpha,\ \alpha)$. Also suppose there is a distribution ς over the types of consumers in the economy. The aggregate demands are $X(p) = \int x(p, p^T\omega_\alpha,\ \alpha)d\varsigma$ and excess demands are $Z(p) = X(p) - \overline{\omega}$, where $\overline{\omega} = \int \omega_\alpha d\varsigma$ is aggregate endowment. As Hildenbrand and Kirman (1988) originally observed and as Jerison and Quah (2008) emphasise, essentially because both $X(p)$ and $Z(p)$ are homogeneous of degree zero in prices, the 'law of demand holds only in exceptional circumstances'. As Jerinson and Quah (2008) note, one case where it does hold is when all consumers endowments are *collinear*, meaning that for each type α there exists $k \geq 0$ with $\omega_\alpha = k\overline{\omega}$ then if $(p, p') \in \{p \in \Re^{\ell}\colon p^T\overline{\omega} = 1\}$ the law of demand holds for price change in a way that preserves mean income then the law of demand will hold. Collinear endowments is a very strong condition and when it is dropped, not even the assumption of homothetic preferences will lead to the law of demand. However, Quah (1997) shows that if the economy is such that all consumers have *homothetic preferences* and preferences and endowments are *independently distributed*, then this is enough to guarantee the law of demand. Again however, the assumption of independence here is not a comfortable one as the distribution of endowments, and the incomes they generate, can reasonably be thought of as having some sort of influence on tastes. As for the weak axiom on aggregate excess demand, Jerison and Quah (2008) point out that if the *mean Slutsky matrix* $\overline{S}(p) = \int S(p, m, \alpha)d\varsigma$, where $S(p, m, \alpha)$ is the Slutsky matrix for the type α consumers, is negative semi-definite (which follows from utility maximization) and if individual excess demands satisfy Jerison's condition of *non-decreasing dispersion of excess demand* (see Chap. 8 for a discussion), then the aggregate excess demands satisfy a version of the weak axiom, which says $p^T Z(p') \leq 0$ implies that $p'^T Z(p) \geq 0$. Again as the discussion in Chap. 8 shows, there is nothing particularly 'general' about this condition.

Given the evident difficulty in finding combinations of and plausible economic assumptions to structure the Jacobian of the excess demand map, it is interesting to explore an approach to general equilibrium comparative statics that is known as 'qualitative comparative statics' which takes a relatively agnostic view about the underlying microeconomics of the economy and asks instead if there are patterns of shocks which can impact on patterns of excess demand price derivatives to give complete comparative static predictions. We now consider some of what that approach has to offer.

10.2.3. *Qualitative comparative statics*

The starting point of qualitative comparative statics is to ask if there is a sign pattern on $D_pZ(p^{\circ};\, \alpha^{\circ})$ and $D_\alpha Z(p^{\circ};\, \alpha^{\circ})$ that permits a complete signing of $Dp(\alpha^{\circ})$, where a *sign pattern* involves a selection from the symbols $\{+, 0, -\}$. Suppose that in Eq. (10.3) the matrix $D_pZ(p^{\circ}; \alpha^{\circ})$ was 2×2 and had sign pattern $\left[\begin{smallmatrix} - & + \\ - & - \end{smallmatrix}\right]$ and the 2×1 shock matrix $D_\alpha Z(p^{\circ};\, \alpha^{\circ})$ has sign pattern $\left[\begin{smallmatrix} - \\ 0 \end{smallmatrix}\right]$. Then the system becomes: $\left[\begin{smallmatrix} - & + \\ - & - \end{smallmatrix}\right]\left[\begin{smallmatrix} \partial p_1/\partial\alpha \\ \partial p_2/\partial\alpha \end{smallmatrix}\right] = \left[\begin{smallmatrix} - \\ 0 \end{smallmatrix}\right]$. The sign pattern of the solution is determined by $\left[\begin{smallmatrix} \partial p_1/\partial\alpha \\ \partial p_2/\partial\alpha \end{smallmatrix}\right] = \left[\begin{smallmatrix} - & + \\ - & - \end{smallmatrix}\right]^{-1}\left[\begin{smallmatrix} - \\ 0 \end{smallmatrix}\right] = \left[\begin{smallmatrix} + \\ - \end{smallmatrix}\right]$. This example, due to Quirk (1997; p. 131), is one of a comparative static system that has *full sign solvability* because $\partial p_1/\partial\alpha > 0$ and $\partial p_2/\partial\alpha < 0$ emerge as comparative static predictions. It is not always the case that full sign solvability obtains. For instance if the example above is changed to: $\begin{bmatrix} - & 0 \\ - & - \end{bmatrix} \begin{bmatrix} \partial p_1/\partial\alpha \\ \partial p_2/\partial\alpha \end{bmatrix} = \begin{bmatrix} - \\ - \end{bmatrix}$, then as Quirk (1997) shows this system becomes *partially sign solvable* and yields the prediction $\partial p_1/\partial\alpha > 0$ and $\partial p_2/\partial\alpha = ?$. It is not hard to see that there are combinations of signs on the Jacobian of the excess demands and the shock matrix which produce complete indeterminancy. The aim of the qualitative calculus is to describe situations in which it is possible to obtain sign solvability. The main results of the field may be summarised as follows. In an early contribution, Rader (1972a) produced examples where qualitative comparative statics were impossible in economies with two or more complementary factors in production, while Bassett, Habibagahi and Quirk (1967) provided the first characterisation results. Detailed surveys of the field are available in Quirk (1997), Hale *et al.* (1999), Buck and Lady (2005). See also Lang, Moore and Whinston (1995) for an interesting discussion of computational approaches to qualitative calculus. The result which summarises the scope of the qualitative calculus in a general equilibrium context is the following.

Theorem 10.4. *[Hale et al. (1999)] If the numeraire is a substitute for all other goods, the law of demand holds for all goods, excess demands are homogeneous of degree zero and satisfy Walras' Law, all terms in the Jacobian matrix* $D_pZ(p^o, \alpha^0)$ *are non-zero, then comparative statics system* $D_pZ(p^o; \alpha^o)\cdot Dp(\alpha^o) + D_\alpha Z(p^o; \alpha^o)$ *is fully sign solvable if and only if*: (i) $D_pZ(p^o, \alpha^o)$ *is a Metzler matrix and all the entries in the shock matrix* $D_\alpha Z(p^o; \alpha^o)$ *are of the same sign*; *or* (ii) *the off-diagonal entries in the kth row or column of* $D_pZ(p^o; \alpha^o)$ *are negative with all other off-diagonal entries positive and* $D_\alpha Z(p^o; \alpha^o)$ *has one non-zero entry*; *or* (iii) *there is exactly one negative off-diagonal entry in* $D_pZ(p^o, \alpha^o), \partial Z_i/\partial p_j$ *with all other off-diagonal entries in* $D_pZ(p^o, \alpha^o)$ *being positive and* $D_\alpha Z(p^o; \alpha^o)$ *has one non-zero entry in the ith position.*

Proof. Hale *et al.* (1999; pp. 205–215). □

Remark 10.5. This result reinforces an observation we have made before that general equilibrium comparative statics seems to depend on a particular combination of excess demand price responses interacting with a particular pattern of shocks to the economy. In summarising the scope of the qualitative calculus, McKenzie (2008) argues that: "... the gross substitute case and the Morishima case may be shown to be the only sign patterns for the Jacobian matrix of the demand functions with all elements non-zero which allows the inverse matrix to be signed without quantitative information". McKenzie (2008; p. 775). While that is certainly an accurate summary of Part (i) of Theorem 10.4, the qualitative calculus does actually have a slightly wider scope as indicated by Parts (ii) and (iii) of the result above. However it is worth noting that quite special structure is needed for comparative static results to emerge in those cases.

10.2.4. *Samuelson's 'correspondence principle'*

All of the approaches to finding comparative static predictions so far considered involve, one way or another, structuring $D_pZ(p^{\circ}, \alpha^{\circ})$ and $D_{\alpha}Z(p^{\circ}, \alpha^{\circ})$ in order to obtain results. As has been observed many times in the literature, such a strategy is mostly at variance with the Sonnenschein-Mantel-Debreu result and the 'general' ambitions of general equilibrium theory. It is therefore of considerable interest to explore circumstances in which definitive comparative static predictions arise from general equilibrium models without imposing undue additional structure on either the economy or the shocks that impact on it. An approach by Samuelson (1941, 1942, 1947) which he called 'The Correspondence Principle' has, on the face of it, particular appeal as it seems to hold the promise of almost 'condition free' comparative statics.

Samuelson's Correspondence Principle is based on the ingenious idea that assuming equilibrium prices are stable, may imply something about the comparative static properties of equilibrium. As Gandolfo (1987) puts it: "This principle suggests that the ambiguity of certain expressions which appear in the solution of a comparative static system can be removed by *assuming* that the equilibrium is stable and using the dynamic stability conditions." Gandolfo (1987; p. 462). In similar vein, Neary (1980) remarks: "The point of all this is [that] ... what matters is the information which the *hypothesis of stability* itself provides about the comparative static effects ..." Neary (1980; p. 817). In a related remark Balasko (1988) contends "[This general result] Theorem 7.3.9 [establishes that] the properties usually associated with comparative statics directly depend, in fact on the number of equilibria. The 'correspondence principle' stated by Samuelson (1947) which relates uniqueness, stability and the comparative statics of equilibria, anticipates in some sense the results of Theorem 7.3.9." Balasko (1988; p. 190).

In order to develop the Correspondence Principle, Samuelson (1947) argued as follows. Suppose that prices are adjusted according to the process $\mathrm{d}p_i/\mathrm{d}t = Z_i(p(t);\alpha)$ for $i = 1, \ldots, \ell$. Using Taylor's theorem, this system can be linearised in the neighbourhood of an equilibrium price vector p^*, as $D_t p(t) = D_p Z(p^*, \alpha)[p(t) - p^*]$. If p^* is assumed to be (locally) stable with respect to the price adjustment process, then certain restrictions are implied on the eigenvalues[2] of $D_p Z(p^*, \alpha)$. But this is one of the matrices that needs to be restricted in Eq. (10.4) if comparative static predictions are to be obtained. This apparent 'correspondence' between the assumption of stability of equilibrium and the existence of definitive comparative static results lead Samuelson (1947), Neary (1980), Gandolfo (1987), and others such as Kemp (1987), Kemp, Kimura and Tawada (1990), to

[2]For details of what these restrictions are see for example Allingham (1975).

remark on the 'mutually supportive dualistic relationship which exists' between economic dynamics and comparative statics.

On the face of it, such an approach to obtaining comparative static results appears to be very attractive and superior to those approaches that impose structure directly on preferences or technologies (and hence on $D_pZ(p, \alpha)$). This is so because if one is prepared to do equilibrium economics at all, it seems reasonable to suppose that the equilibria being dealt with are stable. Samuelson's own 'egg argument' in support of this point of view is perhaps worth stating at this point. The argument is that although an egg has two equilibria at either pole, as well as a set of equilibria around its equator, we would not expect to see an egg in either of its pole equilibria because those equilibria are unstable. By analogy, if one is prepared to do equilibrium economics then it seems little more to ask that the equilibria under study are assumed to be stable. Allingham (1975) summarises the argument as follows: "It was noted in [earlier] that an egg standing on its head, or an economy in an unstable equilibrium would seldom be observed. This suggests that we may reverse the argument of [earlier], and instead of considering what attributes of the economy ensure its stability, assume the economy to be stable and consider what information, particularly comparative static information this implies. The existence of such implications is known as the correspondence principle." Allingham (1975; p. 91). Looked at this way, the correspondence principle approach to obtaining comparative static results seems to do less violence to the general nature of general equilibrium analysis than do the various alternatives which directly restrict preferences or unduly constrain the nature of the shocks which impinge on the economy.

However, in spite of the clear intuition behind the Correspondence Principle, the logical status, scope and usefulness of the Principle has turned out to be controversial. Patinkin (1965) for instance presented an example in which the desired comparative static expressions: "...remain indeterminate even after we add the condition that the system must be stable. That is, dynamic analysis *does not* provide the necessary additional information about comparative static analysis: the 'correspondence principle' does not work." Patinkin (1965; p. 499, emphasis added). Also, Arrow and Hahn (1971) have asserted that there is a logical error underlying the Correspondence Principle, an error which they argue may be seen as follows. Conditions which are sufficient for the stability of Walrasian equilibrium under tatonnement, such as WARP, Gross Substitutes and Diagonal Dominance,[3] also permit unambiguous comparative static predictions on their own account. Thus, it is the *conditions for* stability rather than the supposition of stability *per se* which gives rise to comparative static predictions. Arrow and Hahn (1971) summarise their argument against the Correspondence Principle in the following terms: "[the conclusion] that an 'intimate connection' between stability and comparative statics [exists] ... is too hasty and the impression delusory [since] all these restrictions share the characteristic that they are not necessary for the task for which they were invented, they are only sufficient and this explains why the correspondence principle 'isn't'." Arrow and Hahn (1971; pp. 320, 321).

Arrow and Hahn's conclusion is arrived at after analysis of the following example. Consider an economy with four goods, in which good 4 is the numeraire. Let the parameter α enter the excess demands of only goods 1 and 4 and let $\partial z_1/\partial\alpha > 0$. If the initial equilibrium is described by $z_1(p,\alpha) = z_2(p) = z_3(p) = z_4(p,\alpha) = 0$ then in order to get the comparative static effects of a change in α we need to solve the system: $(\partial z_1/\partial p)\mathrm{d}p + (\partial z_1/\partial\alpha)\mathrm{d}\alpha = (\partial z_2/\partial p)\mathrm{d}p + 0 = (\partial z_3/\partial p)\mathrm{d}p + 0 = 0.$

[3]See the definitions of these conditions in Chap. 4.

This can be written as: $D_p z(p, \alpha)\mathrm{d}p + \partial z_1(p, \alpha)/\partial\alpha \mathrm{d}\alpha = 0$. Therefore, the comparative static expression is $\mathrm{d}p/\mathrm{d}\alpha = -[D_p z(p, \alpha)]^{-1}(\partial z_1/\partial\alpha)$. Now the supposition of local stability of p means that the roots of the matrix $D_p z(p, \alpha)$ have positive real parts. However, this is not enough to give complete information about $\mathrm{d}p/\mathrm{d}\alpha$ in this case. This leads to the remark that: "The necessary conditions for local stability are too weak for the comparison task. This is even more striking, of course when the number of goods is large and global stability is at stake." Arrow and Hahn (1971; p. 321). Kehoe (1987) has similarly argued that the principle is of limited usefulness claiming that: "He [Samuelson] called this methodology the Correspondence Principle. Unfortunately, however except for very low-dimensional cases ($n = 2$, 3) very few such theorems seem available." Kehoe (1987; p. 136). This point is made also by Kemp (1987) and Evans and Honkapohja (2007).

Given the controversy surrounding the status and applicability of the correspondence principle, we wish to devote some time to exploring its foundations. We, being with a result in Samuelson (1971) who showed that in a two country world, if 'final' world Walrasian equilibrium is globally stable relative to a simultaneous tatonnement of the sort specified in Eq. (8.1) of Chap. 8, then the effect on the terms of trade of an international transfer depends only on the impact of the transfer on excess demands at the initial equilibrium. In particular the terms of trade effect is independent of the size of the transfer. Nor does it depend on the stability of the original equilibrium. Samuelson (1983) formally labelled this result the Global Correspondence Principle. In an extension of Samuelson's work, Bhagwati, Brecher and Hatta (1987) observed that in a two good, two country world the parametric disturbance need not be a transfer of goods or income between agents but would equally well apply to any disturbance in the parameters which define the economy. Kemp *et al.* (1990) formulate the GCP as follows:

Global Correspondence Principle. *If a two-good economy is globally stable, then given any Walrasian disequilibrium the price of each commodity in excess demand will approach a level above that currently prevailing and the price of any good in excess supply will fall to a level below that currently prevailing. So* $\Delta p_2 \cdot \Delta Z_2 > 0$ *if there is a disturbance to the excess demand for goods* 2 *and* 1 *is the numeraire.*

Proof. Kemp *et al.* (1990; p. 2). □

Remark 10.6. As Kemp *et al.* (1990) point out, the beauty of the GCP is that it allows the deduction of intuitively appealing comparative static results once we know the sign of the disturbance to the excess demand for good 2. They also note however that as it stands, the GCP only applies to two commodity models, something also noted by Kehoe (1987). An interesting question is then: does the GCP hold for economies with $n > 2$ goods? In other words is it true that $\Delta p_i \Delta Z_i > 0$ if $\Delta Z_i \neq 0$ when $i = 1, \ldots \ell$, and $\ell > 2$. Kemp *et al.* (1990) show, by means of a counter-example, that the answer to this question is 'no'. They then turn to the problem of finding sufficient conditions (on the excess demand functions) for modified versions of the GCP to hold. The authors note that: "[although] the GCP is invalid for $n > [2] \ldots$ attenuated versions of it, valid for arbitrary n, can be found by restricting the Jacobian of the excess demand functions." Kemp *et al.* (1990; pp. 2, 3). The question addressed here, and not addressed by Kemp *et al.* (1990) is: how restrictive are the conditions on the Jacobian and just how attenuated is the GCP after all?

To answer this question, recall the 'Hicksian chain condition' considered earlier in this chapter. This condition requires that there be at least three non-numeraire commodities and that all goods are normal. It also requires that substitutes of substitutes and complements of complements are substitutes, and substitutes of complements and complements of substitutes are complements.[4] As Allingham (1975; pp. 93, 94) demonstrates, it is *only* in such an economy that the Local Correspondence Principle gives complete comparative static information. From Kemp *et al.* (1990) Corollary 1, it follows that this is also true of the GCP. Two questions then naturally arise: (i) how general is such an economy; and (ii) does the Correspondence Principle add anything that is not already in the structure of the Jacobian as a result of the Hicksian chain condition? In order to answer that question, express the Hicksian condition in the following form.

Definition 10.1 (Murata (1977; p. 152)). The Jacobian matrix of the excess demand map of an economy satisfies the Hicksian chain condition and is a *Morishima matrix* if there exist two non-empty sets R, $S \subset \{1, \ldots, \ell\}$ with $\partial Z_i/\partial p_j > 0$ for i, $j \in (R$ or $S)$ and $\partial Z_i/\partial p_j < 0$ if $i \in R$ and $j \in S$, or vice versa. Equivalently, any square matrix with negative main diagonal where the sign $(\partial Z_i/\partial p_j) = \text{sign}(\partial Z_j/\partial p_i)$ for all $i \neq j$ and sign $[(\partial Z_i/\partial p_k) \cdot (\partial Z_k/\partial p_j)] = \text{sign}(\partial Z_i/\partial p_j)$ for distinct i, j, k is a Morishima matrix.

Remark 10.7. If the Jacobian of the excess demand function is a Morishima matrix, then a pair of commodities i, j can be unambiguously labelled as substitutes if $\partial Z_i/\partial p_j$ is positive and complements if it is negative. Note also that a Morishima matrix is sign symmetric. This fact is of interest in the argument which follows — see Murata (1977) for further discussion.

[4] Allingham (1975) gives the following example of an economy where the Morishima conditions hold: an economy with coffee, tea, milk and cream; and one where they fail: an economy with coffee, cream, whisky and soda — for the standard uses of those commodities.

Proposition 10.2 (Kemp-Kimura-Tawada).[5] *The class of economies for which the LCP or GCP give complete comparative static information is the class for which the Jacobian of the excess demand function is a Morishima matrix. Thus* D_pZ*, being a Morishima matrix is a necessary and sufficient condition for the Local Correspondence Principle and the Global Correspondence Principle to be useful.*

Proof. Kemp *et al.* (1990; p. 4). □

Remark 10.8. This result characterises situations in which the LCP and GCP are applicable. Note, as Allingham (1975) does, that Morishima matrices are not necessarily stable. Consequently: "...the assumption of stability made by the Correspondence Principle is a real restriction." Allingham (1975; p. 93) — see also the discussion of this point in Hale *et al.* (1999; p. 176). In light of this observation, the Patinkin-Arrow and Hahn-Kehoe criticism appears to be too strong since the assumption of stability of equilibrium does add additional information, because Morishima matrices are not necessarily stable. Furthermore, the assumption of stability does allow the extraction of complete comparative static information *in the case where* D_pZ *is a Morishima matrix.* The question remains however as to the generality of the class of environments in which the LCP and GCP are useful. This reduces to a consideration of the question: how general are Morishima matrices? An answer to this question may be obtained by appeal to the ideas and techniques of generic analysis. These techniques were discussed in Chap. 3. As was noted there, one way to summarise the genericity of a condition,

[5] In connection with this result the following remark due to Allingham (1975) is of interest " ...the condition is known as the Hicksian chain rule, a rule which of course need only hold for non-numeraire commodities. Only in normal economies with this property is the correspondence principle helpful ... [or] in the Morishima case the correspondence principle generates complete comparative static information, though that is the only case where it does this." Allingham (1975; pp. 93, 94).

result or abstract object is the following: if in constructing a theoretical object, one has to be *relatively careful*, then it is likely that the object is *not generic* in the space of objects in which it resides. With this in mind we now consider the likelihood of encountering a Morishima matrix in the space of Jacobian matrices.

Proposition 10.3. *Let M be the set of Morishima matrices and let J be the space of Jacobian matricies of excess demand functions in an Arrow-Debreu economy. Then M forms a low probability set in the space J when the economy contains a large number of goods.*

Proof. If there are N commodities in the economy and three possible signs $\{+, 0, -\}$ for the entries in the Jacobian matrix of $Z(p), D_pZ(p)$, then from the Sonnenschein-Mantel-Debreu theorem we know that there are $3^{N\times N}$ equally likely possible sign patterns for this Jacobian. By definition a Morishima matrix can display $3^{N(N-1)/2}$ possible sign patterns because $\partial Z_i/\partial p_j = \partial Z_j/\partial p_i$ for all $i \neq j$ and $\partial Z_i/\partial p_i < 0$. Consider the ratio $K = [3^{N(N-1)/2}]/[3^{N\times N}]$. This measures the likelihood of drawing a Morishima matrix from the set of possible Jacobians of the excess demand function of an economy. Since $K = 3^{-(N\times N+N)\cdot 1/2}$ as N becomes large $\lim_{N\to\infty} K = 0$, so K becomes small in an economy with many commodities. Thus the set of Morishima matrices forms a small set in the set of possible matrices, particularly as the number of commodities increases. □

Remark 10.9. The above argument is one way to see that Morishima matrices are not generic and are therefore ‘unlikely’ to occur in general, unless that structure is implied by other, reasonable, conditions. Another way to consider genericity is to consider how *robust* the Morishima property is to perturbation. The idea for this approach stems from a discussion of the Weak and Strong axioms of revealed preference due to Mas-Colell *et al.* (1995), in which they make the following argument: "We have not focussed on the strong axiom [because] the WA is a robust property, whereas the SA (which, remember, yields the symmetry of the Slutsky matrix) is not: *a priori*, the chances of it being satisfied by a real economy are essentially zero [since] if we perturb every preference slightly and independently across consumers, the negative semi-definiteness of the Slutsky matrix (and therefore the WA) may well be preserved but symmetry (and therefore the SA) will almost certainly not be." Mas-Colell *et al.* (1995; p. 115). Applying the same logic to the case in which the Jacobian of the excess demands has the Morishima property, we see that it too is a symmetric matrix. This symmetry is liable to be disturbed by arbitrary perturbation and hence the Morishima property is not robust to perturbations of the primitives that define the economy and hence not generic. Thus we can conclude that the Correspondence Principle has non-generic foundations and therefore it is of limited value in the task of obtaining comparative static results in general circumstances. However, it is true that the Principle does add some meaningful restrictions in a Morishima environment. This is so because, contrary to the Arrow and Hahn argument against the Principle, Morishima matrices do not imply stability of a sequential tatonnement process.

10.2.5. *Homotopy methods*

Along with a number of other authors, Eaves and Schmedders (1999) argue for 'the inevitable and lasting role which homotopy methods will play in theoretical and applied economics'. Following Mas-Colell *et al.* (1995; p. 597), a homotopy in the context we are wanting is the following one-parameter family of excess demand functions where good ℓ has been chosen as the numeraire: $Z(p,t) = tZ(p) + (1-t)Z^0(p)$, where the excess demand function $Z^0(p)$ is simple and its properties are known and by letting t vary from 0 to 1, it is hoped that something can be learned about the equilibrium properties of $Z(p)$. This potentially ties in nicely with the comparative statics problem where a change in the parameters that define the economy in effect replace one excess demand function with another. To see how it can work, the following example due to Eaves and Schmedders (1999) is useful. Consider a 2×2 exchange economy. Each agent i has a CES utility function $u_i(x_1, x_2) = \sum_{\ell=1}^{2} a_\ell^i (x_\ell^{b^i} - 1)/b^i, a_\ell^i \geq 0$ and $b^i < 1$. The elasticity of substitution is $\eta_i = 1/(1-b^i)$ and i's demands for the two goods are given by the expression $x_\ell^i(p_1, p_2) = [(a_\ell^i)^{\eta_i} \sum_{k=1}^{2} p_k w_k^i / [p_\ell{}^{\eta_i} \sum_{k-1}^{2} (a_k^i)^{\eta_i} p_k^{1-\eta_i}]$. Parameters values for the economy are $b^1 = b^2 = -4, a^1 = (1024, 1), a^2 = (1, 1024), \omega_1 = (10, 1), \omega_2 = (1, 12)$. Prices are allowed to vary in the open set $\mathcal{P} = (0,1) \times (0,1)$ and equilibrium prices for this economy, denoted by $\boldsymbol{E}(1)$, is when $Z(p_1, p_2) = 0$ where:

$$Z(p_1, p_2) = \begin{cases} x_1^1(p_1, p_2) + x_1^2(p_1, p_2) - \omega_1^1 - \omega_2^1 \\ p_1 + p_2 - 1. \end{cases}$$

The homotopy method involves approaching the given economy $\boldsymbol{E}(1)$ from a starting point in an 'easy' or simple economy $\boldsymbol{E}(0)$. The economy $\boldsymbol{E}(t)$ is economy $\boldsymbol{E}(1)$ except that the endowment of the second agent is $\omega_2(t) = t(1, 12)$. As Eaves and Schmedders (1999; p. 1267) show the homotopy $H : \mathcal{P} \times [0,1] \to \Re^2$ is given by:

$$H(p_1, p_2, t) = \begin{cases} H_1^1(p_1, p_2, t) \\ \qquad = x_1^2(p_1, p_2) - \omega_1^1 + t(x_1^2(p_1, p_2)) - \omega_2^1 \\ H_2(p_1, p_2) = p_1 + p_2 - 1, \end{cases}$$

The (unique) equilibrium prices for the economy $\mathcal{E}(0)$ occur when $H_1(p_1,\ p_2, 0) = 0$ and this turns out to be $(p_1,\ p_2) = (0.010136, 0.989864)$. Now deform $\mathcal{E}(0)$ into $\mathcal{E}(1)$ by letting t go from 0 to 1 and 'following' the equilibrium price set until $t = 1$ and produces the equilibrium for the economy of interest $\mathcal{E}(1)$, $(p_1,\ p_2) = (0.951883, 0.048117)$. Now suppose the shock to the economy is a variation in the endowment of agent 1. Denote the economies generated by variations in the endowment of agent 1 by $\mathcal{E}(\omega_1^1(t))$. Suppose to begin $\mathcal{E}(\omega_1^1(t)) = \mathcal{E}(1)$ except that the endowment of the first agent is $\omega_1^1(t) = 10+4t$ for $t \in [0, 1]$. This economy starts at $\mathcal{E}(1)$ and when $t = 0.5$ it is $\mathcal{E}(12)$ and at $t = 1$, $\mathcal{E}(14)$. To get comparative static results, we need to be able to compare the equilibria of the starting point economy $\mathcal{E}(1)$ with those of the end point. Suppose that the new endowment due to agent 1 was $\omega_1 = (12, 1)$, then the equilibria of the economy $\mathcal{E}(12)$ are the ones of interest. However as Kehoe (1991; Example 2.1) and Eaves and Schmedders (1999) note, this economy has three equilibria (0.887076, 0.112924), (0.5, 0.5) and (0.112924, 0.887076), and it is therefore not clear what the comparative static prediction for goods prices should be. So while homotopy methods are very useful, their usefulness will generally be interfered with, at least in the comparative static context by what Mas-Colell *et al.* (1995; p. 620) identify as a 'manifestation of a serious shortcoming in general equilibrium theory — the lack of a theory of equilibrium selection'.[6]

We have considered the major techniques that have been brought to bear in an attempt to derive general equilibrium comparative static results along with a number of the results that the application of those techniques have been able to produce. We conclude this chapter by considering an interesting of class of results which it seems natural to refer to as 'welfare comparative statics'.

[6]Note however the ideas in the direction of providing a theory of equilibrium selection in Eaves and Schmedders (1999; p. 1274).

10.3. Welfare comparative statics

10.3.1. *Competitive perversity, the transfer paradox and manipulation via endowments*

While it is interesting and important to try to discover the comparative static properties of equilibrium prices and quantities, it might reasonably be argued that such an exercise is economically interesting if it tells us something about how utility levels vary in response to changes in the fundamentals of the economy. Indeed, one of the primary purposes of economics is to work out 'ways in which economic circumstances can be improved' and to analyse the welfare effects of proposed courses of action in the management of economic affairs. When welfare improving policies are sought in the context of a general equilibrium model, then the 'welfare comparative static' properties of equilibrium states are of central importance.

One economic policy that has been extensively investigated in the literature from a welfare perspective involves the transfer of endowments, or income, among agents in the economy or among countries in the world. One might reasonably expect the welfare effects of such transfers to be reasonably straightforward. That this is not the case is perhaps testimony to how subtle comparative static effects can be in multi-good, multi-agent economies. In this section, we analyse the welfare effects of certain transfer policies and attempt to develop a general formula that allows the identification of Pareto improvements and Pareto improving policies in a general equilibrium context.

In their discussion of general equilibrium comparative statics, Blad and Keiding (1990, pp. 167–168) give an example of how surprising and counterintuitive price responses can be to variations in the primitives which define the economy. Their example involves an exchange economy in which an increase in the amount of a good available in the economy actually causes its

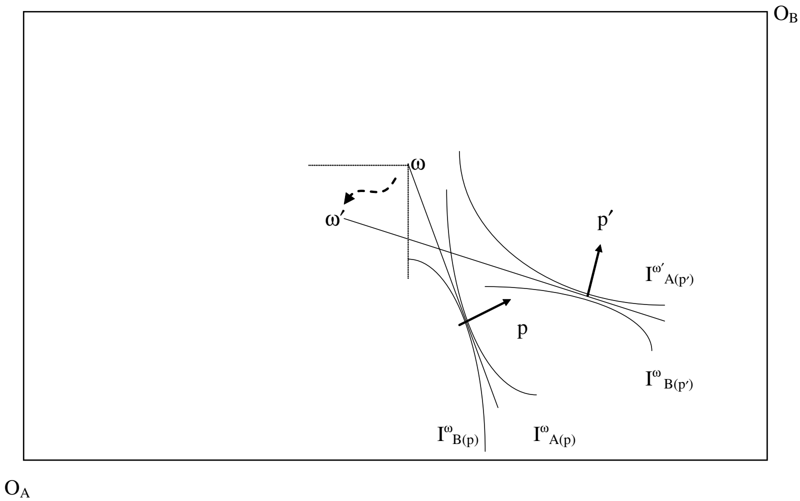

Fig. 10.1. An example of 'competitive perversity' or a 'transfer paradox'.

price to increase. Another example much studied in the literature under the heading of the 'transfer paradox' may be illustrated as follows. Consider a two-by-two exchange economy of the type represented in Fig. 10.1.

The initial endowment in the economy is at ω. Suppose person A transfers some of both of the goods they own to person B so that the new endowment point is now ω'. The 'normal' expectation would be that such a transfer would result in an increase in the welfare of the recipient (B) and a decrease in the welfare of the donor (A). As the example in Fig. 10.1 illustrates, however when the variation in prices which this transfer induces is considered, this need not be the outcome. In particular, even though B is the recipient and A is the donor, the result of this transfer is that when the new equilibrium is established at p', A is strictly better off and B is strictly worse off than they were in the initial equilibrium p — see Woodland (1982; pp. 296–298) for a discussion of this possibility. Apart from being a counterintuitive comparative static result, with considerable policy interest — a theme that will be taken up below — this result is also important to general equilibrium theory because it raises the possibility that equilibrium prices might be open to manipulation via endowment reallocations. If this is indeed the case, it would undermine a key assumption of general equilibrium analysis, namely that agents take prices as given. Considering this point, Safra (1987) notes that the reason for concern with potential manipulation of equilibrium prices via reallocation of endowments stems from the fact that: "...it might well happen that economic agents will find it advantageous to change their endowment holdings and by this increase their utility. Such strategic behaviour *contradicts the fundamental competitive assumption* that agents cannot influence market prices." Safra (1987; p. 516, emphasis added).[7] As Safra's remarks make plain, an important part of general equilibrium theory relies on the idea that agents in the economy behave competitively and that they 'take market prices as given' and beyond manipulation.[8]

There is however quite a lot to be argued if one wants to support the hypothesis of price taking. As Hahn (1982) points out: "The theory has a lively sense of original sin — all people act in their own self interest narrowly defined. But if that is so, will not individuals, or groups of individuals seek to find ways to exert market power? By market power, I mean a situation where an individual's actions can influence market prices. How can we be sure that the hypothesis that individuals act as if prices were given is not in conflict with the postulate that they are rational self-seeking agents? The answer is that we can only be sure if there is no market power for individuals to exploit." Hahn

[7] Safra goes on to argue: "Mathematically, this assumption is equivalent to single agents being negligible relative to the whole economy. For that reason strategic behaviour is strongly connected to the finiteness of the economy. Strategic behaviour of groups of agents, however, can very well be effective in continuum economies. Thus the phenomenon of strategic reallocation of endowments, although more probable in finite economies, is surely not limited to the finite cases."

[8] See Roberts (1987; p. 838) for an extended discussion of this point.

(1982; p. 6). The question which then arises is: what is it that is needed in an economy if there is to be no market power for individuals to exploit? One line of research aimed at answering this question, initiated by Aumann (1964, 1966) and summarised by Khan (1987), resolves the problem by postulating a continuum of atomless agents in the economy. The logic of this approach follows from the observation that the circumstances under which Walrasian equilibrium prices will be taken as given by all agents in the economy are just those circumstances where the equilibrium allocation is not sensitive to the actions of any individual agent.[9] Aumann (1964, 1966) established that the circumstances in which an individual economic agent is economically negligible and unable to manipulate market prices is co-extensive with the set of circumstances in which they were also numerically negligible. His answer to the question 'when is the economy perfectly competitive?' was: 'when it has an uncountable infinity (or a continuum) of agents, all of whom are of Lebesgue measure zero'.[10] This is not a particularly comfortable result for the Walrasian program, because if Aumann's conditions are necessary and sufficient for the economy to be perfectly competitive, as Aumann (1964, 1966) argued they are, then the supposition of price taking in actual *finite* economies is in some doubt.

In light of this, it is interesting to investigate the circumstances under which price taking behaviour will occur in finite economies. To that end, an alternative approach to the problem of price taking behaviour was proposed by Makowski (1980) and Ostroy (1980), and further developed by Makowski and Ostroy (1991). This approach focuses on a so called 'no-surplus' condition as the defining characteristic of an economy. The essence

[9]As Khan (1987) puts it: "An allocation of resources generated under *perfect competition* is an allocation of resources generated by the pursuit of individual self-interest and one which is insensitive to the actions of an single agent." Khan (1987; p. 831).

[10]This way of characterising perfect competition was taken up and refined using nonstandard analysis by Brown and Robinson (1975) and through various asymptotic core equivalence results by Bewley (1973) and Anderson (1986), (see Khan (1987) for details).

of this approach is to suppose that the economy is so arranged that each agent has preferences, endowments and production technology such that the other agents in the economy are as well off when this agent trades as when he or she does not. If this is the case, then a 'no surplus' condition is satisfied.[11] The interesting things about a no surplus allocation from our point of view are as follows: (i) whether the number of agents is large or small, a no-surplus allocation is a Walrasian equilibrium; (ii) the no surplus condition is equivalent to the condition that each agent faces a perfectly elastic demand schedule for the goods he/she sells at Walrasian prices; (iii) almost all economies with small numbers of agents are not no surplus economies. With a large, typically infinite, number of agents and a finite number of commodities, all Walrasian equilibria are no surplus allocations (see Ostroy (1980) and Makowski (1980) for details). The third property of no surplus allocations means that this approach is not so different after all to that of Aumann (1964) since although the no surplus condition *could* hold in a finite economy it appears that it almost never does. As Roberts (1987) puts it: "No-surplus allocations correspond to the economy's having Walrasian equilibria at the same prices with or without a single agent ... An economy is defined as perfectly competitive if the no-surplus condition is met. This can happen with a finite number of agents, but *typically it requires an infinity*." Roberts (1987; p. 839, emphasis added).

Consideration of the ways in which Walrasian equilibrium might be manipulated through reallocations of endowments, leads Safra (1987) to the following taxonomy of cases: redistribution of initial endowment among members of a particular

[11] The situation is summarised by Hahn (1987) as follows: "In an economy with very many agents the market environment of any one of these is independent of the market actions he decides upon. More generally one can characterise the economy as *perfectly competitive* if the removal of any one agent from the economy would leave the remaining agents just as well off as they were before his removal. (The economy is said to satisfy a 'no surplus condition' [in such circumstances])." Hahn (1987; pp. 575, 576).

coalition, (*C-manipulation*); withholding of part of an agent's endowment from the market, (*W-manipulation*); transferring by means of a gift part of one's endowment to another individual, (*G-manipulation*); destroying part of one's initial endowment (*D-manipulation*). Numerous results are available which indicate that these forms of manipulation are possible in a wide variety of circumstances.[12] In what follows, we focus on one particular form of manipulation, namely G-manipulation. This form of manipulation has been much studied as an example of 'competitive perversity' in the mathematical economics literature and as the 'transfer paradox', in the international trade literature. Our point will be to demonstrate that this is one form of manipulation which Walrasian equilibrium is *not* generally open to for a reason not to our knowledge previously noted in the literature. In summary, the reason turns out to be that such behaviour would violate individual rationality if agents were to attempt it. It is therefore one sort of competitive perversity which is unlikely to show up as a result of the deliberate actions of agents in a finite economy.

In order to make our argument, it is necessary to see what is involved in implementing G-manipulation or, equivalently, successfully exploiting a transfer paradox. To begin, consider again the case illustrated in Fig. 10.1, in which agent A gives up some of both goods, but in the new equilibrium experiences a level of utility higher than that experienced in the original equilibrium. This counterintuitive possibility has, as Turunen-Red and Woodland (1988) note, led to the formulation of one of the interesting questions in international trade theory, namely: does a transfer of income (endowment) between two nations (agents) necessarily benefit the recipient at the expense of the donor or can a 'transfer paradox' happen in which the donor

[12] See Postlewaite (1979) and Donsimoni and Polemarchakis (1994).

gains and the recipient loses?[13] The later possibility may open up the potential for donors to manipulate equilibrium prices in their favour by making gifts out of their initial endowments. As Grinols (1987) notes: "Recent theoretical attention devoted to the transfer paradox has included attempts to incorporate it into the body of commercial policy." Grinols (1987; p. 477).

The first investigation of the transfer problem was undertaken by Leontief (1937) who demonstrated the possibility of a transfer paradox by constructing a 2×2 exchange economy similar to that in Fig. 10.1, where a transfer resulted in donor enrichment and recipient immiseration. The relevance of Leontief's work was challenged by Samuelson (1947) who showed that a transfer paradox cannot occur, in a 2×2 model, unless the initial Walrasian equilibrium is unstable. In a straightforward application of the philosophy underlying the Correspondence Principle, he then argued that a transfer paradox of the sort proposed by Leontief would almost never be observed because unstable equilibria are, at best, transitory. Subsequent work showed that the dependence of a transfer paradox on the instability of the initial equilibrium holds for any 2 (agent) $\times$ n (commodity) economy (see Woodland (1982)). However, works by Bhagwati *et al.* (1982), Chichilnisky (1983), Postlewaite and Webb (1984), Kemp and Kojima (1985), Safra (1990) and Rao (1992), Djajic, Lahiri and Raimondos-Moller (1998), have provided examples of m-country, ℓ-commodity worlds, with $m \geq 3$ and $\ell \geq 2$, in which a variety of apparently interesting transfer paradoxes can occur, even at stable Walrasian equilibria. In this context, recall the observation by Grinols (1987), noted above, that the existence of a transfer paradox raises the apparently attractive strategic option for an agent (country) to transfer part of its initial endowment to a third party and in the process manipulate

[13]As our discussion above, and that in Safra (1987) makes clear, there is nothing in the problem which restricts it to being considered only in the context of international transfers. Any transfer between individual agents will do.

equilibrium prices so as to finish up better-off in utility terms. What we propose to show is that although Walrasian equilibria may well be *open* to this form of manipulation, such manipulation will typically not be undertaken by rational self-interested agents. Thus we are led to the conclusion, not previously noted in the literature, that Walrasian equilibria are generally 'strategy proof' to G-manipulation, even in finite economies.

10.3.2. *Conceptual shortcomings of the transfer paradox and G-manipulation*

In his consideration of the transfer paradox and G-manipulation, Safra (1987) concluded that there is no incentive for the potential recipient to accept the transfer if to do so would make it worse off. He therefore claimed that there was a 'conceptual shortcoming' in the literature which studies the transfer paradox. This is so because for a transfer paradox to occur one agent, the recipient, has to take an action which is welfare dominated by an available alternative. Also making Safra's point, Rao (1992) asks: "...why does an agent accept a gift which will end up lowering its welfare level?" Rao (1992; p. 139). Rao's rhetorical question reinforces Safra's point of a conceptual shortcoming in the transfer paradox and G-manipulation literature.

However, not all transfer paradoxes and instances of G-manipulation rely on the recipient losing in order for the donor to gain. Consequently, Safra (1987) and Rao (1992) go only part of the way to identifying a deeper conceptual shortcoming associated with a transfer paradox and G-manipulation. To see this, note that if there exists a transfer which makes the donor better off, then the recipient and/or some other non-participating parties to the transfer are either worse off, no better off, or better off. In the first case, the recipient has no incentive to accept the transfer, as Safra (1987) and Rao (1992) correctly point out. In addition (and this does not seem to have been appreciated in the literature), *in all cases* the potential donor *has no incentive to make the transfer*. This is so because if the potential for a paradoxical transfer exists then, as will be demonstrated, there exists for the donor a known and feasible action which is superior to making the transfer. It follows that when the potential for G-manipulation of Walrasian equilibrium exists, that potential will never *intentionally* be realised via transfers initiated by agents in the economy. This is so either because the intended recipient will refuse to accept the transfer (Safra and Rao's point) or, because the potential donor will realise that to transfer is not individually rational and will therefore not pursue it.

In order to illustrate what we have in mind here consider the following example of G-manipulation due to Postlewaite and Webb (1984). The economy has two goods and three agents who are characterised by the following utilities and endowments:

$$\begin{array}{ll} U^1(x_1,x_2) = 4x_1 + 5x_2 + 3\min(2x_1,x_2) & \omega^1 = (2,0) \\ U^2(x_1,x_2) = x_2 + 2x_1 & \omega^2 = (0,1) \\ U^3(x_1,x_2) = x_1 + 2x_2 + 3\min(x_1,1/2) & \omega^2 = (0,1). \end{array} \quad (10.5)$$

As Postlewaite and Webb (1984) show there is a unique and stable Walrasian equilibrium at prices $p_1 = 1$, $p_2 = 1/2$ with allocations $(3/2,1),(0,1),(1/2,0)$ and welfare levels $U^1 = 14$, $U^2 = 1$, $U^3 = 2$. Suppose that agent 2 makes a gift of 7/16 units of good 2 to agent 1. The post transfer equilibrium prices, allocations and welfare levels are, respectively $p_1 = 1$, $p_2 = 2$, $(5/8,5/4)$, $(7/8,0)$, $(1/2,3/4)$, $U^1 = 12.5$, $U^2 = 1.75$, $U^3 = 3.5$. In this case a transfer paradox has occurred since the donor is better off and the recipient is worse off in the post-transfer equilibrium as compared with the pre-transfer equilibrium.

Two conceptual problems associated with the supposition of manipulation of Walrasian equilibrium prices in this manner, are nicely illustrated in this example. Firstly, as Safra (1987) notes, there is no incentive for agent 1 to accept the transfer in circumstances where to do so would make it worse off. Unless one introduces an arbitrary asymmetry in the calculating abilities of agent 1 compared with those of agent 2, the occurrence of a transfer paradox requires agent 1 to take an action which is not individually rational. If the realisation of a transfer paradox relies on such asymmetry or requires irrational behaviour on the part of agent 1, it loses considerable appeal as an agent initiated possibility. Secondly, and perhaps more importantly, there exists an action which welfare dominates a transfer as far as agent 2 (the potential donor) is concerned. To illustrate what this action is, instead of transferring 7/16 out of its initial endowment to agent 1, let agent 2 submit to the market demands generated by the following problem:

$$\begin{array}{ll} \text{Max } x_2 + 2x_1 & \omega^2_{-T} = (0, 9/16) \\ \text{Max } 4x_1 + 5x_2 \min\,(2x_1, x_2) & \omega_T = (0, 7/16), \end{array}$$

while agents 1 and 3 are left to submit demands based on their original utility functions and initial endowments. Thus, instead of making a transfer to agent 1, agent 2 retains ownership of that part of its endowment which was to be transferred and generates demands out of the proposed transfer which *mimics* agent 1's behaviour. This is precisely designed to reproduce the effect on economy wide excess demand of the actual transfer and hence is designed to influence prices in the same way that an actual transfer would. The action will however also yield an additional consumption benefit to agent 2, provided only that his or her utility function is non-satiated.

Note that it is feasible for agent 2 to undertake this exercise because in order to carry out the original manipulation of equilibrium, it needed to know the characteristics of agent 1, in particular its utility function, in order to be certain that it was transferring goods to the agent whose excess demands would be such as to cause equilibrium prices to move in the desired way. Thus, in a situation where Walrasian equilibrium can be manipulated via a donor enriching, recipient harming transfer, there is no incentive for the *donor* to make the transfer, because that action is utility dominated by a feasible alternative.

Instead of a donor enriching, recipient harming transfer, it may be possible to find a transfer between two agents which leaves them both better off and a third, non-participating agent, worse off. This special case of G-manipulation is usually referred to in the literature as an 'advantageous reallocation' or more colorfully as an 'invisible shakedown'. An occurrence of this type of manipulation via endowments is illustrated in the following example due to Leonard and Manning (1983).[14] Consider a two good, three agent exchange economy parameterised as follows:

$$\text{Agent 1: } U_1(x,y) = x_1^{1/2} \cdot y1^{1/2} \quad \omega_1 = (10,0)$$

$$\text{Agent 2: } U_2(x,y) = x_2^{7/8} \cdot y2^{1/8} \quad \omega_2 = (4,0)$$

$$\text{Agent 3: } U_3(x,y) = 5.25 - 0.047x_3^3 + y_3$$

$$\omega_3 = (0,c) \text{ with } c > 12. \tag{10.6}$$

Let good 2 be the numeraire. As Leonard and Mannin (1983) show the initial equilibrium is $(p_1, p_2) = (1,1)$ and initial consumption levels are $(5,5), (7/2, 1/2), (11/2, c\text{-}11/2)$. A transfer of 4 units of good 1 from agent 1 to agent 2 is proposed. Equilibrium is now $(p_1, p_2) = (3,1)$ and the equilibrium allocations are (3, 9), (7, 3) and $(4, c\text{-}12)$. Agents 1 and 2 are better off since

$$U_1(3,9) = 3^{1/2}9^{1/2} > U_1(5,5) = 5^{1/2}5^{1/2}$$

$$U_2(7,3) = 7^{7/8}3^{1/8} > U_2(7/2,1/2) = 7/2^{7/8}1/2^{1/8},$$

while agent 3 is worse off since $U_3(4, c\text{-}12) = c - 9.758 < U_3(11/2, c\text{-}11/2) = c - 8.069$.

Although this transfer has made both the recipient and the donor better off and is therefore immune to the point made by Safra and Rao, it is not immune to our point. There is still a conceptual shortcoming in the supposition that such a transfer

[14] A similar example is presented in Veendorp (1992).

will occur because agent 1 can improve its welfare even further. Instead of transferring 4 units of good 1 to agent 2, agent 1 can enter the market and mimic the behaviour of agent 2 with the amount it proposed to transfer. That agent 1 is in a position to mimic agent 2 is again clear since under the standard form of the transfer paradox, the donor has to know the characteristics of agent 2 in order to be sure that it is transferring the relevant amount to the appropriate agent in order to engineer the desired change in Walrasian prices. Armed with that much information, however, the donor is also in a position to mimic the proposed recipient's demand behaviour, engineer the appropriate price change *and* enjoy the consumption benefits which flow from trading the retained initial endowment. To illustrate this, suppose that instead of making the transfer, agent 1 submits to the market demands based on the solution to the following problem:

$$\begin{array}{lll} \text{Agent 1:} & \max_{x,y} x_1^{1/2} y_1^{1/2} & \text{for endowment } \omega_1 = (6,0) \\ & \max_{x,y} x^{7/8} y^{1/8} & \text{for endowment } \omega_T = (4,0) \\ \text{Agent 2:} & \max_{x,y} x_2^{7/8} y_2^{1/8} & \text{for endowment } \omega_2 = (4,0) \\ \text{Agent 3:} & \max_{x,y} 5.25 - 0.047 x_3^3 + y_3 & \text{for endowment } \omega_3 = (0,c). \end{array} \tag{10.7}$$

As far as market excess demand is concerned, this will have the same effect on prices as does the transfer of 4 units of good 1 to agent 2 so $p = 3$ is the new equilibrium price. However, the new consumptions are $[(3,9) + (7/2, 3/2)]$, $(7/2, 3/2)$, $(4, c\text{-}12)$ for agents 1, 2, 3, respectively. Since $U_1(13/2, 21/2) > U_1(3,9)$ agent 1 has no incentive to make the transfer to agent 2. Thus even the 'invisible shakedown' form of G-manipulation requires agent 1 to take an action which is not individually rational, since another feasible and utility superior, action exists. This argument may be summarised in a general result as follows. Let $T > 0$ be a transfer from (potential) donor

D to (potential) recipient R. Let p' be the pre-transfer equilibrium price vector, p'' be the post-transfer equilibrium price vector with $U_D(x(p'')) > U_D(x(p'))$. Then there exists a function U_F such that $\max_x U_F$ subject to $p''x = p''T$ will mimic the transfer and deliver additional consumption benefits to D compared with making the transfer, meaning that $U_D[\max_x U_D$ s.t. $p''x = p''(\omega_D - T) + \max_x U_F$ s.t. $p''x = p''T] > U_D(x(p''))$.

Remark 10.10. The idea of establishing the existence of a function U_F which mimics a transfer is that since a transfer paradox relies on an excess demand function Z' with a zero at p' being replaced by Z'' with a zero at p'' due to the transfer of T from D to R, if we can replace Z' by Z'' by introducing U_F instead of the transfer, then we will be able to show that the potential donor can engineer the welfare improving price variation *and* enjoy the consumption generated by maximising U_F. The optimisation problems which generate Z' and Z'' are as follows: let $\boldsymbol{E}$ be a finite exchange economy defined by the set of agents $\{D, R, \ldots, O\}$ who are characterised by utility functions and endowments (U_D, ω_D), $(U_R,\ \omega_R), \ldots, (U_O, \omega_O)$. Then $Z'(p) = [x'_D(p) - \omega_D] + [x'_R(p) - \omega_R] + \cdots + [x'_O(p) - \omega_O]$, where x'_i is generated by the problem: $\max_x U_i(x)$ s.t. $px_i = p\omega_i$ for $i = D, R, \ldots, O$. Suppose that D transfers $T > 0$ to R so that the economy $\boldsymbol{E}_T$ is created. $\boldsymbol{E}_T$ is characterised by the utility functions and endowments $(U_D, \omega_D - T)$, $(U_R, \omega_R + T), \ldots, (U_O, \omega_O)$. The corresponding excess demand function Z'' for this economy is: $Z'' = [x''_D(p) - (\omega_D - T)] + [x''_R(p) - (\omega_R + T)] + \cdots + [x''_O(p) - \omega_O]$, where $x''_D(p)$ is generated by the problem $\max_x U_D(x)$ s.t. $px_D = p(\omega_D - T)$; x''_R is generated by $\max_x U_R(x)$ s.t. $px_R = p(\omega_R + T)$ and the $x''_O(p)$ terms are generated as before.

Using this notation, a transfer paradox occurs if $U_D[x_D(p'')] > U_D[x_D(p')]$, where p' is a Walrasian equilibrium associated with Z' and p'' is a Walrasian equilibrium associated with Z''. We now show that if $\boldsymbol{E}$ generates Z' and $\boldsymbol{E}_T$, generates Z''

then there exists a utility function U_F such that Z'' is also generated by $\boldsymbol{E}_A$, where this "artificial" economy is made up by the utility functions and endowments in (10.8).

$$\begin{aligned}
&(U_D, \omega_D - T),\\
&(U_F, T),\\
&(U_R, \omega_R),\\
&\quad\vdots\\
&(U_O, \omega_O)\\
&Z'' = [x''_D(p) - (\omega_D - T)] + [x''_F(p) - T]\\
&\qquad + [x''_R - \omega_R] + \cdots + [x''_O - \omega_O],
\end{aligned} \tag{10.8}$$

where x''_F is generated by the problem: $\max_x U_F$ s.t. $px = pT$ and $x''_D(p), x''_R(p), x''_O(p)$ are generated as before.

Does such a U_F exist? Suppose that a transfer from D to R has occurred and that a new equilibrium price has been established at p''. For the fictious agent to have the same effect on excess demand as does the transfer, excess demand should not be disturbed by the introduction of this new agent, the deletion of $p''T$ from the income of R and the assignment of $p''T$ to F. In other words, the increase in demand caused by F receiving income $p''T$ should just offset the changed demand of R through losing $p''T$. Let x^T be the demand point when R gets the transfer and x^R be the demand when there is no transfer then what we require is that F's demands replace those of R. To see that this is possible consider Fig. 10.2.

This analysis suggests that we should be able to find a utility function to mimic the effect of the transfer at least in the circumstance where all goods are normal for the potential recipient.

Proposition 10.4. *If all goods are normal for a potential recipient and if a potential donor's utility function*, U_D *is locally non-satiated then there exists a utility function* U_F *that mimics the proposed transfer. Further* $U_D[x(p'') + \max U_F$ *s.t.*

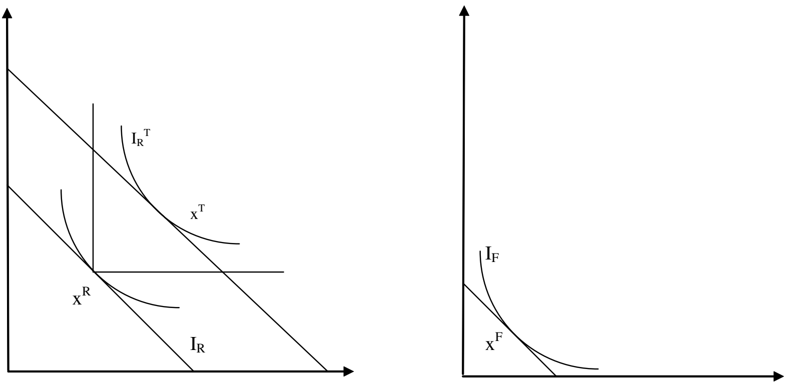

Fig. 10.2. Fictional preferences replicating a transfer paradox.

$p''x = p''T] > U_D[x(p'')]$, *where* $x(p'')$ *is the solution to the problem* $\{\max_x U_D$ *s.t.* $p''x = p''(\omega - T)\}$*.*

Proof. The potential recipient's utility U_R when maximised subject to $p''(\omega_R+T)$ yields the solution x^T and when maximised subject to $p''\omega$ yields the solution x^R. Normality of all goods implies that $x^R \ll x^T$. We can therefore translate the origin to x^R so that U_F becomes U_R with the origin at x^R. Therefore U_F exists. To establish the desired inequality notice that provided $\max_x U_F$ s.t. $p''x = p''T$ occurs at an interior optimum, then given that U_D is not satiated, the inequality follows. $\square$

Remark 10.11. On the basis of our work in this section, it is reasonable to argue that although the literature on the transfer paradox and G-manipulation is large and active, it may be of limited relevance in understanding *agent initiated manipulation* of Walrasian equilibrium. This is so because, as we have seen, the occurrence of a transfer paradox requires either the recipient or the donor (or both) to take an action which is inferior from their point of view to an available alternative. The need for the recipient to take an irrational action if a transfer paradox is to occur has been known since Safra (1987) and Rao (1992). The need for the donor to also behave irrationally appears to be a new observation.

Remark 10.12. Three important words in the above remark are ‘agent initiated manipulation’. Although we have shown that the sort of perverse outcomes studied in the transfer paradox literature are unlikely to occur as a result of agent-initiated transfers, it does not follow that perverse effects flowing from a redistribution may not be observed when such a redistribution is initiated by parties other than consumers in the economy. For instance governments, or their agencies, perhaps operating to implement the Second Fundamental Theorem of Welfare Economics, may inadvertently redistribute endowments in such a way as to cause a transfer paradox. In view of this possibility, it is necessary to investigate the correlation between redistributions and welfare changes in a general context.

10.3.3. *Pareto improvements and the correlation between redistribution and welfare*

Donsimoni and Polemarchakis (1994) have attempted to unify and generalise the available results on the effects of endowment transfers, particularly the nature of the correlation between price and welfare changes and redistributions of endowment in exchange economies. In what follows, we extend the result of Donsimoni and Polemarchakis (1994) to cover the case of a production economy. We also consider an interesting case of endowment redistribution and welfare changes not covered by their theorem. Before doing that however, we present a general formula for the welfare effects of variations in the parameters of the economy.

As was noted at the beginning of the chapter, one of the primary aims of economics is to find ways in which individual welfare can be improved. One apparently value neutral criterion for judging welfare improvement is the criterion suggested by Pareto: *if a policy results in an increase in the welfare of at least one person and no reduction in the welfare of anyone else then such a policy yields a* (*Pareto*) *improvement*. Given this notion, Pareto improving policies, and conditions for their existence, have been sought in a number of contexts. For instance Diewert, Turunen-Red and Woodland (1989) have sought conditions for the existence of Pareto improving tariff and tax changes in the context of a small open economy without public goods. Lockwood (1995) has sought conditions for Pareto improving multilateral tax harmonisation policies in a model where all governments produce public goods. Cornes and Sandler (1998) have characterised Pareto improving redistributions in the context of a closed economy model with public goods. Hammond and Sempere (1995) investigate the problem of turning aggregate gains in welfare into Pareto improvements, a problem also studied by Dixit (1987) and Mandler (1999). Many of these contributions make special assumptions for their analysis to proceed. We now present a formula which characterises Pareto improvements. The *indirect utility function* for consumer i, V^i, expresses a consumer's welfare as a function of prices and income. In particular, letting good be the numeraite $u^i = V^i(p_1, \ldots, p_{\ell-1}, 1, m^i)$. So,

$$\mathrm{d}u^i = \sum_k \partial V^i/\partial p_k \cdot \mathrm{d}p_k + \partial V^i/\partial m^i \cdot \mathrm{d}m^i. \tag{10.9}$$

The effect across the economy of changes in price income is

$$\begin{array}{l}
\mathrm{d}u^1 = \partial V^1/\partial p_1 \cdot \mathrm{d}p_1 + \partial V^1/\partial p_2 \cdot \mathrm{d}p_2 + \cdots + \partial V^1/\partial p_{\ell-1} \cdot \mathrm{d}p_{\ell-1} + \partial V^1/\partial m^1 \cdot \mathrm{d}m^1 \\
\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad\qquad \vdots \qquad \vdots \qquad\qquad \vdots \qquad \vdots \\
\mathrm{d}u^H = \partial V^H/\partial p_1 \cdot \mathrm{d}p_1 + \partial V^H/\partial p_2 \cdot \mathrm{d}p_2 + \cdots + \partial V^H/\partial p_{\ell-1} \cdot \mathrm{d}p_{\ell-1} + \partial V^H/\partial m^H \cdot \mathrm{d}m^H
\end{array} \tag{10.10}$$

Making the obvious definitions and normalising the marginal utility of income to 1, (10.10) can be written as:

$$\underset{(H\times 1)}{\mathrm{d}u} = \underset{(H\times(\ell-1))}{(\partial V/\partial p)} \cdot \underset{((\ell-1)\times 1)}{\mathrm{d}p} + \underset{(H\times 1)}{(\mathrm{d}m)} = \underset{(H\times(\ell-1))((\ell-1)\times 1)(H\times 1)}{V_p \cdot \mathrm{d}p + \mathrm{d}m} . \quad (10.11)$$

In private ownership economies here, consumer income depends on endowments and profit shares interacting with prices. We may write this dependence in general terms as $m = F(p, \alpha)$.[15] So:

$$\mathrm{d}m = F_p \mathrm{d}p + F_\alpha \mathrm{d}\alpha. \quad (10.12)$$

Combining (10.11) and (10.12) yields:

$$\mathrm{d}u = V_p \cdot \mathrm{d}p + F_p \mathrm{d}p + F_\alpha \mathrm{d}\alpha. \quad (10.13)$$

Using Eq. (10.4) with (10.13) yields the following formula for welfare changes:

$$\mathrm{d}u = -[V_p + F_p] \cdot \{[D_p Z(p,\alpha)]^{-1} \cdot D_\alpha Z(p,\alpha) + F_\alpha\}\mathrm{d}\alpha. \quad (10.14)$$

This formula allows Pareto improvements to be identified as those perturbations of the α's such that $\mathrm{d}u \geq 0$ and $\neq 0$ (for a *weak* Pareto improvement), or $\mathrm{d}u \gg 0$ (for a *strict* Pareto improvement).

With this formula established, we now return to the task of extending the analysis of transfers in Donsimoni and Polemarchakis (1994), to the case of a private ownership production economy. To begin, recall the central finding of Donsimoni and Polemarchakis (1994) which is that the welfare and price changes, following a redistribution of endowments in an exchange economy, can follow any pattern. Donsimoni and Polemarchakis (1994) summarise their work in the following terms: "The redistribution of welfare following a redistribution of endowment is arbitrary ... [and our] argument cautions

[15]Then in a private ownership production economy $\mathrm{d}m^i = \mathrm{d}\omega_0^i + \mathrm{d}p \cdot \omega^i + p \cdot \mathrm{d}\omega^i + \sum_j \theta_{ij} \cdot \partial\pi^j(p)/\partial p_j \mathrm{d}p_j$.

against the attempt to evaluate the welfare effects of redistribution policies without due attention to the consequent changes in equilibrium prices." Donsimoni and Polemarchakis (1994; pp. 235, 236). This lesson is also evident in the formula developed in Eq. (10.14) above.

Using the notation in Donsimoni and Polemarchakis (1994), we now extend their analysis to the case of a production economy and in this set up investigate the relationship between endowment redistribution and welfare changes.

10.3.3.1. *Institutional set-up*

Consider a complete market Arrow-Debreu production economy with $L+1$ commodities $\ell \in \{0, 1, \ldots, L\}$. Prices for commodities are $(1, p) \in \{1\} \times \Re^L_{++}$ and commodity 0 is the numeraire good ('money').

10.3.3.2. *Consumers*

Let $u^i : X \to \Re$ be the utility function of consumer i and $\omega^i = (\omega^i_0, \underline{\omega}^i) \in X$ be the endowment of i. Here ω^i_0 is i's endowment of the numeraire good and $\underline{\omega}^i$ denotes i's endowment of non-numeraire goods. Without loss of generality the transfers considered here will be restricted to transfers of the numeraire commodity. Let θ_{ij} be the share of firm j owned by consumer i and let the vector $\theta^i = (\theta_{i1}, \theta_{i2}, \ldots, \theta_{iJ})$ be the list of shares owned by i. In order to maximise utility, consumer i expresses $[z^i_0(p, \omega^i, \theta^i), z^i(p, \omega^i, \theta^i)]$ which are i's excess demands. Then for any good ℓ, including the numeraire, $z^i_\ell = x^i_\ell - \omega^i_\ell$.

Following Donsimoni and Polemarchakis (1994), we say that the behaviour of consumer i is *regular* (*strongly regular*) at $(p, \omega^i_0, \theta_{ij})$ iff there is an open neighbourhood of this point where (i) a utility maximising solution exists, is unique and satisfies the budget constraint $z^i_0 = -pz^i$; and (ii) the excess demand functions are continuous, differentiable and satisfy the following

Slutsky equation $D_p z^i = S^i - v^i \cdot z^{iT} - v^i \theta^i y$. Here S^i is a symmetric and negative semi-definite (negative definite) Slutsky matrix, v^i is a vector of income effects, $v^i = D_{\omega 0} z^i$ and π_p is derivative with respect to p of the aggregate profit function.

The indirect utility function for i, $V^i(p, m^i)$, is assumed to be a continuously differentiable function of p and m^i, where m^i is the income of i. Since $u^i = V^i(p, m^i)$ it follows that $\mathrm{d}u^i = \partial V^i(p, m^i)/\partial p \cdot \mathrm{d}p + \partial V^i(p, m^i)/\partial m^i \cdot \mathrm{d}m^i$. From Roy's identity, we have that $x^i = -[\partial V^i(p, m^i)/\partial p]/[\partial V^i(p, m^i)/\partial m^i]$ so $\mathrm{d}u^i = -x^i \cdot \partial V^i(p, m^i)/\partial m^i \mathrm{d}p + \partial V^i(p, m^i)/\partial m^i \cdot \mathrm{d}m^i$. As before, let the marginal utility of income, $\partial V^i(p, m^i)/\partial m^i$ be 1 and noting that in the context of a private ownership production economy $m^i = \omega_0^i + p \cdot \omega^i + \sum_j \theta_{ij} \pi^j(p)$ we have $\mathrm{d}m^i = \mathrm{d}\omega_0^i + \mathrm{d}p \cdot \omega^i + p \cdot \mathrm{d}\omega^i + \sum_j \theta_{ij} \cdot \partial \pi^j(p)/\partial p \cdot \mathrm{d}p$. Since the transfer is restricted to a transfer of the numeraire commodity, we have the following expression, as a special case of (10.14) for the variation in utility which occurs for i as a result of the transfer:

$$\mathrm{d}u^i = -(z^i - \theta^i \pi_p)^T \mathrm{d}p + d\omega_0^i. \tag{10.15}$$

Equation (10.15) shows that a change in the welfare of i, $(\mathrm{d}u^i)$ depends on the original excess demand of i (z^i, which in part depends on the endowment vector of i, ω^i), the amount of the transfer of the numeraire commodity to or from i ($\mathrm{d}\omega_0^i$), the variation in profits flowing to i ($\theta^i \pi_p$), and the change in equilibrium prices which are caused by that transfer ($\mathrm{d}p$).

10.3.3.3. *The economy*

The economy is a collection of consumers $i \in \{1, \ldots, I\}$ and firms $j \in \{1, \ldots, J\}$. The firms are characterised by convex production possibility sets Y_j which technically restrict the feasible input-output vectors to be such that $y_j \in Y_j$. Again, with a slight abuse of notation the initial allocation of the numeraire in the economy will be denoted by $\omega_0 = (\omega_0^1, \ldots, \omega_0^I)$. Aggregate excess demand for commodities other than the numeraire is $Z(p, \omega_0, \theta) = \sum_{i \in I} z^i(p, \omega_0, \theta) - \sum_j y^j(p)$. A Walrasian equilibrium price vector $p(\omega_0, \theta)$ is such that $Z(p(\omega_0, \theta), \omega_0, \theta) = 0$. A triple $[p(\omega_0), \omega_0, \theta]$ is a *regular Walrasian equilibrium* iff: (i) the behaviour of every individual $i \in I$ is regular at that equilibrium; and (ii) the Jacobian of the excess demand, evaluated at that equilibrium, is of full rank, i.e. $|D_p Z| \neq 0$. If each individual is strongly regular at a regular Walrasian equilibrium, then the equilibrium is *strongly regular* and the aggregate Slutsky matrix $S^A = \sum_i S^i$ is of full rank, i.e. $|S| \neq 0$.

10.3.3.4. *The main result*

A *redistribution* is a variation $\mathrm{d}\omega_0$ in the initial endowment vector ω_0, which satisfies the restriction $\sum_i \mathrm{d}\omega_0^i = 0$. We now want to study the effect of this on equilibrium prices and then on welfare levels of the agents in the economy. From the definition of excess demand and equilibrium in this economy we have that:

$$Z_p \mathrm{d}p + Z_{\omega 0} \mathrm{d}\omega_0 = \left(\sum_i \partial z^i / \partial p \cdot \mathrm{d}p \right) + \left(\sum_i \partial z^i / \partial \omega_0^i \cdot \mathrm{d}\omega_0^i \right)$$
$$- \left(\sum_j \partial y^j(p) / \partial p \cdot \mathrm{d}p \right) = 0. \qquad (10.16)$$

Applying the Slutsky decomposition to (10.16) yields:

$$Z_p \mathrm{d}p + Z_{\omega 0} \mathrm{d}\omega_0 = \sum_i [S^i - (z^i - \theta^i \pi_p) \cdot v^i] \cdot \mathrm{d}p$$
$$+ Z_{\omega 0} \mathrm{d}\omega_0 - \pi_{pp} \cdot \mathrm{d}p = 0. \qquad (10.17)$$

Using the definitions of the aggregate Slutsky matrix and income effects (10.17) yields:

$$S^A - \sum_i (v^i \cdot z^i - v^i \theta^i \pi_p] \cdot \mathrm{d}p + \left(\sum_i v^i \mathrm{d}\omega_0^i \right) - \pi_{pp} \cdot \mathrm{d}p = 0. \qquad (10.18)$$

Since $[p(\omega_0), \omega_0, \theta]$ is a regular Walrasian equilibrium (10.18) can be inverted to yield:

$$\mathrm{d}p = -\left(S^A - \sum_{i \in I} v^i z^i - v^i \theta^i \pi_p\right)^{-1} \cdot \left(\sum_i v^i \mathrm{d}\omega_0^i\right). \quad (10.19)$$

Substitute (10.19) into (10.14) to obtain:

$$\mathrm{d}u^i = (z^i - \theta^i \pi_p) \cdot \left(S^A - \sum_i v^i z^i - v^i \theta^i \pi_p - \pi_{pp}\right)^{-1} \times \left(\sum_i v^i \mathrm{d}\omega_0^i\right) + \mathrm{d}\omega_0^i. \quad (10.20)$$

Now (10.20) holds for each $i = 1, \ldots, I$. Let the $I \times 1$ vector of utility changes in utility levels in the economy be denoted by $\mathrm{d}u$ and the $I \times 1$ vector of endowment changes be $d\omega_0$. Apart from satisfying the adding up restrictions $\sum_i \mathrm{d}u^i = \sum_i \mathrm{d}\omega^i = 0$ (the first of which follows from Pareto optimality of the Walrasian equilibrium in this model and the second from the definition of a redistribution), the vectors $\mathrm{d}u$ and $\mathrm{d}\omega$ are arbitrary. Analogously to the exchange of Donsimoni and Polemarchakis (1994) this is expressed formally as:

Proposition 10.5. *Let $a = (a^1, \ldots, a^I)$ and $b = (b^1, \ldots, b^I)$ be arbitrary $I \times 1$ vectors save for the condition $\sum_i a^i = \sum_i b^i = 0$. If $a^i b^i \neq 0$ for some $i \in I$ then there exists an economy with a strongly regular Walrasian equilibrium where $d\omega^I = a$ and $du^I = b$. In particular, if $H \geq L + 1$, and no individual is in autarky at the initial equilibrium, then there exists an economy in which for any $d\omega_0^H$ the vector du^H is arbitrary.*

Proof. From (10.4) we have:

$$\mathrm{d}p = -[D_p Z(p, \alpha)]^{-1} \cdot D_\alpha Z(p, \alpha). \quad (10.21)$$

From the SMD theorem, we know that provided H (the number of consumers) $\geq L + 1$ (the number of goods) the matrix in square brackets in (10.21) can be replaced by an arbitrary matrix A, subject only to $Ap = p^T A = 0$. Using this observation, (10.16) and (10.21) we have that

$$\mathrm{d}u^h = -z^{hT} \cdot A^{-1} \cdot D_\alpha Z(p, \alpha) + \mathrm{d}\omega_0^h. \tag{10.22}$$

In the present case, the 'shock' matrix $D_\alpha Z(p, \alpha)$ is just $D_{\omega 0H} Z(p, \omega_0^H)$, i.e. the matrix of aggregate income effects caused by a reallocation of endowments. Since utility maximisation does not restrict this matrix it too can be chosen arbitrarily and set equal to B. The expression in (10.22) then becomes

$$\mathrm{d}u^h = -z^{hT} \cdot A^{-1} \cdot B + \mathrm{d}\omega_0^h. \tag{10.23}$$

By virtue of the SMD theorem, we then have that for a given $\mathrm{d}\omega_0^h$ we are able to choose an economy, that is, choose individual excess demands (z^{hT}), price effects (A) and income effects (B), to achieve any desired change in welfare level $\mathrm{d}u^h$ for h. □

Write the economy wide version of Eq. (10.11) as:

$$\underset{(H\times 1)}{\mathrm{d}u^H} = \underset{(H\times L)(L\times 1)}{z^{HT} \cdot \mathrm{d}p} + \underset{(H\times 1)}{\mathrm{d}\omega_0^H}.$$

Then making use of (10.17)–(10.19), this becomes

$$\mathrm{d}u^H = z^{HT}[A]^{-1} \cdot B + \mathrm{d}\omega_0^H \tag{10.24}$$

$$(\mathrm{d}u^H - \mathrm{d}\omega_0^H) = z^{HT}[A]^{-1}B. \tag{10.25}$$

From the SMD theorem when $H \geq L + 1$, we know that A can be chosen arbitrarily. Similarly B is not restricted by the optimisation problem which underlies this expression because it is a matrix of income effects. Provided no individual is in autarky then z^{HT} will be non-zero. Therefore the terms on the RHS of (10.25) can be chosen arbitrarily. This implies that we can find an economy with any desired association between redistribution of endowments and changes in utility.

Remark 10.13. This result is an existence theorem for a particular sort of economy, namely one which exhibits the property that redistribution's of endowment and utility are not correlated. The implication which Donsimoni and Polemarchakis (1994) draw from this is that: "... [since] the information required to determine the redistribution of endowment which yields a particular redistribution of utilities is possibly prohibitive, it would have been desirable to establish some general relation between redistribution's of endowments and welfare. This is not possible." Donsimoni and Polemarchakis (1994; p. 241). Thus while their result does not show that transfers are *necessarily* paradoxical, it does warn that they may well be. It also establishes that there are no general 'rules of thumb' about how utilities will respond to (differential) transfers.

Remark 10.14. Notice that the Donsimoni and Polemarchakis (1994) result, along with the extension presented here, is for differential changes in endowments. It does not follow from this that there are not discrete changes in endowments that might achieve particular distributional objectives. If this were the case, then the results presented in Chap. 6 concerning the achievement of a particular Pareto optimum may be invalid. We now show that, even in the context of a model where a competitive perversity is possible, there still exists a feasible transfer that makes such an outcome impossible. To make this demonstration, reconsider Fig. 10.1, reproduced below and modified as Fig. 10.3.

Suppose instead of making the redistribution from ω to ω', the redistribution is to ω'' on the horizontal axis of the Edgeworth box. If this redistribution is undertaken and provided prices do not become zero or negative, the budget constraint for A is in the cone bounded by $O_A\omega''K$. A typical budget constraint for A is then $A'\omega''$. No matter what equilibrium prices finally prevail in this economy, it is clear that A cannot achieve a utility level greater than $I^{\omega}_{A(p)}$. Thus as a result of this transfer

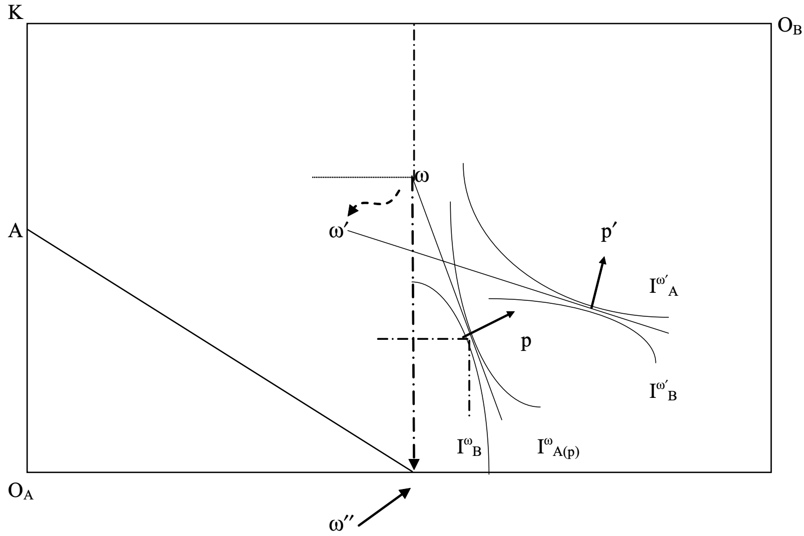

Fig. 10.3. Discrete redistribution.

A cannot be better off (and B cannot be worse off), in the new equilibrium than they were in the original equilibrium. Thus although Donsimoni and Polemarchakis (1994) are correct to argue that *differential* redistributions and welfare changes can be arbitrarily related, it does not necessarily follow that there are no discrete redistributions which support a particular redistribution of welfare.

Remark 10.15. We conclude by considering the following remark due to Donsimoni and Polemarchakis (1994): "If prices are fixed, an individual whose endowment increases gains utility. At competitive [Walrasian] equilibrium, however, prices depend on the distribution of endowments across individuals [and are therefore generally not fixed]." Donsimoni and Polemarchakis (1994; p. 235). It is clear from Eq. (10.11) that Donsimoni and Polemarchakis (1994) are correct to assert that when prices are fixed there can be no competitive perversity (because if $\mathrm{d}p = 0$ then $\mathrm{d}u^i = \mathrm{d}m^i$). What is not true however, is that Walrasian equilibrium prices *necessarily* vary as endowments change, contrary to what Donsimoni and Polemarchakis (1994) implicitly suppose. Smale (1979), for example, has constructed an economy in which it is true: "...that changes in the endowment parameters *don't necessarily affect* a price equilibrium. For example, if (x^*, p^*) is a [Walrasian] equilibrium at an endowment vector ω^* then [in this set up] (x^*, p^*) is also a [Walrasian] equilibrium for ω where $p^*(\omega - \omega_i^*) = 0$ and $\sum_i \omega_i^* = \sum_i \omega_i$. In fact, there is an $m\ell - (m + \ell)$ parameter family of such ω." Smale (1979; p. 547, emphasis added). Intuitively what is happening in Smale's example is that the Walras correspondence (discussed in Chap. 3), has a section which is parallel to the horizontal (endowment) axis. In such a situation and contrary to the contention of Donsimoni and Polemarchakis (1994), a redistribution of endowments will not influence Walrasian equilibrium prices.

10.4. Conclusion

For both methodological and policy reasons, it is highly desirable that general equilibrium models are able to produce unambiguous comparative static predictions about how prices, quantities and utility levels respond to variations in the parameters which define the economy. In large, complex economies in which there are many interconnected markets and multiple feedback loops active, it is not be surprising if there are considerable obstacles in the way of such an undertaking.

In this chapter, we have studied various approaches to obtaining definitive comparative static results. We have also studied 'welfare comparative statics', and have attempted to develop a formula for identifying parameter changes that lead to Pareto improvements; understand something of the welfare effects of endowment redistributions and the related issue of the possible manipulation of equilibria via endowment transfers; and the correlation between endowment redistribution and welfare changes. This is an interesting undertaking in its own right and also because of the implications it potentially carries for attempts to implement the Second Welfare Theorem, a result we discussed at length in the previous chapter.

As far as the manipulation of Walrasian equilibrium is concerned, on the face of it there appears to be a number of ways in which individuals can manipulate Walrasian equilibria in finite economies. Interestingly however, such manipulation does not seem likely to be implemented via the mechanism studied extensively in the literature, namely G-manipulation. This is so because in circumstances where manipulation of this form could occur, there are always better options for the potential manipulator. This seems to be a fundamental reason why the phenomenon studied as the 'transfer paradox' is unlikely to arise, at least as the result of deliberate actions of self-seeking agents in the economy.

However, the fact that a transfer paradox might inadvertently be triggered by agents 'outside' the economy, say government agencies pursuing redistribution policies, leads us to a study of the general relationship between redistribution of endowments and changes in welfare. We found that in the context of a production economy that for differential changes in endowments, the association was arbitrary. Nevertheless, we showed that even when a transfer paradox was possible (because of the nature of the price changes which a transfer would induce), there may still exist a discrete redistribution which would ensure the desired direction of welfare change.

Walras was right to specify the 'high purpose' of definitive comparative statics for the general equilibrium research program. That it is hard to attain this ideal is more a statement about the complexity of general equilibrium systems, rather than any criticism of the vision which suggested that this topic be addressed.

Chapter 11

EMPIRICAL EVIDENCE ON GENERAL EQUILIBRIUM

"... the central weakness of modern economics is, indeed, the reluctance to produce theories that yield unambiguously refutable implications, followed by a general unwillingness to confront those implications with the facts."

M. Blaug

"Economics is an empirical science. Its theories and models therefore stand or fall on the basis of their ability to account for actual economic data."

J. Stock and M. Watson

"... this logic of choice also entails additional properties of the individual demand functions which, when taken into account, give specificity and richness of structure to the general equilibrium model."

Y. Balasko

"Scientific knowledge comes from observation."

E. Malinvaud

11.1. Introduction

At one level, general equilibrium theory is an abstract study of the consequences of economic agents pursuing their own interests. The study is conducted in various institutional contexts, such as the Arrow-Debreu complete markets framework, or the Hicksian temporary equilibrium set-up, which displays varying degrees of market incompleteness, to name just two. In all cases, similar questions are posed and investigated: can the competing interests of agents be reconciled in some sort of market equilibrium? If yes, are equilibrium states numerous or are they relatively few in number? Are equilibrium states optimal in any sense? Can 'market forces' be relied on to guide the economy to equilibrium? Are variations in equilibrium prices and quantities predictable in the face of shocks to the economy? Investigation of these questions has produced the rich and interesting set of results and insights concerning the existence, uniqueness, stability, optimality and comparative statics of equilibrium states, that constitute general equilibrium theory. It could be argued that general equilibrium theory should be left there and recognised for the significant intellectual achievement that it is. However, the general equilibrium way of looking at the economy is not meant to be completely abstract, uninterpretable and divorced from reality. After all, the central players in general equilibrium models are 'consumers' and 'producers' — both of whom have correlates in and are to some extent inspired by, reality. The variables 'explained' by general equilibrium theory namely, prices and quantities of goods produced and consumed, also have readily identifiable real world counterparts. It is therefore perhaps not surprising that the question is asked, how well do general equilibrium models perform when confronted with actual economic data? Consequently, this chapter focuses attention on the capacity of general equilibrium theory to account for actual economic phenomena and its ability to pass a variety of empirical tests of its validity. In Section 2, we introduce some basic ideas associated with testing general equilibrium theory and consider some tests of the theory using microeconomic data. In Section 3, we consider some tests based on macroeconomic data. Section 4 offers some concluding remarks.

11.2. Testing general equilibrium theory

11.2.1. *Preliminaries and basic issues*

As Buck and Lady (2005) remark: "[t]he degree to which economics is an 'empirical' science can be deceptively complicated to pin down. The issue, in summary, concerns the degree to which economics provides hypotheses about its subject matter that can be 'refuted' or following Popper (1934), *falsified*". Buck and Lady (2005; p. 777). In this light, and as Brown and Matzkin (1996) note, general equilibrium theory has been criticised because it seems to lack falsifiable implications or in Samuelson's terms, it does not seem to produce 'meaningful theorems' — see also the discussion in Beed (1991; p. 484). This conclusion arises because, as many authors note, the 'primary source of testable implications of economic theories including general equilibrium theory are its comparative static results' (see for instance Brown and Matzkin (1996; p. 1249) and Echenique (2004; p. 145)). From the work in the previous chapter, it is known how difficult it is to obtain unambiguous (and hence refutable) comparative static predictions. Further, thanks to the Sonnenschein-Mantel-Debreu theorem and Mas-Colell *et al.*'s (1995; p. 616) 'anything goes' principle for comparative statics to which it leads, it might seem that the set of refutable assertions from general equilibrium theory is empty, because general equilibrium comparative static predictions may be consistent with almost *any* pattern of price-quantity responses in the economy. As Brown and Matzkin (1996) note, if this is seen through the prism of Mas-Colell's (1977) result that 'utility maximisation subject to a budget constraint imposes no testable restrictions on the set of equilibrium prices', then the situation might seem hopeless as far as testing general equilibrium theory is concerned. However, this is not so for at least two reasons. Firstly, general equilibrium theory has a basic picture of the economy as a collection of individuals making profit and utility maximising supply and demand decisions in the context of clearing markets. It might therefore be possible to test this basic conception by deriving testable implications of utility and profit maximising behaviour, and also of market clearing. More formally, notice that from the definition of Walrasian equilibrium (see Chap. 2), it is clear that for an economy to be in such a state three things have to hold: (i) consumers have to be utility maximising subject to a budget constraint; (ii) firms have to be profit maximising subject to their technology; and (iii) markets have to clear at prevailing prices. Rejection of any one of these conditions would lead to a rejection of the hypothesis that the sample of data under consideration is consistent with the economy being in a state of Walrasian equilibrium. Therefore, we can test general equilibrium theory (or at least models developed in the theory, such as the Arrow-Debreu model), by looking to see if the microeconomic behaviour of consumers, firms and the observed state of markets is consistent with conditions (i)–(iii). Secondly, it might be possible to by-pass the SMD result (or as Nachbar (2008) puts it 'exploit a loophole' in that result), by considering as Brown and Matzkin (1996) do via testable restrictions linking equilibrium prices with individual endowment profiles. Instead of considering the market excess demand $Z(p)$, where endowments are fixed and prices alone vary, consider the excess demand $Z(p, \hat{\omega})$ as a function of prices and individual endowment profiles. Then look for 'testable restrictions on the equilibrium manifold' — where the *equilibrium manifold* is the set of price-endowment profile pairs, with excess demand zero, i.e. $\{(p, \hat{\omega}) : Z(p, \hat{\omega}) = 0\}$. As Brown and Matzkin (1996) show: "[c]ontrary to the result of Mas-Colell, cited above, we shall show that utility maximisation subject to a budget constraint does impose testable restrictions on the equilibrium manifold." Brown and Matzkin (1996; p. 1250). We will consider both these approaches to testing general equilibrium theory in what follows.

11.2.2. *Testing for general equilibrium using microeconomic data*

11.2.2.1. *Parametric tests of consumer theory*

Suppose consumers are making decisions in a competitive complete markets Arrow-Debreu economy in which there are ℓ goods, prices are $(p_1, p_2, \ldots, p_\ell)$ and individual income is M_i. Denote by $x^i(p_1, p_2, \ldots, p_\ell, M_i)$, with $p_\ell \equiv 1$, consumer i's system of commodity demands and supplies. If this system is generated by utility maximisation subject to a standard budget constraint in an economy in Walrasian equilibrium, then it is well known that consumer behaviour satisfies the following 'Walrasian consumer theory' (WCT) restrictions:

(a) Engel aggregation: $p_1\partial x_1^i/\partial M_i + p_2\partial x_2^i/\partial M_i + \cdots + p_\ell\partial x^i/\partial M_i = 1$.
(b) Cournot aggregation: $p_1\partial x_1^i/\partial p_k + p_2\partial x_2^i/\partial p_k + \cdots + p_\ell\partial x^i/\partial p_k = -x_k^i$ for all $k = 1, \ell$.
(c) Slutsky symmetry: $S = S^T$ where the typical element in S (the Slutsky matrix), is $(\partial x_j^i/\partial p_k)_{U=\text{constant}}$.
(d) Negative semi-definiteness of the Slutsky matrix: $z^T S z \leq 0$ for all $z \in \Re^\ell \backslash (0\}$.
(e) Homogeneity of degree zero: $x^i(p_1, p_2, \ldots, p_\ell, M_i) = x^i(\lambda p_1, \lambda p_2, \ldots, \lambda p_\ell, \lambda M_i), \lambda > 0$.

Since WCT $\Rightarrow$ {(a)–(e)} it follows that the rejection of any one of these restrictions leads to the rejection of WCT. So one way to test this theory is to see if these restrictions show up in actual data. What then is known about the empirical validity (or otherwise) of (a)–(e)?

As Keuzenkamp and Barten (1995) note, studies of consumer behaviour are available in the literature as far back as Davenant (1699), who noticed a negative relationship between the quantity of corn demanded and its price. Davenant did not try to specify any of the restrictions which WCT imposed on consumer behaviour. The first person to attempt that was, according to Keuzenkamp and Barten (1995), Walras (1874). They note that: "According to Walras, the choice of the *numeraire*, the unit in which prices are expressed, is arbitrary. This, of course, is a way of stating the homogeneity condition, and to the best of our knowledge the first time it was done." Keuzenkamp and Barten (1995; p. 108). Walras did not empirically test homogeneity (or any other restriction which follows from WCT). The first informal test of homogeneity appears to be in Schultz (1938). It was Marshak (1943) who proposed the first formal test of any aspect of WCT, and he chose to test for homogeneity. In the process he rejects homogeneity, although as Keuzenkamp and Barten (1995) note, he remarks that no 'sweeping verdict' on the applicability of the null hypothesis should be made on the basis of his test alone. The next serious attempt to assess WCT was due to Stone (1954a) who estimated the demand for 37 categories of food in the United Kingdom for the period 1920–1938 and found that with a few exceptions, homogeneity is not rejected.[1]

Stone (1954b) took his earlier work further and ushered in the 'modern' or systems approach to demand studies with his estimation of the linear expenditure system. In the wake of this study, Barten (1967) performed the first 'systems approach' test of WCT when he performed a joint test of homogeneity and Slutsky symmetry. The result of this test was "... that the homogeneity condition was not well supported by the data, but it was unclear whether the result was robust. The joint test of homogeneity and symmetry ... did not lead to strong suspicion of these conditions taken together." Keuzenkamp and Barten (1995; p. 114). When homogeneity was tested on its own however: "... the homogeneity condition was not well supported

[1] The historical introduction to this section has been heavily informed by Keuzenkamp and Barten (1995).

by the data … [and] homogeneity seemed on shaky grounds." Keuzenkamp and Barten (1995; p. 114–15). This result was further supported by Barten (1969) who tested homogeneity using 16 consumption categories, on data for the Netherlands and using the Rotterdam specification of the demand equations. Byron (1970) replicated Barten's rejection of homogeneity (and symmetry) using Barten's data and a double-log specification of the demand equations.

To guard against the possibility that the rejection was an artifact of the data, numerous other data sets were investigated for congruence with the predictions of WCT. For instance according to Keuzenkamp and Barten (1995; p. 116), homogeneity was rejected for Spanish data using both the Rotterdam specification and the double-log specification of Lluch (1971). Using UK data, Deaton (1974) used five versions of the Rotterdam model, the LES, Houthakker's direct addilog system and a model without substitution effects. For data ranging from 1900 to 1970, Deaton (1974) "… concludes that the null hypothesis of homogeneity is firmly rejected for the system as a whole." Keuzenkamp and Barten (1995; p. 116). Starting with an indirect utility function, Christensen, Jorgensen and Lau (1975) test for Slutsky symmetry and homogeneity and on the basis of their rejection of symmetry (conditional on homogeneity) they remark that although it *might* be the case that the estimating forms are not flexible enough and that the theory is correct after all, their other results "… rule out this alternative interpretation and make possible an unambiguous rejection of the [Walrasian] theory of demand." Christensen *et al.* (1975; p. 381).

At the conclusion of his survey of a large number of studies which test WCT, Deaton (1986) made the following interesting observation: "Although there is some variation in the results through different data sets, different approximating functions, different estimation and testing strategies and different commodity disaggregations, there is a good deal of accumulated evidence rejecting the [Walrasian] restrictions." Deaton (1986; p. 1791). Note however that the mixed and mostly negative results observed by Deaton (1986) concerning tests of the Walrasian restrictions on consumer behaviour might occur for at least the following reasons: (i) WCT is an incorrect way to describe actual economic behaviour; (ii) the functional forms used to conduct parametric tests are not flexible enough to allow the tests to be properly conducted; (iii) the data used to test the theory are not appropriate either because of the level of aggregation present or because there are relevant omitted variables; (iv) there are inappropriate econometric techniques being used and inferences being made.[2]

Considerable effort has been made in the literature to guard against (ii)–(iv), through the development of 'flexible functional forms', experimentation with data sets which display various degrees of disaggregation and through the development and application of appropriate statistical techniques. Without being exhaustive, it is possible to illustrate some of these developments and the implications for the validity of WCT they have produced as follows.

One of the most widely used flexible functional forms is the 'Almost Ideal Demand' (AID) system of Deaton and Muellbauer (1980). In the context of the AID system, numerous tests of the Walrasian theory of consumer behaviour have been made and many times the restrictions implied by WCT have been rejected. However, as Cooper and McLaren (1992) point out, the AID system is not globally regular and the PIGLOG share equations which it generates do not satisfy 'cointegration accounting' unless homogeneity restrictions are relaxed on the price index. This fact provides: "... a possibility that may explain the

[2]See for a discussion of these possibilities, Laitinen (1978), Karagiannis and Mergos (2002).

frequent rejection of homogeneity restrictions in non-regular models such as AID." Cooper and McLaren (1992; pp. 658, 659). The authors find some empirical support for their contention when, using a sample of aggregate Australian data 1954–1989 and four consumption categories, Food, Tobacco and Alcohol, Clothing and Other, they estimate their generalised and more regular version of the AID system, MAIDS, and find that homogeneity is rejected, but that conditional on homogeneity, symmetry is not rejected.[3] Clements and Selvanathan (1994) have argued that when 'new methods' of testing are employed it is possible to reach the conclusion that the hypotheses of homogeneity and symmetry (and also preference independence) are not at such wide variance with the data as was thought to be the case as a result of earlier work. Fox (1996), commenting sceptically on a study of Selvanathan and Selvanathan (1994), which uses some of the 'new methods' referred to by Clements and Selvanathan (1994), remarks that "... Monte Carlo testing of the restrictions implied by demand theory in [their study] finds evidence in support of demand homogeneity [and] symmetry ... This is an interesting result as other studies have rejected these restrictions." Fox (1996; p. 175). As there are mixed outcomes as far as early tests of the Walrasian theory of consumer behaviour is concerned, it is worth inquiring about more recent tests for they might shed light on why these mixed results might be occurring. Holt and Goodwin (1997) and Ryan and Wales (1999) introduce three new flexible consumer demand systems in which expenditures on goods are quadratic functions of income. They estimate two of these systems for a sample of Canadian data on seven categories of consumption for the period 1947–1995 and find that most of the implications of WCT are rejected.

[3]Cooper *et al.* (2001) have also analysed this data set using a Frisch profit function and report that the approach has 'improved regularity properties' relative to earlier work. They do not however explicitly test for properties (a)–(e) in the data.

Nicol (2001) estimates a 'rank-three' demand system for a sample of data taken from the US Consumer Expenditure Survey and he finds that for his sample of data the major implications of WCT are rejected. Fisher, Fleissig and Serletis (2001) compare the empirical performance of eight frequently used flexible functional forms that are either locally flexible, effectively globally regular or asymptotically globally regular. Using a sample of US data they find that for the large majority of cases the implications of WCT fail to hold.

As an illustration of the variation in findings that may result from using different levels of data disaggregation, consider the study of Sabelhaus (1990) who estimated the AID system model using a sample of aggregate US data drawn from the US National Income Accounts for the period 1959–1988. Citing the studies of Christensen *et al.* (1975), Berndt *et al.* (1977) and Deaton and Muellbauer (1980) all of whom reject the Walrasian theory of consumer behaviour, Sabelhaus (1990) observes that such a rejection (a rejection which also occurs in his study) is: "... for the aggregate data consistent with those found by other authors." Sabelhaus (1990; p. 1476). However, when the AID system is estimated for disaggregate data, in particular a cross section of data drawn from the US Consumer Expenditure Survey 1980:1–1986:3, the restrictions implied by WCT are not rejected. This leads Sabelhaus (1990) to suggest that perhaps the rejection found using aggregate data is due to this data being inappropriate to the task of testing the Walrasian restrictions on consumption behaviour, restrictions which after all are restrictions on the demand behaviour of *individual* consumers over highly disaggregated commodities and not on the behaviour of *aggregates*, either goods or consumers.[4] Blundell

[4]There is another interesting implication of these findings concerning the apparent regular failure of the theory when tested on aggregate data. Recall in Chap. 4, we discussed the approach to establishing the uniqueness and stability of equilibrium proposed by Grandmont (1991) and Hildenbrand (1985). It will be recalled that Grandmont's conditions on the distribution of characteristics in the economy lead to the WARP holding *in*

and Robin (1999) estimate a 22-commodity quadratic demand system using household level data from a time series of repeated cross sections. The results are almost all at variance with the implications of WCT.

Karagiannis and Mergos (2002) are concerned to explore, simultaneously, the effects of aggregation, functional forms and econometric technique on tests of WCT. They begin by noting that the use of time-series techniques, particularly cointegration and error correction models have been suggested as ways of proceeding in the estimation of demand systems and as a way of 'resolving the perennial issue of violating the theoretical postulates of homogeneity and symmetry'. In their empirical work, they adopt a linearised AID model and using a sample of Greek food data find that homogeneity is not rejected when a time trend is absent from the model but Slutsky symmetry and homogeneity, or just Slutsky symmetry are rejected. When a time trend is included in the model "...all the theoretical restrictions are rejected ..." Karagiannis and Mergos (2002; p. 142). In their discussion of these results, Karagiannis and Mergos (2002) observe that homogeneity seems sensitive to sample size while Slutsky symmetry seems more sensitive to the aggregation scheme adopted. Obvious ways to proceed in order to test these conjectures, is to extend the sample size and to arrive at consistent commodity aggregation through, perhaps, the 'generalised commodity theorem' of Lewbel (1996). Karagiannis and Mergos (2002) also note that: "On the other hand, rejection of homogeneity and symmetry may also be due to model specification. In our empirical illustration, this is related to a deterministic time trend as well as the linearised AID [and this] source

the aggregate. Now the WARP, if it holds, implies that the (aggregate) Slutsky matrix is negative semi-definite. But as the Deaton (1986) survey makes clear and as the results in Cooper and McLaren (1992) suggest, this is not generally found in aggregate data. This therefore raises an interesting empirical question about the Grandmont (1991) procedure.

of rejecting homogeneity and symmetry deserves further investigation." Karagiannis and Mergos (2002; p. 143). In this context, it would be interesting to test for homogeneity and Slutsky symmetry using the Karagiannis and Mergos data and, say, the MAIDS model of Cooper and McLaren (1992). Noting that 'one of the main difficulties in the estimation (and testing) of demand systems using household data is getting good estimates of price responses', Crawford, Laisney and Preston (2003) develop an estimation approach using 'unit value data'. Applying the technique to a sample of Czech household data they find that symmetry and homogeneity are rejected "... at any reasonable level of significance." Crawford *et al.* (2003; p. 234). However, the authors point out that this outcome might be due to the fact that the demand data was drawn from a sample of married couples. Hence the "... source of this rejection may lie in the misspecification of the unitary model of household preferences." Crawford *et al.* (2003; p. 234). The authors point out that this is consistent with the finding of Browning and Chiappori (1998) 'who reject symmetry for couples but not singles on Canadian data'. Balcombe (2004) begins with the Kuzenkamp and Barten (1995) observation that 'the vast majority of empirical work on demand systems rejects homogeneity and symmetry restrictions'. Taking up the theme that one possible reason for that outcome is 'faulty econometrics', Balcome estimates a small AID system for Greek meat data and using what he argues is superior econometric technique to that historically found in the literature concludes that there is support for both homogeneity and symmetry restrictions. On the basis of this he argues that; "... unless researchers are confident of their testing procedures, they would be unwise to abandon fundamental hypotheses such as symmetry and homogeneity [and] that futher work in this area will tend to lead to considerably lower rejection of these hypotheses when using bootstrapping or more appropriate finite sample critical values than has been the case historically." Balcombe

(2004; p. 461). Buse (1998) is also concerned with the possibility that faulty inference may be responsible for over-rejection of homogeneity. In particular in the context of a linearized version of the AID system he shows that standard exact tests for homogeneity (namely F and Hotelling's T^2) may have 'significant and substantial size distortions'. One possible recourse to solving this problem is to use a size adjusted likelihood ratio statistic, such as that developed by Italianer (1985) to test homogeneity. In the event however, Buse (1998) finds that the behaviour of the LR and T^2 statistics are 'effectively indistinguishable'. In addition it appears that using different approximating price indicies (Stone, Paasche or Laspeyres) is not responsible for homogeneity rejection either. As Buse (1998) puts it: "... as to whether the use of approximating indexes can account for the frequently observed rejection of homogeneity, the answer has to be an unambiguous 'probably not."' Buse (1998; p. 220). He also notes two approaches to accounting for homogeneity rejection due to Attfield (1985) and Ng (1995), but notes that in both cases, the argument is weakened by the authors 'ignoring the consequences of 'Stone index linearization', leaving room for further work to find a 'robust explanation of the over-rejection phenomenon' — see Buse (1998; p. 220) for details.

Noting the work of Attfield (1997) and Ng (1995), Tiffin and Balcombe (2005) motivate their work by observing that 'conventional seemingly unrelated estimation (SUR) of the AID system is subject to small sample bias and distortion in the size of the Wald test for symmetry and homogeneity when the data used in the estimation are cointegrated'. They therefore develop a fully modified seemingly unrelated estimator (FM-SUR) and use bootstrapping techniques to derive small sample properties of the estimator. In an application of their technique to the data used by Blanchiforti *et al.* (1986), they find that: "The Wald statistic obtained with SUR estimation is 13.385 whereas that obtained with FM-SUR is 67.451. Compared with a critical value at the 5% level of 12.592, these results lead to the rejection of homogeneity and symmetry. However, bootstrapping gives 5% critical values of 26.797 and 116.662 for SUR and FM-SUR and the hypothesis is not rejected." Tiffin and Balcombe (2005; p. 266). The authors conclude by arguing that when interest is focussed on testing theoretical restrictions (as it is for us), 'the use of the bootstrap is vital — and equally appropriate to SUR or FM-SUR estimation'. Ogura and Ohtani (2007) begin by noting that while there are studies in which homogeneity and symmetry are not rejected "... these restrictions have been rejected in too many studies." Ogura and Ohtani (2007; p. 497). Their suspicion is that failure to adjust for sample size bias in the test statistics is likely responsible for this outcome. Using a sample of annual Japanese data from the Family Income and Expenditure Survey, a five and ten good disaggregation scheme and Zellner's SUR estimation approach then when an AID system is estimated, they find, in support of their contention "... that the homogeneity restriction is not rejected at the 5% level by the exact test, though it is rejected by the asymptotic Wald test." Ogura and Ohtani (2007; p. 497). The authors also show that their exact test is robust to departures from normality — at least in the direction of an elliptically symmetric distribution "... and the over-rejection for the homogeneity restriction of our test can be more mitigated by using the exact test than by using the test in Laitinen (1978)." Ogura and Ohtani (2007; p. 501).

Using a sample of Norwegian quarterly data, derived from the national accounts and covering the period 1996:1 to 2001:4, Raknerud, Skjerpen and Swensen (2007) estimate a linearised AID system. The model is augmented by the inclusional latent variables representing stochastic trends and seasonality and is estimated in a 'seemingly unrelated time series equations framework (SUTSE)'. The authors find that when tested "The homogeneity restriction is rejected at the 1% level for three of the [nine] commodities and for the system as a whole." Raknerud *et al.* (2007; p. 121). The authors then go on to remark; "However, only the homogeneous model can be given a substantial interpretation in terms of economic theory'. Raknerud *et al.* (pp. 121–22). This is an interesting remark and seems to beg the question: why test the homogeneity restriction in the first place if empirical rejection will in any case be ignored? Further, it overlooks the interesting possibility noted by Deaton (1986) in reflecting on a study of his that used UK housing expenditure data "... that allowing for quantity restrictions using a restricted cost function related to that of the AIDS, removed much of the conflict with homogeneity on post-war British data." Deaton (1986; p. 1824).

11.2.2.2. *Non-parametric tests of consumer theory*

No matter how much ingenuity is invested in specifying flexible functional forms as approximations to underlying direct or indirect utility functions, expenditure functions or consumer demand functions, it is still possible that a parametric approach will miss the true form of the consumer's objective or response functions and thereby bias tests of WCT towards rejection. Non-parametric tests of WCT provide a possible way out of this difficulty by allowing WCT to be tested without specifying any particular functional form for the consumer's objective or response functions. Making this point and suggesting a solution, Lewbel (1995) remarks that: "The outcome of parametric tests may result from incorrect functional forms rather than the truth of economic hypotheses of interest ... nonparametric, moment based, consistent tests of hypotheses involving conditional expectation functions and their deviates [asymptotically] eliminate this problem." Lewbel (1995; p. 396). Lewbel (1995; p. 397) also notes that there is an 'enormous amount of theoretical work on nonparametric testing'. The emphasis in what follows will be on the results of empirical applications of such techniques rather than theory *per se*. However, we note the observation by Chalfant and Zhang (1997) that "... [j]ust as results from parametric methods depend on various assumptions made by the researcher, so do nonparametric results depend on such decisions." Chalfant and Zhang (1997; p. 1176). They illustrate this point by drawing attention to the fact that non-parametric inferences are not necessarily invariant to price and quantity scaling. Thus, although the approach avoids some of the difficulties associated with getting the functional form right, the approach is still capable of delivering "... fragile results [and therefore] nonparametric techniques are not a panacea." Chalfant and Zhang (1997; p. 1176).

Afriat (1967) pioneered the non-parametric approach to testing WCT when he showed that the existence of a utility function that rationalised a set of price-consumption data was equivalent to that data satisfying a Strong Axiom of Revealed Preference like condition that he called 'cyclical consistency'. Since we will make reference to this result again later in the chapter, we record Afriat's theorem, as formulated by Brown and Matzkin (1996; pp. 1252–1253).

Afriat's Theorem. *Consider a sample of N observations on price and consumption bundle pairs $(p^i, x^i)_{i=1,N}$ then the following four conditions are equivalent*: (1) *There exists a non-satiated utility function u, that rationalises the data in the sense for all $i = 1, \ldots, N$ and all x such that $p^i x^i \geq p^i x, u(x^i) \geq u(x)$*; (2) *The data satisfies 'cyclical consistency' in the sense that for all $\{r, s, t, \ldots, q\}, p^r x^r \geq p^r x^s, p^s x^s \geq p^s x^t, \ldots, p^q x^q \geq p^q x^r \Rightarrow p^r x^r = p^r x^s, p^s x^s = p^s x^t, \ldots, p^q x^q = p^q x^r$*; (3) *There exist numbers $U^i, \lambda^i > 0$ such that $U^i \leq U^j + \lambda^j p^j (x^i - x^j)$ for $i, j = 1, \ldots, N$*; (4) *There exists a non-satiated, continuous, concave, monotonic utility function that is consistent with the data.*

Proof. Afriat (1967; pp. 71–74) and Fostel *et al.* (2004). □

Remark 11.1. This result provides a powerful nonparametric way to check a sample of price-consumption data for consistency with utility maximisation.

Varian (1982a) develops a variation on Afriat's theorem that he calls the Generalised Axiom of Revealed Preference (GARP) — a condition which is slightly easier to check on actual data than is Afriat's cyclical consistency condition. He also provided an important application of the techniques to a sample of US data. Varian found that his data was consistent with WCT but he also pointed out that this might be for the spurious reason that the US economy grew year on year throughout his sample period. Using a similar approach, Swofford and Whitney (1987) found that a sample of US quarterly data on per capita consumption goods, leisure and monetary assets was consistent with utility maximisation and hence with the restrictions of WCT. Mattei (1991) uses the nonparametric approach on a sample of Swiss data drawn from monthly household budget surveys 1975–1987 and finds that a WCT is rejected for about half of the households in the sample. Using a sample of BLS consumer interview data for the period 1982–1985, Famulari (1995) found that: (1) households with similar demographics seem to behave as a group in ways consistent with GARP; (2) allowing for measurement errors reduces the rate of violation of GARP when it does occur; and (3) the data suggest that a common utility function which allows for demographics is warranted for this sample of data. In a similar vein, Nicol (2001) used pooled cross-section and times-series data generated by the Canadian economy and found that exact aggregation was rejected but that homogeneity and symmetry restrictions were "relatively less restrictive" for this data. Maki (1992) allowed for taste changes and found that when a consistent set of data drawn from the national accounts was used, the joint restrictions of homogeneity and symmetry are not rejected. These are interesting results which seems to reinforce the contention in Sabelhaus (1990) that it is perhaps the use of inappropriate aggregate data, along with problems associated with the regularity of certain of the flexible functional forms used in estimation which has resulted in the persistent rejections of WCT noted by Deaton (1986). The correction for these deficiencies and the adoption of more robust econometric methods might also be responsible for the somewhat more favourable view taken in surveys such as that of Clements and Selvanathan (1994).

Lewbel (1995) notes that one of the strongest implications of WCT is Slutsky symmetry and that "[t]esting Slutsky symmetry is of great interest because of the importance of consumer rationality in economic analyses ..." Lewbel (1995; p. 379). Using a sample of UK Family Expenditure Survey data for the period 1970–1986, Lewbel tests for Slutsky symmetry. He also controls for spurious violations of Slutsky symmetry by controlling for systematic heterogeneity in preferences (see Lewbel (1995; pp. 390–391) for details). In the end, the results of this study indicate that there is evidence of violations of Slutsky symmetry for all pairs of goods that contain clothing, but no evidence for rejection when clothing is not considered. Reflecting on this result, Lewbel (1995) argues: "Household expenditures are recorded over short time spans (two weeks), so for a semi-durable good like clothing ... measured violations of Slutsky symmetry may arise from differences between when purchases are made and when they are consumed." Lewbel (1995; p. 394). Lewbel also includes a comparison test of Slutsky symmetry using a Quadratic AID system model (QAID). Although there is some variation in the frequency of rejection of Slutsky symmetry between the parametric and non-parametric approaches, rejection when clothing is considered and non-rejection otherwise is generally the case. Indeed Lewbel summarises his findings as: "... using both parametric and non-parametric tests, Slutsky symmetry is rejected for the semi-durable clothing, but not for other goods." Lewbel (1995; p. 397). However, it is important to note, as Lewbel (1995) does, that his symmetry test was performed on one demographically homogeneous group and that before making any general conclusions about the broad applicability of Slutsky symmetry: "It would be useful to replicate the test for other homogeneous groups, or to apply the test to larger and more diverse groups of households ..." Lewbel (1995; p. 397). One issue with nonparametric tests is that as Fleissig *et al.* (2000) and Barnett and Serletis (2008) point out, they are nonstochastic. As a consequence "... violations are all or nothing: either there is a utility function that rationalizes the data or there is not." Barnett and Serletis (2008; p. 218). Tests aimed at allowing the GARP to be tested in a stochastic environment have been developed by Fleissig and Whiney (2005), de Peretti (2005), Jones and de Peretti (2005), although to date, application to real (as opposed to simulated) data sets has, been limited. Finally, the study by Jones and de Peretti (2005) shows that the GARP can be violated by random measurement errors in observed quantity data. They therefore developed a test for GARP in a stochastic framework and compared its performance with that proposed by Varian (1984a). Using a sample of data on a 'large number' of US monetary assets over multiple sample periods spanning 1960 to 1992, they found that their test and that of Varian's "... both supported the null hypothesis of utility maximisation in the majority of the samples that had low numbers of GARP violations ..." Jones and de Peretti (2005; p. 626). This is an interesting finding which strengthens the claim of WCT to be a reasonable description of reality.

11.2.2.3. *Experimental tests of consumer theory*

As ingenious as non-parametric tests are in getting around some of the problems inherent in parametric tests, there remains the problem, pointed out for instance by Sippel (1997), that testing WCT using actual purchase data is 'difficult if not impossible' — see Sippel (1997; pp. 1431–1435) for a discussion of this point. Instead of using such data, Sippel (1997) uses data on consumption behaviour obtained through a controlled experiment that involved real consumption of the goods chosen. The result of the study was that most subjects were found to violate the WARP and that therefore their behaviour was inconsistent with the implications of WCT.

Also, using non-parametric revealed preference tests on experimental data, Mattei (2000) found significant violations of the revealed preference axioms and hence inconsistency with WCT. If 'nearly optimising' behaviour is postulated instead of exact optimisation then, according to Mattei, 'most of these inconsistencies disappear'. However, for this set of data, a random choice model produces almost identical results and it is therefore not clear that these experimental data support even 'approximate' WCT. Andreoni and Miller (2002) test for behaviour consistency with GARP in an experimental situation where agents are also allowed to be altruistic. Their results are striking in that they find "... that subjects exhibit a significant degree of rationally altruistic behaviour. Over 98% of our subjects mode that more constraint with utility maximization. Andreoni and Miller, 2002; p. 737). (see also Andreoni and Miller (2002; pp. 745) for further details). Apart from the obvious interest this has for throwing light on rational behaviour in the face of altruism, this study is also interesting in light of our theoretical work earlier concerning the possible role of altruism in helping to establish conditions necessary for the existence of equilibrium. In particular "... there is a great deal of heterogeneity across subjects. People differ on whether they care about fairness at all, and when they do care about fairness the notion of fairness they employ differs widely…" Andreoni and Miller (2002; p. 745). This empirical work would seem to support the theoretical argument advanced earlier by us, that reliance on voluntary transfers to help establish the existence of equilibrium may not generally be feasible.

11.2.2.4. *Summary of consumer theory tests*

There is an enormous amount of empirical work aimed at testing neoclassical or Walrasian consumer theory. The results presented here are designed to give an account of the main findings available from such tests along with some insight into the means by which they were obtained. It is not possible to summarize this work in the form of a simple statement that 'the theory holds' or 'the theory fails'. What does seem to be true is that a great deal of effort has gone into trying to get the econometrics of the tests 'right', so that rejection (or non-rejection) of the theory does not occur for spurious reasons. As that process has proceeded, it appears that key implications of WCT, such as zero degree homogeneity in prices and income, Slutsky symmetry and GARP, are rejected less often than they were at the hands of earlier, possibly less well-specified, tests. In fact as Barten (2003) notes "… demand homogeneity and Slutsky symmetry have been rejected over and over again [and] do not appear to be easily reproducible in empirical research … [However] [b]y improving the model set-up or the test statistics, rejection can be softened and, in some cases, turned into 'no rejection."' Barten (2003; pp. 7 and 14–15).

On the basis of what we have seen, it seems reasonable to conclude that while WCT is not obviously false, it is not obviously true either. It therefore seems reasonable to maintain an open mind about the empirical relevance of Walrasian equilibrium states, at least until the empirical status of profit maximisation and market clearing have been examined.

11.2.3. *Testing producer behaviour*

11.2.3.1. *Parametric tests*

The literature which concerns itself with testing the implications of Walrasian producer theory (WPT), usually aims at testing the homogeneity, monotonicity, curvature and symmetry implications of Walrasian producer theory. These conditions are analogous (but not identical) to those of WCT and may be stated as follows. If $y(p_1, p_2, \ldots, p_n)$ is a system of producer input demand — output supply equations generated from minimising the cost function $C(w, y)$ or maximising the profit function $\Pi(w, p)$ then it must satisfy:

(a′) $y(p_1, p_2, \ldots, p_n) = y(\lambda p_1, \lambda p_2, \ldots, \lambda p_n)$ for $\lambda > 0$ (homogeneity in prices);
(b′) $\partial y_k/\partial p_k \geq 0; \partial\Pi(w, p)/\partial w \leq 0$ (monotonicity);
(c′) (i) $C(w, y)$ is concave in w; (ii) $\Pi(w, p)$ is convex in p; $\partial C(w, y)/\partial y > 0$ (curvature);
(d′) $\partial y_k/\partial p_j = \partial y_j/\partial p_k$ (symmetry).

Noting what is at stake in testing WPT, Applebaum (1978) remarks that: "When rejecting the null hypothesis, we have in fact rejected the whole set of assumptions identified with the neoclassical [Walrasian] theory of production." Applebaum (1978; p. 98). At the end of his study of the coherence between WPT and a sample of US manufacturing data, Applebaum (1978) remarks: "We find that the theory does not pass the tests for its validity. It performs better with tests for internal consistency, however it still does not pass the test in all cases. Finally, we find that the primal and dual do not yield similar implications, a result which is very disturbing . . . The main conclusion to be drawn is, therefore, that one should be careful in interpretations of empirical results obtained on the basis of neoclassical [Walrasian] production theory." Applebaum (1978; p. 102).

In a subsequent study and using a similar data set, Conrad and Unger (1987) conclude: "The results of our tests are not consistent with the restrictions implied by long-run [Walrasian] equilibrium models. Therefore we conclude that it is incorrect to use the long-run [Walrasian] equilibrium specification." Conrad and Unger (1987; p. 248).

Commenting on the literature aimed at testing WPT, Pencavel and Craig (1994) argue: "There is a literature on testing the implications of profit maximisation ... but the limitations of this research are profound. Typically, tests are applied to observations on aggregations of firms or even aggregations of industries. Sometimes in addition, production is assumed to take place under constant returns to scale." Pencavel and Craig (1994; p. 731).[5] Commenting particularly on studies of the sort which Applebaum (1978) and Unger (1987) have conducted, Pencavel and Craig (1994) argue that they are 'disturbingly unsatisfactory' because of the numerous maintained hypotheses contained in them. In attempting to avoid what they see as profound weaknesses in the available tests of Walrasian producer theory, Pencavel and Craig (1994) set themselves the joint aim of: "... documenting how different types of firms ... respond to changes in the economic environment and whether these responses are consonant with the standard [Walrasian] models of optimisation." Pencavel and Craig (1994; pp. 718–719). The authors argue that their approach to testing WPT is largely free from the shortcomings which they identified in earlier studies primarily because they use data on individual firms which face parametric price changes for both inputs and outputs (so price taking behaviour makes sense) and because no

[5]For example, some of the maintained hypotheses in Applebaum (1978) are constant returns to scale; cost minimising behaviour; perfect competition; long run equilibrium; static optimisation; Hicks neutral technical change; aggregation across inputs.

explicit assumptions are made about returns to scale.[6] The data used consists of price and quantity observations generated by a sample of plywood manufacturing firms operating in the Pacific northwest in a period from the late 1960s to the mid-1980s. As a result of their study, and in contrast with the bulk of the literature, Pencaval and Craig (1994) find that there is some evidence to support both cost minimisation and profit maximisation (see in particular their Tables 5 and 6).

Using an aggregate econometric model for the United Kingdom, Ozanne (1996) reports a violation of (b′) for UK agriculture. In particular, for the sample of data considered by him (1967–1992), there was strong evidence of perverse supply responses in which as agricultural commodity prices fell, supply increased. Focusing on monotonicity, homogeneity and curvature issues, Salvanes and Tjotta (1998) pay particular attention to the cost function estimated in the influential study of the Bell Corporation by Evans and Heckman (1984, 1986). They show that the estimated cost function has negative marginal costs over most of the test region, raising questions about the overall support for WPT reported by Evans and Heckman in their case study.

[6] The output and input equations used in Pencaval and Craig (1994) could however be cause for concern as they are:

$$\log y_{it} = \mu_{1i} + \gamma_1 \log p_{it} + \delta_1 \log r_{it} + \varepsilon_1 \log w_{it} + \xi_{1it}$$
$$\log g_{it} = \mu_{2i} + \gamma_2 \log p_{it} + \delta_2 \log r_{it} + \varepsilon_2 \log w_{it} + \xi_{2it}$$
$$\log l_{it} = \mu_{3i} + \gamma_3 \log p_{it} + \delta_3 \log r_{it} + \varepsilon_3 \log w_{it} + \xi_{it},$$

where y_i is the output of firm i, g_i is the demand for raw materials by firm i, l_i is the demand for labour by firm i, p_i is the output price to firm i, r_i is the price or raw material faced by firm i (saw logs) and w_i is the price of labour faced by firm i. The study does not worry about the flexibility of the functional form chosen to represent the output equation (and indeed implicitly assumes that the production function is Cobb-Douglas), but instead argues that the output equation can be regarded as a first-order approximation to the true output equation. Similar remarks are made about the input demand equations. Own price effects, homogeneity of degree zero and symmetry conditions predicted by a Walrasian theory of the firm can be imposed in the usual fashion by restricting the parameters in this system.

Citing Applebaum (1978) as their motivation and noting his conclusion that WPT does not perform well on data from US manufacturing, Fox and Kivanda (1994) present work which aims to evaluate the empirical performance of WPT using data generated by agricultural, fishery and forestry firms. To that end, they identified every paper that used econometric techniques to estimate cost functions, profit functions or systems of factor demand functions, which was published in the major *Agricultural and Resource Economics* journal between 1978 and 1991. This involved them making a survey of 70 studies which estimated cost, profit or systems of input-output functions and/or tested all or part of WPT.

Fox and Kivanda (1994) found that in the 70 articles surveyed, only 54% tested WPT at all, in spite of the 'falsificationist rhetoric' that they found in this part of the literature. In this subsample, they found that homogeneity was tested only 8.5% of the time and was rejected 50% of the time, monotonicity was tested in 38% of the papers and was rejected only 3.7% of the time, curvature was tested in 45.7% of the papers and was rejected 31.2% of the time, and symmetry was tested in 17% of the papers and was rejected 33.3% of the time. As a result of their study Fox and Kavanda (1994) argue that: "there would appear to be, in our judgment, no basis for a Popperian economist to claim that 'any sensible cost or profit function possesses the properties of [WPT]'." Fox and Kavanda (1994; p. 6).

In a companion piece to Fox and Kavanda (1994), Clark and Cole (1994) raise some additional methodological issues not considered by Fox and Kavanda, including an evaluation of some studies of WPT that use cointegration techniques and also modify slightly the set of implications of WPT that can be tested. Their summary of Fox and Kavanda (1994) is that: "The[ir] results lead to a disturbing conclusion: there seems to be little support for the neoclassical [Walrasian] model from these studies." Clark and Cole (1994; p. 19). After making the methodological changes which they deem necessary, (see Clark and Cole (1994) for details), the authors conclude that: "Our analysis of the assumptions of neoclassical [Walrasian] models focuses on short-run cost minimisation ... In even this case, the neoclassical model appears to be unrealistic in many applications. This suggests ... that the justification for imposing neoclassical properties *a priori* is in general relatively weak." Clark and Cole (1994; p. 26). The results reported by Fox and Kavanda (1994) and Clark and Cole (1994) are particularly interesting because they do not depend on the outcome of a single test of WPT theory but are instead derived from a meta-analysis of a large number of tests of WPT.

Barnett (2002) has raised questions about the reliability of many parametric tests of WPT, which impose curvature globally on technology, but only impose monotonicity locally or not at all. Barnett points out that such a practice is problematic because without satisfaction of both curvature and monotonicity conditions, the second-order conditions for optimisation fail, duality fails and the resulting first-order conditions, input demand and output supply functions become invalid. Commenting particularly on the study by Barnett, Kirova and Pasupathy (1995), which followed the 'widespread practice' of imposing global curvature in a study of the behaviour of financial intermediary firms and manufacturing firms, Barnett (2002) observes that subsequent inspection of the results reported in Barnett, Kirova and Pasupathy (1995) revealed that: "... isoquants, although always satisfying the imposed curvature, often had positive slopes at one or both ends of the isoquants. These positive slopes of isoquants demonstrate violations of monotonicity ... I believe that this problem is relevant to *a large percentage of the currently popular research that similarly imposes only curvature*." Barnett (p. 2). Kumbhakar and Tsionas (2008) take a similar stand when they remark that the estimation of production technology using dual cost or profit functions forces researchers "... to decide whether the cost or profit function should be used. Most often, the decision is in favour of a cost function without much justification from either theoretical or empiral viewpoints." Kumbhaker and Tsionas (2008; p. 147). Since the aim is to ultimately test for profit maximising behaviour, the authors employ the test of Schankerman and Nadiri (1986) on a panel of data consisting of annual observations on the operations of 23 airlines in the US over the period 1971–1986, a period that covers the event of airline deregulation in 1978. In the event they find that 'the profit maximising model is rejected by the data' — see Kumbhaker and Tsionas (2008; pp. 156–63) for details. Apart from casting doubt on a good deal of previous empirical research into the reliability of WPT, these findings and the remarks in Barnett (2002) in particular provide motivation for a consideration of non-parametric tests of WPT, a topic to which we now turn.

11.2.4. *Non-parametric tests of WPT*

Varian (1984b) pioneered the use of the non-parametric techniques in the testing of WPT. Using a sample of US manufacturing data, he found general consistency with the predictions of WPT. Using a similar approach, Mueller (1992) tests WPT using data on agricultural production for each of the 48 contiguous states of the United States over the period 1956–1982. Under the joint hypotheses of profit maximisation, convex technologies and nonregressive technical change, tests were conducted for each state for profit maximisation and CRS. For those states with complete input-output data it was found that observed behaviour was fully consistent with profit maximisation and with the predictions of WPT.

Using more diaggregated data than that employed by Mueller (1992), the study by Tauer (1995) tested the weak axiom of profit maximisation (WAPM) and the weak axiom of cost minimisation (WACM) for a sample of 49 New York dairy farms.

A sample of 11 years of data on each farm was available to test WAPM and WACM, using a non-parametric Malmquist productivity index to control for technological change. In the event, it was found that most farms violated WAPM but that more came close to satisfying WACM (which is a necessary but not sufficient condition for profit maximisation). In particular, over half the farms in the sample were within 10% of cost minimisation. In interpreting such tests, it is important to notice that, as Dasgupta (2005) points out, the WAPM implies behaviour of the sort in condition (b′) in Section 11.2.3.1, but is not implied by it. It is therefore possible to observe firms that behave in a way consistent with Debreu's, 'price variation condition' (see Debreu (1959; p. 47) and Dasgupta (2005; p. 170)) but not be behaving in a way consistent with the WAPM. Ray and Bhadra (1993) modify Varian's WACM to obtain a weak axiom of variable cost minimisation (WAVCM). Using a sample of farms in West Bengal, they find strong evidence against WACM, a result which they claim cannot be rationalised in terms of production uncertainty or risk aversion. However, when the WAVCM is tested, there are many fewer violations, suggesting to the authors that violations of WACM (and by inference WAPM), is due to imperfections in the markets for land and capital rather than to non-cost minimising intentions by farmers. In further work, Ray (1997) notes that Varian's WACM can only be applied when both input price and quantity data are available for individual firms, thus limiting the applicability of this test device. He therefore develops the weak axiom of cost dominance (WACD) and shows that this can be used to test for cost minimising behaviour even when input quantity data is not available. Applying the technique to Nerlove's electricity utilities data it is found that there are significant violations of cost minimisation (and by inference profit maximisation and WPT). Steward (1998) begins by noting that while Varian's WACM approach has many advantages, it possesses a 'potentially critical weakness' which is that "… measurement error and other stochastic influences are not explicitly accounted for…" Steward (1998; p. 617) — an issue also considered in Varian (1985). Such an omission has the potential to invalidate statistical inferences, particularly about firm efficiency and cost minimising behaviour. Steward (1998) therefore develops a bootstrap based test statistic for the WACM (actually for WAVCM) and applies it to a sample of 320 US banks whose production activities in 1993 were taken as data. In the event it was found that WAVCM failed and in fact "… the average bank in the sample could have produced the same output using only 77% of the inputs that were actually used." Steward (1998; p. 619). While acknowledging the difficulties associated with making cross study comparisons, he notes that this rate of inefficiency is lower than as reported in non-parametric studies by Aly *et al.* (1990) and English *et al.* (1993) and in the parametric study by Hunter and Timme (1995).

Ray (2004) also takes up the issue that violations of WACM may not be due to firm inefficiency in choosing input levels but may be due instead to 'measurement error'. Noting that Varian (1985) considered random variations in input quantities as possible sources of measurement error, Ray (2004) develops a statistical test in which random variations in output are allowed. Applying his technique to data from an (effective) sample of 20 US airlines in 1984, it was found that 7 of the 20 (i.e. 35% of the sample), showed no violations of WACM. For a further 6 airlines (30% of the sample), none of the violations was statistically significant, while the remaining 7 airlines showed 'significant violation' of the WACM. However, as Ray (2004) points out "… about 40% of the observed violation of WACM … can be ascribed to chance variation in output rather than to inefficiency." Ray (2004; p. 9). Working in a related direction Silva and Stefanou (2003) present a generalisation of the tests of static cost minimisation in Varian (1984b) is tested to cover the case dynamic cost minimisation. They develop a WADynamicCM and apply it to a balanced panel of Pennsylvanian dairy operators for the period 1986 to 1992. This data for consistency with cost minimisation in three ways: (i) exhaustive pairwise comparisons to check for violations of WADCM; (ii) calculation of the goodness-of-fit measure proposed by Varian (1990) to check the economic significance of any violations; and (iii) a stochastic test to check the statistical significance of any observed violations — see Silva and Stefanou (2003; pp. 19–20) for details. In the event it was found that violations of WADCM occurred in all years, with the average percentage error relative to WADCM ranging from 79.9% to 564.6%, meaning that the departures were economically significant. The deterministic goodness of fit tests also showed high percentages of violations of the WADCM, with some of the violations being 'relatively high'. Finally, the stochastic test found that violations of the WADCM were statistically significant. These outcomes lead to the authors to remark that: "... test results for WADCM indicate inconsistency of the data series with the dynamic cost minimisation hypothesis." Silva and Stefanou (2003; p. 21). The authors note however that apart from the hypothesis of dynamic cost minimisation being an inadequate description of the data, there may be several other reasons for these these outcomes, including: possible non-convexities in dynamic technologies due say to 'lumpiness' in investment; economically inefficient use of variable and/or quasi-fixed factors; important excluded variables, such as unexpected weather conditions; data measurement error, particularly with respect to employment levels of quasi-fixed factors; and finally, the fact that nonparametric tests may be biased towards rejection because necessary as well as sufficient conditions for cost minimisation are being tested. This is an interesting possibility and as Silva and Stefanou (2003; p. 21) point out reveals a 'paradox' that the nonparametric approach is both less and more structured and demanding than the parametric approach. Less for the familiar reason that it does not require the specification of a functional form in order to conduct the test. More because 'first and second order conditions' are incorporated in the analysis, whereas parametric tests include only first order conditions. While these are all important possibilities, it is interesting to note that the test results as they stand did show convincing rejection of the cost minimisation hypothesis.

As was the case in tests of Walrasian consumer theory, tests of Walrasian producer theory have produced mixed results. As is also the case with all empirical work, further research with new empirical techniques and alternative data sets has the potential to upset any tentative conclusion that might be advanced at this point. However, on the basis of the empirical work considered here aimed at testing WPT, it seems reasonable to conclude that it is not at all obvious that Walrasian producer theory provides an empirically adequate description of producer input demand or output supply behaviour. Again, as with the empirical results concerning WCT, this might not be a matter of such great concern if the centrepiece of the notion of Walrasian equilibrium, market clearing, seems to hold empirically. It is to an investigation of this issue that we now turn.

11.2.4.1. *Testing for market clearing: commodity and labour market studies*

There have been a number of studies which aim to directly test for Walrasian style 'market clearing' across the economy, and we consider them below. As a preliminary remark, it is perhaps worth noting the observation in Gilbert and Klemperer (2000) that although "... [e]conomists praise the virtues of price as a mechanism to equate supply and demand ... markets often clear by non-price means [in particular rationing]." Gilbert and Klelmperer (2000; p. 1). They cite various studies which show rationing occurs in markets for commodities as diverse as microprocessors, several metals, electronic parts, metal fasteners, gypsum board, personal computers, semi-conductors, compact disks, titanium dioxide, polypropylene and petrochemicals (see Gilbert and Klemperer (2000; p. 1) for details). They also note a study by Rotemberg and Summers (1990) which finds that a mild form of rationing occurs in which some customers receive the good without delay while others have to wait longer for their order to be filled 'without receiving any compensating price discount'. At a slightly more macro level, Mortensen and Wright (2002) note that labour and housing markets are not obviously in Walrasian states and they ask: "[w]hy does it appear that some markets, say those for labour services and housing, fail to clear?" Mortensen and Wright (2002; p. 1). There maybe something important to be learned from available studies about the empirical relevance (or otherwise) of Walrasian equilibrium states in actual markets. One particularly interesting study that attempts to test for Walrasian market clearing is that due to Rudebusch (1989). As motivation for the study, Rudebusch (1989) makes the following observation: "Although the assumption that markets are continuously cleared by price aids in the formulation of rigorous theoretical models, it is still in doubt as to whether such equilibrium models can be reconciled with the short-run behaviour of the economy." Rudebusch (1989; p. 633). A sketch of the theoretical foundations of Rudebusch (1989) may be made by considering the model constructed by Varian (1977) — see also Hahn (1978) and Babenko and Talman (2006) for developments of Varian's ideas. In Varian's model, there are two agents (consumers and producers) and two goods, a perishable consumption good (C) which has a price p and labour (L) which has a nominal wage v and a real wage $w = v/p$. The technology is described by a production function $f(L)$. Given the real wage, a profit maximising firm will make a *Walrasian demand for labour* $Q_d(w)$. As Varian (1977;

p. 574) notes however, this sort of behavioural hypothesis is very restrictive. Under it firms only focus on real wages — and take no account of other signals that the economy might be generating. To loosen this restriction, Varian (1977) supposes that firms form point expectations about the demand for their product. This *expected demand* is denoted by y and firms then choose a production plan to maximise profit given that output will be less than or equal to y. As Varian (1977; p. 574) points out, this behaviour gives rise to the *constrained demand for labour function*, $Q_d(w, y)$. As for consumers, when faced with w (i.e. v and p), they make plans for labour supply and consumption demand. The Walrasian labour supply function is $Q_s(w)$ and Varian (1977) assumes that all labour income is consumed. Also some proportion of profit income, denoted by $P(w, y)$ is consumed. The Walrasian demand for consumption goods is then $wQ_s(w) + P(w, y)$. However, if households cannot sell their desired amount of labour — meaning that $Q_d(w, y) < Q_s(w)$ — and if we let $Q(w, y) = \min\{Q_s(w), Q_d(w, y)\}$ then the *effective demand* for consumption goods is $Y(w, y) = wQ(w, y)+P(w, y)$. The effect of introducing y here is that as Varian (1977) notes: "... when one introduces a new state variable into the [complete] Walrasian system one allows for the existence of non-Walrasian equilibria [in fact] the implicit function theorem then implies that we will have a continuum of equilibria ... [but] only certain points of this set of equilibria will be equilibria of the complete [Walrasian] system." Varian (1977; pp. 588–89). Using this theoretical framework, Rudebusch (1989) tests for the existence of Walrasian or non-Walrasian states in a sample of US data, looking for evidence of constraints on the decisions of consumers and/or producers, other than those of the Walrasian (i.e. price) type. In particular, to implement his test Rudebusch (1989) supposes that there are two agents, 'households' and 'firms' and three goods, 'labour', 'consumption' and 'investment'. When the household does not face a constraint in the labour market, then its optimisation problem is: $\max_{C,L} U(C, L)$ subject to the budget constraint that $p_c C = wL + n \Rightarrow C_w(p_c, w, n)$ and $L_w(p_c, w, n)$ are the resulting Walrasian or notional consumption demand and labour supply functions. When the household faces a constraint in the labour market, then $L \leq L^u$ is an additional constraint and the solution of this general optimisation problem yields effective consumption demand $C_e(p_c, w, n; L^u)$ and labour supply $L_e = L^u$. Effective demand for goods is equal to the Walrasian demand when labour supply is at its Walrasian level. Thus $C_e(p_c, w, n; L_w(p_c, w, n)) = C_w(p_c, w, n)$. Taking a Taylor expansion of the effective consumption demand function around the unrationed point yields $C_e \approx C_e(p_c, w, n; L_w) + s_c(L^u - L_w)$, where $s_c = \partial C_e/\partial L^u$ is the "spillover effect" of the constraint in the labour market onto the consumption goods market. Firms are characterised by a production function $Y = f(L, KS, E)$ and they generate demands for labour, investment goods and supplies of investment and consumption goods by maximising profits subject to this technology. In order to obtain the effective demands and supplies of firms, it is observed that in general quantity constraints experienced by the firm in any other market will 'spill' over into a given market. Thus effective labour demand is: $L_e^d = L_w^d(w, p_c, p_E) + s_1(I^u - I_w^s) + s_2(C^u - C_w^s)$.

Similar expressions for investment goods demand and supply and consumption goods supply are also developed. The test for Walrasian equilibrium which Rudebusch (1989) conducts involves estimating the general system (i.e. allowing for the possibility of quantity constraints) and then comparing that with the restricted (Walrasian equilibrium) system via a likelihood ratio test. Using a sample of US data 1967:1–1981: Rudebusch (1989) concludes that: "The 5% level of $\chi(3)/2$ is 3.91 and the 1% significance level is 5.65. [Given that the LR statistic is 94.4] clearly we can reject the hypothesis of three markets in equilibrium." Rudebusch (1989; p. 649). Summarising these results he writes: "We find *no evidence of Walrasian equilibrium*". Rudebusch (1989; p. 621, emphasis added).

Focusing on the labour market in the UK between the first and second world wars, Hatton (1988) begins by identifying two 'polar cases'. The first, which is attributed to Beveridge (1944), is that the cause of interwar unemployment was a 'persistent weakness in the demand for labour', which could be taken as evidence of labour market disequilibrium — see Beveridge (1944; p. 89). The second, attributed to Benjmin and Kochin (1979), argues that it is the supply of labour which contracted in the period and that 'the army of the unemployed at the time was a volunteer army' — see Benjamin and Kochin (1979; p. 474). In the later case, unemployment is not evidence of labour market disequilibrium. Using quarterly data, Hatton (1988) estimates a three equation (labour demand, labour force and wage adjustment) model of the UK labour market for the period 1921:1 to 1938:4. In the event, he finds that the data makes it difficult to reject the polar cases. Hatton (1988) observes that "When different views are cast as restrictions on a more general model it proves difficult to reject ... [the] polar cases. The data appears broadly consistent with a story of real wage rigidity, one of benefit-induced unemployment [supply reduction] and, to a lesser extent, one of structural unemployment [demand deficiency]." Hatton (1988; p. 22). It is therefore not clear from this study if the interwar UK labour market was in a Walrasian or a non-Walrasian state. Considering the labour market in the Netherlands, Teulings and Koopmanschap (1989) note that there were large differences in unemployment rates between education levels (something also noted and discussed by us in some detail in Chap. 4). The authors also note that according to the Walrasian theory of how the economy works, these differences should be explained by 'distorted' relative prices or by a lack of human capital. The authors advance and test an alternative hypothesis which is that high unemployment rates for lower education levels is caused by persistent excess supply of labour. Using a sample of data for the Netherlands for the period 1967–1985, they find that in contrast to the Walrasian market-clearing hypothesis, differences in unemployment rates can be explained by persistent excess supply of labour, a finding also consistent with a breakdown in irreducibility considered by us earlier. Also focusing on the labour market, Hall *et al.* (1992) develop a test procedure for serial correlation in discrete switching disequilibrium models. The technique is applied to the UK labour market where it is found that the model outperforms an equilibrium alternative on a number of test criteria.

Hofler and Spector (1993) begin by noting that basically two methods for modelling the determination of employment have been used in the literature. They involve comparing an equilibrium model with a disequilibrium alternative or using a switching regression model. The authors introduce an alternative approach based on the distribution of the error term and on the sign and significance of the real wage coefficient in a reduced form equation for employment. Using a sample of US data for the period 1948–1984, the authors find that: "... the United States labour market has been operating under a fixed wage regime, in which employment is being determined by the short side of the market. Furthermore, the tests also indicate that the real wage *is as likely to be below the equilibrium real wage as it is to be above it.*" Hofler and Spector (1993; p. 123, emphasis added). This is a particularly interesting finding as it indicates that even in a relatively deregulated labour market like that in the Unites States, the economy still appears to have trouble in arriving at the Walrasian equilibrium real wage. Furthermore, on the basis of the evidence in Hofler and Spector (1993), the economy seems to undershoot as often as it overshoots relative to the equilibrium real wage. Arcand and

Brezis (1993) adopt the following approach to testing for market clearing. They take a sample of pre-World War II aggregate data on the US economy — 1892 to 1940 — and examine the flexibility of wages and prices over that period using a simple two-market disequilibrium model and four alternative specifications of a tatonnement adjustment mechanism. A brief sketch of the approach is as follows (for full details see Arcand and Brezis (1993; esp pp. 558–68)). The authors begin by noting that many economists view the long persistence of the Great Depression as being due to 'sluggish adjustment of wages and prices'. They also note that: "A corollary of this is the widely-held view that wage and price flexibility are stabilizing." Arcand and Brezis (1993; p. 555–56). But what if the adjustment processes at work in the economy were not stabilizing? This is a reasonable question because, as we saw in Chap. 8, it is by no means obvious that stabilizing adjustment processes occur naturally and under reasonable conditions. Arcand and Brezis (1993) therefore set up a testing framework in which it is possible to check if the dynamics of the economy were stabilizing, destabilizing, or as they put, it 'neutral'. This latter possibility means that the dynamics of the economy show no tendency towards restoring equilibrium or pushing the economy further away from equilibrium — however they may lead to the economy getting 'stuck' away from Walrasian equilibrium. The four specifications of the adjustment processes considered by Arcand and Brezis (1993) are: (i) a 'PW process' which is a standard Walrasian excess demand driven wage-price adjustment; (ii) a 'YL process' which is a Marshallian quantity adjustment process where supply increases if the 'demand price' exceeds the 'supply price'; (iii) a 'YW process' where the labour market is adjusted by wages and the commodity market is adjusted by quantities; and (iv) a 'PL process' where the labour market is adjusted by quantities and the goods market by prices. In the estimation of the model it is found that the equilibrium restrictions are 'strongly rejected'. However, three of the disequilibrium specifications are not rejected. This leads the authors to argue that: "... based on the dynamics uncovered by our econometric work ... the United States economy was dynamically neutral during the interwar years. The economy may even have been slightly unstable ... The key is that it was not dynamically stable. It was not self-correcting. A series of negative shocks took the economy out of equilibrium: there was nothing pulling it back." Arcand and Brezis (1993; pp. 586–87). The authors' finding leads them to reject 'both statistically and conceptually the equilibrium approach'. As already noted, this is an interesting finding given our theoretical considerations in Chap. 8 which were largely pessimistic about finding plausible adjustment processes that were stabilizing under general conditions.

Dutkowsky (1996) also considers the issue of 'price stickiness', in this case in post-WWII data, pointing out that it is the key focal point in the debate between New Keynesians and those who believe in continuous market clearing — see Dutkowsky (1996; p. 427). Dutkowsky (1996) employs a notion of price stickiness that derives from McCallum (1978), Frydman (1981), Gordon (1990) and Mankiw (1990) which has it that 'prices are sticky if they fail to move sufficiently to clear the market for aggregate goods and services'. Using a sample of aggregate US quarterly data for the period 1973:1 to 1990:3 the estimated models produce "... findings [that] uniformly reject the complete flexibility of prices within the macroeconomy." Dutkowsky (1996; p. 429). This finding carries with it a rejection of Walrasian market clearing, by definition of the notion of price stickiness adopted by Dutkowsky (1996).

Holmes and Hutton (1996) examine annual aggregate US data for the period 1920 to 1989 for evidence of sticky nominal wages and involuntary unemployment. They find significant evidence of both things and point out that these phenomena are inconsistent with 'market-clearing microeconomics' — see

Holmes and Hutton (1996; p. 1581). They also note that, in the model which seems best to describe the data "... this unemployment will not be eliminated by market forces." Holmes and Hutton (1996; p. 1582).

Focussing on the goods market, Rao (1994) estimates a disequilibrium model using a sample of US data for the period 1946–1991. While admitting that there are limitations to the conclusions one can make, as is the case with any empirical study, Rao (1994) nevertheless advances the following as tentative conclusions: "Firstly, the aggregate US goods market is a *disequilibrium market* and therefore the Keynesian rather than the [Walrasian] framework is more appropriate for analysing this market. Secondly, *all the micro markets are unlikely to be in either excess demand or excess supply states in any given time period.* Therefore the variance of excess demand, across the micro markets, is an important explanatory variable of output and price level." Rao (1994; pp. 423–424, emphasis added). Both of these conclusions are very interesting and suggest that Keynesian disequilibrium, rather than Walrasian market clearing, is a feature of the US goods market for the period analysed by Rao (1994).

Manning (1994) observes that it is a common belief that observed wages are above those that would prevail in a competitive equilibrium, particularly when unemployment is also observed. In order to assess this view, he firstly develops an efficiency wage model in which an increase in a binding minimum wage may increase employment. He then develops a general equilibrium-matching model in which there is involuntary unemployment but in which real wages are *below* their market clearing levels. He then considers empirical evidence on unemployment and wage determination to argue that such evidence: "... is just as consistent with this model as with models in which wages are at or above [Walrasian] market-clearing levels." Manning (1994; p. 1). Wang and Zhou (1996) develop a semiparametric technique to estimate the disequilibrium model proposed by Fair and Jaffee (1972). The approach allows a consistent estimation of the model without needing to fully specify the distribution of error terms in both the demand and supply equations, something which has a number of advantages over current techniques for estimating such models. Applying this technique to the Quandt and Rosen (1988) US labour market data, the authors find that the US labour market is described by significant degrees of disequilibrium.

Cole and Ohanian (1999) examine the capacity of a Walrasian market clearing model to account for the behaviour of the US economy during the Great Depression of the 1930s. They find that while the Walrasian model does predict a long deep downturn in response to the large real and monetary shocks that impacted the US economy in the period 1929–1933. However, Walrasian general equilibrium theory predicts a much different recovery from this downturn than the one which actually occurred. In particular, the fact that real output remained at 25–30% below trend during the late 1930s is at variance with the predictions made by a Walrasian model in the face of significant increases in total factor productivity, the money supply and the elimination of bank failures. Palley (1999) constructs a Keynesian general disequilibrium model with 'inside nominal debt'. He shows that in the context of such a model: "... wage adjustment may be insufficient to restore Walrasian equilibrium." Palley (1999; p. 785). Although no formal test of the model is presented, Palley observes that the model is consistent with the stylised facts concerning the US macroeconomy.

Hagan and Mangan (2000) examine Australian unemployment from the mid-1970s to the late-1990s. Their central finding is that although labour demand has been strong over the period, despite this: " ... growth in labour demand has been overshadowed by even greater growth in labour supply, creating a state of *permanent labour market disequilibrium*." Hagan and

Mangan (2000; p. 393, emphasis added). They also comment that the phenomenon is not unique to Australia, ‘but is characteristic of most western economies around the world’. In a departure from the mostly aggregate studies reported above, Weiler (2001) reports a variety of structural labour market models in an effort to understand regional jobless rates. Using both panel and case study data for West Virginia, it is found that “... the non-market clearing dual framework offers the best insights into structural unemployment in West Virginia. The state’s labour markets appear to be both segmented and non-clearing over the long run.” Weiler (2001; p. 587). Such findings cast doubt on the hypothesis of continuous Walrasian market clearing, at least in labour markets. Laroque and Salanie (2002) use individual data from the 1997 French Labour Force Survey, to explain the non-employment of married French women. They find that minimum wages can account for 15% of non-employment for these women and that disincentive effects from welfare measures ‘*may* also be large’, although they are unable to quantify the effect of the welfare measures. Again for this sample of French data the authors conclude that market clearing provides a poor approximation to reality.

The results of tests of the Walrasian market clearing surveyed above are somewhat less ambiguous than that which attempts to test the hypothesis by testing WCT or WPT. Here it seems fairly clear that across a number of methodologies and data sets the hypothesis of market clearing, which is a central part of the definition of Walrasian equilibrium, is generally rejected. It might of course be argued that these tests are conducted in economies where ‘prices were not flexible enough to get the economy to Walrasian equilibrium’ (for, say institutional reasons). That may indeed be the case, as the studies of Weiler (2001), Laroque and Salanie (2002) suggest. However, that does not alter the fact that continuous Walrasian market clearing does not seem to describe the experimental or actual data.

11.2.4.2. *Testing for market clearing: The weak axiom of revealed equilibrium*

Beginning at the same point as non-parametric tests of consumer and producer theory, namely Afriat's (1967) theorem (see the discussion earlier in this chapter), a number of authors have sought to formulate Axiom of Revealed Preference like restrictions on price and individual endowment profile pairs in order to test equilibrium conditions. The seminal paper in this endeavour is Brown and Matzkin (1996), who propose the following *equilibrium inequalities.* Consider an exchange economy with ℓ goods, commodity space $\Re^{\ell}$, I consumers characterised by a consumption set $X_i = \Re^{\ell}_{+}$, an endowment bundle $\omega_i \in \Re^{\ell}_{++}$, and a utility function $U_i : X_i \to \Re$ that is continuous, monotone and concave. Suppose observations are made of a finite number, N, of profiles of individual endowment vectors $\{\omega_i^r\}_{i=1}^{I}$ and market prices p^r where $r = 1, \ldots, N$ but that utility functions and consumption bundles are not observable for individuals. Then as Brown and Matzkin (1996) show, Walrasian equilibrium can be tested in an exchange economy context by checking the consistency of the data on individual endowment profiles and prices with the following restrictions.

There exist $\{\overline{U}_i^r\}_{r=1}^{N}{}_{i=1}^{I}, \{\lambda_i^r\}_{r=1}^{N}{}_{i=1}^{I}, \{x_i^r\}_{r=1}^{N}{}_{i=1}^{I}$ such that:

$$\overline{U}_i^r - \overline{U}_i^s - \lambda_i^s p^s (x_i^r - x_i^s) \leq 0 \qquad (r, s, = 1, \ldots, N; i = 1, \ldots, I) \tag{11.1}$$

$$\lambda_i^r > 0, x_i^r \geq 0 \qquad (r = 1, \ldots, N; i = 1, \ldots, I) \tag{11.2}$$

$$p^r x_i^r = p^r \omega_i^r \qquad (r = 1, \ldots, N; i = 1, \ldots, I) \tag{11.3}$$

$$\sum_{i=1}^{I} x_i^r = \sum_{i=1}^{I} \omega_i^r \quad (r = 1, \ldots, N), \tag{11.4}$$

where $\overline{U}_i^r$ are (unobservable) utility levels, λ_i^r are marginal utilities of income, x_i^r are consumption bundles, p^r are prices and

ω_i^r are endowments. Brown and Matzkin (1996) call an exchange economy *testable* if for every N the above family of polynomial inequalities is satisfied by the observed pairs of profiles of individual endowments ω_i^r and prices p^r if and only if they lie on some equilibrium manifold, i.e. the set of price-endowment profile pairs, with excess demand zero, i.e. $\{(p, \hat{\omega}) : Z(p, \hat{\omega}) = 0\}$, where $\hat{\omega}$ is an endowment profile, that along with prices, is allowed to vary. By application of the Tarski-Seidenberg algorithm in the context of an Arrow-Debreu economy with enough structure to admit the existence of an equilibrium, plus a judiciously chosen example of an exchange economy in which the equilibrium inequalities can be violated (see their Fig. 1, p. 1254), they show that the equilibrium hypothesis is testable, at least in an exchange economy — (see Brown and Matzkin (1996; Theorem 1)). Since individual endowment vectors may be hard to observe, Brown and Matzkin restate the inequalities in (1)–(4) in terms of market prices, p^r, individual consumer incomes M_i^r and aggregate endowment ω^r, at observation r (see their Theorem 2). Using the Chiappori-Rochet version of Afriat's theorem — which derives conditions under which data can be rationalised by a monotone, strictly concave and continuously differentiable utility function see Brown and Matzkin (1996; p. 1253) for details — this leads to the *Weak Axiom of Revealed Equilibrium* for a two person pure exchange economy which may be formulated as follows. Let $\bar{z}_i^r$ (for $r = 1, 2$ and $i = a, b$) denote any vector such that $\bar{z}_i^r \in \text{argmax}_x\{p^s x : p^r x = M_i^r, 0 \leq x \leq \omega^r\}$, where $r \neq s$, meaning that among the budget feasible consumption bundles for i in observation $r, \bar{z}_i^r$ is any of the bundles that cost the most at prices p^s (and $r \neq s$). With this set up Brown and Matzkin (1996) make the following definition:

Definition 11.1 (Weak Axiom of Revealed Equilibrium). A finite set of observations $[\{p^r\}_{r=1}^2, \{M_i^r\}_{r=1}^2{}^b{}_{i=a}, \{\omega^r\}_{r=1}^2]$ on prices, individual incomes and aggregate endowments satisfies the *Weak Axiom of Revealed Equilibrium* if (i) $\forall$ states $r = 1, 2, M_a^r + M_b^r = p^r\omega^r$; (ii) $\forall r, s = 1, 2$ $(r \neq s)$ and $\forall i = a, b$ it is true that $\{(p^s \bar{z}_i^r \leq M_i^s) \Rightarrow (p^r \bar{z}_i^s > M_i^r)\}$; (iii) $\forall r, s = 1, 2$ $(r \neq s), \{(p^s \bar{z}_a^r \leq M_a^s) \wedge (p^s \bar{z}_b^r \leq M_b^s\} \Rightarrow (p^r \omega^s > p^r \omega^r)$.

Remark 11.2. In interpreting these conditions, Brown and Matzkin (1996; p. 1255) point out that: (i) means the sum of individual incomes equals the value of total endowment; (ii) means that it must be the case that at least some of the bundles feasible in observation s and in the budget set for i in state s, cannot be purchased with the income and the prices faced by i in state r; (iii) guarantees that at least one of the pairs of consumption bundles appearing in observations that contain, for each agent, feasible and affordable bundles that could not be purchased at the income-price configuration in state r, are such that they add up to the aggregate endowment. Brown and Matzkin (1996; Theorem 3) then show that a set of observations $[\{p^r\}_{r=1}^2, \{M_i^r\}_{r=1}^2{}_{i=a}^b, \{\omega^r\}_{r=1}^2]$, where p^1 is not simply a scalar multiple of p^2, lies on the equilibrium manifold of some exchange economy if and only if the data satisfy the WARE.

Remark 11.3. Brown and Matzkin's WARE provides a criterion which allows the Walrasian equilibrium hypothesis to be tested, at least in the context of an Arrow-Debreu exchange economy. Various extensions of this methodology have been achieved. First by Brown and Matzkin (1996) who note that the approach can be extended 'to find testable restrictions on the equilibrium manifold of production economies'. Further work in this direction is Carvajal (2005) who shows that the testability results of Brown and Matzkin can be obtained in an economy with aggregate production even if individual production levels are not observable. Brown and Matzkin (1996) also note that their approach can be extended to economies with incomplete futures markets and various sorts of asset structures. Snyder (2004) developes the Brown-Matzkin insight by investigating the questions of how much data is needed in order to test the competitive equilibrium model and why exactly Brown-Matzkin get testable restrictions when the SMD result seems to rule them out. She concludes that being able to observe individual incomes is crucial to the Brown-Matzkin outcome as is the restriction of their search for testable restrictions to a finite domain. By exploiting the 'semi-algebraic' nature of the problem of finding such restrictions, Snyder (2004) provides a general framework for deriving testable conditions. Reinforcing the point made by Snyder (2004) that individual data is essential in the search for testable restrictions, Chiappori *et al.* (2004) derive necessary and sufficient conditions that allow equilibrium prices to be characterised as functions of individual initial endowments. Furthermore, these conditions, when satisfied, allow an economy to be identified, at least generically. However, when only aggregate data is available 'observable restrictions vanish' which leads Chiappori *et al.* (2004) to conclude that 'the availability of individual data is essential for the derivation of testable consequences of the general equilibrium construct'. Interestingly, this is also the conclusion we arrived at in considering direct tests on consumer and producer theory earlier in this chapter. Exploiting the consequences of the assumed optimality of equilibrium allocations and the hypothesis of individual rationality at the heart of standard general equilibrium theory, Bachmann (2006) derives a set of robust testable restrictions in an exchange economy from the joint assumptions of Pareto efficiency and individual rationality. In interesting extensions which move somewhat away from the pure Arrow-Debreu framework, Snyder (1999) derives testable restrictions from a model in which public goods are supplied in a Pareto optimal manner, while Carvajal (2004) considers the extension of Brown-Matzkin to an exchange economy with externalities. Again reinforcing the insight in Snyder (2004), he notes that 'if there is no information on individual choices then in this model the equilibrium concept imposes no restrictions'. In noting that in principle there are many data sets against which the equilibrium restrictions could be tested, Brown and Matzkin (1996; p. 1258) observe there are some practical problems to be overcome in the actual implementation of such tests. They identify allowing for random variations in tastes as being one of the most important such challenges. They note that Brown and Matzkin (1995) consider a random utility model which yields a stochastic family of Afriat inequalities that can in principle 'be identified and consistently estimated'. They nominate the extension of this approach to random exchange economies as a 'significant step in empirically testing the Walrasian hypothesis'. In an interesting contribution, Carvajal (2004) considers the Brown-Matzkin problem in an exchange economy with random preferences and argues that even in such an environment 'general equilibrium theory is falsifiable'.

The work due to and inspired by Brown and Matzkin (1996) is very important at the methodological level. However, to date, there are few published empirical applications of the restrictions which they derive — not withstanding the Brown-Matzkin observation that there are many data sets which could potentially be used for the purpose (see their pp. 1257–58). This is an obvious area for further work, but in the meantime for evaluations of the Walrasian equilibrium hypothesis we need to rely on the empirical work that has actually been conducted.

11.2.4.3. *Testing for market clearing: General equilibrium models and macroeconomic data*

Fluctuations in macroeconomic variables, such as output, employment, prices and wages, have been extensively studied in economics. The reason for this attention is, as Deneckere and Judd (1986) point out because: "The most vexing question in macroeconomics is the issue of why economic activity fluctuates. Our belief as to why economies fluctuate largely determines our attitudes towards social efforts to stabilise economic activity." Deneckere and Judd (1986; p. 1). Standard explanations of business cycles have, until relatively recently, usually relied one way or another on Keynesian or other 'disequilibrium' paradigms. The suggestion that fluctuations in aggregate activity might be a general equilibrium phenomenon is, as Blanchard and Fischer (1989) point out, a fairly recent departure from the standard mode of economic thought. As Blanchard and Fischer (1989) put it:

"For most of the 20th century, especially since the Great Depression, most macroeconomists have looked upon the sharp fluctuations in output and unemployment as *prima facie* evidence of major market imperfections and explored what these imperfections might be. In the last 15 years, however, some have argued that this is a misguided research strategy ... [and] that fluctuations can be explained as the realisation through time of the set of [equilibrium] transactions agreed upon in a complete market Arrow-Debreu economy ..."

Blanchard and Fischer (1989; p. 320). In a similar fashion Chowdhury *et al.* (1994) argue that: "Since the Depression, several theories have been offered as explanations of the causes of business cycles in advanced industrial economies. These can be classified into two groups: equilibrium and disequilibrium theories. Equilibrium business-cycle theories consider short-run deviations of output from trend to be consistent with a state of equilibrium ... In contrast to equilibrium theories, disequilibrium or Keynesian theories view nominal or real rigidities and the resulting failure of prices to clear markets as the main cause of business cycles." Chowdhury *et al.* (1994; p. 527). In light of this situation, an indirect empirical test of the Walrasian hypothesis is provided by considering the empirical adequacy of models of the business cycle that are built on general equilibrium foundations.

There are two major equilibrium models of the business cycle. The first attempts to explain macroeconomic fluctuations in terms of unexpected shocks to the money supply and/or to fiscal policy variables. Models in this class are associated with workers in the New Classical school such as Barro, Lucas, McCallum, Sargent and Wallace, and will be referred to here as 'Nominal Equilibrium Models' of the business cycle (NEMs). This nomenclature has been chosen because it is the unanticipated shocks to nominal variables that are relied on to explain business cycle fluctuations in an environment of continuous market clearing.

The general equilibrium foundations of NEMs is clearly spelled out by Lucas (1975) who writes: "In contrast to conventional macroeconomic models, the model studied below has three distinguishing characteristics: prices and quantities at each point in time are determined in *competitive equilibrium*; the expectations of agents are *rational*, given the information available to them; information is *imperfect*, not only in the sense that the future is unknown, but also in the sense that no agent is perfectly informed as to the current state of the economy." Lucas (1975; p. 1113, emphasis in the original). De Vroey (2007) puts it this way: "The basic methodological precept associated with it [new classical macroeconomics] is what Robert Lucas … has called the 'equilibrium discipline'. According to this, any valid economic reasoning must be based on two premises, that agents behave in an optimizing way and that markets always clear." De Vroey (2007; p. 328).

Models of the second type attempt to explain business cycles in terms of shocks to the 'deep parameters' of the economy, particularly technology and tastes. This type of model is associated with workers such as Kydland, Prescott, Hansen, Eichenbaum and Singleton, and will be referred to as "Real Business Cycle" models (RBCs). The general equilibrium foundations of RBCs is made clear by for instance Stadler (1994) who writes: "... RBC theory views cycles as arising in frictionless, perfectly competitive economies with generally complete markets subject to real shocks ... Thus, RBC theory makes the notable contribution of showing that fluctuations in economic activity are consonant with *competitive general equilibrium environments* in which all agents are rational maximisers." Stadler (1994; p. 1751, emphasis added). With these remarks as background, we now begin the task of assessing how well models which incorporate Walrasian market clearing perform when it comes to explaining the business cycle.

Interest in the empirical adequacy of NEMs, and the light that may throw on the empirical relevance of Walrasian market clearing, raises the question of how the adequacy of such models might be assessed. One way to proceed is suggested by the fact that an implication of this class of model is the so-called 'policy neutrality' or 'Lucas-Sargent-Wallace' (LSW) proposition. As Boschen and Grossman (1983) put it: "The most striking implication of equilibrium models, derived explicitly by Barro (1976), is a neutrality proposition that says that macroeconomic fluctuations — specifically, the time pattern of differences between actual and natural levels of real variables such as aggregate output and employment — evolve independently of those monetary actions that reflect systematic responses to macroeconomic fluctuations." Boschen and Grossman (1983; p. 174).

The policy neutrality proposition is the idea that systematic stabilisation policy will be entirely ineffective in the sense that the probability distribution of output or unemployment around its natural rate, or Walrasian equilibrium value, is independent of the policy adopted by the authorities. For an exposition of the result see, for instance, Pesaran (1982).

The policy neutrality (or ineffectiveness) result sparked a vigorous debate which attempted to understand the theoretical foundations of the result. One contribution to that debate was Tobin (1980) who argued that the proposition followed from the assumption of market clearing, not from the assumption of rational expectations, as is commonly claimed (see Palley (1993) for a discussion of this issue). This makes the empirical fate of NEMs of particular interest to us.

The LSW proposition and NEMs in general stimulated an enormous amount of empirical work. In an influential assessment of the findings of those studies Barro (1984) observed that: "The rational expectations theory of business fluctuations and the empirical work that relates to this theory are surely interesting and suggestive. However, it seems a fair assessment that this research has not provided a definitive analysis of either monetary non-neutrality or of the business cycle more generally." Barro (1984; p. 19). In considering empirical work aimed at testing the LSW proposition and NEMs of the business cycle, Mankiw (1990) argues that: "Although this [New Classical] theory of the business cycle received much attention in the 1970s and 1980s, it has attracted few adherents in more recent years. The reason for decline in popularity is not clear [as] ... there is no completely compelling evidence that explains why this approach has been so widely abandoned." Mankiw (1990; p. 1653). Similar evaluations of the literature may be found in Gray and Spencer (1990), Sephton (1990), Poirier (1991). Not everyone agreed with Mankiw (1990) however, and a number of subsequent studies have attempted to improve on the econometric work that went before in order to get at the truth concerning the applicability of NEMs. Glick and Hutchison (1990) for instance construct what they claim is an 'appropriate' specification of the output equation and US data to find support for the ineffectiveness of anticipated fiscal policy (a finding which they interpret as being consistent with a nominal equilibrium model of the business cycle). Pesaran (1991) develops a novel econometric approach to the estimation of multivariate rational expectations models by adapting a technique proposed by Wickens (1982) and in an application he takes a sample of US data 1955–1985 to test the LSW proposition. The outcome of the test is a rejection of the proposition. Smith and McAleer (1993) study the sensitivity of the LSW proposition when applied to unemployment, to variation in estimation technique using Barro's model as the test bed. In what could be a summary of the entire literature, they conclude that the results for the LSW proposition change according to the sample, the estimation method and the technique chosen for estimating the standard errors of the parameters. Cooley and Hansen (1997), reexamine the role of unanticipated changes in money growth in explaining aggregate fluctuations 'using the methods of quantitative equilibrium business cycle theory'. In particular they construct a stochastic equilibrium growth model which reflects the 'island economy' construction of Lucas (1972, 1975). The authors show that in this model, unanticipated shocks to the money supply: "...can lead to large fluctuations in real economic activity. [However] some aspects of the statistical properties of these fluctuations *differ significantly* from those describing US business cycles." Cooley and Hansen (1997; p. 624, emphasis added). Using a sample of 20 US manufacturing industries, Shelley and Wallace (1998) find that either anticipated or unanticipated money affects output in only 14 cases. Further, using Akaike's FPE criterion, monetary shocks enter most output equations with a lag of three months or less. For only two industries is there any evidence that anticipated or unanticipated money shocks are not neutral at extended lags. This leads the authors to reject the LSW proposition as a feature of this data and leads them to conclude, along with most of the literature that nominal equilibrium models are of limited use when it comes to understanding business cycles. It seems reasonably clear from this empirical evidence that nominal equilibrium models of business cycles and their market clearing equilibrium foundations are not compatible with actual economic data.

As noted earlier, there is a second class of business cycle model which has explicitly general equilibrium foundations, namely the 'real business cycle' class of models. As Katz (1988) notes the RBC approach has its roots in Ricardo and his view that "... cyclical fluctuations do not arise from aggregate shocks, but from slow reallocation of labour across sectors in response to intersectoral shifts in labour demand." Katz (1988; p. 508). Stressing the general equilibrium foundations of RBC models, Rebelo (2005) points out that the models are based on the idea that "... business cycles can be studied using dynamic general equilibrium models. These models feature atomistic agents who operate in competitive markets and form rational expectations about the future." Rebelo (2005; p. 217). Following Stadler (1994), a prototype RBC model in which the economy can be specified, by taking a collection of identical, infinitely lived agents who produce a single good. Each agent has preferences $U_t = \text{Max } E_t[\sum_j \beta^j u(c_{t+j}, l_{t+j})]$, $0 < \beta < 1$, where β is a discount factor, c_t is consumption of the good in t and l_t is leisure in t. Technology is constant returns to scale and is described by the production function $y_t = z_t f(k_t, n_t)$, where y_t is output, k_t is capital carried over from the previous period, n_t is labour and z_t is a strictly positive stochastic parameter which is assumed to follow a stationary Markov process and which 'shocks' total factor productivity. The capital stock evolves according to $k_{t+1} = (1 - \delta)k_t + i_t$, where δ is the depreciation rate and i_t is gross investment. Resource constraints mean that $c_t + i_t = y_t$ and $n_t + l_t = h_t$, where h_t is the total number of hours available to the economy. Agents are assumed to have rational expectations based on the structure of the economy and to know the probability distribution generating z_t as well as the current value of z_t. Therefore, maximising the utility function above subject to these constraints yields a set of first-order conditions which characterise market equilibrium. If the utility function is made to be log-linear, say $u(\bullet) = \theta \log c_t + (1-\theta)\log(1-n_t)$ and the production function is Cobb-Douglas $z_t f(\bullet) = z_t n_t^{\alpha} k_t^{1-\alpha}$ then closed form expressions for consumption and the capital stock are: $c_t = [1-(1-\alpha)\beta] z_t n_t^{\alpha} k_t^{1-\alpha}$ and $k_{t+1} = (1-\alpha)$ $\beta z_t n_t^{\alpha} k_t^{1-\alpha}$. Now shocks to technology, through the action of z_t, will also be propagated through changes in the capital stock and will induce variations in current consumption which have patterns that look very much like those exhibited in actual data. For example in the current set-up, if z_t is an AR(1) process then consumption and the capital stock will follow an AR(2) process and as Stadler (1994) comments: "This is significant because the de-trended quarterly time series of various macroeconomic variables are well described by AR(2) processes for US data." Stadler (1994; p. 1755). Such an outcome is encouraging for the Walrasian program because it seems to show that: "... fairly simple general equilibrium models, in which technical change is stochastic, are capable of capturing many of the cyclical features of economic time series." Stadler (1994; p. 1778).

In an early review of the empirical literature on RBCs, Mullineux and Dickinson (1992) concluded that: "... at this stage, the tests conducted lack power in the relatively small samples available, and hence 'the jury is still out'." Mullineux and Dickinson (1992; p. 350). In the more recent survey by Stadler (1994), it seems that a consensus has emerged that RBC models are not particularly successful when it comes to explaining fluctuations in actual economies for as Stadler notes: "The empirical evidence of what causes business cycles does not give strong support to the proposition that real shocks are responsible for more than a third of output fluctuations." Stadler (1994; p. 1779). Recent additions to the literature are broadly supportive of this position. Stockman and Tesar (1995) report that a standard technology shock driven RBC produces counterfactual implications for co-movements between consumption and prices although they do note that if taste shocks are added to the model, it does get a better fit to the data. Ambler and Paquet (1996) compare the behaviour of a model in which the government optimally chooses public investment and non-military expenditures to maximise the welfare of a representative private agent with the behaviour displayed by the US economy. Their study finds that many of the models' predicted correlations, particularly with output, are not supported by the data. Hartley *et al.* (1997) have observed that the RBC model now dominates research in the New Classical tradition and that these models 'offer the bold conjecture that business cycles are equilibrium phenomena driven by technology shocks'. After a careful review of the available empirical literature, Hartley *et al.* (1997) conclude that: "…on the preponderance of the evidence, the real business cycle model is refuted." Hartley *et al.* (1997; p. 34). As Li (1999) notes, weaknesses of RBC models include: (i) an inability to generate sufficient volatility of hours worked relative to output and average labour productivity; (ii) an overstatement of the contemporaneous correlation between hours worked and productivity; and (iii) the fact that labour productivity leads hours worked over the cycle. In response, he proposes a model with incomplete risk sharing and 'a more realistic treatment of unemployment' and indivisible labour. For a sample of US data 1964:1 to 1994:4, Li (1999) shows that such a modification of a standard RBC does allow this type of model to better account for behaviour of the US labour market than is possible using the standard RBC model.

In two recent responses to the failure of RBC models to seemingly account for macroeconomic reality, Aaland (2001) notes that standard quarterly RBC models are unable to account for empirical regularities in US labour markets. He notes that a weekly version of the model can however give a better account of the data. This occurs because of the theoretical modifications which he makes to the model and the way they interact with the sampling properties of US aggregate data. Also focusing on labour markets, Maffezoli (2001) notes that if the labour market is allowed to be non-Walrasian, in particular to have monopoly labour unions, then an RBC model modified in this way is able to better account for certain features of European macroeconomic data than are RBC models which assume Walrasian labour markets. Gong and Semmler (2004) note that the standard RBC model is based on a general equilibrium framework with competitive markets, flexible wages and prices and continuous market clearing. They also note that such models don't fit important features of the US business cycle (such as those noted earlier). They therefore build a RBC model which has a non-clearing labour market and show that: "[c]alibration for the US economy shows that such model variants will produce a higher volatility in employment, and thus fit the data significantly better than the standard model." Gong and Semmler (2004; p. 17). See also the discussion in Shimmer (2005) who, as Rebelo (2005) notes points out that "... there is still work to be done on producing a model that can replicate the patterns of comovement and volatility of unemployment, vacancies, wages and average labour productivity present in US data." Rebelo (2005; p. 230).

As Hartley *et al.* (1997) note, much of the work aimed at empirically evaluating RBC models involves calibrating an RBC model and then looking at how well the calibrated model can account for 'stylised facts' about macroeconomic fluctuations. Canova *et al.* (1994) object to this practice and argue that RBC models should be evaluated using standard econometric techniques. In particular, in their evaluation of the RBC model proposed by Burnside *et al.* (1993) they work out the restricted VAR representation implicit in the Burnside *et al.* (1993) model and compare it with the unrestricted VAR suggested by the data. The result of their study is that: "... it does not seem as if the essentials of the economy are captured by the Burnside, Eichenbaum and Rebelo (1993) formulation [and] [i]t is worth emphasising here that the rejections of the RBC model using the techniques above are far stronger than those encountered by Burnside, Eichenbaum and Rebelo (1993), where what evidence there was against their model ... was very mild." Canova *et al.* (1994; p. 243). The authors also present a MacKinnon style non-nested J-test of the Burnside *et al.* (1993) model against a Keynesian style multiplier-accelerator alternative and apart from the oil price shock period of the 1970s conclude that: "The evidence from the above equations is that the RBC model rarely adds a great deal to the explanatory power of the [Keynesian] model." Burnside *et al.* (1993; p. 247). Wen (2005) begins with the observation that in post-war US data 'consumption growth Granger causes GDP growth (but not vice versa), and GDP growth Granger causes business investment (but not vice versa)'. As he notes this is a "... causal relationship that cannot be explained by standard RBC models." Wen (2005; p. 2). Wen (2005) therefore investigates whether existing equilibrium business cycle models, driven by demand (instead of technology), shocks can rationalise the data in an economic structure where employment and output respond to demand shocks with a lag — and in particular behind consumption while investment also can't respond to immediately to demand shocks — and must do so with a lag behind output (see Wen (2005; p. 2) for details). The conclusion that Wen (2005) comes to is that this is unlikely and that "[m]ore fundamental modifications of existing models are required in order to fully explain the causal aspects of the business cycle in general equilibrium." Wen (2005; p. 3). He adds the speculative remark that 'Granger causality and the empirical regularities documented here may prove to be a new litmus test for equilibrium business cycle models'. Chari et al. (2000) build a standard general equilibrium model with sticky prices and imperfect competition to see if such models can track the real response to nominal shocks that is found in economic data, but without success. Dotsey and King (2006) however, show that if standard general equilibrium macroeconomic models have a modified production structure in which (i) produced inputs play an important role; (ii) there is significant variation in capacity utilization; and (iii) there is variation in labour supply along the extensive margin then there is 'substantial persistence' and "... otherwise enhances their empirical promise". Dotsey and King (2006; p. 894). Taking a sample of US aggregate data on GDO, Consumption, Investment, Government expenditure, Exports and Imports for the period 1959:1 to 1988:4, Valderrama (2007) observes that the "... conditional distribution of cyclical components of US macroeconomic time series significantly deviate from a normal (Gaussian) distribution and exhibit conditional heteroskedasticity (e.g. ARCH) [or] nonlinearities." Valderrama (2007; pp. 2957–2958). Valderrama notes that 'standard general equilibrium models of business cycles are aimed at explaining the first and second moments of economic time series'. The aim is to see if they can, possibly after suitable modification, capture 'skewness, kurtosis and conditional volatility observed in US data'. In the event it is concluded that "... the RBC model fails to capture the nonlinearities present in the data, which an analysis of selected first and second moments would miss." Valderrama (2007; p. 2981).

11.2.4.4. *Testing for market clearing: Dynamic stochastic general equilibrium models*

The class of models known as 'dynamic stochastic general equilibrium models' attempts to model the time paths taken by aggregate variables in the macro economy, as if they were generated by a complete market Arrow-Debreu economy with optimizing agents and continuous market clearing. As Azariadis and Kaas (2007) put it: "A dynamic general equilibrium (DGE) model is a parsimonious description of a private ownership economy as a stochastic dynamical system in a small space of physical goods and agent characteristics ... Combining first-order conditions for all agents with clearing in all markets, a DGE model reduces economic behaviour to a few stochastic differential or difference equations...” Azariadis and Kaas (2007; p. 14). Since these models have, as a maintained hypothesis the idea of continuous market clearing in all markets, their empirical performance potentially provides some insight into the veracity or otherwise of the market clearing part of the definition of Walrasian equilibrium. Of course the empirical performance of these models also depends on the other hypotheses they include (particularly the first order conditions that come from agent optimization), and any empirical failure of the models may find a cause there. As will be seen however, this does not seems to be the case.

A little over a decade ago, Gali (1996) made the observation that: “ [t]hough the use of dynamic stochastic general equilibrium models (DSGE models) ... have proved useful at understanding some aspects of aggregate fluctuations, the assumption of a Walrasian labour market embedded in most of the examples in the literature has rendered them incapable of accounting for key macroeconomic phenomena...” Gali (1996; p. 839). He proposed a DSGE model in which the assumption of Walrasian equilibrium in the labour market was dropped and showed that a “... calibrated version of the model succeeds in replicating, at least qualitatively, stylized labour market facts...” Gali (1996; pp. 844–845). Also attempting to improve the empirical relevance of DSGE models, Burda and Weder (2002) investigate the contribution that ‘complementarities’ in European labour market may make to explaining unemployment ‘in the context of a DSGE model’. In the event they report that: “... by bringing together both sunspot and technology-driven business cycle models with a non-Walrasian labour market ... the model can reproduce a number of [otherwise elusive] key stylized facts..." Burda and Weder (2002; p. 23). It is interesting to note that this study, like Gali (1996) needs to abandon Walrasian market clearing (at least in the labour market) in order to achieve this outcome. Alternatively, Francis and Ramey (2005) use a DSGE framework and attempt to get an explanation of business cycles out of technology shocks but without making any non-Walrasian 'sticky price' assumptions. Instead they consider a model with Leontief technology and variable capacity utilization and another with habit persistence in consumption and adjustment costs in investment. The authors report success in the sense that they do not have to resort to any sort of 'sticky price' assumption in order to rationalize the data. However the hypotheses that they invoke instead, have limited general appeal and "... do not, however, resurrect the technology-driven RBC hypothesis." Francis and Ramey (2005; p. 1398). Reverting to the 'sticky wage' paradigm, Farmer and Hollenhorst (2006) note that most DSGE models are built around a Walrasian model of the labour market. While these models "... are successful in some dimensions in explaining how unemployment and vacancies move over the business cycle, they cannot account for the observed volatility of unemployment and vacancies." Farmer and Hollenhorst (2006; p. 2). They then show that if the Walrasian market clearing assumption is dropped, 'a DSGE model with *rigid* wages can account for these facts'. However they note that a model where there is some wage flexibility does even better. They conclude that although "... the rigid wage model does better in some dimensions than the flexible wage economy ... [a]n intermediate model in which the real wage adjusts by 19% of the way each quarter towards the flexible wage solution does a much better job." Farmer and Hollenhorst (2006; p. 25). This is an interesting finding which again seems to make the point that Walrasian market clearing, at least in the labour market, does not seem to be a feature of actual economic data — at least when looked at through the prism of DSGE models. To conclude, it is worth noting the observation of Azariadis and Kaas (2007) that "Science expects theories to be both conceptually coherent and consistent with the facts ... Dynamic general equilibrium has had more success with the first requirement than with the second one [and to fix this] we are free to bring to the paradigm macroeconomic frictions of various types, that is, deviations from the strict Arrow-Debreu assumptions of the original DGE framework. "Azariadis and Kaas (2007; pp. 17–18). One of the 'frictions' nominated by the authors for possible inclusion in a DSGE that is aiming to be empirically relevant are 'exogenous price and wage rigidities', which leads back to the suggestion of a non-Walrasian labour market. In any case, it is clear from the argument in Azariadis and Kaas (2007) that DSGE models built on Arrow-Debreu lines and incorporating the hypothesis of Walrasian market clearing are generally not congruent with the data.

The focus of the confrontation between macroeconomic data and the general equilibrium-based macro models considered in the two sections above has often been the labour market, for natural reasons. It is clear from the empirical work reviewed here that different data sets, different empirical methodologies and different estimating techniques come to somewhat different conclusions about some of the details. It seems fair to say however, that the great majority of available studies have difficulty telling a 'continuous market clearing' story as far as labour markets in a number of countries are concerned. A reasonable summary of the situation — particularly if the Debreu (1998) criterion for what market clearing states should look like is adopted — is perhaps well put by Solow (1978): "Deep down I really wish I could believe that Lucas and Sargent are right, because the one thing I know how to do well is equilibrium economics ... It is [however] as plain as the nose on my face that the labour market and many markets for produced goods do not clear in any meaningful sense. Professors Lucas and Sargent say after all, there is no evidence that labour markets do not clear, just the unemployment survey. That seems to me to be evidence ... and I'm not inclined to make up an elaborate story of search or misinformation or anything of the sort [to explain it] ... The notion that excess supply is not there strikes me as utterly implausible." Solow (1978; pp. 206–08) — quoted also in Seidman (2005; p. 133).

11.2.4.5. *Testing for market clearing: The housing market*

While paying attention to the labour market is natural when an attempt is being made to test for market clearing, there is another prominent market in which the test could be conducted, namely the market for housing. Indeed Mortensen and and Wright (2002), after a consideration of some of the relevant literature come to the conclusion that housing markets (and also labour markets) generally do not clear and ask: "Why does it appear that some markets, say those for labour services or housing, fail to clear?" Mortensen and Wright (2002; p. 1). In similar vein Malpezzi (1999) asks: "Why does evidence to date suggest that the market for stocks and bonds is fairly efficient, but the market for housing may not be?" Malpezzi (1999; p. 27). One possibility is that there may be features of housing as a commodity (e.g. its indivisibility), that preclude the existence of Walrasian equilibrium. This seems not to be the case as for instance Weibull (1983) and Gerber (1985) have proved existence theorems for equilibrium in economics with such commodities — although the arguments are slightly different in some respects to the existence arguments encountered earlier in this book. Given that, it seems reasonable to look at what the available empirical studies have to say about market clearing in housing markets. On the basis of a sample of US data on 133 MSAs for the period 1979 to 1996, Malpezzi (1999) found that two factors seemed prominent; (i) the degree of regulation in the market influenced the speed of adjustment; and (ii) price adjustments were linear and in particular that although they "... argued for and tested the possibility that large departures from equilibrium could call forth larger proportional changes in price than small changes ... we cannot reject the null of linear adjustment." Malpezzi (1999; p. 59). Riddel (2000) develops a model of the housing market in Boulder, Colorado for the period 1981:1 to 1995:3 and finds "... support for recent housing-market research that although housing markets may not be efficient in the short-run, a long-run equilibrium does exist." Riddel (2000; p. 772).

After examining a sample of quarterly data generated by the housing market in Singapore over the period 1975 to 1995, Lum (2002) found that 'while demand and supply macrovariables are significant determinants of house prices', it was "... policy variables rather than changes in fundamental determinants [that] were found to significantly impact the adjustment process of price deviations from their long-run equilibrium values ... the empirical findings also suggest that the speed of adjustment of the private housing market in Singapore ... is rather low at 3.4% per quarter ... [so that] even one off [shocks] can have long-lasting disequilibrating effects on the private residential market." Lum (2002; p. 140). Riddel (2004) considers data on the aggregate US housing market for the period 1967 to 1998 and finds 'the market to be characterized by sustained periods of disequilibrium'. In particular her "... results show that the market is characterized by periods of sustained deviation from equilibrium. Market-clearing is an anomaly rather than the status quo." Riddel (2004; p. 134). She goes on to observe that an aggregate study like this is likely to obscure disequilibrium in local markets and that "... one would suppose that regional markets would have experienced even more pronounced spells of excess demand or supply."

Riddel (2004; p. 135). Capozza *et al.* (2004) note that 'numerous asset market studies have found short-run serial correlation and long-run mean reversion in asset prices (see Capozza *et al.* (2004; p. 1)). Using panel data on 62 US metropolitan areas from 1979 to 1995, they estimate the correlation and mean reversion parameters and then 'link the empirical estimates to the implied difference equation' in prices (see Capozza *et al.* (2004; p. 24)). In studying the properties of the implied difference equation and the resulting pice dynamics, they find that "... the dynamics can vary over time and over locations. Most often the coefficients lie in the convergent regions [of the parameter space]; however, there are time periods and locations where the estimates lie in the divergent or explosive region." Capozza *et al.* (2004; p. 3). The importance of this finding is that in such cases, the adjustment processes at work in the housing markets is generally not equilibrating. On examining a sample of Australian quarterly housing data for the period 1970 to 2003, Abelson *et al.* (2005) find that a model which includes income, the unemployment rate, mortgage interest rates, equity prices, the inflation rate and housing supply adequately captures the behaviour of real house prices. The authors also note that 'there are significant lags in adjustment to equilibrium'. In particular "... when real house prices are rising at more than 2 per cent per annum, the housing market adjusts to equilibrium in four quarters. When real prices are flat or falling, the adjustment process takes six quarter." Abelson *et al.* (2005; p. 102). Focusing on regional rather than aggregate national data, Cook (2005) uses cointegration techniques to examine the relationships among housing prices in different regions of the UK. Using a sample of quarterly real house prices covering the period 1973:4 to 2003:1, he finds only 'weak evidence of segmentation' the UK housing market. As far as adjustment from disequilibrium states are concerned, however he finds "... reversion to equilibrium occurring rapidly when prices in the

South [of the UK] decrease ... but more slowly when prices in the south experience a relative increase." Cook (2005; p. 117). Also focusing on UK data, Bramley *et al.* (2008) consider the determinants of price and the nature of disequilibrium in sub-regional and neighbourhood housing markets. Using annual data for the period 1988/89 to 2004/05, the authors aim to test a number of hypotheses, including that 'disequilibrium in local housing markets can be identified from their pricing models' and that 'such local disequilibrium as might exist corrects progressively over time' (see Bramley *et al.* (2008; p. 182)). In the analysis of this data they find, like Cook (2005), that there is little evidence of strong form market segmentation and that they "... do not converge or reach an economic equilibrium even in the medium to longer run. Nevertheless, some qualified support is found for notions of a degree of segmentation and disequilibrium, certainly in particular time periods." Bramley *et al.* (2008; p. 208). This is an interesting finding and one that actually illustrates a point made by us earlier that there appear to be periods of economic history for various countries in which the conditions for the existence and stability of Walrasian equilibrium appear to hold and Walrasian states seem to adequately describe the data. There also appear to be periods of time and particular markets in which Walrasian equilibrium does not seem to be a feature of the data. Arguably, one such time in the US housing market is now, as the data in Figure 11.1 seeks to illustrate.

The graph in Figure 11.1 shows the number of months of supply of existing houses in the US at current rates of sale, monthly for the period 1963:01 to 2008:10 (data and definitions available from US Census Bureau at http://www.census.gov/newhomesales. See also http://calculateriskblogspot.com/). The corresponding data on US house prices is shown in Figure 11.2.

The data that these graphs represent is of interest for at least two reasons. Firstly, because of the light they throw on the Walrasian market clearing hypothesis. If Debreu's (1998)

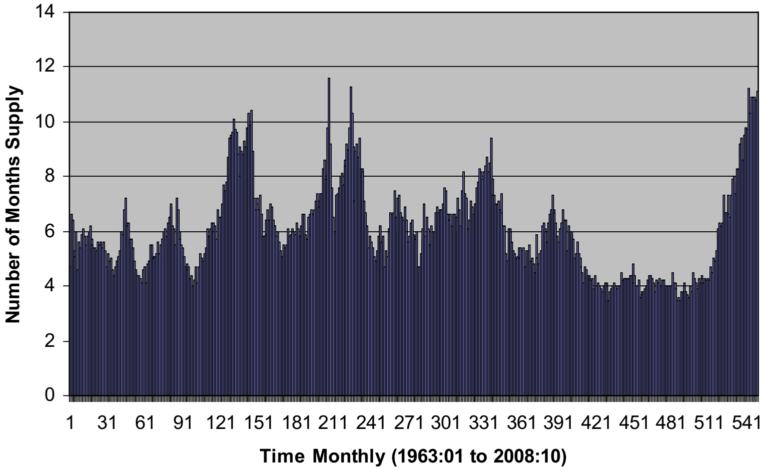

Fig. 11.1. Months of supply at current sale rate.

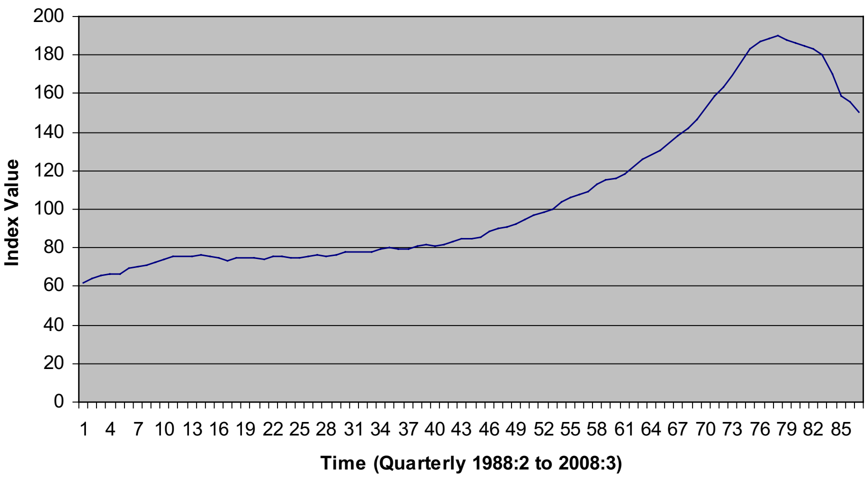

Fig. 11.2. S&P/Case-Shiller US home price index.

criterion is of an 'absence of excessive stocks' is adopted, then this hypothesis would seem to be rejected by US housing data at least some periods of US history — notably since January 2007 to the present. Secondly, the data is also of interest for the following reason. One of the basic insights of general equilibrium theory is that the economy is a *system of interconnected markets.* Consequently, events in one market can have more or less significant impacts in other markets.

A dramatic illustration of this observation is being played out currently in the US and world economy, under the rubric of the 'Global Financial Crisis' (GFC). The basic story of the GFC is by now well documented (sub-prime mortgages, securitization of debt, the proliferation of exotic financial products that became increasingly difficult to value, a flight from these products, and subsequent bank features, a freezing of credit markets and a rapid descent into recession in the general economy). While this seems to have a distinctly exotic 'financial' flavour, it is worth focussing on the basic economics of what happened, and what is still happening, in the foundations of the GFC event. Simply put, an (excess supply) disequilibrium developed in the market for US housing (as illustrated in Figure 11.1). This lead to a significant — but not so far equilibrating — fall in US house prices. The subsequent devaluation in 'mortgage backed securities', the bankruptcy of numerous financial institutions and the rent nationalisation of the US mortgage industry then followed and the GFC was fully underway. The point to note, however, is that it was the functioning of a 'real' market (for housing) that triggered the event. As Makin (2008) put it: ". . . a collapse of the housing market has totally frozen financial markets to a point at which banks are unwilling to lend to each other. . ." Makin (2008; p. 1). In similar vein, Greenspan (2008) observed that ". . . a necessary condition for this crisis to end is a stabilization of home prices in the US, [however] at a minimum, stabilization of home prices is still many months in the future. . ." Greenspan (2008; pp. 1–2). There is a great deal that can be (and is being) written about the GFC. From a general equilibrium point of view, however it nicely emphasises two points that have been themes in this book. Firstly, Hahn's question (and implicit warning), noted in Chap. 1, about where the pursuit of private interest can, in certain circumstances, take deregulated economies. Secondly Saari's remark, discussed in Chap. 8, concerning the absence of any mathematical theory to justify the faith that some people (who perhaps have not studied general equilibrium theory all that closely) have in the 'self-correcting' properties of markets. As an educative phenomenon, the GFC might not be a complete waste of time if a realization occurs that such an outcome appears to be highly conditional at the theoretical level and, at least for some periods of economic history, empirically false.

11.3. Conclusion

Economics aspires to be an empirical science. The centrepiece of economics, namely general equilibrium theory, is sometimes criticised for an apparent inability to produce unambiguous and empirically testable implications. In light of this, the current chapter has two primary aims. Firstly, to point out that, general equilibrium theory can indeed produce 'meaningful theorems' and testable restrictions on data. Secondly, to consider some empirical tests that aim to shed light on how successful the general equilibrium theory has been in its attempts to account for actual economic data. In this regard, we considered available evidence on consumer and producer behaviour, and on market clearing. We also considered the indirect evidence provided by the performance of certain equilibrium based models of the business cycle.

A reasonable overall summary of the available evidence would seem to be that the empirical reliability of the Walrasian hypothesis at the microeceonomic level is mixed and at the macroeconomic level is tending towards being negative. Reasons for this may be due to shortcomings in the econometric aspects of testing procedures, insufficiently flexible estimating equations, or inappropriately aggregated data. However, it may also be worth observing that instances where the theory does appear to be consistent with the data may in fact be instances where the theory is correct, rather than the result of a spurious test outcome. Similarly, when the theoretical predictions do not appear in the data, it may actually be the case that the theory does not provide a good description of the true data generating process. We make this point because there may be economies and periods of economic history, for which Walrasian general equilibrium, and its associated implications, holds. There may likewise be periods of economic history for the same economies for which equilibrium is not a feature of the data. It seems therefore not to be the case that Walrasian equilibrium holds universally across alternative economic structures, including market economics and for all periods of history.

Chapter 12

GENERAL EQUILIBRIUM THEORY IN RETROSPECT

> *"One would have to be far gone in philistine turpitude not to appreciate the towering intellectual achievement which the existence result and the whole Walrasian edifice represents. Nevertheless, this should not blind us to the possibility that what is constructed there has little if nothing to do with the world which we inhabit. Nevertheless, it cannot be denied that the prospect of so many refining the analysis of states which they give no reason to believe have ever or can ever come about is scandalous. It is probably also dangerous."*
>
> F. H. Hahn

12.1. Introduction

General equilibrium theory is interested in whether the operation of a system of interconnected competitive markets, free from government intervention or regulation, will result in a coherent and optimal state of the economy or, alternatively, in chaos. In order to investigate this issue, general equilibrium theory explores broad questions, such as: under what conditions do equilibrium states exist? What are the qualitative properties of equilibrium states? Are equilibrium states consistent with actual economic data?

In this book, results concerning the existence, qualitative properties and empirical relevance of equilibrium states have been presented, examined and discussed. The purpose of this chapter is to briefly review and summarise the earlier work and to make a few concluding remarks concerning the general equilibrium program.

12.2. Equilibrium states and their properties

12.2.1. *The existence of Walrasian equilibrium*

The existence question for Walrasian equilibrium is at the heart of the general equilibrium theory because unless the existence of equilibrium can be established under reasonably general conditions, then as Debreu (1998) for instance points out, general equilibrium theory would be empty. As it happens, there are a number of views in the literature concerning the generality of the circumstances in which equilibrium states exist. These views range from claims that equilibria exist under conditions of 'considerable generality' to claims that conditions for the existence of equilibrium states are 'stringent'.

Given the importance of the existence question for general equilibrium theory, along with the variety of views available in the literature concerning the generality of existence results, effort was devoted to presenting and discussing a number of existence theorems for Walrasian equilibrium. Existence results rely on a number of conditions, however particular attention was focussed on conditions which guarantee the continuity of demand responses and the participation and survival of agents in the economy. Sufficient conditions for demand continuity and agent survival, such as the 'interior endowments' and various 'irreducibility' conditions, were seen to require a quite particular relationship to hold between the primitives which define the economy. This is also true of a necessary condition for the existence of equilibrium and a number of necessary and sufficient conditions for existence. Thus, the existence of equilibrium depends on potentially delicate relationships holding between the primitives that define the economy. As there are no obvious mechanism at work in a standard private ownership economy to guarantee that these relationship conditions are in fact satisfied, it is probably reasonable to err on the side of caution when confronted by claims that Walrasian equilibrium exists in a wide variety of circumstances.

Interestingly, there appear to be features of actual economies that suggest that the sort of relationship conditions needed for the existence of equilibrium may have in fact broken down at various points in economic history. Evidence of this may be seen in the unemployment and non-employment rates in numerous countries; the apparent insensitivity of these rates to significant variations in real wage rates and various labour market reforms; and the apparent division in many economies between 'have' and 'have not' families, with respect to employment.

Conditions under which Walrasian equilibrium exists could arise in actual economies. However, when details of the conditions needed for such an outcome are examined, it is not obvious that an economy will *a priori* necessarily have the structure needed to guarantee existence, or that there is some mechanism at work in the economy to ensure the needed structure. As a consequence, various 'policy interventions' may need to be incorporated into the structure of the economy in order for to guarantee the existence of Walrasian equilibrium states.

12.2.2. *The uniqueness of equilibrium*

There is an extensive literature devoted to the uniqueness question for Walrasian equilibrium. The major conclusion arrived at by the literature is that conditions under which there is just one Walrasian equilibrium are very restrictive. Given that uniqueness is not generally available, it is of interest to know something about the number of equilibrium states that an economy is likely to have. In an early use of generic analysis in economics, Debreu (1970) established that for almost all exchange economies of a particular type, there are finitely many, locally isolated equilibria. Interesting as it is, this result leaves open the possibility that the number of equilibria is nevertheless numerically large, a point made by, for instance, Mas-Colell *et al.* (1995). A result established for exchange economies by Balasko (1988) and extended to the production case by Journee (1992), establishes that the probability of encountering an economy with n equilibria, is inversely proportional to n. It can also be shown that the set of equilibrium prices is negligible relative to the set of all possible prices and therefore possibly not likely to be encountered. This implication is however subject to the important caveat that equilibrium prices are not 'privileged' in the sense that there exist market adjustment processes which select them — and that invites a consideration of the stability question for price adjustment processes.

12.2.3. *The stability of equilibrium*

The stability question is an essential part of general equilibrium theory. Without a convincing argument in favour of convergence to equilibrium of the price adjustment processes at work in the economy, equilibrium states might, in a sense, just as well not exist. This is so because they will never be implemented by processes at work in the economy.

The results available in the literature concerning the convergence of a variety of adjustment processes suggest that it is difficult to tell a convincing story which yields a universal and globally stable adjustment process as the norm. The key reason for this outcome is that if economically intuitive processes, such as a standard tatonnement are considered, then there is a large class of economies for which the process does not converge to equilibrium. Alternatively, processes which are known to converge on general excess demand maps are essentially equivalent to informationally demanding processes of the Global-Newton type. Such schemes require an implausibly large amount of information in order to function. The challenge is then to try to develop stability results which avoid these overwhelming informational requirements without unnecessarily limiting the class of excess demand functions on which convergence can be demonstrated. One way this might be done is to exploit features of the economy other than those contained in the structure of the excess demand functions. To that end we devoted some attention to an interesting stability result proposed by Rader (1996b).

12.2.4. *The optimality of equilibrium*

The optimality question for Walrasian equilibrium is concerned with the capacity of market prices to decentralise optimal allocations of commodities. General equilibrium theory investigates this question and presents its findings as the First and Second Welfare Theorems. These results are important not only in economic theory but have also been widely influential at the policy level.

In light of their fundamental nature, it is important to have an appreciation under which these theorems hold and also some of the circumstances where they fail. Many accounts, particularly of the FWT, draw attention to the relatively small number of hypotheses that it appears to need. However, close inspection reveals that there are often hidden conditions in standard statements of the theorem. We have been concerned to give as complete an account as possible of the conditions under which the theorem holds.

Consideration of the 'deeper' SWT shows that there are interesting directions in which it generalises, relative to the classical conditions under which it is usually stated. It is also true that there are directions in which it fails to generalise. It is also apparent that too much is sometimes claimed on behalf of the theorem at the 'operational' level. In particular, applications and interpretations of the theorem which substitute the word 'achieved' for 'supported' are generally not warranted for reasons to do with a general absence of uniqueness of Walrasian equilibrium and global stability of adjustment processes to equilibrium. This is interesting because it leads to the observation that there are circumstances in which almost all the work associated with arriving at a desired distribution of welfare needs to be done by non-market means, with at most only 'local' work is left to the operation of markets.

12.2.5. *Comparative static properties of equilibrium*

General equilibrium theory views the economy as made up of a number of interconnected markets, between which flow various 'feedback effects'. It is perhaps therefore not surprising that it is difficult to make predictions about how shocks to one or more of the primitives that define the economy will influence equilibrium prices and quantities traded. Nevertheless, the general absence of definitive comparative static results presents a significant problem at the theoretical, applied and methodological levels. Consequently, considerable effort has been devoted in the literature to finding ways in which comparative static results can be obtained, without having to imposing overly restrictive assumptions either on the nature of the shocks to the economy, or on the structure of excess demand functions in the economy. We undertook a study of the major techniques and the principle available results on general equilibrium comparative statics.

We also studied some 'welfare comparative statics' issues, inspired by the curious possibilities thrown up by the 'transfer paradox' and the possibility of equilibrium manipulation through endowment reallocation. While there seem to be a fundamental reason why a transfer paradox is unlikely to arise as a result of the actions of agents in the economy, a transfer paradox like outcome might inadvertently be triggered by agents 'outside' the economy, such as government agencies that are pursuing redistribution policies.

12.2.6. *Empirical evidence on Walrasian equilibrium*

Economics aspires to be an empirical science. As a central part of Economics, general equilibrium theory is sometimes criticised for a perceived inability to produce 'testable implications' and sometimes for an unwillingness to confront those implications with data. Since comparative static predictions are generally the source of testable implications from a theory and since unambiguous comparative static predictions are difficult to come by in general equilibrium systems, it might seem that the situation is hopeless as far as testing general equilibrium theory is concerned. However, this is not true for at least two reasons. Firstly, the hypothesis that the economy is in a state of Walrasian equilibrium has rich implications for the behaviour of consumers, firms and markets, at least in the institutional context of an Arrow-Debreu economy. Secondly, as Brown and Matzkin (1996) show, there are ways to obtain 'general equilibrium restrictions', if the excess demand map is parameterised by prices and endowment profiles and the implied properties of the equilibrium manifold are studied.

A reasonable conclusion to be drawn from the empirical evidence in studies so far available suggest that microeconomic level evidence on Walrasian equilibrium is mixed, while at a macroeconomic level the evidence is almost all negative — with however some notable recent exceptions. Reasons for this outcome may of course reside with faults in testing procedures, with limitations in the functional forms of estimated models, or with inappropriately aggregated data to name a few possibilities.

Attempts have been made here to guard against this by considering a wide range of empirical work which use a large number of empirical methodologies over a number of different data sets displaying varying degrees of disaggregation. It might also be noted that instances where the theory appears to be consistent with the data may be instances where the theory is correct, rather than the result of a spurious test outcome. Similarly when theoretical predictions do not appear to be supported by the data it may actually be the case that the theory does not provide a good description of the true data generation process. The point of this observation is that there appear to be economies and periods of economic history for which Walrasian equilibrium in particulars seems to be a reasonable description of the data. There also appear to be periods of economic history for which Walrasian equilibrium does not seem to hold.

This feature of the literature is interestingly mirrored in the theoretical work presented earlier in this book. Theorems concerning the existence, uniqueness, stability and optimality are contingent on certain conditions holding. That these conditions might hold (or fail) in a given instance may lead to the corresponding general equilibrium property showing up (or not showing up) empirically.

12.3. Conclusion

Economics sets itself the task of studying economic institutions and policy interventions with the aim of promoting human happiness and welfare. General equilibrium theory promotes the objectives of economics by making a detailed study of the equilibrium properties of decentralised or market 'economies'. It is sometimes claimed that general equilibrium theory advocates the institution of markets as the only means by which economic affairs should be managed. If reliable, such claims have wide ranging implications and as a consequence their foundations deserve careful study.

The theoretical foundations of general equilibrium theory have been a primary focus of this book. The major observation to emerge from our work is that the *laissez faire* approach to economic life often ascribed to general equilibrium theory, has a more uncertain theoretical foundation than is commonly acknowledged. It also seems that there is mixed evidence on the empirical relevance of equilibrium states. In spite of this, it is possible to find in the literature many theoretical and applied analyses that are predicated on the Walrasian hypothesis being true. It is also possible to discern a predilection among policy makers in some countries for the policy stance of 'letting the market decide.'

However, when a close study of general equilibrium theory is made and when the results it has established are carefully considered, a more cautious approach to the design of economic institutions and the conduct of economic policy seems warranted. This conclusion seems particularly supported by a study of conditions necessary for the existence of Walrasian equilibrium with 'participation and survival'. It is reinforced by a consideration of the conditions needed for other qualitative properties of equilibrium to hold.

To use Frank Hahn's quoted phrases, the headpiece to this chapter, 'one would have to be far gone in philistine turpitude not to appreciate the towering intellectual achievement that Walrasian general equilibrium theory represents.' The power, elegance and complexity of the theory should not however blind one to the fact that the results obtained by the theory are conditional. The appropriateness of a Walrasian general equilibrium conclusion (or policy prescription) in a particular circumstance requires careful checking, thought and investigation, not a dogmatic application of 'theory', whatever it might be. As Michael Williamson noted in a private communication, the history of the human race is a history of suffering. With the human condition in mind, economics sets itself the task of designing socio-economic institutions (and where necessary policy interventions), that aim to alleviate suffering and promote welfare. The general equilibrium research program fully participates in this task and has been very influential in conditioning thinking about how economic life can best be conducted. In particular, on the basis of the results generated by the program, it is often claimed that the institution of deregulated competitive markets is the best way to address human suffering and to promote economic welfare. This is a profound recommendation and it has wide ranging implications. The foundations of the recommendation therefore deserve careful study. A careful study of the Walrasian program suggests that non-market intervention designed to ensure that the appropriate 'relationship conditions' hold between agents in the economy, particularly between their endowments, preferences and technologies, are needed if markets are to function in the way imagined by neo-c1assical economics. That this idea should emerge from a study of Walrasian general equilibrium theory is perhaps testimony to the vision that Walras had for economics. It may also serve as a cautionary tale of the dangers of accepting supposed implications of the Walrasian program. Implications for the conduct of economic life that flow from any theory or research program should only be accepted once a reasonable congruence between reality and the requirements of the theory has been carefully and rigorously established. Otherwise, the great task that economics sets itself of alleviating unnecessary suffering may, ironically, be subverted by economic structures, institutions and policies that are ill-suited to the reality with which they have to deal.

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