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Some Results for the Moments of Generalized Order Statistics from Linear Exponential Distribution

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بعض النتائج لعزوم الإحصاءات المرتبة المعممة للتوزيع الأسي الخطي

تم إيجاد علاقة صريحة للعزوم المفردة للإحصاءات المرتبة المعممة، كما تم إيجاد علاقة تكرارية لتلك العزوم وذلك للتوزيع الأسي الخطي، ثم تم تخصيص هذه العلاقات لكل من الإحصاءات المرتبة العادية والقيم المسجلة من رتبة K (والقيم المسجلة العادية عند K=1) كحالات خاصة من الإحصاءات المرتبة المعممة. وأخيرا تم تخصيص هذه النتائج للتوزيعين الأسي ورايلي باعتبارهما حالتين خاصتين من التوزيع الأسي الخطي.

Abstract

In this paper, we derive explicit expression for single moments of generalized order statistics from the linear exponential distribution. A recurrence relation for single moments of generalized order statistics has been derived. Explicit expressions and recurrence relations for single moments of ordinary order statistics and k - records (ordinary record values when $k = 1$) have been obtained as special cases. These results are used to establish similar results for exponential and Rayleigh distributions as special cases of linear exponential distribution.

Keyword

Generalized order statistics; Order statistics; Record values; k-records; Moments.

INTODUCTION

In his book (1995) Kamps has introduced the concept of *generalized order statistics* (gOSs). It has been shown that ordinary order statistics (oOSs), k-records (ordinary record values (oRVs) when $k = 1$), sequential order statistics, ordering via truncated distributions and censoring schemes can be discussed as special cases of the gOSs. Kamps's book gave several applications in a variety of disciplines, recurrence relation for moments of order statistics and characterizations. Kamps and Gather (1997), Keseling (1999), Cramer and Kamps (2000), Pawlas and Szynal (2001), Ahmad and Fawzy (2003), AL-Hussaini and Ahmad (2003a,b), among others, have used the gOSs in their work. Kamps (1998) investigated the importance of recurrence relations of oOSs in characterization. Recurrence relations for moments of k-records were investigated, among others, by Grudzien and Szynal (1997), Pawlas and Szynal (1998, 1999).

Suppose that the random variable (rv) X has the linear exponential distribution with distribution function (df) of the form :

$$F(x) \equiv F_X(x;\lambda,\nu) = 1-\exp[-(\lambda x+\nu x^2/2)], \qquad 0 \le x < \infty, \quad (\lambda,\nu \ge 0), \qquad (1.1)$$

and probability density function (pdf) :

$$f(x) = (\lambda+\nu x)\exp[-(\lambda x+\nu x^2/2)], \qquad 0 \le x < \infty, \quad (\lambda,\nu \ge 0). \qquad (1.2)$$

Notice that :

$$f(x) = (\lambda+\nu x)\bar{F}(x), \qquad (1.3)$$

and :

$$\lambda x + vx^2/2 = \frac{1}{2v}[(\lambda = vx)^2 - \lambda^2], \qquad v \ne 0, \qquad (1.4)$$

where $\bar{F}(x) = 1 - F(x)$.

The exponential and Rayleigh distributions are considered as special cases of the linear exponential distribution when $\nu = 0$ and $\lambda = 0$, respectively. The linear exponential distribution has been used in the area of reliability and life-testing, see for example Bain (1974).

Let $X_{1;n,m,k}, X_{2;n,m,k}, \ldots, X_{n;n,m,k}$ be n gOSs from the pdf (1.2), ($n > 1, m$ and k are real numbers and $k \ge 1$). The joint pdf of $X_{1;n,m,k}, \ldots, X_{n;n,m,k}$, is given by Kamps (1995) as follows :

$$f_{1,\ldots,n}(x_1,\ldots,x_n) = C_{n-1}\prod_{i=1}^{n-1}(\bar{F}(x_i))^m f(x_i)(\bar{F}(x_n))^{k-1} f(x_n),$$

where $F^{-1}(0+) < x_1 \le \ldots \le x_n < F^{-1}(1)$ of R^n. The pdf of $X_{r;n,m,k}$ is given as

$$f_{X_{r;n,m,k}}(x)=\frac{C_{r-1}}{\Gamma(r)}g_m^{r-1}(F(x))[\bar{F}(x)]^{\gamma_r-1}f(x),\quad x\in\chi, \tag{1.5}$$

where χ is the domain in which $f_{X_{r;n,m,k}}(x)$ is positive,

$$C_{r-1}=\prod_{i=1}^{r}\gamma_i,\quad \gamma_i=k+(n-i)(m+1),$$

and, for $0<z<1$

$$g_m(z)=\begin{cases}[1-(1-z)^{m+1}]/(m+1), & m\neq -1\\ -ln(1-z) & , \quad m=-1.\end{cases} \tag{1.6}$$

The jth moment of the rth gOS can be obtained, for $j\geq 1$, from (1.5), as

$$E(X_{r;n,m,k}^{j})=\frac{C_{r-1}}{\Gamma(r)}\int_{\chi}x^j g_m^{r-1}(F(x))[\bar{F}(x)]^{\gamma_r-1}f(x)dx. \tag{1.7}$$

The jth moment of oOSs and k-records can be obtained, from (1.7), when $m=0$, $k=1$ and $m=-1$, $k\geq 1$, respectively. We shall use $\mu_{r:n}^{(j)}=E(X_{r:n}^{j})\equiv E(X_{r;n,0,1}^{j})$, $\mu_{(r;k)}^{(j)}=E(X_{U_k(r):U_k(r)+k-1}^{j})\equiv E(X_{r;n,-1,k}^{j})$ and $\mu_{(r)}^{(j)}=E(X_{U(r)}^{j})\equiv E(X_{r;n,-1,1}^{j})$ to denote the jth moment of the rth oOS, k-records and oRV, respectively. In general we shall write, for simplicity, $\mu_{r;n,m,k}^{(j)}=E(X_{r;n,m,k}^{j})$.

In this paper, we establish an explicit expression and recurrence relation for single moments of gOSs from linear exponential distribution. Results for oOSs and oRVs can be deduced as special cases from gOSs. These results have been used to evaluate means of oOSs and oRVs.

EXPLICIT EXPRESSION FOR SINGLE MOMENTS OF gOSs

Now, the *pdf* (1.5) can be written for the linear exponential distribution with *pdf* (1.2) and *df* (1.1) and making use of (1.6), in the following form :

$$fx_{r;n,,m,k}(x)=\begin{cases}\frac{C_{r-1}}{\Gamma(r)(m+1)^{r-1}}(\lambda+vx)\exp\left[-\gamma_r(\lambda x+vx^2/2)\right] \\ \times(1-\exp\left[-(m+1)(\lambda x+vx^2/2)\right])^{r-1}, & m\neq -1 \\ \frac{k^r}{\Gamma(r)}(\lambda+vx)(\lambda x+vx^2/2)^{r-1} \\ \times\exp[-k(\lambda x+vx^2/2)], & m=-1.\end{cases} \tag{2.1}$$

Making use of the binomial expansion and (1.4), we can rewrite (2.1) as :

$$fx_{r;n,m,k}(x)=\begin{cases}\frac{C_{r-1}}{(m+1)^{r-1}}\sum_{j=0}^{r-1}\frac{(-1)^j(\lambda+vx)}{\Gamma(j+1)\Gamma(r-j)}\exp[\frac{\lambda^2\gamma_{r-j}}{2v}] \\ \times\exp\left[-\frac{\gamma_{r-j}}{2v}(\lambda+vx)^2\right], & m\neq -1 \\ \frac{k^r\exp[k\lambda^2/2v]}{(2v)^{r-1}}\sum_{j=0}^{r-1}\frac{(-\lambda^2)^j}{\Gamma(j+1)\Gamma(r-j)}(\lambda+vx)^{2r-2j-1} \\ \times\exp[-(k/2v)(\lambda+vx)^2], & m=-1.\end{cases} \tag{2.2}$$

We can calculate explicit formula for $\mu_{r;n,m,k}$ in the following two cases :

(i) for $m\neq -1, v\neq 0$,

$$E(\lambda+vX_{r:n,m,k})=\frac{C_{r-1}}{(m+1)^{r-1}}\sum_{j=0}^{r-1}\frac{(-1)^j}{\Gamma(j+1)\Gamma(r-j)}\exp[\frac{\lambda^2\gamma_{r-j}}{2v}]$$

$$\times\int_0^{\infty}(\lambda+vx)^2\exp[-\frac{\gamma_{r-j}}{2v}(\lambda+vx)^2]dx. \tag{2.3}$$

Writing :

$$\int_0^{\infty}(\lambda+vx)^t\exp[-\frac{w}{2v}(\lambda+vx)^2(\lambda+vx)^2]dx=I_t(\lambda,wv),$$

Using the transformation $z=\frac{w}{2v}(\lambda+vx)^2$, we can show that :

$$I_t(\lambda,wv)=(2v)^{(t-1)/2}w^{-(t+1)/2}\Gamma(\frac{t+1}{2})IG(\frac{t+1}{2},w\lambda^2/2v), \tag{2.4}$$

where $IG(.,.)$ is the incomplete gamma function in the form :

$$IG(l,z)=\frac{1}{\Gamma(l)}\int_z^{\infty}u^{l-1}\exp^{-u}du$$

So, (2.3) can be rewritten as :

$$E(\lambda+\nu X_{r;n,m,k})=\frac{C_{r-1}}{(m+1)^{r-1}}\sum_{j=0}^{r-1}\frac{(-1)^j}{\Gamma(j+1)\Gamma(r-j)}\exp\left(\frac{\gamma_{r-j}\lambda^2}{2\nu}\right)I_2(\lambda,\nu\gamma_{r-j}),$$

or, equivalently, from (2.4) :

$$\mu_{r;n,m,k}=\sqrt{\pi/2\nu}\,\frac{C_{r-1}}{(m+1)^{r-1}}\sum_{j=0}^{r-1}\frac{(-1)^j\exp\left(\frac{\lambda^2\gamma_{r-j}}{2\nu}\right)}{\Gamma(j+1)\Gamma(r-j)(\gamma_{r-j})^{3/2}}IG(1.5,\lambda^2\gamma_{r-j}/2\nu)-\lambda/\nu. \quad (2.5)$$

(ii) for $m=-1,\nu\neq 0$, we can by the same manner, show that the rth moment of the $k-$records is given by :

$$\mu_{(r;k)}=\sqrt{2/k\nu}\exp(k\lambda^2/2\nu)\sum_{j=0}^{r-1}\frac{(-k\lambda^2/2\nu)^j}{\Gamma(j+1)\Gamma(r-j)}\Gamma(r-j+0.5)$$
$$\times IG(r-j+0.5,k\lambda^2/2\nu)-\lambda/\nu. \quad (2.6)$$

Special cases

(1) If we put $m=0,k=1$ in (2.5), we obtain the explicit moments of the rth oOS of the linear exponential distribution $(\nu\neq 0)$, in the form :

$$\mu_{r:n}=\sqrt{\pi/2\nu}\prod_{i=1}^{r}(n-i+1)\sum_{j=0}^{r-1}\frac{(-1)^j\exp((n-r+j+1)\lambda^2/2\nu)}{\Gamma(j+1)\Gamma(r-j)(n-r+j+1)^{3/2}}$$
$$\times IG(1.5,(n-r+j+1)\lambda^2/2\nu)-\lambda/\nu. \quad (2.7)$$

(2) If we put $m=-1,k=1$ in (2.6), we obtain the explicit moments of the rth oRV of the linear exponential distribution $(\nu\neq 0)$, in the form :

$$\mu_{(r)}=\sqrt{2/\nu}\exp(\lambda^2/2\nu)\sum_{j=0}^{r-1}\frac{(-\lambda^2/2\nu)^j}{\Gamma(j+1)\Gamma(r-j)}$$
$$\times\Gamma(r-j+0.5)IG(r-j+0.5,\lambda^2/2\nu)-\lambda/\nu. \quad (2.8)$$

Note that this result agrees with Raqab (2001) for explicit moments of record values.

Remarks

(1) We can use expressions (2.7) and (2.8) to evaluate means of all oOSs and oRVs. These results have been calculated for $\lambda=0,0.1,0.2$ and $\nu=2,4,6$ for the first five oOSs and oRVs from linear exponential distribution, Tables 1 and 2, respectively.

(2) When $\lambda = 0$, (2.7) and (2.8) give the means of oOSs and oRVs from Rayleigh distribution in the forms

$$\mu_{r:n} = \sqrt{\pi / 2v}\prod_{i=1}^{r}(n-i+1)\sum_{j=0}^{r-1}\frac{(-1)^j (n-r+j+1)^{-3/2}}{\Gamma(j+1)\Gamma(r-j)}, \tag{2.9}$$

and

$$\mu_{(r)} = \sqrt{2/v}\,\Gamma(r+0.5)/\Gamma(r). \tag{2.10}$$

These means can be obtained from Tables 1 and 2 for $\lambda = 0$ and for values of v .

r	n	$v=2$			$v=4$			$v=6$		
		$\lambda=0$	$\lambda=0.1$	$\lambda=0.2$	$\lambda=0$	$\lambda=0.1$	$\lambda=0.2$	$\lambda=0$	$\lambda=0.1$	$\lambda=0.2$
1	1	0.88623	0.73729	0.692698	0.62666	0.53104	0.50800	0.51166	0.43715	0.42157
	2	0.62666	0.50800	0.46539	0.44311	0.36865	0.34635	0.36180	0.30449	0.28932
	3	0.51166	0.40664	0.36547	0.36180	0.29679	0.27504	0.29541	0.24576	0.23090
	4	0.44311	0.34635	0.30632	0.31333	0.25400	0.23269	0.25583	0.21078	0.19617
	5	0.39633	0.30529	0.26624	0.28025	0.22483	0.20391	0.22882	0.18693	0.17253
2	2	1.14580	0.96658	0.92001	0.81020	0.69343	0.66964	0.66153	0.56982	0.55381
	3	0.85664	0.71071	0.66523	0.60574	0.51236	0.48897	0.49458	0.42194	0.40615
	4	0.71731	0.58752	0.54289	0.50722	0.42515	0.40207	0.41414	0.35071	0.33509
	5	0.63024	0.51060	0.46668	0.44564	0.37067	0.34785	0.36387	0.30620	0.29072
3	3	1.29037	1.09452	1.04740	0.91243	0.78396	0.75998	0.74500	0.64376	0.62764
	4	0.99598	0.83389	0.78758	0.70426	0.59957	0.57588	0.57503	0.49318	0.47722
	5	0.84793	0.70290	0.65721	0.59958	0.50687	0.48340	0.48955	0.41747	0.40163
4	4	1.38851	1.18140	1.13400	0.98182	0.84542	0.82135	0.80165	0.69395	0.67778
	5	1.09467	0.921223	0.87449	0.77405	0.66137	0.63753	0.63201	0.54365	0.52761
5	5	1.46196	1.24644	1.19888	1.03376	0.89144	0.86730	0.84407	0.73152	0.71533

Table (1): Means of order statistics from linear exponential distribution

r	$v=2$			$v=4$			$v=6$		
	$\lambda=0$	$\lambda=0.1$	$\lambda=0.2$	$\lambda=0$	$\lambda=0.1$	$\lambda=0.2$	$\lambda=0$	$\lambda=0.1$	$\lambda=0.2$
1	0.78539	0.55536	0.45345	0.73729	0.53104	0.43715	0.69270	0.50800	0.42157
2	1.76715	1.24956	1.02026	1.71960	1.22473	1.00407	1.67697	1.20303	0.98882
3	5.52233	3.90488	3.18832	5.48173	3.88164	3.17346	5.46002	3.86818	3.16222
4	22.5459	15.9449	13.0190	22.5421	15.9301	13.0105	22.6205	15.9552	13.0184
5	114.157	80.7211	65.9085	114.336	80.7572	65.9360	114.977	80.9957	66.0517

Table (2): Means of record values from linear exponential distribution

3. RECURRENCE RELATION FOR SINGLE MOMENTS OF gOSs

Now, we derive the following recurrence relation for single moments of gOSs from df (1.1).

Theorem 3.1

Let X be a rv with df defined by (1.1),then for real m,k with $m \geq -1, k \geq 1$, integers $r, j \geq 1$, the recurrence relation :

$$\mu_{r;n,m,k}^{(j+2)} - \mu_{r-1;n,m,k}^{(j+2)} = \frac{\lambda(j+2)}{v(j+1)} \mu_{r-1;n,m,k}^{(j+1)} - \frac{j+2}{v}\left[\frac{\lambda}{j+1}\mu_{r;n,m,k}^{(j+1)} - \frac{1}{\gamma_r}\mu_{r;n,m,k}^{(j)}\right], \quad (3.1)$$

is satsisfied .

Proof

Let X has the df (1.1), then from (1.7), we have :

$$\mu_{r;n,m,k}^{(j)} = \frac{C_{r-1}}{\Gamma(r)} \int_0^\infty x^j g_m^{r-1}(F(x))[\bar{F}(x)]^{\gamma_r - 1} f(x)dx,$$

or, equivalently, from (1.3) :

$$\mu_{r;n,m,k}^{(j)} = \frac{C_{r-1}}{\Gamma(r)} \int_0^\infty x^j g_m^{r-1}(F(x))[\bar{F}(x)]^{\gamma_r} (\lambda + vx)dx,$$

which can be rewritten as :

$$\mu_{r;n,m,k}^{(j)} = \frac{\lambda C_{r-1}}{\Gamma(r)} \int_0^\infty x^j g_m^{r-1}(F(x))[\bar{F}(x)]^{\gamma_r} dx + \frac{vC_{r-1}}{\Gamma(r)} \int_0^\infty x^{j+1} g_m^{r-1}(F(x))[\bar{F}(x)]^{\gamma_r} dx = \lambda Q_{r;n,m,k}^{j} + vQ_{r;n,m,k}^{j+1}, \quad (3.2)$$

where :

$$Q_{r;n,m,k}^{j} = \frac{C_{r-1}}{\Gamma(r)} \int_0^\infty x^j g_m^{r-1}(F(x))[\bar{F}(x)]^{\gamma_r} dx.$$

Now , we can write :

$$Q_{r;n,m,k}^{j} = \frac{C_{r-1}}{(j+1)\Gamma(r)} \int_0^\infty g_m^{r-1}(F(x))[\bar{F}(x)]^{\gamma_r} d[x^{j+1}].$$

Integrating by parts and then making use of (1.7) , we can show that :

$$Q_{r;n,m,k}^{j} = \frac{\gamma_r}{j+1}\left[\mu_{r;n,m,k}^{(j+1)} - \mu_{r-1;n,m,k}^{(j+1)}\right], \quad (3.3)$$

substituting from (3.3) in (3.2), we can obtain the result (3.1).

Special cases .

(1) If we put $m=0, k=1$ in (3.1), we obtain a recurrence relation for single moments of oOS of the linear exponential distribution in the form :

$$\mu_{r:n}^{(j+2)} - \mu_{r-1:n}^{(j+2)} = \frac{\lambda(j+2)}{v(j+1)} \mu_{r-1:n}^{(j+1)} - \frac{j+2}{v}\left[\frac{\lambda}{j+1}\mu_{r:n}^{(j+1)} - \frac{1}{(n-r+1)}\mu_{r:n}^{(j)}\right]. \quad (3.4)$$

(2) If we put $m = -1, k \geq 1$ in (3.1), we obtain a recurrence relation for single moments of k- records of the linear exponential distribution in the form :

$$\mu_{(r;k)}^{(j+2)} - \mu_{(r-1;k)}^{(j+2)} = \frac{\lambda(j+2)}{v(j+1)} \mu_{(r-1;k)}^{(j+1)} - \frac{j+2}{v}\left[\frac{\lambda}{j+1}\mu_{(r;k)}^{(j+1)} - \frac{1}{k}\mu_{(r;k)}^{(j)}\right]. \quad (3.5)$$

For k=1 in (3.5) , one has the result of oRVs as obtained by Raqab (2001).

(3) When we put $v=0$ in (3.1), (3.4) and (3.5) (after multiplying them by v) , we obtain the corresponding results for the exponential distribution, respectively, as :

$$\mu_{r;n,m,k}^{(j+1)} = \frac{j+1}{\lambda\gamma_r}\mu_{r;n,m,k}^{(j)} + \mu_{r-1;n,m,k}^{(j+1)}$$

$$\mu_{r:n}^{(j+1)} = \frac{j+1}{\lambda(n-r+1)}\mu_{r:n}^{(j)} + \mu_{r-1:n}^{(j+1)}$$

$$\mu_{(r;k)}^{(j+1)} = \frac{j+1}{k\lambda}\mu_{(r;k)}^{(j)} + \mu_{(r-1;k)}^{(j+1)}.$$

Similarly, relations (3.1), (3.4) and (3.5) and for $\lambda = 0$ give the corresponding results for the Rayleigh distribution, respectively, as

$$\mu_{r;n,m,k}^{(j+2)} = \frac{j+2}{v\gamma_r}\mu_{r;n,m,k}^{(j)} + \mu_{r-1;n,m,k}^{(j+2)}$$

$$\mu_{r:n}^{(j+2)} = \frac{j+2}{v(n-r+1)}\mu_{r:n}^{(j)} + \mu_{r-1:n}^{(j+2)}$$

$$\mu_{(r;k)}^{(j+2)} = \frac{j+2}{kv}\mu_{(r;k)}^{(j)} + \mu_{(r-1;k)}^{(j+2)}.$$

Concluding remarks

(1) In this paper general recurrence relation for single moments of $gos's$ from the linear exponential distribution has derived.

(2) The effective application of order statistics techniques requires so many tables, (see for more details Pearson and Hartley (1970, 1972) and Harter and Balakrishnan (1996, 1997)) .

(3) The recurrence relations for moments of $gos's$ are important because: they give general results that can be applied for their special cases, reduce the amount of direct computations, evaluate the higher order moments in terms of the lower order moments and they can be used to characterize distributions.

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