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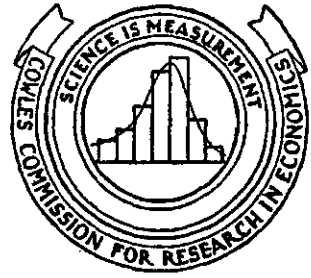
PRICE FLEXIBILITY AND EMPLOYMENT

By

OSCAR LANGE



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PREFACE

This volume is intended as a modest contribution to a much-discussed problem of economic theory and policy. Much of the controversy on the subject is due to differences of assumptions made by various authors. In order to arrive at satisfactory conclusions it is necessary to discuss the problem within the framework of the general theory of economic equilibrium. This theory provides a basis of analysis accepted by exponents of divergent views on our subject. For our purpose, however, the theory of general economic equilibrium had to be restated in a way which explicitly takes account of money. Such restatement leads to the conclusion that substitution between money and goods provides the key for understanding the equilibrating as well as the disequilibrating processes of the economy. The author considers this conclusion as the chief contribution of his study.

The interest in the problem and the recognition of the crucial importance of substitution between money and goods were inspired by Lord Keynes. For the tools of analysis the author is heavily indebted to Professor J. R. Hicks. Professor Hicks has provided the most up-to-date formulation of the theory of general economic equilibrium. He has also enlarged the theory by including an analysis of intertemporal substitution. Professor Paul A. Samuelson has developed a dynamic theory of stability of economic equilibrium of which extensive use has been made in the Appendix. The present volume builds upon the achievements of these three economists.

The author has endeavored to make the presentation as simple as possible. For this reason points of technical detail have been relegated to footnotes, of which there is a considerable number. A special Appendix develops the mathematical theory of stability of economic equilibrium and applies it to the problems of our study. The mathematically prepared reader may find that his understanding of the book will be enhanced by a previous perusal of the Appendix. This Appendix, however, is not a mere restatement in mathematical shorthand of the "literary" part of the volume. Though complementary with it, it covers independent ground.

The manuscript or parts of it have been read by several colleagues, friends, and students, all of whom have made valuable suggestions. They are: Bert Hoselitz, Leonid Hurwicz, Wassily Leontief, A. P. Lerner, J. M. Letiche, Jacob Marschak, Melvin W. Reder, Theodore W. Schultz, Tibor Scitovszky, Jacob Viner, and Abraham Wald. To all of them the author wants to express his thanks. Special thanks are due to Dickson H. Leavens who kindly undertook the editing of this book. The author is also indebted to the Cowles Commission for Research in Economics which has provided the funds for the publication and to the Social Science Research Committee of the University of Chicago which has contributed secretarial help.

OSCAR LANGE

The University of Chicago
October, 1944

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CHAPTER I

INTRODUCTION

THIS MONOGRAPH presents a systematic investigation of the effect of price flexibility, particularly flexibility of prices of factors of production, upon employment and economic stability. According to traditional economic doctrine, unemployment is entirely due to rigidity of factor prices. Hence flexibility of these prices is regarded as desirable and is advocated as a norm of an economic policy which aims at full employment and proper allocation of resources. This view has been subjected to serious criticism. Lord Keynes maintains that, under certain conditions, changes in money wage rates have no effect upon employment but influence only the level of product prices. Some authors even maintain that the relationship is the reverse of what is taught by traditional doctrine, i.e., that a rise in money wage rates increases and a fall in money wage rates decreases employment. The diversity of opinions can be disentangled only by considering the problem within the framework of the general theory of economic equilibrium.[1] By setting the problem in the framework of general-equilibrium theory we are also able to generalize it. For the relation between changes in money wage rates and the demand for labor is but a special case of the relation between the change of the price of and the employment of a factor of production. We shall, therefore, approach the problem in its full generality by considering the effect of price changes upon the employment of any factor whatsoever.

In order to simplify analysis and exposition, we make a number of provisional assumptions: (1) Entrepreneurs and consumers expect current prices to continue over that part of the future which is relevant to their decisions ("static expectations"). (2) Perfect competition reigns throughout the whole economy, i.e., the economy is divided into a (finite) number of atomistic industries, each producing a homogeneous commodity. (3) International trade is absent. These assumptions will be removed successively in the course of the book.

After having established the general effects of price flexibility upon employment and economic stability, we shall investigate how price flexibility affects the readiness with which the economy absorbs shocks coming from changes in the propensity to save, or in the return on investment, or from innovations.

[1] The degree of realism can be increased by studying the problem in terms of sequence analysis which takes account of the time lags in reactions. Such a study, however, in order to be fruitful, must make specific assumptions concerning the time lags in the various reactions. In order that these be realistic, they must be derived from empirical research, i.e., from econometric analysis. As to the dynamic assumptions implicitly underlying equilibrium analysis, see the Appendix.

For the purpose of the present study, price flexibility is defined as follows: The price of a good is said to be flexible if it falls whenever there is excess supply of[2] and rises whenever there is excess demand for the good. In the opposite case, the price is said to be negatively flexible. The price is said to be inflexible, or rigid, if excess supply or excess demand fail to affect it.[3]

[2] By excess supply we mean the excess of supply over demand at a given set of prices; by excess demand we mean the excess of demand over supply at a given set of prices. The term "underemployment" is used as synonymous with excess supply of and the term "bottleneck" is used as synonymous with excess demand for a factor of production.

[3] In addition, it is possible to define the degree of price flexibility in terms of the rate of change of the price per unit of time caused by a given excess supply or demand. Cf. the Appendix, p. 95. The concept of the degree of price flexibility, however, is not needed in the text of our study; it is used only in the Appendix.

CHAPTER II

Partial-Equilibrium Theory

Let there be excess supply (underemployment) of a factor of production.[1] If the price of that factor is flexible, the excess supply causes a fall in the price. In the theory of partial equilibrium, we suppose that the prices of all the other factors as well as the prices of all other products remain constant. In this case it can be shown that, the price of the factor being flexible, any excess supply is absorbed by an increase of the quantity demanded.

The increase of the quantity demanded takes place via two channels. The prices of all other factors being constant, a decline in the price of the underemployed factor induces a substitution of this factor for other factors that are now relatively more expensive. Methods of production change so as to utilize relatively more of the factor the price of which has fallen. The amount of the factor used per unit of output increases. We shall call this the *substitution effect.* But the output of the commodities the production of which utilizes the underemployed factor does not remain constant either. A fall in the price of the factor (while the prices of the other factors remain constant) lowers the marginal-cost schedule. The prices of the products being constant, this results in an increase of output. The increase in output is the greater, the greater the reduction in the marginal-cost schedule, i.e., the greater the proportion of the underemployed factor in the variable cost of producing the product.[2] We shall call this the *expansion effect.* Through these two channels, the substitution of factors and the increase in output, a fall in the price of the underemployed factor leads to an increase of the demand for it.

If the price of the factor is reduced sufficiently, any excess supply will be absorbed. In order that this be achieved, it is even sufficient that only one of the two channels is operating. In a similar way, an excess demand for a factor (a "bottleneck") is made to disappear through a rise in the price. This argument of the partial-equilibrium theory can be presented in terms of the downward-sloping demand curve for a factor of production. Permanent underemployment (and also permanent excess demand) of a factor is possible only as a result of rigidity in its price which prevents the substitution effect and the expansion effect from operating.

The range of validity of partial-equilibrium theory is, however, very limited. This theory assumes that the prices of all other factors and the

[1] A factor of production is here defined as a commodity bought by a firm, i.e., by a unit of economic decision operated to make money profit. A product is defined as a commodity sold by a firm. The same commodity may, of course, be a product to one firm and a factor to another firm.

[2] Marginal cost depends only on the variable cost items. If the underemployed factor enters only into fixed cost items, the increase in output is absent in the short period.

prices of all other products remain constant. This is true only when the factor is used exclusively by a single firm, or by such a small number of firms[2] that they use but a small fraction of the total amount of each of the other factors. Otherwise the attempt to substitute one factor for other ones must affect the prices of other factors and other products. We need, therefore, to study the repercussions of the change in the price of one factor upon the prices of other factors and upon the prices of products. This leads us from partial-equilibrium to general-equilibrium analysis.

[2] The first case is incompatible with the assumption of perfect competition; the other case, though not incompatible in principle, is likely to be so in practice. Thus the assumption of perfect competition in the market for the underemployed factor restricts further the applicability of partial-equilibrium analysis.

CHAPTER III

General-Equilibrium Theory

When the repercussions of the change in the price of a factor upon all other prices in the economy are taken into account, we find that the operation and the intensity of the substitution effect and of the expansion effect depend on the reaction of the other prices. When the price of one factor is lowered, the attempt to substitute this factor for other, now relatively more expensive, factors diminishes the demand for the latter. This, as a rule, causes their prices to fall. The substitution effect can take place only when the prices of the other factors fall *less* than in proportion to the fall in price of the factor which is in excess supply. The elasticity of substitution[1] being given, the substitution effect is the stronger the less the fall in the prices of the other factors. The expansion effect becomes operative only when the prices of the products produced with the factor under consideration fall *less* than in proportion to the fall of the marginal cost corresponding to the old output. The technological conditions underlying the shape of the marginal-cost schedule being given, the expansion effect is the stronger the less the product prices fall relatively to the marginal cost (at the old output). Thus the operation of the substitution effect and of the expansion effect hinges upon whether the prices of the other factors and the prices of the products fall less than in proportion to the price of the underemployed factor or to marginal cost. We shall now investigate the conditions which must be satisfied in order that this be the case.

For this purpose we have to consider the relation between the demand for and supply of goods[2] and the demand for and supply of money. Any demand for a good implies a supply of money in exchange for it, and any supply of a good implies a corresponding demand for money. The stream of money demanded during any period of time in exchange for goods is equal to the aggregate value of all the goods offered for sale during that period. In the same way, the stream of money offered in exchange for goods is equal to the aggregate value of all the goods demanded for purchase. A discrepancy between the two streams indicates a desire of the community to hold more or less money than the stock of money available, i.e., an excess demand for or excess supply of cash balances.[3] Equality of the two streams means that the

[1] The elasticity of substitution represents the technological facility of substituting one factor for another. For a precise definition see R. G. D. Allen, *Mathematical Analysis for Economists* (London: Macmillan and Co., 1938), p. 341.

[2] The term goods is used here as meaning "goods exclusive of money." In the Appendix it is used in a broader connotation which includes money.

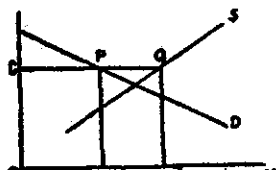
[3] Excess demand for cash balances exists when people (including corporations) wish to hold more than the existing quantity of money; excess supply of cash balances exists when they wish to hold less than the existing quantity of money. These concepts correspond to what is frequently called hoarding and dishoarding. In view, however, of the diversity of meanings attached to the latter word, we prefer to use the terms

community is willing to hold in cash balances exactly the existing stock of money. When demand equals supply for each good in the economy, the two streams of money are equal and the demand for cash balances is equal to the quantity of money in existence. But when excess supply of a good exists, there must be excess demand for cash balances, unless there is sufficient excess demand for some other good or goods.[4] And unless there is sufficient excess supply of some other good or goods, excess demand for a good must be accompanied by excess supply of cash balances.

In order to simplify the subsequent exposition, we notice that excess supply can be considered as negative excess demand, and vice versa. Thus we shall speak only of excess demand for cash balances and of excess supply of goods, this being understood as referring, wherever necessary, also to excess supply of cash balances or excess demand for goods. With this verbal simplification, we can say that the excess demand for cash balances is always equal to the aggregate value of the excess supply of goods.[5]

"excess demand" and "excess supply," which have a well-established connotation in modern equilibrium theory. By money and cash balances we mean not only currency, but also credit money, i.e., bank deposits and other claims which are held for the purpose of making payments (as distinguished from deposits and claims held as investments, that is, for the purpose of earning income).

[4] Underemployment, having been defined by us as excess supply of a factor of production, implies thus the existence of excess demand somewhere else in the economy. This treatment of underemployment differs from the "involuntary unemployment" as defined by Lord Keynes. "Involuntary unemployment" in the Keynesian sense is not an excess supply of labor but an *equilibrium position* obtained by intersection of a demand and a supply curve, the supply curve of labor, however, being infinitely elastic over a wide range with respect to money wages, the point of intersection being to the left of the region where elasticity of supply of labor with respect to money wages becomes finite. Thus "involuntary unemployment," in the Keynesian sense, does not imply excess demand for cash balances, or for other goods, or for both. Demand and supply for cash balances as well as for all other goods are supposed to be in equilibrium in the Keynesian theory. The difference is shown on the adjoining diagram. D is the demand curve and S is the supply curve of the factor. In our treatment "underemployment" consists in the excess supply $AB(=PQ)$, while Lord Keynes considers the line CQS as the supply curve, P as an equilibrium point and $PQ(=AB)$ as involuntary unemployment. A change in the price (OC) appears in the Keynesian theory as a shift of the horizontal part (CQ) of the supply curve. As is easily seen, our treatment is translatable into Keynesian terms and vice versa. The choice is merely a matter of convenience. It seems that our method ties up more easily with general price theory.



[5] This relation can be visualized better by putting it in symbolic form. Let there be n goods in the economy and denote by $p_1, p_2, \cdots, p_n$ their prices and by $S_1, S_2, \cdots, S_n$, respectively, the excess supply of each of them. The S_i's can be positive, negative, or zero. Denote the excess demand for cash balances by X. The relation stated above is

$$X \equiv p_1S_1 + p_2S_2 + \cdots + p_nS_n.$$

This is an identity holding for any values of the p's.

This relation between the excess demand for cash balances and the excess supply of goods enables us to formulate the condition under which a fall in the price of an underemployed factor will be accompanied by a less than proportional fall in the prices of the other factors and of products. The condition is that a proportional fall in all prices in the economy (interest rates being kept constant and bond prices, consequently, being excepted from the general fall[6]) should reduce the excess demand for cash balances to such an extent that substitution of goods for money takes place. Substitution of goods for money implies an increase in the demand for, or a decrease in the supply of, some or all goods. The prices of the goods for which the demand increases, or of which the supply diminishes, rise relatively to the price of the underemployed factor, i.e., by the substitution of goods for money they are kept from falling in the same proportion. If, on the other hand, a proportional fall in all prices causes a substitution of money for goods, there is a decrease in the demand for, or an increase in the supply of, some or all goods. The prices of the goods for which the demand decreases, or of which the supply increases, fall more than in proportion to the price of the underemployed factor.

The effect of a change in the price of a factor of production upon the prices of the other factors and of products thus depends upon the way in which the community reacts to a proportional change in all prices (interest rates remaining constant). It depends on whether the community reacts by a substitution of goods for money or by a substitution of money for goods. This reaction to a proportional change in all prices will be called the *monetary effect* of a general price change. In particular, we shall say that the monetary effect is *positive* when a proportional fall of all prices causes a substitution of goods for money and a proportional rise of all prices induces a substitution of money for goods. When the opposite happens we shall say that the monetary effect is *negative*. Finally, the monetary effect will be said to be *absent*, when there is neither substitution of goods for money nor of money for goods.

Substitution of goods for money occurs when the excess demand for cash balances changes more than in proportion to the change in prices. For this means that the aggregate value of the excess supply of goods changes more than in proportion to the change in prices. If prices fall, this implies that the excess supply of at least some goods diminishes (i.e., demand increases, supply decreases, or both). If prices rise it implies that the excess supply of at least some goods increases. Under these conditions the monetary effect is positive. When the excess demand for cash balances changes less than in proportion to the change in prices, substitution of money for goods occurs as prices fall and substitution of goods for money occurs as prices rise. The

[6] Interest rates, and therefore bond prices (*vide* p. 15 below), must be assumed as constant, for otherwise the prices of goods of different durability could not change in the same proportion.

monetary effect is negative. If the excess demand for cash balances changes in exactly the same proportion as prices, there is no substitution between money and goods and the monetary effect is absent. The monetary effect is thus positive, absent, or negative, according as the excess demand for cash balances changes more than, exactly, or less than proportionally with prices.[7]

When the monetary effect is positive, a fall in the price of an underemployed factor reduces the excess supply. If the fall in the price is sufficiently large, it makes underemployment disappear entirely. Indeed, in this case the prices of the other factors and the prices of the products cannot fall in the same proportion as the price of the underemployed factor. For, should all these prices fall in the proportion indicated, there would occur a substitution of goods for money. There would emerge, as we have seen, an increase in the demand for, or a decrease in the supply of, some or all goods. This increased demand, or decreased supply, would prevent the prices of at least some goods from falling in the same proportion. The prices of these goods, therefore, cannot share in the general proportional fall but must stay relatively higher.

If the increase in demand, or decrease in supply, resulting from the substitution of goods for money, is directed to factors for which the underemployed factor is substitutable, the prices of the former fall less than in proportion to the price of the latter, and the substitution effect takes place. Similarly, if the increase in demand is directed to products which require the underemployed factor in their production, the prices of these products fall less than in proportion to their marginal cost (at the old output) and the expansion effect becomes directly operative.[8] If, instead, the increase in demand is diverted to products which do not utilize the underemployed factor, or to factors for which it is not substitutable, the products just mentioned, or the products produced with the aid of the mentioned factors, rise in price

[7] This follows directly from the identity

$$X \equiv S_1 p_1 + S_2 p_2 + \cdots + S_n p_n$$

explained in footnote 5 above. If all the p's change in the same proportion and X changes more than in proportion to the p's, at least some of the S_i's must change in the same direction as the p's. The monetary effect is positive. Conversely, if X changes less than in proportion to the p's, at least some of the S_i's must change in the opposite direction. The monetary effect is negative. If X changes in the same proportion as the p's, the aggregate money value of the S_i's does the same. The monetary effect is absent.

[8] The increase in demand for the products and the technological conditions determining the shape of the marginal-cost schedules being given, the expansion effect is the stronger the greater the proportion of the underemployed factor in the marginal cost. For the greater this proportion the greater the reduction in marginal cost (at any given output). This fact provided (in somewhat modified form) the basis of the theory of pricing of factors of production of Walras and Cassel. But, as shown below in the text, it is significant only when a positive monetary effect takes place.

relatively to products produced with the aid of the underemployed factors, or of factors for which it is substitutable. This causes a shift in demand (substitution of the relatively cheaper products for the relatively more expensive ones) and the demand for products requiring the underemployed factor, or factors for which it is substitutable, consequently increases.[8] The prices of these products fail to fall in proportion to the marginal cost (at the old output) and the expansion effect becomes operative, too. Thus, under conditions of a positive monetary effect, the fall in the price of a factor secures the operation of the substitution effect, or of the expansion effect, or of both. As a result of the substitution effect and of the expansion effect, the excess supply (underemployment) of the factor disappears.

In a similar way, it can be shown that, when a factor of production is in excess demand and the monetary effect is positive, a rise of its price produces the substitution effect, or the expansion effect, or both, which make the excess demand vanish. For, should all prices rise in the same proportion as the price of the factor which is in excess demand, there would be a substitution of money for goods. The demand for some or all goods would decrease, or the supply would increase. This would prevent the prices of other factors and of products from increasing all in the same proportion.

A positive monetary effect is necessary if the substitution effect and the expansion effect are to operate at all. This is seen immediately by considering the case where the monetary effect is absent. Let the prices of all factors and products change in the same proportion. There will be no substitution of one factor for another or of one product for another, nor will there be any

[8] The demand for the relatively cheaper products may, however, decrease if they are complementary to the products which become relatively more expensive. But it is rather unlikely that the products which require the underemployed factor, or factors for which it is substitutable, should *all* be complementary to the products towards which the increase in demand resulting from excess cash balances is directed. And even if this were the case, there must be some other products which are substitutes for the latter. The demand for them and their prices increase, and if some of them, in turn, are substitutes for the products requiring the underemployed factor (or factors for which it is substitutable) this will tend to cause an increase in the demand for the latter. If the equilibrium is stable at all, this tendency toward an increase in demand must overbalance the opposite tendency resulting from complementarity (cf. J. R. Hicks, *Value and Capital*, Oxford University Press, 1939, pp. 71–72 and 317). In addition, complementary goods are also likely to be "sympathetic," i.e., their very demand schedules are likely to be interrelated in such a way that an upward shift of one schedule is associated with an upward shift of the other (cf. the writer's article, "Complementarity and Interrelations of Shifts in Demand," *Review of Economic Studies*, Vol. 8, October, 1940, pp. 58–63). Thus it seems likely that the increase in demand resulting from excess cash balances is directed towards the whole bunch of complementary products. Situations like the one described in this footnote seem, therefore, to be very infrequent.

change in output because product prices change in exactly the same proportion as factor prices. Only such changes in the demand or supply of any good are possible as are due to a desire to substitute goods for money or vice versa. By hypothesis such substitution is absent in our case. Consequently, the quantities demanded and supplied are not affected by a proportional change in all factor and product prices.[10] The demand and supply schedules are functions of the ratios of the prices of factors and of products.[11] A proportional change of all factor and product prices leaves the quantities demanded and supplied unaffected.

Suppose now that one factor of production is in excess supply and that its price is flexible; the markets of all other factors and of all products are supposed to be in equilibrium. The price of the factor which is in excess supply falls. This causes attempts to substitute this factor for other factors as well as attempts to expand the output of the products which utilize the underemployed factor. The markets of the other factors and of the related products are brought out of equilibrium by the attempts to substitute factors and to expand output. There is now excess supply in these markets. Since demand and supply depend only on the ratios of the prices of factors and of products, equilibrium can be restored in these markets only if the same price ratios obtain as before. This requires that the prices of factors and products which are now in excess supply fall in the same proportion as the price of the factor originally underemployed. Being flexible, these prices will fall in the proportion indicated, thus restoring equilibrium in the corresponding markets. Also all other prices[12] in the economy must fall in the same proportion in order to prevent disequilibrium from arising in any other market.

Neither the substitution effect nor the expansion effect is operative under these conditions. The excess supply of the factor which started the proportional fall in all prices remains exactly the same as it was at the start.[13] In a similar way, an excess demand for a factor would start a proportional rise in all prices and leave the excess demand unchanged. Thus, in the absence of a monetary effect, neither the substitution effect nor the expansion effect can operate;[14] flexibility of factor prices produces only proportional changes

[10] As already indicated, a proportional change of all factor and product prices implies constancy of interest rates. Cf. footnote on p. 7.

[11] Speaking mathematically: All demand and supply functions are homogeneous of zero degree.

[12] Except interest rates, which remain constant.

[13] The argument presupposes that there is but one set of price ratios at which the economy will be in equilibrium, i.e., that the equilibrium position is unique. Otherwise equilibrium might be restored with a different set of price ratios and with a different (higher or lower) level of employment of the factor in question. We disregard, however, the possibility of multiple equilibrium because it seems to be very unlikely in practice.

[14] If the excess supply of or excess demand for the factor is accompanied by disequilibrium in the markets of some other factors or of products, a change in the ratios of the prices might take place and the excess supply of or excess demand for the factor

in all prices, and this leaves, of course, the "real" situation unaffected.[18]

When the monetary effect is negative, the direction of the substitution effect and of the expansion effect is reversed. A proportional fall of all prices will cause a reduction in the demand, or increase in the supply, for some goods at least. If the reduction in demand is directed to factors which are substitutable for the underemployed factor, the prices of the former fall more than in proportion to the price of the latter. There is substitution of other factors for the underemployed factor. If the reduction in demand is directed to products utilizing the underemployed factor, the prices of these products fall more than in proportion to marginal cost (at the old output) and output is contracted. When the reduction in demand is directed to other products, or other factors than those just mentioned, the prices of these products or of the products produced with the other factors fall relatively to the prices of the products which utilize the underemployed factors or factors which are substitutable for it. Demand shifts away from the latter products and their output is contracted.

Thus, when the monetary effect is negative, the substitution effect and the expansion effect are negative, too. In this case, flexibility of factor prices is a source of economic instability. A fall in the price of an underemployed factor diminishes employment of the factor; if, in consequence, the price of the factor falls further, employment diminishes still more, and so on. The

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under consideration might possibly disappear. But the change in the price ratios would throw out of equilibrium the markets which are in equilibrium. This would cause new readjustments of the prices. If the markets initially in disequilibrium are few, the tendency will be to re-establish the price ratios which obtained at the beginning. For only in this way can equilibrium be maintained (or restored) in the major part of the economy. The results will then be as described in the text.

[18] A strict mathematical proof of this proposition is given in Section 4 of the Appendix. Lord Keynes's theory about the effect of change in money wages on employment is a special case of this theorem. His assumption concerning the constancy of the rate of interest is equivalent, under the conditions of his model (cf. footnote 9, page 17), to absence of the monetary effect. In this model the quantity of money changes in such a way that it is always equal to the demand for cash balances at a constant rate of interest and the excess demand for cash balances is constant, namely, always zero. His statement that a reduction of money wages is operative only via a change in the rate of interest is a special case of our statement that the substitution effect and the expansion effect cannot take place without the monetary effect. Dr. A. P. Lerner has thought of the substitution effect as independent of the monetary effect and therefore has restricted Lord Keynes's doctrine by the additional postulate that the elasticity of supply of factors other than labor must be zero in order to exclude an increase in the employment of labor through substitution of labor for other factors. (See his articles: "Mr. Keynes' 'General Theory of Employment, Interest and Money,'" *International Labour Review*, Vol. 34, October, 1936, pp. 435–454, esp. p. 441, and "The Relation of Wage Policies and Price Policies," *American Economic Review*, Vol. 29, March, 1939, Supplement, pp. 158–169, esp. pp. 165–166.) This additional postulate is unnecessary because the substitution effect does not operate independently of the monetary effect.

fall in prices and in employment becomes cumulative. Similarly, excess demand for a factor (a "bottleneck") becomes cumulative and results in a cumulative rise in prices. Equilibrium can be restored only by *raising* the price of the underemployed factor[16] or by lowering the price of a factor for which there is excess demand, i.e., negative price flexibility is required.

We find that the operation of the substitution effect and of the expansion effect depends on the presence and direction of the monetary effect. Flexibility of factor prices guarantees automatic full employment of factors only in the presence of a positive monetary effect. In the absence of a monetary effect, price flexibility produces only proportional movements of all prices, leaving the "real" situation unchanged. In the presence of a negative monetary effect, price flexibility causes cumulative movements of excess supply (or excess demand) and of prices away from equilibrium. A positive monetary effect thus appears as the parachute which stabilizes an economy with flexible factor prices.[17]

[16] This is the case where an increase in money wage rates increases employment.

[17] These results can be expressed, as in partial-equilibrium analysis, in form of an excess-demand curve for the factor of production. But the excess-demand curve is now not a "partial" demand curve, as in partial-equilibrium theory, but a "total" demand curve, which takes into account the repercussions of a change in the price of a factor upon other prices in the economy and the effect these repercussions have upon the demand for the factor. A positive monetary effect is required in order that this "total" excess-demand curve be falling. If the monetary effect is negative the curve is rising; if it is absent the curve is a vertical line.

CHAPTER IV

Analysis of the Monetary Effect

The monetary effect is positive when a proportional change of all prices (interest rates remaining constant) causes a more than proportional change of the excess demand for cash balances; it is negative when it causes a less than proportional change of the latter. In other words, the monetary effect is positive when the *real*[1] excess demand for cash balances diminishes as prices fall and increases as prices rise. It is negative when the reverse is the case and is absent when the real excess demand for cash balances remains unchanged.

The amount of cash balances which individuals and corporations wish to hold is a function of the prices of the goods which can be bought for money. When these prices change, as a rule, the demand for cash balances also changes. If it increases less (or decreases more) than in proportion to prices (which are all assumed to change in the same proportion), the real demand for cash balances diminishes; if it increases more (or decreases less) than in proportion to prices, the real demand for cash balances increases; finally, if it increases (or decreases) in exactly the same proportion as prices, the real demand for cash balances does not change. However, in order to know how the real *excess* demand for cash balances changes, we must know, in addition, the reaction of the real quantity of money in the economy to a proportional change of all prices.

When everybody expects current prices to continue in the future (or at least over that part of the future which is relevant to his decisions), the real demand for cash balances is not affected by a proportional change of all prices (except interest rates). All prices changing in the same proportion, there is no attempt to substitute goods for each other or to change their output. Under the circumstances, a change of the real demand for cash balances can come only from a desire to postpone purchases or to hasten sales, or vice versa. Such desire, however, is precluded by our assumption that current prices are expected to continue in the future and that, therefore, there is no inducement to change the intertemporal structure of planned demand and supply. Consequently, the real demand for cash balances does not change when all prices change in the same proportion, and the nominal demand for cash balances changes in the same proportion as prices.

The real demand for cash balances thus being constant, the behavior of the

[1] Since all prices change in the same proportion, this concept does not involve the use of index numbers. The real excess demand is the excess demand divided by an arbitrary number which changes in the same proportion as prices. Let λ be such a number and let X be the excess demand for cash balances. The real excess demand for cash balances R is then $R = X/\lambda$. In a similar way we define the real demand for cash balances and the real quantity of money.

real *excess* demand for cash balances depends entirely on what happens to the real quantity of money in the economy. If the latter increases, the real excess demand for cash balances diminishes; if it diminishes, the real excess demand for cash balances increases. In order that the monetary effect be positive, the real quantity of money must, therefore, increase as prices fall and decrease as prices rise. If the reverse happens, the monetary effect is negative. Finally, when the real quantity of money in the economy does not change, the monetary effect is absent.

The condition that the real quantity of money be increasing as prices fall and decreasing as prices rise is automatically satisfied when the nominal quantity of money in the economy is constant. In this case, the monetary effect is always positive and flexibility of factor prices automatically maintains or restores full employment and prevents or absorbs excess demand for factors of production. No explicit assumption is made in traditional equilibrium theory about the quantity of money. It seems, however, a fair interpretation of this theory to suppose that it regards the nominal quantity of money as constant.[1a] On this assumption, the conclusion of this theory, that flexibility of factor prices always generates the substitution effect and expansion effect which restore equilibrium of demand and supply, appears to be fully justified. The same conclusion is even justified if the nominal quantity of money diminishes as prices fall or increases as prices rise, provided it does so less than in proportion to the change in prices. The conclusion holds *a fortiori* if the nominal quantity of money changes in the opposite direction to that of prices. Only if the nominal quantity of money decreases or increases in the same proportion as or in greater proportion than prices does the conclusion of the traditional theory not apply. In the first case, the monetary effect is absent and no amount of price flexibility can restore equilibrium; in the second case, the monetary effect is negative and the restoration of equilibrium requires negative price flexibility.

The substitution effect and expansion effect corresponding to a given change in price of a factor of production is the greater the greater the substitution between goods and money when all prices change in the same proportion. For the greater the substitution between goods and money corresponding to a given change of all prices, the less all other prices tend to fall or rise relatively to the fall or rise of the price of the factor of production and the smaller is the change of the price of the factor needed to restore equilibrium. The fluctuations of factor prices required to maintain or restore equilibrium are thus smaller when the nominal quantity of money in the

[1a] This is confirmed by an article of Professor Pigou which appeared after the above was written. See A. C. Pigou, "The Classical Stationary State," *Economic Journal*, Vol. 53, December, 1943, p. 349. Cf. also the reply of M. Kalecki, "Professor Pigou on the Classical Stationary State—A Comment," *ibid.*, Vol. 54, April, 1944, pp. 131–132.

economy is held constant than when it is allowed to vary in the same direction as (though in lesser proportion than) prices, and it is still smaller when the nominal quantity of money varies in the opposite direction to the prices of the factors of production which are in excess demand or excess supply.

Now we shall consider the influence of the monetary effect upon interest rates. For this purpose, we distinguish between two types of goods: commodities and securities. Commodities consist of factors of production and products; securities are claims to future payments, held for the purpose of deriving income from them.[2] Securities are divided into bonds (fixed-income securities) and stocks (securities promising an indefinite income). The redemption price of bonds is fixed in terms of money. Thus our assumption that current prices are expected to continue in the future cannot be extended fully to bond prices, for the price at the date of redemption is fixed by contract.[3] Interest rates vary in the opposite direction to current bond prices, and changes in interest rates can, therefore, be expressed as variations of current bond prices.[4] Our assumption that interest rates are constant, while all other prices in the economy change in the same proportion, implies thus that bond prices remain constant, while prices of commodities and of stocks[5] change.

If the monetary effect is absent, the real excess demand for cash balances remains constant when all commodity and stock prices change in the same proportion. We have seen that, in this case, there is no desire to substitute commodities and stocks for each other or to change their output. As there is neither a desire to substitute commodities and stocks for money, or vice versa, the demand for and supply of commodities and stocks are constant. But the demand for and supply of bonds are not constant. The proportional change of commodity and stock prices changes the real earning power (as expressed in these prices) of bonds. The demand for and supply of bonds

[2] This purpose is the criterion which distinguishes securities from money. Money consists (in addition to legal tender money) of claims held for the purpose of making payments. This distinction, however, is not always clearly drawn because claims may be held for both purposes. The demarcation line between money and securities is, therefore, somewhat arbitrary.

[3] Unless the bonds are perpetual bonds.

[4] Suppose that the bond will be redeemed after n units of time at a price P_n and let the current price of the bond be P_0. Further let r be the fixed income paid to the bond holder per unit of time. The rate of interest i is then determined by the equation

$$P_0 = r \sum_{t=1}^{n} \frac{1}{(1+i)^t} + \frac{P_n}{(1+i)^n}.$$

Since P_n and r are fixed, the interest rate varies in direction opposite to P_0.

[5] Provided the income of stocks is really indefinite. Claims which, from the legal point of view, are classified as stocks, but which, in practice, bear a fixed income (e.g., owing to the corporation's policy of stabilizing dividends) should be treated for our purposes as bonds.

change, consequently, until they represent the same real earning power as before. Since the earnings of bonds are fixed as a nominal amount of money (per unit of time) this requires that the demand for and supply of bonds change in the same proportion as commodity and stock prices (i.e., that the real demand for and supply of bonds be unchanged).[6] Such a change in the demand for and supply of bonds does not involve any substitution between money and bonds, or between bonds and commodities or stocks, because the demand for and supply of bonds change in the same proportion as money incomes. Interest rates are, consequently, unaffected.[7]

If the monetary effect is positive, there is a substitution of goods for money when commodity and stock prices fall and a substitution of money for goods when these prices rise (all in the same proportion). This implies a change in

[6] The demand and supply functions of bonds are thus homogeneous of the first degree in the prices of commodities and stocks. The mathematical proof of this proposition is very simple. Since all commodity and stock prices change in the same proportion, commodities and stocks can be treated as a single good (see Hicks, *op. cit.*, pp. 312–313 and cf. also our Appendix, Section 5). The quantity of this single good will be denoted by x and its price by p_x. Denote by y and p_y the (nominal) quantity and price of bonds, respectively, and let r be the fixed income borne by a dollar's worth of bonds. The real earning of bonds fluctuates in inverse proportion to p_x; the utility derived from bonds is, therefore, a function not of the nominal amount y but of the real amount y/p_x of bonds held. The bondholder maximizes the utility function $u(x, y/p_x)$ subject to the budget equation $p_x(x-\bar{x})+p_y(y-\bar{y})=r\bar{y}$, where $\bar{x}$ and $\bar{y}$ are the initial amounts of x and y, respectively, and are constant; p_x, p_y, and r are constant, too. This leads to the maximum conditions

$$\frac{\partial u}{\partial x}\,dx + \frac{1}{p_x}\,\frac{\partial u}{\partial (y/p_x)}\,dy = 0$$

and

$$p_x dx + p_y dy = 0,$$

from which we derive

$$\frac{\partial u}{\partial (y/p_x)} + \frac{\partial u}{\partial x} = p_y, \quad \text{or} \quad -\frac{\partial x}{\partial (y/p_x)} = p_y.$$

The marginal rate of substitution between "real bonds" and commodities and stocks is thus independent of the price p_x of the latter. Consequently, the real demand for bonds does not change when commodity and stock prices change all in the same proportion; and the demand in dollars changes in the same proportion. This establishes our proposition. The proof can be easily generalized to take care of the existence of several bonds of different kinds and with different prices.

[7] Since the demand and supply functions of bonds are homogeneous of first degree in the prices of commodities and stocks the excess-supply functions are so, too. Using the notation of the preceding footnote, let the excess-supply function of bonds be $B(p_x, p_y)$. The equilibrium condition in the bond market is then expressed by the equation $B(p_x, p_y)=0$. In view of its homogeneity in p_x, $B(p_x, p_y)=p_x F(1, p_y)$ for any value of p_x. The equilibrium condition turns into $F(1, p_y)=0$, i.e., the equilibrium price of bonds (and, consequently, the interest rate) is independent of p_x, the level of commodity and stock prices. This argument, too, can be generalized to include the case of several bonds with different prices.

the demand for or supply of at least some goods, whether commodities, stocks, or bonds (the latter measured in units of real value; the nominal demand for or supply of bonds also changes, as we have seen, when the monetary effect is absent).[8] The change in demand and supply may be (1) confined to commodities, (2) confined to stocks, (3) confined to the real volume of bonds, or it may embrace any combination of the three.[9] If the real demand for or supply of bonds is affected, this leads to a change in bond prices and, consequently, in interest rates.

When a positive monetary effect associated with falling commodity and stock prices causes an increase in demand (or decrease in supply) of some or all commodities, a substitution effect and expansion effect are produced directly because the prices of commodities for which the demand has increased (or the supply of which has decreased) do not fall in the same proportion as the price of the underemployed factor. This result, however, is much less certain to follow when the increase in demand or decrease in supply, caused by the positive monetary effect, is confined to stocks and bonds. In this case the substitution effect and expansion effect are induced only indirectly, as a consequence of a change in stock prices or in interest rates upon the demand for investment goods. Suppose that the increase in demand or decrease in supply is confined to the real amount of bonds. The ensuing fall in interest rates stimulates the demand for investment goods and thus,

[8] Denote the excess supply and the prices of the different commodities and stocks by $S_1, S_2, \cdots, S_n$ and $p_1, p_2, \cdots, p_n$, respectively. Let $B_1, B_2, \cdots, B_m$ and $\pi_1, \pi_2, \cdots, \pi_m$, respectively, be the excess supply and prices of different kinds of bonds. Write X for the excess demand for cash balances. Then (cf. footnote 5 on p. 6, above),

$$X \equiv p_1S_1 + p_2S_2 + \cdots + p_nS_n + \pi_1B_1 + \pi_2B_2 + \cdots + \pi_mB_m.$$

Suppose that the p's all change in the same proportion and that the π's are all constant. If the S's are all homogeneous of zero degree and the B's are all homogeneous of first degree in the variables $p_1, p_2, \cdots, p_n$, X changes in the same proportion as the p's. If X changes more than in proportion to the p's (positive monetary effect), at least some of the S's must change in the same direction as the p's, or some of the B's must change more than in proportion to the p's, or both. A similar result (only inverted as to direction) is obtained if X changes less than in proportion to the p's (negative monetary effect).

[9] The first is the assumption which underlies the old Cambridge theory of cash balances. This theory supposes that any change of the real excess demand for cash balances implies substitution between money and commodities and thus leads directly to changes in commodity prices. The third is the assumption which underlies the model of the *General Theory* of Lord Keynes. Changes in the real excess demand for cash balances imply, according to Lord Keynes, substitution between money and bonds and lead directly only to changes in interest rates. Commodity prices are affected but indirectly, as a result of the influence of interest rates upon investment. Professor Hicks (*op. cit.*, pp. 274 ff.) considers the general case discussed by us in the text, but without distinguishing between bonds and stocks. Cf. the present writer's article, "Complementarity and Interrelations of Shifts in Demand," pp. 62–63.

directly or indirectly, leads to an increase in demand for the underemployed factor. The increase in demand for the factor is the greater the greater the elasticity of investment with respect to reductions in interest rates. If the particular types of investment activity which utilize the underemployed factor, or factors for which the latter is substitutable, are highly inelastic with respect to reductions in interest rates, the effect may be very small; with some friction present in the economy, it may be practically nil. In a similar way, an increase in the demand for or decrease in the supply of stocks may fail to produce sufficient investment activity in the proper fields to absorb the excess supply of the underemployed factor, because investment activity may be just as inelastic with respect to stock prices as with respect to interest rates.

Thus, when the increase in demand resulting from substitution of goods for money is directed to securities rather than to commodities, price flexibility may fail to restore full employment of factors of production, even though the monetary effect is positive. For similar reasons it may fail to remove excess demand for factors of production. A positive monetary effect is, therefore, much less likely to assure automatic restoration of equilibrium through price flexibility when it implies direct substitution between money and securities and the demand for commodities is influenced only indirectly via changes in interest rates or stock prices. The extent to which a monetary effect implies substitution between money and securities rather than between money and commodities depends on the distribution of cash balances in the community. Persons with low incomes and small cash balances are likely to use superfluous cash for the purchase of commodities, while persons (and corporations) with high incomes and large cash balances are likely to use them for the purchase of securities. Thus, when a very large proportion of the community's stock of money is held by persons and corporations with high incomes, the monetary effect is likely to imply a substitution between money and securities rather than between money and commodities.[10] Under

[10] There is some reason to believe that this was the situation of the American economy during the period around 1935. According to the National Resources Committee in 1935, consumers' balances were only 23.5 per cent of the total demand deposits. And probably less than 14 per cent of the total demand deposits were held by consumers with incomes under $5,000, who provided over 88 per cent of consumers' expenditures. On the hypothesis that half of the total currency in the country was held by them, individuals and families with incomes under $5,000 would have held only about 20 per cent of the total quantity of money (demand deposits plus currency). These cash balances, if totally spent, would have increased the total consumers' expenditure by about one-twelfth of its annual rate. Cf. National Resources Committee, *The Structure of the American Economy* (Washington: Government Printing Office, 1939), Part I, pp. 88–89. This seems to indicate that in this period the implication of a change in the real excess demand for cash balances would have been nearer to that assumed in the model of Lord Keynes than to that assumed by the old Cambridge

these circumstances a positive monetary effect may fail to guarantee the restoration of equilibrium.

When considerable friction is present in the economy, the insufficiency of a positive monetary effect to secure the operation of the substitution effect and of the expansion effect (which restore equilibrium between demand for and supply of factors of production) may appear even in cases where there is direct substitution between money and commodities. If the change in demand resulting from the monetary effect is directed to products which do not utilize the underemployed factor or the "bottleneck factor," or to factors for which it is not directly substitutable, and if the chain of substitutions between the commodities the demand for which increases and the factor in question is very long, the substitution effect and the expansion effect, although to be expected according to pure theory, may be hampered to such a degree by friction as to be negligible in practice. Thus, even under a monetary system or policy which produces a positive monetary effect, reliance upon flexibility of prices of factors of production to maintain or restore equilibrium of demand for and supply of the latter is subject to serious limitations. These limitations increase further when we drop the simplifying assumptions made in this and the preceding chapters.

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theory. This conclusion must be qualified on account of the considerable investment financed by corporate saving. A diminution of the real excess demand for cash balances by corporations may lead directly to an increase in the demand for investment goods. However, the qualification made presupposes an imperfect capital market. If the capital market is perfect, as must be assumed at the present stage of our theoretical argument, investment is not influenced by the liquidity position of firms but only by interest rates. The significance of the liquidity position will receive its due consideration at a later stage of our argument.

CHAPTER V

Price Expectations

In the preceding chapters it was assumed that all decisions are based on the expectation that current prices will continue during that part of the future which is relevant to present decisions ("static expectations"). On the basis of this assumption, it has been shown that the real demand for cash balances is constant when all prices (except interest rates) change in the same proportion and that the nature of the monetary effect depends on whether the real quantity of money in the economy increases or decreases. Now we shall drop this simplifying assumption and study in full generality the effect of price expectations.

Both entrepreneurs and consumers plan their purchases and sales over time.[1] The plans concerning the distribution of purchases and sales over time depend, under perfect competition, on the relation between current prices and the discounted present value of *expected* future prices. A rise of the discounted expected prices of a good relatively to its current price causes a shift of planned purchases (or inputs) from the future to the current period and a shift of planned sales (or outputs) from the current period to the future. The reverse holds in case of a relative fall of the discounted expected price. Such shifts will be denoted by the term *intertemporal substitution*, as distinguished from *intratemporal substitution*, i.e., substitution of different goods at the same moment of time. Intertemporal substitution may also take place between different goods, namely, when the discounted expected price of one good changes relatively to the current price of another good. The direction of intertemporal substitution depends on whether a change in the current price of a good causes the discounted expected prices to change more than, exactly, or less than proportionately. In other words, it depends on whether the elasticity of expectation[2] of the discounted price is greater than, equal to, or less than unity.[3]

[1] The length of the period of time over which purchases and sales are planned in advance is here taken provisionally as given. As to how it is determined, see p. 33 below.

[2] The elasticity of expectation is the ratio of the proportional increment of the expected price and the proportional increment in the current price. This concept was introduced by J. R. Hicks, *Value and Capital*, p. 205. If the current price is denoted by p_0 and the price expected to obtain at the moment t by p_t, and if the notation introduced for elasticities by Mr. Champernowne is used, the elasticity of expectation of the price at t is

$$\frac{Ep_t}{Ep_0} = \frac{dp_t}{dp_0} \cdot \frac{p_0}{p_t}.$$

[3] If the discounted expected price of a good depends not only on the current price of that good but also on the current prices of other goods, the elasticity of expectation mentioned in the text has to be interpreted as a *total* (not partial) elasticity. The total elasticity is obtained when the effect of a change in the current price of a good upon

A fall in the current price of a good causes a relative increase or decrease of the discounted expected price (at any given future moment or period) according as the elasticity of expectation is less or greater than unity. Thus, when the elasticity of expectation is less than unity, a fall in the current price causes a shift of planned purchases from the future period to the current period and a shift of planned sales in the reverse direction. The reverse happens when the elasticity of expectation is greater than unity. In the special case when the elasticity of expectation equals unity, any change in the current price is accompanied by an exactly proportionate change of the discounted expected prices and no intertemporal substitution takes place.

Consider now a proportional fall of all current prices except interest rates. As all prices fall in the same proportion there is no intratemporal substitution in current demand or supply.[4] Any change in the (current) real demand for cash balances must thus come from a shift of planned purchases and sales between the present and the future, i.e., from intertemporal substitution. If all elasticities of expectation are unity, intertemporal substitution is ab-

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all other relevant current prices is taken into account. Let q_{rt} be the discounted price of the rth good expected to obtain at the moment t, and let it depend on the current prices $p_{10}, p_{20}, \cdots, p_{n0}$. We have then the relation $q_{rt}=f(p_{10}, p_{20}, \cdots, p_{n0})$ which we call the *expectation function*. The total differential of the expectation function is

$$\frac{dq_{rt}}{dp_{s0}} = \frac{\partial q_{rt}}{\partial p_{10}} \frac{dp_{10}}{dp_{s0}} + \frac{\partial q_{rt}}{\partial p_{20}} \frac{dp_{20}}{dp_{s0}} + \cdots + \frac{\partial q_{rt}}{\partial p_{n0}} \frac{dp_{n0}}{dp_{s0}}.$$

Multiplying both sides by p_{s0}/q_{rt}, we obtain the total elasticity of expectation

$$\frac{dq_{rt}}{dp_{s0}} \frac{p_{s0}}{q_{rt}} = \frac{\partial q_{rt}}{\partial p_{10}} \frac{p_{s0}}{q_{rt}} \frac{dp_{10}}{dp_{s0}} + \frac{\partial q_{rt}}{\partial p_{20}} \frac{p_{s0}}{q_{rt}} \frac{dp_{20}}{dp_{s0}} + \cdots + \frac{\partial q_{rt}}{\partial p_{n0}} \frac{p_{s0}}{q_{rt}} \frac{dp_{n0}}{dp_{s0}}.$$

Each of the items on the right-hand side, for instance, the ith item, can be written in the form

$$\frac{\partial q_{rt}}{\partial p_{i0}} \frac{p_{i0}}{q_{rt}} \frac{dp_{i0}}{dp_{s0}} \frac{p_{s0}}{p_{i0}},$$

which is a product of two elasticities. The first is the partial elasticity of expectation with respect to the current price p_{i0} and will be denoted by Eq_{rt}/Ep_{i0}, the second is the elasticity of reaction of the current price p_{i0} to a change in the current price p_{s0} and will be denoted by Ep_{i0}/Ep_{s0}. The total elasticity of expectation thus is

$$\frac{dq_{rt}}{dp_{s0}} \frac{p_{s0}}{q_{rt}} = \frac{Eq_{rt}}{Ep_{10}} \frac{Ep_{10}}{Ep_{s0}} + \frac{Eq_{rt}}{Ep_{20}} \frac{Ep_{20}}{Ep_{s0}} + \cdots + \frac{Eq_{rt}}{Ep_{n0}} \frac{Ep_{n0}}{Ep_{s0}},$$

i.e., the weighted sum of the partial elasticities of expectation, the elasticities of reaction of the other current prices serving as weights. When all current prices change in the same proportion these weights all equal unity and the total elasticity of expectation is simply the sum of the partial elasticities of expectation.

[4] This does not necessarily imply that there are no changes in the quantities bought or sold in the current period. But such changes as exist are due exclusively to changes in the ratio of current and (discounted) expected future prices and should, therefore, be treated as intertemporal substitution.

sent. In this case the demand for cash balances decreases exactly in the same proportion as prices and the current demand for and supply of each good are unaffected.[5] The real demand for cash balances is constant. If, instead, the elasticities of expectation are all less than unity, a shift of planned purchases from the future to the present and a shift of planned sales from the present to the future takes place. The demand for cash balances, therefore, diminishes more than in proportion to the fall in prices, i.e., the real demand for cash balances decreases. If, on the other hand, the elasticities of expectation are all greater than unity, there is a shift of planned purchases from the present to the future and a shift of planned sales from the future to the present. In consequence, the real demand for cash balances increases. In the same way, it can be shown that a proportional rise of all current prices (except interest rates) results in no change in the real demand for cash balances when the elasticities of expectation are all unity, in an increase of this demand when the elasticities of expectation are all less than unity, and in a decrease of it when they are all greater than unity.

Thus we conclude that the real demand for cash balances varies in the same direction as current prices when all price expectations are inelastic, varies in the opposite direction to prices when all price expectations are elastic, and does not vary at all when price expectations are all of unit elasticity. A special case of expectations of unit elasticity is "static expectations," according to which current prices are supposed to continue in the future. Thus in this case, too, the real demand for cash balances is constant when all prices, except interest rates, change in the same proportion. This has been shown already in the preceding chapter; now it appears to be a special case of a more general condition.[6]

The situation is more complicated when some elasticities of expectation are less than unity and some are greater than unity. This discrepancy may refer to price expectations of different goods or the expectations of the price of the same good at different future dates. In case of such a discrepancy, the intertemporal substitutions resulting from a proportional change of current prices need not be all in the same direction. The real demand for cash balances may, therefore, be affected in either way, according to the *net* effect of the different intertemporal substitutions. When this net effect is to vary the real demand for cash balances in the same direction as the change in current prices, we shall say that price expectations are *prevailingly* inelastic. Correspondingly, we shall say that price expectations are *prevailingly* elastic,

[5] This applies also to bonds, provided the demand and supply is expressed in "real" units. Cf. pp. 15–16, above.

[6] Static expectations imply unit elasticity of expectations but unit elasticity of expectations does not necessarily imply static expectations; it implies only that expected prices vary in the same proportion as current prices.

or of unit elasticity, when the net effect is to vary the real demand for cash balances in the opposite direction to the change in current prices, or not to vary it at all.

In order that the monetary effect be positive, the real *excess* demand for cash balances must diminish when all prices (except bond prices) fall in the same proportion, and must increase when they rise. Or, in other words, the real quantity of money has to increase relatively to the real demand for cash balances whenever prices fall and decrease whenever prices rise. If price expectations are prevailingly of unit elasticity and, consequently, the real demand for cash balances is constant, this requires, as shown in the preceding chapter, that the real quantity of money in the economy increase as prices fall and decrease as prices rise. If price expectations are prevailingly inelastic, the real quantity of money must diminish less than the real demand for cash balances when prices fall and increase less than the latter when prices rise. The reverse must take place if price expectations are prevailingly elastic, i.e., the real quantity of money must increase more than the real demand for cash balances when prices fall and decrease more than the latter when prices rise.

These conditions for a positive monetary effect can be conveniently summarized by means of the concept of the responsiveness of the monetary system. We define the *responsiveness of the monetary system* as the ratio of the (positive or negative) increment in the real quantity of money in the economy to the increment in the real demand for cash balances.[7] Accordingly, the monetary system will be called *responsive* when the real quantity of money increases (or decreases) more than the real demand for cash balances, and it will be called *unresponsive* when the real quantity of money increases (or decreases) less than the real demand for cash balances. When the real quantity of money changes by exactly the same amount as the real demand for cash balances, the monetary system will be said to be *neutral*. In order to extend our definition to the case where the real demand for cash balances is constant, we shall say in this case that the monetary system is responsive, unresponsive, or neutral, according as the real quantity of money increases, decreases, or does not change.[8] The conditions for a positive monetary effect can now be summarized as follows:

Elastic price expectations require a responsive, whereas inelastic price expectations require an unresponsive monetary system. Price expectations of unit

[7] Denote by ΔM the increment in the real quantity of money and by ΔD the increment in the real demand for cash balances. The monetary system is responsive, neutral, or unresponsive according as $\Delta M/\Delta D \gtreqless 1$.

[8] This extension of the definition is necessary because $\Delta M/\Delta D$ has no meaning or is indeterminate when $\Delta D = 0$.

elasticity require a monetary system which is responsive when prices are falling and is unresponsive when prices are rising.

We shall call this the *General Rule.* When not all price expectations are uniformly elastic, inelastic, or of unit elasticity, the elasticity referred to is to be understood as the prevailing elasticity of expectations as defined above. If the relation between the elasticity of price expectations and the responsiveness of the monetary system is the reverse of what is required by the General Rule the monetary effect is negative. If the monetary system is neutral the monetary effect is absent, whatever the elasticity of price expectations.

The effect of a change in the current price of a commodity upon its current demand and supply is the result of the joint action of intertemporal substitution and of intratemporal substitution (and expansion).[9] The first is determined by the elasticity of those price expectations which affect directly the current demand for or supply of the commodity, namely the expectations of the price of the commodity and of the prices of substitute and complementary goods. The second depends on the nature of the monetary effect and on the elasticity of the price expectations which affect the current demand for or supply of other goods. The monetary effect depends on the elasticities of *all* price expectations and on the behavior of the real quantity of money.

Under a neutral monetary system the monetary effect is absent. If all price expectations are of unit elasticity there is no intertemporal substitution and current demand and supply are not affected by a proportional change of all prices except interest rates. As shown in Chapter III, any change in the current price of a factor of production leads then to a change of all other current prices (except interest rates) in the same proportion and the excess supply of or excess demand for each factor remains unchanged.[10] But if

[9] These effects are additive. Let X be the current demand for or supply of the factor; let p_{i0} be the current price of the ith good and let q_{it} be the discounted price of the ith good expected to rule in the tth interval. Suppose that the current demand for or supply of the factor is a function of n current prices and of n discounted prices expected in ν future intervals of time. Let the subscript 0 refer to the "present" and the subscript s to the price of our factor. We have then

$$\frac{dX}{dp_{s0}} = \sum_{i=1}^{n} \frac{\partial X}{\partial p_{i0}} \frac{dp_{i0}}{dp_{s0}} + \sum_{t=1}^{\nu} \sum_{i=1}^{n} \frac{\partial X}{\partial q_{it}} \frac{dq_{it}}{dp_{s0}}.$$

The first term on the right-hand side represents the intratemporal substitution and expansion effect, the second term represents the effect of intertemporal substitution.

[10] Professor Hicks, who after Lord Keynes was the first to point out that no substitution or expansion effect can take place unless the monetary effect is positive, thinks that, when all elasticities of expectation equal unity, the monetary effect is *always* absent. In consequence, with unit elasticities of expectation, neither intertemporal nor intratemporal substitution is possible, and all prices change in the same proportion. Cf. *Value and Capital,* pp. 254–255. As shown in the text, this holds only when

elasticities of price expectations differ from unity, a proportional change of all prices does influence current demand and supply. This causes a change in the relative prices of different goods, and intratemporal substitution and expansion appear, although the monetary effect is absent. The absence of the monetary effect, however, imposes a constraint upon intratemporal substitution and expansion. The aggregate real value of all the (positive and negative) excess supplies of goods in the economy is constant, and, consequently, each change in current demand for or supply of some good is accompanied by a change in the opposite direction in the current demand for or supply of some other good or goods. Thus the changes in current demand and supply are likely to cancel each other and the (positive or negative) intratemporal substitution effect and expansion effect in the market of the factor that is in excess supply or excess demand is not likely to be very pronounced (unless a uniform change of demand for or supply of the other goods in one direction is balanced exclusively by an opposite change in the market of the factor, which could happen only under a very peculiar combination of elasticities of expectation). The current demand for or supply of the factor is, therefore, likely to depend chiefly on the elasticity of the price expectations which influence it directly. If these expectations are inelastic, a fall in the price of an underemployed factor and a rise in the price of a "bottleneck" factor are likely to restore equilibrium; while, if the said expectations are elastic, this is likely to make the excess supply or the excess demand worse.

If the monetary effect is positive, the intratemporal substitution effect and expansion effect must be such as to increase the current demand for (or decrease the current supply of) the factor when its price is reduced and to do the reverse when its price is raised. For if, for instance, current prices fall, a substitution of goods for money takes place. This diminishes the real aggregate value of all the excess supplies of goods and, though some current demand may decrease and some current supply may increase, it must be overbalanced by the increase in current demand and decrease in current supply in other markets. The intratemporal substitution effect and expansion effect must, therefore, lead to an increase in the current demand for (or decrease in current supply of) the factor the price of which has been reduced. To this, however, the effect of direct intertemporal substitution must be added. If the expectations of the prices which influence the current demand for or supply of the factor are inelastic, the intratemporal substitution effect and expansion effect are reinforced by an increase in current demand for (or decrease in current supply of) the factor due to intertemporal substitution.

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the monetary system is neutral. When the monetary system is not neutral, a change in the price of a factor does not produce proportional changes of all other prices and a substitution effect and expansion effect (in either direction) do take place, even if all elasticities of expectation equal unity. Professor Hicks's proposition thus lacks generality.

But if these expectations are elastic, intertemporal substitution tends to diminish current demand for (or to increase current supply of) the factor. The final result is the sum of the effect of intertemporal substitution and of the intratemporal substitution and expansion effect. Unless the factor is so highly specialized that the intertemporal elasticities of substitution outweigh the joint effect of the intratemporal elasticities of substitution and expansion,[11] the intratemporal substitution effect and expansion effect prevail and a fall in the price of an underemployed factor leads to restoration of equilibrium, even though the price expectations which influence directly the demand for the factor be elastic. A similar result is obtained in the case where the price of a "bottleneck" factor is raised.

Elastic expectations of the prices which directly influence the current demand for or supply of the factor may, however, block the restoration of equilibrium when the substitution between money and goods implied in a positive monetary effect is primarily between money and securities. As shown in the preceding chapter, the intratemporal substitution effect and expansion effect may be extremely weak in this case.[12] The result may be that the adverse intertemporal substitution due to elastic expectations of the prices which

[11] By a procedure identical with that in footnote 9 on p. 24 above, the total elasticity of demand for and supply of the factor X can be expressed in the form

$$\frac{dX}{dp_{s0}}\frac{p_{s0}}{X} = \sum_{i=1}^{n}\frac{EX}{Ep_{i0}}\frac{Ep_{i0}}{Ep_{s0}} + \sum_{t=1}^{p}\sum_{i=1}^{n}\frac{EX}{Eq_{it}}\frac{Eq_{it}}{Ep_{s0}}.$$

Here the Eq_{it}/Ep_{s0} are total elasticities of expectation of the prices relevant to the current demand for or supply of the factor. From the theory of demand for or supply of a factor of production, we have

$$\frac{EX}{Ep_{i0}} = K_{i0}S_{si.00} \quad \text{and} \quad \frac{EX}{Eq_{it}} = K_{it}S_{si.0t},$$

where K_{i0} and K_{it} are the proportion of the community's total expenditure spent on the ith good in the current and in the tth interval of time, respectively, $S_{si.00}$ is the elasticity of intratemporal substitution (or expansion) between the factor and the ith good, and $S_{si.0t}$ is the elasticity of intertemporal substitution between the factor (used currently) and the ith good in the tth interval of time ($i=1, 2, \cdots, n$; for $i=s$ the ith good is identical with the factor). The basis of these formulae has been given by J. R. Hicks in *Théorie mathématique de la valeur* (Actualités Scientifiques et Industrielles, No. 580, Paris, Hermann et Cie, 1937), p. 39. We thus obtain

$$\frac{dX}{dp_{s0}}\frac{p_{s0}}{X} = \sum_{i=1}^{n}K_{i0}S_{si.00}\frac{Ep_{i0}}{Ep_{s0}} + \sum_{t=1}^{p}\sum_{i=1}^{n}K_{it}S_{si.0t}\frac{Eq_{it}}{Ep_{s0}},$$

which indicates how the current demand for or supply of the factor depends on the elasticities of intratemporal substitution and expansion and on the elasticities of intertemporal substitution.

[12] If the substitution is between (the real amount of) bonds and money, the intratemporal substitution effect and expansion effect are a result of the influence of a change in interest rates upon investment. Such influence is a result of intertemporal substitution due to a change in interest rates. See on this subject, p. 28 and p. 59 below.

directly influence the current demand for or supply of the factor prevails over intratemporal substitution and expansion. In such a case a fall in the price of an underemployed factor may increase the excess supply of the factor and a rise in the price of a "bottleneck" factor may increase the excess demand for the factor, in spite of the positive monetary effect. The insufficiency of a positive monetary effect is thus even greater than described in the preceding chapter.

On the other hand, inelastic expectations of the prices which influence current demand for or supply of the factor may, under favorable circumstances, stabilize the economy when the monetary effect is negative. This will happen when the factor is so specialized that the intertemporal substitution effect outweighs the intratemporal substitution effect and expansion effect, or when the latter two are very small because the negative monetary effect spends itself in security markets.

We conclude thus that, given the monetary effect, inelastic price expectations always tend to stabilize the economy. If the monetary effect is positive, they reinforce the equilibrating tendency of intratemporal substitution and expansion; if the monetary effect is negative, they counteract their disequilibrating tendency; and if a monetary effect is absent, they act as the chief equilibrating force in the economy. Elastic expectations exert in all these situations a destabilizing influence.

We shall now investigate the effect of changes in interest rates upon intertemporal substitution. Unless the monetary system is neutral, a proportional change of the current prices of commodities and shares leads, as a rule, to a change in interest rates. For instance, when the monetary effect is positive, a fall of all current commodity prices leads, as a rule, to an increase in the real demand for bonds and, therefore, to a fall in interest rates. The degree to which interest rates fall depends on the elasticity of expectation of (discounted) bond prices. The more elastic these prices are, the greater is the increase in the current net demand for bonds and the greater the fall in interest rates. Thus interest rates are the more sensitive to a change in current commodity prices the greater the elasticity of expectation of bond prices.

Intertemporal substitution depends on the elasticity of expectation of *discounted* future prices. The elasticity of expectation of the discounted price of any good is made up of two parts: the elasticity of expectation of the undiscounted price of the good, and the elasticity of reaction (with respect to a change in current prices) of the interest rate at which the price is discounted.[18] Given the expectation of the undiscounted price, the discounted

[18] The exact mathematical relationship is as follows: Let p_t be the undiscounted price expected to obtain at the moment t, and let r_t be the rate of interest (per unit of time) on a loan of duration t. Denote the discount factor by β_t. We have $\beta_t = 1/(1+r_t)^t$ and the discounted expected price is $q_t = \beta_t p_t$. Applying the theorem about the elasticity

price changes in the opposite direction to the rate of interest. A rise in the rate of interest reduces the discounted expected price; a fall in the rate of interest raises the discounted expected price.

If the monetary effect is positive, the real demand for bonds increases as current prices of commodities and stocks fall, and decreases as these prices rise. Interest rates thus vary in the same direction as current commodity and stock prices and the discounted value of expected prices increases as current prices fall and decreases as current prices rise. Given the expectations of the undiscounted future prices, a fall in current prices leads to a shift of planned purchases toward the present and to a shift of planned sales toward the future, while a rise in current prices leads to a reverse shift. The first shift implies an increase in investment; the reverse shift implies a decrease in investment. It is thus through such an intertemporal substitution that a change in interest rates affects investment. This intertemporal substitution exerts a stabilizing influence on the economy. If, however, the monetary effect is negative, interest rates vary in the opposite direction to commodity and stock prices and the intertemporal substitution operates in the opposite direction. The effect on the economy is destabilizing.

Since the change in interest rates resulting from a given change in the current prices of commodities and stocks is the greater the greater the elasticity of expectation of bond prices, we find that highly elastic expectations of bond prices exert a stabilizing influence when the monetary effect is positive and a destabilizing influence when the monetary effect is negative.

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of a product (see R. G. D. Allen, *Mathematical Analysis for Economists*, pp. 252–254), we obtain

$$\frac{Eq_t}{Ep_0} = \frac{Ep_t}{Ep_0} + \frac{E\beta_t}{Ep_0}; \tag{1}$$

i.e., the elasticity of expectation of the discounted price is the sum of the elasticity of expectation of the undiscounted price and of the "elasticity of reaction" of the discount factor with respect to the current price of the good. If necessary, the elasticities can be interpreted as total elasticities. Applying the theorems about the elasticity of a power function and about the elasticity of a sum (see Allen, *ibid.*), we have also

$$\frac{E\beta_t}{Ep_0} = -\frac{E(1+r_t)^t}{Ep_0} = -\frac{tr_t}{1+r_t}\frac{Er_t}{Ep_0},$$

whence

$$\frac{Eq_t}{Ep_0} = \frac{Ep_t}{Ep_0} - \frac{tr_t}{1+r_t}\frac{Er_t}{Ep_0}. \tag{2}$$

Er_t/Ep_0 is the "elasticity of reaction" of the interest rate on loans of duration t to changes in the current price p_0. When the rate of interest remains constant, the second term in (1) and (2) vanishes and the elasticity of expectation of the discounted price is the same as that of the undiscounted price.

CHAPTER VI

Uncertainty

In our discussion of price expectations, we had assumed that entrepreneurs and consumers expect future prices (including bond prices) with certainty, i.e., that the expected prices have definite unique values.[1] Actually, however, price expectations are much less definite. At best, the entrepreneur or consumer expects that a given future price can have a *set of possible values*, some probability corresponding to each of these values. In other words, he is confronted with a probability distribution of possible values of the expected price. We shall say in this case that his price expectations are subject to *uncertainty*.

Some particular value, out of all possible values, may appear to the entrepreneur or consumer as the most probable one and serve as the basis of his expectation. This is the most probable price.[2] The definiteness, however, with which the most probable price is expected actually to occur depends on the *range* of the probability distribution. The greater the range the less definite the expectation of the most probable price. The range can thus be taken as a measure of the degree of uncertainty of the expectation.[3] In most cases the entrepreneur or consumer does not consider the whole range of possible values of the expected price, but disregards the extreme values at both tails of the probability distribution. He does so because the joint probability of these extreme values is too small to bother about. The range with

[1] This does not imply perfect knowledge. The expected prices may differ from the prices which are realized subsequently. The certainty of the expectations is merely *subjective*.

[2] This is the mode of the probability distribution. In case the mode fails to be unique (an extreme example is a rectangular distribution) the mean of the probability distribution (also called mathematical expectation, or actuarial value) must be taken. Some authors use the mean throughout in their treatment of the theory of uncertainty. Cf., for instance, A. C. Pigou, *The Economics of Welfare* (London: Macmillan and Co., 1938, 4 ed.), pp. 773–774; J. R. Hicks, "A Suggestion for Simplifying the Theory of Money," *Economica*, N.S., Vol. 2, February, 1935, pp. 1–19 (but the most probable value is used in *Value and Capital*, pp. 125–126); J. Marschak, "Money and the Theory of Assets," *Econometrica*, Vol. 6, October, 1928, p. 320; H. Makower and J. Marschak, "Assets, Prices and the Monetary Theory," *Economica*, N.S., Vol. 5, August, 1938, p. 272. The most probable value, however, seems to be a more realistic descriptive device because an idea of it can be formed without any computation, by mere ranking. It does not require that the probabilities be measurable.

[3] Most authors use the standard deviation or the coefficient of variation as measure of uncertainty. Cf. Marschak, *op. cit.*, p. 320; Makower and Marschak, *op. cit.*, p. 272. The range, however, seems to us more realistic as a description of the actual evaluation of the degree of uncertainty of price expectations. It does not require an exact knowledge of the whole probability distribution, while the standard deviation or the coefficient of variation does require such a knowledge.

the tail values thus cut off will be called the "*practical range*"[4] and will serve as our measure of the degree of uncertainty of price expectations.

Entrepreneurs and consumers need not, and usually do not, visualize an exact probability distribution of possible prices. For our purpose it is sufficient to assume that each person forms some idea about the most probable value and the "practical range" of the expected price. For instance, an entrepreneur or consumer thinks that the price of some specified good at some given future date will be most probably $100, but in any case not less than $80 and not more than $150. He may believe that there is some slight probability that the price will turn out to be below $80 or above $150, but this is so small as to be negligible in practice and he takes the chances of disregarding such outcomes altogether. Such an assumption seems to be quite realistic.

Entrepreneurs and consumers prefer as a rule more definite to less definite expectations.[5] Consequently, two equal most probable prices are not equivalent if the degree of uncertainty is different. Sellers consider the price which is expected with greater uncertainty as equivalent to a lower most probable price expected with less uncertainty, while buyers consider a price expected with greater uncertainty as equivalent to a higher most probable price expected with less uncertainty. Thus sellers react to greater uncertainty in the same way in which they would react to a lower most probable price and buyers react in the same way as to a higher most probable price. This is the basis on which forward markets operate. Sellers hedge themselves by selling forward at a price below the most probable price they expect; buyers hedge themselves by buying forward at a price above the most probable price they expect.[6] If the forward market relieves the hedging party of all un-

[4] The range between the fifth and ninety-fifth centile, for instance. The "practical range" is analogous to the confidence interval corresponding to a given level of significance, as applied in the theory of statistical estimation.

[5] Up to a point, people may prefer the opposite because they like to gamble. However, the great majority of market transactions are carried on in amounts such that there is definitely a preference for greater certainty of expectations. On this point cf. Pigou, *op. cit.*, p. 776. In addition, people may have a preference for skewness (sellers for positive skewness, buyers for negative skewness) of the probability distribution of price expectations ("long odds"). This explains, for instance, the overcrowding of such occupations as law or girls' jobs in Hollywood.

[6] Thus if the most probable price expected is approximately the same as the spot price, the forward price is below the spot price when the hedging is done by sellers and above the spot price when the hedging is done by buyers. In the language of the English forward markets, a positive difference between forward price and spot price is called "contango" and a negative difference is called "backwardation." This explains the "normal backwardation" as described by Lord Keynes (*A Treatise on Money*, Vol. II, London: Macmillan and Co., 1930, p. 143). There should be a tendency to a "normal contango" when the buyers are the hedgers. This tendency, however, is prevented by the possibility of arbitrage. Cf. N. Kaldor, "A Note on the Theory of the Forward Market," *Review of Economic Studies*, Vol. 7, June, 1940, pp. 198–200.

certainty of price expectation, then the forward price represents the price a definite, certain expectation of which is regarded by the hedger as equivalent to the most probable price actually expected with uncertainty. The difference between the most probable price actually expected and the equivalent price expected with certainty constitutes the *risk premium* of the individual. As more definite expectations are preferred to less definite ones, the risk premium is the greater the greater the degree of uncertainty of the actual expectation.[7]

Thus we can substitute for the most probable prices actually expected with uncertainty equivalent prices expected with certainty.[8] Let us call them the *effective* expected prices. This is the most probable price minus the risk premium.[9] For sellers the risk premium is positive, for buyers it is negative. By means of this device, uncertain price expectations can be reduced to cer-

[7] This can be expressed in terms of indifference curves. Indicate the most probable price along the axis OY and indicate the uncertainty (i.e., dispersion of the probability distribution as measured, for instance, by the "practical range") along the axis OX. The indifference curves as between different most probable prices and different degrees of uncertainty are given in Figures 1 and 2 for sellers and buyers respectively. For

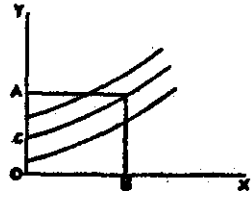

FIGURE 1

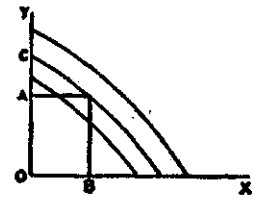

FIGURE 2

sellers the indifference curves are rising because greater uncertainty must be compensated by a greater most probable price. For buyers they are falling because greater uncertainty must be compensated by a lower most probable price. The concavity or convexity of the curves expresses the increasing unwillingness to bear uncertainty. CA or AC expresses the risk premium. It is seen immediately from the diagrams that the risk premium is the greater the greater OB, i.e., the degree of uncertainty of the actual price expectation.

[8] In terms of the diagrams of the preceding footnote, this means replacing the most probable price OA expected with a degree of uncertainty OB by the (subjectively) certain price OC.

[9] The effective price can also be interpreted as the actual price *discounted* for risk. Denoting by ρ the rate of discount for risk, we write (in the notation of the preceding two footnotes)

$$OC = \frac{OA}{1+\rho}.$$

Transforming, we find

$$\rho = \frac{CA}{OC},$$

i.e., the rate of discount for risk is equal to the ratio of the risk premium and of the effective price. For sellers this rate is positive, for buyers it is negative.

tain ones.[10] In consequence, an increase in sellers' uncertainty acts in the same way as a reduction of their expected future selling prices, while an increase in buyers' uncertainty acts in the same way as an increase in their expected future prices of purchase.

Let us define the *effective* elasticity of expectation as that value of the elasticity of expectation of the effective expected prices which is obtained when the risk premium is deducted from the (discounted) most probable price. An increase in sellers' uncertainty lowers the effective elasticity of their expectation when the current price rises and raises it when the current price declines. An increase of buyers' uncertainty raises the effective elasticity of their expectation when the current price rises and lowers it when the current price falls. Changes in the degree of uncertainty can thus be taken care of through a study of their influence upon the effective elasticity of expectation. All the propositions about the effect of the elasticity of expectation upon intertemporal substitution and the demand for cash balances which have been developed in the preceding chapter can be applied to the case of price expectations subjected to uncertainty by making use of the concept of the effective elasticity of expectation.

The introduction of uncertainty into our analysis serves also to determine the length of the period of time over which individuals plan their purchases and sales. This period has been called very aptly the *economic horizon*[11] of the individual. As long as price expectations are (subjectively) certain, the economic horizon is indeterminate. This indeterminateness disappears when uncertainty is allowed for.

[10] This device is also adopted by Professor Hicks in *Value and Capital*, p. 126: "If we are to allow for uncertainty of expectations, . . . we must not take the most probable price as the representative expected price, but the most probable price ± an allowance for the uncertainty of the expectation, that is to say, an allowance for risk." Against this procedure, Dr. A. G. Hart has objected that it is not possible to find such a price as, if expected with certainty, would lead to the same actions of sellers and buyers as the actual expected price subject to uncertainty. See *Anticipations, Uncertainty and Dynamic Planning* (Studies in Business Administration of the University of Chicago, Vol. 11, No. 1, 1941), p. 55. In terms of our diagrams in footnote 7, p. 31, this means that the indifference curves do not reach the axis OY but are either asymptotic to it or indeterminate in its neighborhood. The shape of the indifference curves is, of course, an empirical problem. It may be suggested, however, in reply to Dr. Hart's criticism that the existence of forward markets constitutes an indirect empirical evidence that the indifference curves do reach the OY axis in certain cases at least. Unfortunately, this evidence lacks generality because forward markets exist only for a few goods. If Dr. Hart's criticism should prove empirically justified, we can take, instead of the equivalent price expected with certainty, some price expected with some (arbitrarily chosen) smaller degree of uncertainty which is equivalent to the price expected actually with a greater degree of uncertainty. In terms of our diagrams this would simply mean shifting the origin so that the origin indicates on the OX axis not certainty but any degree of uncertainty chosen for reference.

[11] This term is due to Dr. J. Tinbergen. *Vide* his article, "The Notions of Horizon and Expectancy in Dynamic Economics," *Econometrica*, Vol. 1, 1933, pp. 247–264.

As a rule, the uncertainty of price expectations is the greater the more distant in the future the planned purchase or sale is (at least from a certain date on). Thus the risk premium, which has to be deducted from any given most probable price, increases as the planning of purchases and sales extends farther into the future.[12] Consequently, the effective expected prices of goods to be sold at various future dates decrease, while the effective prices of goods to be bought at various future dates increase. This imposes a limit upon the dates for which any sales or purchases are planned at all.[13] Firms (entrepreneurs) find that, beyond a certain date, the effective expected prices of their products are less than the effective expected marginal costs and that the effective expected marginal value productivities of the factors they plan to employ are less than the effective expected prices of these factors. In a similar way, households (consumers) find that beyond a certain date the effective prices of goods they plan to buy are higher than the effective marginal rates of substitution of the respective goods for money.[14] Thus beyond a certain date the effective expected prices of goods to be sold are too low to induce the planning of sales, while the effective expected prices of the goods to be bought are too high to induce the planning of purchases. No sales or purchases are planned beyond this date. In this way the length of the economic horizon of each individual and corporation is determined.[15]

[12] There is good reason to believe that the risk premium increases at an increasing rate as the date of the planned purchase or sale extends farther into the future. If the uncertainty, as measured by the dispersion of the expectations, e.g., the practical range, increases uniformly with the extension of the date towards the future, the risk premium is bound to increase at an increasing rate because the indifference curves of dispersion and most probable price (cf. Figs. 1 and 2 in footnote 7, p. 31) are convex from below or from above for sellers or buyers, respectively. Thus the risk premium could fail to increase at an increasing rate only if the increase in uncertainty should take place at a sufficiently decreasing rate. But empirically it seems highly unlikely that there is any decrease at all in the rate of increase of uncertainty. The reverse seems much more likely. Our conclusion in the text is quite independent of the fact that the increase in the risk premium takes place at an increasing rate. This fact, however, helps to shorten the length of the economic horizon.

[13] Cf. Hicks, *op. cit.*, p. 225.

[14] This marginal rate of substitution increases as the date of the planned purchases is more distant because the effective expected prices of the factors by the sale of which the household obtains its income decrease and, consequently, the effective expected income decreases, too.

[15] The date at which plans to buy or sell stop may be different with regard to different goods. In such a case the economic horizon has to be defined as the period corresponding to the good the planned purchases or sales of which stop at the latest date. It should also be noticed that the economic horizon, as here defined, does *not* limit the time over which provisions for the future are made. Both households and firms make provision for the future which extend far beyond the period defined as the economic horizon. They do it by acquiring capital assets during the period of planning covered by the economic horizon. The economic horizon is, however, the period over which *specific purchases and sales* of goods (including capital assets of all kinds) are planned, whereas

An economy with flexible factor and product prices is likely to involve greater uncertainty of price expectations than one in which some prices are rigid. In such an economy, the economic horizon of the individuals is, therefore, shorter than in an economy where prices are less flexible. This tends to diminish the effect of intertemporal substitution upon current demand and supply. For this effect is the sum of the intertemporal substitutions between current purchases and sales and the purchases and sales planned at each one of the future dates.[16] Other things being equal, this sum is smaller when the economic horizon is shorter because the number of future dates for which purchases and sales are planned is less. This implies that, given the elasticities of price expectations, the effect of a proportional change of the current prices of commodities upon the real demand for cash balances is likely to be less in a regime where all prices are flexible than in one where some important prices are rigid.

If in such a regime the real quantity of money is held constant, the monetary effect, whether positive or negative, may be rather weak and so may be the (intratemporal) substitution and expansion effects. The monetary effect being weak, greater fluctuations of prices are required to secure automatic maintenance or restoration of equilibrium in the markets of factors of production through flexibility of their prices. But greater price fluctuations mean greater uncertainty of price expectations and thus weaken further the monetary effect and, in consequence, the intertemporal substitution effect. This requires still greater fluctuations of factor prices, and so on. With some friction present, the effects of changes in factor prices may become too weak to be of great practical significance. In practice, the situation may approach one similar to that corresponding to a neutral monetary system. Maintenance (or restoration) of equilibrium of demand for and supply of factors of production may require an active monetary policy, in spite of the fact that the conditions are fulfilled which, in absence of uncertainty, would assure automatic equilibrium without a change in the real quantity of money.

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provision for the future beyond it is made by planning to wind up at the end of this period with a certain amount of assets. Cf. Hicks, *op. cit.*, pp. 193–194 and 229–230. The relationship between the period over which specific transactions are planned and uncertainty, as well as the fact that provision for the future extending beyond it is made in the form of the acquisition of assets, has been pointed out previously by P. N. Rosenstein-Rodan, "The Role of Time in Economic Theory," *Economica*, N.S., Vol. 1, February, 1934, pp. 80–84.

[16] See footnote 9 on p. 24, above.

CHAPTER VII

Imperfect Competition

The preceding analysis of the operation of the substitution effect and of the expansion effect and of their relation to the monetary effect is based on the responses of entrepreneurs to changes in factor prices and in product prices as obtaining under conditions of perfect competition. When the assumption of perfect competition is dropped, it is necessary to consider different patterns of entrepreneurial responses. Under perfect competition, entrepreneurs respond exclusively to prices (which they regard as independent of their individual action). Under monopoly and monopsony, including monopolistic and monopsonistic competition, entrepreneurs respond not to prices but to *schedules* (to demand schedules under monopoly, to supply schedules under monopsony). Under oligopoly and oligopsony, their responses to schedules are based on conjectures as to how other entrepreneurs will react and how this, in turn, will affect the schedules confronting those who contemplate the response.

We shall consider first an economy containing monopolies and monopsonies. In order that the monopolists be confronted with determinate demand schedules to respond to, they must deal with atomistic buyers. In the same way, each monopsonist must deal with atomistic sellers in order to be confronted with a determinate supply schedule.[1] The demand and supply schedules contain among their variables also the prices of other goods beside the price of the good sold or bought by the particular monopolist or monopsonist. But each monopolist and monopsonist must regard the prices of other goods as independent of his own actions.[2]

The nature of economic equilibrium, as well as of disequilibrium, in a monopolistic or monopsonistic market differs from that in a perfectly competitive market. In the latter, disequilibrium consists in excess demand or excess supply. Monopolistic supply, however, is always equal to the demand for the good in question and monopsonistic demand is always equal to supply. A monopolistic or monopsonistic market is in equilibrium when the quantity sold and bought is such that it maximizes the profit of the monopolist or monopsonist. In this case, there is no tendency to change either price or quantity.[3] Disequilibrium occurs in a monopolistic or monopsonistic market when the quantity sold and bought differs from the equilibrium amount. When a monopolist sells more than the equilibrium amount, he will restrict his supply and raise his price. The reverse happens when he sells less than

[1] Bilateral monopoly is thus excluded by our assumptions.

[2] This excludes oligopoly and oligopsony.

[3] In the matter of equilibrium of a system with monopolies and monopsonies, cf. M. W. Reder, "Monopolistic Competition and the Stability Conditions," *Review of Economic Studies*, Vol. 8, February, 1941, pp. 122–125.

the equilibrium amount. We shall denote these cases as *monopolistic underrestriction* and *overrestriction* of supply, respectively. When a monopsonist buys more than the equilibrium amount, he will restrict his demand and lower his price. The reverse happens when he buys less than the equilibrium amount. We shall denote these cases as *monopsonistic underrestriction* or *overrestriction* of demand, respectively. Thus monopolistic underrestriction and monopsonistic overrestriction perform the function of excess demand under perfect competition, while monopolistic overrestriction and monopsonistic underrestriction perform the function of excess supply.[4]

Now let there be either excess supply of (if the market is perfectly competitive) or monopsonistic underrestriction of demand for a factor of production. The price of the factor decreases. This causes an attempt to substitute the factor for other factors. In perfectly competitive factor markets the attempt at substitution takes place because the price of the factor under consideration is now lower relatively to the prices of the other factors. The attempted substitution leads to excess supply of the other factors. In monopsonistic factor markets, the demand for the other factors depends on the ratio of the marginal expenditure[5] for these factors to the price of the factor under consideration.[6] A fall of the latter price creates a situation of monopsonistic underrestriction of the demand for the other factors; the monopsonists find that they are using more of these than the amount which maximizes their profit and they reduce their purchases accordingly. Thus, in the monopsonistic factor markets as well as in the competitive ones prices

[4] Cf. Section 6 in the Appendix. We assume here that the demand schedule for a good is negatively sloped and that the supply schedule is positively sloped. In the case of positively sloped demand schedules and negatively sloped supply schedules, the relation of monopolistic underrestriction, etc., to excess demand and excess supply is reversed.

[5] The marginal expenditure for a factor of production is the increment of the firm's total expenditure for factors resulting from the purchase of an additional unit of the factor. If p is the price of the factor and its elasticity of supply is ϵ, the marginal expenditure is $p(1+1/\epsilon)$. The optimum combination of factors used to produce any given output is that where the marginal value productivities of the factors are proportional to the marginal expenditures. When a factor is bought on a perfectly competitive market, the marginal expenditure for it is equal to its price.

[6] Since oligopsony is excluded by our assumptions (cf. p. 35 above), the factor under consideration must be bought by other firms in a perfectly competitive market. Thus, if this factor is bought by more than one firm, there can be only "partial monopsony" with regard to it. This means that all firms but one must be atomistic buyers of the factor. At any given price set by the monopsonistic firm, the total supply of the factor as well as the demand for the factor by all other firms is determinate. The difference of the two is the supply to the monopsonistic firm at the given price. In this way the supply schedule confronting the monopsonistic firm is determinate. This firm buys the amount which equalizes the marginal expenditure with the marginal value productivity of the factor and sets the price accordingly. The price thus set is the market price confronting all atomistic buyers of the factor.

of factors fall. In monopsonistic markets the marginal expenditure for factors falls as a rule.[7] The resulting reduction in marginal costs causes an attempt to expand the output of products. This leads to a fall of product prices. In perfectly competitive product markets, prices fall because of excess supply. In monopolistic product markets, they fall because the reduction in marginal cost (at the old output) creates a situation of monopolistic overrestriction of output. Monopolistic producers find that their output is less than the output which maximizes their profit and they expand their output and reduce prices. As a rule, marginal revenue also diminishes.[8]

Thus a reduction of the price of an underemployed or of a monopsonistically underrestricted factor causes a fall of the prices of other factors and of products. The effect of the fall in prices upon the demand for or supply of the factor depends on the kind of intratemporal substitution and expansion and on the intertemporal substitution engendered. An intratemporal substitution effect and an expansion effect removing the disequilibrium in the market of the factor under consideration can take place only if the prices of (and the marginal expenditures for) the other factors fall less than in proportion to the decline of the price of this factor and if prices (and the marginal revenues) of the products fall less than in proportion to the reduction in marginal cost (at the old output). It will be shown that this depends on the monetary effect and, in absence of a monetary effect, on intertemporal substitution in the markets of the other factors and of products. In atomistic markets, intertemporal substitution depends on the effective elasticity of discounted price expectations. In nonatomistic markets, instead, it depends on the effective

[7] The marginal expenditure will fall if the marginal-expenditure schedule is positively sloped. As positively sloped supply schedules are assumed in the text, this occurs as a rule. Exceptions, however, are possible in cases where the supply schedule shows a strong curvature concave towards the axis of abscissae. Denoting the supply schedule by $f(x)$, where x is the quantity, we find that the slope of the marginal-expenditure schedule is $2f'(x)+xf''(x)$. By assumption $f'(x)>0$ and $x>0$, the slope of the marginal-expenditure schedule becomes negative when $-f''(x)>2f'(x)/x$. This implies, of course, that $f''(x)<0$. An example of such a situation is given in the adjoining diagram. Quantity is measured on the OX axis and price and marginal expenditure are measured on the OY axis. The continuous line is the supply curve and the dotted line is the marginal-expenditure curve.

[8] Marginal revenue decreases if the marginal-revenue schedule is negatively sloped. This occurs as a rule because the demand schedules are assumed in the text to be negatively sloped. Exceptions are possible when the demand schedule has a strong curvature which is convex towards the axis of abscissae. The conditions are analogous to those given in the preceding footnote, except that $f'(x)<0$ and $f''(x)>0$. An illustration is given in the adjoining diagram in which the continuous line is the demand curve and the dotted line is the marginal-revenue curve.

elasticity of expectation of discounted marginal revenues or expenditures.[9] For brevity, we shall use simply the term "effective elasticity of expectation," meaning the effective elasticity of price expectation or the effective elasticity of expectation of marginal revenue or expenditure according to the requirement of the situation.

We have seen that under a neutral monetary system, and with unit effective elasticities of expectation, the demand and supply schedules are functions of the ratios of the current prices of commodities and stocks and that a proportional change of all these prices leaves the quantities demanded and supplied unaffected. Under these circumstances, equilibrium in the markets of all other factors and of all products can be maintained only if all factor and product prices (and also the prices of stocks) fall in the same proportion as the price of the factor which is in excess supply or of which there is monopsonistic underrestriction. When such a proportional fall of all prices occurs, demand is again equal to supply in all competitive markets, while monopolists and monopsonists sell or buy exactly the same quantity as before, i.e., the equilibrium amount. The latter implies that all marginal revenues and all marginal expenditures also decrease in the same proportion.[10] But, if all prices (except bond prices) change in the same proportion, the excess supply of or the monopsonistic underrestriction of demand for the factor under consideration continues to exist. The price of this factor decreases again and all other prices follow suit proportionately. The initial disequilibrium remains unchanged. In a similar way, an increase in the price of a factor which is in excess demand or of which there is monopsonistic overrestriction of demand, causes a proportional rise of all other prices (bond prices remaining

[9] The elasticity of expectation of marginal revenue (or expenditure) is defined as the ratio of the proportional increment of the expected marginal revenue (or expenditure) and the proportional increment of the current marginal revenue (or expenditure). It may differ from the elasticity of the corresponding price expectation. Denote the price by p, marginal revenue is $p(1-1/\eta)$, where η is the elasticity of demand, and marginal expenditure is $p(1+1/\epsilon)$, ϵ being the elasticity of supply. When the elasticity of the demand or of the supply schedule is expected to change, the elasticity of price expectation differs from the elasticity of expectation of marginal revenue or marginal expenditure. An inelastic expectation of marginal revenue may imply an elastic price expectation when a fall of the current price is associated with the expectation of an increase in the elasticity of the demand schedule. And an elastic expectation of marginal expenditure may imply an inelastic price expectation when the reduction in the current factor price is associated with the expectation of an increase in the elasticity of the supply schedule.

[10] When demand or supply is a function of only the ratios of the prices, a proportional change of all prices implies a change in the same proportion of marginal revenue or marginal expenditure. Denote by p the price and by x the quantity demanded or supplied. Marginal revenue or marginal expenditure is $p+x\partial p/\partial x$. In this expression x remains unchanged if p changes in the same proportion as all other prices. The expression varies then in the same proportion as p.

constant). Thus, under a neutral monetary system and unit effective elasticities of expectation, disequilibrium in the market of a factor of production leads to a cumulative proportional change of all commodity and stock prices, while the "real" situation remains unaltered.

If effective elasticities of expectation differ from unity but the monetary system is neutral, the intratemporal substitution effect and expansion effect depend on intertemporal substitution in the markets of the other factors and of products which increases or decreases current demand (or decreases or increases current supply) and consequently raises or lowers prices (and marginal revenues or expenditures) in these markets. But, because of absence of a monetary effect, the changes in demand or supply in the different markets cancel and the intratemporal substitution effect and expansion effect is weak, as a rule. The effect of a change in the price of a factor of production upon the current demand for or supply of the factor thus depends chiefly on the elasticity of the effective expectations which influence directly the current demand for or supply of the factor. If these expectations are inelastic, current demand changes in the opposite, and current supply changes in the same, direction as the price of the factor, and equilibrium is restored accordingly. If they are elastic, the reverse happens and the disequilibrium is aggravated.

As shown in Chapter III, a positive monetary effect produces, as a rule, an intratemporal substitution effect and expansion effect which tends to restore equilibrium. This holds also when monopolies and monopsonies are present in the economy. The change in current demand for or supply of some or all goods resulting from the substitution between money and goods prevents all other prices, and consequently also the corresponding marginal revenues and expenditures, from changing in the same proportion as the price and the marginal revenue or expenditure of the factor in question. This puts the intratemporal substitution effect and expansion effect in operation and equilibrium is restored, unless adverse intertemporal substitution due to elastic effective expectations influencing the current demand for or supply of the factor outweighs the equilibrating result of the intratemporal substitution effect and expansion effect. The latter may happen when the factor is highly specialized or when the monetary effect spends itself in security markets. A negative monetary effect produces results which are the opposite of those described. It creates a negative intratemporal substitution effect and expansion effect which increases the disequilibrium, unless counteracted by inelastic effective expectations relevant to the current demand for and supply of the factor. Thus, in an economy containing monopolies and monopsonies, flexibility of factor prices operates in the same way as in an economy in which all markets are perfectly competitive.

Under oligopoly and oligopsony, determinate responses of entrepreneurs are possible only when the latter are able to form definite conjectures as to

the reactions of other firms. In the case of monopolistic and monopsonistic competition, such definite conjectures are formed on the basis of the fact that the consequences of each entrepreneur's actions are spread evenly over a large number of other firms and thus become negligible. The other firms are thus assumed not to react at all and the analysis reverts to that of monopoly and monopsony. We need not, therefore, study this case separately. Except for this case, however, determinate responses are possible only on the basis of group behavior.

The uncertainty concerning the reaction of other firms makes each firm afraid "to start the ball rolling." This leads to the establishment of a conventional price (or price structure) and of conventional patterns of behavior which become endowed with the "halo" of ethical norms. Each member of the group is allowed to take actions which do not infringe upon the "rights" of other members, but is penalized for actions which constitute such an infringement. Thus, when an oligopolistic firm raises the price of its product above the conventional level, the other firms in the group do not react, but when it lowers its price below the conventional level, the others follow suit to "keep their own" or to penalize the transgressor against the social consensus.[11] In consequence, the demand schedule confronting each firm has a kink at the level of the conventionally established price and the marginal-revenue schedule is discontinuous at the corresponding output.[12] Under oligopsony, the price paid for a factor may be lowered below the conventional level without the other firms reacting, while an increase of this price above the conventional level "spoils the market" and makes the others follow suit.[13] Thus, at the level of the conventionally established price of the factor, the supply schedule has a kink and the marginal-expenditure schedule is discontinuous at the corresponding input.

Because of the discontinuity of the marginal-revenue schedules under oligopoly, the price of the product and output do not respond to shifts of the marginal-cost curve within a certain range. The range of irresponsiveness is the range of discontinuity of the marginal-revenue curve. Similarly, under

[11] The described pattern of reaction holds when the products of the various firms are substitutes. When they are complementary, the pattern is reversed: a firm may lower the price of its product without provoking reaction of others, but a raise in its price makes the others follow suit. We disregard, in the text, the case of complementarity as not very important in practice.

[12] Cf. Paul M. Sweezy, "Demand under Conditions of Oligopoly," *Journal of Political Economy*, Vol. 47, August, 1939, pp. 568–573; and R. L. Hall and C. J. Hitch, "Price Theory and Business Behaviour," *Oxford Economic Papers*, No. 2, May, 1939, pp. 12–45. The kink is here assumed to be real, not merely imaginary, as in Dr. Sweezy's article.

[13] This is the case when the factors used by the different firms are substitutes; when they are complementary, the pattern is reversed. We disregard in the text the case of complementarity.

oligopsony, the discontinuity of the marginal-expenditure schedules causes a lack of response in the price of the factor and its input to shifts of marginal-value-productivity curve of the factor. The range of irresponsiveness is the range of discontinuity of the marginal-expenditure curve.[14]

The conventional oligopoly or oligopsony price is established either by open or tacit agreement, or by mere custom accepted on the basis of "fairness." The level of the conventional price (at which the demand schedules or the supply schedules have a kink) is determined by the "discipline" of the group, i.e., by the degree to which the individual firms are willing to act in unison as members of the group. The stronger this "discipline" the higher the price of the product (and the greater the degree of monopoly), or the lower the price of the factor (and the greater the degree of monopsony). The degree of monopoly[15] or of monopsony[16] is here not an automatic result of the elasticity of demand or of supply and of the equalization of marginal cost and marginal revenue, or of marginal-expenditure and marginal-value productivity, but depends on the markup for profit consciously chosen by the firms. This markup depends on the "discipline" of the group. An increase in the demand for the products of the firms as a rule strengthens the "discipline" of the oligopolistic group and leads to higher markups. For, when the market is expanding, firms need have little fear that they will get out of step with the rest of the group by increasing their markup and raising their prices. Each such action is likely to be followed by similar actions of other members of the group. In a similar way, an increase

[14] This is illustrated by the following two diagrams. In Fig. 1, ON is the conventional price and OM the corresponding output. The demand curve D has a kink at P and the marginal-revenue curve MR is discontinuous between G and H. Any shift of the marginal-cost curve MC within the range GH fails to affect either price or output.

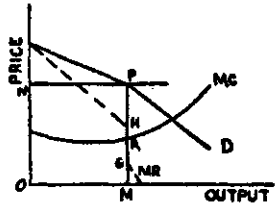

FIGURE 1

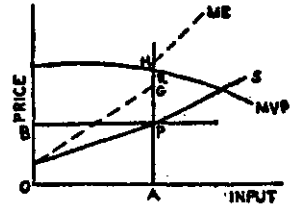

FIGURE 2

In Fig. 2, OB is the conventional price of the factor and OA is the corresponding input. The supply curve has a kink at P and the marginal-expenditure curve ME is discontinuous between G and H. Any shift of the marginal-value-productivity curve MVP between G and H leaves price and input unchanged.

[15] The degree of monopoly is defined as the ratio of the excess of price over marginal cost to the price. In Fig. 1 of footnote 14, it is the ratio RP/MP.

[16] The degree of monopsony is defined as the ratio of the excess of marginal expenditure over price to the price. In Fig. 2 of footnote 14, it is the ratio PR/AP.

in the supply of a factor strengthens the "discipline" of the group. Firms have little fear that, by paying a lower price for the factor, they will get out of step with the rest of the group which will refuse to follow suit. Thus a "sellers' market" for products tends to increase the degree of monopoly, while a "buyers' market" for factors tends to increase the degree of monopsony.

Let us now study the effects of lowering the price of an underemployed or of a monopsonistically underrestricted, factor. Oligopoly may thwart the intratemporal expansion effect and oligopsony may thwart the intratemporal substitution effect. Under oligopoly, output does not respond to shifts of the marginal-cost curve within a certain range. Thus a fall in the prices of (and in the marginal expenditure for) factors may fail to increase output, even if not followed by a proportional decrease in marginal revenue. In such a situation, the expansion effect does not operate. Further, when the monetary effect causes an increase in demand for the products using the underemployed or the monopsonistically underrestricted factor, or factors for which it is substitutable, the effect on output depends on the influence of the increase in demand upon the "discipline" of the oligopolistic group. If the strong "sellers' market" thus created causes a strengthening of this "discipline," the result may be an increase in markups and prices rather than an expansion of output.[17] In both cases, the expansion effect is replaced by a mere increase in the degree of monopoly.

The substitution effect may become inoperative under oligopsony. The essence of the substitution effect consists in the fact that the increase in demand due to a positive monetary effect raises the marginal-value-productivity curves of factors of production and this prevents the prices of (and the marginal expenditures for) these factors from falling in the same proportion as the price of the underemployed or monopsonistically underrestricted factor. This leads to a substitution in favor of the latter. But if the markets of factors for which the latter is substitutable are oligopsonistic, a shift, within a certain range of the marginal-value-productivity curves fails to affect the input of the factors and their prices. There is no substitution effect and the increase in demand resulting from the positive monetary effect is absorbed by an increase in the degree of monopsony.

In a similar way, intertemporal substitution may be thwarted by oligopoly and oligopsony because expected future demand and supply schedules have a kink, too, and planned future output and input are irresponsive (within

[17] Illustrated by Fig. 1 in footnote 14, p. 41, this means that, while the demand curve D is shifted upwards, the kink continues to be vertically above M and the marginal-cost curve continues to pass through the range of discontinuity of the marginal-revenue curve.

limits) to changes in the expected marginal-cost schedules or marginal-value-productivity schedules.

The removal of the assumption of perfect competition thus introduces the following modifications into the picture developed in the preceding chapters. (1) There is no excess demand or excess supply in monopolistic or monopsonistic markets. Disequilibrium in such markets consists in the monopolists selling, or the monopsonists buying, a quantity different from that which maximizes their profit. Monopolistic underrestriction of supply and monopsonistic overrestriction of demand perform in such markets the same function as excess demand performs under perfect competition: they make the price rise. Monopolistic overrestriction of supply and monopsonistic underrestriction of demand perform the same function as excess supply: they make the price fall. These modifications being taken into account, the results concerning the dependence of the intratemporal substitution effect and expansion effect on the monetary effect and concerning the influence of elasticities of expectation upon intertemporal substitution hold also in an economy containing monopolies and monopsonies. (2) Monopolistic competition and monopsonistic competition have exactly the same consequences as monopoly and monopsony, and their analysis is identical with that of the latter two. (3) Oligopoly may thwart the intratemporal expansion effect because the reduction in marginal cost fails to incite a response of output, or because the increased demand resulting from a positive monetary effect is entirely absorbed by an increase in the degree of monopoly through greater "discipline" of the oligopolistic group. (4) Oligopsony may thwart the intratemporal substitution effect because the shift in the marginal-value-productivity curves of factors fails to incite a response of input and is entirely absorbed by an increase in the degree of monopsony. (5) For the same reasons, oligopoly and oligopsony may thwart intertemporal substitution.

The insufficiency of a positive monetary effect to secure restoration of equilibrium, which was shown to occur under certain conditions in a perfectly competitive economy, occurs even more in an economy subjected to oligopolistic and oligopsonistic entrepreneurial responses. Oligopsony and oligopoly may prevent the operation of the intratemporal substitution effect and expansion effect, respectively, even when a positive monetary effect is present. An important way in which this happens is when the positive monetary effect leads to a change in demand for securities rather than for commodities. The direct result is then merely a change in bond prices (interest rates), or in stock prices, or in both. The effect upon output depends on the elasticity of investment with respect to interest rates or stock prices. This elasticity is zero under oligopoly (at least within a wide interval of variation) because of the lack of responsiveness of output to changes in marginal cost. The change in demand for securities is not transmitted into a change in de-

mand for investment goods. The consequence is a mere financial boom or slump, without any effect on output and employment. Thus, in an oligopolistic (and oligopsonistic) economy even more so than under perfect competition, a positive monetary effect of flexible prices of (and marginal expenditures for) factors of production may prove insufficient to secure the maintenance or restoration of equilibrium.

CHAPTER VIII

International Trade

Finally, international trade is taken into consideration. In the first stage of our argument we shall assume that international markets are all atomistic with regard to the different countries, i.e., that prices on these markets are not affected by variations in quantities exported or imported by a single country. It is also assumed provisionally that perfect competition reigns universally in the economy of the country under consideration.

Let the price of an underemployed factor fall in one country. This reduces the marginal cost (at the old output) of the products made with the aid of the underemployed factor, or of factors for which it is substitutable (and which also fall in price). If some of these products are export goods, their prices are determined in international markets and, consequently, remain unchanged. Their marginal cost (at the old output), however, being reduced, an expansion of output of these export goods takes place which increases the demand for the underemployed factor. If some of the factors of production for which the underemployed factor is substitutable are imported from abroad, their prices also do not change because they are determined in international markets. The underemployed factor becomes relatively cheaper in comparison with the imported factors and a substitution of the former for the latter takes place. This increases the demand for the underemployed factor.

As the prices of export goods and of imported factors of production are determined in international markets, they are not affected by changes in domestic demand and, therefore, are also not affected by changes in the real excess demand for cash balances. Thus the expansion of output of exports produced with the underemployed factor, or with substitutable factors, as well as the substitution of the underemployed factor for imported factors operate *independently* of the monetary effect. If the underemployed factor is used exclusively to produce export goods, or if the factors substitutable for it are all imported from abroad, a reduction of its price is, therefore, bound to increase its employment, and if the price is reduced sufficiently, all excess supply of the factor is bound to disappear. Under the same conditions, a rise in price of a "bottleneck" factor is bound to reduce the excess demand for it. In this case, the partial-equilibrium analysis, in terms of the downward sloping demand curve, is applicable and the result is quite independent of the monetary effect.

The situation is more complicated in the case where the underemployed factor (or the "bottleneck" factor) is used to produce both export goods and goods for domestic sale, or in case that not all substitutable factors are imported from abroad. The net expansion effect and the net substitution effect

depend, then, also on what happens to domestic product and factor prices, which, in turn, depends on the nature of the monetary effect.

Let us first study the situation under conditions of a neutral monetary system and of unit effective elasticities of expectations. Under these conditions, the monetary effect is absent. There is no real substitution between goods and money, and the demand schedules and supply schedules of commodities and stocks are functions of only the ratios of prices, i.e., a proportional change of all prices (except interest rates) leaves unaffected the quantities demanded and supplied. The fall in price of the underemployed factor creates a tendency to substitute it for other factors and to expand the output of products produced with it, or with substitutable factors. This causes excess supply of the substitutable factors and also excess supply of the products just mentioned.[1] The result is a fall of the corresponding prices. All other prices fall too, for otherwise there would be excess supply in other markets, because of a shift in demand towards the goods which become relatively cheaper. Equilibrium in the domestic markets requires that all prices fall in the same proportion as the price of the underemployed factor.[2] The general fall in prices, however, is confined to domestic prices because the prices of export goods and of imported factors of production are determined in the international markets. In consequence, there is an expansion of the output of the export goods produced with the underemployed factor, or with factors substitutable for it, and also a substitution of the underemployed factor for substitutable imported factors.

But as all domestic prices fall, nonexport goods become cheaper relatively to export goods. If part of the export goods is bought in the domestic market, a shift in demand from export goods to nonexport goods occurs. This shift acts like a parachute and prevents the prices of (at least) some nonexport goods from falling in proportion to the reduction in marginal cost (at the old output) which results from the fall of prices of domestic factors of production. An expansion of the output of at least some nonexport goods is the consequence. If the nonexport goods, the output of which increases, are produced with the aid of the underemployed factor, there is also an increase in demand for them on that account. Thus, under conditions of a neutral monetary system, there may occur an increase in demand for the underemployed factor in the production of nonexport goods, while no decrease in the demand for the factor coming from this source is possible.

A similar effect takes place when products with domestic substitutes are imported into the country. If the domestic substitutes for the imported products are produced with the aid of the underemployed factor, or with domestic factors substitutable for it, a reduction of the price of the underemployed

[1] Cf. p. 5 above.
[2] Cf. p. 10 above.

factor leads to a replacement of imported products by domestic ones. Consequently, the demand for the underemployed factor increases. If the underemployed factor is not substitutable for the imported factors, a substitution effect in favor of the former takes place, notwithstanding. For, the fall of all domestic prices means that the marginal cost of goods produced with domestic factors decreases relatively to the marginal cost of the goods made with imported factors. This leads to a shift in demand from the latter to the former category of goods. In consequence, the price of all other domestic factors falls less than in proportion to the fall in price of the underemployed factor.

Thus, under conditions of a neutral monetary system, and of unit effective elasticities of expectation, the net intratemporal expansion effect and the net intratemporal substitution effect of a reduction of the price of the underemployed factor are always positive, even if this factor is also used in the production of goods for domestic sale, or if the imported factors are not all substitutable for it. In pure theory this is even the case when the underemployed factor is exclusively used in making products for domestic markets, or when none of the imported factors are substitutable for it. But, in the last-mentioned case, these effects may be rather negligible, particularly when some friction is present. The same result applies *a fortiori* when the monetary effect of the reduction of the price of the underemployed factor is positive. In this case, the intratemporal substitution effect and expansion effect are, as a rule, also stronger than in absence of international trade, because the fact that the prices that are determined in international markets do not change produces a parachute effect *in addition* to the substitution of goods for money. Only when the monetary effect is negative may the net intratemporal expansion effect and the net intratemporal substitution effect fail to be positive, and this only if a substantial part of the underemployed factor is engaged in production for domestic markets or if a substantial part of the factors imported from abroad is not substitutable for the underemployed factor. The nature of the monetary effect depends on the responsiveness of the monetary system and on the elasticities of expectation in the way indicated in the *General Rule*.

Intertemporal substitution depends, as in our preceding analysis, on the effective elasticity of expectation of the prices which affect the current demand for or supply of the underemployed factor. If these expectations are elastic, intertemporal substitution tends to diminish the demand or to increase the supply of the factor. But since an intratemporal substitution effect and expansion effect are much more likely to exist and are also stronger than in absence of international trade, the adverse intertemporal substitution resulting from elastic price expectations is also less likely to prevail over them.

International trade in markets which are atomistic with regard to the different countries thus acts as a stabilizing factor for the economy of a country

with flexible factor prices, provided that this economy is not subject to too strong negative monetary effects and too elastic effective price expectations. This stabilizing action of international trade is the greater, the greater the proportion of the underemployed factor (or of the "bottleneck" factor) engaged in the production of export goods, and the greater the number and importance of factors imported from abroad which are substitutable for the factor under consideration. The more likely is, then, the flexibility of factor prices to maintain automatically full employment of factors and to prevent serious "bottlenecks."

This result holds also when domestic markets operate under conditions of imperfect competition. Let part of the export products be sold in domestic markets under conditions of monopoly, monopolistic competition, or oligopoly. This implies price discrimination (dumping) and is possible only if the home markets are protected. The output of these products is then determined entirely by the competitive prices established in the respective international markets, i.e., it is such that the marginal cost equals the price in the international market. The prices in the home markets and the division of sales between home markets and international markets is such that the marginal revenue derived from the domestic market is equal to the price in the international market.[1] It is seen immediately that the output of these products increases when their marginal cost (at the old output) is reduced because of the fall of the price of the underemployed factor. A reduction of the price of the underemployed factor (or of the factor of which there is monopsonistic underrestriction of demand) also causes an intratemporal substitution of this factor for factors imported from abroad (provided it is substitutable for them).

The results obtained apply also when international markets are not atomistic with regard to single countries, and even if they are subject to imperfect

[1] This is illustrated in Fig. 1 for monopoly and monopolistic competition and in Fig. 2 for oligopoly based on group behavior. The price in the international market is ON, D is the domestic-demand curve, MR is the corresponding marginal-revenue curve, and MC is the marginal-cost curve. The total output is OM, of which OR is sold in the home market at a price equal to RP. A downward shift of the marginal-cost curve causes an increase of the total output but does not affect the domestic sales and price. The increase in output is exported.

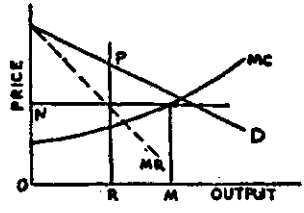

FIGURE 1

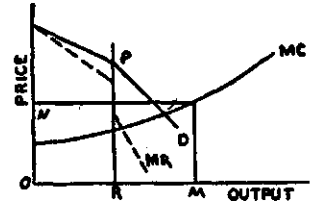

FIGURE 2

competition, provided that the demand and supply curves, and thus also the marginal-revenue and marginal-expenditure curves, in these markets are not affected by a change in the price of the underemployed (or of the "bottleneck") factor or by the resulting changes in domestic prices. The last-mentioned proviso presupposes the complete independence of the monetary effects in different countries. Such an independence, however, is not likely to persist when the international markets cease to be atomistic with regard to single countries. We must, therefore, take into account the international interrelation of monetary effects.

The increase in exports, or the diminution of imports, which results from the reduction of the price of the underemployed (or monopsonistically overutilized) factor, produces an increased inflow (or a decreased outflow) of money into (or from) the country. Other things being equal, this tends to reduce the real quantity of money in the other countries. As a rule,[4] this causes, in the other countries, a lowering of the demand schedules for exports and also (if the factor prices in the other countries are flexible) a lowering of the supply schedules of imports.[5] Equilibrium is reached again when the net flow of money between the countries comes to a stop. The output of export goods in the country which has reduced the price of the underemployed factor will be greater or less than before, according as, in the equilibrium position of the demand schedules[6] for the export goods, the demand price or marginal revenue corresponding to the old output has fallen less or more than the reduction of marginal cost resulting from the fall of the price of the underemployed factor. Imports of factors of production will in the new equilibrium situation be smaller or greater than before, according as, with the new position of the supply schedules, the supply price of (or the marginal expenditure for) the imported factors is, at the old level of imports, reduced less or more than in proportion to the fall of the prices of (or marginal expenditures for) substitutable domestic factors. In a similar way, imports of

[4] Exception has to be made for "inferior" goods, i.e., for goods, the quantity demanded of which increases when the real amount of available cash balances decreases. It should be noticed that this concept of "inferior" goods, though similar, is not identical with that used in the theory of consumers' choice. In this theory the demand is referred to income as one of the independent variables, while here it is referred to cash balances.

[5] When exchange rates are fixed, these changes in the position of the demand or supply schedules take place through a change in the nominal demand or supply prices with which the country under consideration is confronted. In the case of flexible exchange rates, the nominal demand or supply prices quoted in some foreign currency may remain unaltered, but the demand and supply prices in terms of the country's own currency change because of a change of foreign exchange rates.

[6] We speak of an equilibrium position of the demand (and also of the supply) schedules because these schedules depend on the amount of available real cash balances as an additional independent variable. The schedules change their position when the amount of real cash balances in the other countries changes.

products will be smaller or greater than before, according as the price of (or the marginal expenditure for) the imported products (at the old level of imports) has decreased less or more than proportionately to the fall in price of (or marginal expenditure for) domestic substitutes of these products.

If in the new equilibrium exports are greater and imports are smaller than they were before the reduction of the price of the underemployed factor, the real quantity of money in the country is greater than would have been the case were the country not involved in international trade. According to the *General Rule*, this always reinforces a positive monetary effect, weakens a negative monetary effect, or turns an absent monetary effect into a positive one. For if the effective elasticities of (discounted) price expectations[7] are prevailingly less than unity, a proportional fall of all factor and product prices leads to a diminution of the real demand for cash balances, while the real quantity of money in the country increases. If, instead, the effective elasticities of (discounted) price expectations are prevailingly greater than unity, the real demand for cash balances increases, but the real quantity of money is greater than it would have been in the absence of international trade. Finally, if elasticities of expectation are unity, the real demand for cash balances is constant, but the real quantity of money increases. Thus, the influence of international trade upon the economy of a country with flexible factor prices acts here in a stabilizing direction. This influence, however, is reversed, i.e., it is destabilizing, when exports are smaller and imports are greater in the new equilibrium position than before. The influence may be either way, according to the net effect upon the real quantity of money in the new equilibrium, when exports are greater and imports are also greater, or exports are smaller and imports are smaller, too. A similar analysis can be applied to study the consequence of a rise of the price of a "bottleneck" factor.

Thus, when the international interrelation of monetary effects is taken into account, the influence of international trade upon an ecomomy with flexible factor prices may be destabilizing as well as stabilizing. The result depends on the net effect upon the real quantity of money in the country under consideration. This analysis, however, is based on the assumption that the real quantity of money in the different countries adjusts itself "automatically" to effects of international trade. If the other countries follow autonomous monetary policies, they may change their real quantity of money in a way different from the one assumed here. In this case, our analysis must be qualified to the framework of such autonomous monetary policies.

[7] If imperfect competition is present, the effective elasticity of expectation of (discounted) marginal revenues or marginal expenditures has to be taken.

CHAPTER IX

Changes in the Propensity to Consume

Our analysis will now be applied to the study of two special problems which have been in the center of economic controversy for more than a hundred and twenty years. These are the questions of oversaving and of the limitation of investment opportunities. The "orthodox" theory denies that oversaving and limitation of investment opportunities can take place in an economy with flexible prices of factors of production and flexible interest rates. Such an economy is considered to be able to absorb any amount of saving decided upon by income-receivers, and investment opportunities are considered as being unlimited (at least up to the point where all products would become free goods). Insofar as oversaving and limitation of investment opportunities actually do occur, they are explained as being due entirely to rigidity of factor prices, and, according to some, but not all,[1] proponents of this view, also to rigidity of interest rates. This view is, indeed, but a consequence of the more fundamental proposition that underemployment of factors of production is not possible (except as a frictional phenomenon) when factor prices are flexible.

The fallacy of this fundamental proposition, when maintained with full generality, has been established by the preceding analysis. In order that flexibility of factor prices assure automatic full employment of factors of production, a number of very specialized conditions must be satisfied. These conditions refer to the relation between the effective elasticities of expectations and the responsiveness of the monetary system as stated in the *General Rule*, to the type of goods to which the increase in demand implied in the substitution of goods for money is directed, to the sensitivity of investment to changes in interest rates, to the effective elasticities of expectation relevant to the demand for or supply of specialized factors of production, to the nature of entrepreneurial responses as determined by the degree and type of imperfection of competition, and, finally, to the conditions of international trade. Unless all these conditions are satisfied, flexibility of factor prices fails to assure full employment of factors of production as well as to prevent the existence of "bottleneck" factors. It may even become a source of economic instability, i.e., the fall of the price of an underemployed (or monopsonistically underrestricted) factor may lead to a cumulative decrease in the employment of the factor, while a rise of the price of a "bottleneck" factor may

[1] That rigidity of interest rates have here a causative role would be denied by Professor F. H. Knight. His point is based on a denial of the possibility (at least in the long run) of rigid interest rates, because interest rates are considered to adapt themselves to any changes in the rate of return on investment. Cf. Knight, "The Business Cycle, Interest and Money," *Review of Economic Statistics*, Vol. 23, May, 1941, pp. 53–67, esp. pp. 63–65.

result in a cumulative increase in the excess demand for (or monopsonistic overrestriction of) it. We shall see how the principles established by our analysis operate in the case of the two special problems under consideration.

To start with, let us assume an economy where all factor markets are in equilibrium. We classify the commodities (this term excludes securities; cf. p. 15 above) in our economy in four groups: commodities which are only factors of production[2] (i.e., which are sold by households to firms[3]), commodities which are only products[4] (i.e., which are sold by firms to households[5]), commodities which are both products and factors (i.e., which are sold by firms to firms), and commodities which are neither factors nor products (i.e., are sold by households to households). They will be called primary factors,[6] final products, intermediate products (or investment goods), and direct services, respectively. Under modern capitalism (i.e., in an economy where all production is carried on in firms[7]), labor is the only primary factor of production in existence[8] and direct services play a rather subordinate role.

[2] For a definition of factors of production, see footnote 1, on p. 3 above.

[3] By firms we mean units of economic decision operated to obtain money profit.

[4] For a definition of products see footnote 1 on p. 3 above.

[5] By households we mean units of economic decision operated to procure "utility."

[6] The concept of primary factors should not be confused with that of "original" factors used in many (e.g., the Boehm-Bawerkian) treatments of the theory of capital. "Original" factors are distinguished from "capital goods" by the fixity of their supply, whereas the criterion of classification of primary factors is that they are supplied by households, not by firms. Intermediate products are necessarily "capital goods" in the sense of the mentioned theories of capital, but the reverse is not necessarily true, for "capital goods" may be produced and sold by households, e.g., by farms operated to maximize the farmer's (and his family's) utility, not his money profit.

[7] This definition of capitalism is in terms of the purpose for which production is carried on (i.e., for money profit). This is the definition common among writers like Sombart (*Der moderne Kapitalismus*, Vol. I, München, 5th ed., 1922, p. 319), Max Weber (*General Economic History*, trans. F. H. Knight, New York, 1927, pp. 275 ff.), and Brentano (*Der Wirtschaftende Mensch in der Geschichte*, Leipzig, 1923, pp. 211 ff., 253 ff.). This definition can be also found in Marx (cf. *Capital*, Chicago, Charles H. Kerr & Co., 1909, Vol. III, pp. 303–305). It does not exclude the slave capitalism of antiquity and the distinguishing adjective "modern" is added, therefore. It is shown easily that this definition requires separation of labor and the ownership of other productive resources, for an owner of productive resources operating them with his personal labor will not exploit a chance of increasing his money profit if the marginal utility of money income is less than the marginal disutility of the work associated with the exploitation of the chance, i.e., he will behave like a household (maximizing "utility") and not like a firm. When slavery is ruled out, this becomes equivalent with Marx's other definition in terms of the employment of wage labor to operate productive resources owned by other persons (*vide Capital*, Vol. I, pp. 187–189).

[8] This is due purely to institutional reasons. Under slavery, laborers may be bred for profit, and thus be investment goods. Cf. on this subject the illuminating remarks of Professor F. H. Knight, "The Quantity of Capital and the Rate of Interest," *Journal of Political Economy*, Vol. 44, August, 1936, p. 438.

Suppose that, all other things being as they are, the households in the economy decide to diminish their current purchases of final products and direct services. We shall say in this case that the propensity to consume of the community decreases.[9] This means that, at the old output, the demand price for (at least) some final products[10] is reduced, while the demand schedules for all other final products (and direct services) remain unchanged.[11] As long as the prices of the factors of production and of all other goods (including securities) are unchanged, this results, under conditions of perfect competition and as a rule also under monopoly (including monopolistic competition),[12] in a contraction of output of the products for which there has been a decrease in demand. In consequence, the demand for the factors of production used in making these products also decreases.[13] This causes excess supply of or creates a situation of monopsonistic underrestriction of the demand for these factors and a fall of their prices (the prices being assumed to be flexible). Substitution of these factors for other factors and expansion of output of commodities produced with them is attempted and causes a decline of the prices of the other factors. At the same time, marginal costs are reduced on account of lower factor prices. This causes an attempt to expand the output of (at least some) products. The final consequences depend on the nature of the monetary effect and on the elasticities of expectation.

Consider first a neutral monetary system, and suppose that all effective expectations are of unit elasticity. In this case, the monetary effect is nil, irrespectively of the numerical value of the effective elasticities of price ex-

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Insofar, however, as noncapitalist producers are also present in our economy, factors of production other than labor may be primary factors, as, for instance, in the case of products sold to firms by subsistence farmers who operate as households rather than firms (i.e., procure "utility" rather than maximize money profit).

[9] The term "propensity to consume" designates the functional relationship between the current expenditure for final products and direct services on one side and current income and a number of other variables on the other side. A decrease of the propensity to consume means a change of the shape of this function.

[10] It is possible, in theory, that a decrease in the propensity to consume is directed exclusively toward a diminution of the demand for direct services without affecting the demand for final products. This case, however, is practically most unlikely (at least under modern capitalism) and theoretically not very interesting. For these reasons it is disregarded in the text.

[11] A decrease in the propensity to consume may be associated with an upward shift of the demand schedules for some special final products or direct services which are used to compensate for the diminished consumption of the others, provided the real value of the aggregate money expenditure for final products and direct services decreases. We make, however, in the text, the more restrictive assumption according to which this possibility is ruled out in order both to simplify the exposition and to avoid the difficult problem of "real" aggregates.

[12] An exception may occur under monopoly if the decrease in demand is associated with a sufficient increase in the elasticity of the demand schedule.

[13] This is the "principle of derived demand."

pectations. Since intertemporal substitution is excluded, the demand for and supply of each good depend only on the ratios of prices (interest rates being constant).[14] There is excess supply or monopsonistic underrestriction of the demand for the factors for which the factors affected directly by the decrease of the propensity to consume have been substituted, and there is also excess supply or monopolistic overrestriction of output of the products the marginal cost of which has been reduced. The prices (and marginal expenditures and marginal revenues) of these factors and products fall until equilibrium is reached again in the respective markets. But equilibrium requires that the same ratios of prices obtain as before. Thus the prices of the factors and products mentioned fall in the same proportion as the prices of the factors which were affected directly by the change in the propensity to consume.[15] Also all other commodity prices in the economy (and the prices of stocks) fall in the same proportion, for, otherwise, excess supply (or monopsonistic underrestriction and monopolistic overrestriction) would develop in other markets. Thus a proportional fall of all commodity (and stock) prices[16] takes place. All this, however, fails to remove the underemployment of (or monopsonistic underrestriction of the demand for) the factors used to produce the final products for which the demand has been reduced because of the decrease in the propensity to consume. The factors mentioned remain in excess supply (or monopsonistic underrestriction) and their prices fall further. This causes again a proportional fall of all prices, and so on. Thus, with flexible factor prices and with a neutral monetary system, any decrease of the propensity to consume leads to underemployment or monopsonistic underrestriction of the demand for (at least) some factors of production and to a cumulative fall of all prices except interest rates. In a similar way, an increase in the propensity to consume leads to excess demand or monopsonistic overrestriction of the demand for (some or all) factors and to a cumulative rise of all prices. Price flexibility utterly fails to remedy the disequilibrium.

As all commodity and share prices (and all marginal revenues and marginal expenditures) change in the same proportion, the output of all commodities remains the same. Thus, exactly the same amount of each final product and of each investment good is produced as before the change in the propensity to consume. But the demand schedule for (at least) some in-

[14] *Vide* p. 10 and p. 38 above.

[15] The prices of the factors directly made underemployed by the reduction in the propensity to consume all fall in the same proportion. For, should one of them fall in greater proportion than the others, this would cause an attempt to substitute the first factor for other factors and the increased excess supply of the latter would lead to a greater fall in their prices. This argument is based on the assumption that the fall in price is the greater the greater the excess supply, which is implied in the theory of stability of economic equilibrium. Cf. formula (2.6) in the Appendix.

[16] Bond prices remain constant and interest rates thus remain unchanged.

vestment goods, as well as the demand schedule for some final products, is lower or higher, according as the propensity to consume has decreased or increased. Therefore, a decrease in the propensity to consume creates permanent excess supply (or monopolistic underrestriction of output) of (some or all) investment goods, while an increase in this propensity causes permanent excess demand for them. One leads to "oversaving," accompanied by excess supply of investment goods; the other leads to "undersaving," accompanied by excess demand for investment goods. This result is easily explained. Under the conditions assumed, the output of investment goods is fixed. Thus, any change in the demand for investment goods resulting from a change in the propensity to consume causes excess supply or excess demand.

At first sight, it seems, rather strangely, that the disruption of equilibrium is associated with the change of the propensity to consume rather than with its level. This, however, is only apparent. The output of investment goods being fixed, full employment and equilibrium require such a propensity to consume as will provide, for each investment good, a demand just equal to its output.[17] We may call this the *equilibrium propensity to consume*. Any deviation from the equilibrium propensity to consume[18] in either direction creates excess demand or excess supply and results in a cumulative rise or fall of all commodity prices.[19]

If we drop now the assumption of unit elasticities of effective expectations, but retain the assumption of a neutral monetary system, we find that intratemporal substitution and expansion in either direction may take place. However, for reasons indicated in a preceding chapter, the intratemporal substitution effect and expansion effect are not likely to be significant, and the consequences of a change in the propensity to consume depend on the

[17] The concept of "equilibrium consumption" corresponding to any given rate of investment is used by Dr. A. P. Lerner. See his article, "Some Swedish Stepping-Stones in the Theory of Employment," *The Canadian Journal of Economics and Political Science*, Vol. 6, November, 1940, pp. 574–591.

[18] Not only a change in the level (i.e., the total amount of expenditure) of the propensity to consume, but also a change in its composition (i.e., the direction of expenditure) causes disequilibrium. The equilibrium propensity to consume, therefore, implies not only a definite level but also a definite direction of expenditure.

[19] Our result may also be expressed in terms of the Wicksellian theory of the cumulative fall or rise of commodity prices. The output of investment goods being fixed, a deviation from the equilibrium propensity to consume causes *ex ante* saving to exceed, or to fall short of, investment and thus produces a cumulative fall or a cumulative rise of all prices. This can also be put in terms of interest rates. Bond prices, i.e., "money rates," remain constant, but the excess supply of investment goods, resulting from a propensity to consume short of equilibrium requirement, lowers the "natural rate." The consequence is a downward Wicksellian process. In a similar way, a propensity to consume in excess of equilibrium requirement raises the "natural rate" and an upward Wicksellian process takes place.

elasticity of the effective expectations which influence directly current demand for or supply of the factors affected by this change. If these expectations are inelastic, intertemporal substitution diminishes the excess supply of underemployed factors and increases the excess demand for "bottleneck" factors. Price flexibility thus acts to restore equilibrium. If, instead, these expectations are elastic, underemployment or excess demand becomes worse. In this case, price flexibility leads to a cumulative intensification of the disequilibrium caused by a change in the propensity to consume.

CHAPTER X

Changes in the Propensity to Consume (Continued)

Dropping the assumption of a neutral monetary system, we have to consider the effective elasticities of expectations and the elasticity of the monetary system. According to the *General Rule*, a positive monetary effect is produced under an unresponsive monetary system when, with a proportional fall of all current commodity and stock prices, the effective elasticities of (discounted) expectations are prevailingly less than unity, and under a responsive monetary system when they are prevailingly equal to or greater than unity. A negative monetary effect takes place when conditions are reversed.

Consider first the case where the monetary effect is positive. In this case, a proportional fall of all current prices, except interest rates, leads to a diminution of the real excess demand for cash balances. This implies a substitution of goods for money, which acts like a parachute, preventing the prices of other factors and of products (except those for which the demand schedule has been lowered, in consequence of the change in the propensity to consume) from falling all in the same proportion as the prices of the factors of production which became underemployed because of a decrease in the propensity to consume. This parachute action can take place in two ways, or in a combination of the two.

First, the increased demand resulting from the diminution of the real excess demand for cash balances may be directed toward commodities. In view of the fact that the decrease in the propensity to consume diminishes the demand for (at least) some final products and direct services, and that, by assumption, the demand for none of this category of commodities is increased,[1] the increased demand must be directed toward investment goods and primary factors of production. But, as the demand for final products is decreased or unchanged, any increase in the demand for primary factors of production must be for use in the production of investment goods; it is thus

[1] Cf. p. 53 above. This statement, however, holds strictly only in the case that the amount of real excess cash balances available does not enter as an independent variable in the function expressing the propensity to consume. If it does, the demand for the final products and direct services directly affected by the change in the propensity to consume may fall off less than it would otherwise, while the demand for the other final products and direct services actually increases. If the "excess-cash-balance elasticity" of demand for the first-mentioned category of final products and direct services is great enough, it may counterbalance the effect of the diminished propensity to consume. In consequence, the same consumption of final products and direct services which existed before the change in the propensity to consume may be maintained. This corresponds to Professor D. H. Robertson's "abortive saving" (cf. *Money*, New York and Chicago, 1929, pp. 102–103; and *Banking Policy and the Price Level*, London, 1926, pp. 45–47). These complications are disregarded in the text in order to simplify the argument and also because they do not seem to be very important in practice.

but an aspect of the increased demand for investment goods. The prices of (at least some) investment goods, and of the primary factors engaged in their production, fall less than in proportion to the prices of the factors made underemployed by the decrease in the propensity to consume. An intratemporal substitution effect and an expansion effect take place. This goes on as long as the prices of the underemployed factors continue, i.e., as long as there is excess supply of these factors.[2]

If the elasticity of expectation of the effective prices which influence current demand or supply of the factors affected by the decrease in the propensity to consume is unity, the intratemporal substitution effect, and the expansion effect described, lead by their own action to the establishment of a new equilibrium with full employment of all factors of production. If this elasticity differs from unity, intertemporal substitution comes into play. If the expectations mentioned are inelastic, intertemporal substitution increases further current demand for (or decreases further current supply of) the underemployed factors. Full employment is restored with a smaller fall of the prices of these factors than would have been necessary in the absence of intertemporal substitution. But, if the expectations relevant to current demand for or supply of the underemployed factors are elastic, the demand for these factors decreases (and supply increases) because of intertemporal substitution. This counteracts the intratemporal substitution effect and expansion effect. The final result may be either way. If the intertemporal substitution effect is the weaker, full employment is, finally, restored, though the restoration requires a greater fall of the prices of the underemployed factors than would have been necessary otherwise. If, instead, the intertemporal substitution effect proves to be stronger than the intratemporal substitution effect and expansion effect, the excess supply of the factors affected by the decrease in the propensity to consume increases cumulatively.

Second, the increased demand due to the diminution of the real excess cash balances may be directed towards securities rather than towards commodities.[3] Let it be directed toward bonds (measured in real units).[4] Then

[2] Thus, when factor prices are all flexible, the increase in demand for investment goods, resulting from the diminution of the real excess demand for cash balances, is always bound to be greater than the decrease in demand for investment goods and primary factors caused by the decrease in demand for (some or all) final products, i.e., owing to the "principle of derived demand." For, otherwise, excess supply of factors would continue to exist and the prices of these factors would continue to fall. This would diminish further the real excess demand for cash balances and, consequently, the demand for investment goods and primary factors would increase. Finally, there must take place a net increase in the demand for investment goods sufficient to absorb all excess supply of factors of production. The monetary effect thus overbalances the "principle of derived demand."

[3] As to the conditions under which this is likely to happen, cf. p. 18 above.

[4] Cf. p. 16 above.

bond prices rise and there is a fall in interest rates which is the greater, the greater the elasticity of expectation of (discounted) bond prices.[5] The fall in interest rates increases the discounted values of expected commodity prices and thus leads to an intertemporal substitution of purchases toward the present and of sales toward the future. This causes a universal increase of current demand and decrease of current supply of commodities. Excess demand appears and prices rise in markets which were in equilibrium, while previously existing excess supply is diminished (or possibly even made to disappear) and a brake is put on the fall of the corresponding prices. The rise of the prices of the factors which are fully employed causes an intratemporal substitution effect. The rise in price of products which are not in excess supply creates an expansion effect, either directly if they are produced with some of the factors set free by the decrease in the propensity to consume, or indirectly by raising the price of the respective products relatively to the price of products produced with the underemployed factors and causing a substitution of the latter products for the first.

As a rule, the intertemporal substitution is greater in the markets for investment goods than in the markets for final products and direct services. Insofar as the intertemporal substitution indicated is operative in the latter markets, the fall of interest rates may partly reverse the reduction in demand for final products and direct services created by the decrease in the propensity to consume.[6] Such reversal, however, is usually slight.[7] Actual increase in demand is limited to investment goods.[8] Thus, the effect of the fall of interest rates is to increase the current demand for investment goods, and the result is the same as when the diminution of the excess demand for cash balances increases this demand directly.[9] All current prices cannot fall in the

[5] Cf. p. 28 above.

[6] This is equivalent to the proposition of the traditional theory of saving that a fall in interest rates diminishes the "willingness to save." There are, however, exceptions to this rule. If the household plans to be a lender rather than a borrower, the fall in interest rates diminishes the capital value of its assets. The household becomes "poorer" and tends to diminish the current demand for final products and direct services. This diminution may balance, or even overbalance, the intertemporal substitution resulting from an increase in the discounted values of expected prices. In this case, the reversal described in the text will fail to take place and there may be even a further reduction in the demand for final products and direct services. For a precise analysis of the relation between interest rates and "willingness to save," *vide* Hicks, *op. cit.*, pp. 232–35.

[7] Observation shows that the "willingness to save" is but little influenced by changes in interest rates, whatever the direction of this influence.

[8] There may also be an increase in the demand for those final products and direct services for which demand has not decreased in consequence of the change in the propensity to consume. Such increase does not alter our results and is disregarded in the text in order to simplify the exposition.

[9] In this case, too, the monetary effect overbalances the "principle of derived demand," i.e., the increase in the demand for investment goods resulting from the decline

same proportion as the prices of the factors made underemployed by the decrease in the propensity to consume, and the intratemporal substitution effect and expansion effect become operative. This tends to restore full employment, unless prevented by adverse intertemporal substitution resulting from elastic expectations of the effective prices which directly affect current demand for or supply of the underemployed factors.

The same result is obtained when the increased demand resulting from the diminution of the excess demand for cash balances is directed to stocks. An increase in the demand for stocks that comes about because of a diminution of the real excess demand for cash balances, rather than as a consequence of improved expectations of dividends, makes the prices of stocks rise relatively to the earnings expected. This acts exactly like a fall of interest rates. It also leads usually to such a fall because the diminution of the percentage rate of expected earnings implied in a rise of stock prices causes a substitution, under the circumstances described, of (real) bonds for stocks and thus a rise of bond prices.

It should be noticed, however, that the second way described is much less certain to produce the intratemporal substitution effect and expansion effect than the first way. This is so because the intertemporal substitution, resulting from the fall in interest rates, may be very small and practically even negligible. The effect of intertemporal substitution upon current demand and current supply depends on the length of the period over which intertemporal substitutions take place. This is the period over which individuals plan their purchases or sales, i.e., their economic horizon. But the fall in factor prices may increase the uncertainty of price expectations. Such an increase in uncertainty shortens the economic horizons of firms (and also of households).[10] This diminishes the effect of intertemporal substitution upon current demand and supply, not only because the number of intertemporal

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in interest rates is greater than the decrease in demand due to the decreased demand for final products. It must be remembered, however, that this presupposes flexibility of all factor prices. If the prices of some factors (e.g., of primary factors) are rigid, the intratemporal expansion effect associated with any given monetary effect is weaker, because the reduction in marginal cost is less. The failure of marginal cost to fall is the greater (in comparison with a situation where all factor prices are flexible), the greater the decrease in the demand for final products. Thus it may happen that, as the propensity to consume decreases continuously, a point is reached where the decline of interest rates ceases to overcompensate the "principle of derived demand" and the latter starts to overbalance the first. The corresponding propensity to consume may be called the *optimum* propensity to consume. The exact conditions for it are worked out in the author's article, "The Rate of Interest and the Optimum Propensity to Consume," *Economica*, N.S., Vol. 5, 1938. The optimum propensity to consume maximizes the demand for investment goods, but it is not to be necessarily an optimum from the point of view of social policy.

[10] Cf. p. 34 above.

substitutions becomes less, but also because the intertemporal substitutions that fall out are the most important ones.

A change in interest rates affects the discounted values of the expected prices according to their remoteness from the present. The farther ahead in the future the expected price is, the more heavily it is discounted and, accordingly, the greater is the change of its discounted value resulting from any given change in the rate of interest. Thus, the discounted values of prices expected in the near future are but slightly affected by a change in interest rates, and the intertemporal substitution caused by such a change is very small. The important intertemporal substitutions, resulting from changes in interest rates, are those between current purchases and sales and purchases and sales planned in the remote future. As the economic horizon is shortened on account of the increase in uncertainty, these more remote intertemporal substitutions fall out and there remain only the intertemporal substitutions between present purchases and sales and purchases and sales planned in the nearer future. Consequently, the effect of a change in interest rates upon the current demand for investment goods may be very small and (particularly when some friction is present) practically negligible.[11] This explains the well-known insensitiveness of "investment activity" to changes in interest rates. The result is that a fall in the prices of the factors made underemployed by the decrease in the propensity to consume may fail to increase the current demand for investment goods, even when the monetary effect is positive.[12]

There is a further reason that may prevent the increase in the real demand for bonds from affecting the current demand for investment goods in any appreciable way, even if the effect of uncertainty just described does not operate. This is the fact that the increase in demand resulting from a diminution of the real excess demand for cash balances may be directed toward

[11] In the words of Professor Hicks: "Interest is too weak for it to have much influence on the near future; risk is too strong to enable interest to have much influence on the far future." *Op. cit.*, p. 226.

[12] In such a case, a decrease in the propensity to consume leads to a decrease in the demand for investment goods, for the decrease in the demand for these goods resulting from the decrease in demand for (some or all) final products (i.e., due to the "principle of derived demand") is not compensated by an increase in demand caused by the monetary effect. In a similar way, an increase in the propensity to consume results in increased demand for investment goods. The latter effect, however, is likely to happen only up to a certain point after which it is reversed. Under the circumstances described in the text, a continuing increase in the propensity to consume causes a greater and greater diminution of the real excess demand for cash balances and thus goes on raising interest rates. Finally, interest rates will become so high that intertemporal substitution between present purchases and sales and purchases and sales planned in the near future is brought about. In other words, the rise in interest rates will be so great that it will effectively curb the current demand for investment goods, however small the elasticity of this demand with respect to interest rates, short of zero.

short-term bonds (and bills) rather than toward long-term bonds. The main effect of a change in interest rates upon the current demand for investment goods is, as we have seen, through intertemporal substitutions between current purchases and sales and purchases and sales planned in the more distant future. In order to induce such intertemporal substitutions, it is necessary that the corresponding long-term rates of interest, i.e., the rates at which the respective expected prices are discounted, decline. Thus a change in the real demand for short-term bonds can have an effect upon the current demand for investment goods only if the resulting change of short-term interest rates is transmitted into a change of long-term rates. Whether or not such a transmittal takes place depends on the elasticity of expectations of short-term rates.

As bondholders have a choice between holding long-term and short-term bonds (and bills), the effective yield (i.e., after the deduction of the risk premium) of a long-term bond must be equal to the combined effective yield of short-term bonds held over the same period of time, i.e., the effective yield of a loan made for (say) n months must be equal to the combined effective yield of a loan made for only one month and renewed $n-1$ times at the end of each month. Otherwise bondholders would shift from long-term bonds to short-term bonds or vice versa, until the prices of the different kinds of bonds were in proper relation to each other. This means that the long-term rates of interest depend on the current short-term rate and on the short-term rates expected to hold during the period of the long-term loan.[13] A change in short-term rates affects long-term rates in an appreciable degree only if they strongly affect the (effective) short-term rates expected in the future, i.e., if the effective elasticity of expectations of short-term interest rates is sufficiently large.[14] Thus when the effective elasticity of expectations of short-term interest rates is small, an increase in the demand for short-term bonds (and bills) fails to reduce the long-term rates and, consequently, fails also to lead to an increase in the current demand for investment goods.

[13] Let R_n be the effective rate on a loan for n intervals of time, denote by r_1 the current effective rate on a loan for one time-interval and by $r_2, r_3, \cdots, r_n$ the effective rates on such a loan expected to hold during the successive intervals. Then

$$(1 + R_n)^n = (1 + r_1)(1 + r_2) \cdots (1 + r_n).$$

Vide Hicks, *op. cit.*, p. 145.

[14] This follows immediately from the formula in the preceding footnote. If the elasticity of expectation of effective short rates is zero, the effect of a change of the current short rate upon the long rate is negligible, the more so the longer the period of the loan. The effective long rate changes less than, exactly, or more than, in proportion to the current short rate, according as the elasticities of expectation of effective short rates are (all, or "prevailingly") less than, equal to, or greater than unity. Cf. also T. de Scitovszky, "A Study of Interest and Capital," *Economica*, N.S., Vol. 7, August, 1940, pp. 293–317.

Because of the failure of current demand for investment goods to increase, such intratemporal substitution and expansion as may be present are due entirely to the direct influence of price expectations in the different markets. But since there is no substitution between money and commodities, the intratemporal substitution effect and expansion effect are likely to be weak, and if effective expectations are of unit elasticity, they even fall out altogether. The outcome depends, therefore, on direct intertemporal substitution in the markets of the underemployed factors. If the effective price expectations that influence current demand for or supply of these factors are inelastic, excess supply diminishes until equilibrium is restored. If these price expectations are elastic, underemployment, caused by the decrease in the propensity to consume, becomes cumulatively worse, in spite of the positive monetary effect.

If the monetary effect is negative, the result is exactly opposite to that described in the case of a positive monetary effect. A proportional fall of all current commodity prices leads to an increase in the real excess demand for cash balances. This tends to result in a decrease in demand for investment goods. The current prices of investment goods and of primary factors engaged in their production fall more than in proportion to the fall in prices of the underemployed factors, and the intratemporal substitution effect and expansion effect are negative. The excess supply of the factors made underemployed by the decrease in the propensity to consume increases and their prices fall further, unless counteracted by sufficiently inelastic price expectations of the prices which influence current demand for and supply of these factors. This results not only in a cumulative fall in prices (as in the case of a neutral monetary system) but also in a cumulative increase in the excess supply of the factors of production which have been affected by the decrease in the propensity to consume. Flexibility of factor prices makes the economy move farther and farther away from equilibrium. This instability, however, may be mitigated if an increase in the real excess demand for cash balances leads to a decrease in the demand for securities rather than for commodities. The resultant rise in interest rates may fail to diminish the current demand for investment goods, either because increased uncertainty makes this demand insensitive to interest rates, or because the rise is confined to short-term rates not transmitted to long-term rates, and the negative intratemporal substitution effect and expansion effect become insignificant or fail to take place altogether.

As was already indicated in discussing the case of a neutral monetary system, our analysis can also be applied to the study of the consequences of an increase in the propensity to consume and can be generalized so as to include conditions of monopoly and monopsony. If the monetary effect of an increase in the propensity to consume is positive, flexibility of factor prices leads to a new equilibrium, unless the price expectations which influence

current demand and supply of the "bottleneck" factors are highly elastic. It leads cumulatively away from equilibrium, when the monetary effect is negative, unless the said expectations are highly inelastic. This may be modified when results of the monetary effect operate through the security markets rather than directly through the commodity markets. If monopoly and monopsony (including monopolistic and monopsonistic competition) are present, we have to substitute marginal revenues and marginal expenditures for prices in our analysis, wherever necessary. Our results are not affected by it.

The conclusions obtained may, however, be modified considerably in the presence of international trade. If the position of the country in international markets is atomistic, or if, instead, the demand curves for export goods and the supply curves for import goods are not affected by changes in domestic prices, a further stabilizing influence is operative in addition to the monetary effect and inelastic expectations. In this case, equilibrium may be maintained in any economy with flexible factor prices also under conditions of a neutral monetary system and even when the monetary effect is negative, provided the latter is not too strong.[15]

We are now in a position to evaluate the "orthodox" theory, which denies the possibility of "oversaving" (i.e., too low a propensity to consume) when factor prices are flexible. Its reasoning is exactly that of our analysis in the case where the monetary effect is positive. A decrease in the propensity to consume is supposed to diminish the demand for cash balances, which leads to an increase in the demand for investment goods offsetting the reduction of demand for final products. The increase in the demand for investment goods is conceived as taking place either directly (direct investment of "savings"), or indirectly via the markets for securities (indirect investment of "savings" through the capital and money market) and via a consequent fall of interest rates that stimulates the demand for investment goods. This argument is quite correct, provided the monetary effect is positive and, in case the investment of "savings" goes through the security markets, provided the demand for investment goods is responsive to changes in interest rates. A positive monetary effect is obtained by the "orthodox" theory through assuming a constant nominal quantity of money and unit elasticity of price expectations. In this case, the real demand for cash balances is constant, while the real quantity of money increases under a proportional fall of all current commodity prices. Thus the real excess demand for cash balances diminishes.[16] The sensitivity of the demand for investment goods to changes in interest rates is a consequence of disregarding the effects of uncertainty and of assuming unit elasticities of expectations of short-term rates.

[15] *Vide* p. 47 above.
[16] Cf. p. 14 above.

The assumption of unit elasticities of expectations also eliminates intertemporal substitution.

Under these conditions, the "orthodox" theory is perfectly valid. The error of this theory consists in regarding the conditions of its validity as the only possible ones. We have seen that they present but one of many possibilities and that the results and reasoning of the "orthodox" theory are but a special (and empirically none too realistic) case of a more general analysis. There is also a further limitation of the "orthodox" theory as well as of our more general analysis of the problem. This limitation arises from the different pattern of entrepreneurial responses under oligopoly and oligopsony based on group behavior. These have now to be taken into consideration.

The consequences of oligopoly and oligopsony depend largely on whether these types of entrepreneurial responses are also strongly present in the production of final products or whether they are chiefly concentrated in the production of investment goods. If the firms producing the final products for which the demand has changed on account of the change in the propensity to consume are oligopolistic or oligopsonistic, the demand for the factors engaged in producing these products may be only slightly affected, or even not affected at all, by a change in the propensity to consume. In case of oligopoly, the change in demand for final products may, through the effect on the "discipline" of the producing groups, spend itself partly or wholly in changes of prices and of the degree of monopoly. Output will respond only partly, or not at all.[17] In case of oligopsony, the change in demand for the products may fail to be transmitted into a change in the demand for factors, because the latter does not respond to shifts (within a certain range) of the marginal-net-value-productivity curves of the factors. In certain cases, this may also prevent a change in output of final products.[18] Thus oligopoly and oligopsony tend to diminish the changes in output of final products and, even more so, in the employment of factors resulting from changes in the propensity to consume, or even to prevent them altogether. The demand for factors by firms producing final products is more or less stabilized, and all the further consequences of a change in the propensity to consume are either absent or attenuated. Changes in the degree of monopoly and monopsony act here as shock absorbers.

[17] This weakness (or lack) of responsiveness of output is more likely to be operative when the propensity to consume increases than when it decreases. For an increase in demand seems (according to empirical observation) to strengthen the "discipline" of the group while a decrease in demand weakens it.

[18] The latter will happen when the demand of the firm for all factors is oligopsonistic. Otherwise only the demand for the factors subject to the oligopsony of the firm does not change, while the demand for the remaining factors changes in the same direction as the demand for the product. Output changes, but less than in the absence of oligopsony, because, as a result of the fixity of the factors subject to oligopsony of the firm, the marginal cost curves are steeper.

The situation is different when the change in the demand for final products is effectively transmitted into a change of their output and of the demand for the appropriate factors of production, while oligopoly or oligopsony reigns in the production of investment goods. In such a situation, the monetary effect, if present, may fail to produce a change in the output of investment goods as well as intratemporal substitution or expansion effects sufficient to remove the excess supply or excess demand for (or monopsonistic under- or overrestriction of demand for factors. Under oligopoly, the change in demand for investment goods may spend itself, partly or wholly, in price changes and changes in the degree of monopoly; the action of the intratemporal expansion effect thus being weakened or even thwarted altogether. The lack of responsiveness of output to shifts (within a certain range) of the marginal-cost curve under oligopoly tends to produce the same result. Oligopsony may prevent the intratemporal substitution effect from operating because a change in the marginal expenditure for the factors that are in excess supply or excess demand may spend itself, partly or wholly, in changes of their prices and of the degree of monopsony, without affecting much (or even at all) the demand for these factors. Or, when the monetary effect results in a change of interest rates rather than in a direct change in the demand for investment goods, the change of interest rates may fail to induce a change in the current demand for investment goods because of oligopoly and oligopsony, in addition to the causes of such a failure which have been mentioned previously. The lack of responsiveness of output to changes in factor prices (or marginal expenditure) and of input to changes in the marginal net value productivity of factors may prevent intertemporal substitution from arising in consequence of changes in the discounted values of expected prices (or marginal revenues and marginal expenditures).[19]

In the situation described, oligopoly and oligopsony tend to stabilize the output of investment goods and the demand for primary factors engaged in the production of investment goods. If the propensity to consume deviates from the equilibrium level[20] corresponding to the existing output of investment goods, the ensuing excess supply of or excess demand for factors of production may fail to be absorbed through flexibility of their prices, even when such flexibility creates a positive monetary effect and the expectations that influence current demand or supply of the factors are inelastic. On the other hand, flexibility of factor prices is prevented by oligopoly and oligopsony from acting in a destabilizing way when its monetary effect is negative.

[19] Cf. pp. 41–42 above.

[20] *Vide* the definition of the equilibrium propensity to consume on p. 55 above. The argument in the text holds also for deviations of the propensity to consume from its equilibrium composition. Concerning the latter, see footnote 18, on p. 55 above.

CHAPTER XI

Capital Accumulation and Investment Opportunities

By capital accumulation we understand an increase in the stock of investment goods in the economy.[1] Our present problem is to examine the effect of such an increase upon the demand for and output of investment goods.

The demand for an investment good, like that for any other factor of production, is determined by the equalization of the marginal value productivity with the price of (or marginal expenditure for) the good. The marginal physical productivity, which is one of the components of the marginal value productivity,[2] decreases (at least from a certain point on) as the stock of the investment goods utilized in production increases, while the quantities of all other factors utilized remain constant.[3] This is the law of diminishing incremental returns. In consequence, an increase in the stock of (some or all)[4] investment goods that is not accompanied by a proportional increase in the supply of primary factors[5] leads to a decline in the marginal

[1] The concept of capital accumulation used in the text is somewhat narrower than the one which is found in most traditional treatments of the theory of capital. "Capital goods" are usually defined to include also some primary factors, in the sense in which the term is used here (cf. footnote 6 on p. 52). Sometimes durable final products are classified as "capital goods," too. The reason for excluding "produced" primary factors is that none of them are present in a purely capitalist economy, and insofar as they are present empirically this is due to the presence of some noncapitalist producers who are quantitatively rather unimportant. Durable final products are excluded because, being bought by households, the demand for them is regulated by "utility" and not by marginal productivity and, therefore, must be analyzed by the same methods as any demand coming from households.

[2] The marginal value productivity is the marginal physical productivity multiplied by the price, or by the marginal revenue, of the product. When multiplied by the price, it is also called the "value of the marginal product."

[3] Strictly speaking, the marginal physical productivity depends not on the stock of investment goods but on the flow of their services employed in production; for instance, machine-hours. An increase in the flow of these services is, however, always implied in an increase in the stock of investment goods if all investment goods are utilized and if their rate of utilization per unit of time (e.g., the number of hours per week which machinery is utilized in production) decreases, if at all, less than in proportion to the increase in stock. Such an assumption is quite realistic. By making it, we are able to establish a direct relationship between the stock of investment goods and their marginal productivity and thus to simplify our exposition considerably.

[4] It is thus assumed in the text that none of the investment goods decreases in stock. Actually, capital accumulation need not imply this restrictive assumption. All that is necessary is an increase of the aggregate real value of the stock of investment goods in the community. The latter, however, leads straight into the tricky subject of real aggregates, namely the problem: What is to be meant by the aggregate quantity of "real capital" in the community? The whole difficulty is avoided by the assumption made in the text, without loss of any significant aspect of the problem under discussion.

[5] This means, under modern capitalism, the supply of labor (cf. p. 52 above). The

physical productivity of the former.[6] As a rule, this implies a decrease in their marginal value productivity and also, given their prices (or marginal expenditures), a decrease in the demand for them. Such a decrease is always present when the firms operate under conditions of perfect competition on the selling side, for the prices of the products of the investment goods remain prima facie unchanged, while their marginal physical productivity falls. The decrease takes place also under monopoly (and monopolistic competition). An exception occurs when the elasticity of the demand schedules for products increases, i.e., the degree of monopoly decreases, by so much that the resulting increase in marginal revenue balances the decline in marginal physical productivity.

The fall in the demand for investment goods in consequence of an increase in their stock constitutes the "exhaustion of investment opportunities" that many authors claim to be the consequence of capital accumulation when the increase in the supply of primary factors does not keep pace with it. The "orthodox" theory maintains that such an "exhaustion" is impossible if factor prices are all flexible, for the fall in factor prices (including the prices of primary factors) is bound to reduce the cost of production, and thus also the prices of investment goods, to an extent that compensates for the decline in their marginal value productivity. If factor prices are reduced sufficiently, the demand for investment goods can be increased, according to the "orthodox" theory, to any desired extent, whatever the stock of investment goods accumulated. "Investment opportunities" are thus regarded as unlimited.

According to our analysis, the result depends on the kind of monetary effect engendered by the fall of factor prices and on the elasticity of the effective expectations which influence current demand for or supply of the factors. The nature of this dependence has been elaborated in the preceding chapters. When the monetary system is neutral and effective expectations are of unit elasticity, current prices in the economy (except interest rates) fall cumulatively in proportion to the fall in prices of the factors which became underemployed (or monopsonistically underrestricted) on account of the decline of the marginal physical productivity of investment goods. Neither the excess supply of the factors mentioned nor the demand for investment goods is affected thereby. Flexibility of the prices of factors of production thus fails to expand "investment opportunities." It even causes a contraction of the

supply which is here relevant is, of course, the flow of services, i.e., of man-hours of the various kinds of labor. This supply is assumed in the text to increase at a slower rate than the flow of services of investment goods.

[6] The rate of this decline is diminished when the investment goods are co-operant in relation to each other, i.e., when the increase in the stock of one of them (which under our assumptions implies an increase of the flow of its services) increases the marginal productivity of the others (cf. A. C. Pigou, *The Economics of Welfare*, pp. 659–660), but some decline takes place, notwithstanding. In order that there be no decline, labor and investment goods as a group would have to be perfect substitutes, which, obviously, is in contradiction with experience.

latter, when the monetary system is neutral and the expectations that influence current demand for or supply of factors of production are elastic, or when the monetary system is such as to produce, under the condition stated in the *General Rule*, a negative monetary effect, unless the results of the negative monetary effect are outweighed by consequences of inelastic expectations. In this case, all other current prices fall more than in proportion to those of the underemployed factors, with the result that the excess supply of these factors increases and the demand for investment goods is reduced further. This process is also cumulative.

The conclusion of the "orthodox" theory holds only in the case of a monetary system in which a fall of commodity prices causes a positive monetary effect, and even then, only provided the intratemporal substitution effect and expansion effect are not overbalanced by adverse intertemporal substitution resulting from elastic expectations influencing current demand for or supply of factors of production. The diminution of the real excess demand for cash balances implied in the positive monetary effect tends to increase the demand for investment goods either directly, or indirectly through reduction in interest rates. Prices of all other commodities fall less than proportionally to those of the underemployed (or monopsonistically underrestricted) factors, and new "investment opportunities" are thus opened by way of the expansion effect and (both intratemporal and intertemporal) substitution effect which take place in the economy. This tendency, however, may become inoperative when the direct effect of the diminution of the real excess demand for cash balances is primarily to increase the demand for securities. For, in this case, as we have seen, the fall in interest rates may fail to increase the current demand for investment goods (or any other commodities). Further, even if a positive intratemporal substitution effect and expansion effect take place, it may be thwarted or overshadowed by the opposite action of intertemporal substitution due to elastic expectations. Thus, the "orthodox" theory of unlimited investment opportunities under a regime of flexible factor prices is, like the "orthodox" theory of saving, true only in a very special case. It errs in ascribing general validity to what is only one of many possibilities.

This analysis requires some modifications when oligopoly or oligopsony is present. The different type of entrepreneurial responses encountered in this case may prevent the re-establishment of full employment of all factors in spite of a positive monetary effect and of inelastic expectations of the prices (or marginal revenue and expenditures) that influence directly current demand for and supply of factors of production. The necessary expansion effects and (both intratemporal and intertemporal) substitution effects may be weakened or may fail to operate altogether. The intratemporal expansion effect may be thwarted by the lack of response of the output of investment goods which are produced under oligopolistic conditions to the reduction in marginal cost resulting from the fall of factor prices (or marginal expendi-

tures). The same may be done by a strengthening of the group "discipline" among the oligopolistic producers of investment goods in consequence of an crease in demand for these goods due to the diminution of the excess demand for cash balances. The intratemporal substitution effect may be thwarted by the lack of response of the input of factors subjected to oligopsony to shifts in their marginal-value-productivity curves. Intertemporal substitution may fail to take place for any of the reasons just mentioned, or for any combination of them.[7] In the cases described, flexibility of factor prices fails to prevent capital accumulation from leading to excess supply of factors of production because oligopoly and oligopsony provide a barrier against the expansion of investment opportunities.

Under certain conditions, however, this barrier may also act as a barrier against a contraction of "investment opportunities" resulting from the decline in the marginal value productivity of investment goods. Thus, when the demand for investment goods is oligopsonistic, it is not affected by shifts (within a certain range) of their marginal value productivity. The decline in the latter may, therefore, leave unchanged the demand for investment goods, as well as their output and prices. Its only effect is then to diminish the degree of monopsony in the markets for investment goods. An analogous result may happen when the supply of investment goods is oligopolistic. In this case, the decrease in demand for investment goods may ensue in a weakening of the "discipline" of the oligopolistic groups and the prices of investment goods may fall without causing a decrease in output. The demand for the factors of production (i.e., primary factors and other investment goods) that are used to produce the investment goods in question is then unchanged.

Thus the tendency of oligopoly and oligopsony to create rigidity of output and input responses of entrepreneurs, may act as a stabilizing factor cushioning the effects of capital accumulation as well as a factor hampering the adjustment processes normally resulting from a positive monetary effect. The actual result in each particular case depends on the particular type of oligopolistic or oligopsonistic situation described and on how the particular investment good to which it refers is affected by the accumulation of capital. Therefore, it cannot be predicted on a purely theoretical basis. One important practical conclusion, however, can be drawn from our analysis of oligopoly and oligopsony as well as from the fact, already stated, that under monopoly (and monopolistic competition) a sufficiently large decrease in the degree of monopoly may prevent the decline of the marginal physical productivity from causing a decline in the marginal value productivity. The conclusion is that, under all circumstances, a decrease in the degree of monopoly or monopsony facilitates the absorption by the economy of the consequences of capital accumulation.

[7] Cf. p. 42 above.

CHAPTER XII

INNOVATIONS[1]

INNOVATIONS are such changes in production functions, i.e., in the schedules indicating the relation between the input of factors of production and the output of products, as make it possible for the firm to increase the discounted value of the maximum profit obtainable under existing market conditions. The profit under consideration is the total effective profit (i.e., the profit after deduction of risk premiums) which the firm expects to make during the period for which it makes its output and input plans, i.e., during the firm's economic horizon. By market conditions, we mean the effective prices and, under imperfect competition, the effective demand and supply schedules, respectively, of the relevant products and factors. Discounted expected prices and schedules as well as current ones are included.

When the only uncertainty is that of price expectations the effective profit can be calculated by evaluating all prospective receipts and expenses on the basis of the effective expected prices (cf. p. 31 above). In addition to uncertainty of price expectations, however, "technological uncertainty" may be present. The latter consists in uncertainty concerning the quantitative relations between current or future inputs and future outputs. In this case, a "technological risk premium" has to be deducted from the profit calculated on the basis of the effective expected prices. Consequently, the effective profit increases when the technological risk premium decreases. The chief effect of many innovations is to decrease the technological risk premium rather than to increase the expected profit unadjusted for technological uncertainty.[2]

[1] Part of this chapter has been published under the title, "A Note on Innovations," in the *Review of Economic Statistics*, Vol. 25, February, 1943 (Special issue in honor of J. A. Schumpeter), pp. 19–25, and is reproduced here with the kind permission of the editors.

[2] Technological uncertainty arises either when the production function is a stochastic relation between output and input (as, for instance, in agriculture), or when, though the production function is not stochastic, the quantitative input-output relationships are subject to changes because of unforeseen changes in input or output or of the scale of operation of the plant (lack of adaptability and flexibility of the firm's production plan). On the first type of technological uncertainty, cf. G. Tintner, "The Pure Theory of Production under Technological Risk and Uncertainty," *Econometrica*, Vol. 9, July–October, 1941, pp. 305–312; on the second type, see G. Stigler, "Production and Distribution in the Short Run," *Journal of Political Economy*, Vol. 47, June, 1939, pp. 312 ff., and A. G. Hart, "Imputation and the Demand for Productive Resources in Disequilibrium," *Explorations in Economics* (New York: McGraw-Hill, 1936), pp. 264–271. An innovation may consist in a reduction of either type of technological uncertainty. Uncertainty of the first type is reduced by diminishing the variance of the probability distribution of possible outputs corresponding to a given input, e.g., by reducing the dependence of crops on weather conditions. Uncertainty of the second

The economic impact of an innovation depends on the way in which it affects the marginal cost of the current output and the marginal physical productivity of the factors currently employed, as well as on the type of entrepreneurial responses of the firms which adopt the innovation.

The marginal cost of any given current output, as well as the expected marginal cost of any output planned for some future date, may be affected by an innovation in either direction, or not affected at all.[3] If the marginal cost

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type is reduced by increasing the adaptability and flexibility of the plant. Reduction of uncertainty of price expectations (or of expectations of marginal revenue and expenditure) is excluded from our definition of innovations, because the last-mentioned expectations are included in the concept of "existing market conditions."

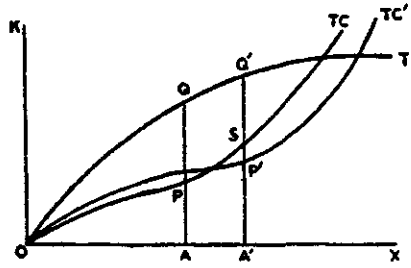
[3] This holds true even in the case where the firm maximizes merely the current profit, as happens when the current profit and the profits expected at later time intervals are independent of each other (for when the profits in two or several subintervals of a period are independent of each other the total profit over the whole period is maximized by maximizing separately the profit in each subinterval). The direction of the change in the current marginal cost depends then on how the innovation affects the total cost of the current output and the current "elasticity of productivity." Let all factors currently employed be increased in the same ratio λ and let x be the current output. The elasticity of productivity is

$$\frac{Ex}{E\lambda} = \frac{dx}{d\lambda}\frac{\lambda}{x}$$

(see Allen, *op. cit.*, p. 263; cf. also S. Carlson, *A Study on the Pure Theory of Production*, (London, 1939) p. 17, and E. Schneider, *Theorie der Produktion* (Wien, Springer, 1934), p. 10. The concept was introduced by Dr. Schneider). According to a theorem established by Dr. Schneider (*op. cit.*, pp. 42–43), we have, for any output x, the relation

$$k(x) = k'(x) \cdot x \cdot \frac{Ex}{E\lambda},$$

where $k(x)$ is the total cost and $k'(x)$ is the marginal cost of the output x. Thus, an innovation reduces or increases the marginal cost of the output x, according as it increases or decreases the elasticity of productivity relatively to the change in total cost it causes. It is clear that the elasticity of productivity may be affected in either direction, or not at all, by an innovation. The same holds for the total cost $k(x)$, except when x is the output which maximizes the firm's profit after adoption of the innovation. In the last-mentioned case, $k(x)$ is always reduced in consequence of an innovation. This can be seen from the following diagram. TR is the total revenue curve and TC is the total cost curve before introduction of the innovation. PQ is the maximum profit obtainable and OA is the corresponding output. After adoption of the innovation, the total cost curve becomes TC' with $P'Q'$ and OA', the corresponding maximum profit and output. From the definition of an innovation, it follows that $P'Q' > PQ$. But $PQ > SQ'$ because PQ is the maximum profit before the introduction of the innovation. Consequently, $P'Q' > SQ'$. But, for any other output than OA' (or, if the cost curves are continuous, for any output not in



of the current output that maximizes the discounted value of the firm's profit before adoption of the innovation is reduced by the innovation, the current output of the firm increases. In the opposite case, it decreases. Similarly, with regard to the output planned for any future date and the corresponding discounted marginal cost. An innovation will be called output-increasing, output-neutral, or output-decreasing at the date t, according as it increases, leaves unchanged, or decreases the output planned for that date.

An innovation increases a firm's current demand for a factor of production, or the demand planned for a certain future date, when the marginal physical productivity of the quantity of the factor used on that date, or planned for that date, before introduction of the innovation, is raised. It diminishes this demand when the opposite is the case. This holds under monopoly and monopsony (including monopolistic and monopsonistic competition) just as well as under perfect competition. The marginal revenues and the marginal expenditures corresponding to the output and input plan preceding the innovation are all given. A change in the marginal physical productivity of the corresponding (current or planned) quantity of a factor thus implies a proportional change of its marginal value productivity.[4] Before introduction of the innovation, the marginal value productivity was equal to the marginal expenditure. Now it exceeds it or falls short of it, and the demand for the factor increases or decreases accordingly. An innovation will be called "using" or "saving" a given factor at the date t, according as it increases or diminishes the demand planned for that date. Thus, an innovation will be labeled, e.g., labor-saving after a year, currently steel-using, etc.[5]

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the neighborhood of OA') total cost need not be less after adoption of the innovation than before the adoption. The argument is independent of the shape of the TR curve and, therefore, holds for imperfect competition as well as for perfect competition. Thus, both the total cost corresponding to OA and the elasticity of productivity at the output OA may be affected by the innovation in either direction. In view of Dr. Schneider's relation, the marginal cost of the output OA may thus be affected in either direction. In the diagram, A' is to the right of A, and the innovation reduces the marginal cost of OA. When the marginal cost of OA is increased or left unchanged, A' is to the left of A or coinciding with A, respectively.

[4] Cf. footnote 2 on p. 67 above.

[5] The classification of innovations as "using" or "saving" a factor given in the text is in terms of the absolute change in the factor's marginal physical productivity. Professor Pigou (*The Economics of Welfare*, p. 674), Professor Hicks (*The Theory of Wages*, London, 1932, pp. 121–122), and Mrs. Robinson ("The Classification of Inventions," *Review of Economic Studies*, Vol. 5, February, 1938, pp. 139–140) have given other classifications which, though differing among themselves, are all in terms of relative changes in the marginal physical productivity (i.e., in terms of changes of the marginal rate of substitution of factors). The difference between our classification and theirs is due to the fact that, whereas we are interested in the effect of innovations upon the demand for and the employment of a factor, Professor Pigou is interested in the effect upon the aggregate real income, and Professor Hicks and Mrs. Robinson in the effect upon the

An innovation which is neither factor-using nor factor-saving will be called factor-neutral.

When an innovation does not "save" any of the factors which the firm employed or planned to employ before its adoption, it is either output-increasing at (at least) some date within the firm's economic horizon, or, instead, it reduces the technological uncertainty[6] attached to the production plan. Under given market conditions, an increase, or lack of change, of the quantities of the different factors entering the firm's production plan, implies an increase, or at best a lack of change, of the discounted value of the total effective cost planned by the firm.[7] An innovation, however, by definition, increases the discounted value of the total effective profit which the firm expects to make during the period covered by its economic horizon. Therefore, the discounted value of the total effective revenue must increase by more than the discounted value of the total effective cost. The market conditions being given, any increase in the first requires either an increase in the output planned for (at least) some date,[8] or a reduction of the technological risk premium. Conversely, an innovation which is not ouput-increasing at all cannot be all-around factor-using or even factor-neutral, unless it causes a decrease in technological uncertainty. It must "save" at least some factor at some date.

Subject to these two restrictions, any combination between the output-increasing or output-decreasing effect and the factor-using or factor-saving nature of an innovation is possible. In particular, an innovation can be at the same time output-increasing at all dates and factor-saving with regard to all factors and dates. Our empirical knowledge seems to indicate that the major part of innovations "use" at least some factors (chiefly investment goods) currently and in the near future, and are output-increasing at some more remote future. The economic effects of such innovations can be divided roughly into two periods: a factor-using period of "gestation" and an output-increasing period of "operation" of the innovation.[9]

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relative shares of the factors under the assumption (common to all three of them), that full employment of all factors is retained or restored after the innovation. Mrs. Robinson's and Professor Hicks's classifications are related and, with the aid of the concept of the elasticity of substitution, translatable one into the other.

[6] See footnote 2 on p. 71 above. Only the technological uncertainty can be reduced by an innovation because the uncertainty of price expectations is included in the concept of "market conditions" which remain unchanged by definition.

[7] We assume that none of the supply schedules of the relevant factors are negatively sloped.

[8] The discounted marginal revenue corresponding to the output planned for each date is considered as not negative, while for some date at least it is assumed to be positive. Since at each date the planned discounted marginal revenue is equal to the planned discounted marginal cost, the first can be negative only when the latter is so.

[9] This has been pointed out by Professor J. A. Schumpeter, who explains on this

In order to find the effect of an innovation upon the output of a commodity and the demand for various factors of production in the whole economy, we have to consider, in addition to the points just discussed, its effect upon the number of firms in an industry. When the industry producing the commodity under consideration operates under conditions of perfect competition, and, in addition, is subject to free entry,[10] the increase in the discounted value of the effective profit attracts new firms into the industry. The influx of new firms continues until the aggregate output of the industry planned for some or all dates increases[11] sufficiently to reduce the discounted value of the effective profit of the firms to zero level.[12] Thus, when free entry is present, any innovation must, with respect to the whole economy, be output-increasing at some date, even though it be exclusively output-decreasing from the point of view of the individual firms.[13] Free entry, by leading to an increase in the number of firms in consequence of an innovation, also exercises a factor-using influence. The net effect of an innovation upon the demand for factors of production by a competitive industry with free entry, however, may be in either direction.

When competition is monopolistic, or monopsonistic, the concept of free entry has no meaning,[14] and it is sufficient to analyze the effects of an innovation upon the decisions of the firm. A superficial analogy to free entry exists when the innovation leads to the establishment of new firms producing new commodities. This case, however, can be treated as the extreme case of output-increasing and factor-using innovations.

Under oligopoly, an innovation cannot be output-increasing unless the diminution of marginal cost caused by it is sufficiently great to induce the firm to break the "discipline" of the group. The last-mentioned case happens when the marginal-cost curve shifts to such an extent as to make it get out

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basis the mechanism of the business cycle, the factor-using period being responsible for the prosperity and the output-increasing period for the recession. Cf. his *Business Cycles: A Theoretical, Historical and Statistical Study of the Capitalistic Process* (New York: McGraw-Hill Co., 1939), Vol. I, pp. 93 ff.

[10] Free entry may be absent even though the competition is perfect in the sense of being atomistic (i.e., no firm being able to influence prices by individual variation of its outputs and inputs).

[11] The demand schedules of the product are all assumed to be negatively sloped.

[12] "Normal" profit is equal to the sum of all the risk premiums. Thus, effective profit, which is profit after deduction of the risk premiums, is zero when profit unadjusted for uncertainty is "normal."

[13] In the special case where the firms maximize only current profit (see footnote 3, p. 72 above), any innovation increases the current output of the industry.

[14] In this case, each firm must be considered as selling a separate product or using separate factors. The concept of an industry thus loses its meaning. Cf. Robert Triffin, *Monopolistic Competition and General Equilibrium Theory* (Harvard University Press, 1940), pp. 81–96.

of the range of discontinuity of the marginal-revenue curve.[15] Thus, only innovations that reduce marginal cost to a great extent can be output-increasing under conditions of oligopoly. It follows that, unless it causes a sufficiently large reduction of marginal cost, or, instead, a decrease in technological uncertainty, an innovation cannot be, under oligopoly, all-around factor-using, or even all-around factor-neutral; it must "save" at least some factor at some date in the firm's production plan. Except with regard to innovations that *greatly* reduce marginal cost, and to innovations that reduce technological uncertainty, oligopoly exerts a selective action against output-increasing and in favor of factor-saving innovations.

Oligopsony favors factor-neutral innovations. For the demand for a factor of production changes under oligopsony only when the marginal-value-productivity curve of the factor is shifted to such an extent as to get out of the range of discontinuity of the marginal-expenditure curve.[16] The demand for factors of production under oligopsony is, therefore, affected only by innovations which produce changes in their marginal physical productivity sufficiently *large* to induce the firm to break the "discipline" of the group. But innovations that do not affect the quantity of factors entering into the firm's production plan must be output-increasing at some date, or, instead, must reduce technological uncertainty. This type of innovation seems to be favored by oligopsonistic conditions.

The type of competition and the entrepreneurial responses associated with it thus exercise an important selective influence upon innovations. Under perfect competition, with free entry of firms, all innovations are output-increasing at some date at least, with regard to the whole industry, but may be either output-increasing or output-decreasing with regard to single firms.[17] Oligopsony favors innovations which are output-increasing with regard to the firm, as well as to the industry,[18] but which at the same time are factor-

[15] Cf. Fig. 1, on p. 41, footnote 14. In order to cause an increase in output, marginal cost must fall below *MG*.

[16] Cf. Fig. 2, on p. 41, footnote 14. The range of discontinuity is here *GH*.

[17] When the innovation is output-decreasing with regard to the firm, it causes, in this case, a deconcentration of the industry.

[18] Unlike the case of monopolistic or monopsonistic competition, the concept of an industry can be applied under conditions of oligopoly or oligopsony. An industry can be defined in the same way as under perfect competition, i.e., as all the firms that produce the same product (or products) or as all the firms that use the same factor. A commodity, whether a factor or a product, is defined as all the "objects" (including services), the prices of which vary in the same proportion (equality of prices is a special case of it). See Section 5, of the Appendix below, and cf. also Triffin, *op. cit.*, p. 138. Oligopolistic or oligopsonistic group behavior establishes a "price structure," i.e., certain ratios of the prices charged by the various sellers or paid by the various buyers, maintained by the "discipline" of the group. Thus all the oligopolists can be considered as selling the same commodity and all the oligopsonists as buying the same commodity, i.e., as forming an industry. As here defined, the extent of an industry coincides with

neutral. Oligopoly favors output-neutral innovations which, with regard to the firm, as well as to the industry, necessarily have factor-saving effects.

When an innovation is limited to a small number of atomistic firms, its consequences can be analyzed by the method of partial-equilibrium theory. This can be done also in the highly unrealistic case when, although the firms are not atomistic or their number small, they are confronted with demand schedules and supply schedules that are absolutely independent of the prices of any other commodities, and, in addition, the prices of their products and of the factors used by them have no influence whatever upon the demand for or supply of any other commodity in the economy. Partial-equilibrium analysis being applicable in both cases, flexibility of factor or product prices secures, after the introduction of the innovation, automatic restoration of equilibrium in the respective markets. But, outside of the narrow range of validity of partial-equilibrium theory, the consequences of innovations depend on the nature of the monetary effect produced.

Consider now an innovation or a "wave" of innovations that spreads over a major part of the economy. Let the innovation "save" currently one or several factors of production, and assume that before introduction of the innovation the economy was in equilibrium. Under the conditions described, the innovation causes excess supply of the factors "saved." Let all prices be flexible, and let us investigate what happens in the markets of the factors "saved" by the innovation. The prices of the factors that are in excess supply fall.[19] If the monetary system is neutral and effective expectations are of unit elasticity, all current prices, except interest rates, fall in the same proportion, without, however, diminishing the excess supply of the factors under consideration. The prices of these factors fall again and a cumulative fall of prices takes place, while the demand and supply situation in the markets remains unaffected.

If the monetary effect is negative, and the effective expectations that influence current demand for and supply of the factors affected by the innovation are not sufficiently inelastic to counteract its consequences, the cumulative fall in prices is accompanied by a cumulative increase in the excess supply of the factors "saved" by the innovation. For, as we have seen in earlier chapters of this study, the prices of other factors and of products fall more

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the extent of the oligopolistic or oligopsonistic groups. It should be noticed, however, that the industry defined in terms of sales of products is not identical with the industry defined in terms of factor purchases. A firm may belong to one industry with respect to its product and to a different industry with respect to each of its factors. If it is a multiproduct firm, it may also belong to a different industry with respect to each of its products. Under perfect competition, however, all firms are alike and belong to the same industry, whether the latter is defined in terms of any of the products or of any of the factors.

[19] They all fall in the same proportion for the reason indicated on p. 15, footnote 54.

than in proportion to the prices (or marginal expenditures) of the "saved" factors. The intratemporal substitution effect and expansion effect are negative, and, by assumption, favorable intertemporal substitution, if present at all, is not strong enough to counteract them.

Full employment of the factors "saved" by the innovation can be restored automatically through a decline of their prices (or marginal expenditures) only under the specific conditions. These conditions require a positive monetary effect, the consequences of which are not outweighed by adverse intertemporal substitution resulting from a high elasticity of the expectations influencing current demand for or supply of the factors affected by the innovation; or, instead, they require such highly inelastic expectations influencing current demand for or supply of factors mentioned as to overbalance the adverse consequences of a negative monetary effect, or the consequences of the absence of a monetary effect. But, as we have seen, the intratemporal substitution effect and expansion effect may fail to operate in spite of a positive monetary effect, if the latter spends itself in security markets. These two effects, as well as intertemporal substitution, may also be thwarted by oligopoly and oligopsony. Again, in such cases full employment is not restored.

If, instead of "saving" factors, the innovation is currently factor-using, excess demand for the factors "used" develops. Under a neutral monetary system, and unit elasticity of effective expectations, this causes a cumulative rise of all prices (except interest rates) but does not diminish the excess demand. The factors "used" by the innovation become permanent "bottlenecks" in the economy. If the monetary system is neutral, but the expectations that influence current demand for or supply of the factor "used" by the innovation are elastic, or if the monetary effect is negative and not overbalanced by great inelasticity of these expectations, excess demand for the "bottleneck" factors increases as well as their prices. The rise in price of "bottleneck" factors causes the excess demand for them to disappear only if either the expectations that influence current demand for or supply of these factors are inelastic while the monetary system is neutral, or if the monetary effect is positive and not thwarted by great elasticity of these expectations. But even in the last-mentioned case, this result may be thwarted when the monetary effect spends itself in security markets, or by the peculiar type of entrepreneurial responses associated with oligopoly or oligopsony.

An interesting case arises in practice when a "wave" of innovations occurs which "uses" factors (particularly investment goods) currently and in the near future and which is output-increasing at a later date. It was already indicated that a major part of innovations seems to be of this type. Let us assume, further, that the monetary effect is negative and that its consequences with regard to current demand for and supply of factors of production are not thwarted by intertemporal substitution resulting from very in-

elastic expectations. The conditions under which the monetary effect will be negative are given by the *General Rule.*[20] They depend on the relation of the effective elasticities of price expectations (or expectations of marginal revenues and expenditures) to the responsiveness of the monetary system. In case of prevailingly inelastic effective expectations, the monetary effect is negative when the monetary system is responsive, while in case of prevailingly elastic expectations or expectations of unit elasticity it is negative when the monetary system is unresponsive. Under the conditions described, the factor-using period of gestation of the innovations produces a cumulative rise in the prices of all commodities (products as well as factors) and of stocks and is accompanied by increasing excess demand for the factors "used" by the innovation (i.e., chiefly investment goods). Later, when the factor-using period of gestation gives way to the output-increasing period of operation of the innovations, excess supply of products develops. The monetary effect being negative, a cumulative fall of all commodity prices (factor prices as well as product prices) takes place, and the excess supply of the products as well as of the factors engaged in their production also increases cumulatively. Innovations of the type under discussion thus produce a characteristic semicyclical pattern.

The upward branch of the semicycle comes to an automatic end because the factor-using character of the innovations is limited to a certain period of time, i.e., to their period of gestation. The output-increasing effect, however, i.e., the period of operation, extends indefinitely into the future as long as the new production functions, introduced with the innovations, continue to form the basis of entrepreneurial production plans. Therefore, if the monetary effect continues to be negative, the downward branch of the semicycle continues indefinitely. The cumulative fall in prices and employment of factors engaged in producing the products that are in excess supply can be reversed only by a change in the nature of the monetary effect.[21] The latter

[20] See p. 24 above.

[21] On the assumption that innovations of the type discussed appear in "waves" or "clusters," we obtain what Professor Schumpeter calls the Pure Model or First Approximation of his theory of the business cycle (*vide op. cit.*, pp. 130 ff.). That the appearance of innovations tends to be distributed over time in "clusters" may, indeed, be deduced from the analysis contained in this section. The discounted value of the effective profit expected by a firm is, other things being equal, the greater the less the uncertainty of price expectations (or expectations of marginal revenues and expenditures), including expectations of interest rates. Thus, many changes of production functions which are technologically possible will be found to increase the discounted value of the maximum effective profit obtainable under given market conditions, when the uncertainty of price expectations is small, but not when it is great. The former periods favor, the latter discourage, innovations. Under conditions of a negative monetary effect, a "wave" of innovations causes an increase in the uncertainty of price expectations, and thus inaugurates a period that is unfavorable to innovations. Should the economy return automatically to a new equilibrium, a period favorable to innovations would ap-

has to become positive. As the expectations are likely to be elastic after the fall in prices continues for some time, this requires that the monetary system be, or become, responsive.[22] But, as we already know, even this may fail to stop the fall of prices, if the monetary effect acts entirely, or chiefly, via changes in interest rates.

The type of innovations just discussed, however, is not likely to occur under a regime of oligopoly and oligopsony, except as "industrial revolutions." Oligopsony, as we have seen, favors factor-neutral innovations. But, such innovations must either be output-increasing at some date, or, instead, diminish technological uncertainty. In neither case do they lead to the semicycle described. If they are output-increasing at a certain date (or dates), they lead, under conditions of a negative monetary effect, to a cumulative fall in prices and an increasing excess supply of the products affected. The fall in prices and excess supply of certain products, however, is not preceded by an excess demand for factors and a cumulative rise in prices, as in the case previously discussed. Oligopoly favors innovations which are output-neutral. Such innovations must either be factor-saving at some date, or diminish technological uncertainty. If they are factor-saving, and the monetary effect is negative, the result is a cumulative increase in the excess supply of the factors "saved" and a cumulative fall of all prices.

Thus, under oligopoly, as well as under oligopsony, innovations are rather more likely to lead only to a cumulative fall in prices and an increasing excess supply of factors or products, respectively, than to a semicycle of the kind described. Such semicycles seem to be characteristic of an economy dominated by the entrepreneurial responses of perfect competition, monopoly, monopsony, monopolistic and monopsonistic competition, rather than of an economy dominated by oligopolistic or oligopsonistic group behavior. The semicycles, however, may appear also in the latter, when the changes in marginal cost or marginal physical productivity are so large that they induce firms to break the "discipline" of the oligopolistic or oligopsonistic group. Such changes are meant by the term "industrial revolutions" used above.

We have studied the effects of innovations in an economy that was in

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pear again. Professor Schumpeter assumes that this is actually the case. But, as shown in the text, the return to a new equilibrium requires a reversal of the monetary effect from a negative to a positive one (and even this may fail to restore equilibrium). Later the monetary effect would have to reverse itself again, i.e., become negative, in order that the new "wave" of innovations start, again, the semicycle described in the text. Professor Schumpeter does not explain these reversals, and, besides, a theory of the business cycle that has to rely upon them implies rather artificial assumptions.

[22] It is possible, however, that price expectations become inelastic after a prolonged fall of prices, because there is a belief in "normal" prices, and the actual prices are regarded as having fallen below "normal." In such a case, the monetary system must be or become, unresponsive.

equilibrium. Now, let us assume that before introduction of the innovation (or innovations) there was excess supply of certain factors, particularly investment goods and primary factors engaged in the production of investment goods, either because of a propensity to consume below equilibrium level,[23] or because of "exhaustion of investment opportunities" resulting from capital accumulation in excess of the increase in the supply of primary factors.[24] Both cases presuppose either that the monetary effect of a fall in factor prices is absent or negative, or, if it is positive, that a diminution of the real excess demand for cash balances fails to increase the demand for investment goods. The latter may happen, either because cash balances are used chiefly to purchase securities and the consequent fall in interest rates fails to stimulate the current demand for investment goods, or because the intratemporal substitution effect and expansion effect are thwarted or overshadowed by adverse intertemporal substitution resulting from high elasticity of the expectations which influence current demand for investment, or, finally, because of oligopoly and oligopsony. Whichever the situation, we shall assume that it is not changed by the innovations. This implies that the innovation does not change the effective elasticities of (discounted) price expectations (or expectations of marginal revenues and expenditures) from prevailingly elastic to prevailingly inelastic ones, or vice versa.

Under the conditions assumed, the effect of innovations depends chiefly on whether they "use" or "save" investment goods and primary factors engaged in the production of investment goods. If they "use" investment goods or the primary factors mentioned, innovations may serve to reduce the excess supply that existed before the introduction of the innovations. It should be noticed, however, that this effect is temporary and only takes place during the factor-using period of gestation of the innovations. Later, during the output-increasing period of operation (which must follow, unless the innovations merely reduce technological uncertainty), an excess supply of products develops. In order that innovations maintain full employment of the factors that otherwise would be in excess supply, an adequate *continuous flow* of innovations that "use" these factors is necessary. In the case, however, that the innovations "save" the factors that are in excess supply, i.e., investment goods and primary factors engaged in the production of the latter, the excess supply is increased by the innovation. Innovations then only render the situation more acute.

Widely held among economists is the opinion that a continuous adequate flow of innovations is an offset against the employment-reducing effects of a fall of the propensity to consume, or of capital accumulation exceeding the increase in the supply of primary factors of production. This opinion is in-

[23] Cf. p. 55 above.

[24] Cf. p. 68 above.

correct when expressed without specification of the nature of the innovations. It is only true in the special case, when the innovations "use" investment goods and primary factors engaged in their production. It is not true, when the innovations are "neutral" with regard to the factors mentioned, and it is wrong *a fortiori* when the factors are "saved" by the innovations. In the last case, the innovations, far from offsetting the diminution of the propensity to consume or the accumulation of capital, are an additional cause of underemployment of factors of production. There is no theoretical nor empirical reason why the innovations should "use" the underemployed factors.

In fact, under oligopsony they tend rather to be factor-neutral, and under oligopoly there is a definite tendency for innovations to be factor-saving. Thus, in an economy in which entrepreneurial responses are oligopsonistic and oligopolistic, innovations cannot be relied upon, except in the case of "industrial revolutions," as an influence offsetting a decline in the propensity to consume or capital accumulation outrunning the supply of primary factors. Under oligopoly, they are likely to aggravate the underemployment of factors, rather than help to alleviate it. This holds even in the case when the innovations would, against our previous assumption, cause such a change in the effective elasticities of expectations as to result in a positive monetary effect. For, we know that oligopsony and oligopoly may thwart the effect of excess cash balances upon the demand for factors of production. Irrespective of the nature of the monetary effect, oligopoly provides a fertile soil for the growth of permanent "technological unemployment."

CHAPTER XIII

The Problem of Policy

We have found that only under very special conditions does price flexibility result in the automatic maintenance or restoration of equilibrium of demand for and supply of factors of production. These conditions require a combination of such a responsiveness of the monetary system and such elasticities of price expectations as produce a positive monetary effect, sensitivity of intertemporal substitution to changes in interest rates (if the positive monetary effect leads to a change in the demand for securities rather than to a direct change in the demand for commodities), absence of highly specialized factors with demand or supply dependent on strongly elastic price expectations, and, finally, absence of oligopolistic or oligopsonistic rigidities of output and input. To a certain extent, the absence of a positive monetary effect may be replaced by the stabilizing influence of foreign trade in an atomistic international market (among the different countries). There are good reasons to believe that these conditions were approximately realized in the long run during a period which extended from the 1840's until 1914. During this period, price flexibility was a workable norm of long-run economic policy.

The feeling of stability and security of the economic order which permeated this period (with possible exceptions during the years 1873–1896), created a strong belief in a "normal" level of certain economic quantities, including prices. Long-range effective price expectations were, therefore, prevailingly inelastic. A prolonged rise or fall of prices was expected to reverse itself, and the real demand for cash balances varied, in the long run, in the same direction as prices. With metallic monetary standards of one kind or another prevalent, the monetary system was, for the most part, unresponsive in the long run. This favored the emergence, in the long run, of positive monetary effects of price changes. In view of the feeling of security and stability dominant among entrepreneurs, the uncertainty of price expectations was small and the demand for investment goods was sensitive to changes in (long-term) interest rates. Entrepreneurial responses were only rarely based on oligopolistic and oligopsonistic group behavior. Under these circumstances, price flexibility led, in the long run, to the operation of the intratemporal substitution effects and expansion effects which automatically stabilized the economy, and intertemporal substitution effects acted with the same result.

This stabilizing function of price flexibility, however, operated only in the long run. Short-run effective price expectations were frequently elastic (or of unit elasticity), and the development of credit money also made it possible for the monetary system to be responsive in the short run. If elastic short-run price expectations happened to coincide with short-run unresponsiveness of the monetary system, or if, conversely, inelastic short-run price expecta-

tions happened to coincide with short-run responsiveness of the monetary system, the monetary effects of price changes were negative.[1] In addition, elastic short-run price expectations frequently exercised a direct destabilizing effect upon current demand for and supply of factors of production, even if the monetary effect was positive. Therefore, price flexibility, particularly flexibility of factor prices, which acted as a stabilizing influence in the long run, failed to do so in the short run. We find accordingly that while remarkably stable in the long run, in the short run the capitalist economy of that period was subjected to strong fluctuations of employment, output, and prices. These fluctuations appeared in the form of the business cycle.

Long-run (but not necessarily short-run[2]) price flexibility thus became universally recognized as a canon of economic policy. It must be mentioned, however, that price flexibility operated as a long-run stabilizing force in the economy, not only because a fortunate combination of circumstances produced the substitution effects and expansion effects, intertemporal and intratemporal, that made such an operation possible, but also because the forces making for excess supply of factors of production were rather weak, and small substitution and expansion effects sufficed to maintain or restore equilibrium. A rapid increase of population, the opening of new countries and continents to capital investment, and the opening of new sources of supply of natural resources prevented the stock of investment goods from growing very much faster than the supply of primary factors of production. This maintained a high marginal productivity of investment goods. The high rate of population increase and the much lower *per capita* income (than at present) maintained the propensity to consume at a high level. The absence of widespread oligopolistic and oligopsonistic group behavior among entrepreneurs prevented the discrimination against factor-using innovations that is exercised under this type of entrepreneurial response. Thus, innovations acted, much more frequently than at present, as a source of *direct* increase in the demand for factors of production. Under the conditions described, the tendencies that might have operated in the direction of oversaving, exhaustion of "investment opportunities" and of "technological unemployment" were rather weak. Such tendencies as were present were easily coped with (except for the period 1873–1896) by the long-run intratemporal substitution

[1] The former coincidence seems to have been characteristic of financial panics, when the real demand for cash balances increased but the real quantity of money, instead of increasing to a greater extent as required in order to produce a positive monetary effect, decreased. The latter coincidence seems to have been the rule during ordinary business recessions, when the fall in prices led to a decrease in the real demand for cash balances, but this failed to result in a positive monetary effect, because a wave of liquidations caused the real quantity of bank money to decrease by an even greater amount.

[2] For instance, speculation was defended by the argument that it stabilizes prices in the short run.

effects and expansion effects, and by the stabilizing intertemporal substitution effects which resulted from price flexibility. Price flexibility was successful as a long-run stabilizer of the economy, not only because the conditions under which it produces the equilibrating effects expected from it by traditional doctrine were approximately fulfilled, but also because the task it faced was an easy one.

In our present capitalist economy, the forces that elicit oversaving, exhaustion of "investment opportunities," and "technological unemployment" have greatly increased in strength. Simultaneously, the conditions which endowed price flexibility with a long-run stabilizing influence upon the economy (in particular the flexibility of prices of factors of production) have largely disappeared. The experiences of two world wars, of political and social upheavals, of war and postwar inflations, of the great depression which fell from what appeared to be a clear sky, in the mind of the business world; all these have shattered the belief in a long-run "normal." Whatever our efforts, this belief will not be restored for a long time; it may take a generation or more to do so. Without specific government intervention designed to influence them, long-range price expectations are likely to be elastic for a long time to come. The great uncertainty of price expectations ("lack of confidence"[3]) that results from the experiences mentioned has made intertemporal substitution and, consequently, the demand for investment goods, highly insensitive to changes in interest rates.[4] The growth of oligopolistic and oligopsonistic groups to a dominant position in present-day capitalism prevents a positive monetary effect of a change in prices from being translated into an increase in output or an increase in demand for factors of production. The disappearance of atomistic conditions in international trade (among the different countries involved) has undermined the stabilizing influence of this trade. All this renders price flexibility inapplicable under present conditions as a norm of either long-run or short-run economic policies.

In view of the fact that long-range price expectations are, under present conditions, likely to be elastic, an automatically working unresponsive monetary system (in the long run) cannot be relied upon to produce positive monetary effects. In order to produce such effects it is necessary to have monetary management planned in accordance with our *General Rule*. Such

[3] Cf. Jacob Marschak, "Lack of Confidence," *Social Research*, Vol. 8, 1941, pp. 41–62.

[4] See pp. 61–62 above. Cf. also, J. E. Meade and P. W. S. Andrews, "Summary of Replies to Questions on Effects of Interest Rates," *Oxford Economic Papers*, No. 1, October, 1938, pp. 14–31; R. S. Sayers, "Business Men and the Terms of Borrowing," *ibid.*, No. 3, February, 1940, pp. 23–31; and P. W. S. Andrews, "A Further Inquiry into the Effects of Rates of Interest," *ibid.*, No. 3, February, 1940, pp. 32–73. See also, J. F. Ebersole, "The Influence of Interest Rates upon Entrepreneurial Decisions in Business—A Case Study," *Harvard Business Review*, Vol. 17, Autumn, 1938, pp. 35–39.

management requires that any increase in the real demand for cash balances associated with falling prices be met by an even larger increase in the real quantity of money. This presupposes the abandonment of revision of the gold standard, as well as of the present banking system, in which creation of credit money is not subject to the control and effective influence of an authority charged with responsibility for the maintenance of the stability of the economy.[5] The adoption of a monetary system, managed according to our *General Rule*, so that price changes may be accompanied by a positive monetary effect, is the first condition of economic stability in our times. This condition, however, is not sufficient, because the positive monetary effect is likely to fail to produce the intratemporal substitution effects and expansion effects necessary for the stabilization of the economy.

If the positive monetary effect results merely in a fall of interest rates, monetary policy must be supplemented by measures which assure directly an increase in the demand for commodities. This requires subsidies given directly to consumers (who will use the money to buy commodities, not securities) or a direct exercise of demand for investment goods by the government (i.e., public investment). Most economists think that such measures have a multiplier effect which causes an initial government expenditure (consumers' subsidy, as well as public investment) to be followed up by an increased demand for commodities by private firms. Such a multiplier effect, however, may be heavily reduced, or even thwarted, by oligopolistic and oligopsonistic group behavior. This will happen under oligopoly, if the initial government expenditure strengthens the discipline of the oligopolistic groups to such an extent that it spends itself largely, or entirely, in an increase of the degree of monopoly. Under oligopsony, the multiplier effect will be thwarted, even without an increase in the discipline of the group, if the shift of the marginal-value-productivity curve, resulting from a change in the demand for the commodity, is confined within the range of discontinuity of the marginal-expenditure curve. Thus, in order to be effective, the government spending must be either on such an enormous scale as to achieve its results even without, or with only a slight, multiplier effect, or it must be associated with a policy that dissolves the oligopolistic and oligopsonistic groups.

The dissolution of these groups cannot take the form of a return to perfect, or even monopolistic and monopsonistic, competition among private firms. The formation of monopolistic and monopsonistic group behavior is not merely the result of "greed for profit." Rules of oligopolistic and oligopsonistic group behavior emerge because, without them, no firm would be able to

[5] The Hundred Per Cent Reserve Plan is one of the means by which the determination of the quantity of money may be effectively concentrated in the hands of an authority charged with the responsibility mentioned.

predict the reaction of other firms to a change of its price.[6] These rules perform the social function of making such prediction possible. Mere removal of these rules would not establish perfect competition, nor even monopolistic or monopsonistic competition, in the Chamberlin sense, for all these forms of entrepreneurial response require that the effects of a change of a firm's price be spread evenly over so many other firms that none of them reacts. Oligopolistic and oligopsonistic group behavior arises where this basic prerequisite of both perfect and monopolistic or monopsonistic competition is not satisfied. Consequently, the mere removal of oligopolistic and oligopsonistic rules would result in general unpredictability of other firms' reactions, a state which may be fittingly described as "oligopolistic (or oligopsonistic) chaos." These rules must, therefore, be replaced by new rules, namely, by rules of public policy, drawn in the interest of the efficient functioning of the economy. In some cases, this may be achieved by subjecting the firms in the oligopolistic or oligopsonistic group to regulation by public agencies that formulate and enforce the new rules. This may work when the group consists of a large number of small firms that were obliged to adopt oligopolistic or oligopsonistic group behavior (frequently with the aid of governmental "fair trade" legislation), because the effects of each firm's price policy are concentrated upon a very small number of other firms. But, in most cases of obnoxious oligopoly and oligopsony, socialization of the respective industries or trades appears to be the only means of securing their operation according to rules compatible with the stability and efficiency of the economy.[7]

Without all the measures described, price flexibility cannot act, under present conditions, as a stabilizing influence in the economy; it may even act as a destabilizing force. But, if these measures are adopted, flexibility of *all* prices proves to be superfluous. The desired positive monetary effect can be obtained directly by means of monetary management. Furthermore, if long-range price expectations are prevailingly elastic, less monetary management, and also less government expenditure, are required when some important price (or prices) in the economy is (or are) rigid, than when all prices are perfectly flexible. For, rigidity of some important[8] price (or prices) will produce directly the intratemporal substitution effects and expansion effects needed to stabilize the economy, without going through a general fall of prices, a consequent increase in the real demand for cash balances, and, finally, a greater increase in the real quantity of money to create a positive

[6] Cf. pp. 39–40 above.

[7] The proper rules of operation to be applied by socialized enterprises are discussed in the present writer's essay *On the Economic Theory of Socialism* (Minneapolis: University of Minnesota Press, 1938).

[8] By "important" price, we mean, in this context, the price of a commodity which is a large item in the community's aggregate expenditure.

monetary effect.[9] Thus, the less flexible some important price (or prices), the less monetary management is required, and the less may the effects of monetary management be thwarted by oligopolistic and oligopsonistic group behavior.[10] This indicates the advisability of fixing the price of some important commodity in the economy.

Since changes of relative prices of goods perform the important function of making the allocation of resources responsive to consumers' demand, only one price should be fixed, and all other prices should be flexible, so that relative prices can adjust themselves to the choices of consumers and to relative marginal costs of production.[11] In order that the price fixing lead to practical results, a commodity must be chosen that stands in substitution and output relations with the greatest number of other commodities. In an industrial country, labor is the most appropriate commodity for this purpose, and fixing of money wage rates[12] will serve best as a means of securing stability of the economy. In an agricultural country, some staple product (e.g., wheat, coffee, sugar) may be chosen for this purpose. In the United States, it may be desirable to fix both money wage rates and the price of some staple agricultural product, with a long-term adjustment of the two, according to changes in the marginal labor-cost of the product chosen. Such price fixing would not only reduce the amount of monetary management required to secure the proper operation of the economy, but would also remove the possibility that flexible prices might turn into a source of instability of the economy, if errors were made in monetary management.

Instead of fixing the price of one important commodity in the economy, one may choose to stabilize the general level of commodity prices. This, too, would secure the operation of the intratemporal substitution effects and

[9] Foreign trade, in an atomistic international market (with regard to the different countries), performs the same function as price rigidity, i.e., it secures the direct operation of substitution effects and expansion effects, without need of a positive monetary effect. Cf. pp. 45–47 above.

[10] An economy, in which oligopolistic and oligopsonistic group behavior has been abolished by socialization of the pertinent industries and trade, can, therefore, "tolerate" a much larger dose of price flexibility than the economy of oligopolistic and oligopsonistic capitalism.

[11] This would preserve what Professor Hansen calls structural price flexibility (cf. *Fiscal Policy and Business Cycles*, Norton and Co., New York, 1941, pp. 313–314). It must be mentioned, however, that such adjustment serves the purpose mentioned only in absence of monopoly and monopsony (including monopolistic and monopsonistic competition), as well as in the absence of oligopoly and oligopsony, i.e., only under perfect competition, or under appropriate "rules of the game," established by socialization (or, in certain cases, also by public regulation of private firms).

[12] This means, in practice, stabilizing an index number of money wage-rates and letting relative wage-rates adjust freely to each other.

expansion effects needed to maintain stability of the economy. It would do so by preventing commodity prices from changing all in the same proportion (or even direction). The stabilization of the general level of commodity prices, however, would require much more monetary management than the fixing of the price of a single commodity (or of the price level of a very small group of commodities), and it would also increase the destabilizing consequences of errors in monetary management. The arguments in favor of the latter are thus similar to some of the arguments in favor of the gold standard and against a "price-index standard." The gold standard implies fixing the money price of one commodity, namely gold, and is similar to our proposal to fix one price in the economy. But gold is a rather poor commodity to choose for this purpose. It has direct substitution and output relations with an extremely small number of other commodities. From the point of view of facilitating the operation of intratemporal substitution and expansion effects and thus securing full employment of factors of production, the gold standard has no more merit than (say) the pepper standard. Notwithstanding, its basic idea, that of fixing the money price of one commodity, is perfectly sound. But the commodity chosen for this purpose must be such as to provide the greatest facility for the operation of intratemporal substitution effects and expansion effects in the economy.

That stabilization of certain prices is necessary, in order to prevent instability of the economy, is commonly recognized during periods of inflationary pressure, as in our present war economy. If we followed the traditional doctrine of regarding price flexibility as a norm of economic policy, we should let prices, including factor prices, such as wages, rise, and rely upon the rise in prices to produce automatically the substitution effects and expansion effects that will absorb excess demand in all markets. We know well that this will not work, unless the decrease in the real demand for cash balances (which, under prevailingly elastic price expectations, results from generally rising prices) is met by an even greater reduction in the real quantity of money, i.e., unless proper management makes the monetary system responsive. We know, furthermore, that this is not sufficient, and that it would not work if the result were merely to raise interest rates. Consequently, the demand for commodities must be reduced directly, by means such as taxation, forced saving, rationing of expenditures, etc. Finally, in order to insure that the decrease in the demand for products is not prevented, by oligopolies and oligopsonies, from being translated into a reduced demand for factors of production, we regulate the output and input directly by means of such measures as licensing of output, priorities on certain factors of production, etc. (this is being done as a substitute for the dissolution of oligopolistic and oligopsonistic groups, which would make the price mechanism applicable also in a war economy). We also know that the amount of monetary manage-

ment necessary in order to prevent inflation and a cumulative increase of the excess demand for commodities can be reduced greatly by fixing important prices, such as money wage-rates,[13] and that this also helps to minimize the consequences of possible errors in monetary management.

The arguments that apply with regard to maintaining the stability of the economy in the face of the danger of a cumulative increase in the excess demand for commodities also apply with regard to maintaining the stability of the economy, in the face of the danger of a cumulative increase in the excess supply of some or all factors of production.

[13] Strangely enough (or, if the sociology of the case is taken into account, not strangely at all), the people who insist upon the necessity of keeping money-wages flexible—in times of depression—are the same ones who demand a ceiling on money wages to prevent inflation.

APPENDIX

The Stability of Economic Equilibrium*

1. The Hicksian Conditions

The theory of stability of economic equilibrium is based on the assumption that an excess demand for a good causes a rise in its price, while an excess supply causes a fall in price. The equilibrium is thus said to be stable when, in the neighborhood of the equilibrium position, a price above the equilibrium price causes excess supply and a price below the equilibrium price causes excess demand. This condition was first stated by Walras. Walras, however, formulated it in a way which limits its applicability to partial-equilibrium analysis. Within the framework of general-equilibrium theory the stability conditions must take into account the repercussions of the change in price of a good upon the prices of other goods as well as the dependence of excess demand (or excess supply) of a good on the prices of the other goods in the system. This has been done by Professor Hicks.[1]

According to Professor Hicks, the economic system is in stable equilibrium if a rise of the price of any good above the equilibrium price causes an excess supply of and a fall of the price below the equilibrium price causes an excess demand for that good, *when the prices of all other goods in the system are so adjusted as to maintain equilibrium in all other markets.* Otherwise the system is either in unstable or in neutral equilibrium. The former is the case when a rise of the price above the equilibrium price produces excess demand and a fall of the price produces excess supply; the latter is the case when no excess demand or excess supply is produced. In both cases adjustment of all other prices maintaining equilibrium in the other markets is presupposed. This formulation of the theory of stability of equilibrium leads to a series of conditions which are best formulated mathematically.

Let there be $n+1$ goods in the economy and let one of them, say the $(n+1)$th, serve as money and *numéraire.* Denote by $p_r (r = 1, 2, \cdots, n)$ the price of the rth good; $p_{n+1} = 1$ by definition. Write further $D_r(p_1, p_2, \cdots, p_n)$ for the demand function and $S_r(p_1, p_2, \cdots, p_n)$ for the supply function of the rth good. We have then n independent excess-demand functions X_r defined by

$$(1.1)\quad X_r(p_1, p_2, \cdots, p_n) \equiv D_r(p_1, p_2, \cdots, p_n) - S_r(p_1, p_2, \cdots, p_n) \qquad (r = 1, 2, \cdots, n).$$

The system is in equilibrium when $X_r = 0$ $(r = 1, 2, \cdots, n)$. The equilibrium is stable when, at the equilibrium point,

* Also issued as Cowles Commission Papers, New Series, No. 8.

[1] See *Value and Capital*, pp. 66 ff. and pp. 315–316.

$$\begin{aligned} &\frac{dX_r}{dp_r} < 0, \\ &\frac{dX_s}{dp_r} = 0 \quad (s \neq r), \end{aligned} \qquad (r \text{ and } s = 1, 2, \cdots, n). \tag{1.2}$$

The inequality indicates negative excess demand (i.e., excess supply) when the price rises above equilibrium, and positive excess demand when the price falls below equilibrium. The equations indicate that the prices in the other markets are adjusted to maintain equilibrium in these markets.[2]

Let us write

$$a_{sr} \equiv \frac{\partial X_s}{\partial p_r} \qquad (r \text{ and } s = 1, 2, \cdots, n). \tag{1.3}$$

We have

$$\frac{dX_s}{dp_r} = a_{s1}\frac{dp_1}{dp_r} + a_{s2}\frac{dp_2}{dp_r} + \cdots + a_{sr} + \cdots + a_{sn}\frac{dp_n}{dp_r} \qquad (s = 1, 2, \cdots, n). \tag{1.4}$$

Exchanging places between dX_s/dp_r and a_{sr}, and taking into account (1.2), we obtain the equations

$$\begin{aligned} -a_{1r} &= a_{11}\frac{dp_1}{dp_r} + a_{12}\frac{dp_2}{dp_r} + \cdots + 0 + \cdots + a_{1n}\frac{dp_n}{dp_r}, \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ -a_{rr} &= a_{r1}\frac{dp_1}{dp_r} + a_{r2}\frac{dp_2}{dp_r} + \cdots - \frac{dX_r}{dp_r} + \cdots + a_{rn}\frac{dp_n}{dp_r}, \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ -a_{nr} &= a_{n1}\frac{dp_1}{dp_r} + a_{n2}\frac{dp_2}{dp_r} + \cdots + 0 + \cdots + a_{nn}\frac{dp_n}{dp_r}. \end{aligned} \tag{1.5}$$

[2] The stability conditions can also be expressed in terms of excess demand for money. The aggregate value of the excess demand in all n markets is the excess supply of money, i.e.,

$$-X_{n+1} = \sum_{s=1}^{n} p_s X_s.$$

All other markets remaining in equilibrium, we have $X_s = 0$ for $s \neq r$ and

$$-\frac{dX_{n+1}}{dp_r} = X_r + p_r\frac{dX_r}{dp_r}.$$

At the equilibrium point $X_r = 0$ and, by virtue of (1.2),

$$\frac{dX_{n+1}}{dp_r} > 0 \qquad (r = 1, 2, \cdots, n). \tag{1.3}$$

Thus the excess demand for money must become positive when the price of a good other than money rises above equilibrium, and become negative when the price falls below equilibrium.

Write

$$(1.6) \qquad J \equiv \begin{vmatrix} a_{11} & a_{12} \cdots a_{1n} \\ a_{21} & a_{22} \cdots a_{2n} \\ \cdots & \cdots \\ a_{n1} & a_{n2} \cdots a_{nn} \end{vmatrix}$$

and denote by J_{rs} the cofactor of a_{rs}. Solving the equations (1.5) we get

$$(1.7) \qquad \frac{dX_r}{dp_r} = \frac{J}{J_{rr}} < 0 \qquad (r = 1, 2, \cdots, n).$$

This is negative because of (1.2).

Amplifying and modifying Professor Hicks's terminology, we introduce the concept of *partial stability* of different order and rank. The system is said to be partially stable of *order* m ($m \leqq n$) if (1.2) is satisfied when only m other prices are adjusted and *the remaining prices are kept constant.*[3] By a procedure analogous to that leading to (1.7) we obtain as a condition of partial stability of order m

$$(1.8) \qquad \left(\frac{dX_r}{dp_r}\right)_{n-m} = \frac{J_{ss,\cdots,s-m}}{J_{ss,\cdots,s-m,rr}} < 0 \quad (r = 1, 2, \cdots, m),$$

where the numerator and the denominator are cofactors of J of order m and $m-1$ respectively. The subscript on the left-hand side indicates which prices are kept constant (namely, $m+1$, $m+2$, $\cdots$, n). The concept of partial stability is always relative to the prices which are kept constant. The system may be partially stable of order m if certain $n-m$ prices are held constant but may fail to be so if $n-m$ other prices are kept constant. When the system is partially stable of order n (n being the number of goods, exclusive of money) we say that it is *totally* stable. The condition (1.8) then turns into (1.7).

The system is said to be stable of *rank* m (and *unstable or neutral of rank* $n-m$) if it is partially stable of order m but not of any higher order. The rank of the stability of the system is thus the highest order of partial stability it possesses. A totally stable system has stability of rank n.

Partial stability of order m is said to be *perfect* when the system shows partial stability of *all* lower orders with respect to *any* prices being held constant. Otherwise the partial stability is said to be imperfect. This definition of perfect partial stability applies also to partial stability of order n, i.e., to total stability. In virtue of (1.8) the condition for perfect stability of order m can be written

[3] In this case (1.2) holds for r and $s = 1, 2, \cdots, m$.

$$(1.9)\quad a_{11} < 0, \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} > 0, \cdots, \text{sign} \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \cdot & \cdot & \cdots & \cdot \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{vmatrix} = \text{sign}\,(-1)^m,$$

the numeration of the goods being, of course, arbitrary. These are the Hicksian conditions for perfect stability.[4]

2. *Dynamic Stability Conditions*

The reader will have noticed that in the mathematical formulation of the theory of stability of economic equilibrium the basic assumption of that theory, namely that excess demand for a good makes its price rise and excess supply makes it fall, does not appear explicitly. This assumption, however, is tacitly implied in the choice of the condition that excess demand should occur when the price is below equilibrium and excess supply should occur when it is above equilibrium. In order to clarify all the implications of stability analysis the basic assumption mentioned must be explicitly introduced into the mathematical formulation of the theory of stability equilibrium. When this is done, stability analysis becomes part of a dynamic theory, as was shown recently by Professor Samuelson.[5] The traditional method of treating the stability of economic equilibrium, as applied by Walras, Marshall, and Hicks, is but an implicit (and therefore imperfect) form of dynamic analysis.

The basic assumption of stability analysis, i.e., that excess demand causes the price to rise and excess supply causes it to fall, can be formulated as follows:

$$(2.1)\qquad \text{sign}\,\frac{dp_r}{dt} = \text{sign}\,X_r \qquad (r = 1, 2, \cdots, n),$$

where dp_r/dt is the rate of change of price over time. Let

$$(2.2)\qquad \frac{dp_r}{dt} = F_r(X_r) \qquad (r = 1, 2, \cdots, n)$$

be a set of functions which satisfy the relations (2.1). Then by (2.1) we have

$$(2.3)\qquad F_r(0) = 0 \qquad (r = 1, 2, \cdots, n),$$

as the equilibrium conditions of the system.

In (2.2) we have a normal system of n differential equations which has the

[4] Professor Hicks limits the concept of perfect stability to total stability. The conditions for perfect stability given by him are thus only for the case $m = n$.

[5] "The Stability of Equilibrium: Comparative Statics and Dynamics," *Econometrica*, Vol. 9, April, 1941, pp. 97–120.

solutions $p_r(t)$ $(r=1, 2, \cdots, n)$.[6] The functions $p_r(t)$ are the adjustment paths of the prices and the equilibrium is stable when these paths lead back to the equilibrium prices, unstable when they lead away from them, and neutral when neither is the case.[7] Expressing all prices in terms of deviations from the equilibrium prices, i.e., putting the latter equal zero, we thus have stable equilibrium when

$$\lim_{t=\infty} p_r(t) = 0 \qquad (r = 1, 2, \cdots, n). \tag{2.4}$$

In order to solve the equations we expand, on the right-hand side of (2.2), F_r and X_r by Maclaurin's theorem and retain only the linear part of the expansion. Expanding F_r, we have

$$\frac{dp_r}{dt} = F'(0)X_r \qquad (r = 1, 2, \cdots, n),$$

and then, expanding X_r, we obtain

$$\frac{dp_r}{dt} = F_r'\sum_{s=1}^{n} a_{rs}p_s \qquad (r = 1, 2, \cdots, n), \tag{2.5}$$

where p_s is expressed as a deviation from the equilibrium price $p_s^0=0$. $F_r' \equiv F_r'(0) = \text{const.}$ and $a_{rs}^0 = a_{rs}(p_1^0, p_2^0, \cdots, p_n^0) = \text{const.}$ We have now a system of linear equations with constant coefficients.

It will be noticed that in view of (2.1)

$$F_r'(0) > 0 \qquad (r = 1, 2, \cdots, n). \tag{2.6}$$

Thus when the functions on the right-hand side of (2.2) are taken as linear in X_r the basic assumption of stability analysis implies necessarily that *the speed of increase of price is the greater the greater the excess demand.* $F_r'(0)$ may serve as a measure of the flexibility of the price p_r. In general it will be said that the price is flexible when $F_r'(0)>0$, inflexible, or rigid, when $F_r'(0)=0$, and negatively flexible when $F_r'(0)<0$. The last two cases are excluded by (2.6).

The solution of the linear system (2.5) is given by the set of functions

$$p_r(t) = \sum_{s=1}^{k} q_{rs}(t)e^{\lambda_s t} \qquad (r = 1, 2, \cdots, n), \tag{2.7}$$

[6] It is assumed that the existence conditions are satisfied. This is always the case when the functions F_r and X_r $(r=1, 2, \cdots, n)$ and their first derivatives are continuous.

[7] These definitions are broader than those on the first page of this Appendix and include the latter as a special case.

where the λ_s ($s=1, 2, \cdots, k$) are the k ($k \leqq n$) distinct roots of the characteristic equation[8]

$$(2.8) \qquad f(\lambda) \equiv \begin{vmatrix} F_1'a_{11}^0 - \lambda & F_1'a_{12}^0 & \cdots & F_1'a_{1n}^0 \\ F_2'a_{21}^0 & F_2'a_{22}^0 - \lambda & \cdots & F_2'a_{2n}^0 \\ \cdots & \cdots & \cdots & \cdots \\ F_n'a_{n1}^0 & F_n'a_{n2}^0 & \cdots & F_n'a_{nn}^0 - \lambda \end{vmatrix} = 0,$$

and the $q_{rs}(t)$ are polynomials in t of degree one less than the multiplicity of the root λ_s.[9] Of the coefficients of the polynomials n are arbitrary and determined by the initial conditions (i.e., by the initial disturbance of equilibrium), the remaining coefficients are found from a system of homogeneous linear equations with matrix of coefficients as given in (2.8).

Let the roots be complex and write

$$(2.9) \qquad \lambda_s = R(\lambda_s) + I(\lambda_s) \qquad (s = 1, 2, \cdots, k),$$

where the two terms on the right-hand side indicate the real and the imaginary part respectively. This includes real roots as a special case in which $I(\lambda_s)=0$. Writing $I(\lambda_s)=\beta i$, we have

$$(2.10) \qquad e^{\lambda_s t} = e^{R(\lambda_s)t}(\cos \beta t + i \sin \beta t).$$

The equilibrium is thus stable, i.e. (2.4) is satisfied, when

$$(2.11) \qquad R(\lambda_s) < 0 \qquad \text{for } s = 1, 2, \cdots, k.$$

This is the stability condition which in the dynamic theory replaces the static condition (1.7). If some $R(\lambda_s)>0$ we get $\lim_{t=\infty} p_r(t) = \pm \infty$ ($r=1, 2, \cdots, n$), and the equilibrium is unstable. If some $R(\lambda_s)=0$ and no $R(\lambda_s)>0$ the equilibrium is neutral.

As in the static theory, we introduce the concepts of *partial stability* of a given order and of rank of stability of the system. The dynamic system is partially stable of *order* m if it is stable when only m prices are allowed to adjust themselves and the other $n-m$ prices are kept constant. This implies that

$$(2.12) \qquad F_r' \equiv 0 \qquad \text{for } r = m+1, \cdots, n$$

and

$$(2.13) \qquad p_s \equiv p_s^0 = 0 \qquad \text{for } s = m+1, \cdots, n.$$

[8] Professor Samuelson (*op. cit.*, pp. 109–110) leaves out the factors F_r' in the characteristic determinant. This can be done only when $F_1' = F_2' = \cdots = F_n'$. His results thus hold only for the special case where the flexibility of all prices in the system is the same.

[9] Thus when λ_s is a simple root the corresponding polynomials $q_{rs}(t)$ ($r=1, 2, \cdots, n$) reduce to constants.

The system of equations (2.5) turns into

$$\frac{dp_r}{dt} = F_r' \sum_{s=1}^{m} a_{rs} p_s \qquad (r = 1, 2, \cdots, m) \tag{2.14}$$

and the solutions become

$$p_r(t) = \sum_{s=1}^{k} q_{rs}(t) e^{\lambda_s t} \quad (r = 1, 2, \cdots, m; k \leqq m). \tag{2.15}$$

The condition for partial stability of order m is given, as before, by (2.11) except that the λ_s are roots of a characteristic equation of order m. The characteristic determinant of this equation is a principal minor of order m of the characteristic determinant in (2.8).

When the dynamic system is partially stable of order n we say that it is *totally* stable. The highest order of partial stability of the system is called the *rank of the stability* of the system.

When the characteristic determinant is symmetric all roots are real.[10] In order that they be all negative it is necessary and sufficient[11] that the Hicksian conditions (1.9) be satisfied. Dynamic partial stability of order m thus requires and implies *perfect* Hicksian stability of the same order. This is clear: symmetry of the characteristic determinant of order m implies (and requires) symmetry of all its principal minors.

3. *Implications of the Validity of the Hicksian Conditions*

The Hicksian conditions for perfect stability are equivalent to the dynamic stability conditions when the characteristic determinant of order m is symmetric. Let us examine the economic meaning of such symmetry. We have from (2.2)

$$F_r' = \frac{d}{dX_r}\left(\frac{dp_r}{dt}\right) \qquad (r = 1, 2, \cdots, m). \tag{3.1}$$

Taking into account (1.3) we obtain

$$F_r' a_{rs} = \frac{\partial}{\partial p_s}\left(\frac{dp_r}{dt}\right) \qquad (r \text{ and } s = 1, 2, \cdots, m). \tag{3.2}$$

[10] We assume that the $F_r' a_{rs}$ are all real, and apply the well-known theorem about the characteristic (or secular) equation proved in the theory of determinants. Cf., for instance, G. Kowalewski, *Einführung in die Determinantentheorie* (Berlin and Leipzig, 1925), pp. 114 ff.; H. W. Turnbull and A. C. Aitken, *An Introduction to the Theory of Canonical Matrices* (London and Glasgow), p. 101. A very simple proof is given by F. R. Moulton, *Differential Equations* (New York, 1930), pp. 298–299.

[11] This is the fundamental theorem about definite Hermitian forms. Cf. Kowalewski, *op. cit.*, p. 199.

The symmetry $F_r'a_{rs} = F_s'a_{sr}$ thus implies

$$\frac{\partial}{\partial p_s}\left(\frac{dp_r}{dt}\right) = \frac{\partial}{\partial p_r}\left(\frac{dp_s}{dt}\right) \qquad (r \text{ and } s = 1, 2, \cdots, m), \tag{3.3}$$

i.e., *the marginal effect of a change in the price p_s upon the speed of adjustment of the price p_r equals the marginal effect of a change in the price p_r upon the speed of adjustment of the price p_s.*[12]

The symmetry of the marginal effect of a change in one price upon the speed of adjustment of another price can be clarified further by a mathematical consideration. The symmetry conditions (3.3) are the sufficient conditions for the integrability of the total differential equation

$$\sum_{r=1}^{m} \frac{dp_r}{dt}\, dp_r = 0. \tag{3.4}$$

When conditions (3.3) hold, there exists a function (or rather a class of functions)[13]

$$P[p_1(t), p_2(t), \cdots, p_m(t)] \tag{3.5}$$

[12] It has been held by some economists that, in order that static equilibrium and stability analysis be applicable, the speed of adjustment must be the same in each market. This view was expressed by S. Kohn ("On the Problems of the Modern Theory of Price and Value," *Economista*, 1925, in Polish); by P. N. Rosenstein-Rodan ("Das Zeitmoment in der Mathematischen Theorie des wirtschaftlichen Gleichgewichtes," *Zeitschrift für Nationalökonomie*, Vol. 1, 1930, pp. 129–142, and "The Role of Time in Economic Theory," *Economica*, N.S., Vol. 1, February, 1934, pp. 90–91); and by Simon Kuznets ("Equilibrium Economics and Business-Cycle Theory," *Quarterly Journal of Economics*, Vol. 44, February, 1930, p. 404). As shown above, this is wrong. The condition of applicability of static analysis is not equality of the speed of price adjustment in each market, but the symmetry of the cross effects of a change in one price upon the speed of adjustment of the other, as indicated in (3.3). This symmetry is similar to the Hotelling conditions in the pure theory of demand or supply without budget limitations ("Edgeworth's Taxation Paradox and the Nature of Demand and Supply Functions," *Journal of Political Economy*, Vol. 40, October, 1932, pp. 591 and 594). These conditions are

$$\frac{\partial D_r}{\partial p_s} = \frac{\partial D_s}{\partial p_r}$$

and

$$\frac{\partial S_r}{\partial p_s} = \frac{\partial S_s}{\partial p_r} \qquad (r \text{ and } s = 1, 2, \cdots, n).$$

If these conditions are satisfied we have, on account of (1.1) and (1.3), $a_{rs} = a_{sr}$ (r and $s = 1, 2, \cdots, n$). If $F_r' = F_s'$ (r and $s = 1, 2, \cdots, n$), this implies the fulfillment of the condition (3.3). Thus, when the flexibility of all prices is the same, the condition of applicability of static-equilibrium and stability analysis is identical with the Hotelling conditions for demand and supply functions.

[13] If P is a solution of the equation then any function $\phi(P)$ such that $\phi'(P) \neq 0$ is also a solution.

such that

$$\frac{dp_r}{dt} = \frac{\partial P}{\partial p_r(t)} \qquad (r = 1, 2, \cdots, m), \tag{3.6}$$

i.e., such that the speeds of adjustments are its partial derivatives. The equation (3.4) can be interpreted as the maximum condition of this function (or class of functions[14]). The adjustment paths $p_r(t)$ $(r=1, 2, \cdots, m)$ are then co-ordinated into a consistent system maximizing this function. The function P may, therefore, be called the *adjustment potential*, and a dynamic system for which an adjustment potential exists will be called an *integrated system; m* will be called the *order of integration* of the system. From (3.3) we see that when the system is integrated of order m it is also integrated in all lower orders. The Hicksian conditions provide the sufficient[15] conditions of (partial) stability (of order m; $m \leqq n$) for integrated (of order m) dynamic systems.

The economic meaning of an integrated system can be illustrated as follows. Suppose that the m adjustment paths $p_r(t)$ $(r=1, 2, \cdots, m)$ are determined by a planning authority that wants to maximize at each moment the total welfare of the community. The adjustment paths must then satisfy the maximum conditions of a function like (3.5). As atomistic competition automatically produces (though not without important qualifications) maximum total welfare within a static system, similarly a dynamic system *may*, under appropriate circumstances, imply the maximization of a potential function which serves as an indicator of total welfare.

4. *Homogeneous Systems*

Consider a system consisting of $n+1$ goods and suppose that the $(n+1)$th good functions as money. Let the excess-demand functions of m goods other than money $(m<n)$ be homogeneous of zero degree in the prices of these goods,[16] and let the excess-demand functions of the remaining $n-m$ goods

[14] The second-order maximum conditions are given by the Hicksian inequalities (1.9). In order to satisfy these, the functions $\phi(P)$ must be restricted to cases where $\phi'(P)>0$.

[15] The conditions of integrability of (3.4) are that $\frac{1}{2}(m-1)(m-2)$ equations of the form

$$\frac{dp_r}{dt}\left[\frac{\partial}{\partial p_s}\left(\frac{\partial p_s}{\partial t}\right) - \frac{\partial}{\partial p_s}\left(\frac{\partial p_t}{\partial t}\right)\right] + \frac{dp_s}{dt}\left[\frac{\partial}{\partial p_r}\left(\frac{\partial p_t}{\partial t}\right) - \frac{\partial}{\partial p_t}\left(\frac{\partial p_r}{\partial t}\right)\right] + \frac{dp_t}{dt}\left[\frac{\partial}{\partial p_s}\left(\frac{\partial p_r}{\partial t}\right) - \frac{\partial}{\partial p_r}\left(\frac{\partial p_s}{\partial t}\right)\right] = 0$$

are satisfied; for this, (3.3) is sufficient but not necessary.

[16] A function $f(x_1, x_2, \cdots, x_m; x_{m+1}, \cdots, x_n)$ is said to be homogeneous of the kth degree *in the variables* $x_1, x_2, \cdots, x_m$ if, for every k, $f(kx_1, kx_2, \cdots, kx_m; x_{m+1}, \cdots, x_n) = k^k f(x_1, x_2, \cdots, x_m; x_{m+1}, \cdots, x_n)$.

other than money be homogeneous of first degree in the same prices. We shall prove that such a system has the following properties:

(1) The excess-demand function for money is homogeneous of first degree in the same m prices.

(2) The system is neutral of rank not less than one and the rank of stability of the system does not exceed $n-1$;

(3) The equilibrium value of one of the m prices in which the excess-demand functions are homogeneous of zero degree is arbitrary and the equilibrium values of the other $m-1$ of these prices are proportional to the arbitrary equilibrium price.

In order to fix ideas assume that the excess-demand functions $X_1, X_2 \cdots, X_m$ are homogeneous of zero degree in the prices $p_1, p_2, \cdots, p_m$ and that the excess-demand functions $X_{m+1}, X_{m+2}, \cdots, X_n$ are homogeneous of first degree in the same variables. We observe that the relation

$$\sum_{r=1}^{m} p_r X_r + \sum_{r=m+1}^{n} p_r X_r + X_{n+1} \equiv 0 \tag{4.1}$$

holds between the $n+1$ excess-demand functions. This relation is an identity in the p's and may be called *Walras' law*.[17] If the prices $p_1, p_2, \cdots, p_m$ are multiplied by an arbitrary number k and the prices $p_{m+1}, \cdots, p_n$, are kept constant, each of the expressions under the summation sign in (4.1) is increased k-fold, for in the first expression the p's are increased k-fold and the X's are unchanged, while in the second expression the p's are unchanged and the X's are increased k-fold. It follows from the identity that X_{n+1} is also increased k-fold. This proves the first property of our system.

Applying Euler's theorem, we have

$$\sum_{s=1}^{m} a_{rs} p_s = 0 \qquad \text{for } r = 1, 2, \cdots, m \tag{4.2}$$

and

$$\sum_{s=1}^{m} a_{rs} p_s = X_r \quad \text{for } r = m+1, m+2, \cdots, n, \tag{4.3}$$

where a_{rs} is defined as in (1.3). Putting the equilibrium prices p_r^0 ($r=1, 2, \cdots, n$) into (4.2) and (4.3) and remembering that $X_r(p_1^0, p_2^0, \cdots, p_n^0) = 0$ ($r=1, 2, \cdots, n$), we obtain

[17] For a special case (the foreign-exchange markets) this relation was known already to Cournot (cf. *Researches into the Mathematical Principles of the Theory of Wealth*, trans. by T. Bacon; New York: Macmillan Co., 1927, pp. 33–34). Walras, however, was the first to give it a general mathematical formulation and to recognize its importance for the theory of prices. See his *Eléments d'économie politique pure* (édition définitive; Paris and Lausanne, 1926), pp. 120–121.

$$\sum_{s=1}^{n} a_{rs}^{0} p_s^{0} = 0 \qquad (r = 1, 2, \cdots, n), \tag{4.4}$$

where $a_{rs}^{0} = a_{rs}(p_1^{0}, p_2^{0}, \cdots, p_n^{0})$.

Consider now the determinant

$$J^{0} = \begin{vmatrix} a_{11}^{0} & a_{12}^{0} \cdots a_{1n}^{0} \\ a_{21}^{0} & a_{22}^{0} \cdots a_{2n}^{0} \\ \cdots\cdots\cdots\cdots \\ a_{n1}^{0} & a_{n2}^{0} \cdots a_{nn}^{0} \end{vmatrix}. \tag{4.5}$$

Multiply the first column by p_1^{0}, add the second column multiplied by p_2^{0}, etc., finally add the mth column multiplied by p_m^{0}. The result is the determinant

$$\begin{vmatrix} \sum_{s=1}^{n} a_{rs}^{0} p_s^{0} & a_{12}^{0} \cdots a_{1n}^{0} \\ \sum_{s=1}^{n} a_{rs}^{0} p_s^{0} & a_{22}^{0} \cdots a_{2n}^{0} \\ \cdots\cdots\cdots\cdots \\ \sum_{s=1}^{n} a_{rs}^{0} p_s^{0} & a_{n2}^{0} \cdots a_{nn}^{0} \end{vmatrix} = p_1^{0} J^{0}. \tag{4.6}$$

On account of (4.4) this determinant vanishes and so does J^{0}, because the origin of the price co-ordinates can always be chosen so that $p_1^{0} \neq 0$. Thus J^{0} is at most of rank $n-1$. The same procedure cannot be repeated with all of the first minors of J^{0} and it is impossible to show that they must all vanish. They may vanish, of course, but need not do so. All that can be asserted is, therefore, that the rank of J^{0} cannot exceed $n-1$.

The determinant

$$D^{0} = \begin{vmatrix} F_1' a_{11}^{0} & F_1' a_{12}^{0} \cdots F_1' a_{1n}^{0} \\ F_2' a_{21}^{0} & F_2' a_{22}^{0} \cdots F_2' a_{2n}^{0} \\ \cdots\cdots\cdots\cdots \\ F_n' a_{n1}^{0} & F_n' a_{n2}^{0} \cdots F_n' a_{nn}^{0} \end{vmatrix} = F_1' F_2' \cdots F_n' J^{0}, \tag{4.7}$$

where $F_r' = F_r'(0) > 0$ $(r = 1, 2, \cdots, n)$ by virtue of (2.6), is at most of the same rank as J^{0}, i.e. $n-1$.

The characteristic equation (2.8) can be written in the polynomial form

$$\lambda^{n} - S_1 \lambda^{n-1} + S_2 \lambda^{n-2} + \cdots + (-1)^{n} S_n = 0, \tag{4.8}$$

where S_r $(r = 1, 2, \cdots, n)$ is the sum of all principal minors of order r in

D^0. D^0 being of rank not higher than $n-1$, at least the last term of the polynomial vanishes and we have

$$\lambda[\lambda^{n-1} - S_1\lambda^{n-2} + S_2\lambda^{n-3} + \cdots + (-1)^{n-1}S_{n-1}] = 0. \tag{4.9}$$

The characteristic equation thus has at least one root $\lambda = 0$ and the system is, therefore, neutral at least of rank one. Since at least one of the roots equals zero, at most $n-1$ roots can have negative real parts, i.e., the order of stability of the system cannot be higher than $n-1$. This proves the second property of our system.

The equilibrium equations are

$$X_r(p_1, p_2, \cdots, p_n) = 0 \qquad (r = 1, 2, \cdots, n). \tag{4.10}$$

In view of the fact that $X_1, X_2, \cdots, X_m$ are homogeneous of zero degree and $X_{m+1}, X_{m+2}, \cdots, X_n$ are homogeneous of the first degree in the variables $p_1, p_2, \cdots, p_m$, the equations can be written in the form

$$\begin{aligned} &\Phi_r\left(1, \frac{p_2}{p_1}, \cdots, \frac{p_m}{p_1}; p_{m+1}, \cdots, p_n\right) = 0 \\ &\qquad\qquad\qquad\qquad\qquad\qquad \text{for } r = 1, 2, \cdots, m, \\ &p_1\Phi_r\left(1, \frac{p_2}{p_1}, \cdots, \frac{p_m}{p_1}; p_{m+1}, \cdots, p_n\right) = 0 \\ &\qquad\qquad\qquad\qquad\qquad\qquad \text{for } r = m+1, m+2, \cdots, n. \end{aligned} \tag{4.11}$$

We see immediately that if the set of prices $p_1^0, p_2^0, \cdots, p_m^0, p_{m+1}^0, \cdots, p_n^0$ is a solution of (4.11),[18] the set of prices $kp_1^0, kp_2^0, \cdots, kp_m^0, p_{m+1}^0, \cdots, p_n^0$, where k is an arbitrary number, is also a solution. This proves the third property of our system.

A practical application of the system under discussion is found by interpreting the goods $1, 2, \cdots, m$ as commodities and stocks and the goods $m+1, m+2, \cdots, n$ as fixed-income-bearing securities. Our system then describes the case where the excess-demand functions of commodities and stocks are homogeneous of zero degree in the prices of commodities and stocks, interest rates (or the prices of fixed-income-bearing securities) being constant. Under these circumstances the demand and supply functions, and, consequently, also the excess-demand functions, of fixed-income-bearing securities are homogeneous of first degree in commodity prices, because if all commodity and stock prices increase k-fold the real earning power of the securities mentioned decreases in inverse proportion and it takes k times as many securities to represent the same real earning power as before.[19] The properties of such a system have been discovered by Lord Keynes in his doc-

[18] The existence of a solution of the equilibrium equations is assumed.

[19] Cf. pp. 16 above.

trine of the effect of changes in money wages upon employment and upon product prices.[20] Lord Keynes's theory presupposes a system in which interest rates are kept constant and in which the demand and supply functions of all commodities are homogeneous of zero degree in money wage rates and commodity prices. Professor Hicks has developed further this doctrine in application to general-equilibrium theory under conditions where all price expectations are of unit elasticity.[21] A mathematical proof of Professor Hicks's conclusions was given by Dr. Mosak.[22] Dr. Mosak uses the Hicksian stability conditions in his proof. His proof is, therefore, restricted to systems in which these conditions are valid. The results established in this section contain those of Keynes, Hicks, and Mosak as special cases.

5. *The Law of Composition of Goods*

The rank of stability of economic equilibrium indicates the maximum number of flexible prices compatible with the stability of the system. To secure stability, the remaining prices must be rigid. Any argument, however, which attaches importance to the number of goods or prices presupposes the existence of a way of classifying goods and determining their number which is not purely arbitrary. From experience we know that there is no unique way of classifying goods. A commodity can be split up into several subcommodities; for instance, wheat into wheat of different grades. On the other hand, several commodities can be combined into one composite commodity. The classification of goods occurring in practical economic life is to a certain degree conventional. In economic science, however, the classification of goods cannot be made on a purely arbitrary basis, because the laws of economics would then be dependent on the particular classification adopted. This would restrict the significance of the propositions of economics to a degree that would make them practically valueless. Each proposition might be changed into its opposite by a mere reclassification of goods. We adopt, therefore, the following *Principle of Invariance:*

The criterion of classification of goods must be such that reclassification of any group of goods in the economic system leaves invariant (1) all propositions of economic theory which relate to the subsystem consisting of the remaining goods, and (2) the formal mathematical structure of the propositions relating to the goods which are reclassified.

In equilibrium and stability theory the criterion required is obtained by

[20] *The General Theory of Employment, Interest and Money* (New York: Harcourt, Brace, 1936), pp. 257–271.

[21] *Op. cit.*, pp. 254–255. It seems, however, that he was not aware of the fact that his analysis and conclusions presuppose a neutral monetary system. Cf. footnote 10 on p. 24 above.

[22] Jacob Mosak, *General-Equilibrium Theory in International Trade*, Cowles Commission Monograph No. 7 (Bloomington, Indiana: Principia Press, 1944), pp. 162–164.

means of the following consideration: Take a system consisting of $n+1$ goods (including money). Let q $(q<n)$ goods be such that their prices vary *always* in the same proportion. Combine these goods into one composite good and define the price of the composite good as a linear combination of the prices of the q goods. Without loss of generality, we can assume that these are the goods $1, 2, \cdots, q$, and the composite good may be represented by the symbol $(1q)$. We have then

$$p_r(t) \equiv b_r p_q(t) \qquad (r = 1, 2, \cdots, q-1). \tag{5.1}$$

where $b_r = \text{const.} > 0$ $(r = 1, 2, \cdots, q-1)$. Denoting the price of the composite good by $p_{(1q)}$, we shall write

$$p_{(1q)}(t) \equiv \sum_{r=1}^{q} w_r p_r(t) \qquad (w_r = \text{const.} > 0). \tag{5.2}$$

Combining (5.1) and (5.2) we find

$$p_r(t) \equiv c_r p_{(1q)}(t) \qquad (r = 1, 2, \cdots, n), \tag{5.3}$$

where

$$c_r = \frac{b_r}{\sum_{s=1}^{q} w_s b_s} > 0 \qquad (b_q = 1). \tag{5.4}$$

The excess demand $X_{(1q)}$ for the composite good $(1q)$ will be defined by the relation

$$p_{(1q)} X_{(1q)} \equiv \sum_{r=1}^{q} p_r X_r. \tag{5.5}$$

Together with (5.2), this leads to the relations

$$X_r = w_r X_{(1q)} \qquad (r = 1, 2, \cdots, q). \tag{5.6}$$

Taking into account (5.3), we write this in the form

$$\begin{aligned} X_r(p_1, p_2, \cdots, p_q; p_{q+1}, \cdots, p_n) &\equiv X_r[c_1 p_{(1q)}, c_2 p_{(1q)}, \cdots, c_q p_{(1q)}; p_{q+1}, \cdots, p_n] \\ &\equiv w_r X_{(1q)}[p_{(1q)}, p_{q+1}, \cdots, p_n]. \end{aligned}$$

Following our previous notation, let us write

$$a_{(1q)s} = \frac{\partial X_{(1q)}}{\partial p_s} \qquad [s = (1q), q+1, q+2, \cdots, n], \tag{5.7}$$

and we obtain the relations

$$\sum_{s=1}^{q} a_{rs}c_s = w_r a_{(1q)(1q)} \qquad (r = 1, 2, \cdots, q). \tag{5.8}$$

$$a_{rs} = w_r a_{(1q)s} \qquad \text{for } s = q+1, q+2, \cdots, n$$

Consider the system of differential equations

$$\frac{dp_r}{dt} = F_r' \sum_{s=1}^{n} a_{rs}{}^0 p_s \qquad (r = 1, 2, \cdots, n), \tag{5.9}$$

i.e., the system (2.5) discussed above. Because of (5.3) and (5.8) this system can be written in the following form:

$$\begin{aligned} \frac{dp_{(1q)}}{dt} &= \frac{F_r' w_r}{c_r}\left[a_{(1q)(1q)}{}^0 p_{(1q)} + \sum_{s=q+1}^{n} a_{(1q)s}{}^0 p_s\right] \\ &\qquad \text{for } r = 1, 2, \cdots, q \\ \frac{dp_r}{dt} &= F_r' \sum_{s=1}^{n} a_{rs}{}^0 p_s \qquad \text{for } r = q+1, q+2, \cdots, n. \end{aligned} \tag{5.10}$$

Since the system (5.10) is equivalent to the system (5.9) the prices of the goods $q+1, q+2, \cdots, n$ are not affected by the combination of the goods $1, 2, \cdots, q$ into a composite good. We see from (5.10) that the differential equations for $s = q+1, q+2, \cdots, n$ are not affected either. The prices $p_1, p_2, \cdots, p_q$ are transformed into $p_{(1q)}$ through multiplication by a constant. By writing

$$F_{(1q)}' = \frac{F_r' w_r}{c_r} \qquad (r = 1, 2, \cdots, q), \tag{5.11}$$

the system (5.10) can be written in the reduced form

$$\frac{dp_r}{dt} = F_r' \sum_{s} a_{rs}{}^0 p_s \qquad [r \text{ and } s = (1q), q+1, q+2, \cdots, n]. \tag{5.12}$$

Comparing this reduced system with the original system (5.9) we find that the first q differential equations in (5.9) are reduced to one equation which retains the mathematical structure of the original equations (i.e., is a linear equation with constant coefficients). We see also that the composite good behaves exactly as if it were a single good and that the composition does not affect the other goods in any way.

The passage from the system (5.9) to the system (5.10) or (5.12) is equivalent to subjecting the system (5.9) to the algebraic transformations

$$\begin{aligned} \frac{dp_r}{dt} &= \frac{c_r}{w_r}\frac{dp_{(1q)}}{dt} \qquad \text{for } r = 1, 2, \cdots, q, \\ \frac{dp_r}{dt} &= \frac{dp_r}{dt} \qquad \text{for } r = q+1, q+2, \cdots, n, \end{aligned} \tag{5.13}$$

and

$$
\begin{aligned}
p_s(t) &\equiv c_s p_{(1q)}(t) && \text{for } s = 1, 2, \cdots, q, \\
p_s(t) &\equiv p_s(t) && \text{for } s = q+1, q+2, \cdots, n.
\end{aligned} \tag{5.14}
$$

These transformations are nonsingular and can be inverted. In economic terms the inverse transformations mean the splitting-up of the composite good $(1q)$ into q separate goods. The inverse transformations change neither the prices of the goods $q+1, q+2, \cdots, n$ nor the corresponding differential equations. The prices of the separated goods $1, 2, \cdots, q$ are obtained by multiplying the price of the composite good by a constant and the corresponding differential equations retain the mathematical structure of the original equation.

Thus the transformations (5.13) and (5.14) as well as their inverses satisfy our *Principle of Invariance.* This consideration leads us to the following criterion of classification of goods:

Any goods the prices of which always vary in the same proportion can be combined into one composite good; and, conversely, any good can be split up into an arbitrary number of separate goods with prices varying always in the same proportion.

We shall call it the *law of composition of goods.* By application of this law the number of goods in the theoretical system can be reduced to a certain minimum. This minimum is attained when no two goods in the system are such that their prices vary always in the same proportion. In this case the theoretical system will be said to be *canonical.* In a canonical system the number of goods is uniquely determined. In a noncanonical system the number of goods is arbitrary and need not even be finite. For any good can be split up into several goods with prices always varying proportionally. By successive application of transformations of this kind the number of goods can be increased indefinitely.

Constant prices are a special case of prices which always vary in the same proportion, namely in the same proportion as the price of money, which equals unity by definition. Thus all goods with rigid prices can be combined with money into one composite good. In a canonical system the introduction of rigid prices is synonymous with a reduction of the number of goods. This suggests an interpretation of the rank of stability of economic equilibrium. Stability of rank $n-q$ of a system containing $n+1$ goods (including money) means that q prices must be kept rigid in order to secure stability. This means that the corresponding canonical systems cannot contain more than $n-q+1$ goods and still be stable. The instability is due to there being q goods too many. In order to secure stability q goods must be combined with money into one composite good. Thus stability short of total stability can be interpreted as indicating an excessive number of goods in the canonical system.

6. *Imperfect Competition*

With some reinterpretation of the economic meaning of symbols, our analysis can be extended to systems containing forms of imperfect competition where sellers or buyers are confronted with determinate and differentiable demand or supply functions. These forms are monopoly and monopsony, monopolistic and monopsonistic competition.[23] This presupposes that each seller deals with atomistic buyers and each buyer deals with atomistic sellers. Each nonatomistic seller or buyer must be regarded as dealing in a separate good. Equilibrium obtains in the system when all prices are such that every seller and every buyer maximizes his profit or utility. If perfectly competitive markets are present, excess demand must vanish in them.

That atomistic buyers and sellers maximize their profit or utility is implied in the construction of their demand and supply functions. The demand and supply functions of the atomistic buyers and sellers being given, the profit or utility U_r, which the nonatomistic seller or buyer of the good r maximizes, can be considered as a function of the prices, i.e., $U_r \equiv U_r(p_1, p_2, \cdots, p_n)$. Of these prices the nonatomistic seller or buyer controls only p_r, and, under the forms of imperfect competition under consideration, he does not take into account a possible influence of a change in p_r upon other prices. We define now for each nonatomistic seller and buyer a function $X_r(p_1, p_2, \cdots, p_n)$, such that

$$(6.1)\quad X_r \equiv \frac{\partial U_r}{\partial p_r} \quad (r \text{ running through any values of the sequence } 1, 2, \cdots, n).$$

We shall call it the *marginal-gain function.*

$X_r = 0$ when the nonatomistic seller or buyer of the good r maximizes his profit or utility. The second-order maximum condition requires that $X_r \gtrless 0$ according as his price is less or greater than the price which maximizes his profit or utility. Thus when $X_r > 0$ the nonatomistic seller or buyer raises his price. He lowers his price when $X_r < 0$. The functions X_r thus conform to the equations (2.1) and, consequently, the differential equations (2.2) and (2.5).[24] In these equations the functions X_r can, therefore, be interpreted as excess-demand functions when the market for the good r is subject to perfect competition, and as marginal-gain functions when competition is imperfect. In this way our analysis can be extended to systems which contain imperfections of competition of the type mentioned. The conclusions of Sections 1–3 and 5 hold fully for such systems.

[23] Oligopoly and oligopsony based on group behavior are excluded because the demand or supply functions, though determinate, are not differentiable at the point of the conventionally established price.

[24] They also satisfy the inequalities (1.2) which are Professor Hicks's conditions for "imperfect" stability.

The properties of homogeneous systems established in Section 4 hold in systems which contain imperfect competition in any of the goods $1, 2, \cdots, m$ (i.e., commodities and stocks), provided the nonatomistic buyers and sellers are firms.

Suppose that the assumptions of Section 4 are satisfied in the atomistic markets. Since in nonatomistic markets excess demand is always zero, irrespective of whether these markets are in equilibrium or not, the corresponding terms in identity (4.1) vanish. This identity is thus restricted to terms relating to atomistic markets and the first property of homogeneous systems follows immediately.

Suppose further that in each atomistic market the demand function confronting the monopolist or the supply function confronting the monopsonist is homogeneous of zero degree in the prices $p_1, p_2, \cdots, p_m$. Denote the demand function or supply function confronting the nonatomistic seller or buyer of the rth good by $D_r(p_1, p_2, \cdots, p_n)$ or $S_r(p_1, p_2, \cdots, p_n)$, respectively. The firm's profit can be expressed in the form

$$U_r(p_1, p_2, \cdots, p_n) \equiv p_r D_r + \sum_{s \neq r} p_s q_s \tag{6.2a}$$

or

$$U_r(p_1, p_2, \cdots, p_n) \equiv -p_r S_r + \sum_{s \neq r} p_s q_s, \tag{6.2b}$$

according as the firm sells or buys the rth good in a nonatomistic market. The q_s are quantities of goods sold or bought in atomistic markets and can be any of the goods $1, 2, \cdots, m$. The q_s which stand for goods bought are negative. Given all prices except p_r, the quantities q_s are chosen so as to maximize the firm's profit. These quantities are thus determined by the set of equations

$$\frac{\partial U_r}{\partial q_s} = \frac{\partial D_r}{\partial q_s}\left(p_r + D_r \frac{\partial p_r}{\partial D_r}\right) + p_s = 0 \tag{6.3a}$$

or $(s \neq r)$.

$$\frac{\partial U_r}{\partial q_s} = -\frac{\partial S_r}{\partial q_s}\left(p_r + S_r \frac{\partial p_r}{\partial S_r}\right) + p_s = 0 \tag{6.3b}$$

In these equations $\partial D_r/\partial q_s$ or $\partial S_r/\partial q_s$ is derived from the firm's transformation function and is the marginal rate of transformation of the sth into the rth good, or vice versa. $\partial p_r/\partial D_r$ or $\partial p_r/\partial S_r$ is the reciprocal of the partial derivative of the demand function or supply function, respectively.

Since D_r or S_r is homogeneous of zero degree in $p_1, p_2, \cdots, p_m$, $\partial p_r/\partial D_r$ or $\partial p_r/\partial S_r$ is homogeneous of first degree in the same variables ($\partial D_r/\partial p_r$ or $\partial S_r/\partial p_r$ is homogeneous of degree -1). The prices p_r and p_s being among the

variables $p_1, p_2, \cdots, p_m$, the equations in (6.3) are invariant under a proportional change of these variables. Consequently, the quantities q_s, which are the solutions of these equations, are not affected by a proportional change in the prices $p_1, p_2, \cdots, p_m$. It follows that the expression (6.2) is homogeneous of first degree in $p_1, p_2, \cdots, p_m$, because the q_s as well as D_r or S_r remain constant when p_r and the p_s all change in the same proportion. The marginal-gain function $X_r \equiv \partial U_r / \partial p_r$ is, therefore, homogeneous of zero degree in $p_1, p_2, \cdots, p_m$. The second and third property of homogeneous system follow from the results of Section 4 by mere reinterpretation of symbols.

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