A Predictive Neutrino Mass Matrix texture on θ_{23}, θ_{13} and $\sum m_i$ based on a broken Symmetry Ansatz

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Abstract

Parametrization of the neutrino mass matrix in terms of well known measured quantities is an attractive way to obtain a phenomenologically viable form. We propose a neutrino mass matrix to predict the value of θ_{23}, θ_{13} and $\sum m_i$ in terms of the charged lepton masses. For the value of $\sum m_i \simeq 0.17 eV$, two of the mixing angles come out as $\theta_{23} \sim 46.08^{\circ}$, and $\theta_{13} \sim 8.69^{\circ}$. However, to accommodate other oscillation parameters we need to add further perturbation of the proposed texture. We also present an illustrative model to realize such texture which is based on type-II seesaw mechanism incorporating the idea of badly broken or approximate symmetry in which the symmetry breaking effect is manifested through the matrix elements. Our proposed texture serves as a leading order approximation of an well descripted neutrino mass matrix.

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1 Introduction

There are different propositions to build a phenomenologically viable neutrino mass matrix following various mixing schemes. Prior to the fact that $\theta_{13} \neq 0$, the Tribimaximal mixing [1, 2] has been widely accepted as a correct description of the neutrino mixing. There are several models invoking different flavour symmetries to obtain the neutrino mass matrix which reconciles with this mixing pattern. After the measurement of nonzero θ_{13}, this mixing has been modified in different ways considering this scheme as a leading order prediction [3]. There are several other mixing schemes namely, Trimaximal mixing [4, 5, 6, 7, 8], Cobimaximal mixing [9, 10, 11, 12], Bilarge mixing [13] etc. to address nonzero θ_{13} as well as other two mixing angles within the experimental ranges. All these mixing schemes are invoked in models with different flavour symmetries to obtain the most elusive structure of the neutrino mass matrix. In this regard, we attempt to parametrize the neutrino mass matrix in terms of some experimentally known quantities. Following we try to write down the neutrino mass matrix in terms of some function of the charged lepton masses and $\sum m_i$. One of the possible way to correlate the neutrino mass matrix in terms of the charged lepton masses is through the invocation of some GUT models. However, in the present work we adopt a different approach, widely investigated earlier, is due to the assumption of broken symmetry ansatz. To demonstrate our proposed texture we consider a type-II seesaw model. One of us has been investigated [14, 15] the idea of badly broken symmetry in the context of the Zee model [16] where the model leads to bimaximal mixing pattern. The crucial difference between the Zee model and the present one is that in the Zee model the diagonal elements are zero due to $SU(2)$ antisymmetry, whereas in the present case the diagonal elements are all nonzero.

In the present work we propose a texture of neutrino mass matrix in terms of the charged lepton masses and a real parameter which is constrained by $\sum m_i$. Thus the proposed texture is completely described by the experimentally measured quantities. The most interesting feature of our proposed matrix is that if we fix the value of the real parameter with $\sum m_i = 0.17\,eV$ (which is the trace of the matrix) the two mixing angles naturally come out as $\theta_{23} = 46.08^\circ$ and $\theta_{13} = 8.69^\circ$ which are well within the 3σ experimental limits of the two mixing angles. We also discussed the perturbation needed to fit other oscillation data. Further, we have demonstrated a model based on type-II seesaw mechanism to realize such texture adhering the ansatz of badly broken symmetry or approximate symmetry. Our plan of the paper is as follows: Section 2 contains the proposed texture and its perturbative form. An illustrative model is given in Section 3. In Section 4 we summarise our conclusions. Masses and mixing angles of the proposed texture is given in Appendix A. Detailed calculation of the matrix element needed for the illustrative model is presented in the Appendix B.

2 Proposed Texture

In the present work, we propose a neutrino mass matrix in terms of the charged lepton masses and $\sum m_i$. Our proposed mass matrix is given by

$$m_\nu^0 = \begin{pmatrix} \frac{k}{6} & \frac{k}{3}\sqrt{\frac{m_e}{m_\mu}} & \frac{k}{3}\sqrt{\frac{m_e}{m_\tau}} \\ \frac{k}{3}\sqrt{\frac{m_e}{m_\mu}} & \frac{k}{6} & \frac{k}{3}\sqrt{\frac{m_\mu}{m_\tau}} \\ \frac{k}{3}\sqrt{\frac{m_e}{m_\tau}} & \frac{k}{3}\sqrt{\frac{m_\mu}{m_\tau}} & \frac{k}{6} \end{pmatrix}. \tag{1}$$

The above mass matrix is always real since the phase of 'k' can be taken out and we consider this texture as a leading order. The parameter k can be easily constrained by the sum of the three neutrino masses as $k = 2\sum m_i$. The expression of the eigenvalues and the three mixing are presented in Appendix A. We consider the experimental inputs as described in Table 1 [17]. The most interesting feature of the above matrix is that for $\sum m_i = 0.17\,eV$, the two mixing angles θ_{23} and θ_{13} naturally come out as $\theta_{23} = 46.08^\circ$ and $\theta_{13} = 8.69^\circ$. Furthermore, the 3σ ranges of the above two mixing angles restrict the value of the sum mass which is depicted in Fig.1. Assuming the value of $\sum m_i \leq 0.23\,eV$ [18, 19], we plot the variation of

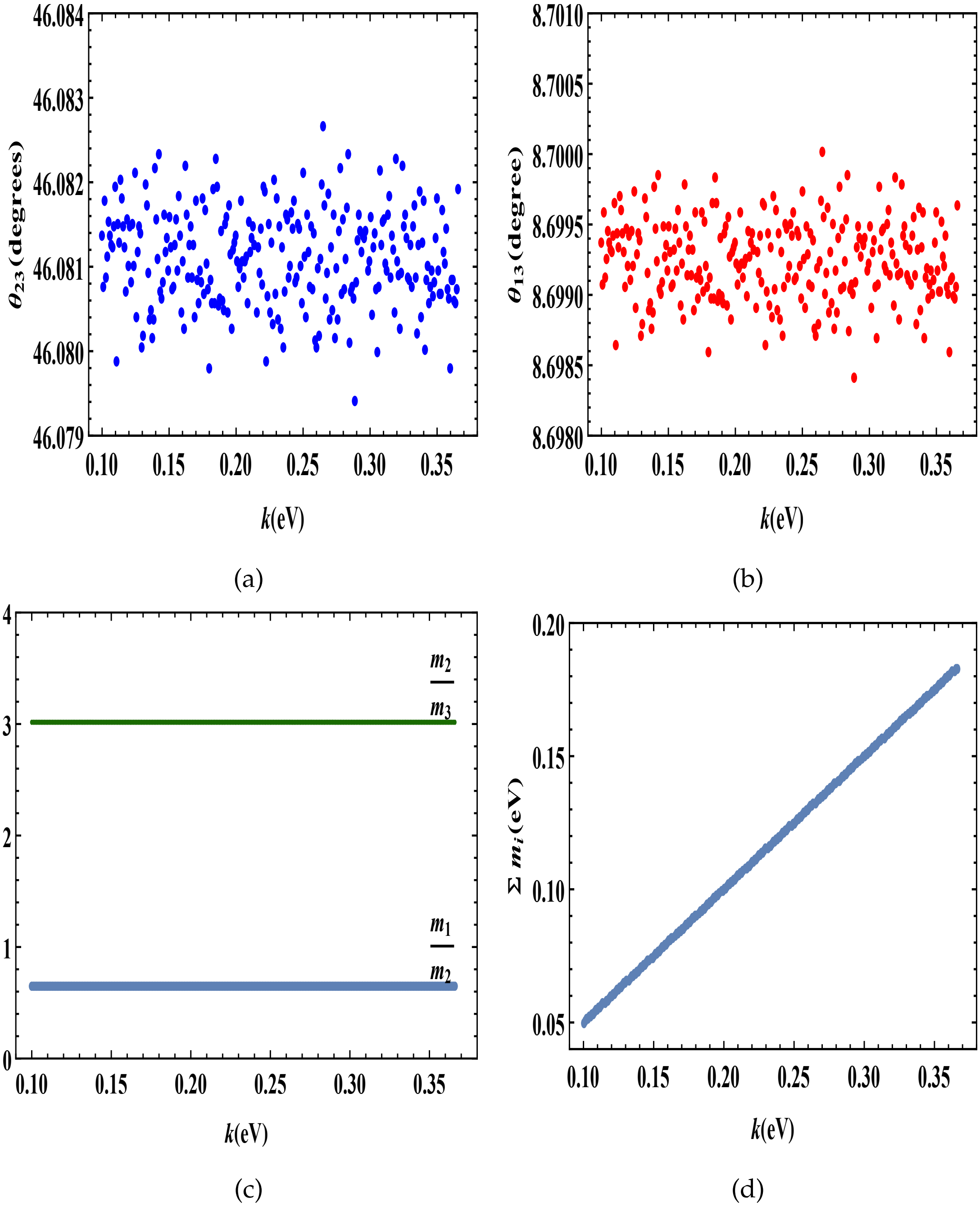


Figure 1: Variation of the parameter k with θ_{13}(a), θ_{23}(b), ratios m_1/m_2 and m_2/m_3(c) and $\sum m_i$(d).

Table 1: Neutrino oscillation parameters.

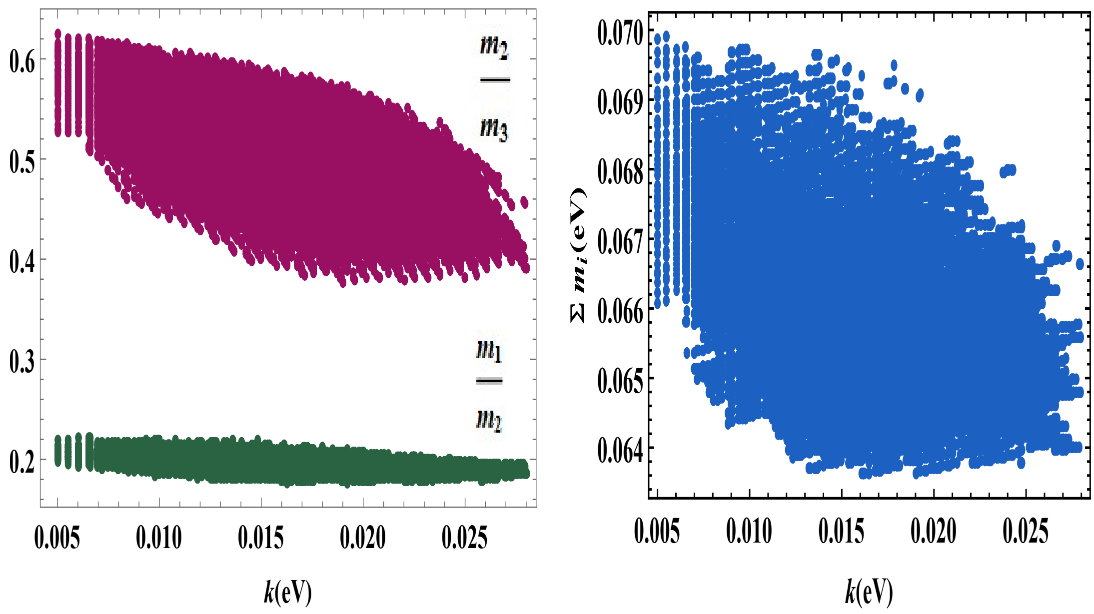
Parameter	θ_{12}°	θ_{23}°	θ_{13}°	$\frac{\Delta m^2_{21}}{10^{-5}\times(eV)^2}$	$\frac{\lvert\Delta m_{3l}\rvert^2}{10^{-3}\times(eV)^2}$
3σ ranges (NO)	31.61-36.27	40.9-52.2	8.22-8.98	6.79-8.01	2.431-2.622
3σ ranges (IO)	31.62-36.27	41.2-52.1	8.27-9.03	6.79-8.01	2.413-2.606
Best fit (NO)	33.82	49.7	8.61	7.39	2.525
Best fit (IO)	33.82	49.7	8.65	7.39	2.512

the parameter k with θ_{13}(Fig.1(a)), θ_{23}(Fig.1(b)). Thus for a variation of θ_{23} and θ_{13} within 3σ ranges restrict the value of the parameter k as $0.1\,eV \leq k \leq 0.35\,eV$ which in turn gives the constraint on $\sum m_i$ as $0.05\,eV \leq \sum m_i \leq 0.18\,eV$. Such correlation between the mixing angles and the sum mass, in our opinion, is very interesting and could be easily tested in the near future. The hierarchy obtained is inverted as shown in Fig 1.(c). In Fig 1.(d), the allowed value of $\sum m_i$ vs k, is presented. Furthermore the value of $|(m_\nu)|_{11}$ is also below the experimental value $|(m_\nu)|_{11} \leq 0.061\,eV$ as given in KamLAND-Zen and EXO-200 experiments [20, 21].

Let us point out the major short comings of the proposed texture: (1) θ_{12} value is coming out too low, (2) the mass squared differences Δm^2_{21} and Δm^2_{23} are also outside the 3σ experimental ranges. Obviously it needs further modification of the above texture. In a most general way, we consider modification of the above matrix texture as

$$m_\nu = \begin{pmatrix} \frac{k}{6}+p_1 & \frac{k}{3}\sqrt{\frac{m_e}{m_\mu}}+p_4 & \frac{k}{3}\sqrt{\frac{m_e}{m_\tau}}+p_5 \\ \frac{k}{3}\sqrt{\frac{m_e}{m_\mu}}+p_4 & \frac{k}{6}+p_2 & \frac{k}{3}\sqrt{\frac{m_\mu}{m_\tau}}+p_6 \\ \frac{k}{3}\sqrt{\frac{m_e}{m_\tau}}+p_5 & \frac{k}{3}\sqrt{\frac{m_\mu}{m_\tau}}+p_6 & \frac{k}{6}+p_3 \end{pmatrix}. \tag{2}$$

There are now seven real parameters($p_1, p_2, p_3, p_4, p_5, p_6$ and k) with which all the neutrino data can be accommodated. However, for numerical estimation, to find a minimal number of necessary parameters, we obtain even if $p_1 = p_4 = 0$, the above matrix can still explain all the neutrino oscillation data. One interesting point is that in this case the hierarchy of neutrino masses become normal($m_3 > m_2 > m_1$)



(a) (b)

Figure 2: Variation of the parameter 'k' with ratios m_1/m_2 and m_2/m_3(a) and $\sum m_i$(b) after adding perturbation.

which is presented in Fig 2.(a). Furthermore, the upper limit of the $\sum m_i$ is too low compared to the experimental upper limit, which is shown in Fig 2.(b). The ranges of the model parameters which satisfy the 3σ ranges of the oscillation data, $\sum m_i$ and $|(m_\nu)|_{11}$ values are given as

$$\begin{aligned}
&p_2 \rightarrow (2.4 - 3.0) \times 10^{-2} eV\\
&p_3 \rightarrow (1.8 - 2.5) \times 10^{-2} eV\\
&p_5 \rightarrow -(1.15 - 1.0) \times 10^{-2} eV\\
&p_6 \rightarrow -(2.65 - 2.25) \times 10^{-2} eV\\
&k \sim (0.5 - 2.8) \times 10^{-2} eV.
\end{aligned} \tag{3}$$

Thus the proposed matrix can accommodate seven experimental constraints with five real parameters. In the next section we present an illustrative model to obtain such type of neutrino mass matrix.

3 Illustrative Model

We consider a model based on type-II seesaw mechanism adhering the idea of badly broken symmetry or approximate symmetry [22]. The philosophy of badly broken symmetry is that some internal symmetries of a transition matrix elements become exact in the large value of a kinematical parameter, following the Goldberger-Treiman relation [23]. Earlier it has been studied in the context of $SU(3)$, chiral $SU(3) \times SU(3)$ groups [24] in the context of hadronic and leptonic currents. In the present work we invoke this idea in the context of a model based on type-II seesaw mechanism. Basically, in this approach, we consider the symmetry breaking effect is proportional to the mass of the charged leptons and the same symmetry breaking parameter is also responsible for the neutrino sector.

Consider the Lagrangian of a type-II seesaw model as

$$\begin{aligned}\mathcal{L} &= f_{ij}\overline{(l_{iL})^c}l_{jL}\Delta + y_{ij}\bar{e}_{iL}e_{jR}\phi + h.c.\\ &= f_{ij}\left(\overline{(\nu_{iL})^c}\nu_{jL}\Delta^0 + \overline{(\nu_{iL})^c}e_{jL}\Delta^+ + \overline{(e_{iL})^c}\nu_{jL}\Delta^+ + \overline{(e_{iL})^c}e_{jL}\Delta^{++}\right) + y_{ij}\bar{e}_{iL}e_{jR}\left\langle\phi^0\right\rangle + h.c.\end{aligned} \tag{4}$$

Here we consider a horizontal symmetry $SU(3)_H$ [25, 26, 27, 28] which is badly broken in the flavor space and the amount of symmetry breaking is designated through the parameter 'k'. The leptons are considered in a triplet representation of the $SU(3)_H$ group and the matrix element is a component of $3 \times 3^* = 1 + 8$ of the same group. We assume the magnitude of the matrix element in the Limit$|p| \to \infty$, is given by

$$y_i \langle e_i(p)|\bar{e}_i e_i|e_i(p)\rangle = k^2 \times Constant. \tag{5}$$

Evaluating the above matrix element given in the l.h.s. of eqn. (5) we get

$$y_i \frac{1}{(2\pi)^3}\frac{m_{e_i}}{E_{e_i}}\overline{u}_{e_i}(p)u_{e_i}(p) = k^2 \times Constant \tag{6}$$

with the normalization condition $\overline{u}_{e_i}(p)u_{e_i}(p) = 1$ and in the $|p| \to \infty$ limit eqn. (6) becomes

$$y_i m_{e_i} = k^2 \times Constant \tag{7}$$

where $|p|$ is included in the constant. Since, $y_i = m_{e_i}/\langle\phi^0\rangle$ we get $m_{e_i}^2 = k^2$.

Next we consider the neutrino part of the Lagrangian and we consider the symmetry breaking as

$$
\begin{aligned}
&f_{ij}\left\{\left\langle\overline{\nu_{iL}}(p)|\overline{(\nu_{iL})^c}\nu_{jL}\Delta^0|\nu_j(p)\right\rangle + \left\langle\nu_{iL}(p)|\overline{(\nu_{iL})^c}e_{jL}\Delta^+|e_j(p)\right\rangle + \right.\\
&\left.\left\langle e_i(p)|\overline{(e_{iL})^c}\nu_{jL}\Delta^+|\nu_j(p)\right\rangle + \left\langle e_i(p)|\overline{(e_{iL})^c}e_{jL}\Delta^{++}|e_j(p)\right\rangle\right\} + i \to j \propto \sqrt{k_i k_j}K
\end{aligned}
\tag{8}
$$

where 'K' is a dimensionless parameter.

Evaluating l.h.s. of eqn. (8), we get

$$
\begin{aligned}
&f_{ij}\left\{m_{\nu_i} + m_{e_j} + m_{e_i} + m_{\nu_j} + i \to j\right\}\\
&= f_{ij}.3(m_{e_j} + m_{e_i})
\end{aligned}
\tag{9}
$$

where we have neglected the neutrino masses compared to the charged lepton masses. Thus, we get from eqn.(5) and eqn. (9) as

$$
3f_{ij}(m_{e_j} + m_{e_i}) = \sqrt{m_{e_i} m_{e_j}}K \tag{10}
$$

which gives

$$
f_{ij} = \frac{\sqrt{m_{e_i} m_{e_j}}K}{3(m_{e_j} + m_{e_i})}. \tag{11}
$$

Explicit calculation of the evaluation of the above matrix elements are given in the Appendix B .

The above expression of f_{ij} leads to the following tree level matrix of neutrino as

$$
m_\nu = \begin{pmatrix}
\frac{K}{6} & \frac{K}{3}\sqrt{\frac{m_e}{m_\mu}} & \frac{K}{3}\sqrt{\frac{m_e}{m_\tau}}\\
\frac{K}{3}\sqrt{\frac{m_e}{m_\mu}} & \frac{K}{6} & \frac{K}{3}\sqrt{\frac{m_\mu}{m_\tau}}\\
\frac{K}{3}\sqrt{\frac{m_e}{m_\tau}} & \frac{K}{3}\sqrt{\frac{m_\mu}{m_\tau}} & \frac{K}{6}
\end{pmatrix}\left\langle\Delta^0\right\rangle \tag{12}
$$

and with $\langle\Delta^0\rangle \sim 1\,eV$ and $K = 0.34$ we get the required angles $\theta_{13} \sim 8.69^\circ$, $\theta_{23} \sim 46.08^\circ$ as obtained earlier. Further perturbation of the above mentioned model can be implimented in many different ways, such as, through higher dimensional operators or through the invocation of type-I+II mechanism etc. which will be investigated elsewhere.

4 Concluding Summary

In the quest towards understanding of an elusive structure of neutrino mass matrix compatible with the extant data, in the present work, we have attempted to parametrize the neutrino mass matrix in terms of some functions of known experimental quantities. Precisely, in the present case in terms of the charged lepton masses and $\sum m_i$. The texture admits 3σ experimental values of θ_{23} and θ_{13} angles and thereby constraint $\sum m_i$ as $\sum m_i \leq 0.18\, eV$. The hierarchy of the neutrino mass is inverted. Since the other oscillation data, such as, θ_{12}, Δm^2_{21}, Δm^2_{32} are coming out outside the present experimental limits, the present texture can be considered as a leading order texture of the neutrino mass matrix. Further modification of the texture by adding five extra parameters leads to a complete description of the neutrino mass matrix. But we agree, in the present work those perturbation parameters are incorporated in an ad-hoc way.

Next, to realize such correlation between the neutrino mass matrix with the masses of the charged leptons is implemented through the motivation of badly broken or approximate symmetry ansatz. We demonstrate in the context of a type-II seesaw model, invoking the broken symmetry ansatz.

We further optimistically seek for a texture which can be written completely in terms of the known experimental quantities and will also try to demonstrate in the context of a well descripted model. Origin of such badly broken symmetry in that context will also be envisaged elsewhere.

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Appendix A. Explicit expressions of the neutrino masses and mixing angles of the proposed texture.

In this section we present explicit expressions of the neutrino masses and mixing angles of our proposed neutrino mass matrix. The expressions for mass eigenvalues and mixing angles are given by

$$
\begin{aligned}
m_1 &= \frac{k}{6}\left[1+\frac{4.2^{1/3}B}{m_\mu m_\tau A}+\frac{A}{3.2^{1/3}m_\mu m_\tau}\right],\\
m_2 &= \frac{k}{6}\left[1-\frac{2.2^{1/3}(1+i\sqrt{3})B}{m_\mu m_\tau A}-\frac{(1-i\sqrt{3})A}{6.2^{1/3}m_\mu m_\tau}\right],\\
m_3 &= \frac{k}{6}\left[1-\frac{2.2^{1/3}(1-i\sqrt{3})B}{m_\mu m_\tau A}-\frac{(1+i\sqrt{3})A}{6.2^{1/3}m_\mu m_\tau}\right],\\
\theta_{23} &= tan^{-1}\left[\frac{\left|4\sqrt{\frac{m_e}{m_\mu}}\sqrt{\frac{m_\mu}{m_\tau}}-2\sqrt{\frac{m_e}{m_\mu}}\left(1-\frac{6m_3}{k}\right)\right|}{\left|-4\sqrt{\frac{m_e}{m_\mu}}\sqrt{\frac{m_\mu}{m_\tau}}+2\sqrt{\frac{m_e}{m_\tau}}\left(1-\frac{6m_3}{k}\right)\right|}\right],\\
\theta_{12} &= tan^{-1}\left[\left|\frac{\left(4m_\mu-m_\tau+2m_\tau\left(\frac{6m_2}{k}\right)-m_\tau\left(\frac{6m_2}{k}\right)^2\right)}{\left(-\sqrt{\frac{m_e}{m_\tau}}+2\sqrt{\frac{m_e}{m_\mu}}\sqrt{\frac{m_\mu}{m_\tau}}+\sqrt{\frac{m_e}{m_\tau}}\left(\frac{6m_2}{k}\right)\right)}\right|\right.\\
&\quad\left.\times\left|\frac{\left(-\sqrt{\frac{m_e}{m_\tau}}+2\sqrt{\frac{m_e}{m_\mu}}\sqrt{\frac{m_\mu}{m_\tau}}+\sqrt{\frac{m_e}{m_\tau}}\left(\frac{6m_1}{k}\right)\right)}{\left(4m_\mu-m_\tau+2m_\tau\left(\frac{6m_1}{k}\right)-m_\tau\left(\frac{6m_1}{k}\right)^2\right)}\right|\right],\\
\theta_{13} &= sin^{-1}\left[\left|\frac{\left(4m_\mu-m_\tau+2m_\tau\left(\frac{6m_3}{k}\right)-m_\tau\left(\frac{6m_3}{k}\right)^2\right)}{\left(-\sqrt{\frac{m_e}{m_\tau}}+2\sqrt{\frac{m_e}{m_\mu}}\sqrt{\frac{m_\mu}{m_\tau}}+\sqrt{\frac{m_e}{m_\tau}}\left(\frac{6m_3}{k}\right)\right)}\right|\right]
\end{aligned}
\tag{13}
$$

where

$$
\begin{aligned}
B &= \left(m_e m_\mu^2 m_\tau + m_\mu^3 m_\tau + m_e m_\mu m_\tau^2\right), \\
A &= \left(432\sqrt{\frac{m_e}{m_\mu}} m_\mu^3 \sqrt{\frac{m_e}{m_\tau}}\sqrt{\frac{m_\mu}{m_\tau}} m_\tau^3 + \right. \\
&\left. \sqrt{186624 m_e^2 m_\mu^6 m_\tau^4 - 6912(m_e m_\mu^2 m_\tau + m_\mu^3 m_\tau + m_e m_\mu m_\tau^2)^3}\right)^{1/3}.
\end{aligned} \tag{14}
$$

Appendix B. Evaluation of the matrix elements given in Section 3.

We consider three lepton doublets (l_1,l_2,l_3) are as a triplet under $SU(3)_H$ so that the term $\bar{e}_i e_j$ is a component of $3 \times 3^* = 1 + 8$.

We define

$$
\psi(x) = \frac{1}{(2\pi)^{3/2}} \int d^3p \sqrt{\frac{m}{E}} \left[\sum_{s=1,2} b_s(p) \overline{u}_s(p) e^{-ipx} + \sum_{s=1,2} d_s^\dagger(p) v_s(p) e^{ipx} \right] \tag{15}
$$

with

$$
\left\{ b_s(p), b_{s'}^\dagger(p') \right\} = \delta_{ss'}(p - p') \tag{16}
$$

and

$$
\begin{aligned}
|p, s\rangle &= b_s^\dagger(p)\, |0\rangle \\
\langle p', s' | p, s \rangle &= \delta_{ss'}(p - p').
\end{aligned} \tag{17}
$$

Thus we get

$$
\begin{aligned}
\psi(x)\, |p, s\rangle &= \frac{1}{(2\pi)^{3/2}} \int d^3p' \sqrt{\frac{m}{E}} \overline{u}'_s(p) e^{-ip'x} b_{s'}(p') b_s^\dagger(p)\, |0\rangle \\
&= \frac{1}{(2\pi)^{3/2}} \sqrt{\frac{m}{E}} \overline{u}_s(p) e^{-ip'x}\, |0\rangle .
\end{aligned} \tag{18}
$$

Therefore the matrix element comes out as

$$
\begin{aligned}
&\left\langle e_i(p) | y_i \bar{e}_i e_j | e_j(p) \right\rangle \\
&= y_{ii} \frac{1}{(2\pi)^3} \frac{m_i}{E_i} \overline{u_i^e}(p) u_i^e(p)
\end{aligned} \tag{19}
$$

and using the normalization condition $\overline{u_i^e}(p) u_i^e(p) = 1$ and the relation $E = |p| + m^2/2|p|$ in the Limit $|p| \to \infty$ we get

$$
y_i m_{e_i} = k^2 \times Constant \tag{20}
$$

which is given in eqn. (7). The neutrino part of the Lagrangian can be evaluated as

$$\begin{aligned} & f_{ij} \left\langle \overline{\nu_{iL}}(p) | \overline{(\nu_{iL})^c} \nu_{jL} | \nu_j(p) \right\rangle + i \leftrightarrow j \\ & = \frac{1}{(2\pi)^3} \sqrt{\frac{m_{\nu_j}}{E_j}} \sqrt{\frac{m_{\nu_i}}{E_i}} \overline{(u_{\nu_{iL}})^c} (u_{\nu_j})_L + i \leftrightarrow j. \end{aligned} \tag{21}$$

Now in the Limit $|p| \to \infty$ we get

$$\lim_{|p| \to \infty} \overline{(u_{\nu_{iL}})^c} (u_{\nu_j})_L = \frac{m_{\nu_j}}{2\sqrt{m_{\nu_i} m_{\nu_j}}} \tag{22}$$

so that we get from Eqn. (21)

$$\frac{1}{(2\pi)^3} f_{ij} m_{\nu_i} \tag{23}$$

Similarly evaluating the other terms we get the result given in eqn. (9).