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# Measurement of Heat and Mass Flow Characteristics of Nanofluid in a Porous Parallel-Plate Channel by Darcy-LTNE/LTE, Brinkman-LTNE/LTE Models

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## Abstract

In current study, fully developed flow of  $\text{Al}_2\text{O}_3$ -water nanofluid with forced convection heat transfer in channel is investigated with different models of heat and mass distribution. The channel is filled with porous media of open-celled Cu metal foams. The Darcy and Brinkman models are used for the mass flow; however, the heat transfer distribution is examined through the local thermal equilibrium (LTE) and the local thermal non-equilibrium (LTNE) models. Exact solutions for Darcy-LTE, Brinkman-LTE, Darcy-LTNE, and Brinkman-LTNE models are obtained. Temperature profiles by these different models are discussed under effect of nanoparticle concentration and compare the profiles with each other.

**Keywords:** nanofluid, porous media, Darcy-LTE, Brinkman-LTE, Darcy-LTNE, Brinkman-LTNE models

## 1. Introduction

Studies on heat transfer in porous media have been augmented rapidly. The porous media likely filled with metallic foams with open cells, metallic sintered fiber felts, or metallic lattice frame structures have high thermal conductivity and have much importance in different engineering areas such as heat transport enhancement, thermal storage, solar thermal utilization, modeling of biological tissues, etc. There are two main models, the local thermal equilibrium (LTE) and the local thermal non-equilibrium (LTNE), which can be used to represent heat transfer phenomena in a porous medium. The LTE model can be used effectively to examine the heat transfer in porous media when the temperature difference between the fluid phase and that of the solid phase is small. But not in all situations the differences between temperatures are ignored. In these cases, the influence of the interfacial surface and interstitial convective heat transfer coefficient became major factors of heat exchange among the fluid and solid phases. In such cases, LTNE model needs to be utilized.

A number of researchers investigated heat transfer through LTE model in porous media that are made of low-conductive materials. Poulikakos and Kazmierczak [1] present a fully developed convective flow in a channel with partially filled porous matrix. They modeled the problem of heat and mass flow in the porous medium by using the Brinkman-LTE model to see the effect of parameters on the flow field and on the heat. Vafai and Kim [2] investigated the forced convection flow in a porous medium channel through Forchheimer-LTE model and found a significant increase in the rate of heat transfer as the inertia parameter increases especially for high- to medium-permeability porous media. Gong et al. [3] studied the influence of the thermal conductivity, permeability, and the porous material's thickness on heat transfer by using a Brinkman-LTE model in annular duct. Their obtained results display that heat transfer is improved by increasing either the thermal conductivity or the permeability. Cheng and Hsu [4] studied the heat and mass flow in an annulus enclosed with porous media by using Brinkman-LTE model under the influence of permeability and porosity. Mitrovic and Maletic [5] got the results for heat and mass flow in a parallel-plate porous channel by using the LTE model with respect to asymmetrical conditions. Sheikholeslami et al. [6] investigated the free convection flow in permeable enclosure by using Darcy-LTE model. In another study, he used the Darcy-LTE model for free convection flow in porous cavity [7].

In highly thermal conductive porous media, solid has usually higher thermal conductivity three to five times more than the fluid. In this case, the LTE model no longer satisfies the requirements of modeling. Then two-energy equation LTNE model is used to measure accurate temperature distribution in fluid and solid phases. Kuznetsov [8] got the results of heat transport phenomena in narrow annulus by using Darcy-LTNE model. Xu et al. [9] studied the Forchheimer-LTE/Forchheimer-LTNE models for heat transfer in pore channel having different thermal conductivities of fluid and solid phases and found the maximum heat distribution in the case of LTE model as compared to LTNE model. Lu et al. [10] described the force convection flow in cylinder fill with pore spume by using Brinkman-LTNE models. Zhao et al. [11] discussed the heat and mass flow through porous media in annulus by using Brinkman-LTNE. Ouyang et al. [12] briefly studied the heat transport phenomena in equidistant-plate channel fill with porous matrix. Xu et al. [13] investigated the mass flow by Brinkman model and LTNE model for heat transfer in equidistant plates filled with mini-spume. Shaikh and Memon [14] provided the numerical results for heat transport in round pipe with or without pore medium by using Darcy-Brinkman-Forchheimer models along LTNE model. Sheikholeslami and Houman [15] reported the transportation of fluid inside a porous cavity through LTNE model.

An important heat transfer fluid is nanofluid that is used in industries because of high rates of heat transfer. The main purpose of nanofluids is to achieve great enhancement in thermal or rheological properties. In a continuation of achieving better thermal conductivity and thermal performance of nanomaterial, many studies have been done [16–20]. Recently, Sheikholeslami [21] demonstrated the nanofluid flow in a porous enclosure by Darcy law model. In another study, he investigated the nanofluid flow in a porous media through non-Darcy law model [22]. In current study, keeping in mind these thermal properties of nanofluid, its fully developed convective flow in a parallel-plate channel filled with highly porous media is investigated. For thermal transport in porous media, LTE and LTNE models are utilized with Darcy and Brinkman models. In addition, the temperature profiles under nanoparticle concentration for both different approaches of heat transfer with constant heat flux at wall are calculated and compared.

## 2. Modeling and formulation

Consider the fully developed forced convection flow of nanofluid through a parallel-plate porous channel filled with open-celled metallic foams. The schematic diagram of the channel is shown in **Figure 1**.

Two infinite plates with height  $2H$  are exposed by constant heat flux  $q_w$ . In this problem, Darcy-LTE, Brinkman-LTE, Darcy-LTNE, and Brinkman-LTNE models are employed for mass flow and heat transfer process in porous media with hydraulically and thermally fully developed conditions.

For fully developed fluid flow, momentum equation with Brinkman term is

$$0 = -\frac{dp}{dx} + \frac{\mu_e}{\varepsilon} \frac{d^2u}{dy^2} - \frac{\mu_{nf}}{K} u. \quad (1)$$

Here,  $u$  is the velocity,  $\mu_e$  is the effective viscosity,  $\mu_{nf}$  is the viscosity of nanofluid,  $K$  is the permeability, and  $\varepsilon$  is the porosity.

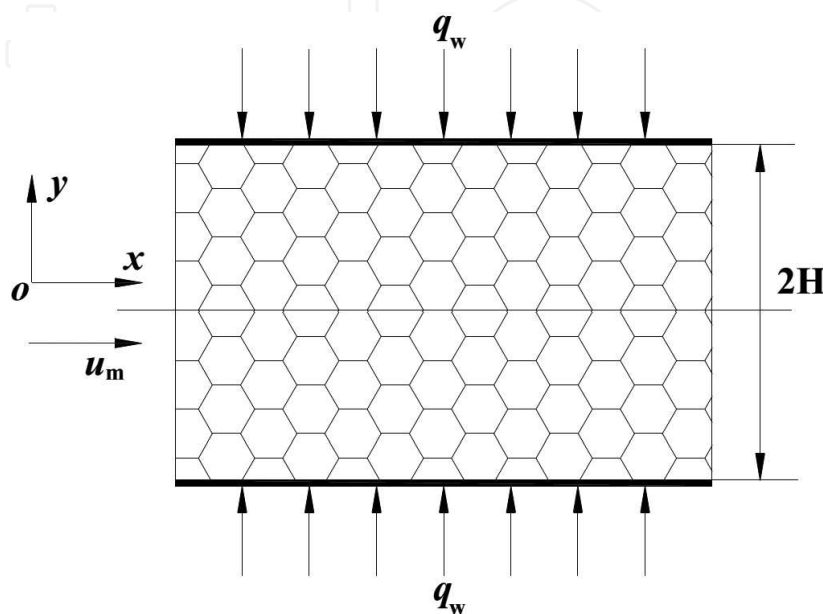
Temperature distribution in porous media owns two basic models LTE and LTNE. The LTE model containing one energy equation that treats the same value of temperature for fluid and solid phases is given in Eq. (2), while the LTNE model having two-energy equations that treats the different values of the temperatures for solid and fluid phases is shown in Eqs. (3) and (4):

$$(\rho C_p)_e u \frac{\partial T_f}{\partial x} = k_e \frac{\partial^2 T_f}{\partial y^2}, \quad (2)$$

$$0 = k_{se} \frac{\partial^2 T_s}{\partial y^2} - hA(T_s - T_f), \quad (3)$$

$$(\rho C_p)_e u \frac{\partial T_f}{\partial x} = k_{fe} \frac{\partial^2 T_f}{\partial y^2} + hA(T_s - T_f). \quad (4)$$

In the above equation,  $T_f$  is the temperature of fluid phase,  $T_s$  is the temperature of solid phase,  $h$  is the heat transfer coefficient,  $A$  is the specific surface area,  $\rho C_p$  is



**Figure 1.**  
 Geometry of problem.

the heat capacity,  $k_e$  is the effective thermal conductivity,  $k_{fe}$  is the fluid phase's thermal conductivity, and  $k_{se}$  is the solid phase's thermal conductivity.

For corresponding boundary conditions, the temperature of the solid and the fluid at the wall interface will be the same, and velocity is considered to be zero:

$$\begin{aligned} y = H : u = 0, T_f = T_s = T_w, \\ y = -H : u = 0, T_f = T_s = T_w, \end{aligned} \quad (5)$$

where  $T_w$  is the temperature at interface.

The total heat flux  $q_w$  is shared among the solid and fluid phases' subject to their temperature gradients and effective conductivities at the wall:

$$\begin{aligned} y = H : k_{fe} \frac{\partial T_f}{\partial y} + k_{se} \frac{\partial T_s}{\partial y} = q_w, \\ y = -H : k_{fe} \frac{\partial T_f}{\partial y} + k_{se} \frac{\partial T_s}{\partial y} = q_w. \end{aligned} \quad (6)$$

The boundary condition cross-ponding to symmetry condition can be used as

$$y = 0 : \frac{\partial u}{\partial y} = 0, \frac{\partial T_f}{\partial y} = \frac{\partial T_s}{\partial y} = 0 \quad (7)$$

In obtaining the analytical solutions of the above governing equations, the following dimensionless parameters are employed:

$$\begin{aligned} Y = \frac{y}{H}, U = \frac{u}{u_m}, P = \frac{K}{\mu_f u_m} \frac{dp}{dx}, \theta_s = \frac{T_s - T_w}{q_w H / k_s}, \\ \theta_f = \frac{T_f - T_w}{q_w H / k_s}, s = \frac{K}{H^2}, C = \frac{k_f}{k_s}, D = \frac{h A H^2}{k_s}. \end{aligned} \quad (8)$$

## 2.1 Darcy-LTNE model

In this case, Darcy model for fluid flow and LTNE model for temperature distribution in solid and fluid phases are utilized. In the Darcy model, the velocity distribution is taken to be uniformed. So, the energy equations for solid and fluid phases are dimensionless as

$$\frac{k_{se}}{k_s} \frac{d^2 \theta_s}{dY^2} - D(\theta_s - \theta_f) = 0, \quad (9)$$

$$C \frac{k_{fe}}{k_f} \frac{d^2 \theta_f}{dY^2} + D(\theta_s - \theta_f) = 1. \quad (10)$$

## 2.2 Darcy-LTE model

In this case, the energy equation for the LTE model is normalized as

$$C \frac{k_e}{k_f} \frac{d^2 \theta_f}{dY^2} = 1 \quad (11)$$

### 2.3 Brinkman-LTNE model

In this part, the Brinkman and LTNE models for heat and mass transfer are utilized. The dimensionless governing equations are obtained as

$$\frac{1}{\varepsilon} \frac{\mu_e}{\mu_f} \frac{d^2 U}{dY^2} - s^2 \left( \frac{\mu_{nf}}{\mu_f} U + P \right) = 1, \quad (12)$$

$$\frac{k_{se}}{k_s} \frac{d^2 \theta_s}{dY^2} - D(\theta_s - \theta_f) = 0, \quad (13)$$

$$C \frac{k_{fe}}{k_f} \frac{d^2 \theta_f}{dY^2} + D(\theta_s - \theta_f) = U. \quad (14)$$

### 2.4 Brinkman-LTE model

For the Brinkman and LTE models, governing equations in dimensionless form are

$$\frac{1}{\varepsilon} \frac{\mu_e}{\mu_f} \frac{d^2 U}{dY^2} - s^2 \left( \frac{\mu_{nf}}{\mu_f} U + P \right) = 1, \quad (15)$$

$$C \frac{k_e}{k_f} \frac{d^2 \theta_f}{dY^2} = U. \quad (16)$$

The dimensionless boundary conditions are

$$\begin{aligned} Y = 1 : \quad & U = 0, \theta_f = \theta_s = 1, \\ y = 0 : \quad & \frac{dU}{dY} = 0, \frac{d\theta_f}{dY} = \frac{d\theta_s}{dY} = 0, \\ y = -1 : \quad & U = 0, \theta_f = \theta_s = 1. \end{aligned} \quad (17)$$

### 2.5 Physical properties

In the above equations, the effective viscosity  $\mu_e$  is defined as

$$\mu_e = (1 + 2.5\varepsilon)\mu_{nf}, \quad (18)$$

where

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}. \quad (19)$$

In the above equation,  $\phi$  is the nanoparticle volume fraction and  $\mu_f$  is the viscosity of base fluid.

Since the heat is transferred via nanofluid in porous media, the effective thermal conductivity is as follows:

$$k_e = \frac{k_m k_{nf}}{\varepsilon k_m + (1 - \varepsilon) k_{nf}}, \quad (20)$$

$$k_{fe} = \varepsilon k_{nf}, \quad (21)$$

$$k_{se} = (1 - \varepsilon) k_{nf}. \quad (22)$$

In the above equation,  $k_{nf}$  is given as

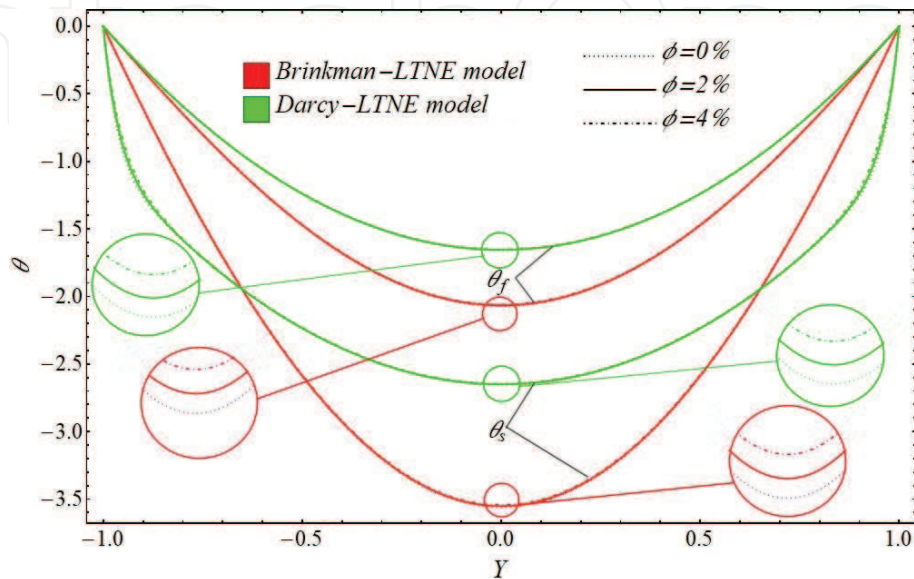
$$k_{nf} = \frac{k_p + 2k_f + 2(k_p - k_f) \phi}{k_p + 2k_f - (k_p - k_f) \phi} k_f, \quad (23)$$

where  $k_f$  is the thermal conductivity of fluid,  $k_p$  is the thermal conductivity of nanoparticle, and  $k_m$  is the thermal conductivity of medium.

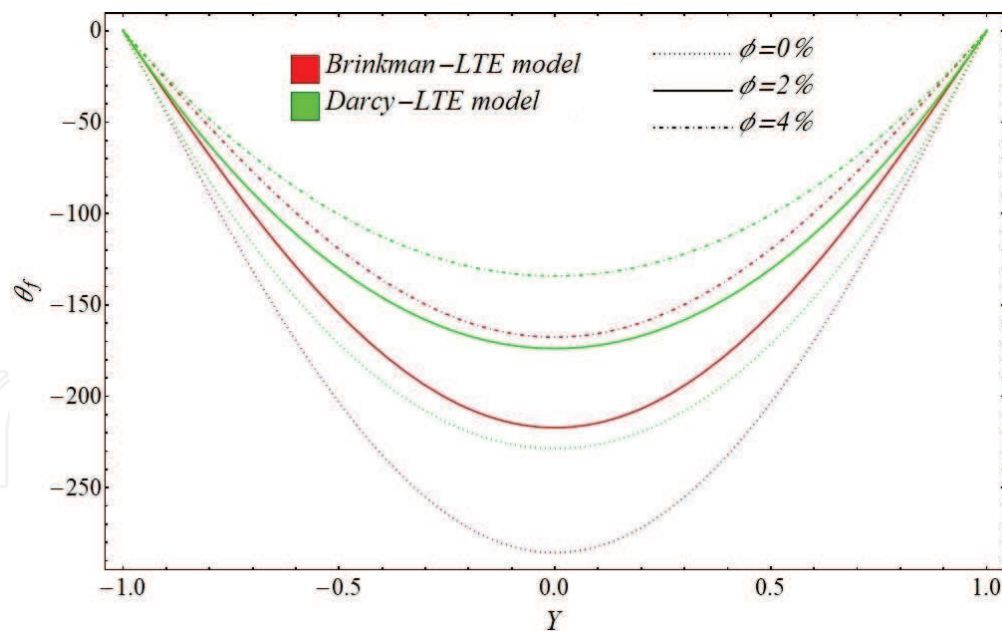
### 3. Results and discussion

The behavior of nanoparticle concentration on temperature distributions in solid and fluid phases is displayed in section. For fluid phase, consider the nanofluid which is repaired by water and alumina oxide nanoparticles. The porous medium is taken as solid phase that is made by open-celled copper metallic foams. The governing equations for physical problem are demonstrated by taking Brinkman-LTNE/Brinkman-LTE and Darcy-LTNE/Darcy-LTE models and converted into non-dimensionless form to find its exact solution. The exact solutions of these equations are obtained by using computational software Mathematica 9. To see the effects of nanoparticle concentration, the values of other embedding parameters are taken to be fixed as  $\varepsilon = 0.7$ ,  $s = 0.64$ ,  $C = 0.0015$ , and  $D = 1$ .

The influence of nanoparticle concentration on the temperature profiles of fluid and solid phases for Brinkman-LTNE and Darcy-LTNE models is shown in **Figure 2**. It shows that the temperature profiles of fluid and solid are increased due to improvement in thermal physical properties of fluids especially thermal conductivity through increasing the concentrations of nanoparticles. Here negative sign shows that fluid transfers the heat to the wall. In this regard, temperature profiles of solid phase in both models are increased because of increasing the temperature of fluid phase by nanoparticle concentrations. The temperature variation for Brinkman-LTNE and Darcy-LTNE models has similar trend, but the temperature profile of Brinkman-LTNE models is found maximum as compared to the Darcy-LTNE models.



**Figure 2.** Effect of nanoparticle concentrations on the temperature profiles of Brinkman-LTNE and Darcy-LTNE models.



**Figure 3.**  
 Effect of nanoparticle concentrations on the temperature profile of Brinkman-LTE and Darcy-LTE models.

The temperature profiles for Brinkman-LTE and Darcy-LTE models with effects of nanoparticle concentration are displayed in **Figure 3**. In **Figure 3**, it is seen that the temperature profiles for both models are amplified by increasing the nanoparticle concentrations. In comparison of models, it is noted that the effects of nanoparticle concentrations are dominant in the case of LTE as compared to LTNE. Moreover, the temperature at wall for LTE and LTNE models is the same but with maximum boost in the case of LTNE models at center. Moreover, the heat transfer with LTNE approach is smaller as compared to LTE approach because of thermal resistance due to solid phase.

#### 4. Conclusions

A fully developed heat and mass flow of  $\text{Al}_2\text{O}_3$ -water nanofluid in a parallel-plate channel filled with porous media of Cu material is investigated by using Darcy-LTE/Darcy-LTNE and Brinkman-LTE/Brinkman-LTNE models. It is found that the distribution of temperature is improved in both approaches of heat transfer by using nanofluid. But temperature distribution is overestimated in LTE approach as compared to LTNE approach. This overestimate results are due to neglecting the difference between thermal conductivities of fluid and solid phase.

#### Nomenclature

$u$	velocity
$\mu_e$	effective viscosity
$\mu_{nf}$	viscosity of nanofluid
$\mu_f$	viscosity of base fluid
$\varepsilon$	porosity
$T_f$	temperature of fluid
$T_s$	temperature of solid phase
$T_w$	temperature at interface



$\rho C_p$	heat capacity
$\phi$	nanoparticle volume fraction
$K$	permeability
$k_e$	effective thermal conductivity
$k_m$	thermal conductivity of medium
$k_{fe}$	thermal conductivity of fluid phase
$k_{se}$	thermal conductivity of solid phase
$k_p$	thermal conductivity of nanoparticle
$k_f$	thermal conductivity of base fluid
$q_w$	heat flux
$h$	heat transfer coefficient
$H$	height
$A$	specific surface area

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