Chapter 5

Fault Residuals Based on Distributed Discrete-Time Linear Kalman Filtering

Dušan Krokavec and Anna Filasová

Additional information is available at the end of the chapter

http://dx.doi.org/10.5772/intechopen.80296

Abstract

The chapter is concerned with the application of distributed discrete-time linear Kalman filtering with decentralized structure of sensors in fault residual generation. Two variants of distributed Kalman filtering algorithms are introduced, giving the incidence of equivalent functional realization structure of fault residual filters. The obtained solutions use Kalman filter innovations in a nonstandard way to generate residuals with significantly higher dynamic signal range. The obtained results, offering structures for fault detection filter realization, are illustrated with a numerical example to note the effectiveness of the approach.

Keywords: linear noisy systems, Kalman filtering, innovation sequences, fault residual filters, distributed computing

1. Introduction

The castigatory principal aspect for designing a fault-tolerant control (FTC) structure is a functionality of diagnostic operations that solve the fault detection and isolation (FDI) tasks. These techniques most commonly use residuals generated by fault detection filters (FDF), followed by the residual signal evaluation within decision functions. Guarantying adequate sensitivity to faults, the accessory objective is to create residuals with minimal sensitivity to noises. Kalman filtering is an optimal state estimation process applied to a dynamic system that involves random noises, giving a linear, unbiased, and minimum error variance recursive algorithm to optimally estimate the unknown state of a dynamic system from noisy data taken from sensors [1, 2].

Practically, a bank of Kalman filters is used to achieve sensor and actuator fault detection applied to a steady-state system, while the statistical characteristics of the system are not required to be known after a fault has occurred [3, 4]. In these methods, the faults are assumed to be known, and the Kalman filters are designed for such kind of sensor or actuator faults. Another approach based on Kalman filtering is the analysis of the innovation sequence, since faults displace its zero mean and change its covariance matrix [5]. The associated problem is quick detection of changes in these parameters from their nominal values. Evidently, research in Kalman filter based-FDI is the subject of wide range of other publications (see, e.g., [6–9] and the reference therein). Other applications can be found in [10].

The state estimation obtained by the Kalman filter prediction-correction equations at every time instant can be solved almost optimally and substantially faster by applying a distributed approach [11–14]. With this setup can be exploited the fact that the correction error can be decaying exponentially with time instant sequence to reach the optimal values [15–18].

The chapter exploits a variant of distributed methods to apply the distributed correction stage filtering equations on each sensor level as well as an approach based on quasi-parallel central computation. Benefiting from the distributed Kalman filtering algorithm, two residually equivalent signal structures are presented for the discrete-time linear noisy systems.

The outline of this chapter is as follows: Section 1 delineates the problem and draws the basic starting points of solutions. Dealing with the discrete-time noisy systems description, the equations describing Kalman filters for uncorrelated measurement and system noises are traced out in Section 2, to delineate distributed approaches in Kalman filter design, suitable for supporting the fault residual generation, presented in Section 3. Section 4 gives a numerical example, illustrating the properties of the proposed method, and Section 5 presents some concluding remarks.

Throughout the chapter, the notations are narrowly standard in such a way that $\boldsymbol{x}^T$ and $\boldsymbol{X}^T$ denote the transpose of vector $\boldsymbol{x}$ and matrix $\boldsymbol{X}$, respectively, and $diag[\cdot]$ denotes a block diagonal matrix—for a square matrix $\boldsymbol{X} > 0$ means that $\boldsymbol{X}$ is a symmetric positive definite matrix. The symbol $\boldsymbol{I}_n$ indicates the nth order unit matrix; $\mathbb{R}$ denotes the set of real numbers; $\mathbb{R}^n$ and $\mathbb{R}^{n\times r}$ refer to the set of all n-dimensional real vectors and $n \times r$ real matrices, respectively; and $\mathcal{Z}_+$ is the set of all positive integers.

2. Discrete-time linear Kalman filter

In this section, one version of the Kalman filtering concept is applied for the discrete-time linear multi inputs and multi outputs (MIMO) plants with the system and output noises of the form

$$\boldsymbol{q}(i+1) = \boldsymbol{F}\boldsymbol{q}(i) + \boldsymbol{G}\boldsymbol{u}(i) + \boldsymbol{v}(i), \tag{1}$$

$$\boldsymbol{y}(i) = \boldsymbol{C}\boldsymbol{q}(i) + \boldsymbol{o}(i), \tag{2}$$

where $\boldsymbol{q}(i) \in \mathbb{R}^n$, $\boldsymbol{u}(i) \in \mathbb{R}^r$, and $\boldsymbol{y}(i) \in \mathbb{R}^m$ are vectors of the system state, input and measurement output variables, respectively; $\boldsymbol{v}(i) \in \mathbb{R}^n$ and $\boldsymbol{o}(i) \in \mathbb{R}^m$ are vectors of the system and measurement noise; and $\boldsymbol{F} \in \mathbb{R}^{n \times n}$, $\boldsymbol{G} \in \mathbb{R}^{n \times r}$, and $\boldsymbol{C} \in \mathbb{R}^{m \times n}$ are conditioned by $1 \le m, r \le n$. Kalman filter is used only for diagnostic purposes. Zero-mean Gaussian white noise processes are considered such that

$$E\left\{\begin{bmatrix} \boldsymbol{v}(i) \\ \boldsymbol{o}(i) \end{bmatrix}\right\} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}, \tag{3}$$

$$E\left\{\begin{bmatrix} \boldsymbol{v}(i) \\ \boldsymbol{o}(i) \end{bmatrix}\begin{bmatrix} \boldsymbol{v}^T(k) & \boldsymbol{o}^T(k) \end{bmatrix}\right\} = \begin{bmatrix} \boldsymbol{Q} & \boldsymbol{S} \\ \boldsymbol{S}^T & \boldsymbol{R} \end{bmatrix}\delta_{ik}, \tag{4}$$

where $E\{\cdot\}$ is the a statistical averaging operator,

$$\delta_{ik} = \begin{cases} 1 & \text{if } i = k, \\ 0 & \text{if } i \neq k, \end{cases} \tag{5}$$

is the Kronecker delta-function and the covariance matrices $\boldsymbol{Q} \in \mathbb{R}^{n \times n}$ and $\boldsymbol{R} \in \mathbb{R}^{m \times m}$ are symmetric positive definite matrices.

It is assumed that the deterministic system initial state $\boldsymbol{q}(0) = \boldsymbol{q}_0$ is independent of $\boldsymbol{v}(i)$ and $\boldsymbol{o}(i)$ in the sense that

$$E\{\boldsymbol{q}_0\boldsymbol{v}^T(i)\} = \boldsymbol{0}, \quad E\{\boldsymbol{q}_0\boldsymbol{o}^T(i)\} = \boldsymbol{0} \quad \text{for all } i \tag{6}$$

and that the system and measurement noises are uncorrelated, i.e., $\boldsymbol{S} = \boldsymbol{0}$.

Determining the optimal system state vector estimate, $\boldsymbol{q}_e(i|i-1)$ denotes the predicted estimation of the system state vector $\boldsymbol{q}(i)$ at the time instant i in the dependency on all noisy output measurement vector sequence $\{\boldsymbol{y}(j),\ j = 0, 1, \ldots, i-1\}$ up to time instant $i-1$; $\boldsymbol{q}_e(i|i)$ is the corrected estimation of the system state vector $\boldsymbol{q}(i)$ at the time instant i in the dependency on all noisy output measurement sequence $\{\boldsymbol{y}(j),\ j = 0, 1, \ldots, i\}$ up to time instant i; and $\boldsymbol{e}(i|i-1) = \boldsymbol{q}(i) - \boldsymbol{q}_e(i|i-1)$ and $\boldsymbol{e}(i|i) = \boldsymbol{q}(i) - \boldsymbol{q}_e(i|i)$ are prediction and correction errors.

Definition 1. *[19] If the Kalman filter, associated with the plant (1), (2) with uncorrelated system and measurement noises, is defined by the set of equations*

$$\boldsymbol{q}_e(i|i-1) = \boldsymbol{F}\boldsymbol{q}_e(i-1|i-1) + \boldsymbol{G}\boldsymbol{u}(i-1), \tag{7}$$

$$\boldsymbol{q}_e(i|i) = \boldsymbol{q}_e(i|i-1) + \boldsymbol{J}(i)\big(\boldsymbol{y}(i) - \boldsymbol{y}_e(i|i-1)\big), \tag{8}$$

$$\boldsymbol{y}_e(i|i-1) = \boldsymbol{C}\boldsymbol{q}_e(i|i-1), \tag{9}$$

$$\boldsymbol{y}_e(i|i) = \boldsymbol{C}\boldsymbol{q}_e(i|i), \tag{10}$$

then with $\boldsymbol{q}_e(0|0) = \boldsymbol{q}_0$, $\boldsymbol{P}(0|0) = \boldsymbol{Q}^\circ$, *where* $\boldsymbol{Q}^\circ \in \mathbb{R}^{n \times n}$ *is a positive definite matrix, yielding*

$$\boldsymbol{J}(i) = \boldsymbol{P}(i|i-1)\boldsymbol{C}^T\left(\boldsymbol{R} + \boldsymbol{C}\boldsymbol{P}(i|i-1)\boldsymbol{C}^T\right)^{-1}, \tag{11}$$

$$P(i|i-1) = FP(i-1|i-1)F^T + Q, \tag{12}$$

$$P(i|i) = (I - J(i)C)P(i|i-1), \tag{13}$$

where

$$P(i|i-1) = E\{e(i|i-1)e^T(i|i-1)\}, \tag{14}$$

$$P(i|i) = E\{e(i|i)e^T(i|i)\}, \tag{15}$$

are the covariance matrices of prediction and correction errors and $J(i) \in \mathbb{R}^{n\times m}$ *is the Kalman filter gain matrix, all at time instant* i.

The discrete-time Kalman filter equations can be algebraically manipulated into a variety of forms [6, 16, 20]. From the point of view of distributed filtration, it is necessary to achieve such form of the equation for calculating the Kalman gain $J(i)$ that yields the matrix C from the matrix inversion operation (see (11)). If the system and measurement noises are uncorrelated, then for the Kalman filter gain, one can propose the following:

Lemma 1. *If the system and measurement noises are uncorrelated, then the Kalman filter gain and the correction error covariance matrix can be computed using (12) and*

$$J(i) = P(i|i)C^T R^{-1}, \tag{16}$$

$$P^{-1}(i|i) = P^{-1}(i|i-1) + C^T R^{-1} C. \tag{17}$$

Proof. Substituting (11) into (13), one can obtain that

$$P(i|i) = P(i|i-1) - P(i|i-1)C^T\left(R + CP(i|i-1)C^T\right)^{-1}CP(i|i-1). \tag{18}$$

Exploiting the Sherman-Morrison-Woodbury formula of the form [21].

$$\left(A + BDB^T\right)^{-1} = A^{-1} - A^{-1}B\left(D^{-1} + B^T A^{-1} B\right)^{-1} B^T A^{-1}, \tag{19}$$

where square invertible matrices A, D, and a matrix B of appropriate dimensions are such that $\left(A + BDB^T\right)$ is invertible, with

$$A = P(i|i-1), \quad B = P(i|i-1)C^T, \quad D = -\left(R + CP(i|i-1)C^T\right)^{-1}, \tag{20}$$

yields, since the covariance matrices are positive definite,

$$P^{-1}(i|i) = P^{-1}(i|i-1) - P^{-1}(i|i-1)P(i|i-1)C^T E^{-1} CP(i|i-1)P^{-1}(i|i-1), \tag{21}$$

where

$$E = -R - CP(i|i-1)C^T + CP(i|i-1)P^{-1}(i|i-1)P(i|i-1)C^T = -R. \tag{22}$$

Then, evidently,

$$P^{-1}(i|i) = P^{-1}(i|i-1) - C^T E^{-1} C \tag{23}$$

and (31) implies (17).

Premultiplying the left side by $P^{-1}(i|i)$ and postmultiplying the right side by $P^{-1}(i|i-1)$, (13) gives

$$P^{-1}(i|i-1) = P^{-1}(i|i) - P^{-1}(i|i)J(i)C \tag{24}$$

and comparing (17) and (24), it can be seen that

$$P^{-1}(i|i)J(i)C = C^T R^{-1} C. \tag{25}$$

Thus, (25) implies (16). This concludes the proof.

Note, since $C^T R^{-1} C$ is at least a positive semi-definite matrix, it is evident from (17) that $P(i|i)$ is never larger than $P(i|i-1)$. Moreover, the result is an unbiased filter with the estimates of minimum error variances. More details can be found in [12, 22].

Corollary 1. *Considering that $q_e(i|i-1)$ is known and $q_e(i|i)$ is the best estimate of $q(i)$ that minimizes the cost criterion*

$$\begin{aligned} T(i) = {} & \left(q(i) - q_e(i|i-1)\right)^T P^{-1}(i|i-1)\left(q(i) - q_e(i|i-1)\right) \\ & + (y(i) - Cq(i))^T R^{-1}(y(i) - Cq(i)). \end{aligned} \tag{26}$$

Then, evaluating (26) it follows, with the optimal setting of a state vector estimate $q(i) = q(i|i)$, that the minimum expected cost is given by

$$\frac{dT(i)}{dq(i)^T} = P^{-1}(i|i-1)\left(q(i|i) - q_e(i|i-1)\right) - C^T R^{-1}(y(i) - Cq(i|i)) = 0, \tag{27}$$

which implies

$$\begin{aligned} \left(P^{-1}(i|i-1) + C^T R^{-1} C\right) q_e(i|i) &= P^{-1}(i|i-1) q_e(i|i-1) + C^T R^{-1} y(i) \\ &= \left(P^{-1}(i|i-1) + C^T R^{-1} C\right) q_e(i|i-1) \\ &\quad + C^T R^{-1}\left(y(i) - Cq_e(i|i-1)\right). \end{aligned} \tag{28}$$

Therefore, using the above relations, at the ith step Eq. (28) gives

$$\begin{aligned} q_e(i|i) &= q_e(i|i-1) + \left(P^{-1}(i|i-1) + C^T R^{-1} C\right)^{-1} C^T R^{-1}\left(y(i) - Cq_e(i|i-1)\right) \\ &= q_e(i|i-1) + P(i|i) \times C^T R^{-1}\left(y(i) - Cq_e(i|i-1)\right). \end{aligned} \tag{29}$$

Pre-multiplying the left side by $P(i|i)$ and post-multiplying the right side by $P(i|i-1)$ then it follows from (17)

$$P(i|i-1) = P(i|i) + P(i|i) C^T R^{-1} C P(i|i-1), \tag{30}$$

which can be proved recursively as follows

$$\boldsymbol{P}(i|i) = \left(\boldsymbol{I}_n - \boldsymbol{P}(i|i)\boldsymbol{C}^T\boldsymbol{R}^{-1}\boldsymbol{C}\right)\boldsymbol{P}(i|i-1). \tag{31}$$

Comparing (29) with the covariance matrix of the filtering error given by (13), it is evident that

$$\boldsymbol{J}(i) = \boldsymbol{P}(i|i)\boldsymbol{C}^T\boldsymbol{R}^{-1} \tag{32}$$

which is identical to (16).

On the other side, substituting (11) into (13), one can write

$$\boldsymbol{P}(i|i) = \boldsymbol{P}(i|i-1) - \boldsymbol{P}(i|i-1)\boldsymbol{C}^T\left(\boldsymbol{R} + \boldsymbol{C}\boldsymbol{P}(i|i-1)\boldsymbol{C}^T\right)^{-1}\boldsymbol{C}\boldsymbol{P}(i|i-1) \tag{33}$$

and using the Sherman-Morrison-Woodbury formula, Eq. (27), it follows

$$\boldsymbol{P}^{-1}(i|i) = \boldsymbol{P}^{-1}(i|i-1) - \boldsymbol{C}^T\left(-\boldsymbol{R} - \boldsymbol{C}\boldsymbol{P}(i|i-1)\boldsymbol{C}^T + \boldsymbol{C}\boldsymbol{P}(i|i-1)\boldsymbol{C}^T\right)^{-1}\boldsymbol{C} \tag{34}$$

and so, evidently, (34) gives (17).

3. Fault residual generation using distributed Kalman filtering

The obtained equations, Eqs. (16) and (17), allow the use of the open form of the Kalman filter equations if

$$\boldsymbol{R} = E\left\{\boldsymbol{o}(i)\boldsymbol{o}^T(i)\right\} = \mathrm{diag}[R_1 \quad R_2 \quad \cdots \quad R_m]. \tag{35}$$

Writing separately,

$$\mathbf{y}^T(i) = \left[y_1(i) \quad y_2(i) \quad \cdots \quad y_m(i)\right], \tag{36}$$

$$\boldsymbol{u}^T(i) = [u_1(i) \quad u_2(i) \quad \cdots \quad u_r(i)], \tag{37}$$

$$\boldsymbol{C}^T = [\boldsymbol{c}_1 \quad \boldsymbol{c}_2 \quad \ldots \quad \boldsymbol{c}_m], \quad \boldsymbol{G} = \left[\boldsymbol{g}_1 \quad \boldsymbol{g}_2 \quad \ldots \quad \boldsymbol{g}_r\right], \tag{38}$$

then, (7)–(11), (16), and (27) imply

$$\boldsymbol{q}_e(i|i-1) = \boldsymbol{F}\boldsymbol{q}_e(i-1|i-1) + \sum_{h=1}^{w}\boldsymbol{g}_h u_h(i-1), \tag{39}$$

$$\boldsymbol{q}_e(i|i) = \boldsymbol{q}_e(i|i-1) + \sum_{h=1}^{m}\boldsymbol{j}_h(i)\left(\boldsymbol{y}_h(i) - \boldsymbol{c}_h^T\boldsymbol{q}_e(i|i-1)\right), \tag{40}$$

$$y_{ej}(i|i-1) = \boldsymbol{c}_h^T\boldsymbol{q}_e(i|i-1), \tag{41}$$

$$y_{ej}(i|i) = \boldsymbol{c}_h^T \boldsymbol{q}_e(i|i), \tag{42}$$

$$\boldsymbol{j}_h(i) = \boldsymbol{P}(i|i)\boldsymbol{c}_h R_h^{-1}, \tag{43}$$

$$\boldsymbol{P}^{-1}(i|i) = \boldsymbol{P}^{-1}(i|i-1) + \sum_{h=1}^{m} \boldsymbol{c}_h R_h^{-1} \boldsymbol{c}_h^T, \tag{44}$$

$$\boldsymbol{P}(i|i-1) = \boldsymbol{F}\boldsymbol{P}(i-1|i-1)\boldsymbol{F}^T + \boldsymbol{Q}. \tag{45}$$

It is evident from the above given formulation that the relation of (40) gives the possibility to compute corrections from the data obtained at all sensor nodes.

Theorem 1. *Defining the residual vector as*

$$\boldsymbol{r}^T(i) = [\, z_1(i) \quad z_2(i) \quad \dots \quad z_m(i)\,], \tag{46}$$

where

$$z_h(i) = \boldsymbol{y}_h(i) - \boldsymbol{c}_h^T \boldsymbol{q}_{ec}(i|i-1), \tag{47}$$

then

$$\boldsymbol{q}_{ec}(i|i-1) = \boldsymbol{F}\boldsymbol{q}_{ec}(i-1|i-1) + \sum_{h=1}^{r} \boldsymbol{g}_h u_h(i-1), \tag{48}$$

$$\boldsymbol{q}_{ed}(i|i-1) = \boldsymbol{F}\boldsymbol{q}_{ed}(i-1|i-1), \tag{49}$$

$$\boldsymbol{q}_{ec}(i|i) = \boldsymbol{q}_{ec}(i|i-1), \tag{50}$$

$$\boldsymbol{q}_{ed}(i|i) = \boldsymbol{q}_{ed}(i|i-1) + \boldsymbol{P}(i|i) \sum_{h=1}^{m} \boldsymbol{c}_h R_h^{-1} \left(z_h(i) - \boldsymbol{c}_h^T \boldsymbol{q}_{ed}(i|i-1) \right), \tag{51}$$

while the filter gain matrices, as well as recurrences of the covariance matrices are given by (43)–(45).

Proof. Considering that there are components of the system state vector estimate that are dependent on the control signal as well as ones that are not dependent on the control signals, since the correction step does not depend on the control inputs, (40) can be rewritten as

$$\boldsymbol{q}_e(i|i) = \boldsymbol{q}_{ec}(i|i-1) + \boldsymbol{q}_{ed}(i|i-1) + \sum_{h=1}^{m} \boldsymbol{j}_h(i) \left(\boldsymbol{y}_h(i) - \boldsymbol{c}_h^T \left(\boldsymbol{q}_{ec}(i|i-1) + \boldsymbol{q}_{ed}(i|i-1) \right) \right). \tag{52}$$

Prescribing that

$$\boldsymbol{q}_e(i|i) = \boldsymbol{q}_{ed}(i|i) + \boldsymbol{q}_{ec}(i|i), \tag{53}$$

Eqs. (52) and (53) can be separated as

$$\boldsymbol{q}_{ec}(i|i) = \boldsymbol{q}_{ec}(i|i-1), \tag{54}$$

$$\boldsymbol{q}_{ed}(i|i) = \boldsymbol{q}_{ed}(i|i-1) + \sum_{h=1}^{m} \boldsymbol{j}_h(i)\left(\boldsymbol{y}_h(i) - \boldsymbol{c}_h^T\left(\boldsymbol{q}_{ec}(i|i-1) + \boldsymbol{q}_{ed}(i|i-1)\right)\right) \tag{55}$$

and using (47), (54) gives (50), and (55) implies (51).

Substituting (53) in (39) yields

$$\boldsymbol{q}_e(i|i-1) = \boldsymbol{F}\left(\boldsymbol{q}_{ed}(i-1|i-1) + \boldsymbol{q}_{ec}(i-1|i-1)\right) + \sum_{h=1}^{w} \boldsymbol{g}_h u_h(i-1) \tag{56}$$

and, evidently, (56) implies (48) and (49). This concludes the proof. □

Remark 1. *If Eqs. (46)–(51) are analyzed from a computational point of view, it is clear that their structures support autonomous parallel calculations only with a single interaction defined by Eq. (47). However, the cost for this parallelism is additional computation at each step, but the directional properties of the components of the residual vector are advantageous in the case of single sensor faults. The directional sensor residual property derives indirectly from relationship (44). Since every component* $\boldsymbol{z}_h(i)$ *carries with it the measurement noise* $o_h(i)$ *if* $\boldsymbol{q}_{ed}(i|i-1)$ *is used for LQG control, it will be no noise at the state control law input.*

In principle, it is possible to define the residue generation by results of the local system state correction at Kalman filtration at the time instant i.

Theorem 2. *Defining the residual vector as*

$$\boldsymbol{r}^T(i) = [\, z_1(i) \quad z_2(i) \quad \dots \quad z_m(i)\,], \tag{57}$$

where

$$z_h(i) = \boldsymbol{y}_h(i) - \boldsymbol{c}_h^T \boldsymbol{q}_{ec}(i|i-1), \tag{58}$$

then

$$\boldsymbol{q}_{edj}(i|i-1) = \boldsymbol{F}\boldsymbol{q}_{edj}(i-1|i-1), \tag{59}$$

$$\boldsymbol{q}_{edj}(i|i) = \boldsymbol{q}_{edj}(i|i-1) + \boldsymbol{j}_h(i)\Big(z_h(i) - \boldsymbol{c}_h^T \boldsymbol{q}_{edj}(i|i-1)\Big), \tag{60}$$

$$\boldsymbol{j}_h(i) = \boldsymbol{P}_h(i|i)\boldsymbol{c}_h R_h^{-1}, \tag{61}$$

$$\boldsymbol{P}_h^{-1}(i|i) = \boldsymbol{P}^{-1}(i|i-1) + \boldsymbol{c}_h R_h^{-1} \boldsymbol{c}_h^T, \tag{62}$$

while the predicted system state at the time instant i is computed centrally and the filtered full system state is covered by the equations

$$\begin{aligned} \boldsymbol{q}_{ed}(i|i) = & \sum_{h=1}^{m} \boldsymbol{P}(i|i)\boldsymbol{P}_h^{-1}(i|i)\boldsymbol{q}_{edj}(i|i) \\ & - \sum_{h=1}^{m} \boldsymbol{P}(i|i)\boldsymbol{P}^{-1}(i|i-1)\boldsymbol{F}\boldsymbol{q}_{edj}(i-1)|i-1)) \\ & + \boldsymbol{P}(i|i)\boldsymbol{P}^{-1}(i|i-1)\boldsymbol{F}\boldsymbol{q}_{ed}(i-1|i-1), \end{aligned} \tag{63}$$

$$\boldsymbol{q}_{ec}(i|i) = \boldsymbol{q}_{ec}(i|i-1) \tag{64}$$

at the time instants $i \in \mathcal{Z}_+$.

Proof. The correction step for the Kalman filter in Eq. (51) can be prescribed locally for the *j*th node as

$$\boldsymbol{q}_{edj}(i|i) = \boldsymbol{q}_{edj}(i|i-1) + \boldsymbol{j}_h(i)(\boldsymbol{z}_h(i) - \boldsymbol{z}_{dh}(i|i-1)), \tag{65}$$

where

$$\boldsymbol{z}_h(i) = \boldsymbol{y}_h(i) - \boldsymbol{c}_h^T\boldsymbol{q}_{ec}(i|i-1), \tag{66}$$

$$\boldsymbol{z}_{dh}(i|i-1) = \boldsymbol{c}_h^T\boldsymbol{q}_{edj}(i|i-1), \tag{67}$$

$$\boldsymbol{j}_h(i) = \boldsymbol{P}_h(i|i)\boldsymbol{c}_h R_h^{-1}, \tag{68}$$

$$\boldsymbol{P}_h^{-1}(i|i) = \boldsymbol{P}^{-1}(i|i-1) + \boldsymbol{c}_h R_h^{-1}\boldsymbol{c}_h^T. \tag{69}$$

Substituting (66), rearranging and postmultiplying the left side by $\boldsymbol{P}_h^{-1}(i|i)$, (65) implies

$$\boldsymbol{P}_h^{-1}(i|i)\Big(\boldsymbol{q}_{edj}(i|i) - \boldsymbol{q}_{edj}(i|i-1)\Big) = \boldsymbol{c}_h R_h^{-1}(\boldsymbol{z}_h(i) - \boldsymbol{z}_{dh}(i|i-1)), \tag{70}$$

$$\boldsymbol{c}_h R_h^{-1}\boldsymbol{z}_h(i) = \boldsymbol{c}_h R_h^{-1}\boldsymbol{c}_h^T\boldsymbol{q}_{edj}(i|i-1) + \boldsymbol{P}_h^{-1}(i|i)\Big(\boldsymbol{q}_{edj}(i|i) - \boldsymbol{q}_{edj}(i|i-1)\Big), \tag{71}$$

respectively. Since (69) gives

$$\boldsymbol{c}_h R_h^{-1}\boldsymbol{c}_h^T = \boldsymbol{P}_h^{-1}(i|i) - \boldsymbol{P}^{-1}(i|i-1), \tag{72}$$

with a simple elimination after inserting (72), (71) gives

$$\begin{aligned} \boldsymbol{c}_h R_h^{-1}\boldsymbol{z}_h(i) & = \boldsymbol{P}_h^{-1}(i|i)\Big(\boldsymbol{q}_{edj}(i|i) - \boldsymbol{q}_{edj}(i|i-1)\Big) \\ & \quad + \boldsymbol{P}_h^{-1}(i|i)\boldsymbol{q}_{edj}(i|i-1) - \boldsymbol{P}^{-1}(i|i-1)\boldsymbol{q}_{edj}(i|i-1) \\ & = \boldsymbol{P}_h^{-1}(i|i)\boldsymbol{q}_{edj}(i|i) - \boldsymbol{P}^{-1}(i|i-1)\boldsymbol{q}_{edj}(i|i-1). \end{aligned} \tag{73}$$

Combining (49) and (51) results in

$$\boldsymbol{q}_{ed}(i|i) = \boldsymbol{F}\boldsymbol{q}_{ed}(i-1|i-1) + \boldsymbol{P}(i|i)\sum_{h=1}^{m}\boldsymbol{c}_h R_h^{-1}\big(\boldsymbol{z}_h(i) - \boldsymbol{c}_h^T\boldsymbol{F}\boldsymbol{q}_{ed}(i-1|i-1), \tag{74}$$

which can be written as

$$\boldsymbol{q}_{ed}(i|i) = \sum_{h=1}^{m} \boldsymbol{P}(i|i)\boldsymbol{c}_h R_h^{-1} \boldsymbol{z}_h(i) + \left(\boldsymbol{I}_n - \sum_{h=1}^{n} \boldsymbol{P}(i|i)\boldsymbol{c}_h R_h^{-1} \boldsymbol{c}_h^T \right) \boldsymbol{F}\boldsymbol{q}_{ed}(i-1|i-1). \tag{75}$$

Pre-multiplying the left side of (44) by $\boldsymbol{P}(i|i)$ leads to

$$\boldsymbol{I}_n - \sum_{h=1}^{w} \boldsymbol{P}(i|i)\boldsymbol{c}_h R_h^{-1} \boldsymbol{c}_h^T = \boldsymbol{P}(i|i)\boldsymbol{P}^{-1}(i|i-1) \tag{76}$$

and considering (76), relation (75) takes the form

$$\boldsymbol{q}_{ed}(i|i) = \sum_{h=1}^{m} \boldsymbol{P}(i|i)\boldsymbol{c}_h R_h^{-1} \boldsymbol{z}_h(i) + \boldsymbol{P}(i|i)\boldsymbol{P}^{-1}(i|i-1)\boldsymbol{F}\boldsymbol{q}_{ed}(i-1|i-1). \tag{77}$$

Thus, the substitution of (73) into (77) gives

$$\begin{aligned} \boldsymbol{q}_{ed}(i|i) = {} & \sum_{h=1}^{m} \boldsymbol{P}(i|i)\boldsymbol{P}_h^{-1}(i|i)\boldsymbol{q}_{edj}(i|i) - \sum_{h=1}^{m} \boldsymbol{P}(i|i)\boldsymbol{P}^{-1}(i|i-1)\boldsymbol{q}_{edj}(i|i-1) \\ & + \boldsymbol{P}(i|i)\boldsymbol{P}^{-1}(i|i-1)\boldsymbol{F}\boldsymbol{q}_{ed}(i-1|i-1) \end{aligned} \tag{78}$$

and with the notation

$$\boldsymbol{q}_{edj}(i|i-1) = \boldsymbol{F}\boldsymbol{q}_{edj}(i-1|i-1), \tag{79}$$

(78) implies (62). This concludes the proof.

Remark 2. *It is clear that each of Eqs. (59)–(62) is only bound to the jth node and therefore such correction can be done locally for each sensor. Conversely, the system state prediction and the residual vector must be computed globally by Eqs. (39), (53), (57), (58), (63), and (64), respectively.*

Remark 3. *Obviously, under the above conditions, the distributed realization of the Kalman filter correction step is optimal in the sense of criterion (26), and therefore the structure of the fault residual generator based on distributed Kalman filtration is optimal.*

4. Illustrative examples

4.1. Example 1

To eliminate specific system dependencies, the Schur discrete-time linear strictly positive system [23] is used for demonstration of the Kalman filtering technique in residual signals construction. The considered system can be put in the system class (1)–(4), with the sampling period $t_s = 0.8s$, with uncorrelated system and measurement Gaussian noise and the noise covariance matrices

http://dx.doi.org/10.5772/intechopen.80296

$$\boldsymbol{R} = \text{diag}\ [0.003\ \ 0.04], \quad \boldsymbol{Q} = 0.002\boldsymbol{I}_4$$

while the system matrix parameters are

$$\boldsymbol{F} = \begin{bmatrix} 0.7650 & 0.6267 & 0.6058 & 0.0510 \\ 0.1048 & 0.1083 & 0.0813 & 0.0098 \\ 0.1484 & 0.1419 & 0.1171 & 0.0150 \\ 0.1709 & 0.2286 & 0.1603 & 0.1998 \end{bmatrix}, \boldsymbol{G} = \begin{bmatrix} 0.0241 & 0.0139 \\ 0.0151 & 0.0013 \\ 0.0109 & 0.0056 \\ 0.0142 & 0.0032 \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} 0.0001 & 0 & 1 & 0 \\ 0.0000 & 0 & 0 & 1 \end{bmatrix}.$$

Since the discrete-time stochastic linear strictly positive system is stable, the system control law in simulations is defined for the forced mode control as $\boldsymbol{u}(i) = \boldsymbol{W}\boldsymbol{w}_o$, where

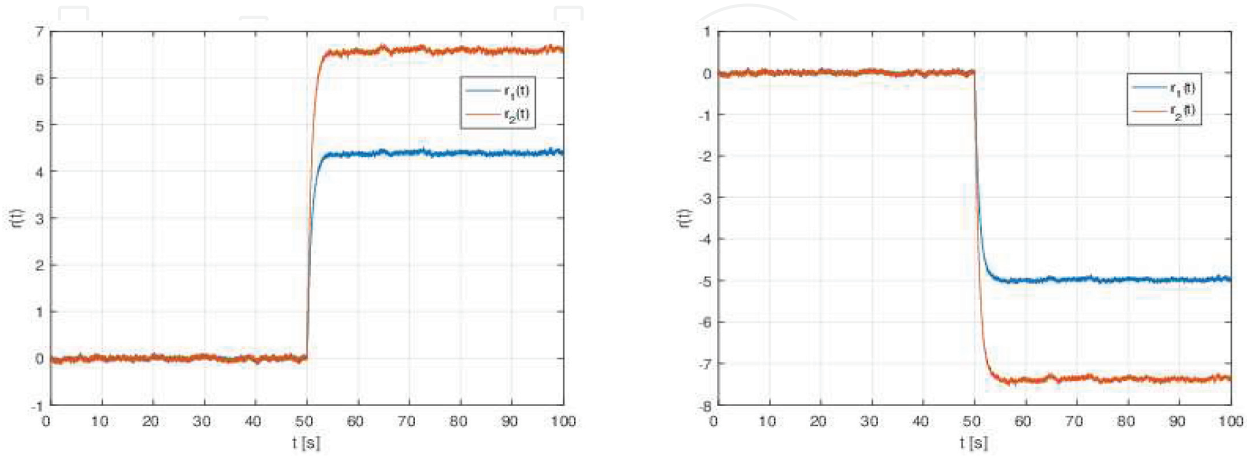
$$\boldsymbol{W} = \begin{bmatrix} -117.3841 & 79.3124 \\ 280.8078 & -187.1829 \end{bmatrix}, \quad \boldsymbol{w}_o = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}$$

and the initial conditions for the Kalman filter are

$$\boldsymbol{q}_e(0|0) = \boldsymbol{0}, \quad \boldsymbol{P}(0|0) = \boldsymbol{I}_4.$$

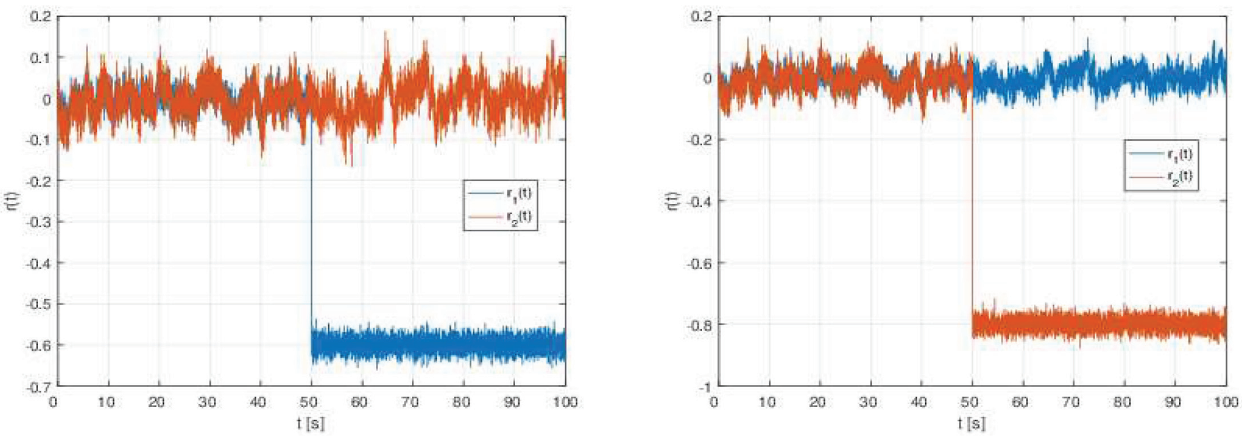
Using the given initial conditions, **Figures 1** and **2** display the residuals obtained by the residual filter generated by the distributed Kalman filter defined in (46)–(51), reflecting single actuator and sensor faults, starting at the time instant $t = 50\ \ s$. The time scale is discrete with the sampling period $T = 0.8s$.

Evidently, the residual trajectories indicate that the proposed residual filter generates directional signals in the event of single sensor faults, and has a significantly higher dynamic signal range in the event of single faults of the actuators, as compared to the residual presented using the standard Kalman filter.



a) b)

Figure 1. Residual responses to single faults: (a) the first actuator and (b) the second actuator.



a) b)

Figure 2. Residual responses to single faults: (a) the first sensor and (b) the second sensor.

4.2. Example 2

To produce another example that demonstrates achievable performances of the presented design method, the sign-indefinite Schur discrete-time linear system is used, where [24]

$$\boldsymbol{F} = \begin{bmatrix} 1.1039 & -0.2360 & -0.0563 & -0.0229 \\ 0.1063 & 0.7971 & -0.0575 & -0.0109 \\ 0.0100 & -0.0211 & 0.9401 & -0.0476 \\ 0.0599 & -0.0843 & -0.0111 & 0.9633 \end{bmatrix}, \boldsymbol{G} = \begin{bmatrix} 0.1957 & 0.2878 \\ 0.0976 & 0.1921 \\ 0.0969 & 0.0939 \\ 0.0012 & 0.0982 \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$t_s = 0.05s$, and the Gaussian noise covariance matrices are $\boldsymbol{R} = \text{diag}\ [0.003\ \ 0.04]$ and $\boldsymbol{Q} = 0.002\boldsymbol{I}_4$. To force the desired system output values, it is prescribed

$$\boldsymbol{W} = \begin{bmatrix} -2.1250 & 0.9375 \\ 1.8750 & -0.5625 \end{bmatrix}, \quad \boldsymbol{w}_o = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}, \quad \boldsymbol{q}_e(0|0) = \boldsymbol{0}, \quad \boldsymbol{P}(0|0) = \boldsymbol{I}_4.$$

Figures 3 and **4** present the residual responses of the residual filter based on distributed Kalman filtering, from which it is clear that the used principle, especially when compared to the achievable responses with the alternative system presented in Example 1, is operational. Evidently, the residual filter behavior is also acceptable for the system parameters in this example and the system noise environment. The step-like single faults start and continue from the time instant $t = 25\ \ s$, the time scale is discrete with the sampling period $T = 0.05s$.

4.3. Example 3

Following the above-mentioned procedures, Example 3 verifies their effectiveness for the linear discrete-time system with the parameters

http://dx.doi.org/10.5772/intechopen.80296

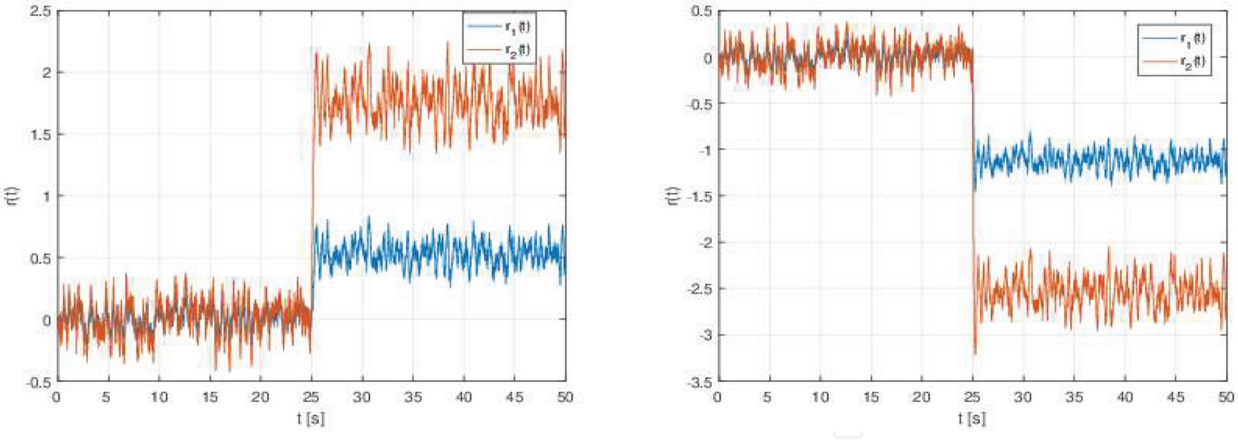
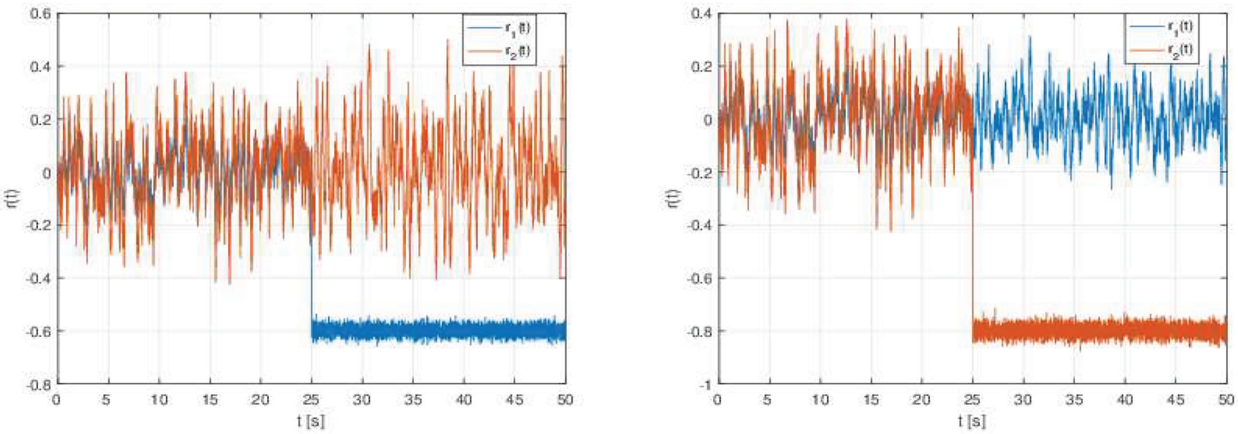


Figure 3. Residual responses to single faults: (a) the first actuator and (b) the second actuator.



a) b)

Figure 4. Residual responses to single faults: (a) the first sensor and (b) the second sensor.

$$\mathbf{F} = \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 \\ 0.7 & 0 & 0 & 0.1 \\ 0.2 & 0.8 & 0.2 & 0.3 \\ 0 & 0 & 0.5 & 0.2 \end{bmatrix}, \boldsymbol{g} = \begin{bmatrix} 0.1 \\ 0 \\ 0.3 \\ 0 \end{bmatrix}, \boldsymbol{g}_f = \begin{bmatrix} 0 \\ 0 \\ 0.3 \\ 0 \end{bmatrix}, \boldsymbol{c}^T = [1 \quad 0 \quad 0 \quad 1], \boldsymbol{c}_f^T = [0 \quad 0 \quad 0 \quad 1],$$

where $\boldsymbol{F}$ is a left-stochastic matrix [25], $t_s = 0.05s$, and the Gaussian noise covariance matrices are $R = 0.003$ and $\boldsymbol{Q} = 0.002\boldsymbol{I}_4$. The behavior of the system is changed by the state-feedback control

$$u(i) = -\boldsymbol{k}^T \boldsymbol{q}(i) + W w_o$$

where

$$\boldsymbol{k}^T = [0.2982 \quad 1.0731 \quad 0.3711 \quad 0.6412], \quad W = 1.1834, \quad w_o = 0.6,$$

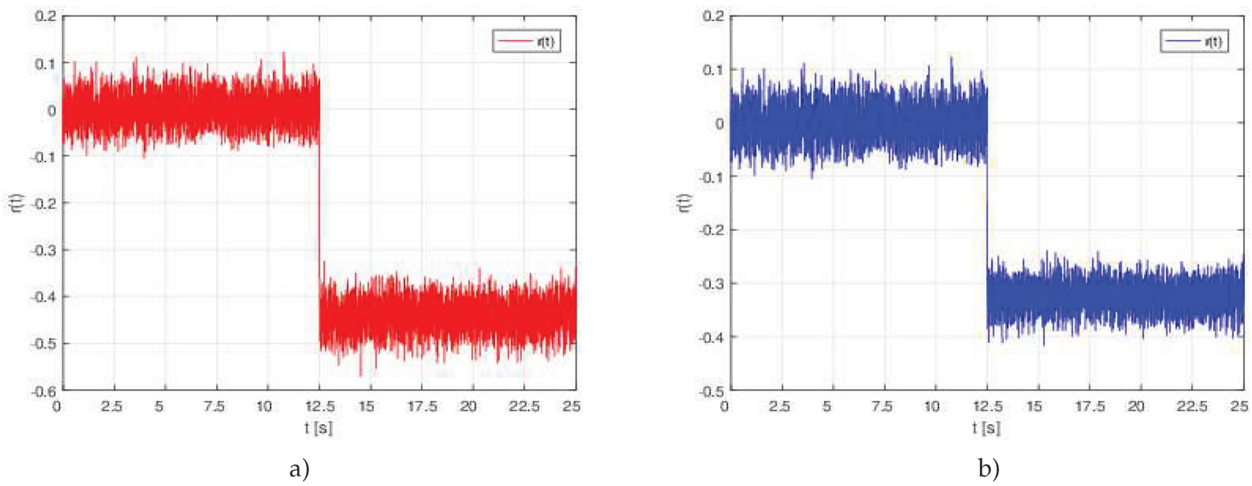


Figure 5. Residual responses to single faults: (a) the actuator and (b) the sensor.

which provides Schur matrix F_c as follows

$$\boldsymbol{F}_c = \boldsymbol{F} - \boldsymbol{g}\boldsymbol{k}^T = \begin{bmatrix} 0.0702 & 0.0927 & 0.2629 & 0.3359 \\ 0.7000 & 0 & 0 & 0.1000 \\ 0.1105 & 0.4781 & 0.0887 & 0.1076 \\ 0 & 0 & 0.5000 & 0.2000 \end{bmatrix}$$

Note, the steady states of $\boldsymbol{F}_c$ are absorbing states.

The single fault effects in residuals, when using the proposed algorithm of distributed Kalman filtering (46)–(51) with setting $\boldsymbol{F} = \boldsymbol{F}_c$, $\boldsymbol{q}_e(0|0) = \boldsymbol{0}$ and $\boldsymbol{P}(0|0) = \boldsymbol{I}_4$ are shown in **Figure 5**. The time scale is discrete with the time sample interval $T = 0.1$s.

From the figures, we find that the fault responses are satisfactory by using the proposed method also for this system and noise environment.

Analyzing all examples, exactly the same responses are reached using the same parameters as before and assuming the same fault patterns if the residuals are evaluated exploiting formulas (57)–(64) or (46)–(51). It is given by the equivalent principles of distributed computing that are bound by the equivalent relationships (44) and (62), respectively. As a result, in this particular point of view, the proposed distributed algorithm has only one common matrix component, $\boldsymbol{P}^{-1}(i|i-1)$, which has to be transmitted to every separated sensor before carrying out the state correction filtering step at every time instant. Since the correction step at time instant i is done in dependency on the measured value at the same time instant $\boldsymbol{y}(i)$, it is clear that the shorter the computation at the correction step, the smaller the time-delay introduced into the fault detection system responses.

http://dx.doi.org/10.5772/intechopen.80296

5. Concluding remarks

Realization forms for fault detection residual structures, based on distributed Kalman filtering destined for noisy discrete-time linear systems, were derived in this chapter. The main idea deals with introducing a distributed sensor measurement noise corrector step of a Kalman filter, applied in such a way to be locally uncorrelated with other sensor measurements. Two different algorithmic supports, a parallel decentralized Kalman filter and a locally distributed Kalman filter, are constructed to generate fault residuals. Both solutions are discussed in detail to demonstrate the condition of their equivalency. The problem accomplishes the manipulation in the manner giving guaranty of asymptotic stability of a local fault residual detection filter. Simulated example is included to illustrate the applicability of the proposed methods, encouraging the results that are obtained. Note, since the Kalman filter is based on the nominal system parameters $\boldsymbol{G}$ and $\boldsymbol{C}$, it cannot estimate system states and outputs starting for faulty regimes with modified matrices $\boldsymbol{G}_f$ and $\boldsymbol{C}_f$, respectively.

From the point of cloud-based distributed systems, to combine appropriately the network and computational resources, a locally distributed Kalman filter seems to be naturally adaptable, also with cross-correlated sensor noises. Of course, no theoretical justification for this affirmation is presented in the chapter. This is seen as an area for future research by the authors.

Acknowledgements

The work presented in this chapter was supported by VEGA, the Grant Agency of the Ministry of Education and the Academy of Science of Slovak Republic, under Grant No. 1/0608/17. This support is very gratefully acknowledged.

Author details

Dušan Krokavec* and Anna Filasová

*Address all correspondence to: dusan.krokavec@tuke.sk

Department of Cybernetics and Artificial Intelligence, Faculty of Electrical Engineering and Informatics, Technical University of Košice, Košice, Slovakia

References

[1] Hassan MF, Salut G, Singh MG, Titli A. A decentralized computational algorithm for the global Kalman filter. IEEE Transactions on Automatic Control. 1978;**23**(2):262-268

[2] Song E, Zhu Y, Zhou J, You Z. Optimal Kalman filtering fusion with cross-correlated sensor noises. Automatica. 2007;**43**(8):1450-1456

[3] Hajiyev C, Caliskan F. Sensor/actuator fault diagnosis based on statistical analysis of innovation sequence and robust Kalman filtering. Aerospace Science and Technology. 2000;**4**(6):415-422

[4] Hu J, Xiao L. Multi-sensor fault diagnosis of aircraft engine based on Kalman filter group. In: Proceedings of 2016 Chinese Intelligent Systems Conference, Vol. 1; 22–23 October 2016; Xiamen, China; Lecture Notes in Electrical Engineering 404. Singapore: Springer Science + Business Media; 2016. pp. 363-379

[5] Mehra RK, Peschon J. An innovations approach to fault detection and diagnosis in dynamic systems. Automatica. 1971;**7**(512):637-640

[6] Petersen IE, Savkin AV. Robust Kalman Filtering for Signals and Systems with Large Uncertainties. Boston: Birkhäuser; 1999. 207 p

[7] Kobayashi T, Simon DL. Evaluation of an enhanced bank of Kalman filters for in-flight aircraft engine sensor fault diagnostics. Transactions of the ASME. 2005;**127**(3):497-502

[8] Xue W, Guo YQ. Application of Kalman filters for the fault diagnoses of aircraft engine. In: Kordic V, editor. Kalman Filter. In Tech: Rijeka; 2010. pp. 349-362

[9] Liu X, Xue N, Yuan Y. Aircraft engine sensor fault diagnostics using an on-line OBEM update method. PLoS One. 2017;**12**(2):1-23

[10] Zhu Y, Zhou J, Shen X, Song E, Luo Y. Networked Multisensor Decision and Estimation Fusion Based on Advanced Mathematical Methods. Boca Raton: CRC Press; 2013. 437 p

[11] Duda Z. State estimation in a decentralized discrete time LQG control for a multisensor system. Archives of Control Sciences. 2017;**27**(1):29-39

[12] Filasová A, Krokavec D. On distributed discrete-time Kalman filtering in large linear time-invariant systems. In: Proceedings of the 21st International Conference on Process Control PC; 6–9 June 2017; Štrbské Pleso, Slovakia. 2017. pp. 291-296

[13] Mahmoud MS. Decentralized Control and Filtering in Interconnected Dynamical Systems. Boca Raton: CRC Press; 2017. 612 p

[14] Govaers F. Distributed Kalman filter. In: Serra GL, editor. Kalman Filters. Theory for Advanced Applications. London: IntechOpen; 2018. 314 p

[15] Speyer JL. Computation and transmission requirements for a decentralized Linear-Quadratic-Gaussian control problem. IEEE Transactions on Automatic Control. 1979; **24**(2):266-269

[16] Berg TM, Durrant-Whyte HF. General decentralized Kalman filters. In: Proceedings of the American Control Conference, Vol. 2; 29 June–1 July 1994; Baltimore, USA. 1994. pp. 2273-2274

[17] Zhu Y. Multisensor Decision and Estimation Fusion. New York: Kluwer Academic Publishers; 2003. 236 p

[18] Marelli D, Zamani M, Fu M, Ninness B. Distributed Kalman filter in a network of linear systems. Systems & Control Letters. 2018;**119**:71-77

[19] Chui CK, Chen G. Kalman Filtering with Real-Time Applications. Berlin: Springer-Verlag; 2009. 230 p

[20] Krokavec D. Decentralized optimal filtering of the large-scale system state. In: Proceedings of the 2nd IFAC Workshop New Trends in Design of Control Systems; Smolenice, Slovakia, 7–10 September 1997; Volumes 30(21). 1997. pp. 445-450

[21] Horn RA, Johnson CR. Matrix Analysis. New York: Cambridge University Press; 2013. 644 p

[22] Bryson AE, Ho YC. Applied Optimal Control. Optimization, Estimation, and Control. New York: Taylor & Francis; 1975. 482 p

[23] Krokavec D, Filasová A. Stabilization of discrete-time LTI positive systems. Archives of Control Sciences. 2017;**27**(4):575-594

[24] Xue D, Chen Y, Atherton DP. Linear Feedback Control. Analysis and Design with MATLAB. Philadelphia: Society for Industrial and Applied Mathematics; 2007. 370 p

[25] Canto B, Canto R, Kostova S. Stabilization of positive linear discrete-time systems by using a Brauer's theorem. The Scientific World Journal. 2014;**2014**:1-6