

# We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

4,500

Open access books available

118,000

International authors and editors

130M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities



WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index  
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?  
Contact [book.department@intechopen.com](mailto:book.department@intechopen.com)

Numbers displayed above are based on latest data collected.  
For more information visit [www.intechopen.com](http://www.intechopen.com)



---

# Basic Design Methods of Heat Exchanger

---

Cüneyt Ezgi

Additional information is available at the end of the chapter

<http://dx.doi.org/10.5772/67888>

---

## Abstract

Heat exchangers are devices that transfer energy between fluids at different temperatures by heat transfer. These devices can be used widely both in daily life and industrial applications such as steam generators in thermal power plants, distillers in chemical industry, evaporators and condensers in HVAC applications and refrigeration process, heat sinks, automobile radiators and regenerators in gas turbine engines. This chapter discusses the basic design methods for two fluid heat exchangers.

**Keywords:** log-mean temperature difference (LMTD), effectiveness-number of transfer units ( $\epsilon - NTU$ ), dimensionless mean temperature difference ( $\Psi - P$ ) and  $(P_1 - P_2)$  effectiveness-modified number of transfer units ( $\epsilon - NTU_o$ ), reduced length and reduced period ( $\Lambda - \pi$ )

---

## 1. Introduction

Heat exchangers (HE) are devices that transfer energy between fluids at different temperatures by heat transfer. Heat exchangers may be classified according to different criteria. The classification separates heat exchangers (HE) in recuperators and regenerators, according to construction is being used. In recuperators, heat is transferred directly (immediately) between the two fluids and by opposition, in the regenerators there is no immediate heat exchange between the fluids. Rather this is done through an intermediate step involving thermal energy storage. Recuperators can be classified according to transfer process in direct contact and indirect contact types. In indirect contact HE, there is a wall (physical separation) between the fluids. The recuperators are referred to as a direct transfer type. In contrast, the regenerators are devices in which there is intermittent heat exchange between the hot and cold fluids through thermal energy storage and release through the heat exchanger surface or matrix. Regenerators are basically classified into rotary and fixed matrix models. The regenerators are referred to as an indirect transfer type.

This chapter discusses the basic design methods for two fluid heat exchangers. We discuss the log-mean temperature difference (LMTD) method, the effectiveness  $\varepsilon$  – NTU method, dimensionless mean temperature difference ( $\Psi - P$ ) and  $(P_1 - P_2)$  to analyse recuperators. The LMTD method can be used if inlet temperatures, one of the fluid outlet temperatures, and mass flow rates are known. The  $\varepsilon$  – NTU method can be used when the outlet temperatures of the fluids are not known. Also, it is discussed effectiveness-modified number of transfer units ( $\varepsilon - NTU_o$ ) and reduced length and reduced period ( $\Lambda - \pi$ ) methods for regenerators.

## 2. Governing equations

The energy rate balance is

$$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + \sum_i \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e \right) \quad (1)$$

For a control volume at steady state,  $\frac{dE_{cv}}{dt} = 0$ . Changes in the kinetic and potential energies of the flowing streams from inlet to exit can be ignored. The only work of a control volume enclosing a heat exchanger is flow work, so  $\dot{W} = 0$  and single-stream (only one inlet and one exit) and from the steady-state form the heat transfer rate becomes simply [1–3]

$$\dot{Q} = \dot{m}(h_2 - h_1) \quad (2)$$

For single stream, we denote the inlet state by subscript 1 and the exit state by subscript 2.

For hot fluids,

$$\dot{Q} = \dot{m}(h_{h1} - h_{h2}) \quad (3)$$

For cold fluids,

$$\dot{Q} = \dot{m}(h_{c2} - h_{c1}) \quad (4)$$

The total heat transfer rate between the fluids can be determined from

$$\dot{Q} = UA\Delta T_{lm} \quad (5)$$

where  $U$  is the overall heat transfer coefficient, whose unit is  $W/m^2 \text{ } ^\circ\text{C}$  and  $\Delta T_{lm}$  is log-mean temperature difference.

## 3. Overall heat transfer coefficient

A heat exchanger involves two flowing fluids separated by a solid wall. Heat is transferred from the hot fluid to the wall by convection, through the wall by conduction and from the wall to the cold fluid by convection.

$$UA = U_o A_o = U_i A_i = \frac{1}{R_t} \quad (6)$$

where  $A_i = \pi D_i L$  and  $A_o = \pi D_o L$  and  $U$  is the overall heat transfer coefficient based on that area.  $R_t$  is the total thermal resistance and can be expressed as [1]

$$R_t = \frac{1}{UA} = \frac{1}{h_i A_i} + R_w + \frac{R_{fi}}{A_i} + \frac{R_{fo}}{A_o} + \frac{1}{h_o A_o} \quad (7)$$

where  $R_f$  is fouling resistance (factor) and  $R_w$  is wall resistance and is obtained from the following equations.

For a bare plane wall

$$R_w = \frac{t}{kA} \quad (8)$$

where  $t$  is the thickness of the wall

For a cylindrical wall

$$R_w = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi Lk} \quad (9)$$

The overall heat transfer coefficient based on the outside surface area of the wall for the unfinned tubular heat exchangers,

$$U_o = \frac{1}{\frac{r_o}{r_i} \frac{1}{h_i} + \frac{r_o}{r_i} R_{fi} + \frac{r_o}{k} \ln\left(\frac{r_o}{r_i}\right) + R_{fo} + \frac{1}{h_o}} \quad (10)$$

where  $R_{fi}$  and  $R_{fo}$  are fouling resistance of the inside and outside surfaces, respectively.

or

$$U_o = \frac{1}{\frac{r_o}{r_i} \frac{1}{h_i} + R_{ft} + \frac{r_o}{k} \ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o}} \quad (11)$$

where  $R_{ft}$  is the total fouling resistance, given as

$$R_{ft} = \frac{A_o}{A_i} R_{fi} + R_{fo} \quad (12)$$

For finned surfaces,

$$\dot{Q} = \eta h A \Delta T \quad (13)$$

where  $\eta$  is the overall surface efficiency and

$$\eta = 1 - \frac{A_f}{A} (1 - \eta_f) \quad (14)$$

where  $A_f$  is fin surface area and  $\eta_f$  is fin efficiency and is defined as

$$\eta_f = \frac{\dot{Q}_f}{\dot{Q}_{f,\max}} \quad (15)$$

Constant cross-section of very long fins and fins with insulated tips, the fin efficiency can be expressed as

$$\eta_{f,\text{long}} = \frac{1}{mL} \quad (16)$$

$$\eta_{f,\text{insulated}} = \frac{\tanh(mL)}{mL} \quad (17)$$

where  $L$  is the fin length.

For straight triangular fins,

$$\eta_{f,\text{triangular}} = \frac{1}{mL} \frac{I_1(2mL)}{I_0(2mL)} \quad (18)$$

For straight parabolic fins,

$$\eta_{f,\text{parabolic}} = \frac{2}{1 + \sqrt{(2mL)^2 + 1}} \quad (19)$$

For circular fins of rectangular profile,

$$\eta_{f,\text{rectangular}} = C \frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) - K_0(mr_1)I_1(mr_{2c})} \quad (20)$$

where the mathematical functions  $I$  and  $K$  are the modified Bessel functions and

$$m = \sqrt{2h/kt} \quad (21)$$

where  $t$  is the fin thickness.

and

$$C = \frac{2r_1/m}{r_{2c}^2 - r_1^2} \quad (22)$$

where

$$r_{2c} = r_2 + t/2 \quad (23)$$

For pin fins of rectangular profile,

$$\eta_{f, \text{pin, rectangular}} = \frac{\tanh mL_c}{mL_c} \quad (24)$$

where

$$m = \sqrt{4h/kD} \quad (25)$$

and corrected fin length,  $L_c$ , defined as

$$L_c = L + D/4 \quad (26)$$

where  $L$  is the fin length and  $D$  is the diameter of the cylindrical fins. The corrected fin length is an approximate, yet practical and accurate way of accounting for the loss from the fin tip is to replace the fin length  $L$  in the relation for the insulated tip case.

$A$  is the total surface area on one side

$$A = A_u + A_f \quad (27)$$

The overall heat transfer coefficient is based on the outside surface area of the wall for the finned tubular heat exchangers,

$$U_o = \frac{1}{\frac{A_o}{A_i} \frac{1}{\eta_i h_i} + \frac{A_o R_{fi}}{A_i \eta_i} + A_o R_w + \frac{R_{fo}}{\eta_o} + \frac{1}{\eta_o h_o}} \quad (28)$$

where  $A_o$  and  $A_i$  represent the total surface area of the outer and inner surfaces, respectively.

## 4. Thermal design for recuperators

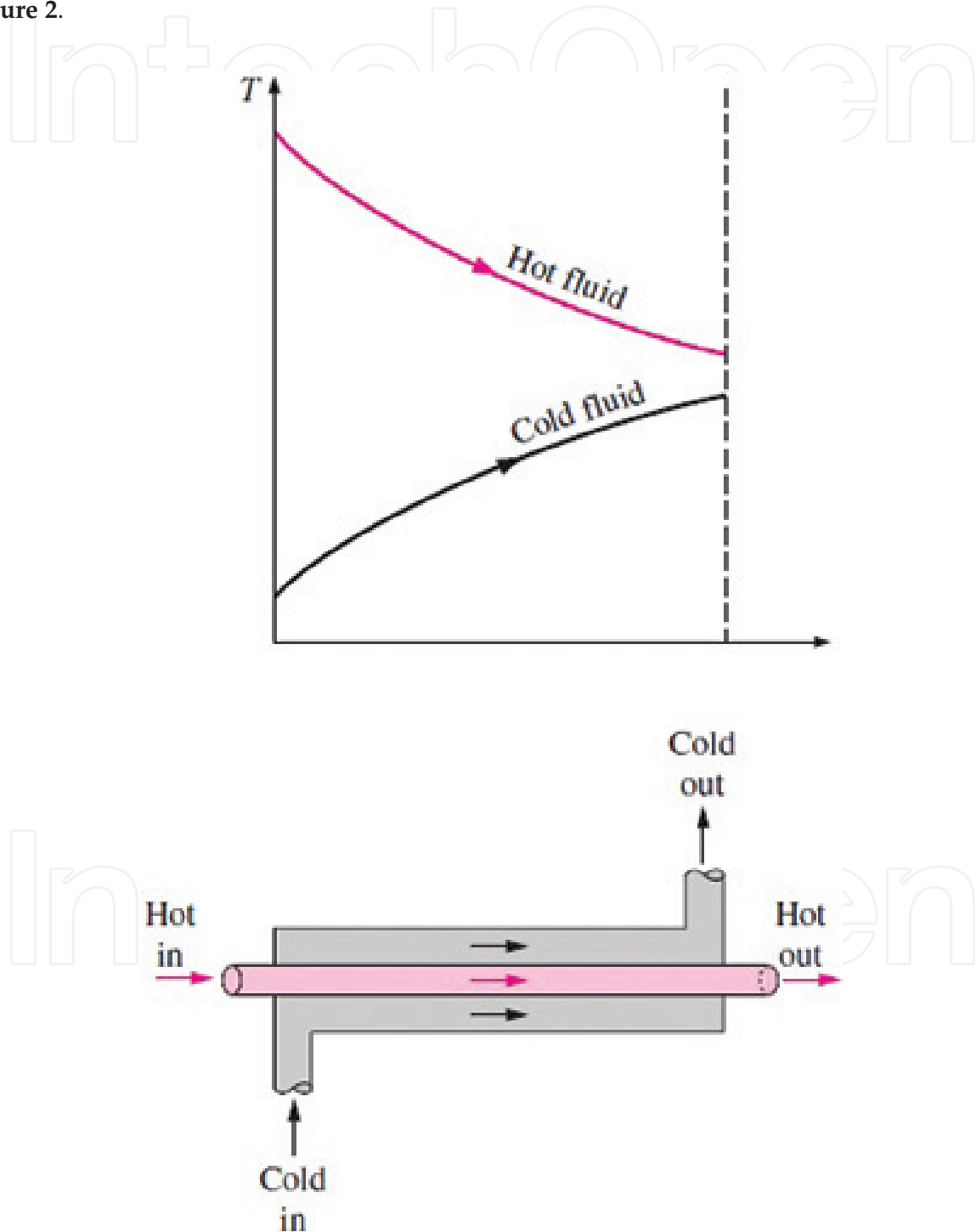
Four methods are used for the recuperator thermal performance analysis: log-mean temperature difference (LMTD), effectiveness-number of transfer units ( $\epsilon - \text{NTU}$ ), dimensionless mean temperature difference ( $\Psi - P$ ) and ( $P_1 - P_2$ ) methods.

### 4.1. The log-mean temperature difference (LMTD) method

The use of the method is clearly facilitated by knowledge of the hot and cold fluid inlet and outlet temperatures. Such applications may be classified as heat exchanger design problems; that is, problems in which the temperatures and capacity rates are known, and it is desired to size the exchanger.

#### 4.1.1. Parallel and counter flow heat exchanger

Two types of flow arrangement are possible in a double-pipe heat exchanger: parallel flow and counter flow. In parallel flow, both the hot and cold fluids enter the heat exchanger at the same end and move in the same direction, as shown in **Figure 1**. In counter flow, the hot and cold fluids enter the heat exchanger at opposite end and flow in opposite direction, as shown in **Figure 2**.



**Figure 1.** Parallel flow in a double-pipe heat exchanger.

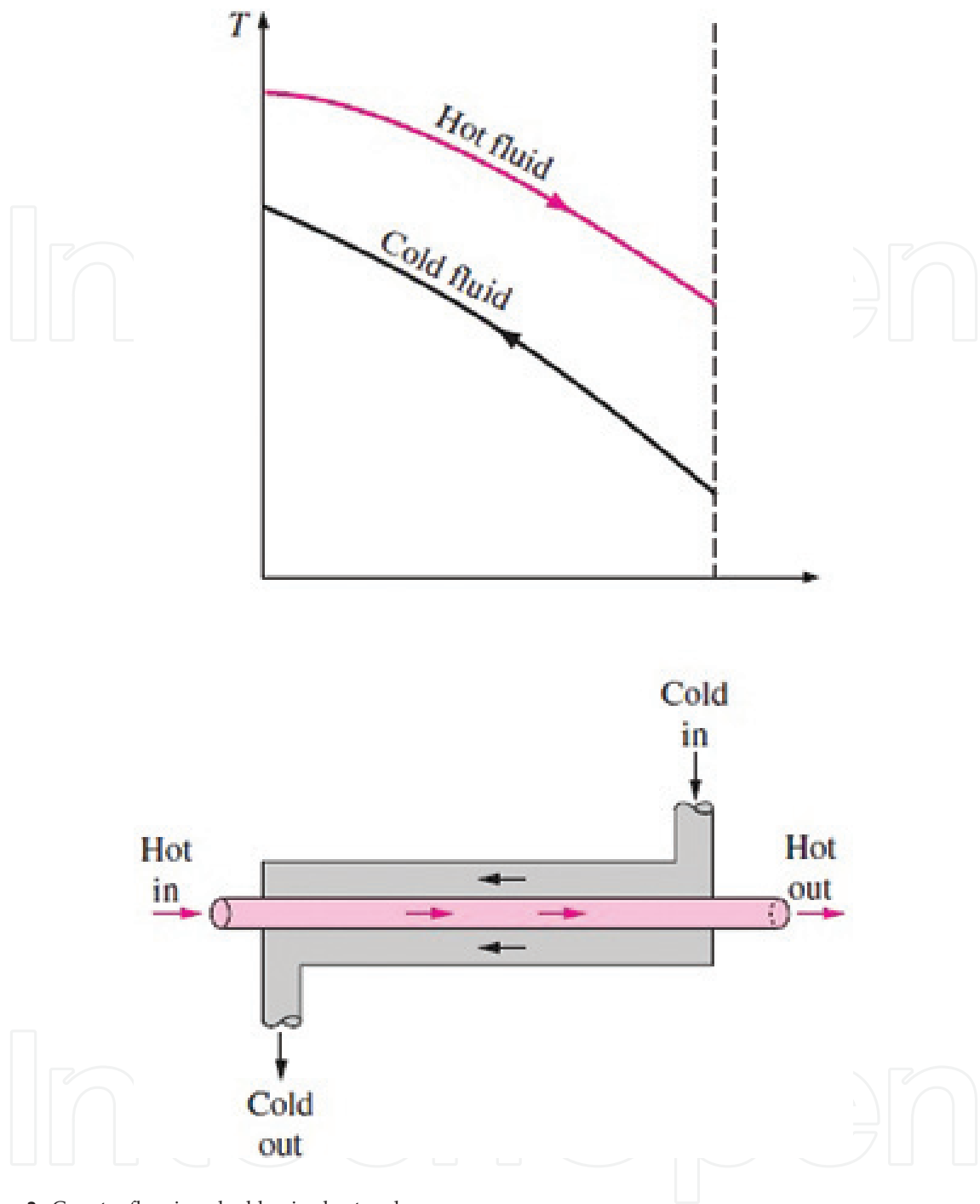


Figure 2. Counter flow in a double-pipe heat exchanger.

The heat transfer rate is

$$\dot{Q} = UA\Delta T_{lm} \tag{29}$$

where  $\Delta T_{lm}$  is log-mean temperature difference and is

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} \tag{30}$$



Then,

$$\dot{Q} = UA \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} \quad (31)$$

where the endpoint temperatures,  $\Delta T_1$  and  $\Delta T_2$ , for the parallel flow exchanger are

$$\Delta T_1 = T_{hi} - T_{ci} \quad (32)$$

$$\Delta T_2 = T_{ho} - T_{co} \quad (33)$$

where  $T_{hi}$  is the hot fluid inlet temperature,  $T_{ci}$  is the cold fluid inlet temperature,  $T_{ho}$  is the hot fluid outlet temperature and  $T_{co}$  is the cold fluid outlet temperature.

The endpoint temperatures,  $\Delta T_1$  and  $\Delta T_2$ , for the counter flow exchanger are

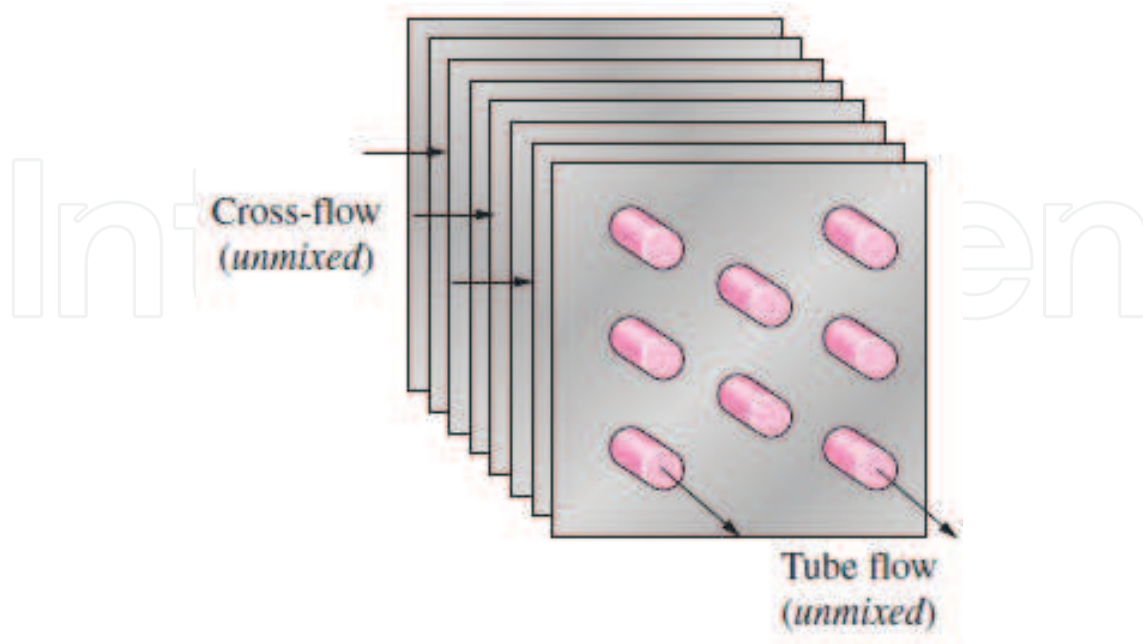
$$\Delta T_1 = T_{hi} - T_{co} \quad (34)$$

$$\Delta T_2 = T_{ho} - T_{ci} \quad (35)$$

#### 4.1.2. Multipass and cross-flow heat exchanger

In compact heat exchangers, the two fluids usually move perpendicular to each other, and such flow configuration is called cross-flow. The cross-flow is further classified as unmixed and mixed flow, depending on the flow configuration, as shown in **Figures 3** and **4**.

Multipass flow arrangements are frequently used in shell-and-tube heat exchangers with baffles (**Figure 5**).



**Figure 3.** Both fluids unmixed.

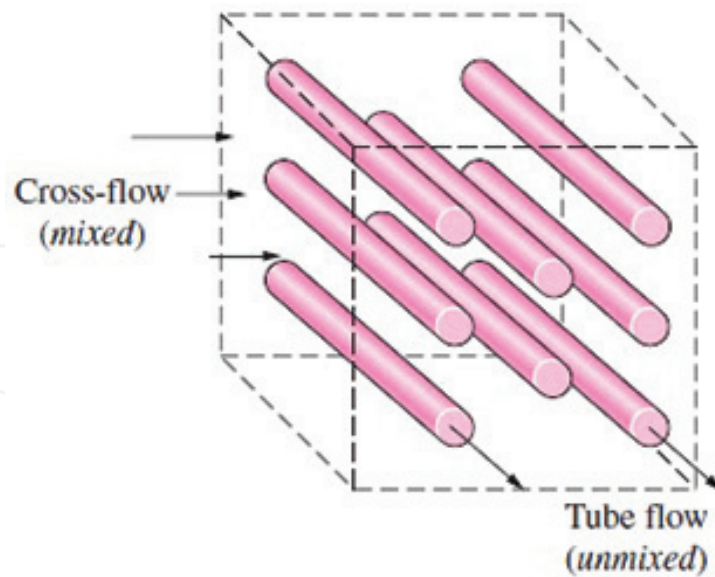


Figure 4. One fluid mixed and one fluid unmixed.

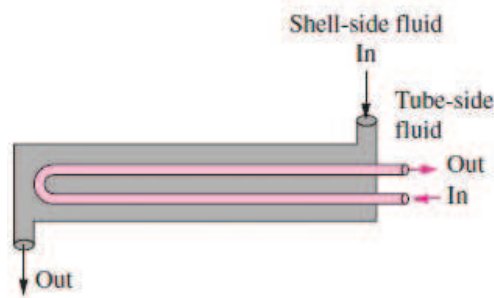


Figure 5. One shell pass and two tube passes.

Log-mean temperature difference  $\Delta T_{lm}$  is computed under assumption of counter flow conditions. Heat transfer rate is

$$\dot{Q} = UAF\Delta T_{lm,cf} \quad (36)$$

where  $F$  is a correction factor and non-dimensional and depends on temperature effectiveness  $P$ , the heat capacity rate ratio  $R$  and the flow arrangement.

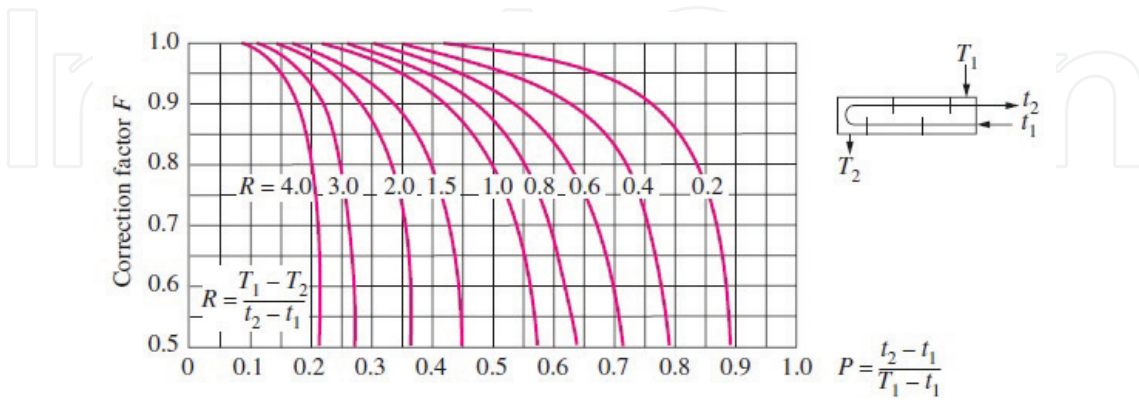
$$P = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} \quad (37)$$

$$R = \frac{T_{h1} - T_{h2}}{T_{c2} - T_{c1}} \quad (38)$$

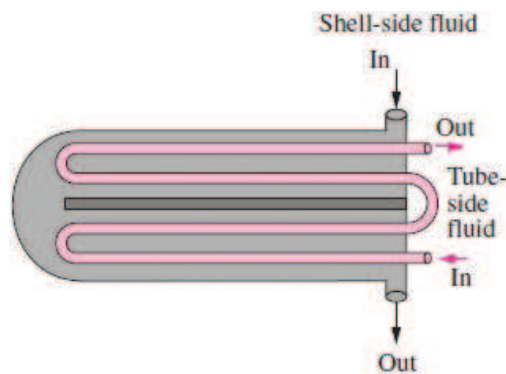
The value of  $P$  ranges from 0 to 1. The value of  $R$  ranges from 0 to infinity. If the temperature change of one fluid is negligible, either  $P$  or  $R$  is zero and  $F$  is 1. Hence, the exchanger

behaviour is independent of the specific configuration. Such would be the case if one of the fluids underwent a phase change.

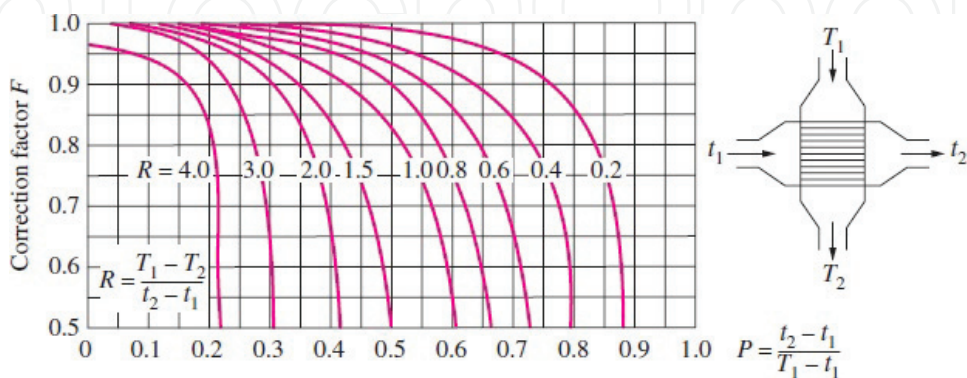
Correction factor  $F$  charts for common shell-and-tube and cross-flow heat exchangers are shown in **Figures 6–10**.



**Figure 6.** One shell pass and any multiple of two tube passes.



**Figure 7.** Two shell passes and four-tube passes.



**Figure 8.** Single pass cross flow with one fluid mixed and the other unmixed.

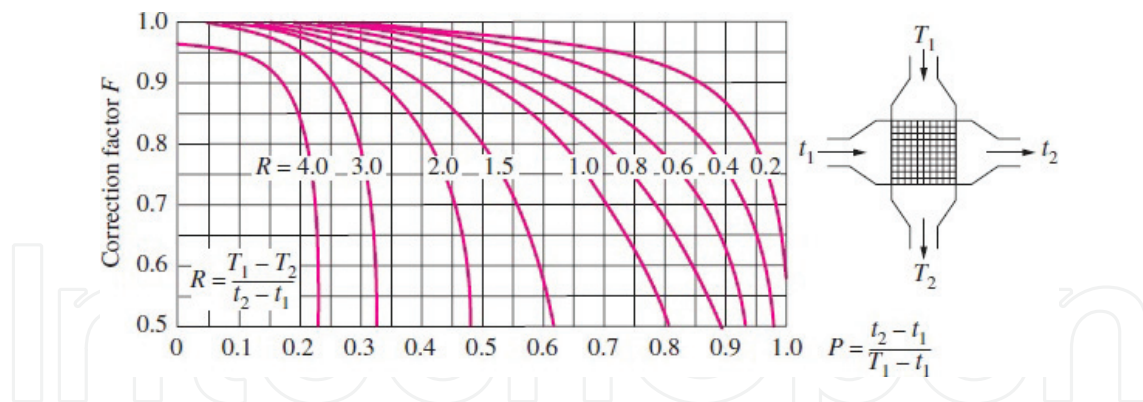


Figure 9. Single pass cross flow with both fluids unmixed.

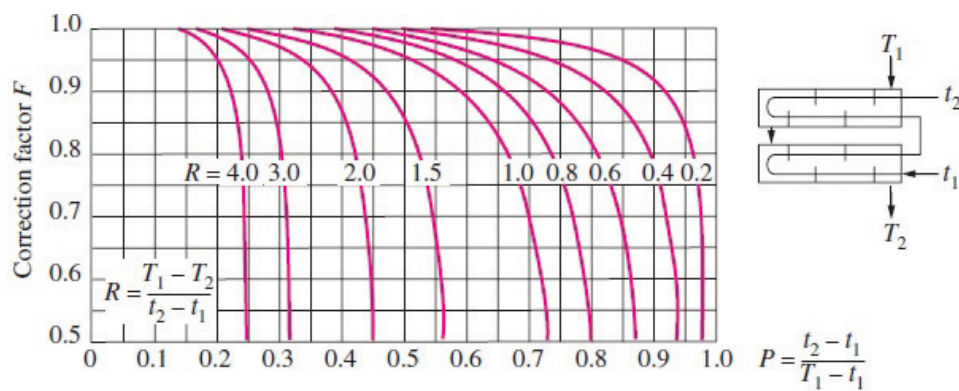


Figure 10. Two shell passes and any multiple of four tube passes.

#### 4.1.3. The procedure to be followed with the LMTD method

1. Select the type of heat exchanger.
2. Calculate any unknown inlet or outlet temperatures and the heat transfer rate.
3. Calculate the log-mean temperature difference and the correction factor, if necessary.
4. Calculate the overall heat transfer coefficient.
5. Calculate the heat transfer surface area.
6. Calculate the length of the tube or heat exchanger

#### 4.2. The $\epsilon$ – NTU method

If the exchanger type and size are known and the fluid outlet temperatures need to be determined, the application is referred to as a performance calculation problem. Such problems are best analysed by the NTU-effectiveness method [4, 5].

Capacity rate ratio is

$$C^* = \frac{C_{\min}}{C_{\max}} \quad (39)$$

where  $C_{\min}$  and  $C_{\max}$  are the smaller and larger of the two magnitudes of  $C_h$  and  $C_c$ , respectively, and  $C_h$  and  $C_c$  are the hot and cold fluid heat capacity rates, respectively.

Heat exchanger effectiveness  $\varepsilon$  is defined as

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}} \quad (40)$$

where

$$\dot{Q}_{\max} = (\dot{m}c_p)_c(T_{h1} - T_{c1}) \text{ if } C_c < C_h \quad (41)$$

or

$$\dot{Q}_{\max} = (\dot{m}c_p)_h(T_{h1} - T_{c1}) \text{ if } C_h < C_c \quad (42)$$

where  $C_c = \dot{m}_c c_{pc}$  and  $C_h = \dot{m}_h c_{ph}$  are the heat capacity rates of the cold and the hot fluids, respectively, and  $\dot{m}$  is the rate of mass flow and  $c_p$  is specific heat at constant pressure.

Heat exchanger effectiveness is therefore written as

$$\varepsilon = \frac{C_h(T_{h1} - T_{h2})}{C_{\min}(T_{h1} - T_{c1})} = \frac{C_c(T_{c2} - T_{c1})}{C_{\min}(T_{h1} - T_{c1})} \quad (43)$$

The number of transfer unit (NTU) is defined as a ratio of the overall thermal conductance to the smaller heat capacity rate. NTU designates the non-dimensional heat transfer size or thermal size of the exchanger [4, 5].

$$\text{NTU} = \frac{UA}{C_{\min}} = \frac{1}{C_{\min}} \int U dA \quad (44)$$

In evaporator and condenser for parallel flow and counter flow,

$$C^* = \frac{C_{\min}}{C_{\max}} = 0 \quad (45)$$

and

$$\varepsilon = 1 - e^{-\text{NTU}} \quad (46)$$

The effectivenesses of some common types of heat exchangers are also plotted in **Figures 11–16**.

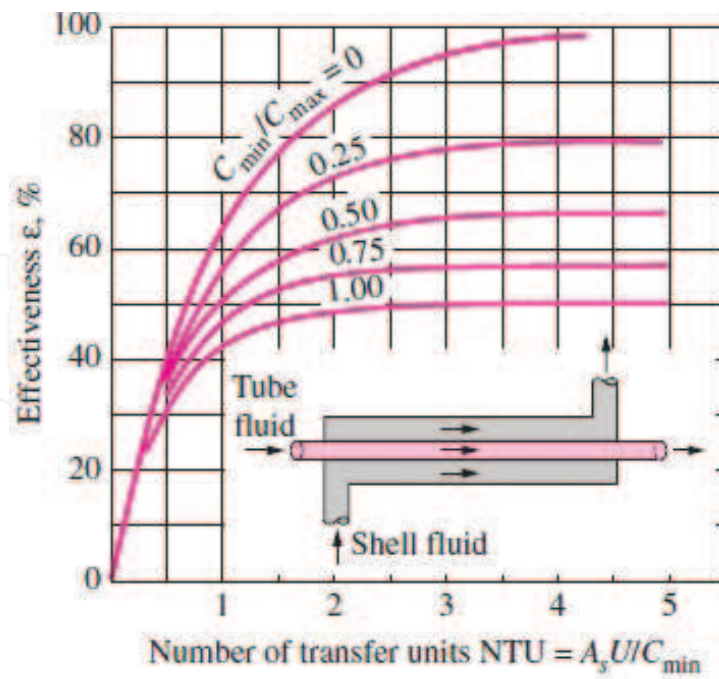


Figure 11. Effectiveness of parallel flow.

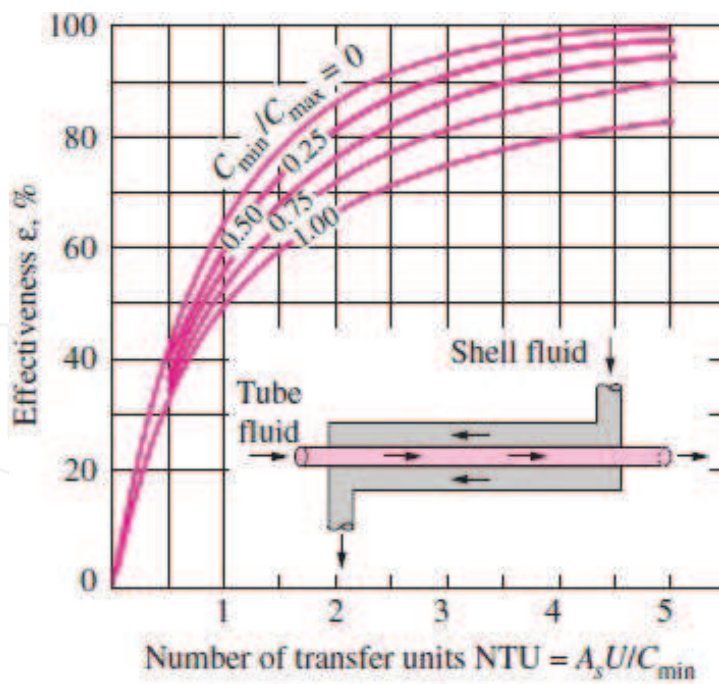


Figure 12. Effectiveness of counter flow.

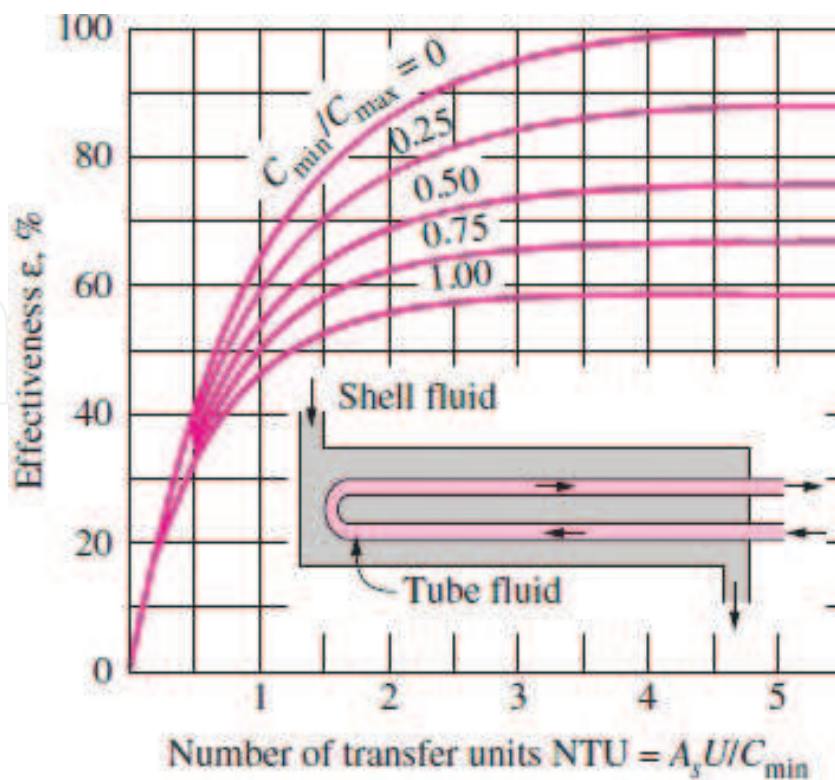


Figure 13. Effectiveness of one shell pass and 2, 4, 6, ... tube passes.

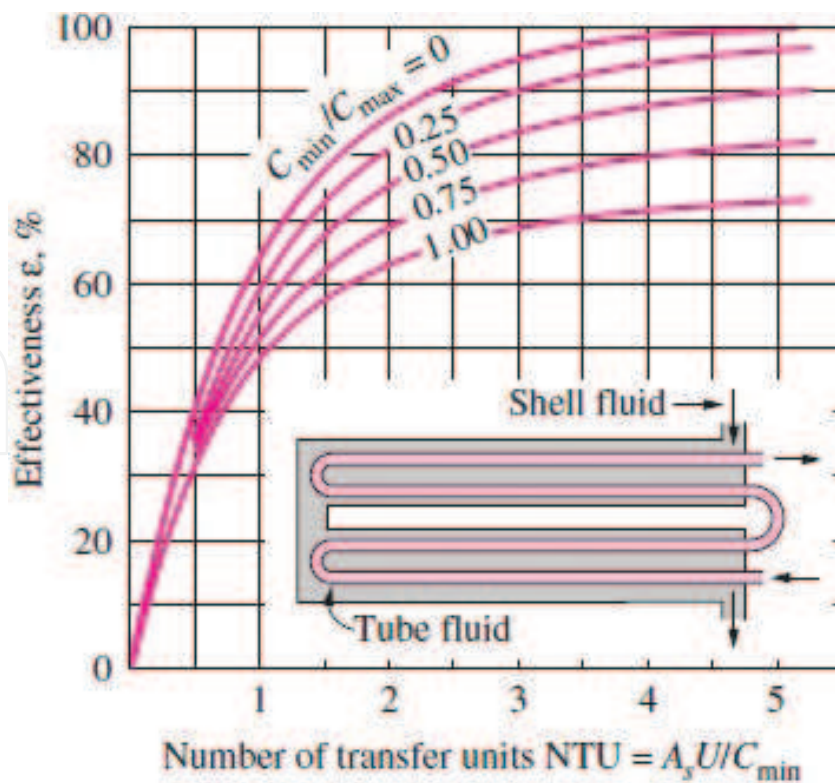


Figure 14. Effectiveness of two shell passes and 4, 8, 12, ... tube passes.

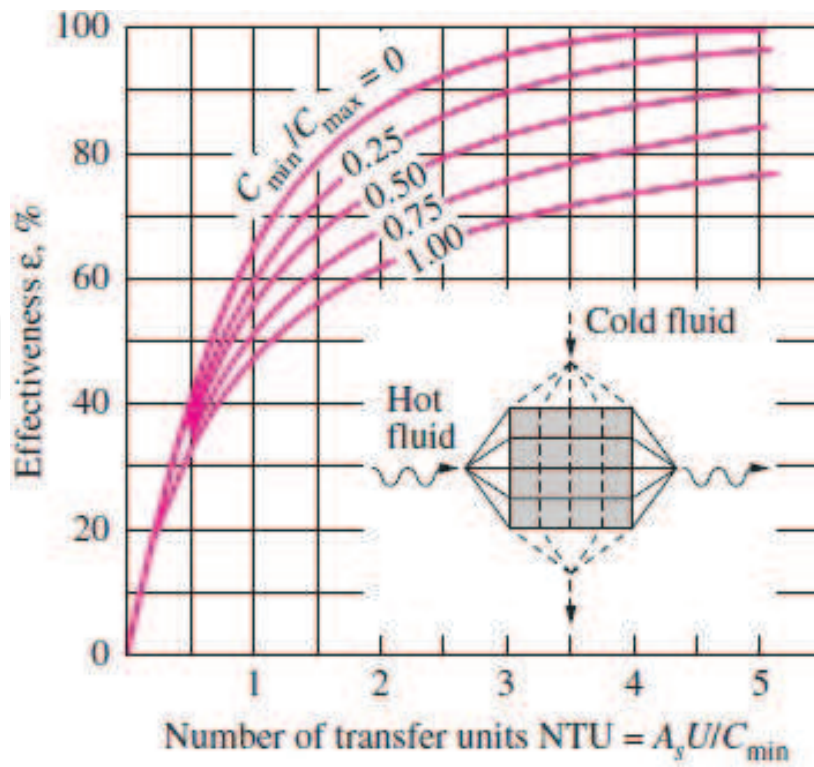


Figure 15. Effectiveness of cross flow with both fluids unmixed.

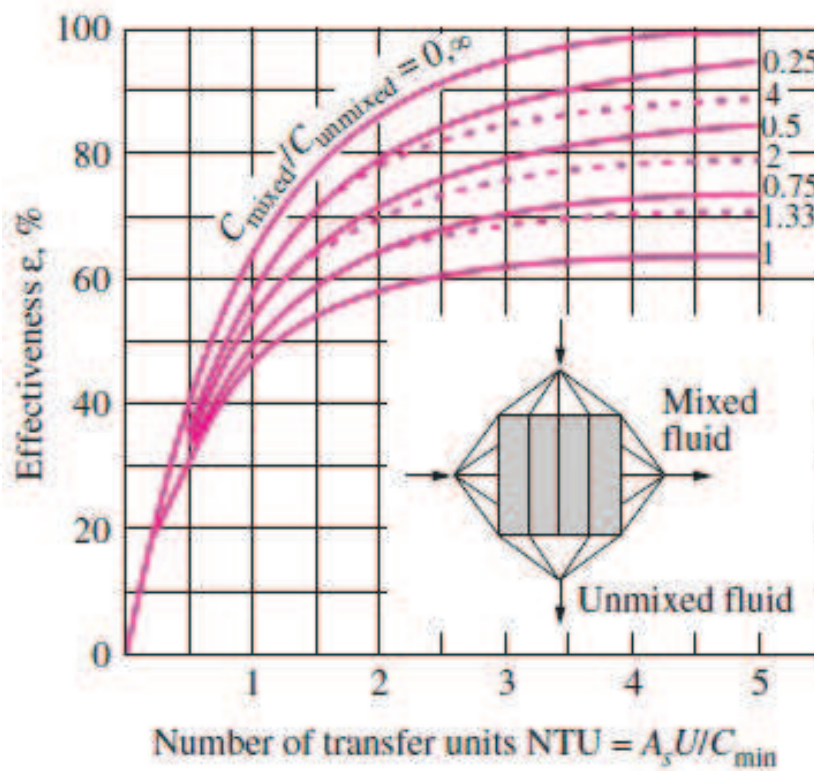


Figure 16. Effectiveness of cross flow with one fluid mixed and the other unmixed.



#### 4.2.1. The procedure to be followed with the $\varepsilon - NTU$ method

a. For the rating analysis:

1. Calculate the capacity rate ratio
2. Calculate NTU.
3. Determine the effectiveness.
4. Calculate the total heat transfer rate.
5. Calculate the outlet temperatures.

b. For the sizing problem:

1. Calculate the effectiveness.
2. Calculate the capacity rate ratio.
3. Calculate the overall heat transfer coefficient.
4. Determine NTU.
5. Calculate the heat transfer surface area.
6. Calculate the length of the tube or heat exchanger

#### 4.3. The $\Psi - P$ method

The dimensionless mean temperature difference is [4]

$$\psi = \frac{\Delta T_m}{T_{hi} - T_{ci}} = \frac{\Delta T_m}{\Delta T_{\max}} \quad (47)$$

$$\psi = \frac{\varepsilon}{NTU} = \frac{P_1}{NTU_1} = \frac{P_2}{NTU_2} \quad (48)$$

where  $P$  is the temperature effectiveness and the temperature effectivenesses of fluids 1 and 2 are defined as, respectively

$$P_1 = \frac{T_{1,o} - T_{1,i}}{T_{2,i} - T_{1,i}} \quad (49)$$

$$P_2 = \frac{T_{2,i} - T_{2,o}}{T_{2,i} - T_{1,i}} \quad (50)$$

$$\psi = \begin{cases} \frac{FP_1(1 - R_1)}{\ln \left[ \frac{(1 - R_1 P_1)}{(1 - P_1)} \right]} & \text{for } R_1 \neq 1 \\ F(1 - P_1) & \text{for } R_1 = 1 \end{cases} \quad (51)$$

where 1 and 2 are fluid stream 1 and fluid stream 2, respectively, and  $R$  is the heat capacity ratio and defined as

$$R_1 = \frac{C_1}{C_2} = \frac{T_{2,i} - T_{2,o}}{T_{1,o} - T_{1,i}} \quad (52)$$

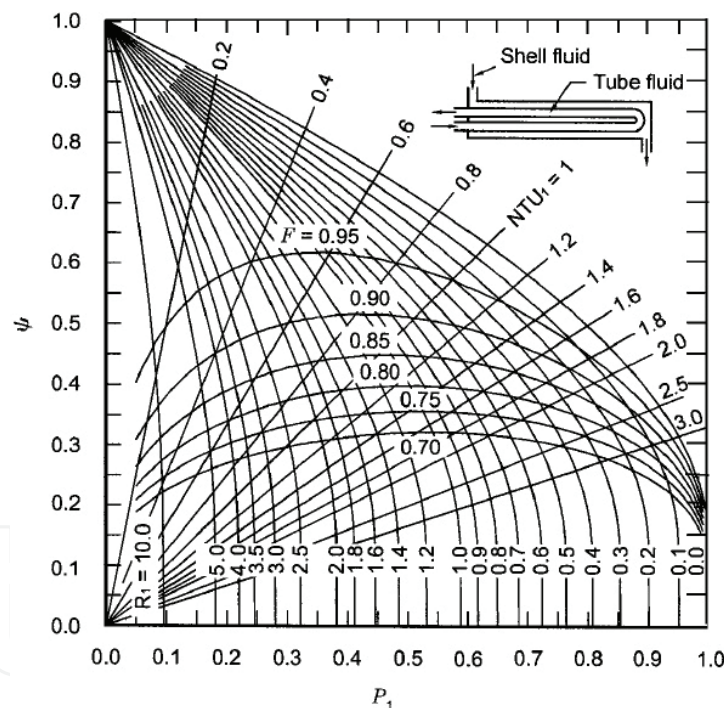
$$R_2 = \frac{C_2}{C_1} = \frac{T_{1,o} - T_{1,i}}{T_{2,i} - T_{2,o}} \quad (53)$$

$$R_1 = \frac{1}{R_2} \quad (54)$$

Non-dimensional mean temperature difference as a function for  $P_1$  and  $R_1$  with the lines for constant values of  $NTU_1$  and the factor is shown in **Figure 17**.

The heat transfer rate is given by

$$q = UA\Psi(T_{hi} - T_{ci}) \quad (55)$$



**Figure 17.** Non-dimensional mean temperature difference as a function for  $P_1$  and  $R_1$ .

#### 4.3.1. The procedure to be followed with the $\Psi - P$ method

1. Calculate  $NTU_1$ .
2. Calculate  $F$  factor.
3. Calculate  $R_1$  with the lines for constant values of  $NTU_1$  and the  $F$  factor superimposed in **Figure 17**.

4. Plot the dimensionless mean temperature  $\Psi$  as a function of  $P_1$  and  $R_1$  in **Figure 17**.
5. Calculate the heat transfer rate.

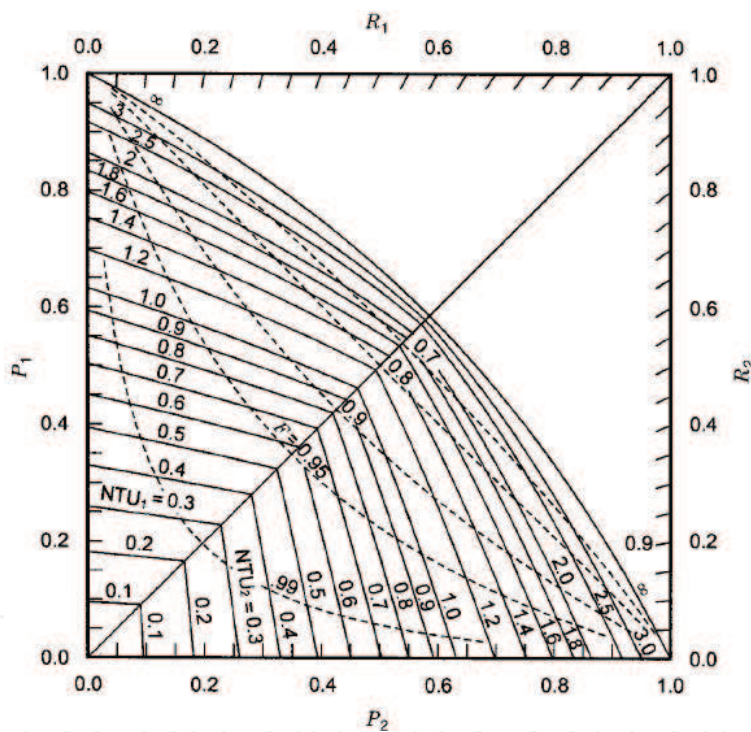
#### 4.4. The $P_1 - P_2$ method

The dimensionless mean temperature difference is [4]

$$\psi = \frac{\varepsilon}{NTU} = \frac{P_1}{NTU_1} = \frac{P_2}{NTU_2} \quad (56)$$

$P_1 - P_2$  chart for 1–2 shell and tube heat exchanger [2] with shell fluid mixed is shown in **Figure 18**.

where 1 and 2 are one shell pass and two tube passes, respectively.



**Figure 18.**  $P_1 - P_2$  chart for 1–2 shell and tube heat exchanger with shell fluid mixed.

##### 4.4.1. The procedure to be followed with the $P_1 - P_2$ method

1. Calculate  $NTU_1$  or  $NTU_2$ .
2. Calculate  $R_1$  or  $R_2$ .
3. Plot  $P_1$  as a function of  $R_1$  with  $NTU_1$  or  $P_2$  as a function of  $R_2$  with  $NTU_2$  in **Figure 18**.
4. Calculate the dimensionless mean temperature  $\Psi$ .
5. Calculate the heat transfer rate.

## 5. Thermal design for regenerators

Two methods are used for the regenerator thermal performance analysis:  $\varepsilon - NTU_o$  and  $\Lambda - \pi$  methods, respectively, for rotary and fixed matrix regenerators.

### 5.1. The $\varepsilon - NTU_o$ method

The  $\varepsilon - NTU_o$  method was developed by Coppage and London in 1953. The modified number of transfer units is [4]

$$NTU_o = \frac{1}{C_{\min}} \left[ \frac{1}{\frac{1}{(hA)_h} + \frac{1}{(hA)_c}} \right] \quad (57)$$

$$C^* = \frac{C_{\min}}{C_{\max}} \quad (58)$$

$$C_r^* = \frac{C_r}{C_{\min}} \quad (59)$$

$$C_r = M_w c_w N \quad (60)$$

where  $c_w$  is the specific heat of wall material,  $N$  is the rotational speed for a rotary regenerator and  $M_w$  is matrix mass and determined as

$$M_w = A_{rc} H_r \rho_m S_m \quad (61)$$

where  $A_{rc}$  is the rotor cross-sectional area,  $H_r$  is the rotor height,  $\rho_m$  is the matrix material density and  $S_m$  is the matrix solidity.

The convection conductance ratio is

$$(hA)^* = \frac{(hA) C_{\min}}{(hA) C_{\max}} \quad (62)$$

Most regenerators operate in the range of  $0.25 \leq (hA)^* < 4$ . The effect of  $(hA)^*$  on the regenerator effectiveness can usually be ignored.

$A$  is the total matrix surface area and given as

$$A = A_{rc} H_r \beta F_{rfa} \quad (63)$$

where  $A_{rc}$  is the rotor cross-sectional area,  $H_r$  is the rotor height,  $\beta$  is the matrix packing density and  $F_{rfa}$  is the fraction of rotor face area not covered by radial seals.

The hot and cold gas side surface areas are proportional to the respective sector angles.

$$A_h = \left( \frac{\alpha_h}{360^\circ} \right) A \quad (64)$$

$$A_c = \left( \frac{\alpha_c}{360^\circ} \right) A \tag{65}$$

where  $\alpha_h$  and  $\alpha_c$  are disk sector angles of hot flow and cold flow in degree, respectively.

The regenerator effectiveness is

$$\varepsilon = \frac{q}{q_{\max}} \tag{66}$$

$$q_{\max} = C_{\min}(T_{hi} - T_{ci}) \tag{67}$$

5.1.1. The counter flow regenerator

The regenerator effectiveness for  $\varepsilon \leq 0.9$  is

$$\varepsilon = \varepsilon_{cf} \left( 1 - \frac{1}{9C_r^{*1.93}} \right) \tag{68}$$

where  $\varepsilon_{cf}$  is the counter flow recuperator effectiveness and is determined as

$$\varepsilon_{cf} = \frac{1 - \exp[-NTU_o(1 - C^*)]}{1 - C^* \exp[-NTU_o(1 - C^*)]} \tag{69}$$

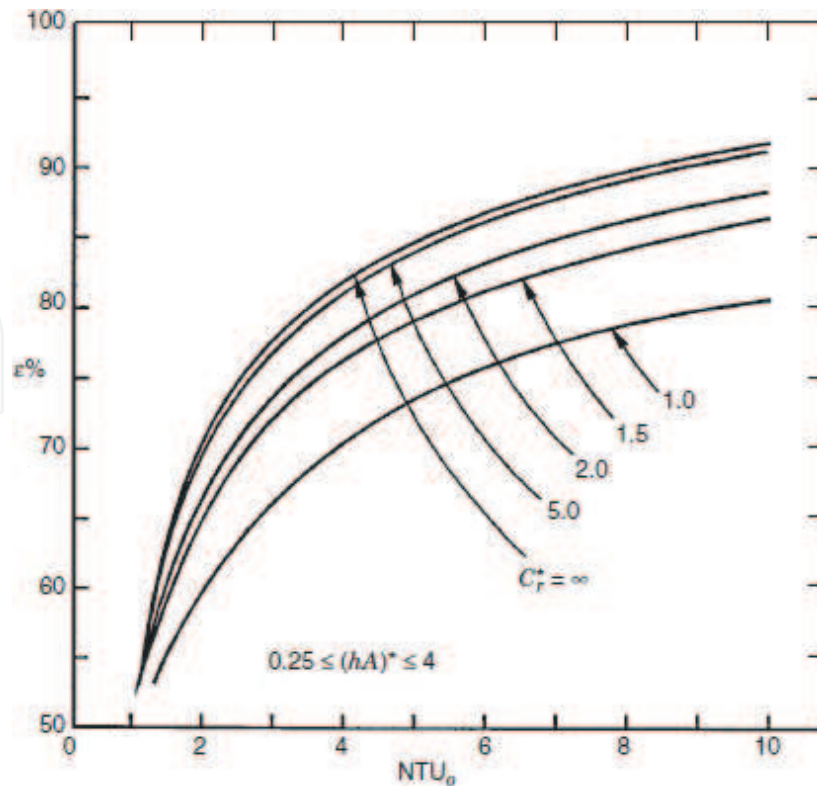
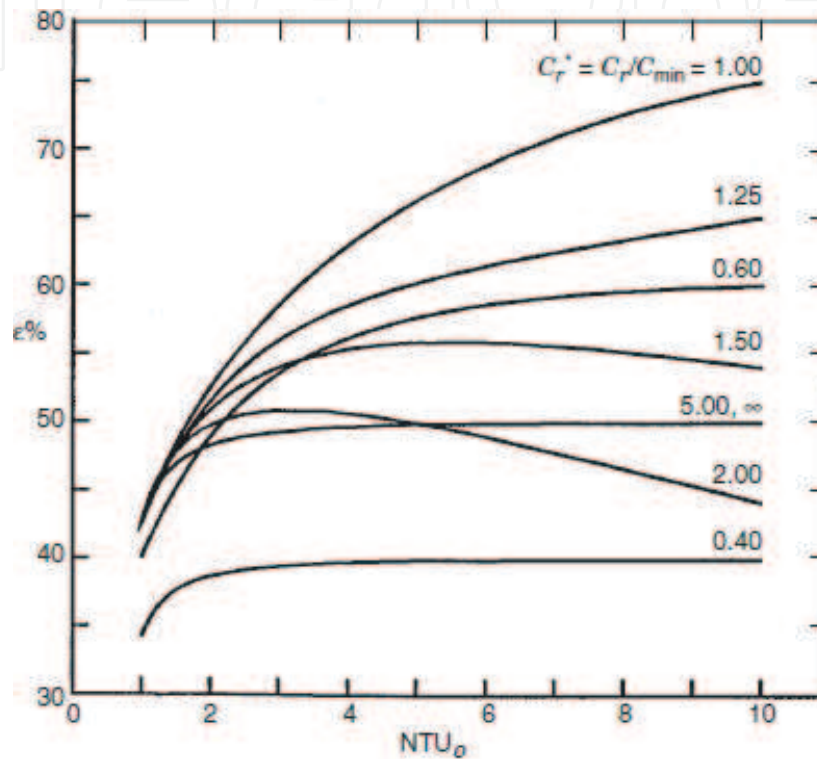


Figure 19. The counter flow regenerator effectiveness as a function of  $NTU_o$  and for  $C^* = 1$ .

The counter flow regenerator effectiveness as a function of  $NTU_o$  and for  $C^* = 1$  is presented in **Figure 19**. The regenerator effectiveness increases with  $C_r^*$  for given values of  $NTU_o$  and  $C^*$ . The range of the optimum value of  $C_r^*$  is between 2 and 4 for optimum regenerator effectiveness.

### 5.1.2. The parallel flow regenerator

The parallel flow regenerator effectiveness as a function of  $NTU_o$  and for  $C^* = 1$  and  $(hA)^* = 1$  is presented in **Figure 20**.



**Figure 20.** The parallel flow regenerator effectiveness as a function of  $NTU_o$  and for  $C^* = 1$  and  $(hA)^* = 1$ .

### 5.1.3. The procedure to be followed with the $\epsilon - NTU_o$ method

1. Calculate the capacity rate ratio.
2. Calculate  $(hA)^*$ .
3. Calculate  $(C_r)^*$ .
4. Calculate  $NTU_o$ .
5. Determine the effectiveness.
6. Calculate the total heat transfer rate.
7. Calculate the outlet temperatures.

## 5.2. The $\Lambda - \pi$ method

This method is generally used for fixed matrix regenerators. The reduced length designates the dimensionless heat transfer or thermal size of the regenerator. The reduced length is [4]

$$\Lambda = bL \quad (70)$$

The reduced lengths for hot and cold sides, respectively, are

$$\Lambda_h = \left( \frac{hA}{C} \right)_h = ntu_h \quad (71)$$

$$\Lambda_c = \left( \frac{hA}{C} \right)_c = ntu_c \quad (72)$$

The reduced period is

$$\pi = cP_h \text{ or } cP_c \quad (73)$$

where  $b$  and  $c$  are constants.

The reduced periods for hot and cold sides, respectively, are

$$\pi_h = \left( \frac{hA}{C_r} \right)_h \quad (74)$$

$$\pi_c = \left( \frac{hA}{C_r} \right)_c \quad (75)$$

Designations of various types of regenerators are given in **Table 1**. For a symmetric and balanced regenerator, the reduced length and the reduced period are equal on the hot and cold sides:

$$\Lambda_h = \Lambda_c = \Lambda = \Lambda_m = \frac{hA}{\dot{m}c_p} = ntu \quad (76)$$

$$\pi_h = \pi_c = \pi = \pi_m = \frac{hAP}{M_w c_w} \quad (77)$$

The actual heat transfer during one hot or cold gas flow period is

$$Q = C_h P_h (T_{hi} - T_{ho}) = C_c P_c (T_{co} - T_{ci}) \quad (78)$$

The maximum possible heat transfer is

$$Q_{\max} = (CP)_{\min} (T_{hi} - T_{ci}) \quad (79)$$

The effectiveness for a fixed-matrix regenerator is

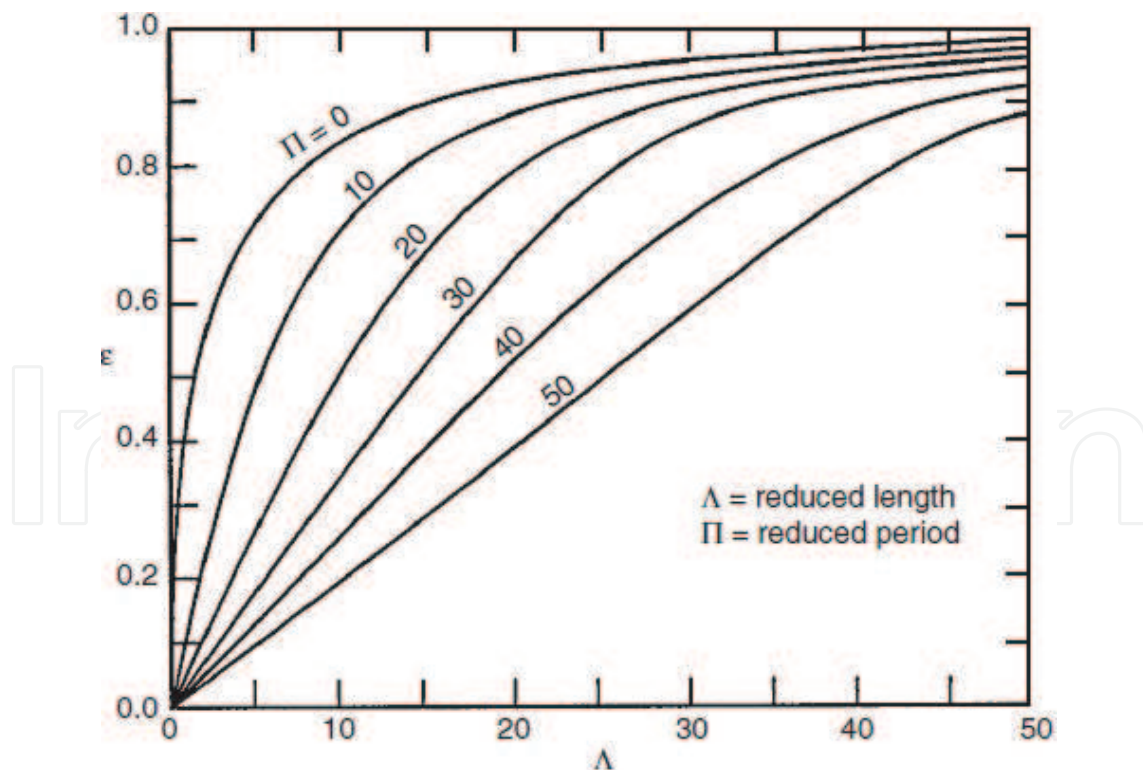
$$\varepsilon = \frac{Q}{Q_{\max}} = \frac{(CP)_h (T_{hi} - T_{ho})}{(CP)_{\min} (T_{hi} - T_{ci})} = \frac{(CP)_c (T_{co} - T_{ci})}{(CP)_{\min} (T_{hi} - T_{ci})} \quad (80)$$

The effectiveness chart for a balanced and symmetric counter flow regenerator is given in **Figure 21**.

The effectiveness chart for a balanced and symmetric parallel flow regenerator is given in **Figure 22**.

Regenerator	
Balanced	$\frac{\Lambda_h}{\Pi_h} = \frac{\Lambda_c}{\Pi_c}$ or $\gamma = 1$
Unbalanced	$\frac{\Lambda_h}{\Pi_h} \neq \frac{\Lambda_c}{\Pi_c}$
Symmetric	$\pi_h = \pi_c$
Unsymmetric	$\pi_h \neq \pi_c$
Symmetric and balanced	$\Lambda_h = \Lambda_c, \pi_h = \pi_c$
Unsymmetric and balanced	$\frac{\Lambda_h}{\Pi_h} = \frac{\Lambda_c}{\Pi_c}$
Long	$\Lambda/\Pi > 5$

**Table 1.** Designation of various types of regenerators for  $\Lambda - \Pi$  method.



**Figure 21.** The effectiveness chart for a balanced and symmetric counter flow regenerator.



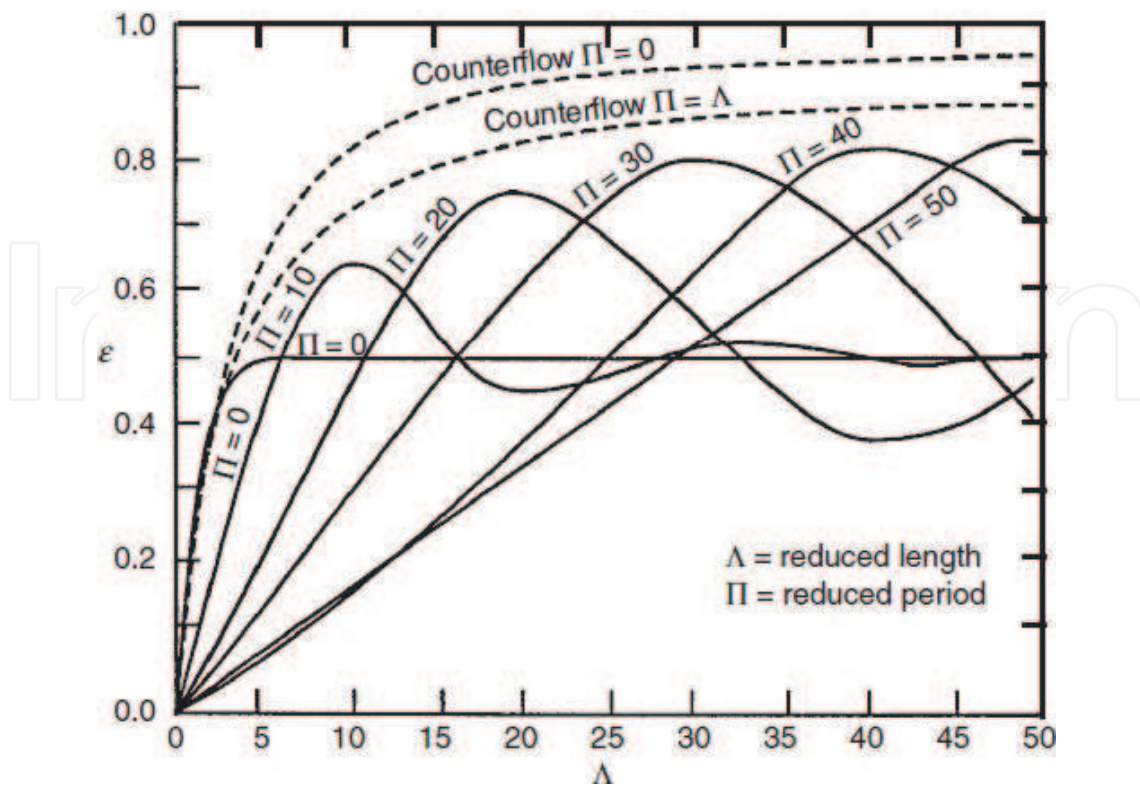


Figure 22. The effectiveness chart for a balanced and symmetric parallel flow regenerator.

### 5.2.1. The procedure to be followed with the $\Lambda - \pi$ method

1. Calculate the reduced length.
2. Calculate the reduced period.
3. Calculate  $C^*$ .
4. Calculate  $(C_r)^*$ .
5. Calculate  $NTU_o$ .
6. Determine the effectiveness.
7. Calculate the total heat transfer rate.
8. Calculate the outlet temperatures.

## 6. Conclusion

This chapter has discussed the basic design methods for two fluid heat exchangers. The design techniques of recuperators and regenerators, which are two main classes, were investigated.

The solution to recuperator problem is presented in terms of log-mean temperature difference (LMTD), effectiveness-number of transfer units ( $\epsilon - NTU$ ), dimensionless mean temperature

difference ( $\Psi - P$ ) and ( $P_1 - P_2$ ) methods. The exchanger rating or sizing problem can be solved by any of these methods and will yield the identical solution within the numerical error of computation. If inlet temperatures, one of the fluid outlet temperatures, and mass flow rates are known, the LMTD method can be used to solve sizing problem. If they are not known, the ( $\varepsilon - NTU$ ) method can be used. ( $\Psi - P$ ) and ( $P_1 - P_2$ ) methods are graphical methods. The ( $P_1 - P_2$ ) method includes all major dimensionless heat exchanger parameters. Hence, the solution to the rating and sizing problem is non-iterative straightforward.

Regenerators are basically classified into rotary and fixed matrix models and in the thermal design of these models two methods: effectiveness-modified number of transfer units ( $\varepsilon - NTU_o$ ) and reduced length and reduced period ( $\Lambda - \pi$ ) methods for the regenerators. ( $\Lambda - \pi$ ) method is generally used for fixed matrix regenerators.

## Nomenclature

$A$	Total heat transfer surface area of heat exchanger, total heat transfer surface area of all matrices of a regenerator, $m^2$
$A_f$	Fin surface area, $m^2$
$A_{rc}$	Rotor cross-sectional area, $m^2$
$C$	Flow stream heat capacity rate, W/K
$C_w$	Matrix heat capacity rate, W/K
$c_p$	Specific heat at constant pressure, J/kgK
$c_w$	Specific heat of wall material, J/kgK
$d, D$	Diameter, m
$E$	Total energy, kJ
$F_{rfa}$	Fraction of rotor face area not covered by radial seals.
$H_r$	Rotor height
$h$	Specific enthalpy, kJ/kg
$k$	Thermal conductivity, W/mK
$L$	Length of heat exchanger, m
$\dot{m}$	Mass flow rate, kg/s
$M_w$	The total mass of all matrices of a regenerator, kg
$N$	Rotational speed for a rotary regenerator, rev/s, rpm
NTU	Number of transfer units
$ntu_c$	Number of transfer units based on the cold fluid side
$ntu_h$	Number of transfer units based on the hot fluid side
$P$	Temperature effectiveness for one fluid stream
$\dot{Q}$	Heat transfer rate, kW
$r$	Tube radius, m
$R$	Thermal resistance, $m^2K/W$
$R_f$	Fouling resistance, fouling factor, $m^2K/W$
$S_m$	Matrix solidity
$T$	Temperature, $^{\circ}C, K$

$T_c$	Cold fluid temperature, °C, K
$T_h$	Hot fluid temperature, °C, K
$t$	Wall thickness, m
$\Delta T_{lm}$	Log-mean temperature difference, °C, K
$U$	Overall heat transfer coefficient, W/m <sup>2</sup> K
$V$	Velocity, m/s
$\dot{W}$	Power, kW
$z$	Elevation, m

### Greek symbols

$\beta$	packing density for a regenerator, m <sup>2</sup> /m <sup>3</sup>
$\Delta$	Difference
$\varepsilon$	Effectiveness
$\rho_m$	Matrix material density, kg/m <sup>3</sup>
$\eta$	Efficiency
$\Lambda$	Reduced length for a regenerator
$\pi$	Reduced period for a regenerator

### Subscripts

$c$	Cold fluid
$cf$	Counter flow
$cv$	Control volume
$e$	Exit conditions
$f$	Fin, finned, friction
$h$	Hot
$i$	Inlet conditions, inner, inside
$lm$	logarithmic mean
$max$	Maximum
$min$	Minimum
$o$	Outer, outside, overall
$u$	Unfinned
1	Initial or inlet state, fluid 1
2	Final or exit state, fluid 2

## Author details

Cüneyt Ezgi

Address all correspondence to: [cuneytezgi@beykent.edu.tr](mailto:cuneytezgi@beykent.edu.tr)

Department of Mechanical Engineering, Faculty of Engineering and Architecture, Beykent University, Istanbul, Turkey

## References

- [1] Cengel Y, Ghajar A. Heat and Mass Transfer: Fundamentals and Applications. 5th ed. McGraw-Hill Education; UK, 2014. 992 p, ISBN-13: 978-0073398181
- [2] Kakaç S, Liu H, Pramuanjaroenkij A. Heat Exchangers: Selection, Rating, and Thermal Design. 3rd ed. CRC Press; USA, 2012. 631 p, ISBN-13: 978-1439849903
- [3] Moran MJ, Shapiro HN, Bruce R, Munson BR, DeWitt DP. Introduction to Thermal Systems Engineering: Thermodynamics, Fluid Mechanics, and Heat Transfer. 2nd ed. Wiley; USA, 2002. 576 p, ISBN-13: 978-0471204909
- [4] Shah RK, Sekulic DP. Fundamentals of Heat Exchanger Design. 1st ed. Wiley; USA, 2002. 750 p, ISBN-13: 978-0471321712
- [5] Thulukkanam K. Heat Exchanger Design Handbook, Second Edition (Mechanical Engineering). 2nd ed. CRC Press; USA, 2013. 1260 p, ISBN-13: 978-1439842126

