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OPTIMAL LOAD FLOW STUDY - UTILIZING O.R. TECHNIQUES

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THE UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

OPTIMAL LOAD FLOW STUDY - UTILIZING O.R. TECHNIQUES

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

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degree of

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TECHNIQUES

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ABSTRACT

The purpose of this research is to present a new operational research (OR) optimal load flow approach. This new approach is implemented using nonlinear programming methods. As conventional and existing O.R. models, this model incorporates network performance variables such as bus voltages as well as topological and elemental constraints.

This research includes a discussion of the performance constraints of the network and implementation into the O.R. model. In addition, it also explores the development and sensitivity of the model. Two examples of a small network problem and an IEEE 39 bus system are implemented and solved. Detail discussions about the ability of the model to work under a wide variety of situations such as loss of generation, loss of lines, various faults, and voltage regulation are also included in this research.

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CHAPTER ONE

INTRODUCTION

Over the past thirty years an enormous amount of effort has been expended in research and development in the load flow area. Hundreds of papers have been published on methods of load flow calculation. Perhaps the most recurrent question arising in this field is : which is the best method to choose for a given application? The answer is rarely easy. The relative properties and performances of different load flow methods can be influenced substantially by the types and sizes of problems to be solved, by the computing facilities available and by the precise details of implementation. The final choice is almost inevitably a compromise between the various criteria by which load flow methods are to be compared with each other.

O.R. techniques are certainly the most promising tool for solving this type of problems. The L.P. solution technique applied to solve optimal real power generation is already well known. However, the same technique is much more difficult to apply to reactive power optimization, due to the non-linearity of relations between power flow and bus voltages or between reactive power injections and voltages.

State of the art

In the past, some O.R. methods using sensitivity relationships and gradient search approaches have emerged to solve this complex problem. Peschon et al. (1) developed the power flow sensitivity and cost sensitivity relationship to optimize the real and reactive power generations in the system. They also presented a linear programming approximation to the optimization problem of minimizing the production costs.

Dopaz et al. (2) presented a method of minimizing the production cost by coordinating real and reactive power allocation in the system. The procedure at first determines the real power dispatch based on the Lagrangian multipliers and then proceeds to optimize the reactive power allocation by a gradient approach. The objective function, which is system loss reduction, yields the required gradient vector.

Hano et al. (3) presented a method of controlling the system voltage and reactive power distribution in the system. They determined the required sensitivity relationships between controlled and controllable variables, and loss sensitivity indices, and then employed a direct search technique to minimize the system losses.

Dommel and Tinney (4) developed and presented a nonlinear optimization technique to determine the optimal power flow solution. They minimized a nonlinear objective function of production costs or losses using Kuhn-Tucker conditions.

Savulesu (5) presented an approach to determine loss sensitivity , reactive power transmittance and steady-state stability indices. Based on these indices, he employed a suitable search procedure to move toward the required optimal system conditions.

Since most of the above mentioned algorithms use a gradient search technique, there is no coordinated variable control over the system performance.

Narital and Hamman (6) use the sensitivity analysis of power systems as an optimization technique called the "Method of Box" to minimize the voltage deviations from their desired values. As a secondary step, they minimize the system losses.

Shoults and Chen (7) found the adjustments to the transformer taps and the generator terminal voltage required to restore the reactive power flows in lines and the load bus voltages to their desired values.

These methods, however, are not suitable for optimal scheduling of reactive power flows and voltages in the system.

Hobson (8) developed a method of finding the network constrained reactive power control. He used incremental transmission line and transformer models and linearized network equations. Then the problem was solved by a special L.P. technique by giving priorities to generators in the system. This method seems to maintain only soft limits on transformer taps, generator voltages, generator reactive power, etc..

Mamandur and Chenoweth (9) developed a new L.P. and N.R. inter-reactive and repetitive method to improve the voltage profile and to minimize the system losses. They used the dual linear programming technique to evaluate the new status for the state variables (e.g. P, Q, V, etc.). Then a conventional N.R. load flow is performed. This completes one iteration of the VAR control problem. Iterations are repeated until the constraints are satisfied. Since this method used L.P. technique to evaluate the system performance, it linearizes some non-linearity of the state variables, and injects some reactive flow errors. Although the LP-NR repetition will reduce the errors, the computer time will be increased.

Chamorel and Germond (10) presented a method based on the active-reactive decoupling. First an active power flow optimization is performed, then the constraints are modified for reactive optimization. The difficulty of this method is reactive power optimization because each voltage modification leads to general modification of loads, reactive losses and penalty factors.

Shoults and Sun (11) developed another decoupling technique to solve power flow optimization. They decomposed the power flow problem into two suboptimal problems (i) real power optimization with system voltage assumed constant,

(ii) reactive power optimization with real power generation and bus phase angles assumed constant. Then using non-linear programming technique (SUMT) solves these two sub-problems alternatively.

Burchett et al. (12) presented a similar method as (11), except they used different non-linear programming technique to reach the optimal.

Since, all three algorithms used decoupling techniques, they assume the ratio R/X of transmission line is low. If this assumption does not hold the results will be inaccurate.

The need for a more realistic and accurate model

The electric utility industry has doubled its energy output almost every decade since its inception in 1882. As a result it is one of the largest single industries in the United States. In addition, the electric utilities, with a few exceptions, make up a single electrically contiguou system covering the entire continental United States.

Therefore, in planning a future power system, it is extremely important to study how the projected transmission network will really perform under certain forecasted load levels and contingency conditions.

The proposed model is based on the system performance as well as the topology. Bus voltages are assigned to be the control variables , which governs the system to satisfy an acceptable performance index. The solution method utilizes non-linear programming techniques to solve the optimal load flow problem generally and in several contingency situations; such as loss of generation, loss of lines, various faults and voltage regulation.

<u>Chapters overview</u>

Chapter one Introduction.

Chapter two Development of the model.

Chapter three Model implementation.

Chapter four Summary and conclusion.

That where possible equations are expressed in the standard mathematical forms. Example equations are expressed in the Fortran notations to more clearly show model mechanization.

CHAPTER TWO

DEVELOPMENT OF THE MODEL

Any power system load flow model is composed of three parts, (i) real power flow for each system line, (ii) reactive power flow for each line, and (iii) real and imaginary solution of the bus voltage. The bus voltage variables are the dominant variables of the system solution, since they determine the real and reactive power flows of the network, as in the following derivation.

Derivation of the power flow equation

(a) Real power flow

$$P_i = E_i \, I_{ij} \, \text{Cos}\phi_i$$

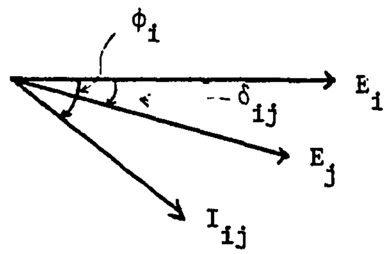

Figure 2.1 Vector diagram

$$I_{ij} = \frac{E_i - E_j}{Z_{ij}}$$

$$= \frac{E_i}{Z_{ij}} \angle -\phi_z - \frac{E_j}{Z_{ij}} \angle -\delta_{ij} - \phi_z$$

$$I_{ij} \, \text{Cos}\phi_i = \frac{E_i}{Z_{ij}} \text{Cos}\phi_z - \frac{E_j}{Z_{ij}} \text{Cos}(\phi_z + \delta_{ij})$$

$$P_{ij} = E_i I_{ij} \cos\phi_i = \frac{E_i^2}{Z_{ij}} \cos\phi_z - \frac{E_i E_j}{Z_{ij}} \cos(\phi_z + \delta_{ij})$$

$$= \frac{E_i^2}{Z_{ij}} x \frac{R_{ij}}{Z_{ij}} - \frac{E_i E_j}{Z_{ij}} \sin(90 - \phi_z - \delta_{ij})$$

$$= - \frac{E_i E_j}{Z_{ij}} \sin(-\delta_{ij} + \alpha_z) + \frac{E_i^2 R}{Z_{ij}^2} \quad , \quad \alpha_z = 90 - \phi_z$$

$$= \frac{E_i E_j}{Z_{ij}} \sin(+\delta_{ij} - \alpha_z) + \frac{E_i^2 R}{Z_{ij}^2} \qquad (3.1)$$

$$\delta_{ij} = \mathrm{Tan}^{-1} \frac{IV_i}{RV_i} - \mathrm{Tan}^{-1} \frac{IV_j}{RV_j}$$

$$\sin\delta_{ij} = \sin\left(\mathrm{Tan}^{-1} \frac{IV_i}{RV_i} - \mathrm{Tan}^{-1} \frac{IV_j}{RV_j}\right)$$

$$= \sin\left(\mathrm{Tan}^{-1} \frac{IV_i}{RV_i}\right) \cos\left(\mathrm{Tan}^{-1} \frac{IV_j}{RV_j}\right)$$

$$- \cos\left(\mathrm{Tan}^{-1} \frac{IV_i}{RV_i}\right) \sin\left(\mathrm{Tan}^{-1} \frac{IV_j}{RV_j}\right)$$

$$= \frac{IV_i}{E_i} x \frac{RV_j}{E_j} - \frac{RV_i}{E_i} x \frac{IV_j}{E_j}$$

$$\cos\delta_{ij} = \cos\left(\tan^{-1}\frac{IV_i}{RV_i} - \tan^{-1}\frac{IV_j}{RV_j}\right)$$

$$= \frac{RV_i}{E_i} \times \frac{RV_j}{E_j} + \frac{IV_i}{E_i} \times \frac{IV_j}{E_j}$$

$$P_{ij} = \frac{E_i^2 R_{ij}}{Z_{ij}^2} + \frac{E_i E_j}{Z_{ij}} \sin(\sigma_{ij} - \alpha_z),\ \alpha_z = \tan^{-1}\frac{R}{X}$$

$$= \frac{E_i^2 R_{ij}}{Z_{ij}^2} + \frac{E_i E_j}{Z_{ij}} (\sin\sigma_{ij} \cos\alpha_z - \cos\sigma_{ij} \sin\alpha_z)$$

$$= \frac{E_i^2 R_{ij}}{Z_{ij}^2} + \frac{E_i E_j}{Z_{ij}^2} (X \sin\sigma_{ij} - R \cos\sigma_{ij})$$

$$= \frac{E_i^2 R_{ij}}{Z_{ij}^2} + \frac{E_i E_j}{Z_{ij}^2} \left[X_{ij} \left(\frac{IV_i RV_j - RV_i IV_j}{E_i E_j} \right) - R_{ij} \left(\frac{RV_i RV_j - IV_i IV_j}{E_i E_j} \right)\right]$$

$$= \frac{E_i^2 R_{ij}}{Z_{ij}^2} + \frac{X_{ij}(IV_i RV_j - RV_i IV_j) - R_{ij}(RV_i RV_j + IV_i IV_j)}{Z_{ij}^2}$$

$$P_{ji} = \frac{E_j^2 R_{ij}}{Z_{ij}^2} + \frac{X_{ij}(IV_j RV_i - RV_j IV_i) - R_{ij}(RV_j RV_i + IV_j IV_i)}{Z_{ij}^2}$$

(b) Reactive power flow

$$Q'_{ij} = - \frac{E_i^2 X_{ij}}{Z_{ij}^2} + \frac{E_i E_j}{Z_{ij}} \mathrm{Cos}(\sigma_{ij} - \alpha_z), \quad \alpha_z = \mathrm{Tan}^{-1} \frac{R}{X}$$

$$= - \frac{E_i^2 X_{ij}}{Z_{ij}^2} + \frac{E_i E_j}{Z_{ij}} (\mathrm{Cos}\sigma_{ij} \, \mathrm{Cos}\alpha_z + \mathrm{Sin}\sigma_{ij} \, \mathrm{Sin}\alpha_z)$$

$$= - \frac{E_i^2 X_{ij}}{Z_{ij}^2} + \frac{E_i E_j}{Z_{ij}^2} (X \, \mathrm{Cos}\sigma_{ij} + R \, \mathrm{Sin}\sigma_{ij})$$

$$= + \frac{E_i^2 X_{ij}}{Z_{ij}^2} - \frac{X_{ij} (RV_i RV_j + IV_i IV_j) + R_{ij} (IV_i RV_j - RV_i IV_j)}{Z_{ij}^2}$$

$$Q_{ij} = Q'_{ij} - \frac{V_{ij}}{2 X_{cij}}$$

Model Formulation

Based on above equations, the bus voltage variables are utilized as instant bridges between the real and reactive power flow as well as control variables on the system performance. The mathematical relations among these three variables are original and interfaced. There is no linearization involved; thus the inaccuracy due to the linearization ,occurring in many other optimal load flow techniques, is nonexistant in this model. The system performance is controlled by various objective functions and line capacity constraints. Instead of using tap changing variables, this model formulates voltage regulation constraints, and keeps the voltage drop of the designated lines within the preset value. The objective function of this technique will give the best system voltage profile for the least system losses. Furthermore, unlike most optimal load flow techniques, this model does not include a swing bus, because the optimal solution is produced upon a total system capacity for all source and sink buses including the swing bus.

Objective - As in all optimization problems, one of the difficulties is to define the objectives of power flow optimization, which will serve as the optimization objective function. The following objectives can be mentioned :

- Minimizing system real power losses.
- Minimizing system reactive power losses.
- Rescheduling the generating units in case of failure of one or more system elements (line, transformer or generator) to avoid an unsecure state (in this research, the generation failure and loss of a line have been looked into.)
- Existing load expansion.
- Minimizing real power generation cost.

In this research, although the model is capable of doing them, the last two objectives have not been addressed.

Constraints - serving as a feasible region for the optimization, constraints are determined by the relation with the physical properties of system elements. Based on the different purposes, the constraints in this model are separated into two major sets and one optional set.

(1) Real power flow constraints - This set of constraints using voltage variables and the Kirchhoff's Law distributes the real power among the whole system. At the generation buses, the generation capacity limit will be kept. At the load buses, the demand will also be kept.

(2) Reactive power flow constraints - Like the real power flow constraints, these constraints use voltage variables and the Kirchhoff's Law and distribute the reactive power among the whole system. Depending on the system instant situation, the generation buses will have the capability to either allow the generator sending or receiving VARS, but it is limited within a range. At the load bus, depending on the nature of the load, it will be formed either to send or receive VARS, and most importantly, this model includes the charging VARS in the network to make the result realistic.

Another unique feature of this model is that voltage profile or voltage regulation constraints are optional. If the objective function is to minimize the system loss, it will seek the over all system minimal losses and produce the entire system voltage profile at minimal losses. If a particular line voltage profile constraint is put in, either that constraint will be redundant or the over all system losses will be converged to a suboptimal value or even divergent. So unless a specific requirement is made to certain lines voltage profile restriction, putting voltage regulation constraint into this model is not recommended, because it will not improve the over all system losses.

The model

Prior to the introduction of the formal mathematical model the following notations are defined :

A. Notation

1. List of solution variables

S : the real part of the bus voltage at the node i.

T : the reactive part of the bus voltage at the node i

2. Known system data

B : real power demand at the node i.

G : real power source at the node i.

Z : the magnitude of the line impedance.

I : line reactance.

R : line resistance.

Y : line charging admittance.

H : line capacity of the line i - j.

C : upper limit of the transmitting generator reactive power.

D : lower limit of the generator receiving reactive power.

E : voltage regulation factor.

F : reactive load demand or reactive power supply from the load center.

3. Set definitions.

G : generator buses.

L : load buses.

N : network (non-generator, non-load) buses.

M : total system buses.

B. Objective function - The intent of the user may be formed in the following ways :

1. Minimize the total reactive power losses.

$$\text{Min} \sum_{ij}\sum \{ (S_i^2 + T_i^2 + S_j^2 + T_j^2 - 2S_iS_j - 2T_iT_j) \times \frac{I_{ij}}{T_{ij}^2} \} , \; i<j , \; \begin{matrix} i\varepsilon M \\ j\varepsilon M \end{matrix}$$

2. Minimize the total real power losses.

$$\text{Min} \sum_{ij}\sum \{ (S_i^2 + T_i^2 + S_j^2 + T_j^2 - 2S_iS_j - 2T_iT_j) \times \frac{R_{ij}}{T_{ij}^2} \} , \; i<j , \; \begin{matrix} i\varepsilon M \\ j\varepsilon M \end{matrix}$$

3. Maximize the total real power received by the load buses.

$$\text{Min} \sum_{ij}\sum \{ (-\frac{I_{ij}}{T_{ij}^2} \times (T_iS_j - S_iT_j) + \frac{R_{ij}}{T_{ij}^2} \times (S_i^2 + T_i^2 - S_iS_j - T_iT_j), \qquad \begin{matrix} i\varepsilon M \\ j\varepsilon M \end{matrix}$$

C. Constraints.

1. All the real transmitted power from the source bus will be less than or equal to the source real power capacity.

$$\sum_j \{ \frac{I_{ij}}{T_{ij}^2} \times (T_iS_j - S_iT_j) + \frac{R_{ij}}{T_{ij}^2} \times (S_i^2 + T_i^2 - S_iS_j - T_iT_j)\} \leq A_i , \qquad \begin{matrix} i\varepsilon G \\ j\varepsilon M \end{matrix}$$

2. All the real power received by the load bus will be greater than or equal to the real power demand at that bus.

$$-\sum_j \{ \frac{I_{ij}}{T_{ij}^2} \times (T_iS_j - S_iT_j) + \frac{R_{ij}}{T_{ij}^2} \times (S_i^2 + T_i^2 - S_iS_j - T_iT_j)\}$$

(continue) $\geq B_i$, $i \varepsilon L$, $j \varepsilon M$

3. At an intermediate (non-source and non-sink) bus, the sum of all the real power transmitted will equal to zero.

$$-\sum_j \{ \frac{I_{ij}}{T_{ij}^2} \times (T_i S_j - S_i T_j) + \frac{R_{ij}}{T_{ij}^2} \times (S_i^2 + T_i^2 - S_i S_j - T_i T_j)\} = 0, \quad i \varepsilon N,\ j \varepsilon M$$

4. The generator can either transmit or receive reactive power; however, there is an upper and lower limit on the reactive power transmitted.

$$\pm\sum_j \{ \frac{I_{ij}}{T_{ij}^2} \times (S_i^2 + T_i^2 - S_i S_j - T_i T_j) - \frac{R_{ij}}{T_{ij}^2} \times (T_i S_j - S_i T_j)$$

$$- \frac{A_{ij}}{2} \times (S_i^2 + T_i^2)\} \lessgtr C_i \text{ or } D_i , \quad i \varepsilon G,\ j \varepsilon M$$

5. The load bus can either transmit or receive reactive power. All the reactive power received by the load bus should be greater than or equal to the amount of reactive power capable of being transmitted or received (positive means receive, negative means transmit).

$$-\sum_j \{ \frac{I_{ij}}{T_{ij}^2} \times (S_i^2 + T_i^2 - S_i S_j - T_i T_j) - \frac{R_{ij}}{T_{ij}^2} \times (T_i S_j - S_i T_j)$$

$$- \frac{A_{ij}}{2} \times (S_i^2 + T_i^2)\} \geq \pm F_i , \quad i \varepsilon L,\ j \varepsilon M$$

6. At the intermediate (non-source and non-sink) bus, the

sum of all the reactive power that can be transmitted will be equal to zero.

$$-\Sigma\{ \frac{I_{ij}}{T_{ij}^2} \times (S_i^2 + T_i^2 - S_iS_j - T_iT_j) - \frac{R_{ij}}{T_{ij}^2} \times (T_iS_j - S_iT_j)$$

$$- \frac{A_{ij}}{2} \times (S_i^2 + T_i^2)\} = 0 , \qquad i \varepsilon N,\ j \varepsilon M$$

7. (optional) Voltage regulation constraint of the system can be controlled by choice of the factor e.

$$S_i^2 + T_i^2 + S_j^2 + T_j^2 - 2 \times (S_iS_j + T_iT_j) \leq E_i \times (S_i^2 + T_i^2),$$

$$i \varepsilon M,\ j \varepsilon M$$

$$S_i^2 + T_i^2 + S_j^2 + T_j^2 - 2 \times (S_iS_j + T_iT_j) \leq E_j \times (S_j^2 + T_j^2),$$

8. (optional) The line constraints are as follows :

$$\frac{I_{ij}}{T_{ij}^2} \times (T_iS_j - S_iT_j) + \frac{R_{ij}}{T_{ij}^2} \times (S_i^2 + T_i^2 - S_iS_j - T_iT_j) \leq H_{ij} ,$$

$$i \neq j , \qquad i \varepsilon M,\ j \varepsilon M$$

The above equations show that this model only deals with two different kinds of variables: real part of bus voltage and reactive part of bus voltage. Real power flows and reactive power flows will be calculated from the output of the model independently (see Appendix 1.). Therefore a lot of variables and constraints are reduced, so the computation speed is improved tremendously.

Sensitivity analysis of the model

One of the major difficulties of the nonlinear programming is that the results produced by the model can merely be a local optimum. In order to decide whether it

indeed is a global optimum or not, further analysis of the results is needed. Another difficult reason for nonlinear programming is that there are a lot of different methods to solve nonlinear problems. It is always helpful to analyze the problem first, then choose the method.

Generally, if the objective function is convex and the subjective constraints form a convex set, then there exists a global optimum. If either the objective function or the constraint set is not convex, then the situation is more complicated, but there are still some rules can be followed.

Rule (1) Minimize $f(x)$

$$\text{S.T.}\quad g_i(x) \leqslant 0$$

$$g_j(x) \geqslant 0$$

Let x be a feasible solution, and let $I=\{i : g_i(\bar{x})=0\}$, $J=\{j : g_j(\bar{x})=0\}$, suppose that f is pseudoconvex at $\bar{x}$, g_i is quasiconvex and differentiable at x for each $i \in I$, and g_j is quasiconcave and differentiable at x for each $j \in J$. Furthermore, suppose that the Kuhn-Tucker condition hold true at x; that is, there exists non-negative scalar u_i for $i \in I$, and non-positive scalar u_j for $j \in J$, such that $\nabla f + \Sigma u_i g_i + \Sigma u_j g_j = 0$. Then $\bar{X}$ is a global optimal solution to the problem.

Rule (2) Minimize $f(x)$

$$\text{S.T.}\quad g_i(x) \leqslant 0$$

$g_j(x) \geq 0$

This rule is similar to rule 1, except that some authors prefer to use the multipliers $u_i = -u_i < 0$, $u_j = -u_j > 0$. In this case, the Kuhn-Tucker conditions are hold as

$\nabla f - \sum_i u_i g_i - \sum_j u_j g_j = 0.$

Rule (3) minimize $f(x)$

S.T. $g_i(x) \leq 0$

$g_j(x) \geq 0$

$h_k(x) = 0$

The rule for inequality constraints in rule 3 are the same as (1), the differences exist at equality constraints which the previous two types did not include. Assume the Kuhn-Tucker conditions hold at x, that is $\nabla f + \sum_i u_i g_i + \sum_j u_j g_j + \sum_k v_k h_k = 0$. Let $M = \{k, v > 0\}$ and $N = \{k, v < 0\}$. Further suppose that h is quasiconvex at $\bar{x}$ for $k \in M$, and h is quasiconcave at $\bar{x}$ for $k \in N$. Then $\bar{x}$ is a global optimal solution to the problem.

Rule (4) minimize $f(x)$

S.T. $g_i(x) \leq 0$

$g_j(x) \geq 0$

$h_k(x) = 0$

Rule 4 is not much different from rule 3, except that the multipliers are chosen negative of the rule 3, so the Kuhn-Tucker conditions are hold as $\nabla f - \sum_i u_i g_i - \sum_j u_j g_j - \sum_k v_k h_k = 0$.

To prove convexity, it must prove that let $f: s \in E$ be twice differentiable on S, and the Hessian matrix is positive semidefinite at each point in S. To prove quasiconvexity at x is to show $f[\lambda\bar{x}+(1-\lambda)x] \leqslant$ maximum $\{f(x), f(\bar{x})\}$ for each $\lambda\varepsilon(0, 1)$ and each $x\varepsilon S$. To prove quasiconcavity at $\bar{x}$ is to show $f[\lambda\bar{x}+(1-\lambda)x] \geqslant$ minimum $\{f(x), f(\bar{x})\}$ for each $\lambda\varepsilon(0, 1)$ and each x S. To prove function f pseudoconvexity at $\bar{x}$ is to show if $\nabla f(\bar{x})^t (x - \bar{x}) \geqslant 0$ for $x\varepsilon S$ implies that $f(x) \geqslant f(\bar{x})$. Otherwise if the function is convex then it is pseudoconvex.

The following examples will illustrate some characteristics of the objective function and constraints.

(a) Objective function

(1) $f(x) = S_i^2 + T_i^2 + S_j^2 + T_j^2 - 2S_iS_j - 2T_iT_j$

So f(x) is twice differentiable.

$$\begin{pmatrix} 2 & -2 & 0 & 0 \\ -2 & 2 & 0 & 0 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & -2 & 2 \end{pmatrix}$$

$H-\lambda I = 0$, are eigenvalues, I is an identity matrix, $\lambda = 0, 0, 4, 4$, so the Hessian matrix is positive semidefinite, from the conditions stated above, f(x) is a convex function.

(2) $f(x) = S_1^2 + T_1^2 + S_2^2 + T_2^2 - 2S_1S_2 - 2T_1T_2 + 40 \times (S_1^2 + T_1^2 + S_3^2 + T_3^2 - 2S_1S_3 - 2T_1T_3)$

f(x) is twice differentiable.

$$\begin{pmatrix} 82 & -2 & -80 & 0 & 0 & 0 \\ -2 & 2 & 0 & 0 & 0 & 0 \\ -80 & 0 & 80 & 0 & 0 & 0 \\ 0 & 0 & 0 & 82 & -2 & -80 \\ 0 & 0 & 0 & -2 & 2 & 0 \\ 0 & 0 & 0 & -80 & 0 & 80 \end{pmatrix}$$

$H - \lambda I = 0$, are eigenvalues, I is an identity matrix, $\lambda = 0.0, 0.0, 2.981, 2.981, 161.01888, 161.01888$, so the Hessian matrix is positive semidefinite, from the conditions shown above, f(x) is a convex function.

(b) Constraints

$$g(x) = T_iS_j - S_iT_j + 2 \times (S_i^2 + T_i^2 - S_iS_j - T_iT_j)$$

g(x) is twice differentiable.

$$\begin{pmatrix} 4 & -2 & 0 & -1 \\ -2 & 0 & 1 & 0 \\ 0 & 1 & 4 & -2 \\ -1 & 0 & -2 & 0 \end{pmatrix}$$

$H - \lambda I = 0$, are eigenvalues, I is an identity matrix, $\lambda = -1, -1, 5, 5$. The Hessian matrix failed to prove either positive or negative semidefinite.

These examples show that although the objective function can be proved convex in most cases, the constraint set has shown no general convexity for the load flow problems. Therefore, it has to be studied individually, which will be done in the next chapter.

CHAPTER THREE

MODEL IMPLEMENTATION

In order to demonstrate the capability of the model, two sample problems will be solved, the optimality of the solution will be checked and results will be compared with the results from existing network solution technique and from some linearized O.R. technique. It will also be proved in this chapter that the model is able to handle a list of various situations, e.g. loss of line, loss of generation, bus fault, line fault, and voltage regulation.

The first problem is a five bus system chosen from Computer Methods in Power System Analysis [13]. The system consists of two generator buses and three load buses. A single line diagram of the system is shown in figure 3.1, while the generator data and other network details of the system are given in Table 3.1 and 3.2.

Problem formulation

By utilizing the above data and the developed model, this five bus problem is transformed into a series of mathematical expressions. First consider the objective function, the model stated there are three different choices for it. Minimizing the total reactive power losses is

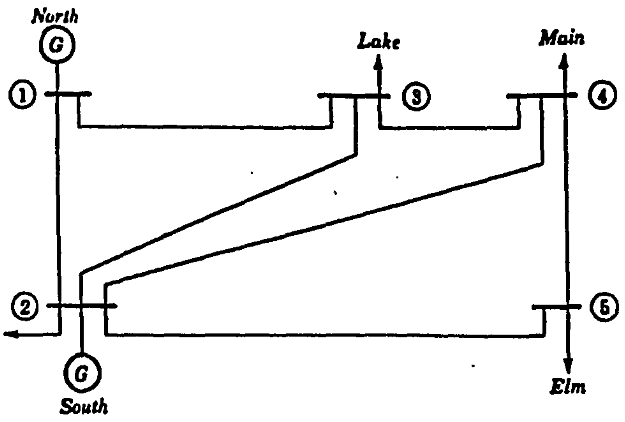

Figure 3.1 five bus system

Table 3.1 bus data

Bus code p	Assumed bus voltage	Generation		Load	
		Megawatts	Megavars	Megawatts	Megavars
1	1.06 + j0.0	0	0	0	0
2	1.0 + j0.0	40	30	20	10
3	1.0 + j0.0	0	0	45	15
4	1.0 + j0.0	0	0	40	5
5	1.0 + j0.0	0	0	60	10

Table 3.2 line data

Bus code p-q	Impedance z_{pq}	Line charging $y'_{pq}/2$
1-2	0.02 + j0.06	0.0 + j0.030
1-3	0.08 + j0.24	0.0 + j0.025
2-3	0.06 + j0.18	0.0 + j0.020
2-4	0.06 + j0.18	0.0 + j0.020
2-5	0.04 + j0.12	0.0 + j0.015
3-4	0.01 + j0.03	0.0 + j0.010
4-5	0.08 + j0.24	0.0 + j0.025

chosen to be the object of this problem, and the equation is:

```
g(11)=18.75*x(1)**2+32.5*x(3)**2+38.75*x(5)**2+38.75*x(7)**2
     +11.25*x(9)**2+18.75*x(2)**2+32.5*x(4)**2+38.75*x(6)
     **2+38.75*x(8)**2+11.25*x(10)**2-30.0*x(1)*x(3)-7.5*
     x(1)*x(5)-10.0*x(3)*x(5)-10.0*x(3)*x(7)-15.0*x(3)*x(9)
     -60.0*x(5)*x(7)-7.5*x(7)*x(9)-30.0*x(2)*x(4)-7.5*x(2)*
     x(6)-10.0*x(4)*x(6)-10.0*x(4)*x(8)-15.0*x(4)*x(10)-
     60.0*x(6)*x(8)-7.5*x(8)*x(10)
```

Note that it should have a similar form of objective function for minimizing real power losses, except the coefficients will be different. Since the goal is to minimize the total system reactive power losses, those optional constraints can be omitted, and the constraint set is composed of real and reactive power flow. Since the system has no non-generator or non-load bus, intermediated constraints can be omitted.

(a) Real power flow

```
g(1)=6.25*x(1)**2+6.25*x(2)**2-5.0*x(1)*x(3)-1.25*x(1)*x(5)+
     15.0*x(2)*x(3)-15.0*x(1)*x(4)+3.75*x(2)*x(5)-3.75*x(1)*
     x(6)-5.0*x(2)*x(4)-1.25*x(2)*x(6)
g(2)=10.83333*x(3)**2+10.83333*x(4)**2-5.0*x(1)*x(3)-1.66667
     *x(3)*x(5)-1.66667*x(3)*x(7)-2.5*x(3)*x(9)+15.0*x(1)*
```

```
     x(4)-15.0*x(3)*x(2)+5.0*x(5)*x(4)-5.0*x(3)*x(6)+5.0
     *x(7)*x(4)-5.0*x(3)*x(8)+7.5*x(9)*x(4)-7.5*x(3)*x(10)-
     5.0*x(2)*x(4)-1.66667*x(4)*x(6)-1.66667*x(4)*x(8)-2.5
     *x(4)*x(10)
g(3)=-12.9167*x(5)**2-12.9167*x(6)**2+1.25*x(1)*x(5)+1.66667
     *x(3)*x(5)+10.0*x(5)*x(7)-3.75*x(1)*x(6)+3.75*x(5)*x(2)
     -5.0*x(3)*x(6)+5.0*x(5)*x(4)-30.0*x(7)*x(6)+30.0*x(5)
     *x(8)+1.25*x(2)*x(6)+1.66667*x(4)*x(6)+10.0*x(6)*x(8)
g(4)=-12.9167*x(7)**2-12.9167*x(8)**2+1.66667*x(3)*x(7)+10.0
     *x(5)*x(7)+1.25*x(7)*x(9)-5.0*x(3)*x(8)+5.0*x(7)*x(4)-
     30.0*x(5)*x(8)+30.0*x(7)*x(6)-3.75*x(9)*x(8)+3.75*x(7)*
     x(10)+1.66667*x(4)*x(8)+10.0*x(6)*x(8)+1.25*x(8)*x(10)
g(5)=-3.75*x(9)**2-3.75*x(10)**2+2.5*x(3)*x(9)+1.25*x(9)
     *x(7)-7.5*x(3)*x(10)+7.5*x(9)*x(4)-3.75*x(7)*x(10)+3.75
     *x(9)*x(8)+2.5*x(4)*x(10)+1.25*x(8)*x(10)
```

(b) Reactive power flow

```
g(6)=18.695*x(1)**2+18.695*x(2)**2-15.0*x(1)*x(3)-3.75*x(1)
     *x(5)-5.0*x(3)*x(2)+5.0*x(1)*x(4)-1.25*x(5)*x(2)+1.25*
     x(1)*x(6)-15.0*x(2)*x(4)-3.75*x(2)*x(6)
g(7)=32.415*x(3)**2+32.415*x(4)**2-15.0*x(1)*x(3)-5.0*x(3)*
     x(5)-5.0*x(3)*x(7)-7.5*x(3)*x(9)-5.0*x(1)*x(4)+5.0*x(3)
     *x(2)-1.66667*x(5)*x(4)+1.66667*x(3)*x(6)-1.66667*x(7)
     *x(4)+1.66667*x(3)*x(8)-2.5*x(9)*x(4)+2.5*x(3)*x(10)-
     15.0*x(2)*x(4)-5.0*x(4)*x(6)-5.0*x(4)*x(8)-7.5*x(4)*
     x(10)
```

```
g(8)=-38.695*x(5)**2-38.695*x(6)**2+3.75*x(1)*x(5)+5.0*x(3)*
     x(5)+30.0*x(5)*x(7)+1.25*x(1)*x(6)+1.66667*x(3)*x(6)+
     10.0*x(7)*x(6)-1.25*x(5)*x(2)-1.66667*x(5)*x(4)-10.0*
     x(5)*x(8)+3.75*x(2)*x(6)+5.0*x(4)*x(6)+30.0*x(6)*x(8)
g(9)=-38.695*x(7)**2-38.695*x(8)**2+5.0*x(3)*x(7)+30.0*x(5)*
     x(7)+3.75*x(7)*x(9)+1.66667*x(3)*x(8)+10.0*x(5)*x(8)+
     1.25*x(9)*x(8)-1.66667*x(7)*x(4)-10.0*x(7)*x(6)-1.25*
     x(7)*x(10)+5.0*x(4)*x(8)+30.0*x(6)*x(8)+3.75*x(8)*x(10)
g(10)=-11.21*x(9)**2-11.21*x(10)**2+7.5*x(3)*x(9)+3.75*x(7)*
     x(9)+2.5*x(3)*x(10)+1.25*x(7)*x(10)-2.5*x(9)*x(4)-1.25
     *x(9)*x(8)+7.5*x(4)*x(10)+3.75*x(8)*x(10)
```

O.R. analysis of the problem

The Hessian matrix of the objective function can be formulated in following way:

$$
\begin{pmatrix}
37.5 & -30 & -7.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-30 & 65 & -10 & -10 & -15 & 0 & 0 & 0 & 0 & 0 \\
-7.5 & -10 & 77.5 & -60 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -10 & -60 & -77.5 & -7.5 & 0 & 0 & 0 & 0 & 0 \\
0 & -15 & 0 & -7.5 & 22.5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 37.5 & -30 & -7.5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -30 & 65 & -10 & -10 & -15 \\
0 & 0 & 0 & 0 & 0 & -7.5 & -10 & 77.5 & -60 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -10 & -60 & 77.5 & -7.5 \\
0 & 0 & 0 & 0 & 0 & 0 & -15 & 0 & -7.5 & 22.5
\end{pmatrix}
$$

The eigenvalues are equal to 0.00011, 0.00022, 24.99982, 24.99997, 29.27783, 29.27788, 87.68541, 87.68552, 138.03539, 138.03583. Since the objective function is twice differentiable and the Hessian matrix is positive definite, it is strictly convex.

The Hessian matrix of each constraint can be formed as following :

(1) Constraint no. 1

$$\begin{pmatrix} 12.5 & -5 & -1.25 & 0 & -15 & -3.75 \\ -5 & 0 & 0 & 15 & 0 & 0 \\ -1.25 & 0 & 0 & 3.75 & 0 & 0 \\ 0 & 15 & 3.75 & 12.5 & -5 & -1.25 \\ -15 & 0 & 0 & -5 & 0 & 0 \\ -3.75 & 0 & 0 & -1.25 & 0 & 0 \end{pmatrix}$$

The eigenvalues equal to -11.20528, -11.20527, 0.0, 0.0, 0.0, 23.70514, 23.70518. Since Hessian matrix failed to prove semi-positive definite or semi-negative definite, therefore constraint no.1 is neither convex nor concave.

(2) Constraint no. 2

0	-5	0	0	0	0	15	0	0	0
-5	21.667	-1.667	-1.667	-2.5	-15	0	-5	-5	-7.5
0	-1.667	0	0	0	0	5	0	0	0
0	-1.667	0	0	0	0	5	0	0	0
0	-2.5	0	0	0	0	7.5	0	0	0
0	-15	0	0	0	0	-5	0	0	0
15	0	5	5	7.5	-5	21.667	-1.667	-1.667	-2.5
0	-5	0	0	0	0	-1.667	0	0	0
0	-5	0	0	0	0	-1.667	0	0	0
0	-7.5	0	0	0	0	-2.5	0	0	0

The eigenvalues are equal to -11.19884, -11.19881, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 32.86534, 32.86536, so it is neither convex nor concave.

(3) Constraint no. 3

$$\begin{pmatrix} 0 & 0 & 1.25 & 0 & 0 & 0 & -3.75 & 0 \\ 0 & 0 & 1.667 & 0 & 0 & 0 & -5 & 0 \\ 1.25 & 1.667 & -25.83 & 10 & 3.75 & 5 & 0 & 30 \\ 0 & 0 & 10 & 0 & 0 & 0 & -30 & 0 \\ 0 & 0 & 3.75 & 0 & 0 & 0 & 1.25 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 & 1.667 & 0 \\ -3.75 & -5 & 0 & -30 & 1.25 & 1.667 & -25.83 & 10 \\ 0 & 0 & 30 & 0 & 0 & 0 & 10 & 0 \end{pmatrix}$$

The eigenvalues are equal to -47.70515, -47.70494, 0.0, 0.0, 0.0, 0.0, 21.8716, 21.8718, so it is neither convex nor concave.

(4) Constraint no. 4

$$\begin{pmatrix} 0 & 0 & 1.667 & 0 & 0 & 0 & -5 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 & -30 & 0 \\ 1.667 & 10 & -25.83 & 1.25 & 5 & 30 & 0 & 3.75 \\ 0 & 0 & 1.25 & 0 & 0 & 0 & -3.75 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 & 1.667 & 0 \\ 0 & 0 & 30 & 0 & 0 & 0 & 10 & 0 \\ -5 & -30 & 0 & -3.75 & 1.667 & 10 & -25.83 & 1.25 \\ 0 & 0 & 3.75 & 0 & 0 & 0 & 1.25 & 0 \end{pmatrix}$$

The eigenvalues are equal to -47.70514, -47.7049, 0.0, 0.0, 0.0, 0.0, 21.87166, 21.87175, so it is neither convex nor concave.

(5) Constraint no. 5

$$\begin{bmatrix} 0 & 0 & 2.5 & 0 & 0 & -7.5 \\ 0 & 0 & 1.25 & 0 & 0 & -3.75 \\ 2.5 & 1.25 & -7.5 & 7.5 & 3.75 & 0 \\ 0 & 0 & 7.5 & 0 & 0 & 2.5 \\ 0 & 0 & 3.75 & 0 & 0 & 1.25 \\ -7.5 & -3.75 & 0 & 2.5 & 1.25 & -7.5 \end{bmatrix}$$

The eigenvalues are equal to -13.35142, -13.35138, 0.0, 0.0, 5.85143, 5.85143, so it is neither convex nor concave.

(6) Constraint no. 6

$$\begin{bmatrix} 37.39 & -15 & -3.75 & 0 & 5 & 1.25 \\ -15 & 0 & 0 & 5 & 0 & 0 \\ -3.75 & 0 & 0 & -1.25 & 0 & 0 \\ 0 & 5 & -1.25 & 37.39 & -15 & -3.75 \\ 5 & 0 & 0 & -15 & 0 & 0 \\ 1.25 & 0 & 0 & -3.75 & 0 & 0 \end{bmatrix}$$

The eigenvalues are equal to -8.96589, -8.87181, 0.0, 0.0, 0.0, 40.2616, 46.35579, so it is neither convex nor concave.

(7) Constraint no. 7

0	-15	0	0	0	0	-5	0	0	0
-15	64.83	-5	-5	-7.5	5	0	1.667	1.667	2.5
0	-5	0	0	0	0	-1.667	0	0	0
0	-5	0	0	0	0	-1.667	0	0	0
0	-7.5	0	0	0	0	-2.5	0	0	0
0	5	0	0	0	0	-15	0	0	0
-5	0	-1.667	-1.667	-2.5	-15	64.83	-5	-5	-7.5
0	1.667	0	0	0	0	-5	0	0	0
0	1.667	0	0	0	0	-5	0	0	0
0	2.5	0	0	0	0	-7.5	0	0	0

The eigenvalues are equal to -5.25179, -5.25179, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 70.08154, 70.08171, so it is neither convex nor concave.

(8) Constraint no. 8

$$\begin{bmatrix} 0 & 0 & 3.75 & 0 & 0 & 0 & 1.25 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 & 1.667 & 0 \\ 3.75 & 5 & -77.39 & 30 & -1.25 & -1.667 & 0 & 0 \\ 0 & 0 & 30 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & -1.25 & 0 & 0 & 0 & 3.75 & 0 \\ 0 & 0 & -1.667 & 0 & 0 & 0 & 5 & 0 \\ 1.25 & 1.667 & 0 & 10 & 3.75 & 5 & -77.39 & 30 \\ 0 & 0 & -10 & 0 & 0 & 0 & 30 & 0 \end{bmatrix}$$

The eigenvalues are equal to -89.10028, -89.10013, 0.0, 0.0, 11.71033, 11.71043, so it is neither convex nor concave.

(9) Constraint no. 9

$$\begin{pmatrix} 0 & 0 & 5 & 0 & 0 & 0 & 1.667 & 0 \\ 0 & 0 & 30 & 0 & 0 & 0 & 10 & 0 \\ 5 & 30 & -77.39 & -1.25 & -1.667 & -10 & 0 & -1.25 \\ 0 & 0 & -1.25 & 0 & 0 & 0 & 1.25 & 0 \\ 0 & 0 & -1.667 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & -10 & 0 & 0 & 0 & 30 & 0 \\ 1.667 & 10 & 0 & 1.25 & 5 & 30 & -77.39 & 3.75 \\ 0 & 0 & -1.25 & 0 & 0 & 0 & 3.75 & 0 \end{pmatrix}$$

The eigenvalues are equal to -89.12573, -88.9503, 0.0, 0.0, 0.0, 0.0, 11.56047, 11.73605, so it is neither convex nor concave.

(10) Constraint no. 10

$$\begin{pmatrix} 0 & 0 & 7.5 & 0 & 0 & 2.5 \\ 0 & 0 & 3.75 & 0 & 0 & 1.25 \\ 7.5 & 3.75 & -22.42 & -2.5 & -1.25 & 0 \\ 0 & 0 & -2.5 & 0 & 0 & 7.5 \\ 0 & 0 & -1.25 & 0 & 0 & 3.75 \\ 2.5 & 1.25 & 0 & 7.5 & 3.75 & -22.42 \end{pmatrix}$$

The eigenvalues are equal to -25.48544, -25.48543, 0.0, 0.0, 3.06547, 3.06547, so it is neither convex nor concave.

From these analyses, it is found that although the objective function of this problem is convex, its constraint set failed to show convex or concave, so there is no general convexity or concavity existing for the problem. To solve it, another useful conception in optimization is the notion of convexity or concavity at a point. In some cases, the requirement of a convex or concave function may be too strong and really not essential. Instead, convexity or concavity at a point may be all that is needed. So the next step is to choose an algorithm, to find the Kuhn-Tucker point, and to check the convexity or concavity at that point.

The algorithm and results

Since the problem is not a convex , so it can not be applied to a special convex programming, and it has to go to some generalized non-linear programming techniques. The generalized reduced gradient algorithm (GRG)[14] is a generalized non-linear programming, and an extension of the Wolf algorithm [15] to accommodate both a non-linear objective function and non-linear constraints. In essence, the method defines new variables that are normal to some of the constraints and transforms the gradient to this new basis.

In this research, a program called GRG2 [16,17] has been used to implement the presented problem. GRG2 is a revised GRG method, and uses the generalized reduced gradient technique to solve constrained non-linear problems of the following form:

Minimize or maximize f(x)

Subject to :

$$L_i < g_i(x) < U_i \text{ , } i=1,..,m$$

$$L_j < x_j > U_j \text{ , } j=1,..,n$$

Where x is a vector of n real variables, g(x) are m constraints, f(x) is the objective function.

To use GRG2, it is required to prepare a subroutine which computes the values of g(x). It is also needed to provide the bounds on the constraints. GRG2 uses first partial derivatives of each g(x) with respect to each x variable. These are automatically computed by a finite difference approach.

This GRG2 program operates in two phases. If the starting point is infeasible, optimization terminates either with a message that the problem is infeasible or with a feasible solution. In the former case a different starting point should be tried since the problem may actually have feasible solutions. If the start point is feasible, it will definitely stay in the feasible region. One of the advantages of staying in feasible solution is although the solution might not be optimal, it still is a reasonable solution.

After implementing the five bus system to GRG2 at the range of $0 < S_i < 1.09$, $-0.15 < T_i < 0$, the resultant bus voltages and line flows are placed in the following Tables.

Table 3.3 resultant bus voltage

bus no.	bus voltage
1	1.09+j0
2	1.0768628-j0.049845636
3	1.0523333-j0.086923925
4	1.0512788-j0.092621771
5	1.044449-j0.10620502

It was mentioned in chapter three that in order to

Table 3.4 Resultant line flows and system reactive losses

from bus i to bus j	real (MW)	reactive (MVAR)
1 - 2	88.6573	-9.251
1 - 3	40.6622	0.5826
2 - 1	-87.3288	6.1858
2 - 3	24.6699	3.1006
2 - 4	27.906	2.495
2 - 5	54.7543	6.694
3 - 1	-39.5404	-2.9748
3 - 2	-24.3405	-6.6665
3 - 4	18.8774	-5.3594
4 - 2	-27.4919	-5.8046
4 - 3	-18.8439	3.2312
4 - 5	6.3324	-2.4277
5 - 2	-53.6975	-7.0333
5 - 4	-6.3035	-3.0254
The total reactive power losses is 12.939 MVAR		

Table 3.5 resultant bus voltage in polar form

bus no.	magnitude	angle
1	1.09	0.0
2	1.078016	-2.6502
3	1.055917	-4.72198
4	1.055351	-5.03498
5	1.049835	-5.80618

Table 3.6 program status

constraint	status	largrange multiplier
1	FREE	0.0
2	UPPERBND	NEGATIVE
3	LOWERBND	POSITIVE
4	LOWERBND	POSITIVE
5	LOWERBND	POSITIVE
6	FREE	0.0
7	FREE	0.0
8	LOWERBND	POSITIVE
9	LOWERBND	POSITIVE
10	LOWERBND	POSITIVE

save the computing time, a independent small program is put into effect to calculate the bus voltage magnitude and angle, and line flows from the bus voltage. The results are included in Table 3.4, 3.5 and 3.6.

The results obtained from GRG2 is called a Kuhn-Tucker point, and it is not necessarily the global optimum. Since this five bus problem failed to prove a general convexity or concavity, it has to use point examining criteria in order to check the global optimum. The techniques for point convexity and point concavity were mentioned in the chapter three, the rules for global optimal check were also developed in the previous chapter. From Table 3.6, there are several things which can be observed to help select the rule. They are as follows:

(1) Constraint 1 and 6 are not binded.

(2) Constraint 2 and 7 reach the upper bounds, and their largrange multipliers are non-positive.

(3) Constraint 3, 4, 5, 8, 9, and 10 reach the lower bounds, and their largrange multipliers are non-negative.

Therefore, to check the global optimum, rule 2 should apply here. Thus, constraint 1 and 6 do not need analysis; constraint 2 and 7 should be quasiconvex and differentiable at the K-T point; constraint 3, 4, 5, 8, 9, and 10 should be quasiconcave and differentiable at the K-T point, and the objective function should be pseudoconvex at the K-T point.

In order to check the quasiconvexity and quasiconcavity at x, two approaches described in chapter three are:

(1) quasiconvexity $f[\lambda\bar{x}+(1-\lambda)x] \leq \max\{f(x), f(\bar{x})\}$, $\lambda \in (0,1)$, (3.1)

(2) quasiconcavity $f[\lambda\bar{x}+(1-\lambda)x] \geq \min\{f(x), f(\bar{x})\}$, $\lambda \in (0,1)$, (3.2)

A random feasible point $\bar{x}$ is selected, so that $f[\lambda\bar{x}+(1-\lambda)x]$, $\lambda \in (0,1)$ can be calculated. The results of each constraints are in the Table 3.7.

Table 3.7 shows that binded constraints no. 2 and 7 are quasiconvex at the K-T point, and binded constraints no. 3, 4, 5, 8, 9, and 10 are quasiconcave at the K-T point. Furthermore all the constraints are differentiable at the K-T point, and it has already been proved that the objective function is convex, hence the rule 2 is satisfied, so the solution is a global optimal solution.

The second testing problem is a IEEE 39 bus system[18]. The system consists of 10 generator buses and 17 load buses. A one line diagram of the system is shown in figure 3.2, while the generator data, line data, and other network details of the system are given in Table 3.8 and 3.9

<u>Problem formulation</u>

The procedures of formulating this 39 bus system into the model are similar to the previous problem, however, this problem is much bigger in size and is more complex. First consider the objective function. Like the previous example,

Table 3.7 quasiconvexity or quasiconcavity check at the K-T point.

1	2	3	4	5	6	7	8	9	10
1.29320	.20000	.45000	.40000	.60001	-.08668	.18534	.15000	.05001	.10004
1.35524	.19998	.45000	.40000	.65732	-.03068	.18977	.15002	.05002	.14411
1.39660	.19998	.45000	.39999	.69526	.00665	.19272	.15002	.05001	.17266
1.43796	.19998	.44999	.39999	.73298	.04399	.19564	.15000	.05000	.20053
1.50000	.20000	.44999	.39998	.78917	.09999	.20000	.14999	.04997	.24109

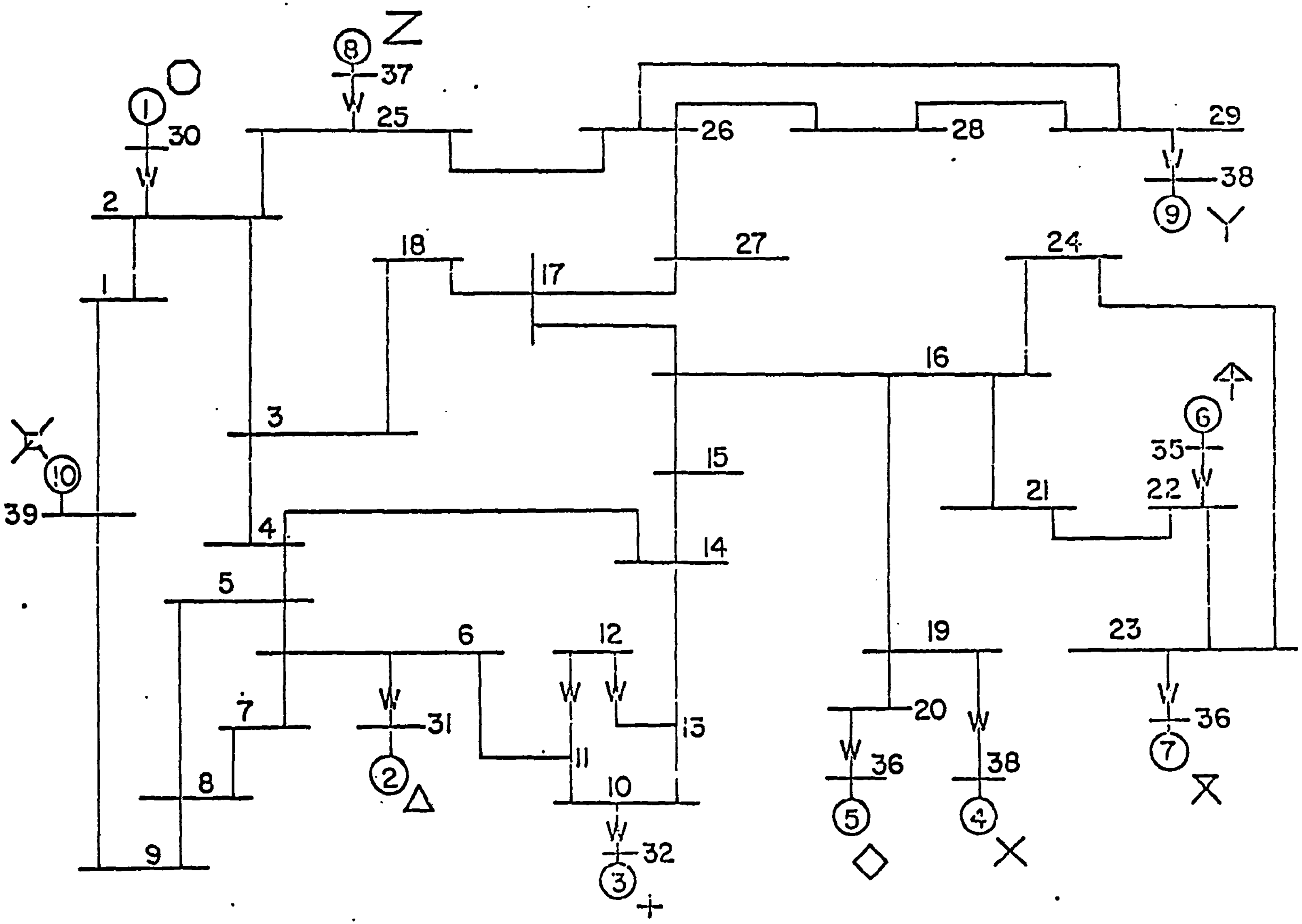

Figure 3.2 Thirty nine bus system

TEST SYSTEM LINE DATA
in p.u. on 100 MVA Base

Table 3.8

BUS	BUS	RESISTANCE	REACTANCE	SUSCEPTANCE	TAP	PHASE
1	2	.00350	.04110	.69870	.0000	.00
1	39	.00100	.02500	.75000	.0000	.00
2	3	.00130	.01510	.25720	.0000	.00
2	25	.00700	.00860	.14600	.0000	.00
3	4	.00130	.02130	.22140	.0000	.00
3	18	.00110	.01330	.21380	.0000	.00
4	5	.00080	.01280	.13420	.0000	.00
4	14	.00080	.01290	.13820	.0000	.00
5	6	.00020	.00260	.04340	.0000	.00
5	8	.00080	.01120	.14760	.0000	.00
6	7	.00060	.00920	.11300	.0000	.00
6	11	.00070	.00820	.13890	.0000	.00
7	8	.00040	.00460	.07800	.0000	.00
8	9	.00230	.03630	.38040	.0000	.00
9	39	.00100	.02500	1.20000	.0000	.00
10	11	.00040	.00430	.07290	.0000	.00
10	13	.00040	.00430	.07290	.0000	.00
13	14	.00090	.01010	.17230	.0000	.00
14	15	.00180	.02170	.36600	.0000	.00
15	16	.00090	.00940	.17100	.0000	.00
16	17	.00070	.00890	.13420	.0000	.00
16	19	.00160	.01950	.30400	.0000	.00
16	21	.00080	.01350	.25480	.0000	.00
16	24	.00030	.00590	.06800	.0000	.00
17	18	.00070	.00820	.13190	.0000	.00
17	27	.00130	.01730	.32160	.0000	.00
21	22	.00080	.01400	.25650	.0000	.00
22	23	.00060	.00960	.18460	.0000	.00
23	24	.00220	.03500	.36100	.0000	.00
25	26	.00320	.03230	.51300	.0000	.00
26	27	.00140	.01470	.23960	.0000	.00
26	28	.00430	.04740	.78020	.0000	.00
26	29	.00570	.06250	1.02900	.0000	.00
28	29	.00140	.01510	.24900	.0000	.00
12	11	.00160	.04350	.00000	1.0060	.00
12	13	.00160	.04350	.00000	1.0060	.00
6	31	.00000	.02500	.00000	1.0700	.00
10	32	.00000	.02000	.00000	1.0700	.00
19	33	.00070	.01420	.00000	1.0700	.00
20	34	.00090	.01800	.00000	1.0090	.00
22	35	.00000	.01430	.00000	1.0250	.00
23	36	.00050	.02720	.00000	1.0000	.00
25	37	.00060	.02320	.00000	1.0250	.00
2	30	.00000	.01810	.00000	1.0250	.00
29	38	.00080	.01560	.00000	1.0250	.00
19	20	.00070	.01380	.00000	1.0600	.00

TEST SYSTEM BUS DATA

Table 3.9

BUS	TYPE	VOLTS	ANGLE	LOAD MW	LOAD MVAR	GEN MW	GEN MVAR
1	0	1.0475	-9.370	.00	.00	.00	.00
2	0	1.0489	-6.600	.00	.00	.00	.00
3	0	1.0304	-9.650	322.00	2.40	.00	.00
4	0	1.0038	-10.470	500.00	184.00	.00	.00
5	0	1.0050	-9.310	.00	.00	.00	.00
6	0	1.0074	-8.620	.00	.00	.00	.00
7	0	.9967	-10.810	233.80	84.00	.00	.00
8	0	.9957	-11.320	522.00	176.00	.00	.00
9	0	1.0281	-11.120	.00	.00	.00	.00
10	0	1.0170	-6.210	.00	.00	.00	.00
11	0	1.0125	-7.030	.00	.00	.00	.00
12	0	1.0000	-7.040	8.50	88.00	.00	.00
13	0	1.0142	-6.920	.00	.00	.00	.00
14	0	1.0117	-8.580	.00	.00	.00	.00
15	0	1.0158	-8.970	320.00	153.00	.00	.00
16	0	1.0322	-7.550	329.40	32.30	.00	.00
17	0	1.0339	-8.550	.00	.00	.00	.00
18	0	1.0313	-9.400	158.00	30.00	.00	.00
19	0	1.0500	-2.920	.00	.00	.00	.00
20	0	.9909	-4.340	680.00	103.00	.00	.00
21	0	1.0321	-5.140	274.00	115.00	.00	.00
22	0	1.0500	-.690	.00	.00	.00	.00
23	0	1.0450	-.890	247.50	84.60	.00	.00
24	0	1.0377	-7.430	308.60	-92.20	.00	.00
25	0	1.0575	-5.430	224.00	47.20	.00	.00
26	0	1.0521	-6.680	139.00	17.00	.00	.00
27	0	1.0379	-8.700	281.00	75.50	.00	.00
28	0	1.0501	-3.170	206.00	27.60	.00	.00
29	0	1.0500	-.410	283.50	26.90	.00	.00
30	1	1.0475	-4.380	.00	.00	250.00	145.10
31	1	.9820	.000	9.20	4.60	563.30	205.50
32	1	.9831	1.790	.00	.00	650.00	205.70
33	1	.9972	2.290	.00	.00	632.00	109.10
34	1	1.0123	.850	.00	.00	508.00	167.00
35	1	1.0493	4.270	.00	.00	650.00	211.30
36	1	1.0635	6.960	.00	.00	560.00	100.50
37	1	1.0278	1.350	.00	.00	540.00	.70
38	1	1.0265	6.650	.00	.00	830.00	22.80
39	1	1.0300	-10.920	1104.00	250.00	1000.00	86.00

the choice is to minimize the system reactive power losses as the goal to demonstrate the model. Therefore, in the constraint set, those optional constraints are also neglected. (The detail mathematical formulation of this system see Appendix 2.).

The results of the O.R. analysis are similar to the previous problem too. Since the objective function is convex, it is positive definite. The constraint set does not form a convex set, because all of the constraints failed to show convexity or concavity, for instance constraint no. 31, its eigenvalues equal to -40.0, -40.0, 40.0, 40.0, it is neither semi-positive definite nor semi- negative definite, so it is not convex or concave. The similar results will happen to the other constraints.

Since the pre-analysis does not yield many special characters of the system, the five bus problem , a general and powerful non-linear programming code, has to be used, and a K-T point convexity or concavity check will be performed afterwards. The choice of programming code still is GRG2, and the results are in Table 3.10, 3.11, and 3.12.

This problem includes intermediate nodes, and from Table 3.12 it shows the multipliers are non-positive when $g(x) < 0$, the multipliers are non-negative when $g(x) > 0$, so rule (4) should be applied. The objective function has already been proved convex, hence it is pseudoconvex at the K-T point and satisfies the rule 3. A feasible point is

Table 3.10 resultant bus voltages in polar form

bus no.	magnitude	angle
1	1.080566	-8.1548
2	1.07746	-5.7335
3	1.06071	-8.43349
4	1.036155	-9.15705
5	1.037672	-8.02113
6	1.039851	-7.35731
7	1.029972	-9.42774
8	1.029198	-9.90412
9	1.062895	-9.76046
10	1.050386	-5.14108
11	1.045664	-5.89711
12	1.027795	-5.9166
13	1.047298	-5.81423
14	1.04399	-7.38933
15	1.045629	-7.80983
16	1.060452	-6.49299
17	1.063955	-7.44311
18	1.061498	-8.22489
19	1.073384	-2.06018
20	1.069121	-3.25178

(continue.)

21	1.058776	-4.22368
22	1.07422	0.0
23	1.070567	-0.20678
24	1.065601	-6.38534
25	1.087637	-4.52947
26	1.086807	-5.75489
27	1.070817	-7.61212
28	1.089483	-2.54372
29	1.09	0.0
30	1.092072	-3.52961
31	1.06978	0.00317
32	1.081517	1.43078
33	1.088563	2.30764
34	1.090237	1.19352
35	1.093429	4.53883
36	1.089268	7.2037
37	1.084616	1.56726
38	1.092929	6.19346
39	1.065036	-9.59073

Table 3.11 line flows

from bus i to bus j	real *100MW	reactive *100MVAR
1 - 2	-1.179095	-0.200546
1 - 39	1.179117	0.200689
2 - 1	1.183391	-0.562477
2 - 3	3.648416	0.815858
2 - 25	-2.331772	0.568304
2 - 30	-2.499969	-0.821699
3 - 2	-3.632467	-0.924591
3 - 4	0.723791	1.058176
3 - 18	-0.311249	-0.15687
4 - 3	-0.72157	-1.265182
4 - 5	-1.66536	-0.074239
4 - 14	-2.613258	-0.501568
5 - 4	1.667426	-0.036992
5 - 6	-4.844216	-0.492715
5 - 8	3.176741	0.530187
6 - 5	4.848617	0.503095
6 - 7	4.265453	0.853376
6 - 11	-3.413409	-0.477719
6 - 31	-5.700514	-0.878196
7 - 6	-4.254893	-0.812478
7 - 8	1.917262	-0.026879

(continue)

8 - 5	-3.168968	-0.579005
8 - 7	-1.915875	-0.039855
8 - 9	-0.135546	-1.148189
9 - 8	0.137532	0.763184
9 - 39	-0.137545	-0.763163
10 - 11	3.449878	0.814545
10 - 13	3.051015	0.447898
10 - 32	-6.500778	-1.261777
11 - 6	3.421057	0.416278
11 - 10	-3.445299	-0.845392
11 - 12	0.02417	0.428641
12 - 11	-0.023901	-0.42131
12 - 13	-0.061075	-0.458514
13 - 10	-3.047554	-0.490891
13 - 12	0.061399	0.467323
13 - 14	2.986093	0.023293
14 - 4	2.618482	0.436309
14 - 13	-2.978765	-0.129441
14 - 15	0.360292	-0.306816
15 - 14	-0.360059	-0.08991
15 - 16	-2.839769	-1.439335
16 - 15	2.847899	1.334614
16 - 17	2.05795	-0.637266
16 - 19	-4.524557	-0.328331
16 - 21	-3.270048	0.247432

(continue)

16 - 24	-0.405825	-0.942635
17 - 16	-2.055118	0.521861
17 - 18	1.893717	0.09524
17 - 27	0.161662	-0.615882
18 - 3	0.311345	-0.082691
18 - 17	-1.891482	-0.218027
19 - 16	4.553718	0.337678
19 - 20	1.742593	0.26113
19 - 33	-6.296221	-0.598024
20 - 19	-1.740706	-0.223947
20 - 34	-5.059337	-0.806389
21 - 16	3.277763	-0.403321
21 - 22	-6.017829	-0.747266
22 - 21	6.043933	0.912329
22 - 23	0.456154	0.274515
22 - 35	-6.500022	-1.185407
23 - 22	-0.455972	-0.483891
23 - 24	3.515585	-0.086615
23 - 36	-5.534654	-0.276233
24 - 16	0.406089	0.870969
24 - 23	-3.491833	0.052654
25 - 2	2.367126	-0.695973
25 - 26	0.778595	-0.344268
25 - 37	-5.385707	0.568506
26 - 25	-0.77695	-0.245524
26 - 27	2.658219	0.829176

(continue)

26 - 28	-1.389862	-0.356816
26 - 29	-1.881269	-0.396116
27 - 17	-0.161416	0.252744
27 - 26	-2.648726	-1.008376
28 - 26	1.396934	-0.489033
28 - 29	-3.456943	0.212964
29 - 26	1.898565	-0.633219
29 - 28	3.471193	-0.35497
29 - 38	-8.204756	0.66182
30 - 2	2.499969	0.929669
31 - 6	5.700514	1.647344
32 - 10	6.500778	2.0567
33 - 19	6.320523	1.091021
34 - 20	5.080004	1.219722
35 - 22	6.500022	1.726401
36 - 23	5.54805	1.005018
37 - 25	5.400583	0.006685
38 - 29	8.25038	0.227832
39 - 1	-1.177577	-1.025408
39 - 9	0.137568	-0.594682
The total reactive power losses are 84.9205 MVAR, real power losses are 38.789 MW		

Table 3.12 system status

constraint no.	system status	largrange multiplier
1 , 40	EQUALITY	NEGATIVE
2 , 41	EQUALITY	NEGATIVE
3 , 42	LOWERBND	POSITIVE
4 , 43	LOWERBND	POSITIVE
5 , 44	EQUALITY	NEGATIVE
6 , 45	EQUALITY	NEGATIVE
7 , 46	LOWERBND	POSITIVE
8 , 47	LOWERBND	POSITIVE
9 , 48	EQUALITY	NEGATIVE
10 , 49	EQUALITY	NEGATIVE
11 , 50	EQUALITY	NEGATIVE
12 , 51	LOWERBND	POSITIVE
13 , 52	EQUALITY	NEGATIVE
14 , 53	EQUALITY	NEGATIVE
15 , 54	LOWERBND	POSITIVE
16 , 55	LOWERBND	POSITIVE
17 , 56	EQUALITY	NEGATIVE
18 , 57	LOWERBND	POSITIVE
19 , 58	EQUALITY	NEGATIVE
20 , 59	LOWERBND	POSITIVE
21 , 60	LOWERBND	POSITIVE
22 , 61	EQUALITY	NEGATIVE
23 , 62	LOWERBND	POSITIVE

(continue)

24 , 63	LOWERBND	POSITIVE
25 , 64	LOWERBND	POSITIVE
26 , 65	LOWERBND	POSITIVE
27 , 66	LOWERBND	POSITIVE
28 , 67	LOWERBND	POSITIVE
29	LOWERBND	POSITIVE
30	UPPERBND	NEGATIVE
31	UPPERBND	NEGATIVE
32 , 71	UPPERBND	NEGATIVE
33 , 72	UPPERBND	NEGATIVE
34	UPPERBND	NEGATIVE
35	UPPERBND	NEGATIVE
36	FREE	0.0
37 , 76	UPPERBND	NEGATIVE
38	FREE	0.0
39	UPPERBND	NEGATIVE
68	FREE	0.0
69	FREE	0.0
70	FREE	0.0
73	FREE	0.0
74	FREE	0.0
75	UPPERBND	NEGATIVE
77	UPPERBND	NEGATIVE
78	LOWERBND	POSITIVE
79	EQUALITY	NEGATIVE
80	EQUALITY	NEGATIVE

** A : quasiconvex
B : quasiconcave

Table 3.13 quasiconvexity or quasiconcavity check at the K-T point.

1	2	3	4	5	6	7	8	9	10
.00000	.00000	3.22000	5.00000	.00000	-.00001	2.33800	5.22000	.00000	.00001
.00000	.00000	3.22012	5.00019	.00001	-.00001	2.33812	5.22031	.00000	.00000
.00000	.00000	3.22015	5.00022	.00001	-.00001	2.33814	5.22037	.00000	.00001
.00000	.00000	3.22012	5.00019	.00001	-.00001	2.33813	5.22030	.00000	.00001
.00000	.00001	3.22000	5.00000	.00001	-.00001	2.33800	5.22000	.00000	.00001
A,B	A,B	B	B	A,B	A,B	B	B	A,B	A

11	12	13	14	15	16	17	18	19	20
.00001	.08500	.00000	.00001	3.20000	3.29400	.00000	1.58000	.00000	6.80000
.00001	.08499	.00000	.00000	3.20008	3.29409	.00000	1.58005	.00000	6.80007
.00000	.08499	-.00001	.00000	3.20010	3.29411	.00000	1.58006	.00000	6.80008
.00000	.08499	-.00001	.00001	3.20008	3.29409	.00000	1.58005	.00000	6.80007
-.00001	.08499	-.00001	.00001	3.20000	3.29400	.00000	1.58000	.00000	6.80000
A	A,B	A,B	A	B	B	A,B	B	A,B	B

21	22	23	24	25	26	27	28	29	30
2.74000	.00000	2.47500	3.08600	2.24000	1.39000	2.81000	2.06000	2.83500	2.50000
2.74004	.00000	2.47503	3.08608	2.24008	1.39005	2.81009	2.06008	2.83511	2.49989
2.74005	.00000	2.47503	3.08610	2.24010	1.39006	2.81011	2.06009	2.83513	2.49987
2.74004	.00000	2.47503	3.08609	2.24008	1.39005	2.81008	2.06008	2.83510	2.49989
2.74000	.00000	2.47500	3.08600	2.24000	1.39000	2.81000	2.06000	2.83500	2.50000
B	A,B	B	B	B	B	B	B	B	A

31	32	33	34	35	36	37	38	39	40
5.70050	6.50080	6.32050	5.08000	6.50000	5.54806	5.40060	8.25039	1.04000	.00000
5.70038	6.50078	6.32050	5.07993	6.49539	5.56378	5.40057	8.24356	1.04001	-.00001
5.70028	6.50075	6.32048	5.07992	6.49230	5.57420	5.40055	8.23889	1.04001	-.00001
5.70018	6.50053	6.32035	5.07993	6.48919	5.58456	5.40039	8.23413	1.04001	.00000
5.70000	6.50000	6.32001	5.08000	6.48452	5.60000	5.40000	8.22681	1.04000	.00000
A,B	A,B	A,B	A	A,B	A,B	A,B	A,B	B	A

41	42	43	44	45	46	47	48	49	50
.00001	.02399	1.83998	.00000	-.00002	.83999	1.76602	.00000	.00002	.00003
.00001	.02406	1.84019	.00006	-.00005	.84011	1.76633	.00001	.00001	.00003
.00000	.02409	1.84022	.00001	-.00003	.84015	1.76636	.00001	.00000	.00003
-.00001	.02405	1.84021	.00005	-.00002	.84013	1.76627	.00000	.00000	-.00003
.00000	.02401	1.84002	-.00002	.00000	.83997	1.76600	.00001	.00003	-.00003
A	B	B	A	A	B	B	A,B	A	A,B
51	52	53	54	55	56	57	58	59	60
.88000	.00000	.00003	1.53001	.32300	.00001	.30001	-.00002	1.03001	1.14999
.88005	-.00003	.00002	1.53010	.32305	.00000	.30002	-.00003	1.03001	1.15003
.88005	-.00007	.00002	1.53011	.32306	-.00001	.30004	-.00001	1.03004	1.15003
.88004	-.00003	.00001	1.53011	.32306	.00000	.30003	.00000	1.03005	1.15002
.88000	-.00004	.00002	1.52999	.32300	.00004	.30000	.00002	1.03000	1.15000
B	A	A	B	B	A	B	A	B	B
61	62	63	64	65	66	67	68	69	70
.00000	.84599	-.92201	.47201	.17001	.75501	.27601	.32642	.92978	1.64734
.00000	.84600	-.92201	.47203	.17002	.75506	.27603	.30906	1.08673	1.78361
-.00001	.84600	-.92198	.47203	.17003	.75509	.27603	.29753	1.19109	1.87424
-.00002	.84600	-.92197	.47203	.17004	.75507	.27602	.28607	1.29522	1.96468
-.00002	.84599	-.92200	.47200	.17001	.75500	.27600	.26899	1.45100	2.10000
A,B	B	B	B	B	B	B	A,B	A,B	A,B
71	72	73	74	75	76	77	78	79	80
2.05699	1.09101	1.21973	1.72670	1.00500	.00699	.22800	1.61999	.00006	.00016
2.05699	.94952	1.35481	1.84270	1.00491	.00694	.22790	1.83360	-.00663	-.00136
2.05700	.85604	1.44487	1.91998	1.00490	.00693	.22788	1.97283	-.00788	-.00160
2.05700	.76323	1.53493	1.99722	1.00491	.00695	.22790	2.10952	-.00666	-.00128
2.05700	.62531	1.67001	2.11301	1.00500	.00700	.22800	2.30978	.00000	.00015
A,B	A,B	A,B	A,B	A	A	A	A,B	A	A

(Continue)

randomly selected, so that the quasiconvexity or quasiconcavity of each constraint at the K-T point can be verified. The results are in the Table 3.13.

Since the objective function satisfies rule 3 , and all the constraints are differentiable and satisfy rule 3, so the K-T point is a global optimal solution.

<u>Comparisons of the model</u>

(a) Nonlinear (GRG2) method vs linearized method

Since this research focuses on non-linear programming, comparisons of the accuracy and complexity between the GRG2 and a linearized method are worthwhile to consider. In this linearized method, real and reactive power flow are also treated as variables; a separable programming algorithm is introduced to linearize the model, and a very powerful linear package called MPSX is used to execute the model. The results of the comparisons of the five bus system are the following.

Table 3.14 structure and operation comparisons.

	linear	G.R.G
Zloss	0.16045	0.126817
Time	6.76sec	0.34sec
Iteration	360	20
Constraint	133	10
Variables	396	10

Table 3.15 bus voltage

	linear	G.R.G
1	1.1 + j0	1.1 + j0
2	1.08637-j0.04967	1.08693-j0.049332
3	1.06108-j0.0863	1.06287-j0.08616
4	1.06-j0.09197	1.06183-j0.0918
5	1.05374-j0.10551	1.05506-j0.105246

The reasons that the executing time of the non-linear model is faster than the linearized model are two: (1) The non-linear model only consists of voltage variables, after the program reaches the optimal solution, then the voltage variables will be substituted into the power system equations to calculate the real and reactive power flow. All these post optimal calculation takes only one iteration, thus it is much faster than allowing put real and reactive power flows as variables in the model. (2) Not including separable programming technique, this model has less constraints and variables, therefore, it is faster.

Also comparing the losses from Table 3.14, the results from the model is more accurated and realistic, because this technique does not linearize the network.

(b) This model vs Gauss-Siedel method

Most of the existing load flow packages have used either Gauss-Siedel or Newton- Raphson or their revised versions; however, in several contingent situations the accuracy and ability of the conventional load flow methods are very limited. These problems could be overcome if this model were used.

Following are several comparisons of the accuracy of the results between this model and the conventional method [18] under different contingencies. In order to have a justified comparison, the swing bus voltage is alway assigned to be the same bus voltage of the GRG2 method.

(1) Comparison of the results under normal situation

Table 3.16 results of the five bus system by Gauss-Siedel method

no	magnitude	angle
1	1.09	0.0
2	1.08	-2.7
3	1.058	-4.7
4	1.058	-5.1
5	1.052	-5.8
The system reactive power losses are 12.94 MVAR		

After comparing Table 3.4 with table 3.16, and table 3.9 with table 3.17, this model clearly shows several advantages. (1) The system reactive losses are less in both

Table 3.17 the results of the 39 bus system by Gauss-Siedel method

no	magnitude	angle
1	1.069	-8.3
2	1.062	-5.9
3	1.054	-8.6
4	1.047	-9.4
5	1.059	-8.4
6	1.064	-7.7
7	1.052	-9.7
8	1.05	-10.2
9	1.066	-10.0
10	1.07	-5.6
11	1.067	-6.3
12	1.054	-6.3
13	1.064	-6.2
14	1.054	-7.7
15	1.04	-8.0
16	1.048	-6.6
17	1.05	-7.6
18	1.05	-8.4
19	1.055	-2.0
20	0.994	-3.4
21	1.043	-4.3

(continue)

22	1.056	0.1
23	1.051	-0.1
24	1.052	-6.5
25	1.068	-4.4
26	1.063	-5.7
27	1.051	-7.7
28	1.056	-2.2
29	1.054	0.5
30	1.053	-3.5
31	1.07	0.0
32	1.04	1.6
33	0.997	3.2
34	1.012	1.8
35	1.049	5.1
36	1.064	7.8
37	1.027	2.3
38	1.027	7.6
39	1.056	-9.8
The system reactive losses are 933.51 MVAR		

cases, because the model has an objective function which is minimizing the reactive losses, and there is no way to put such a function on the conventional load flow methods. (2) There is no swing bus assigned in the model, all bus voltages are given a range initially, and from that point on, the model will generate the optimal bus voltage data for the whole system; thus this is more flexible than most conventional methods. (3) In most conventional methods, unless the choice of the swing bus is better or additional required transformer tap changing is installed, the ability for the conventional methods to reach an optimal solution is greatly reduced; however, the effect of the tap changing, because of the objective function and flexibility, is automatically included in this model.

(2) Comparison of results under loss of generation

Assume generator bus 31 of the 39 bus system suddenly loses 195MW of its capacities. In the conventional methods, the losses will be supplied by the swing bus. In contrast, this model,using minimal cut off real load as an objective function, will show the minimal amount of the load to be cut and where to cut them. The results of this contingent situation are shown in Appendix 3, and it is found that bus7 and bus 12 have to cut some amount of their real load in this model in order to meet the crisis, this gives a more realistic result.

(3) Comparison of results under loss of a line

Assume line 4-14 of the 39 bus system is cut off. In the conventional methods, all the additional load will be picked up by the swing bus; however, this model being more realistic, uses the same technique as (2), and locates the minimal amount of the load that needed to be cut. The results of this contingent situation are in Appendix 4, it shows that in the model the bus 7 has to cut some of its real power load or bus 23, 28, and 29 have to cut some amount of their reactive load, in order to save the system.

Various capabilities of the model

In additional to loss of a line and loss of generation, this model can also deal with various situations. They are (1) minimal real power losses, (2) bus fault, (3) line fault, (4) voltage regulation. They are discussed in the following paragraphs:

(1) Minimal real power losses

While the procedure for this is similar to minimizing reactive power losses, the objective function is different. The results of the same 39 bus system under the object of minimizing real losses is shown in Appendix 5, so that the comparisons can be made against the results of minimal reactive losses (see Table 3.10 and 3.11) and the results of the Gauss-siedel method(see Table 3.17). The results of minimizing real power losses are almost the same as minimizing reactive power losses; the reactive losses of the former is slightly larger than the latter, the real losses of the former is slightly smaller than the latter. So minimizing the real losses will also minimize the reactive losses and vise versa. Since it has already demonstrated that the results of minimal reactive power losses are more accurate than the results of Gauss-Siedel method in the previous section, the results of minimizing the real power losses are more accurate than the results of Gauss-Siedel method too.

(2) Bus fault

Assume bus no. 3 of the five bus system is faulted, the fault impedance is 1.0+j0.0 ohms. This model will be able to locate minimal load which should be cut off. In addition, it will lower the bus voltages to meet the crisis. The results are in Table 3.18, 3.19 and 3.20.

(3) Line fault

Assume line 1 - 2 of the five bus system is shorted in the middle, this model is able to do the same things as in the bus fault, and the results are in Table 3.21, 3.22 and 3.23.

(4) Voltage regulation

Chapter three states that voltage regulation is an unique optional constraint set which conventional methods have no ability to include. It also stated that with this constraint set installed, they will either be redundant or increase the total system losses or the results sometimes will be infeasible. The five bus example illustrates the regulation of the voltage drop of the line 2 - 5.

Table 3.18 line flow and system losses under bus fault, zf=1.0+j0.0

From bus i to bus j	real x100MW	reactive x100MVAR
1 - 2	0.994614	0.021611
1 - 3	0.505398	0.078387
2 - 1	-0.960231	0.048089
2 - 3	0.340444	0.060727
2 - 4	0.33495	0.048674
2 - 5	0.484843	0.042247
3 - 1	-0.468746	0.005158
3 - 2	-0.326966	-0.040672
3 - 4	-0.038573	-0.069161
4 - 2	-0.322061	-0.03048
4 - 3	0.03869	0.059862
4 - 5	-0.01032	-0.029381
5 - 2	-0.467199	-0.005335
5 - 4	0.010386	0.005089
The system reactive power losses are 34.569MVAR, real losses are 58.759MW		

Table 3.19 bus voltage under bus fault, zf=1.0+j0.0

no	magnitude	angle
1	0.759147	0.0
2	0.733981	-6.0678
3	0.69298	-12.50099
4	0.696323	-12.44001
5	0.703459	-12.31182

Table 3.20 the cut demand on every load bus under the bus fault

bus no.	real	reactive
3	9.60455	4.53642
4	10.6264	5.0
5	14.3185	10.0
total	34.5494	19.53642

Table 3.21 line flow and system losses under the line 1 - 2 fault.

From bus i to bus j	real x100MW	reactive x100MVAR
1 - 3	0.021291	0.004077
2 - 3	0.037977	0.020104
2 - 4	0.039229	0.020151
2 - 5	0.056387	0.026394
3 - 1	-0.018346	0.004194
3 - 2	-0.031721	-0.001888
3 - 4	0.005091	-0.002268
4 - 2	-0.032642	-0.000942
4 - 3	-0.00506	0.002169
4 - 5	-0.00148	-0.001195
5 - 2	-0.047668	-0.000674
5 - 4	0.001506	0.000768
The system reactive power losses are 42.177MVAR, real losses are 14.059MW		

Table 3.22 bus voltage under line 1 - 2 fault

no	magnitude	angle
1	0.113317	0.0
2	0.133600	-0.00733
3	0.098382	-25.26415
4	0.098542	-26.2952
5	0.102102	-24.70547

Table 3.23 the cut demand on every load bus under line 1 - 2 fault.

bus no.	real	reactive
3	40.5012	15.0
4	36.0802	5.0
5	55.3783	10.0
total	131.9587	30.0

Originally the percentage of the voltage regulation of that line is 6.19% at bus 5 end and 6.03% at bus 2 end (see Table 4.3). If the factors are more than 6.19 %, then these constraints are redundant. If the factors are assigned to 6%, the system will not be convergent, because 6% voltage regulation on line 2 - 5 at bus 5 end will lower bus 2 voltage, which creates extra power flow from bus 1 to bus 2. Since line 2 - 5 is regulated, then this extra power can not be supplied to load bus 5, therefore the demand on bus 5 can not be met. If the factor at the bus 2 end is 6% and at the bus 5 end is 6.16%, the system will converge; the voltage drop of line 2 - 5 will be regulated, yet the total system losses are increased. The results are shown in the Table 3.24 and table 3.25. After comparing these Tables with table 3.4 and table 3.5, it can be seen that when the voltage drop of the line 2 - 5 is regulated, the total system losses are higher.

Table 3.24 bus voltage under voltage regulation on line 2 - 5

no	magnitude	angle
1	1.09	0.0
2	1.075059	-3.16454
3	1.053494	-5.11431
4	1.052809	-5.45559
5	1.046958	-6.30258

Table 3.25 line flow and system losses under voltage regulation on line 2 - 5.

From bus i to bus j	real x100MW	reactive x100MVAR
1 - 2	1.060688	-0.088002
1 - 3	0.439318	0.008705
2 - 1	-1.041703	0.074642
2 - 3	0.232401	0.031861
2 - 4	0.267604	0.025601
2 - 5	0.541851	0.06729
3 - 1	-0.426223	-0.026869
3 - 2	-0.22944	-0.068291
3 - 4	0.20561	-0.054925
4 - 2	-0.263763	-0.059361
4 - 3	-0.20521	0.03394
4 - 5	0.068952	-0.024524
5 - 2	-0.531439	-0.070956
5 - 4	-0.068608	-0.029557
Total system reactive power losses are 15.0109MVAR		

Summary of the comparisons

In this chapter, two example problems were presented and solved. Accuracy, efficiency and capability comparisons have been made from the results of this model verses some existing models iincluding both network solution and O.R. solution. The comparisons show this model is superior to most existing ones in many ways.

(1) Normal situation

Comparing to some O.R. solutions, the results of this model show less losses and higher efficiency (see Table 3.14). Comparing to the network solution, the results of this model show that the bus voltage value is very closed to the network solution, but the losses are less (see Table 3.26, and 3.27)

Table 3.26 the differences of the results of the five bus system and network solution.

bus no.	voltage different (%)
1	0.0
2	0.1837037
3	0.1968756
4	0.2503733
5	0.205806
Difference of the system reactive power losses are .0077237 %	

(2) Contingency situation

The results of this model show that it is capable of handling various contingency situations, which most existing models have difficulties to deal with.

Table 3.27 the differences of the results of the 39 bus system and network solution.

bus no.	voltage difference (%)
1	1.0819497
2	1.4826739
3	0.6366161
4	1.0358229
5	2.0139728
6	2.2696486
7	2.0939248
8	1.9811336
9	.2912688
10	1.8330934
11	1.9996325
12	2.486249
13	1.5697422
14	.9497157
15	.5412561
16	1.1881685

(continue)

17	1.3290474
18	1.0950567
19	1.7425689
20	7.5574427
21	1.5125636
22	1.7253737
23	1.861752
24	1.2928654
25	1.8386694
26	2.2396095
27	1.885537
28	3.1707413
29	3.4155612
30	3.7105448
31	.0205664
32	3.9920211
33	9.1838503
34	7.7309346
35	4.2353616
36	2.3748081
37	5.6101265
38	6.4195776
39	.8556879
Difference of the system reactive power losses are 9.0309677 %	

CHAPTER FOUR

SUMMARY AND CONCLUSION

In the beginning chapter, this research has shown the needs for a new optimal load flow model, then this new model, based on the electrical and socialogical concerns, was carefully developed in the following chapter. It showed that the new O.R. model does not linearize the electrical phenonmeon, and it only contains bus voltage variables which ties the real power flow and the reactive power flow together. In the third chapter, this new model was implemented by two examples to prove its wide range of capabilities. The detail results and various comparisons with existing model shown in the same chapter.

This new model demonstrated its effectiveness and potential for load flow studies. It differs from most conventional methods by not having a swing bus and the tap changing effect is included automatically, which leads to a better and more accurated solution. This model also proved that it is able to do various things that most conventional methods can not do or are very limited in doinq. For instance:

(1) Loss of line - this model locates the minimal amount of load which is to be cut off, but most conventional

methods, because of the swing bus, are not able to do this.

(2) Loss of generation - the model handles this situation similar to loss of a line.

(3) Bus fault - the swing bus prohibits most conventional methods to search for minimal load cut, but this model is able to do that.

(4) Line fault - this model locates the minimal amount of load which is to be cut off and lowers the system voltage automatically.

(5) Voltage regulation -most conventional methods are not able to do it at all, but in this model voltage regulation can be installed optionally to meet a line specific requirement. This will reduce the voltage drop and losses of that particular line, but it will not improve the over all system losses.

Furthermore, this model is capable of doing things which are not illustrated in this research, e.g. load expansion, line capacity limitation, and minimal cost of the power losses.

As a non-linear model, this model used the GRG2 nonlinear package to execute the program. The objective function is a convex function, but the constraint set does not form a convex set, so the result is not necessarily the global optimum. Therefore; a point quasiconvexity or quasiconcavity was introduced in chapter three in order to check the optimality of the solution. Since this is a primal type of problem, the solution will always be a feasible one, even though the global optimum might not be reached in some cases.

Future studies

Because of non-linearity, the only disadvantage of this model is that the execution time is slower than the conventional method. As a consequence, a better non-linear algorithm would help solve the time problem, and a better non-linear algorithm will also help to search for the global optimum.

If the zero-one integer variables were included, this model could be expanded as a system planning tool. It would be able to give the line flow and bus voltage data; however, an additional advantage of this model would be to indicate where to build more lines and substations in order to meet the forecast.

Another area this model might be able to break into is stability, because if the governing equations in the stability study can be transformed into O.R. type constraints, then the stability problem can be solved by operational research techniques.

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APPENDIX ONE

Computer code for the calculation of the P and Q.

```
c     ieee 39 buses system (feasible initial solution, slack bus is 31)
      dimension pf(39,39),qf(39,39),vb(39),ang(39),rl(39,39),
     1xl(39,39),ca(39,39),x(78)
      a=0.
      do 101 m=1,78
      read(5,339)x(m)
  101 continue
      read(5,340)nn,d1,d2,vr,zloss
      do 131 i=1,39
      do 132 j=1,39
      rl(i,j)=0.
      xl(i,j)=0.
      ca(i,j)=0.
  132 continue
  131 continue
      write(6,331)
      write(6,341)nn,d1,d2,vr,zloss
      do 133 i=1,39
      read(5,332)kk,kl,km,kn,kj
      ni=i+39
      write(6,333)
      write(6,336)
      do 134 j=1,39
      nj=j+39
      if(j.eq.kk.or.j.eq.kl.or.j.eq.km.or.j.eq.kn.or.j.eq.kj)go to 431
      go to 134
  431 if(j.gt.i)go to 432
      rl(i,j)=rl(j,i)
      xl(i,j)=xl(j,i)
      ca(i,j)=ca(j,i)
      go to 433
  432 read(5,334)rl(i,j),xl(i,j),ca(i,j)
      a=a+rl(i,j)*(x(i)**2+x(j)**2+x(ni)**2+x(nj)**2-2.*x(i)*x(j)-2.*
     1x(ni)*x(nj))
  433 pf(i,j)=xl(i,j)*(x(ni)*x(j)-x(i)*x(nj))+rl(i,j)*(x(i)**2+x(ni)**2
     1        -x(i)*x(j)-x(ni)*x(nj))
      qf(i,j)=xl(i,j)*(x(i)**2+x(ni)**2-x(i)*x(j)-x(ni)*x(nj))-rl(i,j)
     1        *(x(ni)*x(j)-x(i)*x(nj))-ca(i,j)*(x(i)**2+x(ni)**2)
      write(6,335)i,j,pf(i,j),qf(i,j)
  134 continue
  133 continue
      write(6,333)
      write(6,342)a
      write(6,333)
      write(6,337)
      do 135 k=1,39
      nk=k+39
      vb(k)=sqrt(x(k)**2+x(nk)**2)
      ang(k)=57.29578*atan(x(nk)/x(k))
      write(6,338)k,vb(k),ang(k)
  135 continue
  331 format(///1x,'***************************************************
     1***********'/1x,'the final report of epri 39 buses system')
  332 format(5i5)
  333 format(/1x,'*****************************************************
     1**********')
  334 format(3f15.5)
  336 format(/1x,'the line flow report')
```

```
335 format(/1x,'from bus',2x,i2,2x,'to bus',2x,i2,3x,'real power'
   1       ,f11.6,3x,'reactive power',f11.6)
337 format(/1x,'the bus voltage report')
338 format(/1x,'the bus no.',2x,i2,3x,'magnitude',1x,f10.6,3x,
   1       'angle',1x,f10.5)
339 format(f12.7)
340 format(i5,4f10.5)
341 format(/1x,'the slack bus is',2x,i2/1x,'real power limit',1x,f10.5
   1/1x,'reactive power limit',1x,f10.5/1x,'real part of the bus volta
   2ge is less than',2x,f6.3/1x,'************************************
   3****************************'//1x,'the optimal value of the total
   4 reactive power losses under the stated condition is',2x,f10.5)
342 format(/1x,'the associated system real power losses',2x,f10.5)
    stop
    end
```

APPENDIX TWO

Mathematical formulation of the IEEE 39 bus system for GRG2 technique.

```
c     ieee 39 buses system (feasible initial solution, slack bus is 31)
c     reactive supply greater than demand
      real*8 z(20000)
      data ncore/20000/
      call grg(z,ncore)
      stop
      end
c
c
      subroutine gcomp(g,x)
c
      real*8 g(1),x(1)
c
c     constraints
c
      g(1)=24.15573*(x(40)*x(2)-x(1)*x(41))+2.05706*(x(1)**2+x(40)**2
     1    -x(1)*x(2)-x(40)*x(41))+39.9361*(x(40)*x(39)-x(1)*x(78))
     2    +1.59744*(x(1)**2+x(40)**2-x(1)*x(39)-x(40)*x(78))
      g(2)=24.15573*(x(41)*x(1)-x(2)*x(40))+2.05706*(x(2)**2+x(41)**2
     1    -x(1)*x(2)-x(40)*x(41))+65.73792*(x(41)*x(3)-x(2)*x(42))
     2    +5.65956*(x(2)**2+x(41)**2-x(2)*x(3)-x(41)*x(42))
     3    +69.94144*(x(41)*x(25)-x(2)*x(64))+56.92908*(x(2)**2+x(41)**2
     4    -x(2)*x(25)-x(41)*x(64))+55.24862*(x(41)*x(30)-x(2)*x(69))
      g(3)=-65.73792*(x(42)*x(2)-x(3)*x(41))-5.65956*(x(3)**2+x(42)**2
     1    -x(3)*x(2)-x(42)*x(41))-46.77412*(x(42)*x(4)-x(3)*x(43))
     2    -2.85476*(x(3)**2+x(42)**2-x(3)*x(4)-x(42)*x(43))-74.67715
     3    *(x(42)*x(18)-x(3)*x(57))-6.17631*(x(3)**2+x(42)**2-x(3)
     4    *x(18)-x(42)*x(57))
      g(4)=-46.77412*(x(43)*x(3)-x(4)*x(42))-2.85476*(x(4)**2+x(43)**2
     1    -x(4)*x(3)-x(43)*x(42))-77.82101*(x(43)*x(5)-x(4)*x(44))
     2    -4.86381*(x(4)**2+x(43)**2-x(4)*x(5)-x(43)*x(44))-77.22239
     3    *(x(43)*x(14)-x(4)*x(53))-4.78899*(x(4)**2+x(43)**2-x(4)
     4    *x(14)-x(43)*x(53))
      g(5)=77.82101*(x(44)*x(4)-x(5)*x(43))+4.86381*(x(5)**2+x(44)**2
     1    -x(5)*x(4)-x(44)*x(43))+382.353*(x(44)*x(6)-x(5)*x(45))
     2    +29.41177*(x(5)**2+x(44)**2-x(5)*x(6)-x(44)*x(45))+88.83249
     3    *(x(44)*x(8)-x(5)*x(47))+6.34518*(x(5)**2+x(44)**2-x(5)*x(8)
     4    -x(44)*x(47))
      g(6)=382.353*(x(45)*x(5)-x(6)*x(44))+29.41177*(x(6)**2+x(45)**2
     1    -x(6)*x(5)-x(45)*x(44))+108.2353*(x(45)*x(7)-x(6)*x(46))
     2    +7.05882*(x(6)**2+x(45)**2-x(6)*x(7)-x(45)*x(46))+121.069
     3    *(x(45)*x(11)-x(6)*x(50))+10.33515*(x(6)**2+x(45)**2-x(6)
     4    *x(11)-x(45)*x(50))+40*(x(45)*x(31)-x(6)*x(70))
      g(7)=-108.2353*(x(46)*x(6)-x(7)*x(45))-7.05882*(x(7)**2+x(46)**2
     1    -x(7)*x(6)-x(46)*x(45))-215.7599*(x(46)*x(8)-x(7)*x(47))
     2    -18.76173*(x(7)**2+x(46)**2-x(7)*x(8)-x(46)*x(47))
      g(8)=-88.83249*(x(47)*x(5)-x(8)*x(44))-6.34518*(x(8)**2+x(47)**2
     1    -x(8)*x(5)-x(47)*x(44))-215.7599*(x(47)*x(7)-x(8)*x(46))
     2    -18.76173*(x(8)**2+x(47)**2-x(8)*x(7)-x(47)*x(46))-27.43806
     3    *(x(47)*x(9)-x(8)*x(48))-1.7385*(x(8)**2+x(47)**2-x(8)*x(9)
     4    -x(47)*x(48))
      g(9)=27:43806*(x(48)*x(8)-x(9)*x(47))+1.7385*(x(9)**2+x(48)**2
     1    -x(9)*x(8)-x(48)*x(47))+39.9361*(x(48)*x(39)-x(9)*x(78))
     2    +1.59744*(x(9)**2+x(48)**2-x(9)*x(39)-x(48)*x(78))
      g(10)=230.563*(x(49)*x(11)-x(10)*x(50))+21.44772*(x(10)**2
     1     +x(49)**2-x(10)*x(11)-x(49)*x(50))+230.563*(x(49)*x(13)-x(10)
     2     *x(52))+21.44772*(x(10)**2+x(49)**2-x(10)*x(13)-x(49)*x(52))
     3     +50*(x(49)*x(32)-x(10)*x(71))
```

```
 g(11)=121.069*(x(50)*x(6)-x(11)*x(45))+10.33515*(x(11)**2+x(50)**2
1     -x(11)*x(6)-x(50)*x(45))+230.563*(x(50)*x(10)-x(11)*x(49))
2     +21.44772*(x(11)**2+x(50)**2-x(11)*x(10)-x(50)*x(49))
3     +22.95745*(x(50)*x(12)-x(11)*x(51))+0.84441*(x(11)**2+x(50)
4     **2-x(11)*x(12)-x(50)*x(51))
 g(12)=-22.95745*(x(51)*x(11)-x(12)*x(50))-0.84441*(x(12)**2+x(51)
1     **2-x(12)*x(11)-x(51)*x(50))-22.95745*(x(51)*x(13)-x(12)
2     *x(52))-0.84441*(x(12)**2+x(51)**2-x(12)*x(13)-x(51)*x(52))
 g(13)=230.563*(x(52)*x(10)-x(13)*x(49))+21.44772*(x(13)**2+x(52)
1     **2-x(13)*x(10)-x(52)*x(49))+22.95745*(x(52)*x(12)-x(13)
2     *x(51))+0.84441*(x(13)**2+x(52)**2-x(13)*x(12)-x(52)*x(51))
3     +98.22992*(x(52)*x(14)-x(13)*x(53))+8.75316*(x(13)**2+x(52)
4     **2-x(13)*x(14)-x(52)*x(53))
 g(14)=77.22239*(x(53)*x(4)-x(14)*x(43))+4.78899*(x(14)**2+x(53)
1     **2-x(14)*x(4)-x(53)*x(43))+98.22992*(x(53)*x(13)-x(14)
2     *x(52))+8.75316*(x(14)**2+x(53)**2-x(14)*x(13)-x(53)*x(52))
3     +45.76804*(x(53)*x(15)-x(14)*x(54))+3.79643*(x(14)**2+x(53)
4     **2-x(14)*x(15)-x(53)*x(54))
 g(15)=-45.76804*(x(54)*x(14)-x(15)*x(53))-3.79643*(x(15)**2+x(54)
1     **2-x(15)*x(14)-x(54)*x(53))-105.4166*(x(54)*x(16)-x(15)
2     *x(55))-10.09308*(x(15)**2+x(54)**2-x(15)*x(16)-x(54)*x(55))
 g(16)=-105.4166*(x(55)*x(15)-x(54)*x(16))-10.09308*(x(16)**2
1     +x(55)**2-x(16)*x(15)-x(55)*x(54))-111.6688*(x(55)*x(17)
2     -x(16)*x(56))-8.78294*(x(16)**2+x(55)**2-x(16)*x(17)-x(55)
3     *x(56))-50.93911*(x(55)*x(19)-x(16)*x(58))-4.17962*(x(16)
4     **2+x(55)**2-x(16)*x(19)-x(55)*x(58))-73.81486*(x(55)*x(21)
5     -x(16)*x(60))-4.37421*(x(16)**2+x(55)**2-x(16)*x(21)-x(55)
6     *x(60))-169.0544*(x(55)*x(24)-x(16)*x(63))-8.59599*(x(16)
7     **2+x(55)**2-x(16)*x(24)-x(55)*x(63))
 g(17)=111.6688*(x(56)*x(16)-x(17)*x(55))+8.78294*(x(17)**2+x(56)
1     **2-x(17)*x(16)-x(56)*x(55))+121.069*(x(56)*x(18)-x(17)
2     *x(57))+10.33515*(x(17)**2+x(56)**2-x(17)*x(18)-x(56)*x(57))
3     +57.4789*(x(56)*x(27)-x(17)*x(66))+4.31922*(x(17)**2+x(56)
4     **2-x(17)*x(27)-x(56)*x(66))
 g(18)=-74.67715*(x(57)*x(3)-x(18)*x(42))-6.17631*(x(18)**2+x(57)
1     **2-x(18)*x(3)-x(57)*x(42))-121.069*(x(57)*x(17)-x(18)*x(56))
2     -10.33515*(x(18)**2+x(57)**2-x(18)*x(17)-x(57)*x(56))
 g(19)=50.93911*(x(58)*x(16)-x(19)*x(55))+4.17962*(x(19)**2+x(58)
1     **2-x(19)*x(16)-x(58)*x(55))+72.2778*(x(58)*x(20)-x(19)
2     *x(59))+3.66627*(x(19)**2+x(58)**2-x(19)*x(20)-x(58)*x(59))
3     +70.25182*(x(58)*x(33)-x(19)*x(72))+3.46312*(x(19)**2+x(58)
4     **2-x(19)*x(33)-x(58)*x(72))
 g(20)=-72.2778*(x(59)*x(19)-x(20)*x(58))-3.66627*(x(20)**2+x(59)
1     **2-x(20)*x(19)-x(59)*x(58))-55.41701*(x(59)*x(34)-x(20)
2     *x(73))-2.77085*(x(20)**2+x(59)**2-x(20)*x(34)-x(59)*x(73))
 g(21)=-73.81486*(x(60)*x(16)-x(21)*x(55))-4.37421*(x(21)**2+x(60)
1     **2-x(21)*x(16)-x(60)*x(55))-71.19609*(x(60)*x(22)-x(21)
2     *x(61))-4.06835*(x(21)**2+x(60)**2-x(21)*x(22)-x(60)*x(61))
 g(22)=71.19609*(x(61)*x(21)-x(22)*x(60))+4.06835*(x(22)**2+x(61)
1     **2-x(22)*x(21)-x(61)*x(60))+103.7614*(x(61)*x(23)-x(22)
2     *x(62))+6.48508*(x(22)**2+x(61)**2-x(22)*x(23)-x(61)*x(62))
3     +69.93007*(x(61)*x(35)-x(22)*x(74))
 g(23)=-103.7614*(x(62)*x(22)-x(23)*x(61))-6.48508*(x(23)**2+x(62)
1     **2-x(23)*x(22)-x(62)*x(61))-28.45899*(x(62)*x(24)-x(23)
2     *x(63))-1.78885*(x(23)**2+x(62)**2-x(23)*x(24)-x(62)*x(63))
3     -36.75229*(x(62)*x(36)-x(23)*x(75))-0.67559*(x(23)**2+x(62)
4     **2-x(23)*x(36)-x(62)*x(75))
```

```
 g(24)=-169.0544*(x(63)*x(16)-x(24)*x(55))-8.59599*(x(24)**2+x(63)
1     **2-x(24)*x(16)-x(63)*x(55))-28.45899*(x(63)*x(23)-x(24)
2     *x(62))-1.78885*(x(24)**2+x(63)**2-x(24)*x(23)-x(63)*x(62))
 g(25)=-69.94144*(x(64)*x(2)-x(25)*x(41))-56.92908*(x(25)**2+x(64)
1     **2-x(25)*x(2)-x(64)*x(41))-30.65883*(x(64)*x(26)-x(25)
2     *x(65))-3.03741*(x(25)**2+x(64)**2-x(25)*x(26)-x(64)*x(65))
3     -43.07464*(x(64)*x(37)-x(25)*x(76))-1.114*(x(25)**2+x(64)
4     **2-x(25)*x(37)-x(64)*x(76))
 g(26)=-30.65883*(x(65)*x(25)-x(26)*x(64))-3.03741*(x(26)**2+x(65)
1     **2-x(26)*x(25)-x(65)*x(64))-67.41573*(x(65)*x(27)-x(26)
2     *x(66))-6.42055*(x(26)**2+x(65)**2-x(26)*x(27)-x(65)*x(66))
3     -20.92484*(x(65)*x(28)-x(26)*x(67))-1.89825*(x(26)**2+x(65)
4     **2-x(26)*x(28)-x(65)*x(67))-15.86802*(x(65)*x(29)-x(26)
5     *x(68))-1.44716*(x(26)**2+x(65)**2-x(26)*x(29)-x(65)*x(68))
 g(27)=-57.4789*(x(66)*x(17)-x(27)*x(56))-4.31922*(x(27)**2+x(66)
1     **2-x(27)*x(17)-x(66)*x(56))-67.41573*(x(66)*x(26)-x(27)
2     *x(65))-6.42055*(x(27)**2+x(66)**2-x(27)*x(26)-x(66)*x(65))
 g(28)=-20.92484*(x(67)*x(26)-x(28)*x(65))-1.89825*(x(28)**2+x(67)
1     **2-x(28)*x(26)-x(67)*x(65))-65.66074*(x(67)*x(29)-x(28)
2     *x(68))-6.08775*(x(28)**2+x(67)**2-x(28)*x(29)-x(67)*x(68))
 g(29)=-15.86802*(x(68)*x(26)-x(29)*x(65))-1.44716*(x(29)**2+x(68)
1     **2-x(29)*x(26)-x(68)*x(65))-65.66074*(x(68)*x(28)-x(29)
2     *x(67))-6.08775*(x(29)**2+x(68)**2-x(29)*x(28)-x(68)*x(67))
3     -63.93443*(x(68)*x(38)-x(29)*x(77))-3.27869*(x(29)**2+x(68)
4     **2-x(29)*x(38)-x(68)*x(77))
 g(30)=55.24862*(x(69)*x(2)-x(30)*x(41))
 g(31)=40.*(x(70)*x(6)-x(31)*x(45))
 g(32)=50.*(x(71)*x(10)-x(32)*x(49))
 g(33)=70.25182*(x(72)*x(19)-x(33)*x(58))+3.46312*(x(33)**2+x(72)
1     **2-x(33)*x(19)-x(72)*x(58))
 g(34)=55.41701*(x(73)*x(20)-x(34)*x(59))+2.77085*(x(34)**2+x(73)
1     **2-x(34)*x(20)-x(73)*x(59))
 g(35)=69.93007*(x(74)*x(22)-x(35)*x(61))
 g(36)=36.75229*(x(75)*x(23)-x(36)*x(62))+0.67559*(x(36)**2+x(75)
1     **2-x(36)*x(23)-x(75)*x(62))
 g(37)=43.07464*(x(76)*x(25)-x(37)*x(64))+1.114*(x(37)**2+x(76)
1     **2-x(37)*x(25)-x(76)*x(64))
 g(38)=63.93443*(x(77)*x(29)-x(38)*x(68))+3.27869*(x(38)**2+x(77)
1     **2-x(38)*x(29)-x(77)*x(68))
 g(39)=-39.9361*(x(78)*x(1)-x(39)*x(40))-1.59744*(x(39)**2+x(78)
1     **2-x(39)*x(1)-x(78)*x(40))-39.9361*(x(78)*x(9)-x(39)*x(48))
2     -1.59744*(x(39)**2+x(78)**2-x(39)*x(9)-x(78)*x(48))
 g(40)=24.15573*(x(1)**2+x(40)**2-x(1)*x(2)-x(40)*x(41))-2.05706
1     *(x(40)*x(2)-x(1)*x(41))+39.9361*(x(1)**2+x(40)**2
2     -x(1)*x(39)-x(40)*x(78))-1.59744*(x(40)*x(39)-x(1)*x(78))
3     -0.72435*(x(1)**2+x(40)**2)
 g(41)=24.15573*(x(2)**2+x(41)**2-x(1)*x(2)-x(40)*x(41))-2.05706
1     *(x(41)*x(1)-x(2)*x(40))+65.73792*(x(2)**2+x(41)**2
2     -x(2)*x(3)-x(41)*x(42))-5.65956*(x(41)*x(3)-x(2)*x(42))
3     +69.94144*(x(2)**2+x(41)**2-x(2)*x(25)-x(41)*x(64))
4     -56.92908*(x(41)*x(25)-x(2)*x(64))+55.24862*(x(2)**2
5     +x(41)**2-x(2)*x(30)-x(41)*x(69))-0.55095*(x(2)**2+x(41)
6     **2)
 g(42)=-65.73792*(x(3)**2+x(42)**2-x(3)*x(2)-x(42)*x(41))+5.65956
1     *(x(42)*x(2)-x(3)*x(41))-46.77412*(x(3)**2+x(42)**2
2     -x(3)*x(4)-x(42)*x(43))+2.85476*(x(42)*x(4)-x(3)*x(43))
3     -74.67715*(x(3)**2+x(42)**2-x(3)*x(18)-x(42)*x(57))
4     +6.17631*(x(42)*x(18)-x(3)*x(57))+0.3462*(x(3)**2+x(42)**2)
 g(43)=-46.77412*(x(4)**2+x(43)**2-x(3)*x(4)-x(42)*x(43))+2.85476
```

```
1     *(x(43)*x(3)-x(4)*x(42))-77.82101*(x(4)**2+x(43)**2
2     -x(4)*x(5)-x(43)*x(44))+4.86381*(x(43)*x(5)-x(4)*x(44))
3     -77.22239*(x(4)**2+x(43)**2-x(4)*x(14)-x(43)*x(53))
4     +4.78899*(x(43)*x(14)-x(4)*x(53))+0.2469*(x(4)**2+x(43)**2)
 g(44)=77.82101*(x(5)**2+x(44)**2-x(5)*x(4)-x(44)*x(43))-4.86381
1     *(x(44)*x(4)-x(5)*x(43))+382.353*(x(5)**2+x(44)**2
2     -x(5)*x(6)-x(44)*x(45))-29.41177*(x(44)*x(6)-x(5)*x(45))
3     +88.83249*(x(5)**2+x(44)**2-x(5)*x(8)-x(44)*x(47))
4     -6.34518*(x(44)*x(8)-x(5)*x(47))-0.1626*(x(5)**2+x(44)**2)
 g(45)=382.353*(x(6)**2+x(45)**2-x(6)*x(5)-x(45)*x(44))-29.41177
1     *(x(45)*x(5)-x(6)*x(44))+108.2353*(x(6)**2+x(45)**2
2     -x(6)*x(7)-x(45)*x(46))-7.05882*(x(45)*x(7)-x(6)*x(46))
3     +121.069*(x(6)**2+x(45)**2-x(6)*x(11)-x(45)*x(50))
4     -10.33515*(x(45)*x(11)-x(6)*x(50))+40.*(x(6)**2
5     +x(45)**2-x(6)*x(31)-x(45)*x(70))-0.14765*(x(6)**2+x(45)**2)
 g(46)=-108.2353*(x(7)**2+x(46)**2-x(7)*x(6)-x(46)*x(45))+7.05882
1     *(x(46)*x(6)-x(7)*x(45))-215.7599*(x(7)**2+x(46)**2
2     -x(7)*x(8)-x(46)*x(47))+18.76173*(x(46)*x(8)-x(7)*x(47))
3     +0.0955*(x(7)**2+x(46)**2)
 g(47)=-88.83249*(x(8)**2+x(47)**2-x(8)*x(5)-x(47)*x(44))+6.34518
1     *(x(47)*x(5)-x(8)*x(44))-215.7599*(x(8)**2+x(47)**2
2     -x(8)*x(7)-x(47)*x(46))+18.76173*(x(47)*x(7)-x(8)*x(46))
3     -27.43806*(x(8)**2+x(47)**2-x(8)*x(9)-x(47)*x(48))
4     +1.7385*(x(47)*x(9)-x(8)*x(48))+0.303*(x(8)**2+x(47)**2)
 g(48)=27.43806*(x(9)**2+x(48)**2-x(9)*x(8)-x(48)*x(47))-1.7385
1     *(x(48)*x(8)-x(9)*x(47))+39.9361*(x(9)**2+x(48)**2
2     -x(9)*x(39)-x(48)*x(78))-1.59744*(x(48)*x(39)-x(9)*x(78))
3     -0.7902*(x(9)**2+x(48)**2)
 g(49)=230.563*(x(10)**2+x(49)**2-x(10)*x(11)-x(49)*x(50))
1     -21.44772*(x(49)*x(11)-x(10)*x(50))+230.563*(x(10)
2     **2+x(49)**2-x(10)*x(13)-x(49)*x(52))-21.44772*(x(49)*x(13)
3     -x(10)*x(52))+50.*(x(10)**2+x(49)**2-x(10)*x(32)
4     -x(49)*x(71))-0.0729*(x(10)**2+x(49)**2)
 g(50)=121.069*(x(11)**2+x(50)**2-x(11)*x(6)-x(50)*x(45))-10.33515
1     *(x(50)*x(6)-x(11)*x(45))+230.563*(x(11)**2+x(50)**2
2     -x(11)*x(10)-x(50)*x(49))-21.44772*(x(50)*x(10)-x(11)*x(49))
3     +22.95745*(x(11)**2+x(50)**2-x(11)*x(12)-x(50)*x(51))
4     -0.84441*(x(50)*x(12)-x(11)*x(51))-0.1059*(x(11)**2+x(50)**2)
 g(51)=-22.95745*(x(12)**2+x(51)**2-x(12)*x(11)-x(51)*x(50))
1     +0.84441*(x(51)*x(11)-x(12)*x(50))-22.95745*(x(12)**2+x(51)
2     **2-x(12)*x(13)-x(51)*x(52))+0.84441*(x(51)*x(13)-x(12)
3     *x(52))
 g(52)=230.563*(x(13)**2+x(52)**2-x(13)*x(10)-x(52)*x(49))-21.44772
1     *(x(52)*x(10)-x(13)*x(49))+22.95745*(x(13)**2+x(52)
2     **2-x(13)*x(12)-x(52)*x(51))-0.84441*(x(52)*x(12)-x(13)
3     *x(51))+98.22992*(x(13)**2+x(52)**2-x(13)*x(14)-x(52)*x(53))
4     -8.75316*(x(52)*x(14)-x(13)*x(53))-0.1226*(x(13)**2+x(52)**2)
 g(53)=77.22239*(x(14)**2+x(53)**2-x(14)*x(4)-x(53)*x(43))-4.78899
1     *(x(53)*x(4)-x(14)*x(43))+98.22992*(x(14)**2+x(53)**2
2     -x(14)*x(13)-x(53)*x(52))-8.75316*(x(53)*x(13)-x(14)*x(52))
3     +45.76804*(x(14)**2+x(53)**2-x(14)*x(15)-x(53)*x(54))
4     -3.79643*(x(53)*x(15)-x(14)*x(54))-0.33825*(x(14)**2+x(53)
5     **2)
 g(54)=-45.76804*(x(15)**2+x(54)**2-x(15)*x(14)-x(54)*x(53))
1     +3.79643*(x(54)*x(14)-x(15)*x(53))-105.4166*(x(15)**2
2     +x(54)**2-x(15)*x(16)-x(54)*x(55))+10.09308*(x(54)*x(16)
3     -x(15)*x(55))+0.2685*(x(15)**2+x(54)**2)
 g(55)=-105.4166*(x(16)**2+x(55)**2-x(16)*x(15)-x(55)*x(54))
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     1      +10.09308*(x(55)*x(15)-x(16)*x(54))-111.6688*(x(16)**2
     2      +x(55)**2-x(16)*x(17)-x(55)*x(56))+8.78294*(x(55)*x(17)-x(16)
     3      *x(56))-50.93911*(x(16)**2+x(55)**2-x(16)*x(19)-x(55)
     4      *x(58))+4.17962*(x(55)*x(19)-x(16)*x(58))-73.81486
     5      *(x(16)**2+x(55)**2-x(16)*x(21)-x(55)*x(60))+4.37421*(x(55)
     6      *x(21)-x(16)*x(60))-169.0544*(x(16)**2+x(55)**2-x(16)
     7      *x(24)-x(55)*x(63))+8.59599*(x(55)*x(24)-x(16)*x(63))
     8      +0.466*(x(16)**2+x(55)**2)
      g(56)=111.6688*(x(17)**2+x(56)**2-x(17)*x(16)-x(56)*x(55))-8.78294
     1      *(x(56)*x(16)-x(17)*x(55))+121.069*(x(17)**2+x(56)**2
     2      -x(17)*x(18)-x(56)*x(57))-10.33515*(x(56)*x(18)-x(17)*x(57))
     3      +57.4789*(x(17)**2+x(56)**2-x(17)*x(27)-x(56)*x(66))
     4      -4.31922*(x(56)*x(27)-x(17)*x(66))-0.29385*(x(17)**2
     5      +x(56)**2)
      g(57)=-74.67715*(x(18)**2+x(57)**2-x(18)*x(3)-x(57)*x(42))+6.17631
     1      *(x(57)*x(3)-x(18)*x(42))-121.069*(x(18)**2+x(57)**2
     2      -x(18)*x(17)-x(57)*x(56))+10.33515*(x(57)*x(17)-x(18)*x(56))
     3      +0.17285*(x(18)**2+x(57)**2)
      g(58)=50.93911*(x(19)**2+x(58)**2-x(19)*x(16)-x(58)*x(55))-4.17962
     1      *(x(58)*x(16)-x(19)*x(55))+72.2778*(x(19)**2+x(58)**2
     2      -x(19)*x(20)-x(58)*x(59))-3.66627*(x(58)*x(20)-x(19)*x(59))
     3      +70.25182*(x(19)**2+x(58)**2-x(19)*x(33)-x(58)*x(72))-3.46312
     4      *(x(58)*x(33)-x(19)*x(72))-0.152*(x(19)**2+x(58)**2)
      g(59)=-72.2778*(x(20)**2+x(59)**2-x(20)*x(19)-x(59)*x(58))+3.66627
     1      *(x(59)*x(19)-x(20)*x(58))-55.41701*(x(20)**2+x(59)**2-x(20)
     2      *x(34)-x(59)*x(73))+2.77085*(x(59)*x(34)-x(20)*x(73))
      g(60)=-73.81486*(x(21)**2+x(60)**2-x(21)*x(16)-x(60)*x(55))+4.3742
     1      *(x(60)*x(16)-x(21)*x(55))-71.19609*(x(21)**2+x(60)**2
     2      -x(21)*x(22)-x(60)*x(61))+4.06835*(x(60)*x(22)-x(21)*x(61))
     3      +0.25565*(x(21)**2+x(60)**2)
      g(61)=71.19609*(x(22)**2+x(61)**2-x(22)*x(21)-x(61)*x(60))-4.06835
     1      *(x(61)*x(21)-x(22)*x(60))+103.7614*(x(22)**2+x(61)
     2      **2-x(22)*x(23)-x(61)*x(62))-6.48508*(x(61)*x(23)-x(22)
     3      *x(62))+69.93007*(x(22)**2+x(61)**2-x(22)*x(35)-x(61)
     4      *x(74))-0.22055*(x(22)**2+x(61)**2)
      g(62)=-103.7614*(x(23)**2+x(62)**2-x(23)*x(22)-x(62)*x(61))
     1      +6.48508*(x(62)*x(22)-x(23)*x(61))-28.45899*(x(23)**2
     2      +x(62)**2-x(23)*x(24)-x(62)*x(63))+1.78885*(x(62)*x(24)
     3      -x(23)*x(63))-36.75229*(x(23)**2+x(62)**2-x(23)*x(36)
     4      -x(62)*x(75))+0.67559*(x(62)*x(36)-x(23)*x(75))
     5      +0.2728*(x(23)**2+x(62)**2)
      g(63)=-169.0544*(x(24)**2+x(63)**2-x(24)*x(16)-x(63)*x(55))
     1      +8.59599*(x(63)*x(16)-x(24)*x(55))-28.45899*(x(24)**2
     2      +x(63)**2-x(24)*x(23)-x(63)*x(62))+1.78885*(x(63)*x(23)-x(24)
     3      *x(62))+0.2145*(x(24)**2+x(63)**2)
      g(64)=-69.94144*(x(25)**2+x(64)**2-x(25)*x(2)-x(64)*x(41))+56.9291
     1      *(x(64)*x(2)-x(25)*x(41))-30.65883*(x(25)**2+x(64)**2
     2      -x(25)*x(26)-x(64)*x(65))+3.03741*(x(64)*x(26)-x(25)*x(65))
     3      -43.07464*(x(25)**2+x(64)**2-x(25)*x(37)-x(64)*x(76))
     4      +1.114*(x(64)*x(37)-x(25)*x(76))+0.3295*(x(25)**2+x(64)**2)
      g(65)=-30.65883*(x(26)**2+x(65)**2-x(26)*x(25)-x(65)*x(64))+3.0374
     1      *(x(65)*x(25)-x(26)*x(64))-67.41573*(x(26)**2+x(65)**2
     2      -x(26)*x(27)-x(65)*x(66))+6.42055*(x(65)*x(27)-x(26)*x(66))
     3      -20.92484*(x(26)**2+x(65)**2-x(26)*x(28)-x(65)*x(67))
     4      +1.89825*(x(65)*x(28)-x(26)*x(67))-15.86802*(x(26)**2
     5      +x(65)**2-x(26)*x(29)-x(65)*x(68))+1.44716*(x(65)*x(29)-x(26)
     6      *x(68))+1.2809*(x(26)**2+x(65)**2)
```

```
      g(66)=-57.4789*(x(27)**2+x(66)**2-x(27)*x(17)-x(66)*x(56))+4.31922
     1     *(x(66)*x(17)-x(27)*x(56))-67.41573*(x(27)**2+x(66)**2
     2     -x(27)*x(26)-x(66)*x(65))+6.42055*(x(66)*x(26)-x(27)*x(65))
     3     +0.2806*(x(27)**2+x(66)**2)
      g(67)=-20.92484*(x(28)**2+x(67)**2-x(28)*x(26)-x(67)*x(65))
     1     +1.89825*(x(67)*x(26)-x(28)*x(65))-65.66074*(x(28)**2
     2     +x(67)**2-x(28)*x(29)-x(67)*x(68))+6.08775*(x(67)*x(29)-x(28)
     3     *x(68))+0.5146*(x(28)**2+x(67)**2)
      g(68)=-15.86802*(x(29)**2+x(68)**2-x(29)*x(26)-x(68)*x(65))
     1     +1.44716*(x(68)*x(26)-x(29)*x(65))-65.66074*(x(29)**2
     2     +x(68)**2-x(29)*x(28)-x(68)*x(67))+6.08775*(x(68)*x(28)-x(29)
     3     *x(67))-63.93443*(x(29)**2+x(68)**2-x(29)*x(38)-x(68)
     4     *x(77))+3.27869*(x(68)*x(38)-x(29)*x(77))
     5     +0.639*(x(29)**2+x(68)**2)
      g(69)=55.24862*(x(30)**2+x(69)**2-x(30)*x(2)-x(69)*x(41))
      g(70)=40.*(x(31)**2+x(70)**2-x(31)*x(6)-x(70)*x(45))
      g(71)=50.*(x(32)**2+x(71)**2-x(32)*x(10)-x(71)*x(49))
      g(72)=70.25182*(x(33)**2+x(72)**2-x(33)*x(19)-x(72)*x(58))-3.46312
     1     *(x(72)*x(19)-x(33)*x(58))
      g(73)=55.41701*(x(34)**2+x(73)**2-x(34)*x(20)-x(73)*x(59))-2.77085
     1     *(x(73)*x(20)-x(34)*x(59))
      g(74)=69.93007*(x(35)**2+x(74)**2-x(35)*x(22)-x(74)*x(61))
      g(75)=36.75229*(x(36)**2+x(75)**2-x(36)*x(23)-x(75)*x(62))-0.67559
     1     *(x(75)*x(23)-x(36)*x(62))
      g(76)=43.07464*(x(37)**2+x(76)**2-x(37)*x(25)-x(76)*x(64))-1.114
     1     *(x(76)*x(25)-x(37)*x(64))
      g(77)=63.93443*(x(38)**2+x(77)**2-x(38)*x(29)-x(77)*x(68))-3.27869
     1     *(x(77)*x(29)-x(38)*x(68))
      g(78)=-39.9361*(x(39)**2+x(78)**2-x(39)*x(1)-x(78)*x(40))+1.59744
     1     *(x(78)*x(1)-x(39)*x(40))-39.9361*(x(39)**2+x(78)**2
     2     -x(39)*x(9)-x(78)*x(48))+1.59744*(x(78)*x(9)-x(39)*x(48))
     3     +0.975*(x(39)**2+x(78)**2)
c
c     function
c
      g(79)=24.15573*(x(1)**2+x(40)**2+x(2)**2+x(41)**2-2.*x(1)*x(2)-2.
     -*x(40)*x(41))+39.9361*(x(1)**2+x(40)**2+x(39)**2+x(78)**2-2.*x(1)
     -*x(39)-2.*x(40)*x(78))+65.73792*(x(2)**2+x(41)**2+x(3)**2+x(42)**2
     --2.*x(2)*x(3)-2.*x(41)*x(42))+69.94144*(x(2)**2+x(41)**2+x(25)**2
     -+x(64)**2-2.*x(2)*x(25)-2.*x(41)*x(64))+55.24862*(x(2)**2+x(41)**2
     -+x(30)**2+x(69)**2-2.*x(2)*x(30)-2.*x(41)*x(69))+46.77412*(x(3)**2
     -+x(42)**2+x(4)**2+x(43)**2-2.*x(3)*x(4)-2.*x(42)*x(43))+74.67715
     -*(x(3)**2+x(42)**2+x(18)**2+x(57)**2-2.*x(3)*x(18)-2.*x(42)*x(57))
     -+77.82101*(x(4)**2+x(43)**2+x(5)**2+x(44)**2-2.*x(4)*x(5)-2.*x(43)
     -*x(44))+77.22239*(x(4)**2+x(43)**2+x(14)**2+x(53)**2-2.*x(4)*x(14)
     --2.*x(43)*x(53))+382.353*(x(5)**2+x(44)**2+x(6)**2+x(45)**2-2.
     -*x(5)*x(6)-2.*x(44)*x(45))+88.83249*(x(5)**2+x(44)**2+x(8)**2+x(47
     -)**2-2.*x(5)*x(8)-2.*x(44)*x(47))+108.2353*(x(6)**2+x(45)**2+x(7)
     -**2+x(46)**2-2.*x(6)*x(7)-2.*x(45)*x(46))+121.069*(x(6)**2+x(45)**
     -2+x(11)**2+x(50)**2-2.*x(6)*x(11)-2.*x(45)*x(50))+40.*(x(6)**2+x(4
     -5)**2+x(31)**2+x(70)**2-2.*x(6)*x(31)-2.*x(45)*x(70))+215.7599
     -*(x(7)**2+x(46)**2+x(8)**2+x(47)**2-2.*x(7)*x(8)-2.*x(46)*x(47))
     -+27.43806*(x(8)**2+x(47)**2+x(9)**2+x(48)**2-2.*x(8)*x(9)-2.*x(47)
     -*x(48))+39.9361*(x(9)**2+x(48)**2+x(39)**2+x(78)**2-2.*x(9)*x(39)
     --2.*x(48)*x(78))-x(79)
      g(80)=230.563*(x(10)**2+x(49)**2+x(11)**2+x(50)**2-2.*x(10)*x(11)
     --2.*x(49)*x(50))+230.563*(x(10)**2+x(49)**2+x(13)**2+x(52)**2-2.*
     -x(10)*x(13)-2.*x(49)*x(52))+50.*(x(10)**2+x(49)**2+x(32)**2+x(71)*
     -*2-2.*x(10)*x(32)-2.*x(49)*x(71))+22.95745*(x(11)**2+x(50)**2+x(12
```

```
     -)**2+x(51)**2-2.*x(11)*x(12)-2.*x(50)*x(51))+22.95745*(x(12)**2
     -+x(51)**2+x(13)**2+x(52)**2-2.*x(12)*x(13)-2.*x(51)*x(52))+98.2299
     -2*(x(13)**2+x(52)**2+x(14)**2+x(53)**2-2.*x(13)*x(14)-2.*x(52)*x(5
     -3))+45.76804*(x(14)**2+x(53)**2+x(15)**2+x(54)**2-2.*x(14)*x(15)-2
     -*x(53)*x(54))+105.4166*(x(15)**2+x(54)**2+x(16)**2+x(55)**2-2.*x(1
     -5)*x(16)-2.*x(54)*x(55))+111.6688*(x(16)**2+x(55)**2+x(17)**2+x(56
     -)**2-2.*x(16)*x(17)-2.*x(55)*x(56))+50.93911*(x(16)**2+x(55)**2
     -+x(19)**2+x(58)**2-2.*x(16)*x(19)-2.*x(55)*x(58))+73.81486*(x(16)
     -**2+x(55)**2+x(21)**2+x(60)**2-2.*x(16)*x(21)-2.*x(55)*x(60))+
     -169.0544*(x(16)**2+x(55)**2+x(24)**2+x(63)**2-2.*x(16)*x(24)-2.*x(
     -55)*x(63))+121.069*(x(17)**2+x(56)**2+x(18)**2+x(57)**2-2.*x(17)*
     -x(18)-2.*x(56)*x(57))+57.4789*(x(17)**2+x(56)**2+x(27)**2+x(66)**2
     --2.*x(17)*x(27)-2.*x(56)*x(66))+72.2778*(x(19)**2+x(58)**2+x(20)
     -**2+x(59)**2-2.*x(19)*x(20)-2.*x(58)*x(59))+70.25182*(x(19)**2+x(5
     -8)**2+x(33)**2+x(72)**2-2.*x(19)*x(33)-2.*x(58)*x(72))-x(80)
      g(81)=55.41701*(x(20)**2+x(59)**2+x(34)**2+x(73)**2-2*x(20)*x(34)-
     -2*x(59)*x(73))+71.19609*(x(21)**2+x(60)**2+x(22)**2+x(61)**2-2*x(2
     -1)*x(22)-2*x(60)*x(61))+103.7614*(x(22)**2+x(61)**2+x(23)**2+x(62)
     -**2-2*x(22)*x(23)-2*x(61)*x(62))+69.93007*(x(22)**2+x(61)**2+x(35)
     -**2+x(74)**2-2*x(22)*x(35)-2*x(61)*x(74))+28.45899*(x(23)**2+x(62)
     -**2+x(24)**2+x(63)**2-2*x(23)*x(24)-2*x(62)*x(63))+36.75229*(x(23)
     -**2+x(62)**2+x(36)**2+x(75)**2-2*x(23)*x(36)-2*x(62)*x(75))+30.658
     -83*(x(25)**2+x(64)**2+x(26)**2+x(65)**2-2*x(25)*x(26)-2*x(64)*x(65
     -))+43.07464*(x(25)**2+x(64)**2+x(37)**2+x(76)**2-2*x(25)*x(37)-2*
     -x(64)*x(76))+67.41573*(x(26)**2+x(65)**2+x(27)**2+x(66)**2-2*x(26)
     -*x(27)-2*x(65)*x(66))+20.92484*(x(26)**2+x(65)**2+x(28)**2+x(67)**
     -2-2*x(26)*x(28)-2*x(65)*x(67))+15.86802*(x(26)**2+x(65)**2+x(29)**
     -2+x(68)**2-2*x(26)*x(29)-2*x(65)*x(68))+65.66074*(x(28)**2+x(67)**
     -2+x(29)**2+x(68)**2-2*x(28)*x(29)-2*x(67)*x(68))+63.93443*(x(29)**
     -2+x(68)**2+x(38)**2+x(77)**2-2*x(29)*x(38)-2*x(68)*x(77))+x(79)+x(
     -80)
      return
      end
//lked.sysin dd dsn=ea0043.grg.obj,disp=old
//go.sysin dd *
   80   81
name ieee 39 buses system
rows
 e   1     0.0
 e   2     0.0
 g   3     3.22
 g   4     5.0
 e   5     0.0
 e   6     0.0
 g   7     2.338
 g   8     5.22
 e   9     0.0
 e  10     0.0
 e  11     0.0
 g  12     0.085
 e  13     0.0
 e  14     0.0
 g  15     3.2
 g  16     3.294
 e  17     0.0
 g  18     1.58
 e  19     0.0
 g  20     6.8
 g  21     2.74
 e  22     0.0
```

g	23	2.475
g	24	3.086
g	25	2.24
g	26	1.39
g	27	2.81
g	28	2.06
g	29	2.835
l	30	2.5
l	31	5.7008
l	32	6.51
l	33	6.3205
l	34	5.08
l	35	6.5
l	36	5.6
l	37	5.4006
l	38	8.3
g	39	1.04
e	40	0.0
e	41	0.0
g	42	0.024
g	43	1.84
e	44	0.0
e	45	0.0
g	46	0.84
g	47	1.766
e	48	0.0
e	49	0.0
e	50	0.0
g	51	0.88
e	52	0.0
e	53	0.0
g	54	1.53
g	55	0.323
e	56	0.0
g	57	0.3
e	58	0.0
g	59	1.03
g	60	1.15
e	61	0.0
g	62	0.846
g	63	-0.922
g	64	0.472
g	65	0.17
g	66	0.755
g	67	0.276
g	68	0.269
l	69	1.451
l	70	2.1
l	71	2.057
l	72	1.091
l	73	1.67
l	74	2.113
l	75	1.005
l	76	0.007
l	77	0.228
g	78	1.62
e	79	0.0
e	80	0.0

```
end
bounds
 r   1  39   0.0               1.09
 r  40  69   -0.25             0.0
 r  70  77   0.0               0.25
 r  78       -0.25             0.0
 g  79       0.0
 g  80       0.0
end
initial
separate
     1    1.0372955
     2    1.0574767
     3    1.0343415
     4    1.0069168
     5    1.0109224
     6    1.0152323
     7    0.9976871
     8    0.99448366
     9    1.0140103
    10    1.030481
    11    1.0243039
    12    1.0063022
    13    1.0262932
    14    1.0199604
    15    1.0223909
    16    1.0412665
    17    1.0414622
    18    1.036458
    19    1.0596739
    20    1.0599007
    21    1.0462078
    22    1.0675683
    23    1.0626191
    24    1.047215
    25    1.0695653
    26    1.0665716
    27    1.0469693
    28    1.0735929
    29    1.0754144
    30    1.0842141
    31    1.0645418
    32    1.0657641
    33    1.0685404
    34    1.09
    35    1.0883983
    36    1.0721631
    37    1.0693344
    38    1.0716231
    39    1.0075499
    40    -0.15058421
    41    -0.10810876
    42    -0.15663458
    43    -0.16594766
    44    -0.14550843
    45    -0.13380395
    46    -0.16952209
    47    -0.17774838
```

```
    48    -0.17791118
    49    -0.094162621
    50    -0.10767591
    51    -0.10612918
    52    -0.10631416
    53    -0.13492473
    54    -0.14303749
    55    -0.12074215
    56    -0.13882546
    57    -0.15296544
    58    -0.038349017
    59    -0.061175254
    60    -0.078543037
    61    0.0
    62    -0.0034997948
    63    -0.11932808
    64    -0.0859941
    65    -0.10973339
    66    -0.14294843
    67    -0.048815033
    68    0.0
    69    -0.068051678
    70    0.00005910088
    71    0.028767894
    72    0.045607408
    73    0.021941672
    74    0.086859624
    75    0.13933975
    76    0.031152123
    77    0.11916878
    78    -0.17339433
    79    1.9175105
    80    2.2085374
end
print
ipr      4
end
met
min
end
go
stop
//
/*eof
```

APPENDIX THREE

The results under the loss of generation contengency

- bus 31 lost 195 MW of its capacity.

****
the final report of epri 39 buses system under loss of generation on bus 31
, amount 195 MW
the real demand will be cut   2.21748 at the bus   7
the real demand will be cut   .08500 at the bus 12
the total real demand is cut by amount of   2.30248

****

the line flow report

from bus 1 to bus 2 real power -1.027031 reactive power -.345300

from bus 1 to bus 39 real power 1.027003 reactive power .345226

****

the line flow report

from bus 2 to bus 1 real power 1.030287 reactive power -.412431

from bus 2 to bus 3 real power 3.675242 reactive power 1.123990

from bus 2 to bus 25 real.power -2.205508 reactive power .612267

from bus 2 to bus 30 real power -2.500011 reactive power -1.324285

****

the line flow report

from bus 3 to bus 2 real power -3.658019 reactive power -1.211748

from bus 3 to bus 4 real power .580099 reactive power 1.445126

from bus 3 to bus 18 real power -.142014 reactive power -.257054

****

the line flow report

from bus 4 to bus 3 real power -.576791 reactive power -1.626276

from bus 4 to bus 5 real power -2.251665 reactive power .100447

from bus 4 to bus 14 real power -2.171607 reactive power -.314419

****

the line flow report

from bus 5 to bus 4 real power 2.255627 reactive power -.175220

from bus 5 to bus 6 real power -4.713789 reactive power -.320382

from bus 5 to bus 8 real power 2.458094 reactive power .495814

****

```
the line flow report
from bus    6  to bus   5   real power   4.718124   reactive power    .331976
from bus    6  to bus   7   real power   2.912684   reactive power    .750422
from bus    6  to bus  11   real power  -1.929802   reactive power    .193316
from bus    6  to bus  31   real power  -5.700471   reactive power  -1.274131
*************************************************************************
the line flow report
from bus    7  to bus   6   real power  -2.907378   reactive power   -.784810
from bus    7  to bus   8   real power   2.786535   reactive power   -.055648
*************************************************************************
the line flow report
from bus    8  to bus   5   real power  -2.453145   reactive power   -.577309
from bus    8  to bus   7   real power  -2.783475   reactive power    .011699
from bus    8  to bus   9   real power    .016803   reactive power  -1.200449
*************************************************************************
the line flow report
from bus    9  to bus   8   real power   -.014498   reactive power    .837103
from bus    9  to bus  39   real power    .014396   reactive power   -.837407
*************************************************************************
the line flow report
from bus   10  to bus  11   real power   1.965224   reactive power    .072423
from bus   10  to bus  13   real power   2.584904   reactive power   -.191625
from bus   10  to bus  32   real power  -4.549999   reactive power    .120550
*************************************************************************
the line flow report
from bus   11  to bus   6   real power   1.932372   reactive power   -.306623
from bus   11  to bus  10   real power  -1.963727   reactive power   -.131630
from bus   11  to bus  12   real power    .031002   reactive power    .436612
*************************************************************************
```

the line flow report

from bus 12 to bus 11 real power -.030705 reactive power -.428538

from bus 12 to bus 13 real power .030713 reactive power -.451345

****

the line flow report

from bus 13 to bus 10 real power -2.582311 reactive power .144132

from bus 13 to bus 12 real power -.030383 reactive power .460302

from bus 13 to bus 14 real power 2.612631 reactive power -.605563

****

the line flow report

from bus 14 to bus 4 real power 2.175317 reactive power .231250

from bus 14 to bus 13 real power -2.606455 reactive power .496235

from bus 14 to bus 15 real power .431292 reactive power -.726677

****

the line flow report

from bus 15 to bus 14 real power -.430473 reactive power .351899

from bus 15 to bus 16 real power -2.769728 reactive power -1.882381

****

the line flow report

from bus 16 to bus 15 real power 2.778949 reactive power 1.793717

from bus 16 to bus 17 real power 1.914813 reactive power -.532786

from bus 16 to bus 19 real power -4.522977 reactive power -.535990

from bus 16 to bus 21 real power -3.161054 reactive power .016841

from bus 16 to bus 24 real power -.303602 reactive power -1.064213

****

the line flow report

from bus 17 to bus 16 real power -1.912349 reactive power .415910

from bus 17 to bus 18 real power 1.723977 reactive power .199375

from bus 17 to bus 27 real power .188264 reactive power -.616127

```
********************************************************************************
the line flow report
from bus  18  to bus   3   real power    .142053   reactive power    .022654
from bus  18  to bus  17   real power  -1.722050   reactive power   -.322420
********************************************************************************
the line flow report
from bus  19  to bus  16   real power   4.552889   reactive power    .560207
from bus  19  to bus  20   real power   1.743032   reactive power    .031721
from bus  19  to bus  33   real power  -6.295877   reactive power   -.591376
********************************************************************************
the line flow report
from bus  20  to bus  19   real power  -1.741161   reactive power    .005152
from bus  20  to bus  34   real power  -5.058849   reactive power  -1.035053
********************************************************************************
the line flow report
from bus  21  to bus  16   real power   3.168329   reactive power   -.175064
from bus  21  to bus  22   real power  -5.908401   reactive power   -.975376
********************************************************************************
the line flow report
from bus  22  to bus  21   real power   5.934202   reactive power   1.138691
from bus  22  to bus  23   real power    .565848   reactive power    .415991
from bus  22  to bus  35   real power  -6.500024   reactive power  -1.553672
********************************************************************************
the line flow report
from bus  23  to bus  22   real power   -.565537   reactive power   -.621061
from bus  23  to bus  24   real power   3.412546   reactive power    .028871
from bus  23  to bus  36   real power  -5.586204   reactive power   -.253675
********************************************************************************
the line flow report
```

```
from bus  24  to bus  16   real power    .303916   reactive power    .995064
from bus  24  to bus  23   real power  -3.389818   reactive power   -.072756
*******************************************************************************
the line flow report
from bus  25  to bus   2   real power   2.238300   reactive power   -.740054
from bus  25  to bus  26   real power    .907000   reactive power   -.311589
from bus  25  to bus  37   real power  -5.385448   reactive power    .578938
*******************************************************************************
the line flow report
from bus  26  to bus  25   real power   -.904732   reactive power   -.260012
from bus  26  to bus  27   real power   2.631509   reactive power    .843363
from bus  26  to bus  28   real power   8.225460   reactive power  -2.178658
from bus  26  to bus  29   real power  -1.804272   reactive power   -.396619
*******************************************************************************
the line flow report
from bus  27  to bus  17   real power   -.187998   reactive power    .261201
from bus  27  to bus  26   real power  -2.621962   reactive power  -1.016112
*******************************************************************************
the line flow report
from bus  28  to bus  26   real power  -7.962867   reactive power   4.080522
from bus  28  to bus  29   real power -32.442535   reactive power  17.813532
*******************************************************************************
the line flow report
from bus  29  to bus  26   real power   1.820507   reactive power   -.619273
from bus  29  to bus  28   real power  33.830330   reactive power  -3.162879
from bus  29  to bus  38   real power  -8.252879   reactive power    .691537
*******************************************************************************
the line flow report
from bus  30  to bus   2   real power   2.500011   reactive power   1.451203
*******************************************************************************
```

the line flow report

from bus 31 to bus 6 real power 5.700471 reactive power 2.099735

****

the line flow report

from bus 32 to bus 10 real power 4.549999 reactive power .280088

****

the line flow report

from bus 33 to bus 19 real power 6.320484 reactive power 1.090545

****

the line flow report

from bus 34 to bus 20 real power 5.079997 reactive power 1.458006

****

the line flow report

from bus 35 to bus 22 real power 6.500024 reactive power 2.112389

****

the line flow report

from bus 36 to bus 23 real power 5.600009 reactive power 1.004697

****

the line flow report

from bus 37 to bus 25 real power 5.400611 reactive power .007349

****

the line flow report

from bus 38 to bus 29 real power 8.300029 reactive power .227692

****

the line flow report

from bus 39 to bus 1 real power -1.025552 reactive power -1.146758

from bus 39 to bus 9 real power -.014364 reactive power -.472941

****

the associated system real power losses 2.00241

****

the bus voltage report

the bus no.	1	magnitude	1.066277	angle	-7.50195
the bus no.	2	magnitude	1.068388	angle	-5.36932
the bus no.	3	magnitude	1.047169	angle	-8.12789
the bus no.	4	magnitude	1.014633	angle	-8.68439
the bus no.	5	magnitude	1.014671	angle	-7.07264
the bus no.	6	magnitude	1.016435	angle	-6.39507
the bus no.	7	magnitude	1.007727	angle	-7.86702
the bus no.	8	magnitude	1.006775	angle	-8.59128
the bus no.	9	magnitude	1.043073	angle	-8.75101
the bus no.	10	magnitude	1.016952	angle	-5.04027
the bus no.	11	magnitude	1.015747	angle	-5.50656
the bus no.	12	magnitude	.997001	angle	-5.54333
the bus no.	13	magnitude	1.016646	angle	-5.65967
the bus no.	14	magnitude	1.019807	angle	-7.14379
the bus no.	15	magnitude	1.030508	angle	-7.70669
the bus no.	16	magnitude	1.049537	angle	-6.41276
the bus no.	17	magnitude	1.052281	angle	-7.31358
the bus no.	18	magnitude	1.049095	angle	-8.03740
the bus no.	19	magnitude	1:066551	angle	-1.92393
the bus no.	20	magnitude	1.065235	angle	-3.13595
the bus no.	21	magnitude	1.050716	angle	-4.18847
the bus no.	22	magnitude	1.069180	angle	.00000
the bus no.	23	magnitude	1.064191	angle	-.25779
the bus no.	24	magnitude	1.055397	angle	-6.33604
the bus no.	25	magnitude	1.077470	angle	-4.18284
the bus no.	26	magnitude	1.075535	angle	-5.63363
the bus no.	27	magnitude	1.059257	angle	-7.51027

the bus no.	28	magnitude	1.178189	angle	-23.90643
the bus no.	29	magnitude	1.078770	angle	.00000
the bus no.	30	magnitude	1.091645	angle	-3.14580
the bus no.	31	magnitude	1.057112	angle	1.22667
the bus no.	32	magnitude	1.018520	angle	.00000
the bus no.	33	magnitude	1.081779	angle	2.49954
the bus no.	34	magnitude	1.090287	angle	1.31478
the bus no.	35	magnitude	1.093421	angle	4.56030
the bus no.	36	magnitude	1.082739	angle	7.31346
the bus no.	37	magnitude	1.074315	angle	2.03090
the bus no.	38	magnitude	1.081552	angle	6.36256
the bus no.	39	magnitude	1.047484	angle	-8.77957

APPENDIX FOUR A

The results under the loss of a line contengency - demand cut restricts to real load.

****
the final report of epri 39 buses system under loss of line 4 - 14

the real demand will be cut .00008 at the bus 4
the real demand will be cut .19252 at the bus 7
the total real demand is cut by amount of .19260

****

the line flow report

from bus 1 to bus 2 real power -1.145573 reactive power -.346356

from bus 1 to bus 39 real power 1.145505 reactive power .346117

****

the line flow report

from bus 2 to bus 1 real power 1.149711 reactive power -.383734

from bus 2 to bus 3 real power 3.849408 reactive power 1.074661

from bus 2 to bus 25 real power -2.498600 reactive power .631432

from bus 2 to bus 30 real power -2.499965 reactive power -1.321400

****

the line flow report

from bus 3 to bus 2 real power -3.830449 reactive power -1.136346

from bus 3 to bus 4 real power 1.597879 reactive power 1.561219

from bus 3 to bus 18 real power -.987633 reactive power -.449581

****

the line flow report

from bus 4 to bus 3 real power -1.591374 reactive power -1.684356

from bus 4 to bus 5 real power -3.408525 reactive power -.155460

****

the line flow report

from bus 5 to bus 4 real power 3.417818 reactive power .169189

from bus 5 to bus 6 real power -6.382701 reactive power -.689876

from bus 5 to bus 8 real power 2.965308 reactive power .521109

****

```
the line flow report
from bus   6  to bus   5   real power   6.390856   reactive power    .751894
from bus   6  to bus   7   real power   4.319199   reactive power    .904726
from bus   6  to bus  11   real power  -5.009881   reactive power   -.880859
from bus   6  to bus  31   real power  -5.700517   reactive power   -.776038
**************************************************************************
the line flow report
from bus   7  to bus   6   real power  -4.307644   reactive power   -.841250
from bus   7  to bus   8   real power   2.162364   reactive power    .001415
**************************************************************************
the line flow report
from bus   8  to bus   5   real power  -2.958065   reactive power   -.567611
from bus   8  to bus   7   real power  -2.160484   reactive power   -.057372
from bus   8  to bus   9   real power   -.101687   reactive power  -1.141066
**************************************************************************
the line flow report
from bus   9  to bus   8   real power    .103810   reactive power    .783145
from bus   9  to bus  39   real power   -.103812   reactive power   -.783274
**************************************************************************
the line flow report
from bus  10  to bus  11   real power   4.921218   reactive power   1.344311
from bus  10  to bus  13   real power   1.579262   reactive power   -.119429
from bus  10  to bus  32   real power  -6.500791   reactive power  -1.227010
**************************************************************************
the line flow report
from bus  11  to bus   6   real power   5.027611   reactive power    .945799
from bus  11  to bus  10   real power  -4.911303   reactive power  -1.314010
from bus  11  to bus  12   real power   -.116223   reactive power    .368531
**************************************************************************
```

the line flow report

from bus 12 to bus 11 real power .116453 reactive power -.362280

from bus 12 to bus 13 real power -.201446 reactive power -.517663

****

the line flow report

from bus 13 to bus 10 real power -1.578312 reactive power .052808

from bus 13 to bus 12 real power .201936 reactive power .530987

from bus 13 to bus 14 real power 1.376486 reactive power -.582944

****

the line flow report

from bus 14 to bus 13 real power -1.374661 reactive power .421217

from bus 14 to bus 15 real power 1.374857 reactive power -.420368

****

the line flow report

from bus 15 to bus 14 real power -1.371564 reactive power .070569

from bus 15 to bus 16 real power -1.828684 reactive power -1.601423

****

the line flow report

from bus 16 to bus 15 real power 1.833428 reactive power 1.465746

from bus 16 to bus 17 real power 2.868456 reactive power -.150318

from bus 16 to bus 19 real power -4.522822 reactive power -.556908

from bus 16 to bus 21 real power -3.134884 reactive power -.009107

from bus 16 to bus 24 real power -.338136 reactive power -1.071028

****

the line flow report

from bus 17 to bus 16 real power -2.863214 reactive power .069587

from bus 17 to bus 18 real power 2.573266 reactive power .440260

from bus 17 to bus 27 real power .290114 reactive power -.509156

****

the line flow report

from bus 18 to bus 3 real power .988747 reactive power .232151

from bus 18 to bus 17 real power -2.568875 reactive power -.532775

****

the line flow report

from bus 19 to bus 16 real power 4.552819 reactive power .582724

from bus 19 to bus 20 real power 1.743080 reactive power .008701

from bus 19 to bus 33 real power -6.295845 reactive power -.590819

****

the line flow report

from bus 20 to bus 19 real power -1.741207 reactive power .028222

from bus 20 to bus 34 real power -5.058792 reactive power -1.058204

****

the line flow report

from bus 21 to bus 16 real power 3.142049 reactive power -.150469

from bus 21 to bus 22 real power -5.882009 reactive power -.999286

****

the line flow report

from bus 22 to bus 21 real power 5.907652 reactive power 1.160193

from bus 22 to bus 23 real power .336706 reactive power .432386

from bus 22 to bus 35 real power -6.244387 reactive power -1.593303

****

the line flow report

from bus 23 to bus 22 real power -.336493 reactive power -.638824

from bus 23 to bus 24 real power 3.447667 reactive power .046207

from bus 23 to bus 36 real power -5.586172 reactive power -.253354

****

the line flow report

from bus 24 to bus 16 real power .338459 reactive power 1.003006

from bus 24 to bus 23 real power -3.424432 reactive power -.081426

****

the line flow report

from bus 25 to bus 2 real power 2.540880 reactive power -.744397

from bus 25 to bus 26 real power .603902 reactive power -.317560

from bus 25 to bus 37 real power -5.385185 reactive power .589345

****

the line flow report

from bus 26 to bus 25 real power -.602878 reactive power -.257066

from bus 26 to bus 27 real power 2.528756 reactive power .735110

from bus 26 to bus 28 real power -1.412019 reactive power -.305339

from bus 26 to bus 29 real power -1.903959 reactive power -.343215

****

the line flow report

from bus 27 to bus 17 real power -.289883 reactive power .157642

from bus 27 to bus 26 real power -2.519965 reactive power -.912027

****

the line flow report

from bus 28 to bus 26 real power 1.419617 reactive power -.500953

from bus 28 to bus 29 real power -3.479588 reactive power .225174

****

the line flow report

from bus 29 to bus 26 real power 1.922391 reactive power -.629134

from bus 29 to bus 28 real power 3.494592 reactive power -.347904

from bus 29 to bus 38 real power -8.251959 reactive power .708146

****

the line flow report

from bus 30 to bus 2 real power 2.499965 reactive power 1.450882

****

the line flow report

from bus 31 to bus 6 real power 5.700517 reactive power 1.589313

****

the line flow report

from bus 32 to bus 10 real power 6.500791 reactive power 2.057337

****

the line flow report

from bus 33 to bus 19 real power 6.320483 reactive power 1.090624

****

the line flow report

from bus 34 to bus 20 real power 5.079994 reactive power 1.482237

****

the line flow report

from bus 35 to bus 22 real power 6.244387 reactive power 2.113314

****

the line flow report

from bus 36 to bus 23 real power 5.599991 reactive power 1.005115

****

the line flow report

from bus 37 to bus 25 real power 5.400614 reactive power .007250

****

the line flow report

from bus 38 to bus 29 real power 8.299953 reactive power .227762

****

the line flow report

from bus 39 to bus 1 real power -1.143800 reactive power -1.122127

from bus 39 to bus 9 real power .103842 reactive power -.497689

****

the associated system real power losses .42640

****

the bus voltage report

the bus no.	1	magnitude	1.054129	angle	-8.95493
the bus no.	2	magnitude	1.057251	angle	-6.52612
the bus no.	3	magnitude	1.036496	angle	-9.48373
the bus no.	4	magnitude	1.000437	angle	-11.24389
the bus no.	5	magnitude	1.005236	angle	-8.76148
the bus no.	6	magnitude	1.008367	angle	-7.83097
the bus no.	7	magnitude	.997774	angle	-10.06154
the bus no.	8	magnitude	.996771	angle	-10.63366
the bus no.	9	magnitude	1.031680	angle	-10.55000
the bus no.	10	magnitude	1.026736	angle	-4.44340
the bus no.	11	magnitude	1.019226	angle	-5.57179
the bus no.	12	magnitude	1.003695	angle	-5.25561
the bus no.	13	magnitude	1.026482	angle	-4.81434
the bus no.	14	magnitude	1.030212	angle	-5.59161
the bus no.	15	magnitude	1.032991	angle	-7.22001
the bus no.	16	magnitude	1.048438	angle	-6.38250
the bus no.	17	magnitude	1.047457	angle	-7.71734
the bus no.	18	magnitude	1.041913	angle	-8.80635
the bus no.	19	magnitude	1.065869	angle	-1.88798
the bus no.	20	magnitude	1.064850	angle	-3.10206
the bus no.	21	magnitude	1.049924	angle	-4.17369
the bus no.	22	magnitude	1.068680	angle	.00000
the bus no.	23	magnitude	1.063663	angle	-.14666
the bus no.	24	magnitude	1.054358	angle	-6.29518
the bus no.	25	magnitude	1.068287	angle	-5.18273
the bus no.	26	magnitude	1.067387	angle	-6.16690

the bus no.	27	magnitude	1.052605	angle	-8.00064
the bus no.	28	magnitude	1.068772	angle	-2.77336
the bus no.	29	magnitude	1.069292	angle	-.11243
the bus no.	30	magnitude	1.080721	angle	-4.25648
the bus no.	31	magnitude	1.037280	angle	.00000
the bus no.	32	magnitude	1.058241	angle	2.42914
the bus no.	33	magnitude	1.081103	angle	2.54110
the bus no.	34	magnitude	1.090302	angle	1.34914
the bus no.	35	magnitude	1.093198	angle	4.38355
the bus no.	36	magnitude	1.082222	angle	7.43196
the bus no.	37	magnitude	1.064991	angle	1.13973
the bus no.	38	magnitude	1.071976	angle	6.36429
the bus no.	39	magnitude	1.035289	angle	-10.41854

APPENDIX FOUR B

The results under the loss of a line contengency - demand cut includes the reactive load.

```
******************************************************************************
the final report of epri 39 bu   ses system

the reactive demand will be cut    0.269000   at the bus 23
the reactive.demand will be cut    0.846000   at the bus 28
the reactive demand will be cut    0.0583357  at the bus 29
the total real demand is cut by amount of 0.0000

******************************************************************************

the line flow report

from bus   1  to bus   2   real power  -1.174668   reactive power   -.521115

from bus   1  to bus  39   real power   1.174621   reactive power    .520901

******************************************************************************

the line flow report

from bus   2  to bus   1   real power   1.179366   reactive power   -.162777

from bus   2  to bus   3   real power   3.851047   reactive power   1.046141

from bus   2  to bus  25   real power  -2.529979   reactive power    .433156

from bus   2  to bus  30   real power  -2.499984   reactive power  -1.315850

******************************************************************************

the line flow report

from bus   3  to bus   2   real power  -3.831315   reactive power  -1.086566

from bus   3  to bus   4   real power   1.698618   reactive power   1.905130

from bus   3  to bus  18   real power  -1.087285   reactive power   -.842558

******************************************************************************

the line flow report

from bus   4  to bus   3   real power  -1.689808   reactive power  -1.978519

from bus   4  to bus   5   real power  -3.310161   reactive power    .138532

******************************************************************************

the line flow report

from bus   5  to bus   4   real power   3.319523   reactive power   -.115005

from bus   5  to bus   6   real power  -6.354269   reactive power   -.769001

from bus   5  to bus   8   real power   3.034696   reactive power    .883918

******************************************************************************
```

```
the line flow report
from bus   6  to bus   5   real power   6.362962   reactive power    .840982
from bus   6  to bus   7   real power   4.417631   reactive power   1.538347
from bus   6  to bus  11   real power  -5.080104   reactive power  -1.173854
from bus   6  to bus  31   real power  -5.700529   reactive power  -1.205459
*****************************************************************************
the line flow report
from bus   7  to bus   6   real power  -4.403684   reactive power  -1.429844
from bus   7  to bus   8   real power   2.065812   reactive power   -.316255
*****************************************************************************
the line flow report
from bus   8  to bus   5   real power  -3.026102   reactive power   -.900786
from bus   8  to bus   7   real power  -2.063913   reactive power    .266601
from bus   8  to bus   9   real power   -.130115   reactive power  -1.131794
*****************************************************************************
the line flow report
from bus   9  to bus   8   real power    .132457   reactive power    .806322
from bus   9  to bus  39   real power   -.132449   reactive power   -.806322
*****************************************************************************
the line flow report
from bus  10  to bus  11   real power   4.989776   reactive power   1.657676
from bus  10  to bus  13   real power   1.510979   reactive power   -.479074
from bus  10  to bus  32   real power  -6.500798   reactive power  -1.178549
*****************************************************************************
the line flow report
from bus  11  to bus   6   real power   5.100054   reactive power   1.273913
from bus  11  to bus  10   real power  -4.978599   reactive power  -1.609310
from bus  11  to bus  12   real power   -.121387   reactive power    .335380
*****************************************************************************
```

the line flow report

from bus 12 to bus 11 real power .121596 reactive power -.329706

from bus 12 to bus 13 real power -.206599 reactive power -.550284

****

the line flow report

from bus 13 to bus 10 real power -1.509981 reactive power .417260

from bus 13 to bus 12 real power .207182 reactive power .566151

from bus 13 to bus 14 real power 1.302878 reactive power -.983228

****

the line flow report

from bus 14 to bus 13 real power -1.300619 reactive power .835507

from bus 14 to bus 15 real power 1.300470 reactive power -.835843

****

the line flow report

from bus 15 to bus 14 real power -1.296712 reactive power .506072

from bus 15 to bus 16 real power -1.903059 reactive power -2.035066

****

the line flow report

from bus 16 to bus 15 real power 1.909489 reactive power 1.921415

from bus 16 to bus 17 real power 3.005102 reactive power .282052

from bus 16 to bus 19 real power -4.521090 reactive power -.711440

from bus 16 to bus 21 real power -3.265735 reactive power -.386409

from bus 16 to bus 24 real power -.421687 reactive power -1.428848

****

the line flow report

from bus 17 to bus 16 real power -2.999154 reactive power -.350395

from bus 17 to bus 18 real power 2.674338 reactive power .864834

from bus 17 to bus 27 real power .324578 reactive power -.515015

****

the line flow report

from bus 18 to bus 3 real power 1.089126 reactive power .642850

from bus 18 to bus 17 real power -2.669074 reactive power -.942803

****

the line flow report

from bus 19 to bus 16 real power 4.551880 reactive power .752510

from bus 19 to bus 20 real power 1.743587 reactive power -.168374

from bus 19 to bus 33 real power -6.295525 reactive power -.584558

****

the line flow report

from bus 20 to bus 19 real power -1.741670 reactive power .206156

from bus 20 to bus 34 real power -5.058270 reactive power -1.235342

****

the line flow report

from bus 21 to bus 16 real power 3.273698 reactive power .244428

from bus 21 to bus 22 real power -6.013767 reactive power -1.394751

****

the line flow report

from bus 22 to bus 21 real power 6.041428 reactive power 1.592237

from bus 22 to bus 23 real power .422026 reactive power -.031658

from bus 22 to bus 35 real power -6.463422 reactive power -1.559608

****

the line flow report

from bus 23 to bus 22 real power -.421929 reactive power -.177628

from bus 23 to bus 24 real power 3.533233 reactive power .437173

from bus 23 to bus 36 real power -5.586289 reactive power -.259469

****

the line flow report

from bus 24 to bus 16 real power .422277 reactive power 1.366587

from bus 24 to bus 23 real power -3.508368 reactive power -.445135

****

the line flow report

from bus 25 to bus 2 real power 2.573584 reactive power -.537718

from bus 25 to bus 26 real power .570583 reactive power -.548818

from bus 25 to bus 37 real power -5.384529 reactive power .614143

****

the line flow report

from bus 26 to bus 25 real power -.569425 reactive power -.005743

from bus 26 to bus 27 real power 2.494503 reactive power .758633

from bus 26 to bus 28 real power -1.413591 reactive power -.450291

from bus 26 to bus 29 real power -1.901502 reactive power -.472839

****

the line flow report

from bus 27 to bus 17 real power -.324305 reactive power .173417

from bus 27 to bus 26 real power -2.485659 reactive power -.928102

****

the line flow report

from bus 28 to bus 26 real power 1.421328 reactive power -.338046

from bus 28 to bus 29 real power -3.481351 reactive power .120211

****

the line flow report

from bus 29 to bus 26 real power 1.920106 reactive power -.477628

from bus 29 to bus 28 real power 3.496469 reactive power -.238732

from bus 29 to bus 38 real power -8.251567 reactive power .716159

****

the line flow report

from bus 30 to bus 2 real power 2.499984 reactive power 1.450939

****

the line flow report

from bus 31 to bus 6 real power 5.700529 reactive power 2.100172

****

the line flow report

from bus 32 to bus 10 real power 6.500798 reactive power 2.056935

****

the line flow report

from bus 33 to bus 19 real power 6.320488 reactive power 1.090957

****

the line flow report

from bus 34 to bus 20 real power 5.079985 reactive power 1.669654

****

the line flow report

from bus 35 to bus 22 real power 6.463422 reactive power 2.112664

****

the line flow report

from bus 36 to bus 23 real power 5.599992 reactive power 1.004906

****

the line flow report

from bus 37 to bus 25 real power 5.400596 reactive power .007109

****

the line flow report

from bus 38 to bus 29 real power 8.300005 reactive power .228381

****

the line flow report

from bus 39 to bus 1 real power -1.172505 reactive power -1.235073

from bus 39 to bus 9 real power .132513 reactive power -.384822

****

the associated system real power losses .45280

the associated system reactive power losses 9.59000

****

the bus voltage report

the bus no.	1	magnitude	1.022797	angle	-9.40101
the bus no.	2	magnitude	1.034125	angle	-6.81437
the bus no.	3	magnitude	1.013480	angle	-9.91075
the bus no.	4	magnitude	.969438	angle	-11.86798
the bus no.	5	magnitude	.970500	angle	-9.27700
the bus no.	6	magnitude	.973961	angle	-8.28459
the bus no.	7	magnitude	.957070	angle	-10.72474
the bus no.	8	magnitude	.957607	angle	-11.32584
the bus no.	9	magnitude	.994215	angle	-11.17412
the bus no.	10	magnitude	.996921	angle	-4.64029
the bus no.	11	magnitude	.987832	angle	-5.84929
the bus no.	12	magnitude	.973278	angle	-5.50263
the bus no.	13	magnitude	.998247	angle	-5.02456
the bus no.	14	magnitude	1.006249	angle	-5.82124
the bus no.	15	magnitude	1.018371	angle	-7.46481
the bus no.	16	magnitude	1.038140	angle	-6.59023
the bus no.	17	magnitude	1.033391	angle	-8.00553
the bus no.	18	magnitude	1.024365	angle	-9.15713
the bus no.	19	magnitude	1.058738	angle	-2.03536
the bus no.	20	magnitude	1.060026	angle	-3.26988

the bus no.	21	magnitude	1.044752	angle	-4.27113
the bus no.	22	magnitude	1.069140	angle	.00000
the bus no.	23	magnitude	1.068247	angle	-.20103
the bus no.	24	magnitude	1.046176	angle	-6.48101
the bus no.	25	magnitude	1.047286	angle	-5.47386
the bus no.	26	magnitude	1.053953	angle	-6.47500
the bus no.	27	magnitude	1.038746	angle	-8.32907
the bus no.	28	magnitude	1.062383	angle	-3.04805
the bus no.	29	magnitude	1.064432	angle	-.36510
the bus no.	30	magnitude	1.058061	angle	-4.44420
the bus no.	31	magnitude	1.015500	angle	.00000
the bus no.	32	magnitude	1.028864	angle	2.64205
the bus no.	33	magnitude	1.074034	angle	2.45300
the bus no.	34	magnitude	1.088609	angle	1.20018
the bus no.	35	magnitude	1.093423	angle	4.53474
the bus no.	36	magnitude	1.086800	angle	7.31292
the bus no.	37	magnitude	1.043645	angle	1.10840
the bus no.	38	magnitude	1.067074	angle	6.17136
the bus no.	39	magnitude	.999715	angle	-10.99553

APPENDIX FIVE

The results of the minimum of real power loss.

```
********************************************************************************
the final report of epri 39 buses system

the slack bus is  31
real power limit    5.70050
reactive power limit    2.10000
real part of the bus voltage is less than   1.090
********************************************************************************

the optimal value of the total  real power losses under the stated condition

is .38784

********************************************************************************

the line flow report

from bus   1  to bus   2   real power  -1.179095   reactive power   -.200546

from bus   1  to bus  39   real power   1.179121   reactive power    .200688

********************************************************************************

the line flow report

from bus   2  to bus   1   real power   1.183391   reactive power   -.562477

from bus   2  to bus   3   real power   3.648416   reactive power    .815858

from bus   2  to bus  25   real power  -2.331772   reactive power    .568304

from bus   2  to bus  30   real power  -2.499975   reactive power   -.821699

********************************************************************************

the line flow report

from bus   3  to bus   2   real power  -3.632467   reactive power   -.924591

from bus   3  to bus   4   real power    .723791   reactive power   1.058176

from bus   3  to bus  18   real power   -.311226   reactive power   -.156875

********************************************************************************

the line flow report

from bus   4  to bus   3   real power   -.721570   reactive power  -1.265182

from bus   4  to bus   5   real power  -1.665360   reactive power   -.074239

from bus   4  to bus  14   real power  -2.613298   reactive power   -.501559

********************************************************************************

the line flow report

from bus   5  to bus   4   real power   1.667426   reactive power   -.036992
```

from bus 5 to bus 6 real power -4.844216 reactive power -.492715

from bus 5 to bus 8 real power 3.176741 reactive power .530187

****

the line flow report

from bus 6 to bus 5 real power 4.848617 reactive power .503095

from bus 6 to bus 7 real power 4.265453 reactive power .853376

from bus 6 to bus 11 real power -3.413409 reactive power -.477719

from bus 6 to bus 31 real power -5.700514 reactive power -.878196

****

the line flow report

from bus 7 to bus 6 real power -4.254893 reactive power -.812478

from bus 7 to bus 8 real power 1.917262 reactive power -.026879

****

the line flow report

from bus 8 to bus 5 real power -3.168968 reactive power -.579005

from bus 8 to bus 7 real power -1.915875 reactive power -.039855

from bus 8 to bus 9 real power -.135546 reactive power -1.148189

****

the line flow report

from bus 9 to bus 8 real power .137532 reactive power .763184

from bus 9 to bus 39 real power -.137541 reactive power -.763163

****

the line flow report

from bus 10 to bus 11 real power 3.449878 reactive power .814545

from bus 10 to bus 13 real power 3.051039 reactive power .447893

from bus 10 to bus 32 real power -6.500820 reactive power -1.262302

****

the line flow report

from bus 11 to bus 6 real power 3.421057 reactive power .416278

from bus 11 to bus 10 real power -3.445299 reactive power -.845392

from bus 11 to bus 12 real power .024173 reactive power .428640

****

the line flow report

from bus 12 to bus 11 real power -.023903 reactive power -.421310

from bus 12 to bus 13 real power -.061075 reactive power -.458515

****

the line flow report

from bus 13 to bus 10 real power -3.047579 reactive power -.490892

from bus 13 to bus 12 real power .061399 reactive power .467323

from bus 13 to bus 14 real power 2.986032 reactive power .023302

****

the line flow report

from bus 14 to bus 4 real power 2.618522 reactive power .436303

from bus 14 to bus 13 real power -2.978704 reactive power -.129454

from bus 14 to bus 15 real power .360321 reactive power -.306821

****

the line flow report

from bus 15 to bus 14 real power -.360088 reactive power -.089905

from bus 15 to bus 16 real power -2.839737 reactive power -1.439346

****

the line flow report

from bus 16 to bus 15 real power 2.847866 reactive power 1.334623

from bus 16 to bus 17 real power 2.057845 reactive power -.637243

from bus 16 to bus 19 real power -4.524578 reactive power -.328324

from bus 16 to bus 21 real power -3.270078 reactive power .247440

from bus 16 to bus 24 real power -.405969 reactive power -.942607

****

the line flow report

from bus 17 to bus 16 real power -2.055013 reactive power .521840

from bus 17 to bus 18 real power 1.893819 reactive power .095221

from bus 17 to bus 27 real power .161699 reactive power -.615888

****

the line flow report

from bus 18 to bus 3 real power .311322 reactive power -.082683

from bus 18 to bus 17 real power -1.891583 reactive power -.218000

****

the line flow report

from bus 19 to bus 16 real power 4.553740 reactive power .337675

from bus 19 to bus 20 real power 1.742585 reactive power .261131

from bus 19 to bus 33 real power -6.296228 reactive power -.598024

****

the line flow report

from bus 20 to bus 19 real power -1.740699 reactive power -.223947

from bus 20 to bus 34 real power -5.059330 reactive power -.806390

****

the line flow report

from bus 21 to bus 16 real power 3.277794 reactive power -.403325

from bus 21 to bus 22 real power -6.017829 reactive power -.747266

****

the line flow report

from bus 22 to bus 21 real power 6.043933 reactive power .912329

from bus 22 to bus 23 real power .456154 reactive power .274515

from bus 22 to bus 35 real power -6.500022 reactive power -1.185407

****

the line flow report

from bus 23 to bus 22 real power -.455972 reactive power -.483891

from bus 23 to bus 24 real power 3.515572 reactive power -.086614

from bus 23 to bus 36 real power -5.534646 reactive power -.276233

****

the line flow report

from bus 24 to bus 16 real power .406232 reactive power .870942

from bus 24 to bus 23 real power -3.491821 reactive power .052650

****

the line flow report

from bus 25 to bus 2 real power 2.367126 reactive power -.695973

from bus 25 to bus 26 real power .778582 reactive power -.344265

from bus 25 to bus 37 real power -5.385702 reactive power .568506

****

the line flow report

from bus 26 to bus 25 real power -.776937 reactive power -.245528

from bus 26 to bus 27 real power 2.658254 reactive power .829168

from bus 26 to bus 28 real power -1.389853 reactive power -.356819

from bus 26 to bus 29 real power -1.881263 reactive power -.396119

****

the line flow report

from bus 27 to bus 17 real power -.161453 reactive power .252757

from bus 27 to bus 26 real power -2.648761 reactive power -1.008362

****

the line flow report

from bus 28 to bus 26 real power 1.396925 reactive power -.489032

from bus 28 to bus 29 real power -3.456943 reactive power .212964

****

the line flow report

from bus 29 to bus 26 real power 1.898558 reactive power -.633219

from bus 29 to bus 28 real power 3.471193 reactive power -.354970

from bus 29 to bus 38 real power -8.204783 reactive power .661821

****

the line flow report

from bus 30 to bus 2 real power 2.499975 reactive power .929670

****

the line flow report

from bus 31 to bus 6 real power 5.700514 reactive power 1.647344

****

the line flow report

from bus 32 to bus 10 real power 6.500820 reactive power 2.057254

****

the line flow report

from bus 33 to bus 19 real power 6.320531 reactive power 1.091021

****

the line flow report

from bus 34 to bus 20 real power 5.079998 reactive power 1.219722

****

the line flow report

from bus 35 to bus 22 real power 6.500022 reactive power 1.726401

****

the line flow report

from bus 36 to bus 23 real power 5.548043 reactive power 1.005013

****

the line flow report

from bus 37 to bus 25 real power 5.400578 reactive power .006685

****

the line flow report

from bus 38 to bus 29 real power 8.250407 reactive power .227838

****

the line flow report

from bus 39 to bus 1 real power -1.177581 reactive power -1.025408

from bus 39 to bus 9 real power .137564 reactive power -.594682

****

the associated system reactive power losses 8.49237

****

the bus voltage report

the bus no.	1	magnitude	1.080566	angle	-8.15479
the bus no.	2	magnitude	1.077460	angle	-5.73350
the bus no.	3	magnitude	1.060710	angle	-8.43349
the bus no.	4	magnitude	1.036155	angle	-9.15705
the bus no.	5	magnitude	1.037672	angle	-8.02113
the bus no.	6	magnitude	1.039851	angle	-7.35731
the bus no.	7	magnitude	1.029972	angle	-9.42774
the bus no.	8	magnitude	1.029198	angle	-9.90412
the bus no.	9	magnitude	1.062895	angle	-9.76046
the bus no.	10	magnitude	1.050386	angle	-5.14108
the bus no.	11	magnitude	1.045664	angle	-5.89711
the bus no.	12	magnitude	1.027795	angle	-5.91661
the bus no.	13	magnitude	1.047298	angle	-5.81424
the bus no.	14	magnitude	1.043990	angle	-7.38930
the bus no.	15	magnitude	1.045629	angle	-7.80984
the bus no.	16	magnitude	1.060452	angle	-6.49301

the bus no.	17	magnitude	1.063955	angle	-7.44308
the bus no.	18	magnitude	1.061498	angle	-8.22490
the bus no.	19	magnitude	1.073384	angle	-2.06018
the bus no.	20	magnitude	1.069121	angle	-3.25178
the bus no.	21	magnitude	1.058776	angle	-4.22368
the bus no.	22	magnitude	1.074220	angle	.00000
the bus no.	23	magnitude	1.070567	angle	-.20678
the bus no.	24	magnitude	1.065601	angle	-6.38532
the bus no.	25	magnitude	1.087637	angle	-4.52947
the bus no.	26	magnitude	1.086807	angle	-5.75487
the bus no.	27	magnitude	1.070817	angle	-7.61213
the bus no.	28	magnitude	1.089483	angle	-2.54372
the bus no.	29	magnitude	1.090000	angle	.00000
the bus no.	30	magnitude	1.092072	angle	-3.52961
the bus no.	31	magnitude	1.069780	angle	.00317
the bus no.	32	magnitude	1.081527	angle	1.43076
the bus no.	33	magnitude	1.088563	angle	2.30765
the bus no.	34	magnitude	1.090237	angle	1.19352
the bus no.	35	magnitude	1.093429	angle	4.53883
the bus no.	36	magnitude	1.089268	angle	7.20369
the bus no.	37	magnitude	1.084616	angle	1.56726
the bus no.	38	magnitude	1.092929	angle	6.19348
the bus no.	39	magnitude	1.065036	angle	-9.59074

APPENDIX SIX

The results of the minimum of reactive power loss.

```
***************************************************************************
the final report of epri 39 buses system

the slack bus is  31
real power limit    5.70050
reactive power limit    2.10000
real part of the bus voltage is less than   1.090
***************************************************************************

the optimal value of the total reactive power losses under the stated

condition is     8.49205

***************************************************************************

the line flow report

from bus   1  to bus   2   real power  -1.179095   reactive power   -.200546

from bus   1  to bus  39   real power   1.179117   reactive power    .200689

***************************************************************************

the line flow report

from bus   2  to bus   1   real power   1.183391   reactive power   -.562477

from bus   2  to bus   3   real power   3.648416   reactive power    .815858

from bus   2  to bus  25   real power  -2.331772   reactive power    .568304

from bus   2  to bus  30   real power  -2.499969   reactive power   -.821699

***************************************************************************

the line flow report

from bus   3  to bus   2   real power  -3.632467   reactive power   -.924591

from bus   3  to bus   4   real power    .723791   reactive power   1.058176

from bus   3  to bus  18   real power   -.311249   reactive power   -.156870

***************************************************************************

the line flow report

from bus   4  to bus   3   real power   -.721570   reactive power  -1.265182

from bus   4  to bus   5   real power  -1.665360   reactive power   -.074239

from bus   4  to bus  14   real power  -2.613258   reactive power   -.501568

***************************************************************************
```

the line flow report

from bus 5 to bus 4 real power 1.667426 reactive power -.036992

from bus 5 to bus 6 real power -4.844216 reactive power -.492715

from bus 5 to bus 8 real power 3.176741 reactive power .530187

****

the line flow report

from bus 6 to bus 5 real power 4.848617 reactive power .503095

from bus 6 to bus 7 real power 4.265453 reactive power .853376

from bus 6 to bus 11 real power -3.413409 reactive power -.477719

from bus 6 to bus 31 real power -5.700514 reactive power -.878196

****

the line flow report

from bus 7 to bus 6 real power -4.254893 reactive power -.812478

from bus 7 to bus 8 real power 1.917262 reactive power -.026879

****

the line flow report

from bus 8 to bus 5 real power -3.168968 reactive power -.579005

from bus 8 to bus 7 real power -1.915875 reactive power -.039855

from bus 8 to bus 9 real power -.135546 reactive power -1.148189

****

the line flow report

from bus 9 to bus 8 real power .137532 reactive power .763184

from bus 9 to bus 39 real power -.137545 reactive power -.763163

****

the line flow report

from bus 10 to bus 11 real power 3.449878 reactive power .814545

from bus 10 to bus 13 real power 3.051015 reactive power .447898

from bus 10 to bus 32 real power -6.500778 reactive power -1.261777

```
******************************************************************************
the line flow report
from bus  11  to bus   6  real power   3.421057  reactive power   .416278
from bus  11  to bus  10  real power  -3.445299  reactive power  -.845392
from bus  11  to bus  12  real power    .024170  reactive power   .428641
******************************************************************************
the line flow report
from bus  12  to bus  11  real power   -.023901  reactive power  -.421310
from bus  12  to bus  13  real power   -.061075  reactive power  -.458514
******************************************************************************
the line flow report
from bus  13  to bus  10  real power  -3.047554  reactive power  -.490891
from bus  13  to bus  12  real power    .061399  reactive power   .467323
from bus  13  to bus  14  real power   2.986093  reactive power   .023293
******************************************************************************
the line flow report
from bus  14  to bus   4  real power   2.618482  reactive power   .436309
from bus  14  to bus  13  real power  -2.978765  reactive power  -.129441
from bus  14  to bus  15  real power    .360292  reactive power  -.306816
******************************************************************************
the line flow report
from bus  15  to bus  14  real power   -.360059  reactive power  -.089910
from bus  15  to bus  16  real power  -2.839769  reactive power -1.439335
******************************************************************************
the line flow report
from bus  16  to bus  15  real power   2.847899  reactive power  1.334614
from bus  16  to bus  17  real power   2.057950  reactive power  -.637266
from bus  16  to bus  19  real power  -4.524557  reactive power  -.328331
```

from bus 16 to bus 21 real power -3.270048 reactive power .247432

from bus 16 to bus 24 real power -.405825 reactive power -.942635

****

the line flow report

from bus 17 to bus 16 real power -2.055118 reactive power .521861

from bus 17 to bus 18 real power 1.893717 reactive power .095240

from bus 17 to bus 27 real power .161662 reactive power -.615882

****

the line flow report

from bus 18 to bus 3 real power .311345 reactive power -.082691

from bus 18 to bus 17 real power -1.891482 reactive power -.218027

****

the line flow report

from bus 19 to bus 16 real power 4.553718 reactive power .337678

from bus 19 to bus 20 real power 1.742593 reactive power .261130

from bus 19 to bus 33 real power -6.296221 reactive power -.598024

****

the line flow report

from bus 20 to bus 19 real power -1.740706 reactive power -.223947

from bus 20 to bus 34 real power -5.059337 reactive power -.806389

****

the line flow report

from bus 21 to bus 16 real power 3.277763 reactive power -.403321

from bus 21 to bus 22 real power -6.017829 reactive power -.747266

****

the line flow report

from bus 22 to bus 21 real power 6.043933 reactive power .912329

from bus 22 to bus 23 real power .456154 reactive power .274515

from bus 22 to bus 35 real power -6.500022 reactive power -1.185407

****

the line flow report

from bus 23 to bus 22 real power -.455972 reactive power -.483891

from bus 23 to bus 24 real power 3.515585 reactive power -.086615

from bus 23 to bus 36 real power -5.534654 reactive power -.276233

****

the line flow report

from bus 24 to bus 16 real power .406089 reactive power .870969

from bus 24 to bus 23 real power -3.491833 reactive power .052654

****

the line flow report

from bus 25 to bus 2 real power 2.367126 reactive power -.695973

from bus 25 to bus 26 real power .778595 reactive power -.344268

from bus 25 to bus 37 real power -5.385707 reactive power .568506

****

the line flow report

from bus 26 to bus 25 real power -.776950 reactive power -.245524

from bus 26 to bus 27 real power 2.658219 reactive power .829176

from bus 26 to bus 28 real power -1.389862 reactive power -.356816

from bus 26 to bus 29 real power -1.881269 reactive power -.396116

****

the line flow report

from bus 27 to bus 17 real power -.161416 reactive power .252744

from bus 27 to bus 26 real power -2.648726 reactive power -1.008376

****

the line flow report

from bus 28 to bus 26 real power 1.396934 reactive power -.489033

from bus 28 to bus 29 real power -3.456943 reactive power .212964

****

the line flow report

from bus 29 to bus 26 real power 1.898565 reactive power -.633219

from bus 29 to bus 28 real power 3.471193 reactive power -.354970

from bus 29 to bus 38 real power -8.204756 reactive power .661820

****

the line flow report

from bus 30 to bus 2 real power 2.499969 reactive power .929669

****

the line flow report

from bus 31 to bus 6 real power 5.700514 reactive power 1.647344

****

the line flow report

from bus 32 to bus 10 real power 6.500778 reactive power 2.056700

****

the line flow report

from bus 33 to bus 19 real power 6.320523 reactive power 1.091021

****

the line flow report

from bus 34 to bus 20 real power 5.080004 reactive power 1.219722

****

the line flow report

from bus 35 to bus 22 real power 6.500022 reactive power 1.726401

****

the line flow report

from bus 36 to bus 23 real power 5.548050 reactive power 1.005018

****

the line flow report

from bus 37 to bus 25 real power 5.400583 reactive power .006685

```
*****************************************************************************
the line flow report
from bus  38  to bus  29   real power   8.250380   reactive power    .227832
*****************************************************************************
the line flow report
from bus  39  to bus   1   real power  -1.177577   reactive power  -1.025408
from bus  39  to bus   9   real power    .137568   reactive power   -.594682
*****************************************************************************
the associated system real power losses       .38789
*****************************************************************************
the bus voltage report
the bus no.   1   magnitude   1.080566   angle   -8.15479
the bus no.   2   magnitude   1.077460   angle   -5.73350
the bus no.   3   magnitude   1.060710   angle   -8.43349
the bus no.   4   magnitude   1.036155   angle   -9.15705
the bus no.   5   magnitude   1.037672   angle   -8.02113
the bus no.   6   magnitude   1.039851   angle   -7.35731
the bus no.   7   magnitude   1.029972   angle   -9.42774
the bus no.   8   magnitude   1.029198   angle   -9.90412
the bus no.   9   magnitude   1.062895   angle   -9.76046
the bus no.  10   magnitude   1.050386   angle   -5.14108
the bus no.  11   magnitude   1.045664   angle   -5.89711
the bus no.  12   magnitude   1.027795   angle   -5.91660
the bus no.  13   magnitude   1.047298   angle   -5.81423
the bus no.  14   magnitude   1.043990   angle   -7.38933
the bus no.  15   magnitude   1.045629   angle   -7.80983
the bus no.  16   magnitude   1.060452   angle   -6.49299
the bus no.  17   magnitude   1.063955   angle   -7.44311
```

the bus no.	18	magnitude	1.061498	angle	-8.22489
the bus no.	19	magnitude	1.073384	angle	-2.06018
the bus no.	20	magnitude	1.069121	angle	-3.25178
the bus no.	21	magnitude	1.058776	angle	-4.22368
the bus no.	22	magnitude	1.074220	angle	.00000
the bus no.	23	magnitude	1.070567	angle	-.20678
the bus no.	24	magnitude	1.065601	angle	-6.38534
the bus no.	25	magnitude	1.087637	angle	-4.52947
the bus no.	26	magnitude	1.086807	angle	-5.75489
the bus no.	27	magnitude	1.070817	angle	-7.61212
the bus no.	28	magnitude	1.089483	angle	-2.54372
the bus no.	29	magnitude	1.090000	angle	.00000
the bus no.	30	magnitude	1.092072	angle	-3.52961
the bus no.	31	magnitude	1.069780	angle	.00317
the bus no.	32	magnitude	1.081517	angle	1.43078
the bus no.	33	magnitude	1.088563	angle	2.30764
the bus no.	34	magnitude	1.090237	angle	1.19352
the bus no.	35	magnitude	1.093429	angle	4.53883
the bus no.	36	magnitude	1.089268	angle	7.20370
the bus no.	37	magnitude	1.084616	angle	1.56726
the bus no.	38	magnitude	1.092929	angle	6.19346
the bus no.	39	magnitude	1.065036	angle	-9.59073