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A HEURISTIC SIMULATION AND OPTIMIZATION ALGORITHM FOR  
LARGE SCALE NATURAL GAS STORAGE VALUATION UNDER  
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SCHOOL OF INDUSTRIAL AND SYSTEMS ENGINEERING

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# Table of Contents

<b>ACKNOWLEDGMENTS</b> .....	<b>IV</b>
<b>TABLE OF CONTENTS</b> .....	<b>VIII</b>
<b>LIST OF TABLES</b> .....	<b>X</b>
<b>LIST OF FIGURES</b> .....	<b>XI</b>
<b>ABSTRACT</b> .....	<b>XII</b>
<b>CHAPTER 1: INTRODUCTION</b> .....	<b>1</b>
1.1. MOTIVATION OF THE RESEARCH.....	1
1.2. SIGNIFICANCE OF THE RESEARCH .....	5
1.3. CONTRIBUTION OF THE RESEARCH .....	5
<b>CHAPTER 2: LITERATURE REVIEW</b> .....	<b>8</b>
2.1. GENERAL CONCEPTS.....	8
2.1.1. <i>Brief History of Natural Gas</i> .....	8
2.1.2. <i>Natural Gas Supply Chain</i> .....	9
2.1.3. <i>Systems Modeling, Simulation, and Optimization</i> .....	14
2.2. NATURAL GAS STORAGE VALUATION .....	17
<b>CHAPTER 3: NATURAL GAS STORAGE FACILITY MODELING</b> .....	<b>22</b>
3.1. OVERVIEW .....	22
3.2. UNDERGROUND STORAGE .....	25
3.3. ABOVEGROUND STORAGE .....	30
3.4. GAS STORAGE LOSS.....	34
3.5. GAS STORAGE TREND.....	35
<b>CHAPTER 4: PROBLEM FORMULATION</b> .....	<b>38</b>
4.1. PROBLEM DEFINITION.....	38
4.2. DEFINITION OF TERMS AND MATHEMATICAL MODEL .....	39
4.3. MODEL COMPLEXITY.....	46
<b>CHAPTER 5: SOLUTION APPROACH</b> .....	<b>54</b>
5.1. GENERAL FRAMEWORK .....	54
5.2. HEURISTIC APPROACH .....	55
<b>CHAPTER 6: COMPUTATIONAL RESULTS AND ANALYSIS</b> .....	<b>60</b>
6.1. PRICE SIMULATION .....	60
6.2. DECISION SCENARIOS .....	61
6.3. MODEL PERFORMANCE.....	66
6.4. MODEL BEHAVIOR.....	68
6.5. COMPARISON OF THE HEURISTIC WITH OTHER MODELS.....	72



<b>CHAPTER 7: CONCLUSION AND FUTURE RESEARCH .....</b>	<b>80</b>
7.1. CONCLUSIONS.....	80
7.2. FUTURE RESEARCH DIRECTION .....	83
7.2.1. <i>Interdependency modeling</i> .....	83
7.2.2. <i>Pipelines storage optimization</i> .....	87
7.2.3. <i>Cycling</i> .....	91
7.2.4. <i>A meta-heuristic algorithm</i> .....	92
<b>REFERENCES .....</b>	<b>98</b>

## List of Tables

Table 1. Sample price simulation input parameters .....	61
Table 2. Sample optimization output data.....	61

## List of Figures

Figure 1. Natural gas supply chain schematic diagram.....	10
Figure 2. Distribution of underground storage facilities in the U.S.....	25
Figure 3. Gas injection-withdrawal cycle .....	36
Figure 4. Salt cavern and non-salt cavern injection withdrawal pattern.....	36
Figure 5. A schematic salt cavern storage facility .....	42
Figure 6. Experimental approach .....	54
Figure 7. Procedural optimization approach .....	59
Figure 8. Injection decision space for fifty realizations.....	62
Figure 9. Withdrawal decision space for fifty realizations .....	63
Figure 10. Withdrawal-injection decisions .....	64
Figure 11. Inventory level space for fifty realizations .....	65
Figure 12. Cumulative profit graph.....	66
Figure 13. Computation time comparison.....	67
Figure 14. Solution quality comparison.....	68
Figure 15. Stopping criteria scenario 1 .....	69
Figure 16. Stopping criteria scenario 2 .....	70
Figure 17. Injection-withdrawal curves sample 1 .....	71
Figure 18. Injection-withdrawal curves sample 2.....	71
Figure 19. Comparison of inventory levels for all the scenarios .....	76
Figure 20. Comparison of gas injection decision for all the scenarios .....	77
Figure 21. Comparison of gas withdrawal decision for all the scenarios .....	77
Figure 22. Cash flow comparison for all the scenarios.....	78
Figure 23. Scenario 1 and Scenario 2 decisions.....	79
Figure 24. An example of gas supply chain interdependency modeling .....	86
Figure 25. The United States natural gas pipeline network as of 2013.....	88
Figure 26. Natural gas pipeline compressor stations illustration .....	90
Figure 27. Sources and uses of natural gas in the U.S. as of 2009 .....	91
Figure 28. Crossover process .....	96
Figure 29. Imitation process.....	96

## **Abstract**

Natural gas storage valuation is an optimal scheduling of natural gas storage facilities. It is a complex predictive decision making research problem since it involves the financial decisions and the physical storage facility characteristics. The challenge arises from large scale stochastic input data sets and complex mathematical models. Research in the literature has been heavily focused on the financial facet of the valuation with little emphasis on the physical storage facility characteristics. The mathematical models and the solution approaches provided in the literature so far are also either overly simplified or are only relevant for very small scale problems. The contribution of this research is on the physical storage facility characteristics in combination with the financial aspect of the natural gas storage valuation.

A large scale stochastic non-linear natural gas storage valuation problem that includes underground and aboveground storage facilities is formulated and solved efficiently. A new heuristic simulation and optimization natural gas storage valuation algorithm that handles a very complex and large size problems is proposed. The algorithm (i) decreases significantly the computation time from hundreds of days to fractions of a second, (ii) provides a reasonable solution quality, and (iii) incorporates all the possible underground and aboveground physical gas storage facility complexities.

The research has both practical applications and mathematical significance. Practically, natural gas storage facility managers can use the models developed in

this research as decision support tools to make a predictive storage decision under uncertainty within a reasonable time. Mathematically, a novel perspective to solving a non-linear natural gas storage facilities valuation problem is provided. Such an approach can be used in a variety of applications; for instance, the algorithm can be applied to a high penetration of renewables to electric power grid and fluid flow network optimization among others.

## **Chapter 1: Introduction**

In this research, a comprehensive natural gas storage valuation mathematical model that has not been addressed in the literature is formulated. Since the model is very complex to solve as a single problem, a new heuristic algorithm is developed to decouple the problem into two sub optimization problems and solved hierarchically. The algorithm reduces the computational complexity of the model significantly at a reasonable solution quality. It also behaves in a similar fashion when compared to other models in the literature. The details of the research are presented in seven chapters.

In the first introductory chapter, motivation, significance, and the contribution of the research to the state of the art is presented. Then review of the literature and basic concepts needed to understand the research is presented in the second chapter. The core of the research formulation for the underground and aboveground storage facilities characteristics is discussed in the third chapter. The research problem is formulated in chapter four followed by the research methodology in the fifth chapter. A case study problem is solved and interpreted in terms of the research requirements in chapter six. The major research findings and directions for future research are presented in chapter seven.

### **1.1. Motivation of the Research**

The availability of the energy resources has been one of the grand challenges of our century (Bardi, 2013). The nations of our earth strive to provide the required amount of energy to keep their economy growing (Hossain, 2012). On the contrary,

the supply of the energy resources is dwindling (Spreng, Flüeler, Goldblatt, and Minsch, 2012). This necessitates the optimal use of the available resources accounting for the social, political, geographical, and environmental impacts while researching for new types of energy sources (Anadon, Bunn, Gallagher, and Jones 2009; Holdren, 1999). In addition, scientists and political leaders suggest the need for rigorous researches to transition from the use of fossil fuels to renewable energy sources. (State of the Union, 2013; Moniz et al., 2011; Greving and Gasterra , 2009; Liang, Ryvak, Sayeed, and Zhao, 2012; Biner, Boles, Cwagenberg, Gates, and Ilayian, 2014). Among all fossil fuels, natural gas has been proposed by the research scientists to be used as the transition fuel from the consumption of fossil fuels to renewables because of the following reasons.

Natural gas is environmental friendly. It is easy to use for cooking, relatively clean compared to other fossil fuels such as oil and coal. For example, coal releases 227 pounds of carbon dioxide to atmosphere per one million Btu (British thermal unit) consumption. But natural gas emits 117 pounds of carbon dioxide to provide the same amount of energy (Brown, Krupnick, and Walls, 2009; Trembath, Luke, Shellenberger, and Nordhaus, 2013). The greenecon.net presents as follows

*Natural gas, because of its low carbon content and high fuel efficiency, achieves lower carbon dioxide emissions than oil, propane, or coal. Natural gas produces 46% less carbon dioxide than coal and 10% less than oil.*

Natural gas also provides energy efficient solutions when used for systems like modern condensing boiler technology. Its large proportion of hydrogen to

carbon content improves the efficiency level of natural gas relative to oil, propane, coal, and wood (Beér, 2007).

Natural gas is abundant in wide geographic locations throughout the world. As of 2013, it is estimated that the natural gas reserve worldwide is about 5146 trillion cubic ft. In the US alone, the available natural gas reserve can last for hundred more years at the current consumption rate (Laherrère, 2004; Bary, Crotofino, Prevedel, Berger, Brown, Frantz, and Ren, 2002).

Natural gas is flexible enough to easily transport from one location to another either in gaseous form or by converting into liquid or solid. As a result, it is suitable to use as a backup energy source to fill the power generation gap created by inconsistent power production of renewable energy sources such as wind and solar (Moniz et al., 2011; Thomas and Dawe, 2003; Greving and Gasterra, 2009). Moniz et al. (2011) describe the importance as follows.

*An additional gas-fired capacity will be needed as backup if variable and intermittent renewable, especially wind, are introduced on a large scale. Policy and regulatory steps are needed to facilitate adequate capacity investment for system reliability and efficiency. These increasingly important roles for natural gas in the electricity sector call for a detailed analysis of the interdependencies of the natural gas and power generation infrastructures.*

Advantages of natural gas include the potential of easily constructing infrastructures to alleviate high transmission congestion and the capability of generators to accommodate the picking demand (Balat, 2009). The flexibility and the ease of the use of natural gas will also have the possibility of leading to an



innovation of new production process. In addition, natural gas has political significance to the United States since the country's demand will increase by 50% in 2025 (Reiten, 2003). Issues of import independencies and secure energy supply will also arise (Moniz et al., 2011; Hughes, 2011).

This requires that the natural gas supply chain operation be efficient (Selot, 2008; Tomasgard, Rømo, Fodstad, and Midthun, 2007). The storage of natural gas is a critical component of the natural gas supply chain that needs to be optimized to balance the gas supply and demand (Hoagie, Amorer, Wang, and Economides, 2013; Wang and Economides, 2012). Storage allows stable gas flow rate by keeping excess production, and filling the gap created by inconsistent power generation of inconsistent renewable energy sources such as wind (Moniz, Jacoby, Meggs, Armstrong, Cohn, Connors, ... and Kaufman, 2011). There will also be the need for new natural gas storage facilities as the existing facilities are not capable of accommodating the increase in gas inventory level because of unconventional shale gas discoveries (Bowker, 2007). In addition, it is not uncommon to see storage facilities that have poor schedule optimization schemes. For example, Baker/Cedar Creek Field in Montana is the largest storage field in the United States but it has not be efficiently used because of the dwindling production in the nearby gas fields (EIA, 2014). However, the storage scheduling task, otherwise called the storage valuation, is a very complex problem because of the high gas price volatility, the type and characteristics of the storage facility, the geographic distribution of storage facilities, the cycling effect, and the complexity of storage facility characteristics.

Unlike other physical products, the amount of gas that can be withdrawn or injected to a facility varies non-linearly based on the amount of inventory in the facility.

### **1.2. Significance of the Research**

Natural gas working storage capacity increased nearly by 2 percent in the Lower 48 states of the US between November 2011 and November 2012 (EIA 2014). Demonstrated maximum volume increased 1.8 percent to 4,265 billion cubic feet (Bcf). Design capacity increased 2.0 percent to 4,575 Bcf. By 2020, 650 Bcf of additional natural gas working capacity will be needed. In addition, the annual investment in storage of \$10 and \$20 billion is required for the next fifteen years (Boogert and Jong, 2008). The U.S. Department of Energy proposed five major areas of research to cope up with the storage need increases. One of the research areas is the innovative modeling of natural gas storage injection and withdrawal techniques (Levin, 2011). This research addresses a new solution approach to resolve the complexity associated with the injection and withdrawal technique that will contribute to the nation's research goals.

### **1.3. Contribution of the Research**

This research focuses on the integrated underground and aboveground storage facilities characteristic problem formulation and solution approach. The aboveground storages are the pipelines and the storage tanks. The underground storage facilities are the salt caverns, the depleted reservoirs, and the aquifers. Salt caverns are far away from majority of the demand regions. However, they require low base gas, and have high deliverability and high injection rates. The inventory

turnover capability of salt cavern is usually 6 to 12 times per year. They are also capable of daily production and injection. Depleted reservoirs are widely available in the United States. However, they require high base gas compared to salt caverns. On the other hand, aquifer storage facilities are located close to customers but they require the highest amount of base gas relative to salt caverns and depleted reservoirs.

The main challenge for the gas storage facilities managers is to make optimal decisions of how much gas to inject, hold, or withdraw to maximize profit over a wide time window for any combination of the storage facilities they manage. Since making storage decisions over a wide time window and multiple storage facilities will increase to the complexity of the problem, there is a need for an efficient way of solving the problem. The natural gas storage valuation optimization models developed in the literature so far are either overly simplified or are impossible to solve for large scale problems. In this research, I develop a new algorithm that incorporate the complexities of natural gas storage facilities and reliable to solve a large scale problem. The algorithm significantly decreases the computation time from hundreds of days to a fraction of a second for hundreds of stochastic parameter realizations. The algorithm also provides a reasonable solution quality. This approach is simple and robust with regard to gas storage valuation industry application. Simple in a sense that a systematic method to solve a stochastic dynamic non-linear large scale gas storage valuation problem within few seconds is proposed. The algorithm developed is robust because it provides a consistent solution quality.

The gas storage valuation models developed in the literature so far are not comprehensive to capture the combinations of the underground storage facilities and the aboveground storage facilities. I formulate a flexible mathematical model that combines the various underground storage facilities and aboveground storage facilities in this research. This will provide storage facility managers the flexibility to easily switch from the use of one type of facility to another, or to optimize the storage decision for all the underlying facilities simultaneously. The algorithm developed also provides an optimal selection of the storage facilities.

None of the mathematical models developed so far worked on combinations of different storage facilities, including the cycling effects of the storage facilities. The algorithm I develop decompose the decision making strategies based on cycling effect of the storage facilities. For example, if a storage option required is just for few hours, the pipeline facility is explored first. The decision maker can also decide the sequence of the storage facilities they want to use for a specific time period.

## **Chapter 2: Literature Review**

In this chapter, the general concepts of natural gas, a brief description of systems modeling, simulation, optimization, and review of related researches are presented.

### **2.1. General Concepts**

#### *2.1.1. Brief History of Natural Gas*

Natural gas is a colorless and odorless, non-renewable fossil fuel. It contains about 95% of methane in its pure form. Other compositions of natural gas are ethane, propane, butane, pentane, nitrogen, carbon dioxide, water vapor, and traces of other gases (Katz, 1959; Ludtke, 1986). The discovery of natural gas dates back to Middle East in Iran between 6000 and 2000 BC. It was then recognized around 900 BC in China. However, China drilled the first natural gas well in 211 BC. In Europe, the Great Britain scientists were pioneers to discover natural gas in 1659. But it appeared on European market after 1790. William Hart was named the father of natural gas in the United States after he dug the first natural gas well in 1821 (Katz, 1959; Ludtke, 1986; Mokhatab, William , and Speight, 2006).

Natural gas was named a ‘trouble maker’ before fully recognized as a usable energy source. Coal and oil miners used to halt well drilling operations and evacuate the workers when gas came out of the drilling wells. Lighting also contributed to the discovery of natural gas. People were wondering why gas leaks were ignited by lightning. The use of natural gas started significantly in 19<sup>th</sup> century after the availability of crude oil had started dwindling. But the consumption was limited to

small radius of production areas because of the unavailability of good infrastructure to transport over long distances. After World War II, with technological capability to produce leak preventing couplings, transmission pipelines, and reliable storage systems helped the distribution to long distances (Mokhatab et al., 2006).

Natural gas is mainly consumed in industry, electric power, residential, transportation, and commercial centers. According to Future of Natural Gas report by Moniz et al. (2011), natural gas consists 23.4% of the total energy supply in 2009. Major portion of the supply is used in industry (32%), residential and commercial (35%), and electric power (30%); while only 3% is used in transportation sector.

#### *2.1.2. Natural Gas Supply Chain*

The success of a company depends on an effective design of its supply chain network (Klibi and Martel, 2009). A well designed supply chain links maximize the overall value generated (Award, 2010). The values are optimized across the whole process from the supplier to the end users, which includes production, transportation, storage, and distribution to customers. Supply chain consists of all bodies that are directly or indirectly involved in the product movement/service delivery process, to satisfy a customer demand (Chopera and Meindl, 2007). Similar to any other industry, natural gas has a complex supply chain network that needs to be optimized (Tomasgard, Rømo, Fodstad, and Midthun, 2007; Midthun, Bjørndal, and Tomasgard, 2009). The major elements of the natural gas supply chain include exploration, drilling, gathering, processing, transmission, storage, metering, and distribution to end users. A visual representation of Chesapeake natural gas supply

chain taken from Moniz et al. (2011) report is presented in Figure 1 below, followed by brief descriptions of each component.

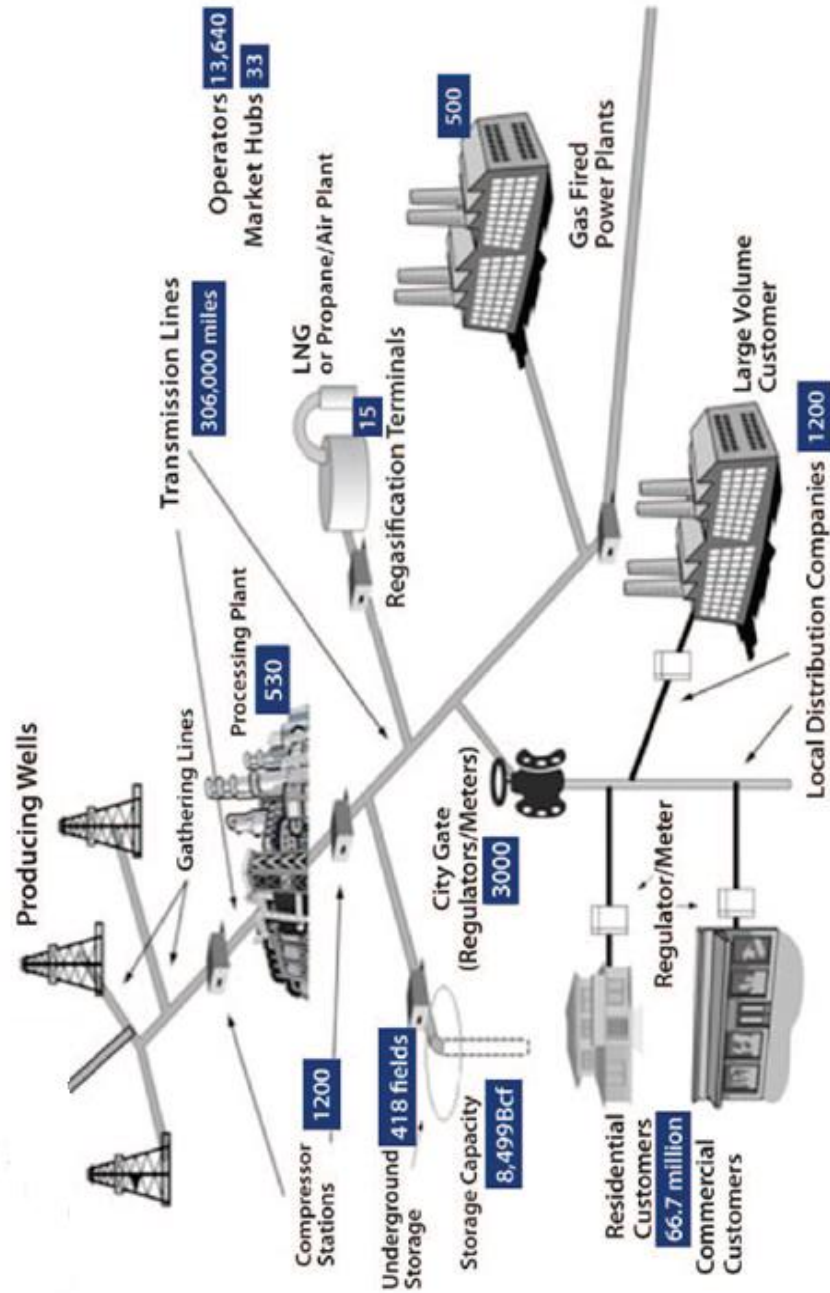


Figure 1. Natural gas supply chain schematic diagram

Source: Moniz et al. (2011)

### *Exploration*

Exploration is the process of searching for natural gas deposits before making drilling decisions. The search is based on geological surveys and geophysics. Geologists study the structure of a surface for an indication of the presence of gas deposits. Then they use devices such as seismic exploration, onshore seismology, offshore seismology, magnetometers, and gravimeters to further investigate the presence of gas. Precision of the exploration depends on the type of the instrument used, and the number of samples taken. This requires a detailed decision analysis techniques before making a drilling decision (Kaufman, 1963; Pirson, 1963; Bielak and Steeb, 1999).

### *Drilling*

After an indication of the natural gas deposit presence is confirmed, a gas well is drilled to bring up the gas for processing. Factors such as onshore or offshore drilling, the geological formation of the deposit, and the drilling technology affect the drilling performance. Drilling is not without risk mostly because of exploration false alarms. Sometimes the exploration analysis results indicate the availability of gas deposit when there is actually no gas reserve in the underlying area. There might also be way lesser than the amount of gas reserve estimated (Mokhatab, William , and Speight, 2006; Skogdalen and Vinnem, 2012; Li, Yu, Liu, and Gao, 2008).

### *Gathering*

Once the gas is extracted out of the ground, it is sent to processing centers. The main concern in this stage of the supply chain in that the gathering system may



not be efficient. There are several ways of gathering systems of which one is through pipelines. If pipeline is used for gathering, the pipeline network that connects the gas wells to the processing plant must be optimized based on the pipeline capacity and distance (Peretti and Toth, 1982; Johnson, Gagnolet, Ralls, and Stevens, 2011; Mokhatab and Poe, 2012).

### *Processing*

Processing is the method of converting the natural gas into a useable form by removing the toxic substances that are mixed with the unprocessed gas before use by consumers. Location of a natural gas processing plant is an important issue to consider in addition to the location of end users, the current and the future discovery of gas deposit, the environmental factors, and geographical locations before making a processing plant location decision (Devold, 2006; Kidnay, Parrish, McCartney, 2011; Baker and Lokhandwala, 2008).

### *Transmission*

The term “transmission” is more often used to describe the transportation of processed gas to distribution centers and customers through pipelines or tanks even though it is used for gas gathering as well. Gas is mostly transported via pipelines through interstate or intrastate to satisfy customers’ demand (Gunes, 2013; Mokhatab et al., 2006; Contesse, Ferrer, and Maturana, 2005). Refer to the Future Research Directions of this research for a potential research area of natural gas transmission through pipelines.

### *Storage*

Storage plays a vital role in natural gas supply chain network. The Federal Energy Regulatory Commission report describes the importance of gas storage as a key component of the natural gas grid that helps to maintain reliability of gas supplies during periods of high demand. Storage can help local distribution centers to maintain adequate supply during periods of heavy demand by supplementing pipeline capacity, and can serve as backup supply in case of interruptions in wellhead production. On the other hand, excess gas is stored when there is surplus production (FERC, 2011; Katz and Tek, 1981; Quinn and MacDonald, 1992; Mokhatab and Poe, 2012).

### *Marketing and Distribution*

The final end point of the natural gas supply chain is the customer, similar to any other product. The major users of natural gas are: residential, commercial, industry, electric power, and transportation. The main challenges in distribution are demand forecast, location of local distribution companies, and distribution network design (Chin and Vollmann, 1992; Guldman and Wang, 1999; Muthuraman, Aouam, and Rardin, 2008).

### *Metering*

The flow of gas through pipeline is measured to verify the amount of gas that is delivered to customers. The measurements are commonly taken by gas measuring device such as orifice meters. However, the accuracy of the measurement depends on the type of the device used. A great care should be taken since a one percent error in

measurement accuracy of natural gas in a pipeline that delivers 300 MMcf per day at 45 cents per Mcf may cost a half million dollar a year(Kouba,1986; Scelzo, 2001; Ragle, Hayes, King, and Johnson, 2001).

### *2.1.3. Systems Modeling, Simulation, and Optimization*

A system is an integration of activities that interact to accomplish a specific task. A model is a representation of a system. Systems modeling is the representation of a real systems using physical model or mathematical model. A physical model is usually building a blueprint of the system. A mathematical model comprises of computer simulation and analytical solutions. Mathematical models can represent either deterministic or stochastic systems, or both. For a given system, one can either experiment with the actual system or experiment with a model of the system. (Law and Kelton, 1991; Maria, 1997).

Deterministic systems models are used under the assumption that all the system variables can be determined at the moment of decision making. This type of models are relatively easy to solve. However, in real life, system parameters are usually difficult to determine because of system noise, non-linearity, and high dimensionality. Under such conditions, stochastic based decisions are more relevant. Stochastic decisions are described by stochastic processes. A stochastic process is defined as the mathematical abstraction of an empirical process whose development is governed by probabilistic laws such as the Poisson and exponential distributions (Gross and Harris, 1962; Lasota and Mackey, 1985).

### *Simulation Modeling*

Simulation is the act of mimicking the operation of real world system in a computerized laboratory environment. Simulation plays a vital role in evaluating systems performance in complex systems where it is very difficult to apply analytical methods. Analytical approach is mathematical models such as linear programming, differential equation or probability distribution that gives us 'exact' solution to a specific problem. Solving complex problems using analytical approaches may require lots of simplified assumptions. But the final solution might be sufficient or 'inferior' for implementation. On the other hand, there is no guarantee that simulation outputs arrive at best solution; but it is possible to increase the precision by increasing the number of simulation run (Roberts, Andersen, Deal, Garet, and Shaffer, 1983; Vangheluwe, 2004; Chen and Lee, 2010).

Simulation is classified into three categories: static or dynamic, deterministic or stochastic, and discrete or continuous simulation. Static simulation is the representation of a system at a particular point in time using techniques such as Monte Carlo models. Dynamic simulation represents a system as it evolves over a period of time. A good example of dynamic simulation can be a conveyor system in a factory. The inputs to simulation models are either determined before use or random values that are generated using some probabilities. The simulation model which has a probabilistic input is classified as stochastic simulation; whereas, the models that does not contain any random values are categorized under deterministic simulation. The customer arrival to a shopping store is a good example of a stochastic simulation. When the dependent variables of a simulation changes

instantaneously at specified point in simulation time such as a bank teller status change, it is termed as discrete simulation. On the other hand, continuous simulation deals with the variable change continuously over a period of time. The change in speed of an accelerating car with respect to time can be one example of continuous simulation. Most systems are modeled based on combination of two or more of the above simulation categories (Law and Kelton, 1991; Pritsker and O'Reilley, 1999; Winston, 2004).

Representation of a system performance is based on the effectiveness of the simulation modeling. Gross and Harris (1985) classify simulation modeling into three main phases: data generation, bookkeeping, and output analysis. The data generation involves the arrivals, the service rates, the length of queue, and the throughput of the system. The bookkeeping phase is associated with updating and monitoring when new events such as arrival and departure occur in a system.

### *Optimization*

Dating back to the invention of linear programming by George Dantzig in 1947, optimization tools have become very popular to solve complex problems to an optimal or near optimal solutions. The classes of exact optimization algorithms solve a problem optimally. While a group of heuristic algorithms provide a near optimal solutions. Some of the exact algorithms include simplex algorithm, Branch-and-Bound algorithm, Cutting-plane algorithm; and heuristic algorithms include tabu search, genetic algorithms, and simulated annealing. They are used in several areas of research to optimize supply chain performance, such as distribution network optimization. The exact algorithms are used for linear and integer problems that can

be solved in polynomial time. Problems that cannot be solved in polynomial time using exact methods are solved by heuristic algorithms to find a near optimal solution in a reasonable time. Most optimization problems in energy industry are one of those problems that cannot be solved in polynomial time (Pardalos, Rebennack, and Scheidt, 2009; Dantzig, 1963; Gill, Murray, and Wright, 1981; Rardin and Uzsoy, 2001).

## **2.2. Natural Gas Storage Valuation**

Natural gas storage valuation is the planning of natural gas storage facilities (Felix and Weber, 2012). Natural gas storage valuation is a very difficult optimization problem since is highly affected by financial aspects and physical aspects of natural gas storage facility characteristics. The financial aspects include the gas price dynamics and the gas storage lease policies. The physical aspect of the storage valuation are the complex storage facilities constraints. These are the inventory balance dynamic constraint, the non-linear flow rate constraint, the storage facility capacity constraint, and the supply and demand uncertainties. The injection and withdrawal capacities vary non-linearly based on the storage inventory level. Gas storage valuation takes into account gas stock level (Makassikis et al., 2007). The stock level is directly affected by the injection and withdrawal injection rates (Ikoku, 1980; Thomson et al., 2009). There are several techniques used to determine how much natural gas inventory to hold, inject or withdraw from a storage facility. The common techniques in the literature are Monte-Carlo simulation, partial differential equations, binomial and trinomial trees (Holland, 2007). Partial differential equations and binomial trees provide precise solutions but cannot handle

large size problems. Monte-Carlo simulation can be used to analyze large size problems but it only provides good (not optimal) solution. Increasing the simulation accuracy requires large number of runs which can be very expensive and time consuming.

Guldmann (1983) is one of the pioneer to explore the valuation of natural gas storage valuations combined with purchases and service reliability. Guldmann implemented a chance-constraint programming approach to optimize the level of gas storage for a single depleted reservoir, a single supplier, and multiple consumers. The suppliers are pipeline transmission companies, and the supply on any given day does not exceed an average of historic data. The consumer demand is considered to be uncertain. The reservoir inventory level vs. withdrawal/injection characteristics is modeled as a linear relationship based on historic storage data. Regression models are developed to estimate gas demand taking the monthly weather uncertainty into consideration. The information provided to the decision makers, natural gas distribution companies, is storage decision scenarios along with their respective costs. Several years later, Guldmann extended the research to multiple suppliers (Guldmann, 1986).

A year after Guldmann, Caton used a simulation model to estimate daily gas storage valuation. However, Gulman explored the valuation of liquefied natural gas (LNG) and pipeline storage decision making requirements. Temperature and historic demand data are used to develop a linear regression model. The model is simulated to forecast future demand. Random numbers are generated to estimate the over-

forecast and the under-forecast situations, but the details of performance evaluation is not presented (Caton, 1984).

Thompson and his colleagues proposed a new gas storage valuation technique for a single salt cavern storage facility. They took into consideration the fluid dynamics and thermodynamics effect of storage facility in their formulation and implemented using partial-integro-differential equation. They predicted gas prices to decide how much gas to store. However, it is impossible to solve a large scale problem because of the complexity of the algorithm (Thompson et al., 2009)

Holland developed a similar storage optimization model based on pre-print version of Thompson et al. formulation preprint research paper. But Holland used the combination of stochastic simulation and support vector machine for the price prediction. He used Monte-Carlo simulation for the long term price prediction, and support vector regression for short term price prediction. The optimization algorithm is based on integer programming. The storage facility complexities are overlooked in the research (Holland, 2007).

Carmona and Ludkovski considered the valuation of natural gas on finite horizon which is similar to Thompson et al. (2009) pri-print version. They focused on timing optionality of storage to construct optimal switching problem with inventory. They used Monte-Carlo regressions. Assuming the gas can be bought and sold on the spot market, they attempted to maximize profit given the operational constraints (Carmona and Ludkovski, 2005).



Because of the complex storage valuation, Makassikis et al. developed parallel computing algorithm based on stochastic control algorithm. Since the gas storage valuation is a stochastic dynamic programming problem, parallelism is not an easy method to approach the problem (Makassikis, Supelec, Vialle, and Warin 2007)

Boogert and Jong used Monte-Carlo method for gas storage valuation for a single salt cavern storage facility. They included in their model the effect of price dynamics and the physical storage facility characteristics. After running experiments for multiple injection and withdrawal rates, they concluded that fifty price realizations are enough to capture price realization for each simulation run (Boogert and De Jong, 2008).

Lai, Margot, and Secomandi (2010) compared an approximate dynamic programming with a practice-based heuristic algorithm. They developed an efficient approach for a gas storage valuation which can also be applied to other commodities but they did not take into account the effect of inventory levels on injection and withdrawal rates, and the gas loss. The idea of the research is heavily based on the comparison of the optimal and heuristic natural gas storage valuation of Secomandi (2008).

From the financial aspects of the storage valuation, many research have been carried out to predict the natural gas price. Some of the techniques are neural network (Doris, 1999), regression models (Solomon, 2001), principal component analysis (Bjerksund, Stensland, and Vagstad, 2008) time series and non-parametric

approaches (Mishra, 2012). Based on historic natural gas data, researchers proposed different models in different time periods. For example, Pindyck (2001) used 75 years of data (1919-1996) to see what type of model is more reliable for long term gas price prediction. He suggested that energy prediction models should incorporate mean reversion to stochastic changes. From 1991-2004, German (2007) confirmed the Pindyck's conclusion. The pricing is determined based on gas energy demand and supply forecast. Factors that affect supply include but not limited to natural gas production amount uncertainty, amount of gas import and export, and natural gas availability in storage. For example, Street, Barroso, Chabar, Mendes, & Pereira (2008) developed a gas supply contract pricing model associating with thermal power plant as a supply dependent. Gas demand may also be affected by sources of energy prices such as oil, change weather condition including extreme events, and economic growth (Wong-Parodi et al., 2006; Herbert 1993Mu, 2007). Since most of the natural gas consumed in the United States is produced within the country, US natural gas price is highly affected by domestic production. Hurricane and other severe weather conditions also affect natural gas supply. During winter seasons and extreme summer seasons, the gas demand goes up and the gas prices likewise (Rogel-Salazar and Sapsford, 2014). Then when the weather is leveled the demand goes down which takes the price down. Prices also fluctuate when consumers switch from the use of one form of energy to another.

## **Chapter 3: Natural Gas Storage Facility Modeling**

### **3.1. Overview**

Natural gas is stored aboveground or underground depending on the availability and the needs of the storage facilities. Pipelines and tanks are the most common aboveground storage facilities. The aboveground storages are mostly located in highly congested regions. Salt cavern, depleted reservoir, and aquifers are the widely used underground storage facilities. Underground storages are usually used to satisfy long term demands while aboveground storages are for short term demands.

Storage facility modeling techniques used in the literature are Wymouth equation, flow rate equation, gas law principles, and Bernoulli's principles among others (Ikoku, 1980; Scelzo, 2001; Coelho and Pinho, 2007). There are several factors taken into consideration for the modeling purpose. These are base gas requirement, working gas capacity, deliverability, injection rate, formation, and property of the storage facility materials.

Base gas is used as cushion in underground storage facilities. It maintains reservoir pressure though out the facility service period. The common misconception is that base gas is sometimes referred as a safety inventory. However, base gas is added to the facility at the beginning of the facility usage and removed at the end of the usage. For some facilities it is possible to recover portion of the base gas if not all of it at the end of the facility usage as some of the gas inside the facility may escape through porous structure of the storage facility. Different facilities have different

base gas requirements. Base gas for salt cavern comprises of 20-30% of the total storage capacity. For depleted reservoirs it is up to 50% while it is 50-80% for aquifers (Gumrah, Izgec, Gokcesu, and Bagci, 2005; Azin, Nasiri, and Entezari, 2008).

Working gas capacity is the space available for gas inventory that can be depleted and replenished during the storage period. The working gas inventory at any time is the difference between the total gas available in a storage facility minus the base gas.

Injection rate is the rate at which gas is added to a storage facility. The maximum injection rate of a facility can be determined by experimentation. Withdrawal rate is the rate at which gas can be removed from a storage facility. Similar to the maximum injection rate, the maximum withdrawal rate can also be determined experimentally. In the literature, the gas withdrawal rate is usually referred to as production rate or deliverability. From the basic principles of fluid dynamics, injection rate reaches maximum when the storage facility is empty, keeping constant other factors such as temperature and force of injection. On the contrary, withdrawal rate attains its maximum when the storage facility is at the maximum gas holding capacity (Menezes, 2001; Kuye and Ezuma, 2008).

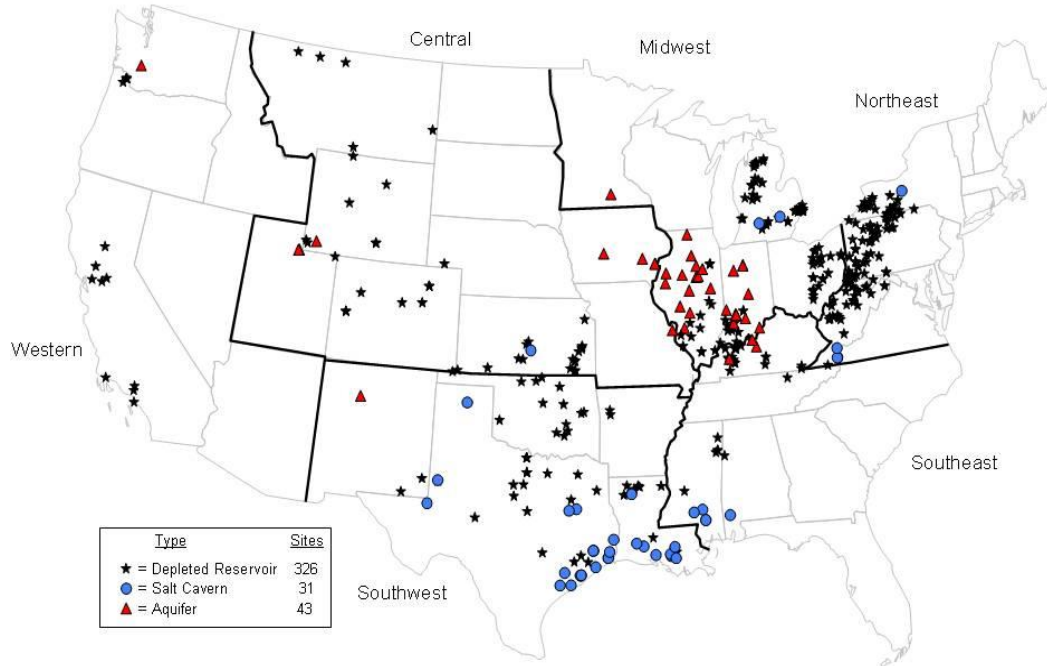
For storage facility contractors, the scheduling of a facility has a consequential effect on the profit when they make decisions such as how much gas to inject, hold or withdraw every day to maximize profit over a storage time window.

The storage facility is also required to be emptied at the end of the lease; and the stored gas has to meet demand.

As of 2013, about 400 underground natural gas storage sites were operational in the United States. In the same year, four new storage facilities were added to Michigan, Mississippi, Pennsylvania, and West Virginia one for each. About 18 existing storage facilities were expanded and two were closed (EIA, 2014). The storage facilities for each of the six US regions are shown on Figure 2 below. The distributions are: Central (49), Midwest (121), Northeast (110), Southeast (34), Southwest (66), and Western (20). The star symbols show the depleted reservoir, the circular dots show the salt caverns, and the triangular dots show the aquifers. Cumulatively, there are about 550 underground storage facilities worldwide (Bary et al., 2002).

Midwest region storage facilities serve Illinois, Indiana, Michigan, and Ohio. This region has the largest number of storage facilities in the States, which is 121. Pennsylvania and New York are the major transit area for Northeast storage facility. They do have also largest underground storage capacity, including West Virginia. These regions provide more supply from storage facilities. As a result, there are more pipelines that leaves the states than those enter into the states. Most of the storage facilities are near to production fields, and used to balance the production flow when market demand fluctuates. Midwest region has the largest storage facility compared to all the other regions. This is because of large population size, cold winters, and

large natural gas pipeline systems. The Southwest Region is the second because of large salt formation of high natural gas production level (EIA, 2014).



Source: Energy Information Administration, Office of Oil & Gas, Natural Gas Division Gas, Gas Transportation Information System, December 2008.

**Figure 2. Distribution of underground storage facilities in the U.S.**

### 3.2. Underground Storage

Very large portion of natural gas is stored in underground storage facilities. As mentioned earlier, these underground storage facilities are the salt cavern, the aquifer, and the depleted reservoir. The storage capabilities vary from facility to facility. Hence, the storage facility owners or contractors consider the benefits and the drawbacks of the facilities to decide which facility to use. The major storage facility attributes are injection and deliverability rate, base gas requirement, and

distance from market. Ikoku (1980) puts the relationship between some of the attributes as follows.

*A major consideration in storage operation is the relationship between cushion gas and working gas. Because cushion gas provides the reservoir energy to support storage deliveries and maintain proper deliverability rates, it is a necessary segment of the reservoir. The ratio of base gas to working gas varies from reservoir to reservoir, and from time to time, depending upon operating conditions.*

Depleted reservoirs are widely available. There are about 326 in the United States as of 2011. Eighty two percent of the storage facilities were of this category (EIA, 2014). These facilities are converted to storage after production of either oil or gas. They have infrastructures already built during the gas extraction, which can be used for storage. But they require high base gas. The other storage facility is salt domes/caverns. These are mainly located in the gulf coast, far from market. But they have low base gas requirements and high deliverability and injection rates. "It has deliverability since there is no pore compared to depleted reservoirs. In addition, salt formation has moderately high strength and deforms plastically to close fractures that could otherwise cause gas leaks. Its porosity and permeability to liquid and gaseous hydrocarbons are near zero, so stored gas cannot escape," (Barajas and Civan, 2014). The flexibility of a salt cavern facility to respond to short term supply and demand is the major advantage to consider. Owner of this facility can use the 6-12 cycles per year advantage. On the other hand, aquifer storage facilities are located close to customers but they require high base gas which is about 80%, and this cannot be recovered. It also requires high control during storage and withdrawal.

Various components of storage come together to model a storage facility. These components are underground reservoir, injection wells, withdrawal wells, injection-withdrawal wells, observation wells, gathering system, compressor facility, metering facility, dehydrator, and transmission line to pipelines (Flanigan, 1995; Buschbach and Bond, 1974; Knepper, 1997).

Researchers model underground storage facilities based on the aforementioned characteristics with minor differences from one research to another research problem. For example, the capacity of depleted oil reservoirs for gas may be modeled as follows. Refer to (Katz, 1959; Flanigan, 1995; Aminian & Mohaghegh, 2009) for more information.

Given the following variables,

$\Delta N$  = oil produced in bbl

$V$  = volume of gas to replace oil produced, Mcf

$P$  = reservoir pressure, psia

$T$  = reservoir temperature, °R

$Z$  = compressibility factor for gas

$B_0$  = formation volume factor

The formation volume factor is the ratio of the reservoir volume to the volume of the residual oil remaining after pressure has been depleted to atmospheric and the oil cooled to 60 Fahrenheit degrees.

Then

$$V = \frac{5.16 \Delta N B_0}{1000} \left( \frac{P}{14.65} \right) \left( \frac{520}{TZ} \right) = 0.199 \frac{\Delta N P B_0}{TZ}$$



### *Steady and unsteady state flow in reservoir*

Flow through porous media in a reservoir may be treated as steady- state when conditions do not change with time or as unsteady-state when conditions do change with time. Pressure depletion of a gas field upon a gas withdrawal is unsteady-state phenomenon; however, under certain conditions, steady-state flow formulas find considerable use. Such condition might include flow in areas adjacent to a producing well or flow from a well that is said to have become stabilized (Janson, 2013; Sarkar, Toksoz, and Burns, 2002)

### *Steady state flow equation*

The following is based on Darcy's law. "The ability of porous media to conduct fluids through their interstices is known as permeability. The unit of permeability is called a darcy; 0.001 darcy is termed a millidarcy (Coelho, and Pinho, 2007). The permeability is represented by K. In predicting the capacity of oil wells for gas injection or production, the flow rate and flowing bottom hole pressure on the oil well are desired. The flow equations for oil and gas can be combined as follows.

$$\frac{hK}{\ln(r_2/r_1)} = \frac{Q_0 B u_0}{0.00708(P_1 - P_2)}$$

Where  $Q_0$  = oil flow rate, bbl/day( of stock-tank oil)

$B$  = formation volume factor

$u_0$  = oil viscosity, centipoises

$P_1$  = reservoir pressure, psia

$P_2$  = flowing bottom hole pressure, psia

$hK$  = thickness-permiability product, millidarcy-ft

$r_2$  = well bore radius, ft

$r_1$  = reservoir radius, ft

$$\frac{hK}{\ln(r_2/r_1)} = \frac{QzTu_g}{0.00703(P_1^2 - P_2^2)}$$

$Q$  = gas flow rate, Mcf/day

$Z$  = compressibility factor at average reservoir conditions

$T$  = reservoir temperature. oR

$u_g$  = gas viscosity

Combining the oil and gas equations,

$$Q = \frac{0.0993Q_0Bu_0(P_1^2 - P_2^2)_g}{zTu_g(P_1 - P_2)_0}$$

Similarly, the storage of gas in aquifers can be formulated. When gas is stored in an aquifer, gas pressure in the reservoir must be maintained higher than the original water pressure to force any water into the aquifer (Wattenbarger, Startzman, and Gajdica, 1988). During the initial injection gas into a water well, pressure from 100 to 300 psi above the water reservoir may be required to start gas entering the porous rock. Once gas has started flowing, the usual flow considerations for gas apply. The rate at which gas can be injected at some fixed gas-bubble pressure is determined by the aquifer behavior in the unsteady-state process. The solution to the unsteady-state flow equation is as follows.

$$q = 6.283\phi C_w R_g^2 h (P_g - P_f) Q_t$$

$q$  = water movement from gas bubble, cu ft.

$\phi$  = fractional porosity of formation

$C_w$  = compressibility of water including formation, volumes/(volume)(psi)

$R_g$  = radius of gas bubble, ft

$h$  = formation thickness, ft

$P_g$  = pressure on gas bubble

$P_f$  = initial water pressure

$Q_t$  = a function of dimensionless time  $t_D$  obtained from standard tables

$t_D$  = dimension less time =  $\frac{0.00632 Kt}{\phi\mu C_w R_g^2}$

$K$  = permeability of formation, millidarcys

$t$  = time from beginning of pressure maintenance, days

$\mu$  = water viscosity, centipoises

To use the equations, the compressibility of water and the reduction of pore volume with pressure decrease are needed. Solution gas increases the compressibility of water by 20 percent for each 20 cu ft/bbl dissolved in water (Flanigan, 1995).

### 3.3. Aboveground Storage

Very small amount of gas is stored aboveground. This can be stored in pipelines or storage tanks as mentioned earlier.

#### *Pipelines*

Pipelines are used for intra-state and inter-state transportation of natural gas. They can also be used for storage, usually to satisfy short term demands. The pressure in the pipeline is reduced when there is no demand. On the other hand, the pipelines operate at maximum capacity during high demand seasons. “Gas pipeline systems are often used as temporary natural gas storage facilities. Intermediate natural gas compression stations enable the pressure in the main pipeline system to be raised appreciably (from 300 to 1000 psia with a corresponding rise in the amount

of gas stored in the pipes). Quantities of natural gas which are not required at that moment by utilities are in this way stored in pipeline systems. When natural gas demand increases, then stored gas can be supplied to utilities by lowering the pressure in the pipeline system (from 1,000 to 350 psia, as a typical example). Pipeline storage is important for compensating peaks in demand which have time intervals of few hours” (Ikoku, 1980). During colder seasons pipeline plan to maintain more than normal line pack. In addition, compressor operations also need to be optimized.

The pipeline capacity can be modelled by Weymouth equation. Weymouth states “the storage capacity of natural gas pipeline as the difference between the gas contents of the pipeline under packed and unpacked conditions. Packed is when withdrawal from the line is at minimum and the discharge pressure is maximum. Unpacked when withdrawals are maximum and pressure is a minimum for a constant supply of gas to the line” (Katz, 1959). Hence, the storage capacity of a gas-transmission line can be calculated by using the following formula for the content of a natural gas pipeline under condition of isothermal flow. A steady flow equation.

$Q_h$  = gas flow rate. Ft<sup>3</sup>/hr at  $P_b$  and  $T_b$

$T_b$  = base temperature, °R.

$p_b$  = base pressure, psia

$p_1$  = inlet pressure, psia

$p_2$  = outlet pressure, psia

$D$  = inside diameter of pipe, in.

$G$  = gas specific gravity (air = 1)

$T$  = average flowing temperature, °R.

$f$  =Moody friction factor

$L$  = length of pipe, miles

$\bar{Z}$  = gas deviation factor at average flowing temperature and average pressure.

$L$  = volume of gas

$$Q_h = 3.23 \frac{T_b}{p_b} \left[ \frac{(p_1^2 - p_2^2) D^5}{f G T L \bar{Z}} \right]^{0.5}$$

Where  $\left(\frac{1}{f}\right)^{0.5}$  = transmission factor, and Weymouth proposed that  $f$  varies as a function of diameter in inches as follows.

$$f = \frac{0.032}{D^{1/3}}$$

Substituting  $f$  into the original equation will give us the following.

$$Q_h = 18.062 \frac{T_b}{p_b} \left[ \frac{(p_1^2 - p_2^2) D^{16/3}}{G T L \bar{Z}} \right]^{0.5}$$

Or it can be written as

$$p_1^2 - p_2^2 = L \frac{Q_h^2 p_b^2 G T \bar{Z}}{326.24 T_b^2}$$

Then,

$$p_1 = \sqrt{L \frac{Q_h^2 p_b^2 G T \bar{Z}}{326.24 T_b^2} + p_2^2}$$

It is easy to show the daily flow rate ( $Q_d$ ) as well.

$$Q_d = 433.488 \frac{T_b}{p_b} \left[ \frac{(p_1^2 - p_2^2) D^{16/3}}{G T L \bar{Z}} \right]^{0.5}$$

The volume of gas ( $dV$ ) contained in any increment of length  $dL$  will be  $dV = 5,280 dLA$ , which is an equation for the cubic feet of gas in the section at pressure  $p$  and temperature of flow  $T$ ,  $A$  being the cross-sectional area of the pipe in square feet. Reducing the above equation into base conditions gives

$$dV = 5,280 dL A \frac{p_1 T_b}{p_b T}$$

then the total quantity of the gas in the line will be

$$V = \int_0^L dV = 5,280 A \frac{T_b}{p_b T} \int_0^L p_1 dL$$

$$A = \frac{\pi D^2}{4 \cdot 144}$$

Integrating the above equation and substituting the value of  $p_1$  and  $A$ , we come up with the following equation.

$$V = 19.20 \frac{D^2 T_b L}{p_b T} \left[ p_1 + p_2 - \frac{p_1 p_2}{p_1 + p_2} \right]$$

The above equation gives the quantity of gas measured at base conditions stored in the pipeline for any given flow condition. To determine the storage capacity of a simple pipeline, the pressures at both ends are determined by both packed and unpacked flow conditions. The difference between the two quantities is the storage capacity of the pipeline.

Let

$V$  = volume at unpacked condition

$V'$  = volume at packed condition

$p$  = pressure unpacked condition ( $p_1, p_2$ )

$p'$  = pressure packed condition ( $p_1', p_2'$ )

Storage capacity is  $= V' - V$  where ,

$$V = 19.20 \frac{D^2 T_b L}{p_b T} \left[ p_1 + p_2 - \frac{p_1 p_2}{p_1 + p_2} \right], \text{ and } V' = 19.20 \frac{D^2 T_b L}{p_b T} \left[ p'_1 + p'_2 - \frac{p'_1 p'_2}{p'_1 + p'_2} \right]$$

The maximum pipeline inventory carrying capacity at time t,  $I_t$  is

$$0 \leq I_t \leq 19.20 \frac{D^2 T_b L}{p_b T} \left[ \left[ p'_1 + p'_2 - \frac{p'_1 p'_2}{p'_1 + p'_2} \right] - \left[ p_1 + p_2 - \frac{p_1 p_2}{p_1 + p_2} \right] \right]$$

Maximum injection capacity

$$J_t \leq 433.488 \frac{T_b}{p_b} \sqrt{\frac{(p_{t(inlet)}^2 - p_{t(inside)}^2) D^{16/3}}{G T_t L \bar{Z}}}$$

Maximum withdrawal capacity

$$W_t \leq 433.488 \frac{T_b}{p_b} \sqrt{\frac{(p_{t(inside)}^2 - p_{t(outlet)}^2) D^{16/3}}{G T_t L \bar{Z}}}$$

### *Natural Gas Storage Tanks*

Natural gas is stored either in the form of liquid, liquefied natural gas (LNG) or solid as hydrates. LNG are usually used in conjunction with pipelines, usually as a backup for pipelines. The gas is cooled to about -260 degree F. The LNG to gas ratio is about 1:600 (Levine, 2011). Storage tanks are used in highly populated areas where any of the underground storage usages are not viable.

### **3.4. Gas Storage Loss**

The loss of gas from a storage facility depends on the type of facility. There are several factors that contribute to the loss of natural gas from a storage facility. Gas sometimes leaks from old well castings to other formation of the facilities. There is also gas loss through cap rocks but not common. Gas also leaks through a low

permeability connection to a companion reservoir that is not part of the storage facility, which is usually more likely. In addition, gas leaks in surface equipment and pipelines, and from storage when gas travels past saddle seal. However, in some cases gas that migrated may move back as the pressure in the reservoir is lowered (Flanigan, 1995; Bernard and Holm, 1970).

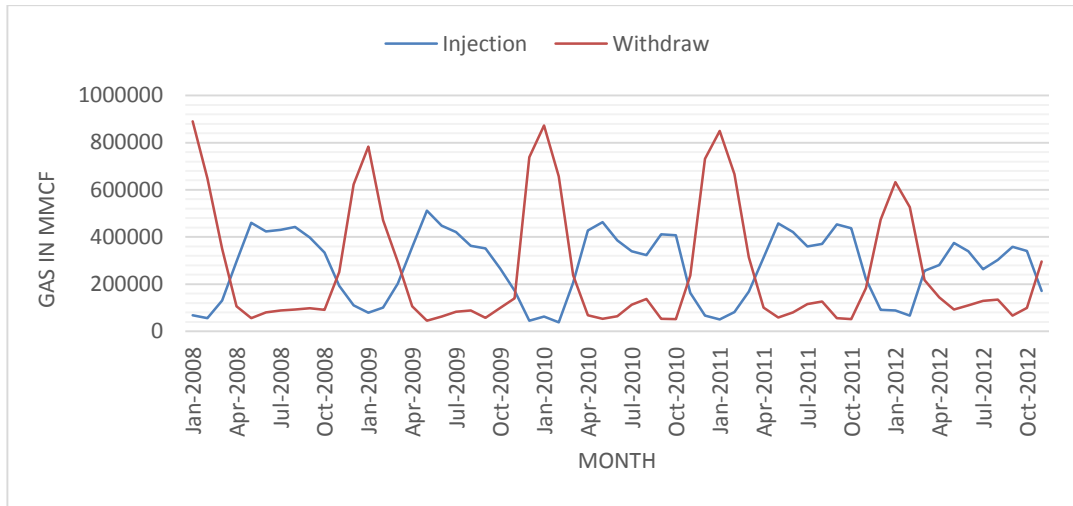
Generally, compared to aboveground storage facilities underground storages have high rate of gas loss. Gas loss for aboveground is mainly to maintain the performance of the facilities such as the energy consumed by the compressors. Many researches are underway to replace the energy consumptions by solar or wind sources. We do not go into the details of the other losses of aboveground facility in this research; however, I present a leak equation for aquifer as given by the following equation (Flanigan, 1995).

$q_1 = 3.74 * 10^{-7} (p_G^2 - 1.6^2)^n$ ; where,  $q_1$  is the daily leak rate in MMcf/D [ $10^6 m^3/d$ ] and  $p_G$  is the maximum storage pressure in psia [MPa]. Exponent  $n$  is assumed to be 1 in most cases.

### **3.5. Gas Storage Trend**

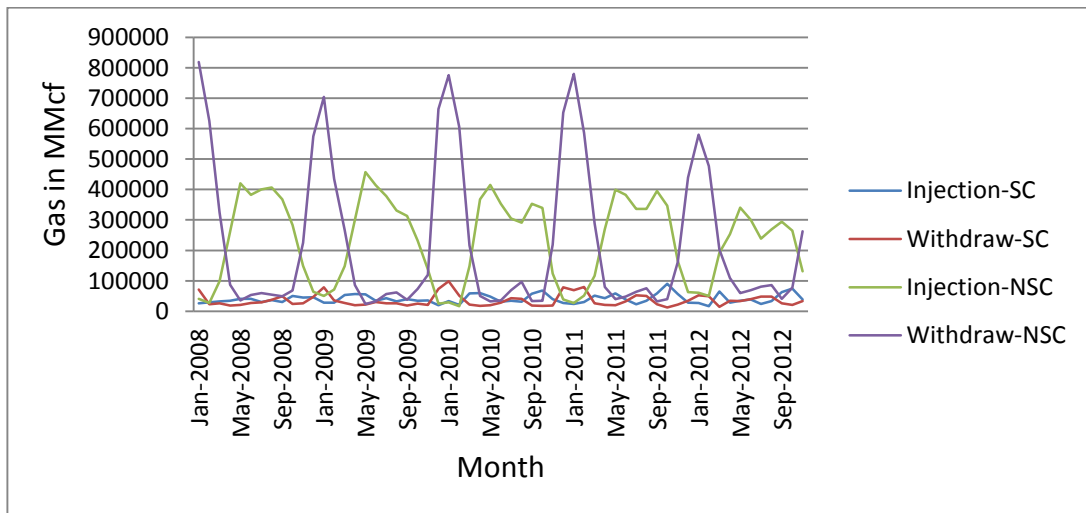
In this section, the US natural gas storage data from 2008 to 2012 are used to present the natural gas storage trend. All the data are monthly basis, and are taken from Energy Information Administration. The injection and withdrawal historic trends are shown by the Figure 3 below.





**Figure 3. Gas injection-withdrawal cycle**

Gas injection increases when withdrawal decreases, and vice-versa. The gas withdrawal is higher around the month of January for all the five years. Injection is high in summer seasons. Figure 3 is for all types of underground storage facilities. It is important to classify into various storage facilities to see the difference. Figure 4 shows the comparison of salt cavern and non-salt cavern (depleted reservoir and aquifer) storage facilities.



**Figure 4. Salt cavern and non-salt cavern injection withdrawal pattern**

Figure 4 shows that non salt cavern (NSC) facilities dominate the salt cavern (SC). We can see more cycles in salt cavern than non-salt cavern storage facilities. Withdrawal rate hits pick in non- salt cavern faster than salt cavern in winter season. Injection rate is slower than withdrawal rate for both groups of storage facilities.

## **Chapter 4: Problem Formulation**

### **4.1. Problem Definition**

The research problem is formulated to support natural gas storage valuation decision over large time window. The valuation problem formulation is for the combination of aboveground and underground storage facilities. A case study for natural gas storage contractors is used in the formulation for a clear understanding of how the model works. However, the generalization of the model is applicable in any gas storage valuation problems, in any capacity.

Basically, natural gas storage facility contractors lease a storage facility for one year, usually from April to March. Then they want to maximize profit over the lease period. Gas is purchased from pipeline transmission companies, and is sold to natural gas distribution companies. The contractors should determine how much gas to buy and inject to a facility, withdraw and sell from a facility, or do nothing to maximize profit over the lease period.

The contractors should follow the contract policy. One such a policy is that the storage facilities have zero working inventories at the beginning of the lease period. Also, the contractor is expected to free up the facility at the end of the lease period; otherwise, any amount left in the storage facility is void.

A comprehensive, flexible mathematical model is developed to valuate single storage facility and multiple storage facilities. The gas storage decision makers can use the model for each facility separately or for any combination of storage facility. Price uncertainty and realistic physical storage characteristics are incorporated.

As of today, underground natural gas storage facilities are widely used. The research problem is formulated based on the three underground storage facilities and pipelines. I leave storage tanks for future research. The formulation is explained in the following section.

#### 4.2. Definition of Terms and Mathematical Model

The definition of terms used in the formulation and the mathematical model are presented in this section. Some other terms are also defined within other sections as needed.

The decision variables are the withdrawal and injection decisions. The inventory level is also another decision variable which depends on the withdrawal and injection rates decisions. All the remaining terms defined in this section are model parameters.

$t$  = time window for which gas storage decision has to be made

$t = 1, 2, \dots, T$

$T$  = lease expiration period

$$T = \begin{cases} T_k^{\text{cav}} & \text{for salt cavern } k, k \in K_{\text{cav}} \\ T_k^{\text{dep}} & \text{for depeleted reservoir } k, k \in K_{\text{dep}} \\ T_k^{\text{aqu}} & \text{for aquifer } k, k \in K_{\text{aqu}} \\ T_k^{\text{pipe}} & \text{for pipeline } k, k \in K_{\text{pipe}} \end{cases}$$

$k$  is a storage facility in set  $K$ , where  $K = \{K_{\text{cav}}, K_{\text{dep}}, K_{\text{aqu}}, K_{\text{pipe}}\}$

$J_{kt}$  = facility k injection decision on period t

$$J_{kt} = \begin{cases} J_{kt}^{\text{cav}} & \text{for salt cavern } k, k \in K_{\text{cav}} \\ J_{kt}^{\text{dep}} & \text{for depeleted reservoir } k, k \in K_{\text{dep}} \\ J_{kt}^{\text{aqu}} & \text{for aquifer } k, k \in K_{\text{aqu}} \\ J_{kt}^{\text{pipe}} & \text{for pipeline } k, k \in K_{\text{pipe}} \end{cases}$$

$W_{kt}$  = facility k withdrawal decision on period t

$$W_{kt} = \begin{cases} W_{kt}^{\text{cav}} & \text{for salt cavern } k, k \in K_{\text{cav}} \\ W_{kt}^{\text{dep}} & \text{for depeleted reservoir } k, k \in K_{\text{dep}} \\ W_{kt}^{\text{aqu}} & \text{for aquifer } k, k \in K_{\text{aqu}} \\ W_{kt}^{\text{pipe}} & \text{for pipeline } k, k \in K_{\text{pipe}} \end{cases}$$

$I_{kt}$  = facility k inventory level on period t

$$I_{kt} = \begin{cases} I_{kt}^{\text{cav}} & \text{for salt cavern } k, k \in K_{\text{cav}} \\ I_{kt}^{\text{dep}} & \text{for depeleted reservoir } k, k \in K_{\text{dep}} \\ I_{kt}^{\text{aqu}} & \text{for aquifer } k, k \in K_{\text{aqu}} \\ I_{kt}^{\text{pipe}} & \text{for pipeline } k, k \in K_{\text{pipe}} \end{cases}$$

$I_{\text{max}(k)}$  = storage facility k maximum capacity

$$I_{\text{max}(k)} = \begin{cases} I_{\text{max}(k)}^{\text{cav}} & \text{for salt cavern } k, k \in K_{\text{cav}} \\ I_{\text{max}(k)}^{\text{dep}} & \text{for depeleted reservoir } k, k \in K_{\text{dep}} \\ I_{\text{max}(k)}^{\text{aqu}} & \text{for aquifer } k, k \in K_{\text{aqu}} \\ I_{\text{max}(k)}^{\text{pipe}} & \text{for pipeline } k, k \in K_{\text{pipe}} \end{cases}$$

$I_{b(k)}$  = base gas requirement for facility k

$$I_{b(k)} = \begin{cases} I_{b(k)}^{\text{cav}} & \text{for salt cavern } k, k \in K_{\text{cav}} \\ I_{b(k)}^{\text{dep}} & \text{for depeleted reservoir } k, k \in K_{\text{dep}} \\ I_{b(k)}^{\text{aqu}} & \text{for aquifer } k, k \in K_{\text{aqu}} \\ I_{b(k)}^{\text{pipe}} & \text{for pipeline } k, k \in K_{\text{pipe}} \end{cases}$$

$L_{kt}$  = facility k gas loss on period t

$$L_{kt} = \begin{cases} L_{kt}^{\text{cav}} & \text{for salt cavern } k, k \in K_{\text{cav}} \\ L_{kt}^{\text{dep}} & \text{for depeleted reservoir } k, k \in K_{\text{dep}} \\ L_{kt}^{\text{aqu}} & \text{for aquifer } k, k \in K_{\text{aqu}} \\ L_{kt}^{\text{pipe}} & \text{for pipeline } k, k \in K_{\text{pipe}} \end{cases}$$

$P_{kt}^s$  = gas price realization s of facillity k on period t

$$P_{kt}^s = \begin{cases} P_{kt}^{s(\text{cav})} & \text{for salt cavern } k, k \in K_{\text{cav}} \\ P_{kt}^{s(\text{dep})} & \text{for depeleted reservoir } k, k \in K_{\text{dep}} \\ P_{kt}^{s(\text{aqu})} & \text{for aquifer } k, k \in K_{\text{aqu}} \\ P_{kt}^{s(\text{pipe})} & \text{for pipeline } k, k \in K_{\text{pipe}} \end{cases}$$

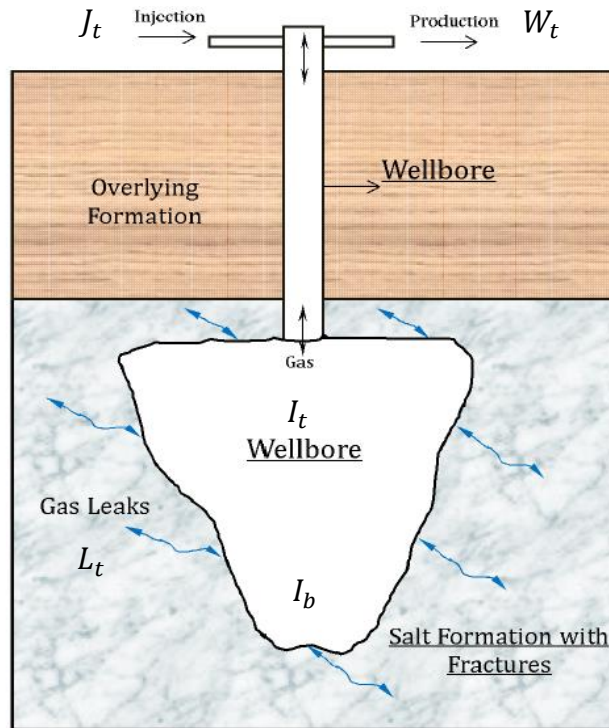
$D_{kt}$  = facility k gas demand on period t

$$D_{kt} = \begin{cases} D_{kt}^{\text{cav}} & \text{for salt cavern } k, k \in K_{\text{cav}} \\ D_{kt}^{\text{dep}} & \text{for depeleted reservoir } k, k \in K_{\text{dep}} \\ D_{kt}^{\text{aqu}} & \text{for aquifer } k, k \in K_{\text{aqu}} \\ D_{kt}^{\text{pipe}} & \text{for pipeline } k, k \in K_{\text{pipe}} \end{cases}$$

$S_{kt}$  = facility  $k$  gas supply on period  $t$

$$S_{kt} = \begin{cases} S_{kt}^{\text{cav}} & \text{for salt cavern } k, k \in K_{\text{cav}} \\ S_{kt}^{\text{dep}} & \text{for depleted reservoir } k, k \in K_{\text{dep}} \\ S_{kt}^{\text{aqu}} & \text{for aquifer } k, k \in K_{\text{aqu}} \\ S_{kt}^{\text{pipe}} & \text{for pipeline } k, k \in K_{\text{pipe}} \end{cases}$$

It is essential to note that when time  $t$  represent a month,  $t=1$  is April and  $t=12$  is March. For days in a year  $t=1$  is April 1<sup>st</sup> and  $t=T$  is March 31<sup>st</sup>. Figure 5 provides a visual representation of a salt cavern storage facility, which we use as a reference to formulate the storage schedule. It is taken from Barajas and Civan (2014). The diagram does not necessary portray the actual shape of a gas storage facility.



**Figure 5. A schematic salt cavern storage facility**

*Source: Barajas and Civan (2014)*

The mathematical formulation is as follows. The objective function is to maximize the expectation of the profit for all the desired facilities over a lease period  $T$ . The cash inflows as gas is withdrawn and sold, which is represented by the positive coefficient value for withdrawal decision variable. However, injection decision variable causes cash outflow since gas is purchased and added to storage facility. Hence, it is represented by the negative coefficient of the objective function equation. Basically only working gas is depleted and replaced, and the decision variables are for the working gas. The base gas remains in the storage facility throughout the lease period to maintain the reservoir pressure.

The outer summation of the objective function represents the number of days left in the lease period. The inner summation indicates the number of storage facilities over which the decisions are to be made.

The first constraint enforces the use or lose contract policy. All the gas purchased and added to all the storage facilities should be sold or used before the lease expires. For a given facility, the sum of the beginning inventory and the gas injection should be equal to the sum of the gas withdrawals and lost gas for all the remaining duration in the lease period. Note that the beginning inventory is zero at the beginning of the lease period.

The second constraint shows that the inventory level at the beginning of a given day is equal to the inventory at the beginning of the previous day plus the injection on the previous day minus the withdrawal and the lost gas on previous day for a facility. This is a typical dynamic inventory constraint.



Constraint 3 enforces that the inventory in storage facility on a given day should be less than the facility's maximum working capacity. Negative inventory level is not allowed.

$$\max \left[ \sum_t^T \sum_{k \in N} P_{kt}^S (a_{kt} W_{kt} - b_{kt} J_{kt}) \right]$$

Subject to

$$I_{kt} + \sum_t^T J_{kt} - \sum_t^T W_{kt} - \sum_t^T L_{kt} = 0; \forall k \quad 1$$

$$I_{kt} - I_{k(t-1)} - J_{k(t-1)} + W_{k(t-1)} + L_{k(t-1)} = 0; \forall k, \forall t \quad 2$$

$$0 \leq I_{kt} \leq I_{\max(k)}, \forall k, \forall t \quad 3$$

$$J_{kt} \leq J_{\max(k)} \sqrt{\frac{I_{b(k)}(I_{\max(k)} - I_{kt})}{I_{\max(k)}(I_{kt} + I_{b(k)})}} \quad 4$$

$$W_{kt} \leq W_{\max(k)} \sqrt{\frac{I_{kt}}{I_{\max(k)}}} \quad 5$$

$$\sum_{k \in N} J_{kt} \leq S_t; \forall t \quad 6$$

$$\sum_{k \in N} W_{kt} = D_t; \forall t \quad 7$$

$$a_{kt}, b_{kt} \in \{0,1\} \quad 8$$

The injection and withdrawal rates vary based on inventory level as shown by constraints 4 and 5 respectively. These are non-linear constraints since the inventory level is also another unknown variable.

Constraints 6 and 7 show that the injection and withdrawal quantities should not exceed the gas supply from pipelines and the gas demand from the distribution companies respectively. The withdrawal or injection decision on a given time window is represented by the binary constraint 8.

Some of the parameters are complex combinations of some other parameters. For example, the gas price  $t$  at any time for each facility is given by

$P_{kt}^s = P_{0i} \exp \left[ \left( \mu - \frac{\sigma^2}{2} \right) t + \sigma Z(t) \right]$ . The detail explanation of the formula derivation is provided at the end of this section.

Similarly, the following formulas are used to determine some of the model parameters.

Maximum injection and withdrawal rate for aquifer and depleted reservoir

$$J_{\max(k)} \approx W_{\max(k)} = 6.283 \phi C_w R_g^2 h (P_g - P_f) Q_t$$

Loss function for aquifers and depleted reservoirs is

$L_{kt} = 3.74 * 10^{-7} (p_G^2 - 1.6^2)^n$ . The loss function is usually assumed to be constant for salt caverns and pipelines.

Maximum injection and withdrawal rate for depleted oil reservoirs

$$J_{\max(k)} \approx W_{\max(k)} = \frac{0.0993 Q_0 B u_0 (P_1^2 - P_2^2)_g}{z T u_g (P_1 - P_2)_0}$$

The maximum inventory carrying capacity for pipeline

$$I_{\max(k)} = 19.20 \frac{D^2 T_b L}{p_b T} \left[ \left[ p'_1 + p'_2 - \frac{p'_1 p'_2}{p'_1 + p'_2} \right] - \left[ p_1 + p_2 - \frac{p_1 p_2}{p_1 + p_2} \right] \right]$$

Maximum injection pipeline capacity

$$J_{\max(k)} = 433.488 \frac{T_b}{p_b} \sqrt{\frac{(p_{t(inlet)}^2 - p_{t(inside)}^2) D^{16/3}}{GT_t L \bar{Z}}}$$

Maximum withdrawal pipeline capacity

$$W_{\max(k)} = 433.488 \frac{T_b}{p_b} \sqrt{\frac{(p_{t(inside)}^2 - p_{t(outlet)}^2) D^{16/3}}{GT_t L \bar{Z}}}$$

### 4.3. Model Complexity

For each storage facility, a three decision variable and eight constraints problem should be solved for all the duration remaining in the lease period. Note that the constraints are not independent. For K number of storage facilities and T lease time window, the problem size we need to solve for each price realization before we make a decision is  $3K(T-(t-1))$  decision variables and  $8K(T-(t-1))$  constraints problem, where  $T-(t-1)$  is the total number of days left in the lease period. For example, solving a 400 underground storage facilities problem on the first day of the lease assuming that there are 252 working days in a year has the following complexity. The number of decision variables for one price realization is  $3*400*(252-(1-1)) = 303,400$ . The number of constraints will be  $8*400*(252-(1-1)) = 806,400$  constraints of which 201,600 are non-linear. The data input for price is  $K*T$  for each run. The supply data are  $K*T$ . The demand data are

$K \cdot T$ . The base gas, maximum injection, maximum withdrawal, inventory capacity data are each  $K$ . The total data input for a single run is  $3 \cdot K \cdot T + 4 \cdot K = 3 \cdot 400 \cdot 252 + 4 \cdot 400 = 304,000$  data points. For 100 realizations, we use a randomly generated data based on the historic data.

The derivation of the non-linear constraints 4 and 5 are based on the gas law, the fluid flow, and the Bernoulli's principles. Please refer to Thompson et al. (2009), Barajas and Civan (2014), and Ikoku (1980) for the details. Below is the summary of the derivations

*Derivation of withdrawal and injection rates*

The injection and withdrawal rates vary non-linearly and are very complicated to formulate. An approximate derivations are provided in this section.

From real gas law under standard condition it is known that  $PV = ZnRT$

$p$  = absolute pressure, psia

$V$  = volume,  $\text{ft}^3$

$T$  = absolute temperature,  $^{\circ}\text{R}$

$n$  = number of lb-moles, where one lb-mole is the molecular weight of the gas (lbs)

$R$  = the universal gas constant which, for the above units, has the value of  $10.732 \text{ ft}^3/(\text{lb-mole } ^{\circ}\text{R})$

$Z$  = the gas deviation factor or the  $Z$ -factor

$$Z = \frac{\text{Actual volume of } n \text{ moles of gas at certain pressure, } p \text{ and Temperature, } T}{\text{Ideal (calculated) volume of } n \text{ moles of gas at same } p \text{ and } T}$$

If  $V_0$  is the gas volume at an atmospheric pressure of 14.7 psia then applying the real gas law equation,  $pV_0 = nRT$ , since  $Z$  is  $\cong 1$  at atmospheric pressure. This gives  $14.7 V_0 = nRT$ ,  $P = \frac{ZnRT}{V}$ . If we let  $k_1 = \frac{ZRT}{V}$ ,  $P = k_1 n$  under fixed temperature and volume. The number of substance in a storage facility is the summation of the base gas and the working inventory. The gas inventory at time  $t$  is  $I_{kt} + I_{bi}$ , which implies  $P = k_1(I_t + I_b)$ .

#### *Withdrawal rate*

From Bernoulli's principle, for a gas leaving a storage facility,

$$P_{inside} = P_{outside} + \frac{1}{2} \rho_{outside} v_{outside}^2$$

$P_{inside}$  = gas pressure inside storage facility

$P_{outside}$  = gas pressure leaving storage facility

$\rho_{outside}$  = density of gas leaving storage facility

$v_{outside}$  = velocity of gas leaving storage facility

Since the pressure inside can be given by,  $P_{inside} = k_1(I_t + I_b)$ . In addition, when working gas inventory falls down to zero, we have only base gas in a particular facility. This is the only condition when pressure inside the facility equals to outside pressure; which is  $P_{inside} = P_{outside} = k_1(0 + I_b)$ .

Note that we use base gas to maintain reservoir pressure. Then substituting  $P_{inside}$  by  $k_1(I_t + I_b)$ , and  $P_{outside}$  by  $k_1(0 + I_b)$  into  $P_{inside} = P_{outside} + \frac{1}{2}\rho_{outside}v_{outside}^2$ , will give us the following

$$k_1(I_t + I_b) = k_1(0 + I_b) + \frac{1}{2}\rho_{outside}v_{outside}^2$$

This implies,  $\rho_{outside} = \sqrt{\frac{2k_1I_t}{\rho_{outside}}}$

We also know that volumetric flow rate of gas leaving a storage facility is given by

$$Q = vA, \text{ where}$$

$v$  = velocity of the gas

$A$  = cross-sectional area

We can replace the withdrawal rate  $W_t$  by  $Q$  as  $W_t = v_{outside}A$ , we also know that

when we withdraw gas we deal with velocity of gas leaving the storage facility,

$v_{outside}$  . When we substitute

$$v_{outside} = \sqrt{\frac{2k_1I_t}{\rho_{outside}}} \text{ in } W_t = v_{outside}A, \text{ and let area } A \text{ be } k_2 \text{ we will find}$$

$$W_t = k_2 \sqrt{\frac{2k_1I_t}{\rho_{outside}}} = \sqrt{\frac{2k_1}{\rho_{outside}}} k_2 \sqrt{I_t}$$

$$\text{Let } k_3 = \sqrt{\frac{2k_1}{\rho_{outside}}} k_2 \sqrt{I_t}$$

$$W_t = k_3 \sqrt{I_t}$$

The maximum flow rate can only be achieved when working gas capacity at time t is equal to the maximum working capacity, i.e.  $I_t = I_{max}$  at  $W_{max}$ . Hence,

$$W_{max} = k_3 \sqrt{I_{max}}, \text{ implies } k_3 = \frac{W_{max}}{\sqrt{I_{max}}}.$$

Therefore the maximum rate at time t is given by

$$W_t = W_{max} \sqrt{\frac{I_t}{I_{max}}}$$

*Injection rate*

Similar to the withdrawal rate derivation,  $P_{inside} = k_1(I_t + I_b)$ .

Given  $\rho = \frac{M}{V}$ , where M and V are mass of gas inside a storage and volume of the storage facility respectively. Mass of gas is proportional to base gas and working gas assuming that the storage facility has fixed volume.

$$\text{Let } M = k_4(I_t + I_b), \text{ hence } \rho = \frac{k_4}{V}(I_t + I_b)$$

Again from Bernoulli's principle, outside pressure is the sum of inside pressure and

$\frac{1}{2}\rho_{inside}v_{inside}^2$ , as opposed to withdrawal rate.

$$P_{outside} = P_{inside} + \frac{1}{2}\rho_{inside}v_{inside}^2$$

Plugging  $\rho = \frac{k_4}{V}(I_t + I_b)$  and  $P_{inside} = k_1(I_t + I_b)$  into  $P_{outside} = P_{inside} + \frac{1}{2}\rho_{inside}v_{inside}^2$ , gives

$$P_{outside} = k_1(I_t + I_b) + \frac{1}{2} \frac{k_4}{V} (I_t + I_b) v_{inside}^2$$

$$v_{inside} = \sqrt{\frac{2VP_{outside}}{k_4} \frac{1}{(I_t + I_b)} - \frac{2Vk_1}{k_4}}$$

$$\text{Let } k_5 = \frac{2VP_{outside}}{k_4}; \text{ and } k_6 = \frac{2Vk_1}{k_4}$$

$$v_{inside} = \sqrt{\frac{k_5}{(I_t + I_b)} - k_6}$$

Again from volumetric flow rate we know that the injection rate at time t is given by

$J_t = v_{inside}A$ . We cannot inject gas when the facility is at its maximum working

capacity, i.e  $J_t = 0$  when  $I_t = I_{max}$ . Hence,

$$J_t = A \sqrt{\frac{k_5}{(I_t + I_b)} - k_6}$$

$$0 = A \sqrt{\frac{k_5}{(I_{max} + I_b)} - k_6}$$

This gives  $k_5 = k_6(I_{max} + I_b)$ , implies

$$J_t = A \sqrt{\frac{k_6(I_{max} + I_b)}{(I_t + I_b)} - k_6}$$

$$J_t = A\sqrt{k_6} \sqrt{\frac{(I_{max} + I_b)}{(I_t + I_b)} - 1}$$

$$\text{Let } k_7 = A\sqrt{k_6}$$

$$J_t = k_7 \sqrt{\frac{(I_{max} - I_t)}{(I_t + I_b)}}$$

On the other hand, injection at time t is maximum when inventory is zero. i.e if  $I_t =$

0 then  $J_t = J_{max}$  as a result,



$$J_{max} = k_7 \sqrt{\frac{(I_{max} + 0)}{(0 + I_b)}}$$

Implies  $k_7 = J_{max} \sqrt{\frac{I_b}{I_{max}}}$ , put in  $J_t$

$$J_t = J_{max} \sqrt{\frac{I_b(I_{max} - I_t)}{I_{max}(I_t + I_b)}}$$

Now we have an expression for injection and withdrawal rate per unit time in terms of inventory level. Base gas,  $I_b$  and  $I_{max}$  are known parameters or can be estimated.

### *Gas price derivation*

Brownian motion with drift equation is used to model the gas price.

Brownian motion represents the random price move either upward or downward at any time  $t$  in the future. It has volatility factor which shows how quickly gas prices change because of severe weather conditions, change in demand, availability of supply, and economic conditions. Mean reversion pricing component is common in gas industry because of seasonality. The gas price goes up and down very quickly. The mean reversion pulls the price back to a long run average gas price. High gas price tends to have negative trend to revert back to the mean; on the other hand, low gas price will have positive trend to bounce back to the average.

The model is given by  $dP = \mu(P, t)dt + \sigma(P, t)dZ$ , where  $P$  is the gas price,  $\mu$  is the drift rate,  $\sigma$  is the volatility, and  $dZ$  is the random term which has the form of  $dZ = \phi\sqrt{dt}$ . The  $\phi$  is a random variable taken from a normal distribution with mean zero and variance one ( $\phi \sim N(0,1)$ ). We assume that the gas price  $P$  is random and goes to  $P + dP$ , as time  $t$  goes to  $t + dt$ ;  $dt$  is an infinitesimal time.

Using Ito's Lemma it is easy to derive the value of P from  $dP = \mu(P, t)dt + \sigma(P, t)dZ$  with the assumption that it follows logarithmic price parameter (no negative price value). Please refer to (Cont and Voltchkova, 2005; Pindyck, 1999; and Forsyth, 2012) for the details.

$$P_t = P_0 \exp \left[ \left( \mu - \frac{\sigma^2}{2} \right) t + \sigma Z(t) \right]$$

Where,

$P_t$  = gas price at time t

$P_0$  = initial gas price. There is not one perfect way to estimate a starting parameter as stated in Pindyck (2001), hence average historic price data are used.

$\mu$  = the drift term which is the percent price mean change

$\sigma$  = volatility which is standard deviation of the percent price change

The expressions for the mean and the change in price are respectively given by

$$\mu = \frac{\sum_{t=1}^T \Delta P}{T-1} \text{ and } \Delta P = P_t - P_{t-1}.$$

## Chapter 5: Solution Approach

### 5.1. General Framework

The mathematical formulation of the problem has no meaning from a decision making point of view because of the stochastic input parameters. There could be infinite number of possible solutions. Therefore, I propose a simulation of finite stochastic parameters realizations followed by optimization of the mathematical model. The optimization model is run for every realization of the parameters. Then the expected value of the outputs is used for the decision making purpose. The pictorial representation of this approach is depicted by Figure 6 below.

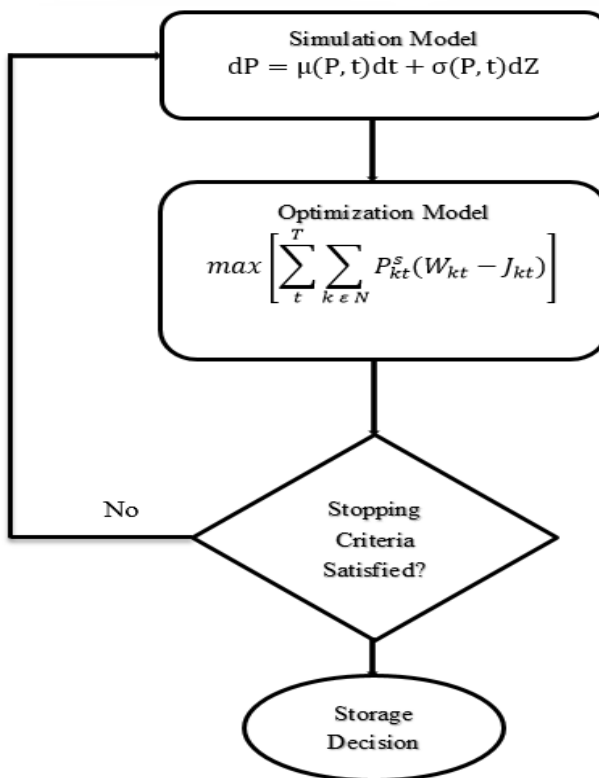


Figure 6. Experimental approach

Percent change in price historic data are used to estimate the drift coefficient and volatility. The optimization model is very difficult to solve as a single problem because of the complexity of the problem explained in the previous chapter. It has very large number of non-linear withdrawal and injection constraints, the integer constraints and the dynamic constraints. The following heuristic approach is proposed to linearize the problem and minimizes the number of non-linear constraints to zero.

## **5.2. Heuristic Approach**

First, the non-linear constraints are linearized by fixing the value of the inventory level variable for the injection and withdrawal rate constraints. However, the inventory level in the inventory balance in the dynamic constraint remains as a decision variable. Then the original problem will become a linear dynamic model. Then solve the linear dynamic model one time. The output values of the model will be average withdrawal decisions, average injection decisions, and the corresponding average inventory levels. I represent the problem I solve here as Problem 1.

Second, solve the first iteration of the original model using the average inventory levels of Problem 1 as input to the model. Then resolve the model until stopping criterion are satisfied. The stopping criterion are the objective function value and the inventory level differences. I represent the problem we solve in the second step as Problem 2.

More specifically, Problem 1 and Problem 2 are given as follows.

*Problem 1:*

$$\max \left[ \sum_t^T \sum_{k \in N} P_{it}^s (a_{kt} W_{kt} - b_{kt} J_{kt}) \right]$$

Subject to

$$I_{kt} + \sum_t^T J_{kt} - \sum_t^T W_{kt} - \sum_t^T L_{kt} = 0; \forall k \quad 1$$

$$I_{kt} - I_{k(t-1)} - J_{k(t-1)} + W_{k(t-1)} + L_{k(t-1)} = 0 \quad 2$$

$$0 \leq I_{kt} \leq I_{\max(k)} \quad 3$$

$$J_{kt} \leq J_{\max(k)} \quad 4$$

$$W_{kt} \leq W_{\max(k)} \quad 5$$

$$\sum_{k \in N} J_{kt} \leq S_t; \forall t \quad 6$$

$$\sum_{k \in N} W_{kt} = D_t; \forall t \quad 7$$

$$a_{kt}, b_{kt} \in \{0,1\} \quad 8$$

Problem 2:

$$\max \left[ \sum_t^T \sum_{k \in N} P_{kt}^S (a_{kt} W_{kt} - b_{kt} J_{kt}) \right]$$

Subject to

$$I_{kt} + \sum_t^T J_{kt} - \sum_t^T W_{kt} - \sum_t^T L_{kt} = 0; \forall k \quad 1$$

$$I_{kt} - I_{k(t-1)} - J_{k(t-1)} + W_{k(t-1)} + L_{k(t-1)} = 0 \quad 2$$

$$0 \leq I_{kt} \leq I_{\max(k)} \quad 3$$

$$J_{kt} \leq J_{\max(k)} \sqrt{\frac{I_{b(k)}(I_{\max(k)} - I_{kt}^{r-1})}{I_{\max(k)}(I_{kt}^{r-1} + I_{b(k)})}} \quad 4$$

$$W_{kt} \leq W_{\max(k)} \sqrt{\frac{I_{kt}^{r-1}}{I_{\max(k)}}} \quad 5$$

$$\sum_{k \in N} J_{kt} \leq S_t; \forall t \quad 6$$

$$\sum_{k \in N} W_{kt} = D_t; \forall t \quad 7$$

$$a_{kt}, b_{kt} \in \{0,1\} \quad 8$$

The summary of the heuristic approach is shown by the Figure 7. The inventory level output of Problem 1 is represented as  $(I_{kt(0)}^*)$ . The stopping criterion are based on objective function value ( $f^*$ ) and the inventory level for the second stage iteration after each run ( $I_{kt}^{r*}$ ), where the iteration number is represented by  $r$ . We terminate the iteration when the average difference of the previous iteration and the current iteration is less than or equals to  $\Delta_f$  and  $\Delta_I$  respectively. The  $\Delta_f$  and  $\Delta_I$

are specified by the modeler.  $J_{kt}^{s(R)}$ ,  $W_{kt}^{s(R)}$ ;  $I_{kt}^{s(R)}$  are the last iteration values for each price realization.

The details of the heuristic algorithm implementation is given below.

For  $s \in$  all price realization scenarios

Solve Problem 1

For  $k \in$  all storage facilities

For  $t$  from (T-t) to T

$$\text{Set: } J_{kt} = f(J_{\max(k)})$$

$$\text{Set: } W_{kt} = f(W_{\max(k)})$$

$$\max_{J_{kt}, W_{kt}, I_{kt}} \text{profit}$$

Set objective function value to  $f^{*(0)}$ ; inventory level to  $I_{kt}^{*(0)}$

Solve Problem 2

$$\text{Initialize: } |f^{*(r-1)} - f^{*(r)}| = \infty; |I_{kt}^{*(r-1)} - I_{kt}^{*(r)}| = \infty; r = 0$$

$$\text{While } |f^{*(r-1)} - f^{*(r)}| \leq \Delta_f \text{ and and } |I_{kt}^{*(r-1)} - I_{kt}^{*(r)}| \leq \Delta_I$$

$$r := r + 1$$

$$\text{Update: } f^{*(r-1)} - f^{*(r)}; I_{kt}^{*(r-1)} - I_{kt}^{*(r)}$$

For  $k \in$  all storage facilities

For  $t$  from (T-t) to T

$$\text{Set: } J_{kt} = f(J_{\max(k)}, I_{b(k)}, I_{\max(k)}, I_{kt}^{r-1})$$

$$\text{Set: } W_{kt} = f(W_{\max(k)}, I_{\max(k)}, I_{kt}^{r-1})$$

$$\max_{J_{kt}, W_{kt}, I_{kt}} \text{profit}$$

$$J_{kt}^{*(s)} = J_{kt}^{s(R)}; W_{kt}^{*(s)} = W_{kt}^{s(R)}; I_{kt}^{*(s)} = I_{kt}^{s(R)}$$

$$W_{kt}^* = E[W_{kt}^{*(s)}]; J_{kt}^* = E[J_{kt}^{*(s)}]; I_{kt}^* = E[I_{kt}^{*(s)}]$$

We begin by solving Problem 1 and store the optimal inventory level and the objective function values. Then solve Problem 2 and store the optimal inventory level and the objective function values again. Iterate over problem 2 while the difference between the current and the previous objective function values, and the difference between the current and the preceding inventory level is greater than a very small number set by the modeler. The inventory level from Problem 1 is used as input to inventory levels of constraints 4 and 5. The inventory level in constraint 3 remains as a decision variable. The algorithm is summarized using Figure 7.

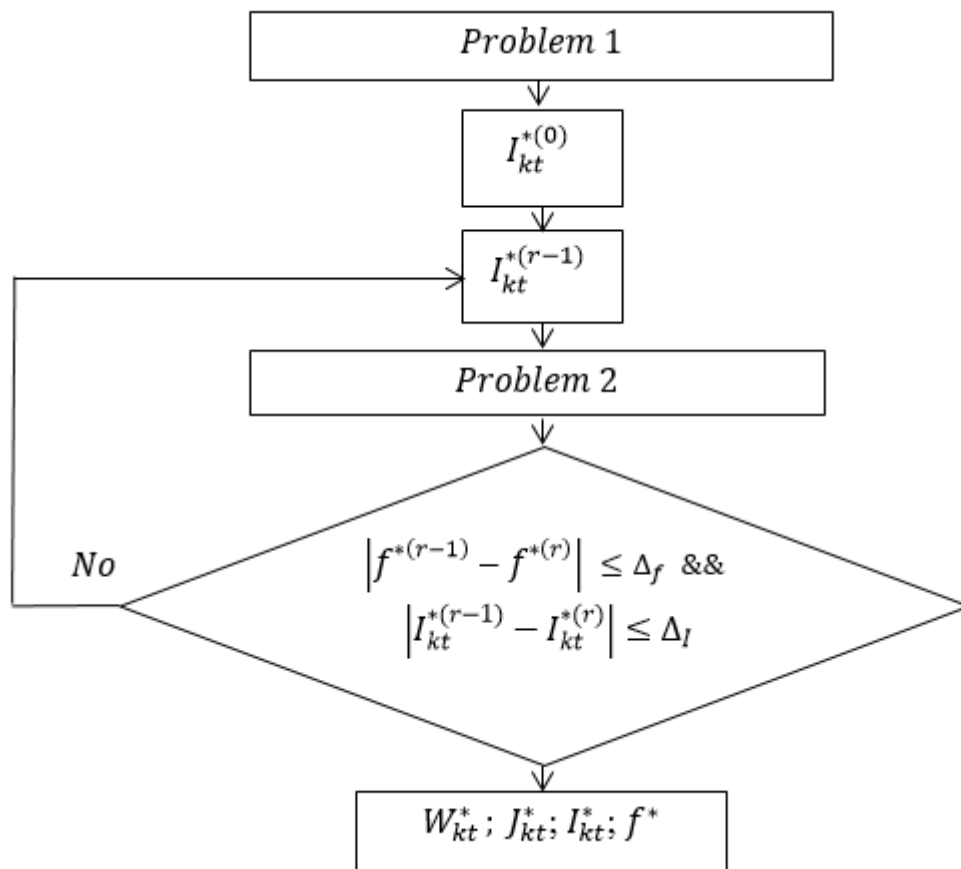


Figure 7. Procedural optimization approach



## **Chapter 6: Computational Results and Analysis**

The heuristic algorithm developed in this research is capable of solving a very large scale problem. One such a problem is 400 underground storage facilities, 0.305 million miles of pipeline, 0.3 million decision variables, 0.8 million constraints for every realization of stochastic parameters. The output data from this large scale model is well over 3000 pages. The reader can contact me for the large scale dataset need. A numerical result for a very small size problem is presented in this section for simplicity. The input data into the model is location specific. Some missing data are either approximated based on the location's historic data or systematically simulated. Energy Information Administration, Naturalgas.org, yahoo finance, and Natural Gas Handbook are sources of the historic data and some of the constant values.

### **6.1. Price Simulation**

The gas price is simulated for each storage facility location for every decision time window. Average price, the sample mean change, and the sample standard deviation of daily price changes for each month were computed. Table 1 below is a monthly sample of such computation of twelve months of a location over 17 years.

The  $P_0$ ,  $\mu$ , and  $\sigma$  represent the average price, the monthly sample mean change, the sample standard deviation of monthly price changes respectively. These data were used as input parameters to the geometric Brownian motion equation for simulation of gas prices which are used as input to the optimization model. The data for each month are used to simulate the price of the same month of the following year for the particular storage facility.

**Table 1. Sample price simulation input parameters**

<i>Month</i>	$P_0$	$\mu$	$\sigma$	<i>Month</i>	$P_0$	$\mu$	$\sigma$
April	4.96	0.0049	0.3015	October	5.01	0.0090	0.5806
May	5.06	0.0056	0.3469	November	5.05	0.0100	0.6745
June	5.16	0.0073	0.5031	December	5.06	0.0133	0.7350
July	4.77	0.0064	0.4867	January	5.20	0.0109	0.8835
August	4.83	0.0049	0.4501	February	5.00	0.0120	0.7085
September	4.57	0.0097	0.6073	March	4.77	0.0083	0.5766

## 6.2. Decision Scenarios

A decision scenario for one month time window is presented in this section. For each time window, a day in this case, fifty realizations of the possible decisions are simulated and optimized. The summary of the results is presented in the Table 2 below. The summary is for the minimum, expected value, and maximum possibilities of the profit scenarios based on minimum, average, and maximum, values of the injection and withdrawal decision scenarios.

**Table 2. Sample optimization output data**

Jt	Wt	Profit Scenario		
		Min	Mean	Max
59.17	0.00	-253.27	-294.62	-347.39
50.48	8.53	-428.86	-504.24	-609.59
36.77	17.77	-502.77	-600.51	-742.47
38.50	15.32	-593.40	-717.95	-905.61
33.53	13.54	-668.12	-818.13	-1039.42
32.36	15.65	-726.67	-901.18	-1160.62
31.57	15.49	-784.54	-981.22	-1284.84
22.49	27.24	-768.70	-957.52	-1245.63
34.09	19.84	-816.27	-1027.95	-1364.73
29.00	22.55	-837.83	-1059.48	-1415.82
30.08	22.78	-861.16	-1095.32	-1479.94
28.48	28.36	-861.52	-1095.88	-1480.94
26.74	27.37	-859.70	-1092.80	-1474.81
17.44	43.69	-784.80	-961.25	-1214.43
15.70	39.97	-715.91	-838.64	-968.03
11.61	48.48	-607.83	-654.28	-616.82

9.39	44.38	-504.09	-479.98	-270.49
8.04	40.41	-409.28	-316.59	89.40
7.28	31.52	-338.45	-193.64	369.20
3.60	27.24	-266.31	-75.91	627.29
0.00	16.20	-222.12	4.43	807.75
		-444.24	8.86	1615.51

Figures 8 and 9 below show the injection and the withdrawal decision spaces respectively. The blue line connecting decisions of each day is the median decision while the black line is the mean value. The red dotted points show the fifty possible outcomes for each day.

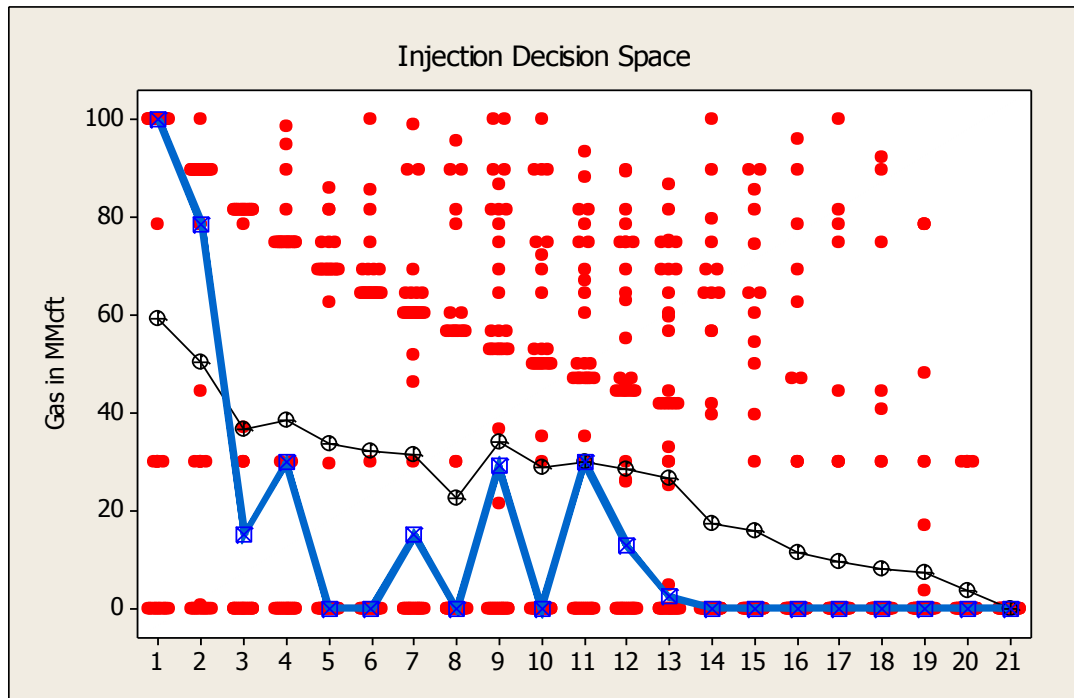


Figure 8. Injection decision space for fifty realizations

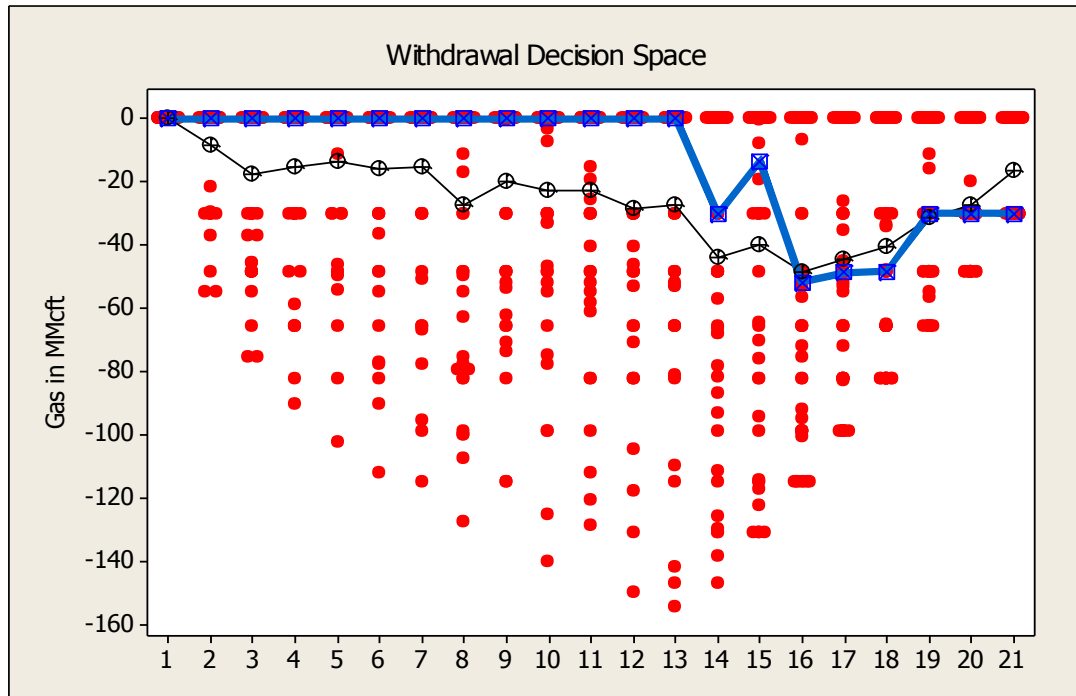
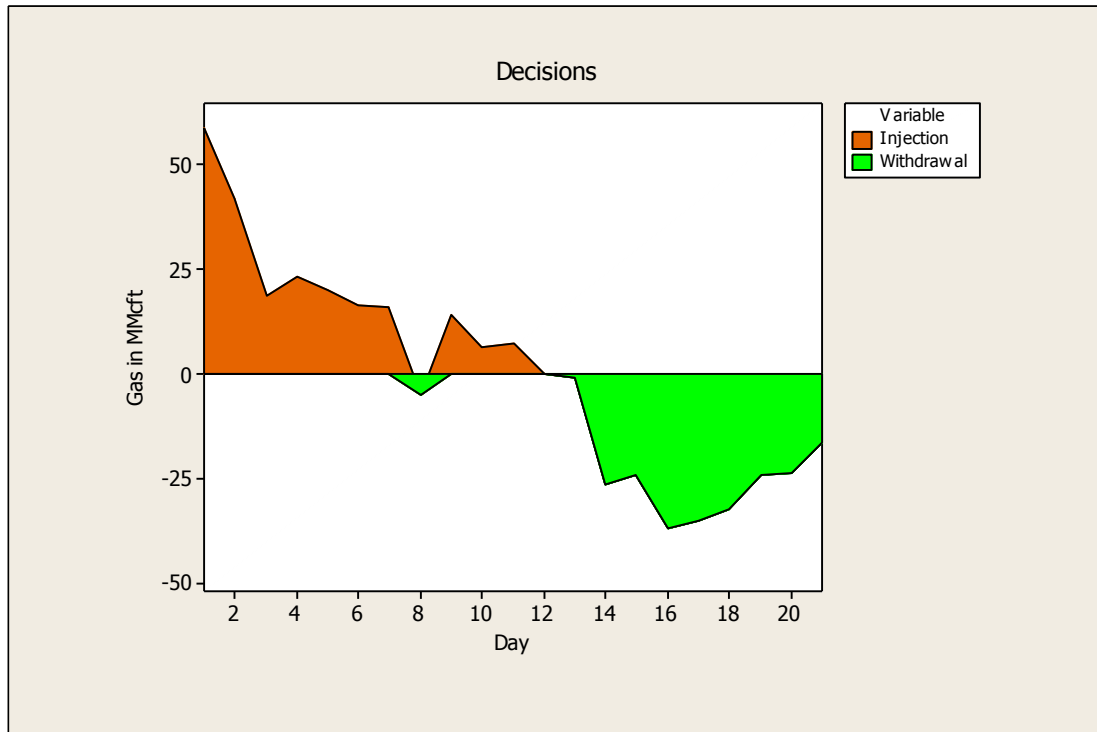


Figure 9. Withdrawal decision space for fifty realizations

The injection and withdrawal decisions for the median values appear to be mutually exclusive. The direction of the resultants of the injection-withdrawal decisions of the expected values are also consistent with the magnitude indicated by the median. This shows that the heuristic algorithm execute the model as desired. From the decision making point of view, the decision maker is assumed to be interested in the resultant of the decisions generated by the expected values. This is depicted by Figure 10 below. The above zero orange region shows the injection decision for days 1 - 5, 9 - 11. The inventory level inside the storage facility increases as we inject gas to the facility. The below zero green surface shows the withdrawal decision for days 8, 13-21 indicating the removal of gas from the storage facility. The ‘do-nothing’ decision option is shown on day 8.



**Figure 10. Withdrawal-injection decisions**

The decision maker uses this algorithm as a support tool. When we run the algorithm, the result we find is similar to Figure 10 above. However, the decision to be made is only for the current time window. Then after the current decision is made, one waits to see the outcome of the decision that has already been made before making a decision for the next time window. The simulation-optimization model is again run for the days remaining in the lease period. In a similar fashion, a decision for one day is made based on the possible outcomes of the days left in the lease period. A similar approach is used by Levary and Dean (1980) for a natural gas flow model under uncertainty in demand for a single natural gas trader.

Storage facility capacity constraint is also one of the decision variables. It is expected that the algorithm provides a result consistent with the lower and upper bounds of the total storage capacity. The lower bound is usually the base gas

inventory level, and the upper bound is the combination of the base gas and the working gas. Inventory level decision space is shown by Figure 11. The inventory level decision space for the scenarios start from zero, increases upto certain level and decrease until it becomes zero again. This is consistent with the mathematical formulation. The storage facility is assumed to be empty at the beginning of the lease period and the use-or-lose policy applies at the end of the lease period. The blue line connects the the expected value of the daily scenarios. The red dots are the fifty realizations for each time window.

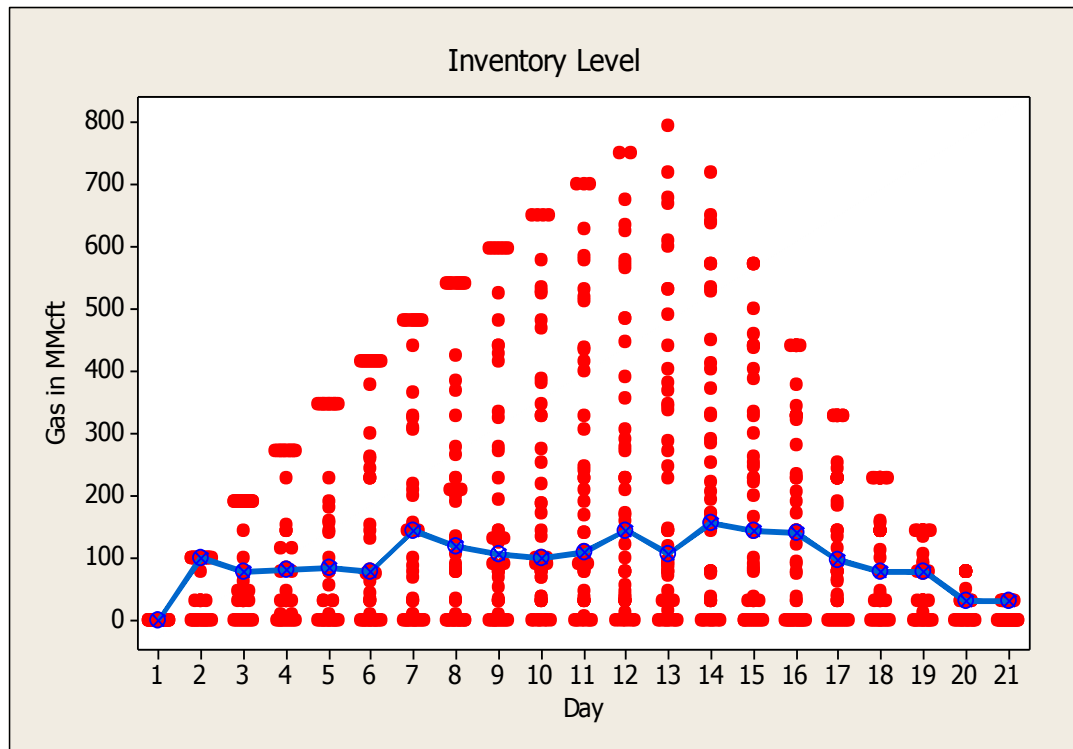


Figure 11. Inventory level space for fifty realizations

The profit scenario is shown by Figure 12 below. The green diamond dots, the red rectangular dots, and the black circular dots represent the maximum profit scenario, the average , and the minimum profit scenarios respectively. These are

based on their respective maximum, average, and minimum injection-withdrawal scenarios.

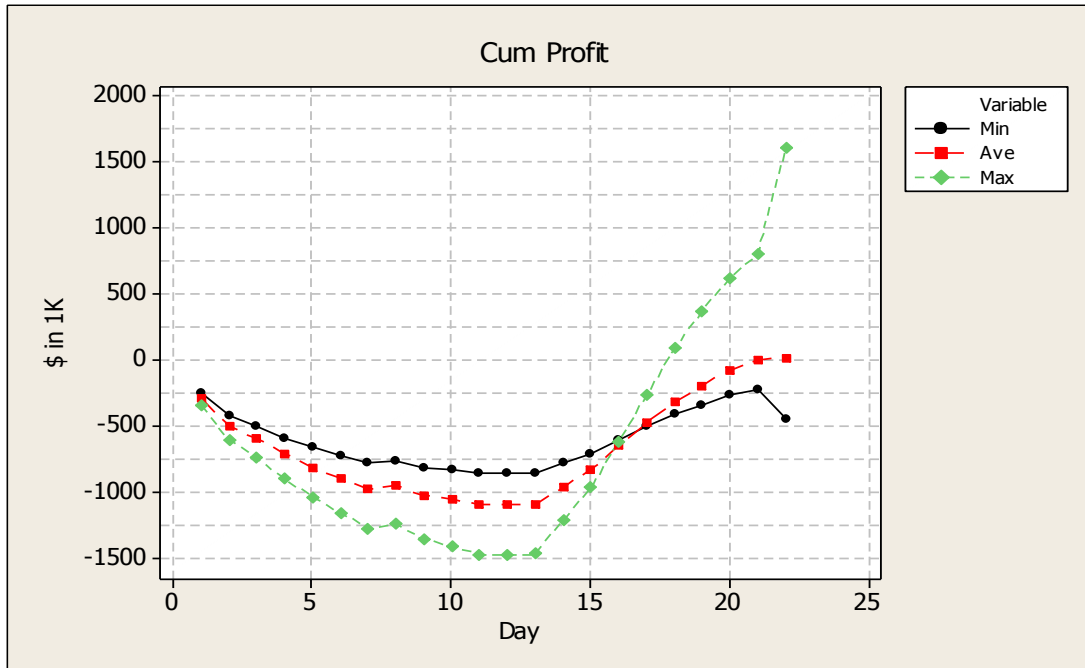
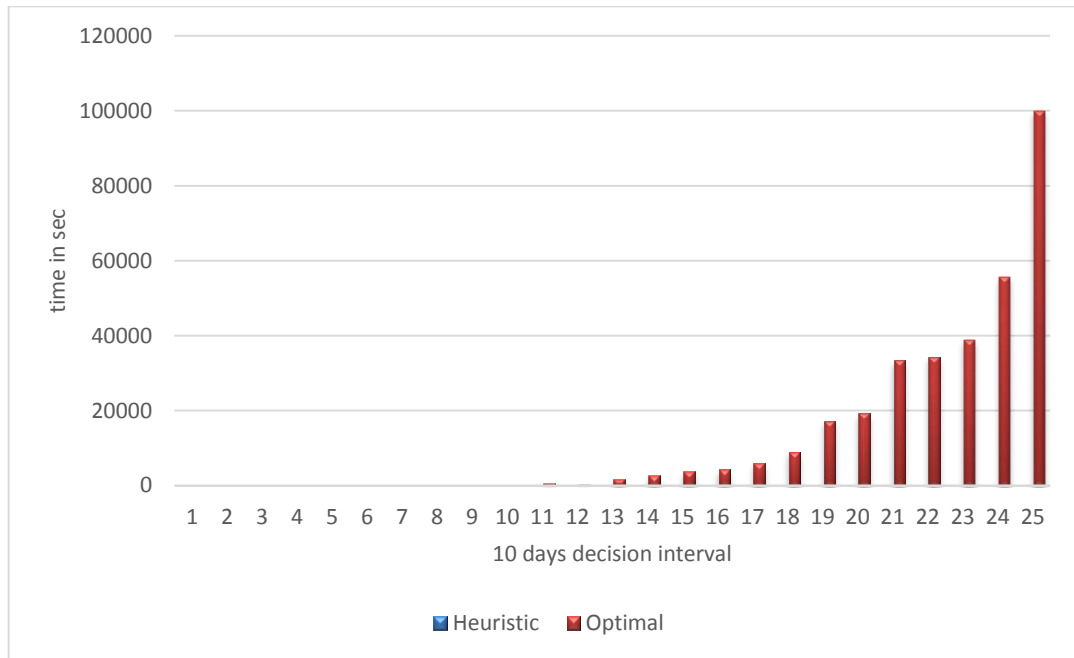


Figure 12. Cumulative profit graph

### 6.3. Model Performance

The performance of the model is evaluated based on computation time and solution quality. A yearlong decision scenario for one facility is used for the performance evaluation.

Figure 13 below shows the computational time comparison of the optimal solution of the model and the heuristic procedural approach. The red bars show the optimal solution computation time. The blue bars show the heuristic solution computation time. The result is only for one realization of price parameter and one salt cavern storage facility. It can be seen that the heuristic algorithm takes fraction of a second while it takes about 27 hours to solve the same problem optimally.



**Figure 13. Computation time comparison**

The solution quality of the heuristic algorithm is reasonably good. For smaller time windows such as a month, the heuristic provides a solution optimality gap as low as 0.05%. The worst case scenario approaches about 10% solution gap. Figure 14 shows the solution quality of the problem solved using the heuristic approach and the optimal approach. The red bars show the optimal solution values. The blue bars show the solution found by the heuristic procedural approach.



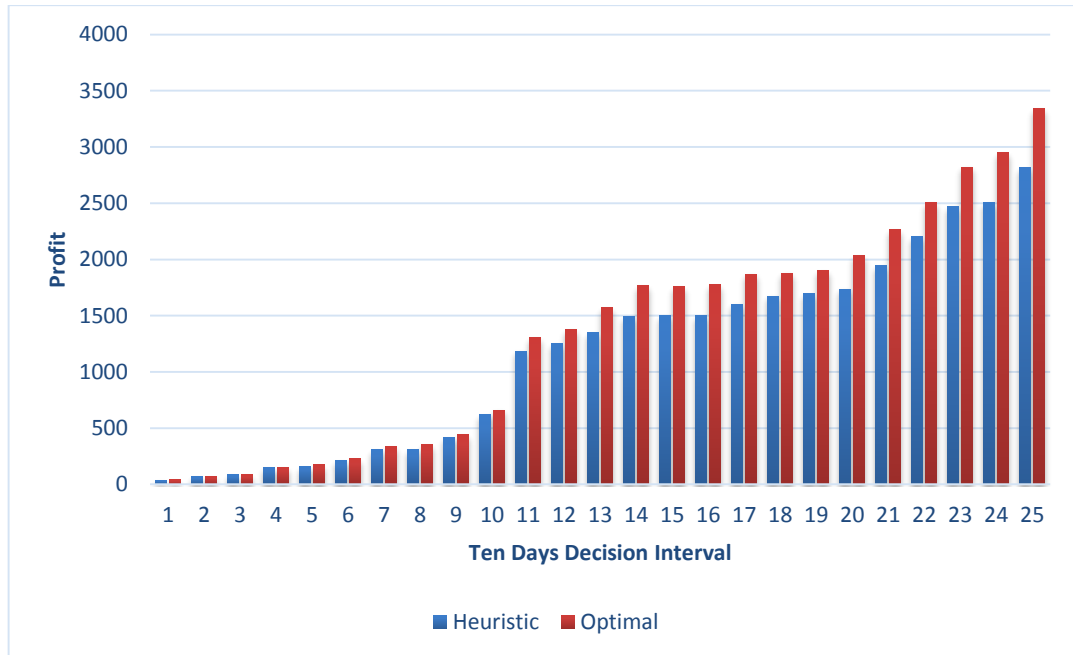


Figure 14. Solution quality comparison

#### 6.4. Model Behavior

Several experiments were carried out to observe the behavior of the stopping criteria and the complexity of the model due to the physical storage facility characteristics.

##### *The stopping criteria*

The stopping criteria used in the algorithm is the sum of the objective value differences and the sum of inventory level differences of the current iterations and the previous iteration, as described in the algorithm section. Figure 15 shows the consecutive objective value difference for the first seventeen iteration. It can be seen that the difference monotonically decreases for the three scenarios used. The scenarios were created based on maximum working capacity and base gas requirements.

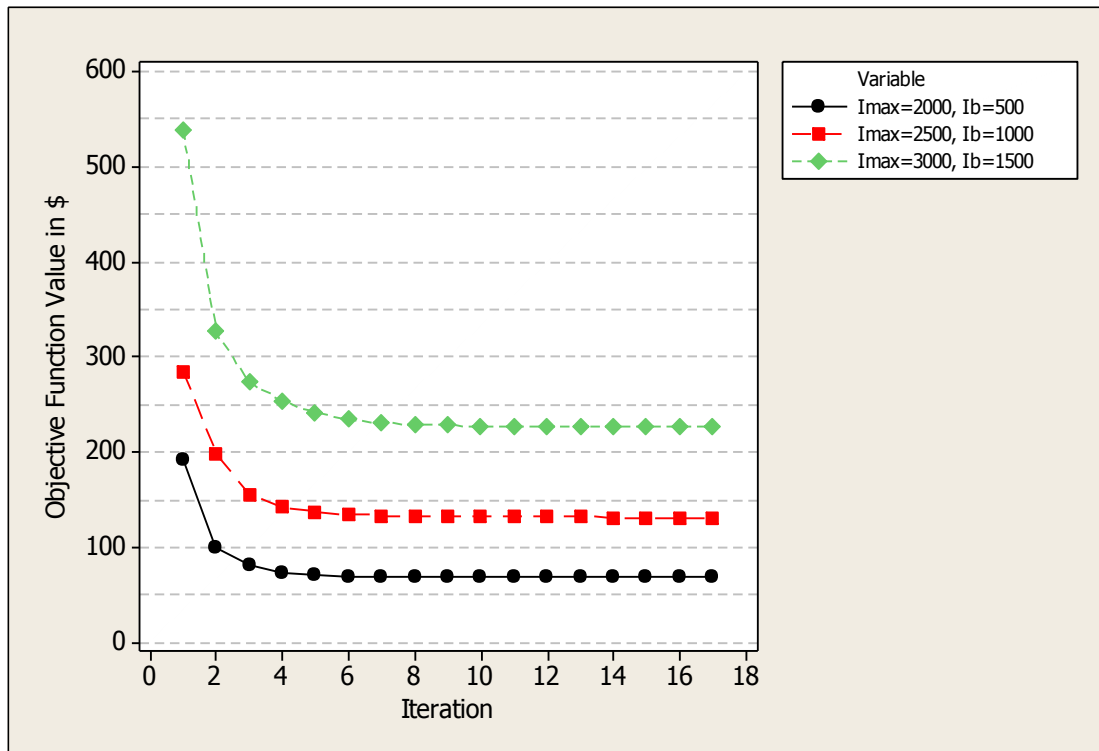


Figure 15. Stopping criteria scenario 1

Figure 16 shows the inventory level difference for the same scenarios used in objective value difference. The inventory level difference smoothly decreases until it reaches the small stopping criteria for all the scenarios experimented.

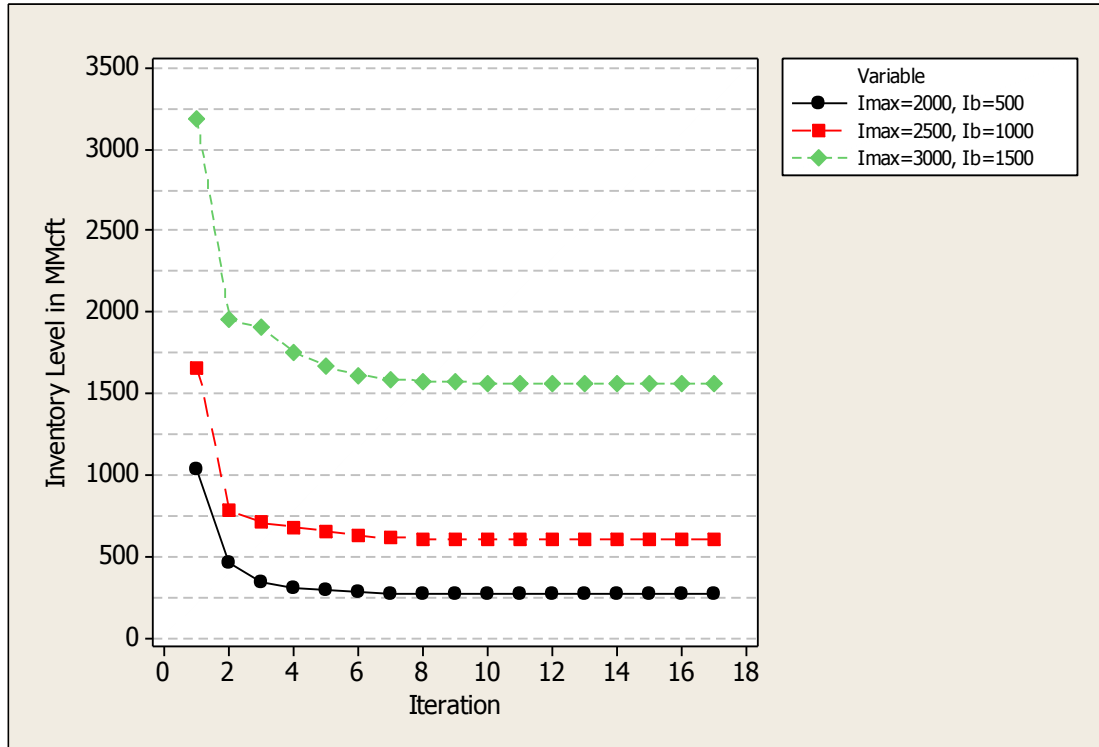


Figure 16. Stopping criteria scenario 2

### Storage facility complexity

Three different scenarios based on maximum working capacities and base gas are created to demonstrate the non-linearity of the problem. Scenario 1 (maximum working capacity of 1500 MMcft and 400MMcft of base gas) Figure 17, Scenario 2 (maximum working capacity of 2000 MMcft and 500MMcft of base gas) Figure 18, and Scenario 3 (maximum working capacity of 3000 MMcft and 500MMcft of base gas). Each scenario is experimented by five scenarios of maximum injection and withdrawal rates. These are maximum injection rate and withdrawal rates of 60 and 200MMcft, 70 and 250 MMcft, 80 and 300 MMcft, 90 and 350 MMcft, and 100 and 400MMcft respectively.

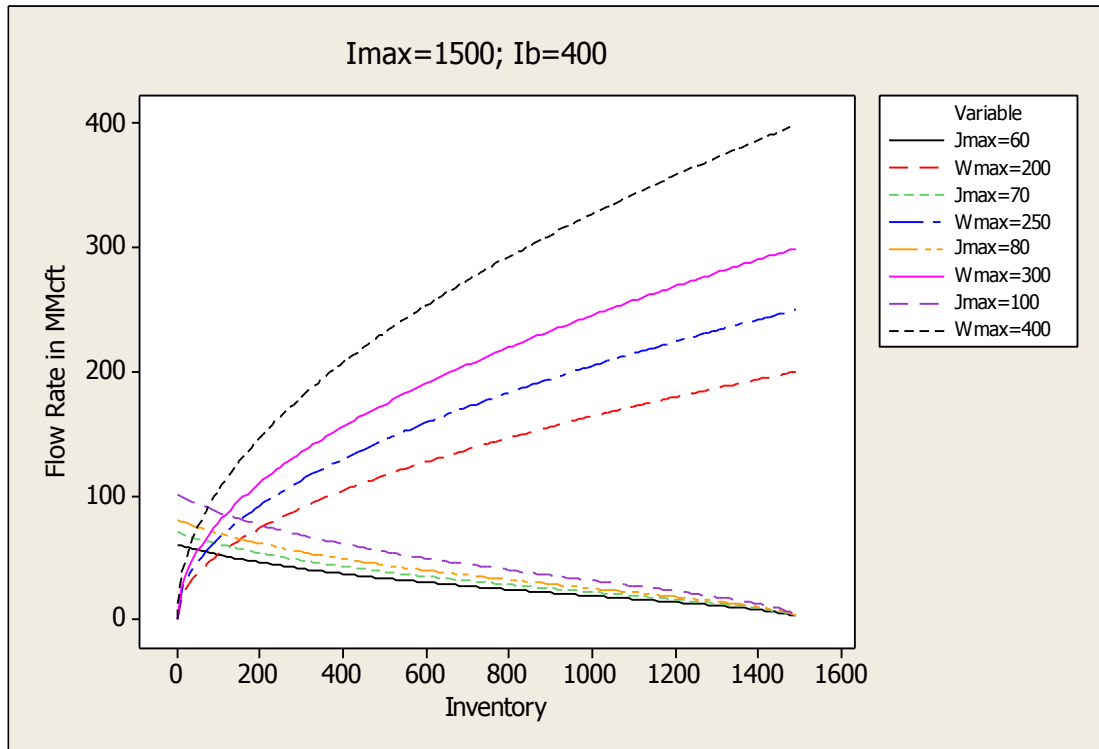


Figure 17. Injection-withdrawal curves sample 1

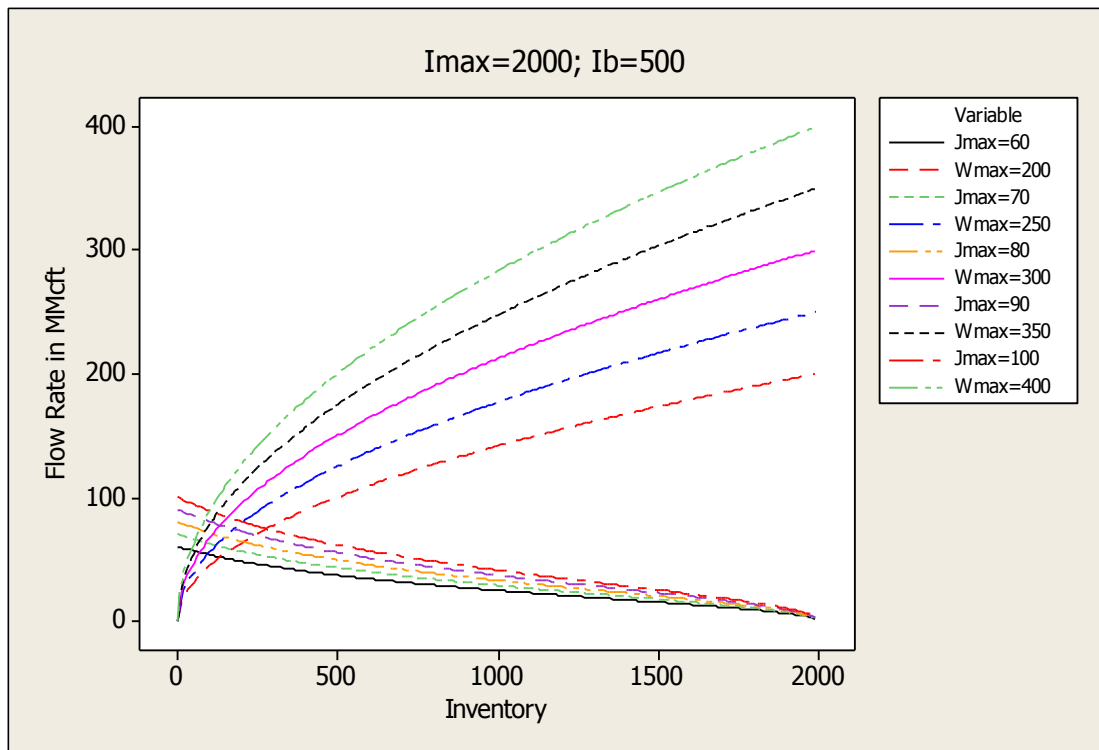


Figure 18. Injection-withdrawal curves sample 2

The upper curves show the withdrawal rates while the lower curves show the injection rates. The injection and withdrawal rates have inverse relationship with the amount of inventory in the storage. At lower amount of inventory the injection rate is higher while the withdrawal rate is lower. For maximum inventory level in the storage the injection rate is lower while the withdrawal rate is higher. The change in increase or decrease of the inventory level results in non-linear increase or decrease of injection and withdrawal levels.

### **6.5. Comparison of the heuristic with other models**

Three other storage schedule optimization scenarios that are being used in the literature are compared to the heuristic to further look into the performance of the algorithm.

*Scenario 1* - Injection and withdrawal rates do not depend on the effect of inventory pressure in storage facility. The facility is required to operate only at maximum injection and withdrawal rates.

*Scenario 2* - Injection and withdrawal rates do not depend on the effect of inventory pressure in storage facility; but injection and withdrawal rates are allowed to operate at any rate less than or equal to maximum operation capabilities.

*Scenario 3* - Injection and withdrawal rates depend on the effect of inventory pressure in storage facility. The heuristic approach developed in this research is based on this scenario.

*Scenario I*

The scenario is formulated as a mixed integer program (MIP) model. A similar problem is formulated and solved by Holland (2007) and being used by a gas company in England. The formulation is given as follows.

$$\max \left[ \sum_t^T \sum_{k \in N} P_{kt}^S (a_{kt} W_{kt} - b_{kt} J_{kt}) \right]$$

Subject to

$$I_{kt} + \sum_t^T J_{kt} - \sum_t^T W_{kt} - \sum_t^T L_{kt} = 0; \forall k$$

$$I_{kt} - I_{k(t-1)} - J_{k(t-1)} + W_{k(t-1)} + L_{k(t-1)} = 0; \forall k, \forall t$$

$$0 \leq I_{kt} \leq I_{\max(k)}, \forall k, \forall t$$

$$J_{kt} = J_{\max(k)}$$

$$W_{kt} = W_{\max(k)}$$

$$\sum_{k \in N} J_{kt} \leq S_t; \forall t$$

$$\sum_{k \in N} W_{kt} = D_t; \forall t$$

$$a_{kt}, b_{kt} \in \{0,1\}$$

The injection and withdrawal rates are equal to their maximum capabilities for the injection withdrawal constraints.

## Scenario 2

In this scenario, the injection and the withdrawal rates do not depend on the effect of inventory level pressure in the storage facility; however, the injection and the withdrawal rates are allowed to operate at any rate less than or equal to the maximum operation capacity. The problem becomes dynamic linear program (DLP) and it is similar to Lai, Margot, and Secomandi (2010) work. The formulation looks like as follows.

$$\max \left[ \sum_t^T \sum_{k \in N} P_{kt}^S (a_{kt} W_{kt} - b_{kt} J_{kt}) \right]$$

Subject to

$$I_{kt} + \sum_t^T J_{kt} - \sum_t^T W_{kt} - \sum_t^T L_{kt} = 0; \forall k$$

$$I_{kt} - I_{k(t-1)} - J_{k(t-1)} + W_{k(t-1)} + L_{k(t-1)} = 0; \forall k, \forall t$$

$$0 \leq I_{kt} \leq I_{\max(k)}, \forall k, \forall t$$

$$J_{kt} \leq J_{\max(k)}$$

$$W_{kt} \leq W_{\max(k)}$$

$$\sum_{k \in N} J_{kt} \leq S_t; \forall t$$

$$\sum_{k \in N} W_{it} = D_t; \forall t$$

$$a_{kt}, b_{kt} \in \{0,1\}$$

### *Scenario 3*

In this scenario, the injection and the withdrawal rates vary based on the inventory level. The problem becomes a dynamic non-linear program (DNLP). This is the foundation of the comprehensive problem formulated in this research. A similar problem was first formulated by Thompson et al. (2009).

A single salt cavern storage facility is used to illustrate the comparison of the scenarios. Some of the data are provided in Thompson et al. (2009). The maximum injection and withdrawal rates are 80MMcft/day and 250 MMcft/day respectively. The maximum working gas inventory is 2000 MMcft. The base gas is 500MMcft.

Figure 19 shows the inventory level comparison of the scenarios. All the scenarios provide a similar inventory increase or decrease trend. However, the inventory level for the MIP scenario appears to be the highest while for the heuristic approach developed in this research shows a lower inventory level.



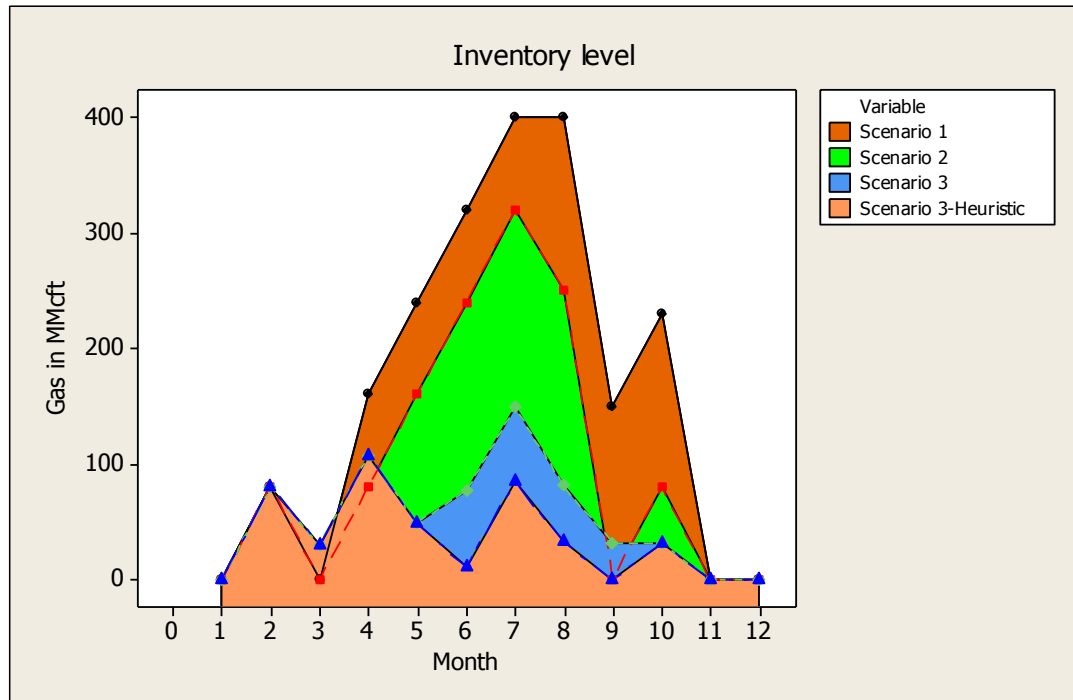


Figure 19. Comparison of inventory levels for all the scenarios

When we look at the injection and withdrawal decisions, a similar pattern is reflected across all the scenarios as shown by Figures 20 and 21. When scenario 1 provides the injection decision, all the remaining scenarios also provide injection decision.

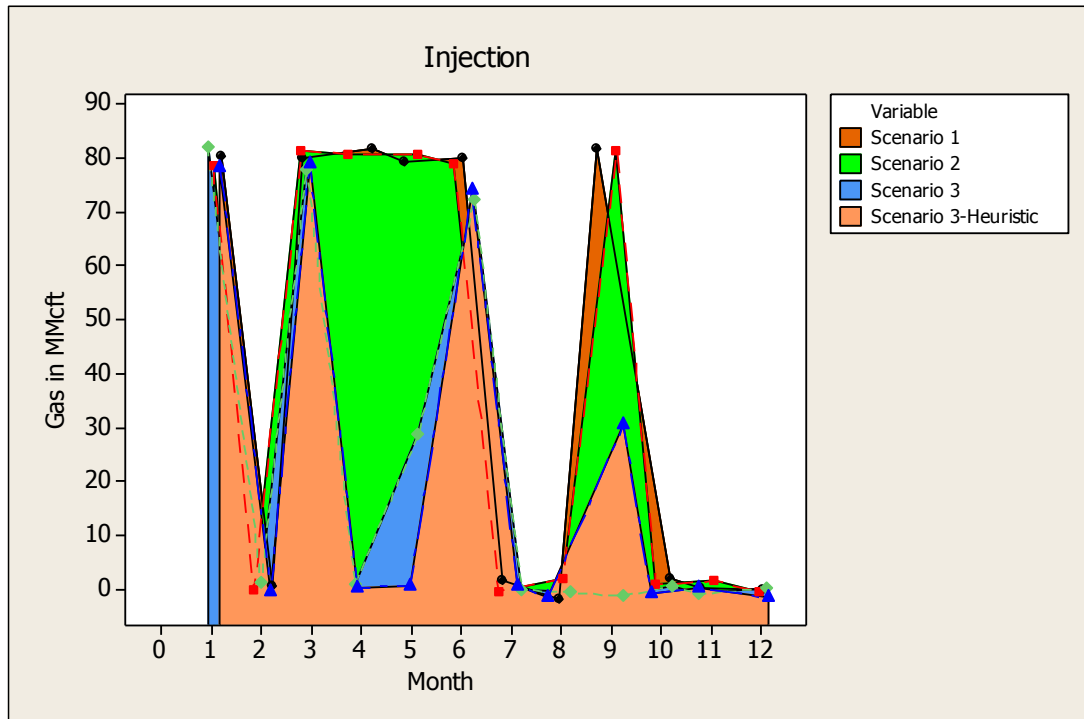


Figure 20. Comparison of gas injection decision for all the scenarios

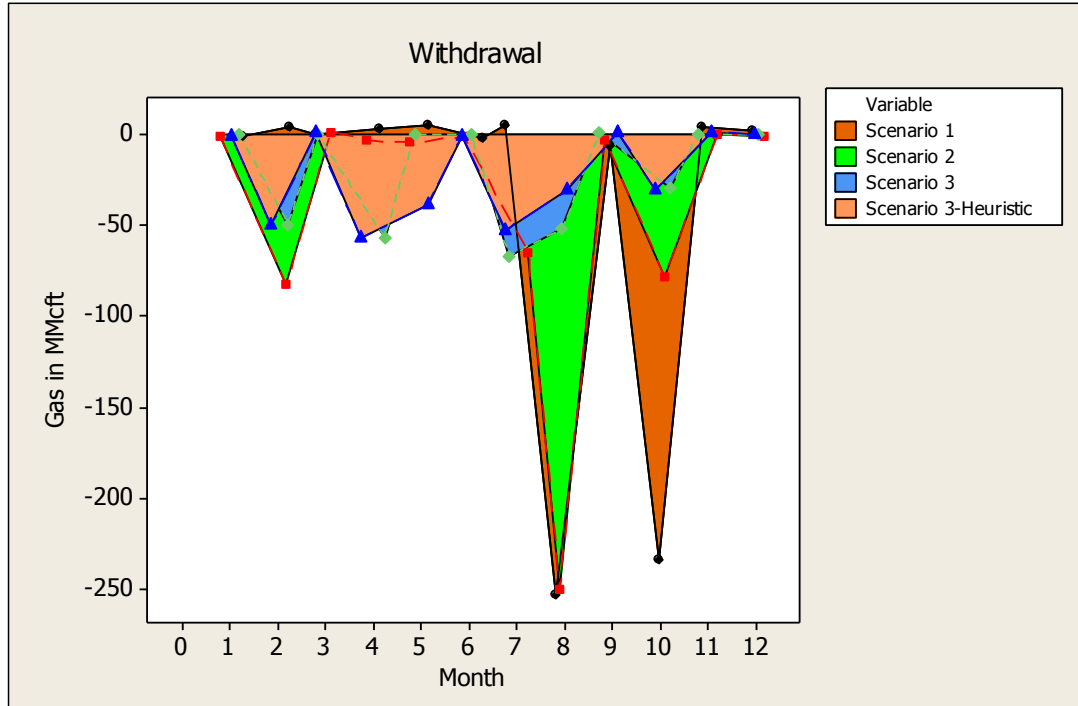


Figure 21. Comparison of gas withdrawal decision for all the scenarios

The cumulative cash flow for all the scenarios is shown by Figure 23 below. More or less, the cash flow again shows a similar increasing or decreasing trend across all the scenarios. This is consistent with the withdrawal and injection decisions. It can be seen that the scenarios provide the best and the conservative profits possible. The heuristic algorithm provides the conservative solution, whereas the MIP provides the maximum solution.

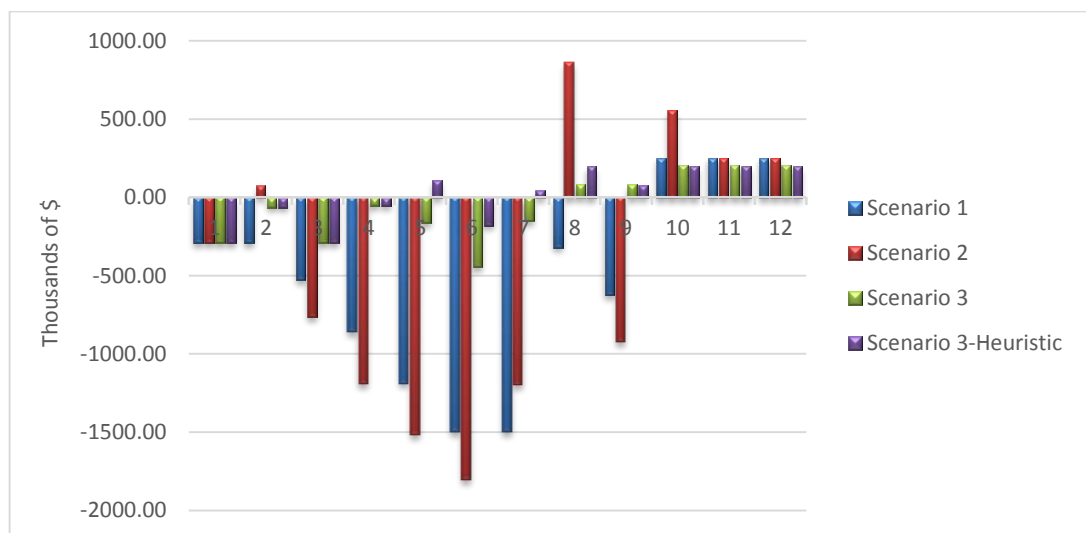


Figure 22. Cash flow comparison for all the scenarios

It is also important to note that scenarios 1 and 2 provide the same profit at the end of the desired storage facility time window for this case example. However, the cash flow is not exactly the same for both scenarios for specific time intervals as shown by Figure 22. This was because the withdrawal decisions made on each month differ in each scenario. It would be an interesting research problem to look at the risk associated with both scenarios. A good risk analysis approach for this type of problem is provided by Koberstein, Wolf, and König (2011). However, this case may change when we solve different size problems.

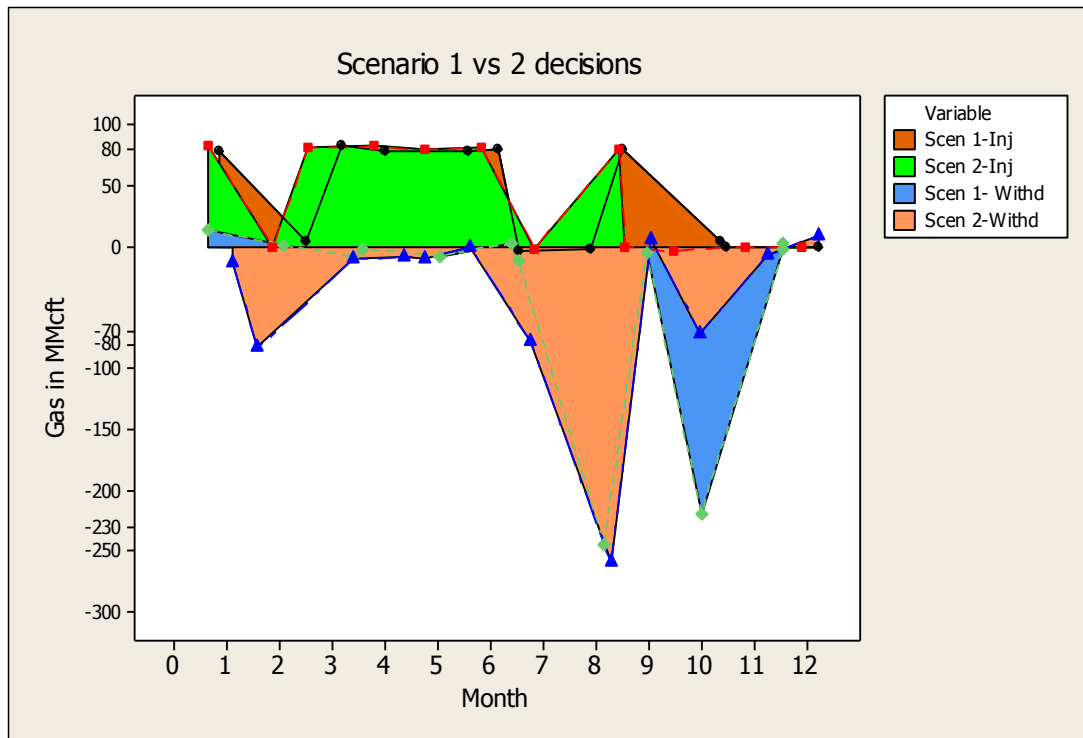


Figure 23. Scenario 1 and Scenario 2 decisions

## **Chapter 7: Conclusion and Future Research**

### **7.1. Conclusions**

Natural gas is a forerunner candidate to be used as a transition fuel as we shift from the consumption of fossil fuels to renewables. Therefore, there is a need to optimize the performance of the gas supply chain. Natural gas storage is the critical component of the supply chain since it is used to balance the gas supply and demand. However the valuation of the natural gas storage is very complex problem since it is highly affected by the financial aspects and physical storage facility characteristics. The complexities arise from stochastic gas price parameters, the non-linear gas injection and withdrawal flow rates which are functions of the square of gas flow velocity, and the type of storage facility.

There are many natural gas storage valuation researches. However, a large portion of them focus on the financial aspect of the storage facility valuation with little emphasis on the complexities of the storage facility physical characteristics. Few of the research that address the physical storage facility characteristics are also applied to very small size problems. In this research, the physical storage facility characteristics in combination with the financial aspect of the natural gas storage valuation is addressed. A comprehensive mathematical storage facility valuation model that includes aboveground and underground storage facilities is formulated and solved efficiently.

The research problem is formulated based on the tenet that natural gas traders lease a natural gas storage facility for a specific time period, usually for one year.

Then they need to know when to buy and inject natural gas to a storage facility, or when to withdraw from a storage facility and sell to maximize profit over the lease time window. A decision for the next day should be made by mid-night of the current day through the realization of the number of days left in the lease period. The problem needs to be solved in several minutes or less. However, the natural gas storage scheduling mathematical models take hundreds of days to solve optimally because of their complexities.

I proposed a heuristic approach that dramatically decreases the computation time from hundreds of days to fraction of a second, that provides a reasonable solution quality, and that incorporates all the possible gas storage facility complexities. In the heuristic approach, I decouple the problem into two stochastic linear problems and solve in two steps using simulation and simplex algorithm. The steps are determined based on the decision variables. There are three decision variables involved in the formulation: injection decision, withdrawal decision, and the inventory level. The inventory level is the function of both injection and withdrawal decisions. In the first step, I solve for the inventory level using simplex algorithm for every realization of price parameter. Then the output of the algorithm is used as input to the second step where I solve for injection and withdrawal decision variables. The second step provides an instance of approximate decision solution to the optimal solution. Hence, it needs to be resolved many times until a specified stopping criteria is satisfied. Since the model has also a stochastic price parameter, a simulation and optimization framework is used to solve the overall problem. The two steps above are run for every realization of price parameter until a

stopping criteria associated with the framework is satisfied. Then the final decision is made based on the expected value of the injection and withdrawal decision variables.

The performance of the model is illustrated using case examples. The heuristic approach provide a solution quality of less than 0.05% for one month decision scenario. It decreases slightly and provides about 10% decline on the worst case scenario. However, it dramatically reduces the computation time from over hundred days to fraction of a second under the worst case solution quality scenario. The convergence of the heuristic to a solution is shown numerically. It converges monotonically for both of the stopping criterion used.

Comparisons of the heuristic approach developed in this research is compared with models in the literature. The models are represented as scenarios. The scenarios have different degrees of complexity. The scheduling decisions made depend on the complexity of the problem. As the complexity of a storage model increases, the decisions become more conservative. It can also be seen that that the injection-withdrawal decisions follow a similar pattern of the storage historic data trend section of this report. The comparison of scenarios 1 and 2 indicate the same maximum profit. However, their withdrawal-injection decisions have slight variations. The decision maker has to look at the various decisions made over time rather than just considering the final profit. Generally, the scenarios give a good look into a gas storage optimization decision making strategies. The scenarios with the minimum and maximum values can be used as lower and upper bounds respectively for an expected profit.

The comprehensive mathematical model and the solution approach developed is more reliable than the models developed in the literature as far as my knowledge is concerned. The main contributions of this research are the formulation of the realistic natural gas storage valuation mathematical model that is applicable for the combination of both aboveground and underground storage facilities, and the efficient heuristic solution approach developed. Such an approach can be used in a variety of applications; for instance, the algorithm can be applied to a high penetration of renewables to electric power grid and fluid flow network optimization among others.

## **7.2. Future Research Direction**

In addition to the storage facilities valuation algorithm proposed in this research, there are several extensions of the research that I would like to address during the next step of my career. These are interdependency modeling, pipelines storage optimization, studying the details of the cycling effect, and exploring other heuristic algorithms for gas storage optimization.

### *7.2.1. Interdependency modeling*

There are several potential research areas for natural gas interdependency modeling such as (1) interdependency modeling of the natural gas supply chain components (2) Interdependency modeling between natural gas and renewables (3) Interdependency modeling between natural gas companies and auto-industries (4) Integration of refueling stations design and natural gas.



### *Interdependency modeling between natural gas and renewables*

The current consumption percentage of solar, wind, hydroelectric, and bio-fuel renewable energies account less than 10% of the total energy in the United States. The capacity of these energy sources gradually increase to replace the usage of fossil fuels. Natural gas will be the preferred type of fossil fuel to exploit during the transition process because of its suitable characteristics. It will also be used as a backup energy source in case of supply interruption even if the renewables are fully developed and replace the consumption of fossil fuels Moniz et al. (2011). These require to model an interdependency between natural gas and the renewables (Keyaerts, Rombauts, Delarue, and D'haeseleer, 2010; Shearer, Bistline Inman and Davis, 2014). Input–Output Inoperability Model will be used as the main research methodology (Santos, 2006).

### *Interdependency modeling between natural gas companies and auto-industries*

According to the U.S. Department of Transportation, transportation service needs increase by 2% each year. The production of automobiles is expected to increase by the same proportion as well. Considering that the automobiles engines will either be hybrids or use natural gas, the collaboration of the natural gas companies and auto industries will improve the both industries performances. This can be achieved through an interdependency modeling between the companies. Input–Output Inoperability Model will be used as the main research methodology.

### *Integration of refueling stations design and natural gas*

The number of natural gas refueling stations increase as most automobiles engines will either be hybrid or use natural gas. The amount of joules produced by one unit of, for example, oil is much greater than one unit of natural gas. Hence, more natural gas refueling stations are required compared to the existing gas stations. A thorough research analysis is required to advise the creation, design, and placement of the stations. In addition, the construction of future homes will provide an opportunity for residents to refuel from their own homes. These will further add a great deal of complexity and uncertainty in the design of the problem. Weiszfeld's Algorithm will be primarily used as the research methodology.

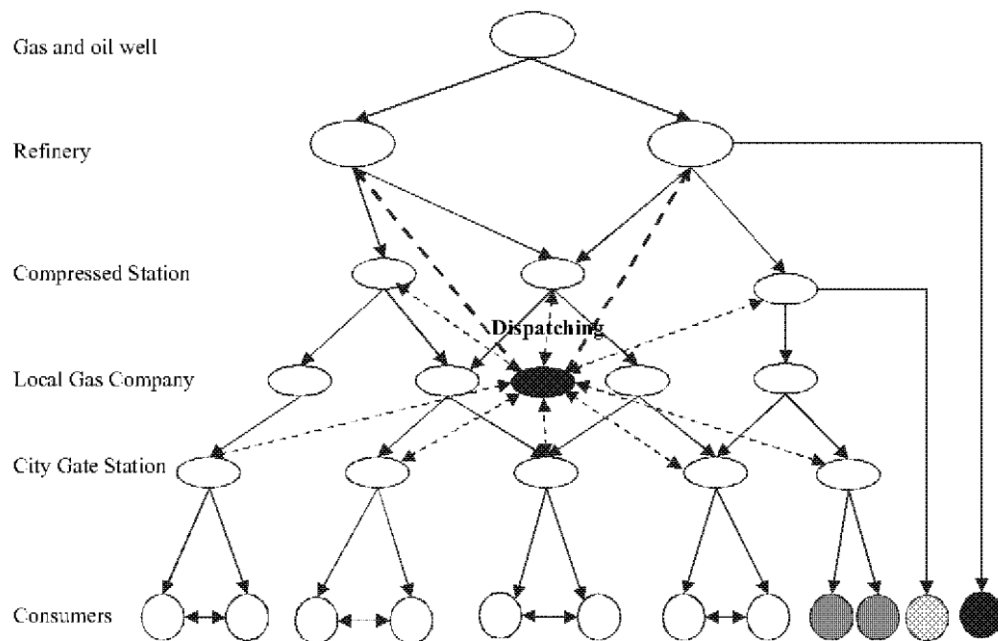
### *Gas Supply Chain Interdependency Modeling*

An integrated system provides a big picture of a system's performance to improve the efficiency. Likewise, integrating the natural gas supply chain upstream-storage-downstream enhances the natural gas supply chain system efficiency. The integration is achieved through the inter-dependency modeling of the upstream and the downstream components of the natural gas supply chain with the storage component. A good model is proposed by Hamedi, Husseini, and Esmaelian (2009) for a six level, multi period natural gas distribution networks. The levels defined were: suppliers, producers (refinery), first kind distributor (the compressor stations), wholesaler (local distribution centers), second kind distributor (city gas station), and consumers. The first level has two types of suppliers: the gas and oil wells that provide raw materials as the first suppliers, and importation of final product as second type of suppliers. The sixth level has four consumer groups: injection oil

wells as type one consumers, domestic and commercial subscribers as type two consumers, power plant as type three consumers, and exportation as type four consumers. The supply chain network showing the relationship between the levels is shown by Figure 24.

They formulated a multi-period mixed integer non-linear programming problem including one month time intervals to design the network. The nonlinear terms were linearized by adding additional constraints. The problem was solved level by level, heuristically, where the solution for the first level is used as an input for the second level, and so on. They were able to solve the problem in reasonable time, with a good precision.

*M. Hamed et al. / Energy Policy 37 (2009) 799–812*



**Figure 24. An example of gas supply chain interdependency modeling**

*Source: Hamed et al.,2009*

Incorporating the details of the gas storage schedule optimization model developed in this research to Hamed et al. (2009) will improve the efficiency of the natural gas supply chain. However, the model becomes very complicated to solve as a single problem. Proposing a systematic approach to solve such a problem will be a good research problem candidate.

### *7.2.2. Pipelines storage optimization*

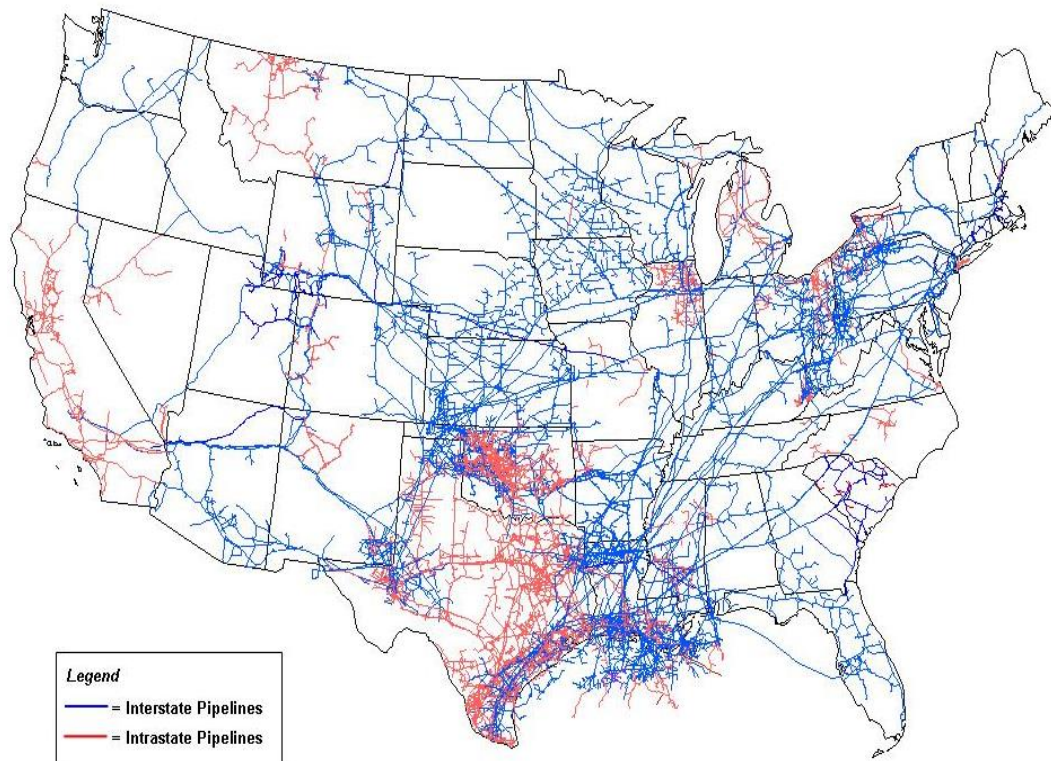
The pipeline storage model addressed in this research is based on horizontal laminar flow. But in real world, the pipelines are angular. One can easily incorporate the pipeline inclination model into the problem we formulate and solve for large scale network.

Currently, the United States has very complicated natural gas transportation infrastructure. As of 2009, there were about 210 natural gas pipeline systems which cover over 305,000 miles. Thirty one states depend on interstate natural gas supply for about 85% of their demand, while interstate constitutes about seventy one percent of the United States pipeline network. Among these, thirty percent of the total U.S. pipelines mileage operates within state borders. They get supply from interstate pipelines and local gas producers, and carry to local customers (EIA, 2014).

Texas is the leading natural gas consuming state. It has also the largest intrastate pipeline network which is 45,000 miles. Texas is still expanding the network due to increase in demand and expansion of natural gas production. The pipeline network is shown in Figure 25 below. The blue and red lines show the interstate and intrastate pipelines respectively. Sixteen of the thirty one major

interstate pipelines emerge from the Southwest states. In most cases some of the pipelines pass through many states before they reach the final delivery point (EIA, 2014; NaguraGas.org).

The desired natural gas pipeline utilization is when a pipeline company operates at its full capacity. But factors such as maintenance services (scheduled and unscheduled), fluctuation in market demand, and problems pertaining to weather fluctuation affects the performance.



**Figure 25. The United States natural gas pipeline network as of 2013**

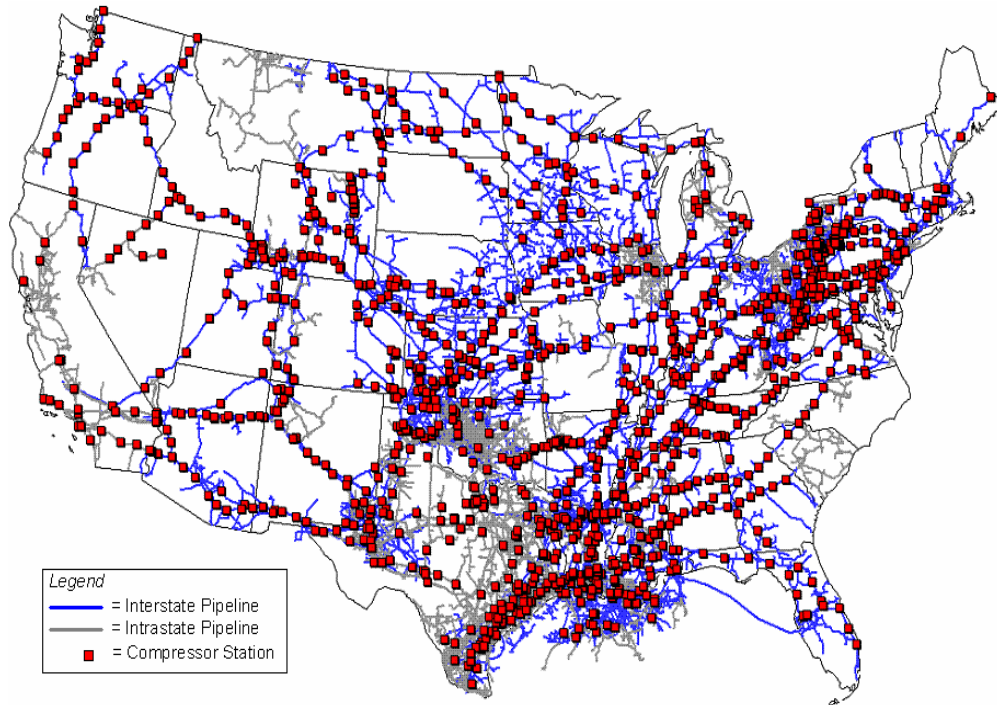
*Source: Energy Information Administration (EIA, 2014)*

This may attract attention to the possibility of reanalyzing the current pipeline network system and see if there could be an alternative design to minimize the interstate pipeline network. It is not achieved by demolishing and rebuilding the

existing pipeline networks; since it may not be feasible. But it provides good information for new network design. For example, Texas is expanding the pipeline network because of demand increase in the state. The analyses can also help a new pipeline extension to new shale gas exploration areas such as to New England, Southeast, Northeast, Midwest, Central, and West United States.

The movement of natural gas cannot be practical without the presence of compressors. Both the inter and intra pipeline systems have more than 1,400 compressor stations which control the forward movement of natural gas in the pipelines throughout the country (see Figure 26). The stations are shown by red rectangular dots. Compressors are used to maintain the required pressure for the natural gas movement along the transmission grids. But the number of compressors depends on the type of transmission grid. If looping transmission grid is used, the number of compressors might be small. “Looping is when one pipeline is laid parallel to another and is often used as a way to increase capacity along a right-of-way beyond what is possible on one line or an expansion of an existing pipeline”(EIA, 2014). The reason is that looping helps as a backup along the transmission grids. It provides the gas for lower level pipes to maintain the pressure. It also helps as a storage device to capture pick demands. The transmission grid is a natural gas mainline with wide diameter. It is usually used for long distance transportation, unlike the lateral transmission. Lateral transmission grids are used for short distance distributions. Most compressors follow chicken-egg-chicken principle. They use the energy to run from the transmission line. But recently

because of environmental concerns, the utilization of electric driven compressors is being adopted (EIA, 2014).



**Figure 26. Natural gas pipeline compressor stations illustration**

*Source: EIA, 2014*

Determining the location of the compressor stations along the pipelines is a big challenge. However, based on state equations of flow of gas in a pipeline, if we know the rate of decrease of the gas pressure along the pipeline, we can boost the pressure back when it falls below the required value. This can be done by continuously monitoring/simulating the gas flow (Munoz, Jimenez-Redondo, Perez-Ruiz, and Barquin, 2003).

### 7.2.3. Cycling

To optimize the combined depleted reservoir and salt cavern, the cycling effect needs to be considered. Usually we add gas to storage from April through October, and sell from October through March. The natural gas traders make purchase during the first seven months and then sell the stored gas for the next five months. Suppose that they purchase at price  $b$ , which is random variable, and we sell at price  $s$  random variable. Assume they buy as much as they can during the first seven months, but the demand is limited and unknown. Assume the demand follows random distribution. It will be an interesting topic to combine the cycling effect to the combination of supply and demand of the different energy sources. The proportion of the different energy sources supply and demand is given by the following picture..

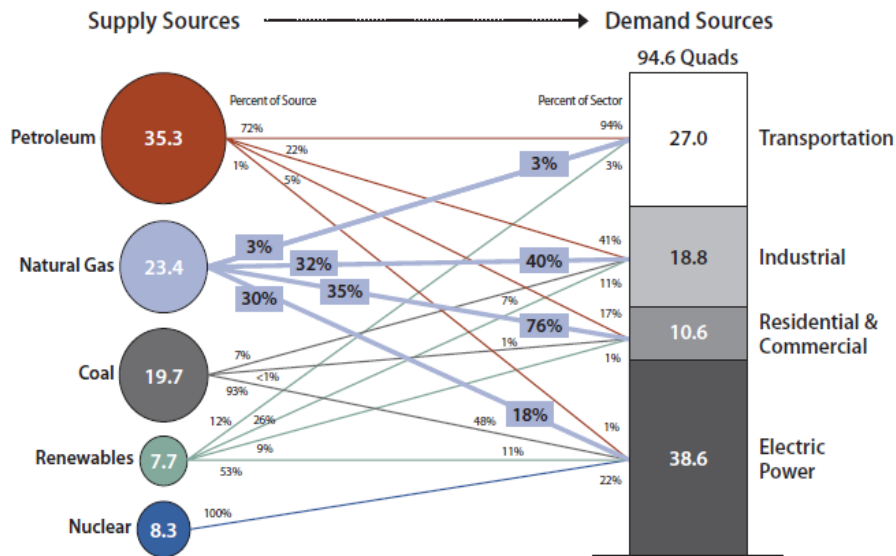


Figure 27. Sources and uses of natural gas in the U.S. as of 2009

Source: Moniz et al., 2011



#### 7.2.4. A meta-heuristic algorithm

This research can also be extended to a comparison of the model developed in this research with the performance of a meta-heuristic genetic algorithm in addition to the global solution. A preliminary formulation and result is presented here, which can be extended to a full comparison of the models.

Given the realization of gas price on a given day, it is required to know how much gas to buy and store or remove from a storage facility and sell to maximize profit over a specific lease period T, subject to the use-or-lose lease policy, inventory level, storage capacity, gas flow rate (injection and withdrawal), gas supply, and gas demand constraints. The problem can be re-formulated as follows.

##### *Objective function*

For the storage facility k on a given day t, the profit is calculated by deducting the revenue generated by gas sell revenue minus the costs as shown below.

$$F(t, k) = revenue(t, k, s) - cost(t, k, s)$$

The revenue is obtained by multiplying the simulated gas price by the withdrawal rate.

$$revenue(t, k, s) = P_{tks} W_{tk}$$

$$cost(t, k, s) = P_{tks} J_{tk} + L_{tk} + C_{tk}$$

The total profit for all storage facilities over a time horizon T is given as follows can be given by the maximization of the following equation.

$$\max \left[ \sum_t^T \sum_i^K P_{tks} a_{tk} W_{tk} - \sum_t^T \sum_i^K P_{tks} b_{tk} J_{tk} \right]$$

*Constraint 1*

The sum of the injections of a given facility over the remaining days in the lease period and the beginning working inventory of the facility should be equal to the sum of the withdrawals of a facility over the lease period and the gas loss over the lease period.

The beginning working inventory and the sum of the injection of a given facility over the remaining days in the lease period should be equal the sum of the withdrawal of a facility over the remaining days in lease period and the gas loss over the lease period. Note that the beginning inventory on the first day of the lease is zero. This constraint is based on the fact that the gas left in the storage facility at the end of the lease is void. All that was purchased should be sold the latest on the last day of the lease. We also assume that the generators that operate the storage facility use the working inventory gas.

$$I_{tk} + \sum_t^T J_{tk} - \sum_t^T W_{tk} - \sum_t^T L_{tk} = 0; \forall k$$

### *Constraint 2*

The inventory at the beginning of a time period  $t$  should be equal to the beginning inventory of the previous day, plus the injection on the previous day, minus the withdrawal on the previous day, minus the lost gas on the previous day.

$$I_{tk} - I_{k(t-1)} - J_{k(t-1)} + W_{k(t-1)} + L_{k(t-1)} = 0$$

### *Constraint 3*

The working inventory on a given day should not exceed the maximum working gas capacity.

$$0 \leq I_{tk} \leq I_{\max(k)}, \forall t, \forall k$$

### *Constraint 4*

Injection rate constraint

$$J_{ki} \leq J_{\max(k)} \sqrt{\frac{I_{b(k)}(I_{\max(k)} - I_{t(k)})}{I_{\max(k)}(I_{t(k)} + I_{b(k)})}}$$

### *Constraint 5*

Withdrawal rate constraint

$$W_{tk} \leq W_{\max(k)} \sqrt{\frac{I_{t(k)}}{I_{\max(k)}}}$$

### *Constraint 6*

The supply constraint. The supply is the forecast for time period t.

$$\sum_{k \in K} J_{tk} \leq S_{tk}, \forall t$$

### *Constraint 7*

The demand constraint. The demand is based on demand forecast for a period time t.

$$\sum_{k \in K} W_{tk} \leq D_{tk}, \forall t$$

### *Fundamentals of the Natural Gas Genetic Algorithm Implementation*

The Genetic algorithm is started with generating the initial population, which is called chromosome (Chu and Beasley, 1997; Walters and Sheble, 1993). The chromosomes of the algorithm is composed of injection and withdrawal decision variables and moreover how much are the injection or withdrawal should be in each period. A binary decision of “0” when there is no injection or withdrawal for a storage facility and “1” when there is an injection decision or a withdrawal decision for a storage facility.

Then the offspring of the population is generated. The crossover and mutation process is on the withdrawal and deposit rate. The Figure 28 represents the crossover result for two chromosomes. The  $I_i$  and  $I'_i$  in the yellow cell and the  $W_i$  and  $W'_i$  in orange cells change their location to produce new offspring (Baker and Ayechev, 2003; Ahn and Ramakrishna, 2002).

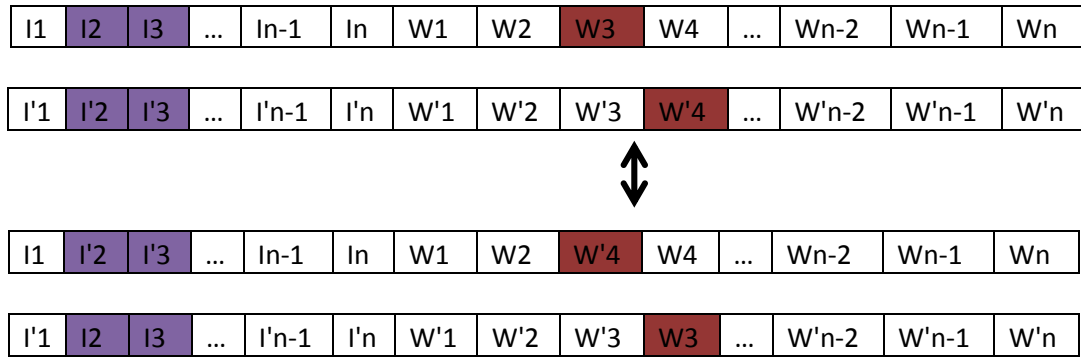


Figure 28. Crossover process

The mutation process happens when the amounts in cells are changed randomly. The Figure 29 represents the mutation. In this figure the value of yellow cells are changed randomly.

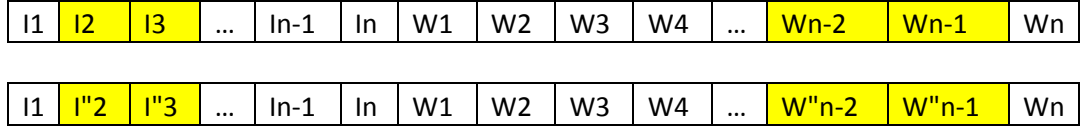


Figure 29. Imitation process

A candidate solution is a string whose length is the product of the number of storage facilities and the number of days left in the lease period. The initial population is created randomly. After trial and error, a population of size 100 chromosomes provides a good result. After running the genetic algorithm with 1000 iterations, the objective function cost of withdrawal or injection is shown in Figure 29. As it can be seen from the graph by increasing the number of iterations we obtain better answers in the feasible area which means that the GA works from a feasible solution. We set the values in such a way that it approaches the objective function to the optimal answer.

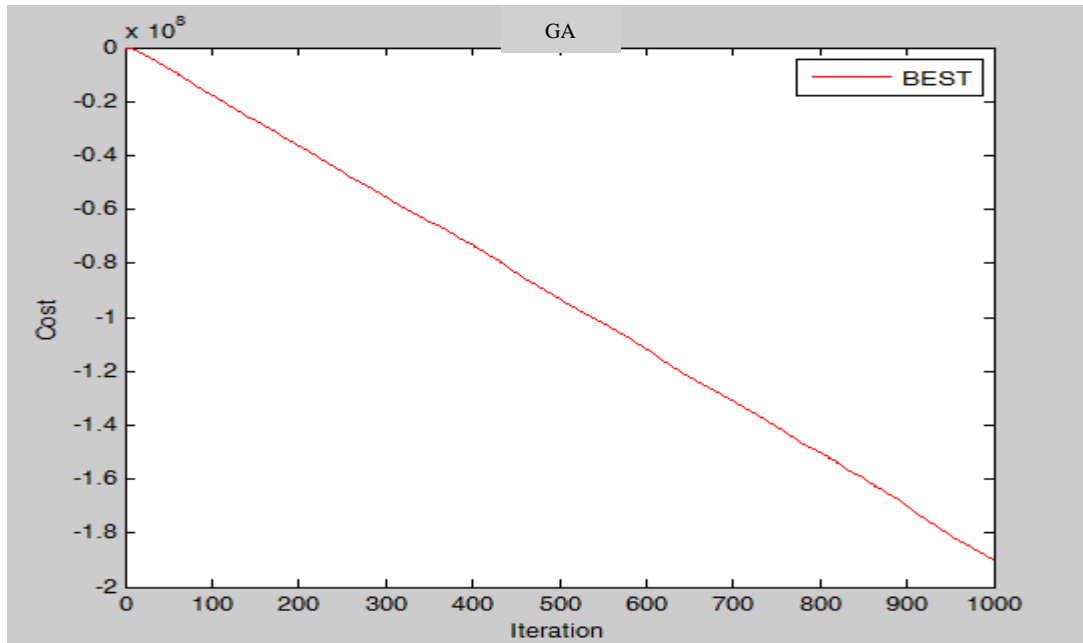


Figure 29. GA algorithm result after 1000 operation

For each chromosome, the inventory level is determined and no longer a variable. The only variables are the injection and withdrawal rates, which makes the problem easier to solve similar to the heuristic approach developed in this research.

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