EXPECTED SECURITY COST OPTIMAL POWER FLOW USING

PARALLEL AND DISTRIBUTED COMPUTATION

By

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CHAPTER 1

INTRODUCTION

It is impossible to predict when a part of the power system will fail. Normally, each component of a power system is equipped with an automatic protection system, which take it out of service if it is operated beyond certain limit or a fault occurs. In any event an outage of one part of the system may push some other parts of the system to exceed their respective limits, which would result in further outages. Due to such cascading outages, a major portion of the system or the entire system may collapse. This situation is called *system blackout.*

In power systems, a contingency could be the loss of one or more transmission lines or the loss of on-line generators. By having redundancy in design and proper flow of power it is possible to prevent cascading failures. A system with such characteristics is considered *secure.* The problem of finding the minimum-cost operating state while maintaining security is called the *security-constrained optimal power flow* (SCOPF) problem. Operating costs can be reduced by taking into account the corrective capabilities of system after the occurrence of a contingency [1]. More economical solution of system security is possible by taking security as an economic cost instead of a constraint. Moreover, consideration of the probabilities of the contingencies would further help to bring down the operating cost. This idea led to the formulation of *expected security cost optimal power flow* (ESCOPF).

Various extensions of the standard economic dispatch problem, to be discussed in Chapter 2, can include power system security and reliability. The *optimal power flow* (OPF) considers both economic dispatch and stability, but it requires heavy

computation [2]. As reference [3] notes

> The security-constrained optimal control of an electric power system generation-transmission network is an extremely difficult task. This difficulty tends to increase with growth in system size, interconnection, and other operating problems.

All forms of OPF have enormous scope and potential for on-line application as well as off-line applications in system planning. For on-line use, computational burden is the primary problem that we must overcome to achieve on-line OPF. This problem can be resolved, up to a certain extent, if computational burden is divided among several or many computers. This will speed up the solution process. The ESCOPF problem can be decoupled into numerous subproblems, which can be distributed among several processors. The solution can then be obtained by solving the problem in parallel fashion, as described in Chapter 3. Application of ESCOPF has been discussed in Chapter 4.

CHAPTER 2

LITERATURE REVIEW

2.1 Economic Dispatch

Economic dispatch is the process of providing the required real power load demand and losses by allocating generation among a set of on-line generating units such that total generation cost is minimized. The objective of the problem is to minimize the total cost of generation subject to supplying the total load demand and losses. Mathematically the problem can be stated as [4]:

$$\min_{\{P_{G_1},\cdots,P_{G_n}\}} \sum_{i=1}^{n} C_i(P_{G_i}) \tag{2.1}$$

subject to:

$$\sum_{i=1}^{n} P_{G_i} = P_D + P_L(P_{G_2},\cdots,P_{G_n}) \tag{2.2}$$

$$P_{G_i}^{\min} \leq P_{G_i} \leq P_{G_i}^{\max}, \qquad \text{where } i \in \{1,\cdots,n\} \tag{2.3}$$

where, C is the total generation cost, P_{G_i} is the real power generation level of the ith generator, P_D represents the total load demand, and P_L is the transmission losses associated with a particular set of generation levels.

The first constraint implies that the sum of the output powers must equal the load demand plus losses. The two inequality constraints imply that the power output of each unit must be greater than or equal to the minimum power permitted and must also be less than or equal to the maximum power permitted on that particular unit.

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In the above formulation, we do not consider P_L to be a function of P_{G_1}, since the slack bus generation P_{G_1} is completely determined by the injections at other buses and the slack bus demand P_{D_1}. Moreover, we have suppressed the dependence of P_L on the loads $\{P_{D_i}\}$, which are assumed to be fixed in quantity and location. The Lagrangian function for the above mentioned problem can be written as:

$$\mathcal{L} = \sum_{i=1}^{n} C_i(P_{G_i}) - \lambda \left[\sum_{i=1}^{n} P_{G_i} - P_D - P_L(P_{G_2}, \cdots, P_{G_n}) \right]. \tag{2.4}$$

Differentiating and setting to zero yields

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial P_{G_1}} &= \mathrm{MC}_1 - \lambda = 0, \text{ and} \\
\frac{\partial \mathcal{L}}{\partial P_{G_i}} &= \mathrm{MC}_i - \lambda \left(1 - \frac{\partial P_L}{\partial P_{G_i}} \right) = 0, \qquad i = 2, \ldots, n \\
\Rightarrow \lambda &= \mathrm{MC}_1, \text{ and} \\
\lambda &= \mathrm{MC}_i \frac{1}{\left(1 - \frac{\partial P_L}{\partial P_{G_i}} \right)}, \qquad i = 2, \ldots, n
\end{aligned}
$$

The quantity λ, the Lagrange multiplier, is known as the *"system lambda"*. It is the cost associated with generating an additional unit of energy.

Let

$$L_i = \frac{1}{\left(1 - \frac{\partial P_L}{\partial P_{G_i}} \right)}, \qquad i = 2, \ldots, n \tag{2.5}$$

$$\text{and} \quad L_1 = 1, \tag{2.6}$$

L_1 and L_i are called the *loss-penalty factors*. Condition for economic dispatch can be written as

$$\lambda = \mathrm{MC}_i \cdot L_i, \qquad i = 1, \ldots, n \tag{2.7}$$

Consider a case when by increasing a generator's output increases system losses,

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then that generator's entire increment of generation is not available for the load. For example, if a generator's output increases by 1MW but losses increase by 0.1MW. The net real power provided by the generator to be used by the load is 0.9MW. So the cost of this increment (1MW) is not the generator's marginal cost but $(MC/0.9)$ instead, or $(MC_i \cdot L_i)$ [4]. So the penalty factor in this case will be 1.111.

Similarly, an increase in a generator's output could result in a decrease in system losses. Such a generator would have a loss penalty factor $L_i < 1$, because the effective cost of incremental generation from such a generator would be less than its actual marginal cost.

2.2 Loadflow

The principal information obtained from the loadflow analysis is the magnitude and phase angle of the voltage at each bus. Using these values, we can calculate real and reactive power flows in transmission lines and transformers, and the line losses.

2.2.1 Flows on transmission systems

Consider a network of transmission lines connecting a set of buses or nodes in a power transmission system. The notation used in this section is illustrated in figure 2.1, following the treatment of [4]. The double-subscript notation $\bar{S}_{ik}$ and $\vec{I}_{ik}$, for $k \neq i$,
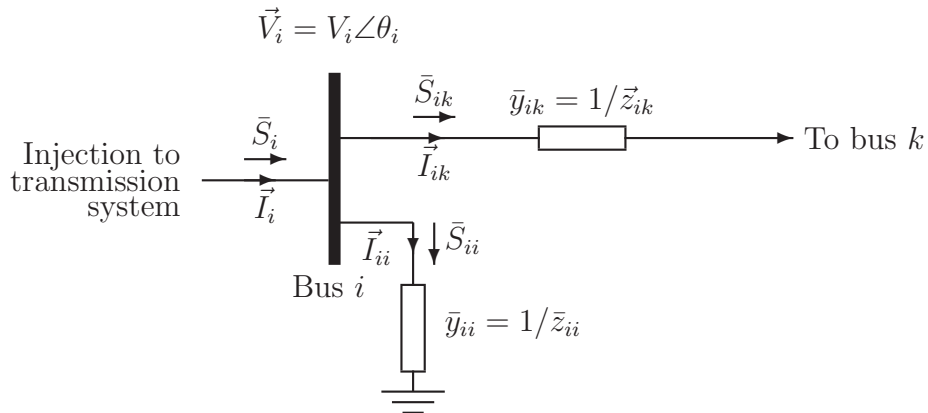
Figure 2.1: Bus i and connection to transmission system [4]

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indicates complex power or phasor current (respectively) flowing from bus i towards bus k along line ik. For $i = k$, the double subscript notations $\bar{S}_{ii}$ and $\vec{I}_{ii}$ indicate complex power and phasor current flowing to ground through the shunt element at bus i. The single subscript notations $\bar{S}_i$ and $\vec{I}_i$ indicate total complex power and phasor current injection into the transmission system at bus i. Generally, the complex power injected at bus i will consist of power generated by any generator(s) at bus i, minus any power consumed at bus i, or

$$\bar{S}_i = \bar{S}_{\mathrm{G}_i} - \bar{S}_{\mathrm{D}_i}. \tag{2.8}$$

Consider a line between bus i and bus k, with a series impedance of $\bar{z}_{ik}$, the series admittance of the line will be defined as

$$\bar{y}_{ik} = \frac{1}{\bar{z}_{ik}}. \tag{2.9}$$

If there is no direct connection between bus i and bus k, then $\bar{y}_{ik} = 0$. Similarly we can define the admittance of any shunt element connecting bus i to ground as $\bar{y}_{ii}$, including line charging[1] for every transmission line connected to the bus i. The current flowing from bus i towards bus k is

$$\vec{I}_{ik} = \frac{1}{\bar{z}_{ik}}(\vec{V}_i - \vec{V}_k) = \bar{y}_{ik}(\vec{V}_i - \vec{V}_k), \tag{2.10}$$

whereas the current flowing through the shunt element $\bar{y}_{ii}$ from bus i to ground is

$$\vec{I}_{ii} = \bar{y}_{ii}\vec{V}_i, \tag{2.11}$$

where $\vec{V}_i$ is the phasor voltage at bus i. If the total number of buses is n, the total

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[1]Line charging is the capacitive effect due to which current flows in the transmission line whether it is loaded or not.

current injected into the power transmission system at bus i is

$$\vec{I_i} = \bar{y}_{ii}\vec{V_i} + \sum_{\substack{k=1 \\ k \neq i}}^{n} \bar{y}_{ik}\left(\vec{V_i} - \vec{V_k}\right),\tag{2.12}$$

or, collecting the $\vec{V_i}$ terms,

$$\vec{I_i} = \vec{V_i}\sum_{k=1}^{n}\bar{y}_{ik} - \sum_{\substack{k=1 \\ k \neq i}}^{n}\bar{y}_{ik}\vec{V_k}.\tag{2.13}$$

The right side of the above equation motivates the following definition:

$$\bar{Y}_{ii} = \sum_{k=1}^{n}\bar{y}_{ik},\tag{2.14}$$

$$\bar{Y}_{ik} = -\bar{y}_{ik}, \quad i \neq k.\tag{2.15}$$

So we can write

$$\vec{I_i} = \sum_{k=1}^{n}\bar{Y}_{ik}\vec{V_k}.\tag{2.16}$$

The definitions in equations (2.14) and (2.15) can be written, in matrix form, as

$$\bar{\mathbf{Y}}_{\text{bus}} = \begin{bmatrix} \bar{y}_{11} + \cdots \bar{y}_{1n} & -\bar{y}_{12} & \cdots & -\bar{y}_{1n} \\ -\bar{y}_{21} & \bar{y}_{21} + \cdots \bar{y}_{2n} & \cdots & -\bar{y}_{2n} \\ \vdots & & \ddots & \vdots \\ -\bar{y}_{n1} & -\bar{y}_{n2} & \cdots & \bar{y}_{n1} + \cdots \bar{y}_{nn} \end{bmatrix}.\tag{2.17}$$

Note that, the diagonal element ii of the $\bar{\mathbf{Y}}_{\text{bus}}$ matrix consists of the sum of all admittances connected to bus i (whether they are series admittances connecting to another bus or are shunt admittances), while the off-diagonal element ik is the negative of the series admittance connecting bus i to bus k. If there is no transmission line connecting bus i with bus k, then $\bar{y}_{ik} = 0$, as mentioned before. Similarly, $\bar{y}_{ii} = 0$ if no

shunt element is present between bus i and ground.

Let us define the bus voltage and bus current injection vectors as

$$\vec{\mathbf{V}} = \begin{bmatrix} \vec{V}_1 \\ \vec{V}_2 \\ \vdots \\ \vec{V}_n \end{bmatrix} \quad \text{and} \quad \vec{\mathbf{I}} = \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \\ \vdots \\ \vec{I}_n \end{bmatrix},$$

The equation (2.16) in matrix form can be written as:

$$\vec{\mathbf{I}} = \bar{\mathbf{Y}}_{\mathrm{bus}} \vec{\mathbf{V}}. \tag{2.18}$$

The matrix $\bar{\mathbf{Y}}_{\mathrm{bus}}$ is called the bus admittance matrix. This matrix represents a transmission line network, and explains how current injections into a network are related to the bus voltage magnitudes and angles.

The complex power injection at bus i can be written in terms of $\bar{\mathbf{Y}}_{\mathrm{bus}}$ matrix elements as

$$\begin{aligned} \bar{S}_i &=& \vec{V}_i \vec{I}_i^* = \vec{V}_i \sum_{k=1}^{n} \bar{Y}_{ik}^* \vec{V}_k^*, \tag{2.19} \\ &=& V_i \sum_{k=1}^{n} V_k Y_{ik} e^{(\theta_i - \theta_k - \delta_{ik})}, \tag{2.20} \end{aligned}$$

where

$$\begin{aligned} \vec{V}_i &=& V_i \angle \theta_i, \ \ \vec{V}_k = V_k \angle \theta_k, \ \text{and} \\ \bar{Y}_{ik} &=& G_{ik} + j B_{ik} = Y_{ik} \angle \delta_{ik}. \end{aligned}$$

We can split equation (2.20) into real and reactive parts, in terms of the bus voltage magnitudes $\{V_i, \ i = 1, \ldots, n\}$ and bus voltage angles $\{\theta_i, \ i = 1, \ldots, n\}$. Then the

real and imaginary parts of equation (2.20) become

$$P_i(\mathbf{V}, \theta) = \sum_{k=1}^{n} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \delta_{ik}), \qquad (2.21)$$

$$Q_i(\mathbf{V}, \theta) = \sum_{k=1}^{n} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \delta_{ik}). \qquad (2.22)$$

2.2.2 The loadflow problem

The *loadflow* or *powerflow* problem is stated below [4, 5, 6]:

- Given a power system described by a $\bar{\mathbf{Y}}_{\mathrm{bus}}$ matrix, and given a set of bus voltage magnitudes, bus voltage angles, and real and reactive power bus injections,

- Determine the other voltage magnitudes and angles and real and reactive power injections.

Let

V_i, θ_i: voltage magnitude and voltage angle at bus i, and

P_i, Q_i: real and reactive power injected at bus i, respectively.

Then in the loadflow problem two of the four quantities V_i, θ_i, P_i, and Q_i at each bus are specified, and the other two are to be determined.

Once all of the variables are determined, power flows on individual transmission lines can be easily found.

2.3 Optimal Power Flow

Generation scheduling provided by economic dispatch may overload some transmission lines or voltage magnitude on any bus may go beyond the permissible range (typically 0.95p.u. to 1.05p.u.). Moreover, economic dispatch does not give any information about the losses in the transmission network. Therefore, after solving

economic dispatch problem, a loadflow study must be performed to ensure the feasibility of the dispatched power and to find the associated losses. In case no limit is violated, using the loss-penalty factors (provided by the solution of loadflow for the current operating state), economic dispatch changes the state so as to minimize cost. In case the results of loadflow show that the economic dispatch overloads any transmission line or violates any bus voltage limit, the system dispatcher would take some units off of the economic dispatch and manually re-dispatch them so the overload is removed [4]. The loadflow problem is solved again. This cycle of solving economic dispatch and loadflow continues until no constraint is violated. Following are the disadvantages associated with this procedure.

- This procedure probably will not result in minimum operating cost

- Manual (operator assisted) cycling can occur between economic dispatch and loadflow

- Cost minimization does not consider adjustment of bus voltages

To avoid these disadvantages optimal power flow (first stated in 1960s, [7]) was introduced, which combines economic dispatch and loadflow into a single problem. Optimal power flow (OPF) is an optimization problem in which economic dispatch, optimization over voltage and reactive injections, and operational limits are combined. The objective function is the cost function, which is minimized subject to a set of equality constraints and a set of inequality constraints. Mathematically, the OPF problem can be written in very general terms as:

$$\min_{Y} C(Y) \tag{2.23}$$

subject to:

$$h(Y) = 0,$$

$$g(Y) \leq 0,$$

where

- Y: A vector of control and state variables,

- $C(Y)$: The objective function (operating cost of the system),

- $h(Y)$: Equality constraints vector function (real and reactive power flow equations),

- $g(Y)$: Inequality constraints vector function (transmission line limits, voltage limits, generation capacity limits).

This is very large nonlinear programming problem. Algorithms leading to its solution have been developed over the decades. Two well-known techniques, Newton's method and Primal-Dual Interior-Point (PDIP), which can lead to the solution of OPF problem, will be discussed in following sections.

2.3.1 Optimization by Newton's Method

Since 1960s Newton's method, one of the so-called active set methods, has been a standard iterative solution algorithm for the power flow problem [8, 9, 10]. This algorithm converges very rapidly near the solution. This method relies on knowing the final active set. But we can only guess until we solve the problem. In this method we have to divide inequality constraints into two groups: binding and non binding. Let us suppose set $\mathcal{A}$ has all of those inequality constraints which appear to be binding at the current iteration and set $\mathcal{A}^c$ has those inequality constraints which do not appear to be binding. The binding inequality constraints, *i.e.* the set $\mathcal{A}$, are

treated as equality constraints. Therefore, the optimization problem stated in (2.23) can be solved by forming a Lagrange function as shown below:

$$\mathcal{L}(Y, \lambda, \mu) = C(Y) + \sum_{i=1}^{n} \lambda_i h_i(Y) + \sum_{i \in \mathcal{A}} \mu_i g_i(Y), \qquad (2.24)$$

where λ_i is the Lagrange multiplier for the ith equality constraint and μ_i is the Lagrange multiplier for the ith binding inequality (equality) constraint. The multiplier μ_i has the same properties as λ_i, except that, for all constraints present in $\mathcal{A}$, $\mu_i > 0$. If $\mu_i > 0$, this implies that the inequality constraint is binding, which means that it is active [4] ($g_i(Y) = 0$ at the optimum). Therefore, $\mathcal{A}$ is also known as the *active set* [11] and this method is also known as Newton's active set method.

To optimize the objective function shown in equation (2.23), a special version of the necessary first-order optimality conditions, called *Karush-Kuhn-Tucker* (KKT) conditions, can be written as shown below:

$$1) \quad \frac{\partial \mathcal{L}}{\partial Y_i} = 0, \qquad (2.25)$$

$$2) \quad \frac{\partial \mathcal{L}}{\partial \lambda_i} = h_i = 0, \qquad (2.26)$$

$$3) \quad \frac{\partial \mathcal{L}}{\partial \mu_{\mathcal{A}}} = g_{\mathcal{A}} = 0, \qquad (2.27)$$

$$4) \quad \mu_{\mathcal{A}} > 0 \qquad (2.28)$$

Note that $g_{\mathcal{A}}$ is a set of binding inequality (active) constraints and $\mu_{\mathcal{A}}$ is the set of corresponding Lagrangian multipliers. The set of above equations can be written in vector form as:

$$\begin{bmatrix} \frac{\partial \mathcal{L}}{\partial Y_i} \\ \frac{\partial \mathcal{L}}{\partial \lambda_i} \\ \frac{\partial \mathcal{L}}{\partial \mu_{\mathcal{A}}} \end{bmatrix} = \begin{bmatrix} \nabla_Y \mathcal{L} \\ \nabla_\lambda \mathcal{L} \\ \nabla_{\mu_{\mathcal{A}}} \mathcal{L} \end{bmatrix} = 0, \qquad (2.29)$$

Let

$$z = \begin{bmatrix} Y \\ \lambda \\ \mu_{\mathcal{A}} \end{bmatrix} \qquad (2.30)$$

So equation (2.29) can be written as:

$$\omega(z) = \nabla_z \mathcal{L}(z) = 0, \qquad (2.31)$$

The nonlinear equation $\omega(z) = 0$ can be solved by the Newton-Raphson method, which is based on the following Taylor series expansion of $\omega(z)$ about an operating point z^p as:

$$\omega(z) = \omega(z^P) + \left.\frac{\partial \omega(z)}{\partial z}\right|_{z=z^p} \cdot (z - z^p) + \text{higher order terms} = 0, \qquad (2.32)$$

Ignoring higher order terms and defining $\Delta z = z - z^p$, the above equation can be rewritten as:

$$\omega(z^p) + \left.\frac{\partial \omega(z)}{\partial z}\right|_{z=z^p} \cdot \Delta z = 0 \qquad (2.33)$$

$$\Rightarrow \quad \left.\frac{\partial \omega(z)}{\partial z}\right|_{z=z^p} \cdot \Delta z = -\omega(z^p) \qquad (2.34)$$

The update vector Δz is called the *Newton step*. It tells how far and in which direction the variables and multipliers should move from this current point z^p to get closer to the solution [12]. Since $\omega(z)$ is the gradient of the Lagrangian function $\mathcal{L}(z)$, equation (2.34) can be written in terms of the Lagrangian function $\mathcal{L}(z)$ as:

$$\left.\frac{\partial^2 \mathcal{L}(z)}{\partial z^2}\right|_{z=z^p} \cdot \Delta z = - \left.\frac{\partial \mathcal{L}(z)}{\partial z}\right|_{z=z^p} \qquad (2.35)$$

Or, simply

$$W \cdot \Delta z = -\omega(z) \tag{2.36}$$

where W denotes the second order derivatives (or the *Hessian matrix*) of $\mathcal{L}(z)$ with respect to z as shown below:

$$W = \left. \frac{\partial^2 \mathcal{L}(z)}{\partial z^2} \right|_{z=z^p} = \begin{bmatrix} \frac{\partial^2 \mathcal{L}(z)}{\partial Y^2} & \frac{\partial^2 \mathcal{L}(z)}{\partial Y \partial \lambda} & \frac{\partial^2 \mathcal{L}(z)}{\partial \mu_Y \partial \mu_{\mathcal{A}}} \\[2mm] \frac{\partial^2 \mathcal{L}(z)}{\partial \lambda \partial Y} & \frac{\partial^2 \mathcal{L}(z)}{\partial \lambda^2} & \frac{\partial^2 \mathcal{L}(z)}{\partial \lambda \partial \mu_{\mathcal{A}}} \\[2mm] \frac{\partial^2 \mathcal{L}(z)}{\partial \mu_{\mathcal{A}} \partial Y} & \frac{\partial^2 \mathcal{L}(z)}{\partial \mu_{\mathcal{A}} \partial \lambda} & \frac{\partial^2 \mathcal{L}(z)}{\partial \mu_{\mathcal{A}}^2} \end{bmatrix}_{z=z^p} \tag{2.37}$$

ω is the gradient of the Lagrangian function as shown below:

$$\omega(z)|_{z=z^p} = \begin{bmatrix} \nabla_Y \mathcal{L} \\[2mm] \nabla_\lambda \mathcal{L} \\[2mm] \nabla_{\mu_{\mathcal{A}}} \mathcal{L} \end{bmatrix}, \tag{2.38}$$

Note that both W and $\omega(z)$ are evaluated at the current operating point point z^p. From equations (2.24),(2.26) and (2.27) it is obvious that

$$\frac{\partial^2 \mathcal{L}(z)}{\partial \lambda^2} = 0, \qquad \frac{\partial^2 \mathcal{L}(z)}{\partial \lambda \partial \mu_{\mathcal{A}}} = 0, \qquad \frac{\partial^2 \mathcal{L}(z)}{\partial \mu_{\mathcal{A}} \partial \lambda} = 0 \ \ and \ \ \frac{\partial^2 \mathcal{L}(z)}{\partial \mu_{\mathcal{A}}^2} = 0,$$

To conveniently write matrix W, let us introduce some notation as:

$$H_Y \quad = \quad \frac{\partial^2 \mathcal{L}(z)}{\partial Y^2} = \text{Hessian of } \mathcal{L}(z) \text{ with respect to Y,} \tag{2.39}$$

$$J_h \quad = \quad \frac{\partial^2 \mathcal{L}(z)}{\partial \lambda \partial Y} = \text{Jacobian of equality constraint functions } h(Y) = \frac{\partial h(Y)}{\partial Y}, \tag{2.40}$$

$$J_{g_{\mathcal{A}}} \quad = \quad \frac{\partial^2 \mathcal{L}(z)}{\partial \mu_{\mathcal{A}} \partial Y} = \text{Jacobian of constraint functions } g_{\mathcal{A}}(Y) = \frac{\partial g_{\mathcal{A}}(Y)}{\partial Y}. \tag{2.41}$$

$$\therefore \qquad W = \begin{bmatrix} H_Y & J_h^T & J_{g_\mathcal{A}}^T \\ J_h & 0 & 0 \\ J_{g_\mathcal{A}} & 0 & 0 \end{bmatrix}$$

Hence, equation (2.36) can be written in matrix form as:

$$\begin{bmatrix} H_Y & J_h^T & J_{g_\mathcal{A}}^T \\ J_h & 0 & 0 \\ J_{g_\mathcal{A}} & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta Y \\ \Delta \lambda \\ \Delta \mu_\mathcal{A} \end{bmatrix} = - \begin{bmatrix} \nabla_Y \mathcal{L} \\ \nabla_\lambda \mathcal{L} \\ \nabla_{\mu_\mathcal{A}} \mathcal{L} \end{bmatrix}, \qquad (2.42)$$

Initially a guess z_o is chosen, $i.e.$

$$z_o = \begin{bmatrix} Y_o \\ \lambda_o \\ \mu_{\mathcal{A}o} \end{bmatrix},$$

Using this initial guess, matrix W and vector $\omega(z)$ are calculated. In case all the elements of the vector $\omega(z)$ are very close to zero, then z_o is the solution. Otherwise we have to solve (2.36) or (2.42) to obtain the Newton step (Δz) as:

$$\Delta z = -W^{-1} \omega(z) \qquad (2.43)$$

Improved guess or estimated solution for the next iteration is found as:

$$z_{new} = z_{old} + \alpha \Delta z, \qquad (2.44)$$

This procedure is repeated until the problem has converged to a solution. The scaling coefficient, some times called the accelerating factor, α is usually 1. However, it can be adjusted to values above or below 1 to speed up convergence or cause convergence

in a divergent case or in our case prevent violation of inequality constraints.

As mentioned earlier that in Newton's method we use only active inequality constraints in the Lagrangian function (2.24). After obtaining an updated set of variables and multipliers, we may find that some constraints in set $\mathcal{A}$ are no longer binding and some constraints in set $\mathcal{A}^c$ have become violated. So after each iteration we must check all the inequality constraints in sets $\mathcal{A}$ & $\mathcal{A}^c$ and change their position as given in [11].

- If the updated μ's of the constraint functions in the current active set are zero or have become negative, then the corresponding constraints must be released from the current active set.

- If other constraint functions evaluated at the updated variables violate their limits, then those constraints must be included in the new active set.

Once the active set has been updated, $\omega(z^{p+1})$ is checked for convergence. There are several criteria for checking convergence of Newton's method. The convergence tolerance may be set to the maximum absolute value of elements in $\omega(z)$ *i.e.* the ∞ norm, or to its Euclidean norm or some other norm [13]. Newton step calculation is repeated, with the improved guess, if the updated z^{p+1} does not yet satisfy the desired convergence criterion.

2.3.2 Optimization by Primal-Dual Interior-Point Method

The primal dual interior point method (PDIP) has been very successful in solving optimization problems in power systems as shown in references [14, 15, 16, 17, 18]. To avoid problems associated with active set methods we introduce a logarithmic barrier function so that we can get rid of the inequality constraints. The barrier function, being infinite on the boundary of the feasible region, forces the current trial solution away from the boundary into the interior of the feasible region. Let us
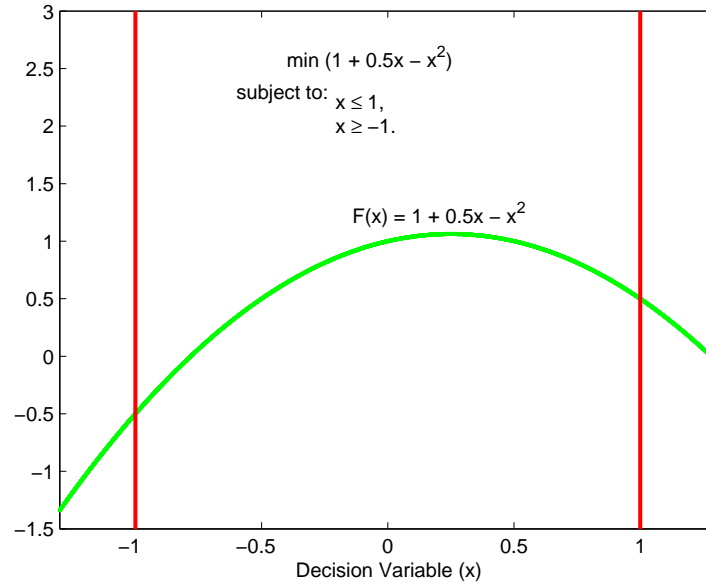
Figure 2.2: An optimization problem

consider the following optimization example:

$$\min_x F(x) = 1 + 0.5x - x^2 \tag{2.45}$$

subject to:
$$x \leq 1,$$
$$x \geq -1.$$

This problem in shown in the Figure 2.2. To write the constraints in standard form we proceed as:

First Constraint $= (x \leq 1) \Rightarrow (1 - x \leq 0) \Rightarrow g_1(x) \leq 0$

$g_1(x) = 1 - x \Rightarrow -g_1(x) = x - 1$

Second Constraint $= (x \geq -1) \Rightarrow (-x - 1 \leq 0) \Rightarrow g_2(x) \leq 0$

$g_2(x) = -1 - x \Rightarrow -g_2(x) = x + 1$

Barrier function is formed as: $B(x, \nu) = F(x) - \nu \sum_{i=1}^{2} \ln(-g_i(x))$,

$\Rightarrow B(x, \nu) = 1 + 0.5x - x^2 - \nu \ln(x-1) - \nu \ln(x+1)$, where ν is a positive number called the barrier parameter. For $\nu = 0.2$, the barrier function is shown in the Figure 2.3. We start with a larger value of ν and as PDIP algorithm converges to the optimum
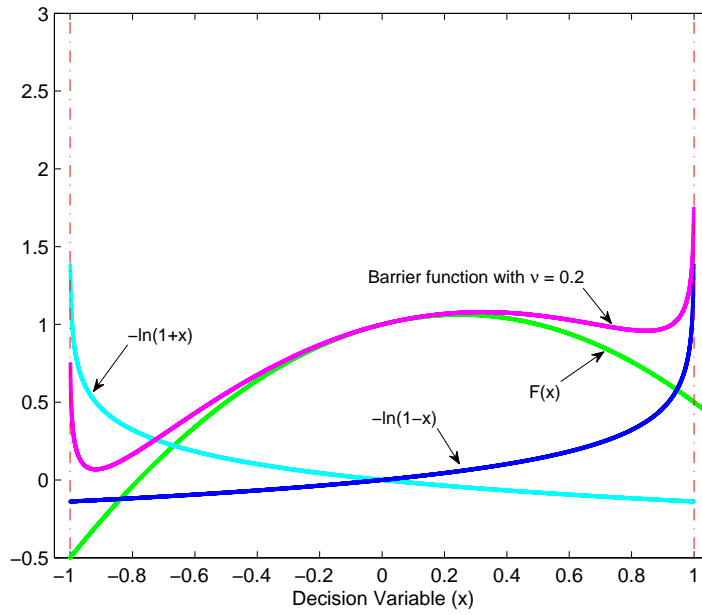
Figure 2.3: Barrier function and its components
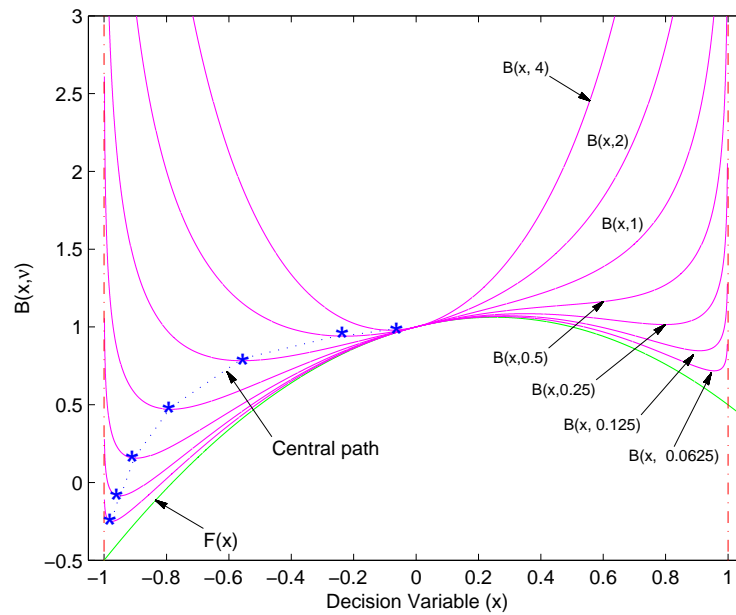
Figure 2.4: Barrier function with different ν

of $B(x, \nu)$, we gradually decrease ν to zero [15]. Figure 2.4 shows barrier function for different values of ν. We can see that the the barrier function forces the current trial solution away from the boundary of the feasible region into the interior of the feasible region. The path, connecting the optimum points of the barrier function for all values of ν is called the *central path* which will be discussed later.

OPF using PDIP algorithm

Let us apply primal-dual algorithm to the OPF problem shown in equation (2.23), which is restated below:

$$\min_{Y} C(Y) \tag{2.46}$$

$$\text{subject to:} \qquad h(Y) = 0,$$

$$g(Y) \leq 0.$$

We eliminate inequality constraints by introducing the following logarithmic barrier function

$$B(Y, \nu) = C(Y) - \nu \sum_{i=1}^{m} \ln(-g_i(Y)), \tag{2.47}$$

With the introduction of the barrier function our problem (2.46) changes to the following problem.

$$\min_{Y} B(Y, \nu) \tag{2.48}$$

$$\text{subject to:} \qquad h_i(Y) = 0 \qquad i = 1, \cdots, n$$

Note that now the optimization is subject to equality constraints only. To solve this problem, we form the Lagrangian function as:

$$\mathcal{L} = B(Y, \nu) + \sum_{i=1}^{n} \lambda_i h_i(Y), \quad (2.49)$$

To optimize $B(Y, \nu)$, we find critical points of the Lagrangian function for our current ν. The first order conditions, *Karush-Kuhn-Tucker* (KKT) conditions, can be written as shown below:

$$1) \quad \frac{\partial \mathcal{L}}{\partial Y_i} = \nabla_Y \mathcal{L} = 0,$$

$$2) \quad \frac{\partial \mathcal{L}}{\partial \lambda_i} = \nabla_\lambda \mathcal{L} = h_i(Y) = 0, \qquad i = 1, \cdots, n$$

The gradient of $\mathcal{L}$ with respect to Y is given below:

$$\nabla_Y \mathcal{L} = \nabla_Y C(Y) + \sum_{i=1}^{n} \lambda_i \nabla_Y h_i(Y) + \nu \sum_{i=1}^{m} \left(\frac{1}{-g_i(Y)} \right) \nabla_Y g_i(Y) \quad (2.50)$$

Let us define two independent variables, s_i (primal slack) and μ_i (dual slack) as shown below:

$$s_i = -g_i(Y), \quad (s_i > 0) \quad i = 1, \cdots, m,$$

$$\mu_i = \frac{\nu}{s_i}, \quad (\mu_i > 0) \quad i = 1, \cdots, m,$$

Therefore, equation (2.50) can be written as:

$$\nabla_Y \mathcal{L} = \nabla_Y C(Y) + \sum_{i=1}^{n} \lambda_i \nabla_Y h_i(Y) + \nu \sum_{i=1}^{m} \left(\frac{1}{s_i} \right) \nabla_Y g_i(Y)$$

or

$$\nabla_Y \mathcal{L} \;=\; \nabla_Y C(Y) + \sum_{i=1}^{n} \lambda_i \nabla_Y h_i(Y) + \sum_{i=1}^{m} \mu_i \nabla_Y g_i(Y) \qquad (2.51)$$

To obtain a more compact representation of equation (2.51), matrices J_h & J_g and vectors λ & μ are introduced as:

$$J_h \;=\; \text{Jacobian of equality constraint functions } h(Y) = \tfrac{\partial h(Y)}{\partial Y}, \qquad (2.52)$$

$$J_g \;=\; \text{Jacobian of inequality constraint functions } g(Y) = \tfrac{\partial g(Y)}{\partial Y} \qquad (2.53)$$

and

$$\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix}, \quad \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_m \end{bmatrix}.$$

So

$$J_h^T \;=\; \begin{bmatrix} \nabla_Y h_1(Y) & \nabla_Y h_2(Y) & \cdots & \nabla_Y h_n(Y) \end{bmatrix}, \text{ and}$$

$$J_g^T \;=\; \begin{bmatrix} \nabla_Y g_1(Y) & \nabla_Y g_2(Y) & \cdots & \nabla_Y g_m(Y) \end{bmatrix}.$$

The vector form of equation (2.51) is:

$$\nabla_Y \mathcal{L} \;=\; \nabla_Y C(Y) + J_h^T \lambda + J_g^T \mu \qquad (2.54)$$

So to optimize the Lagrangian function shown in (2.49) we have to solve following set of equations for a fixed scalar ν, starting with a initial ν and decreasing it gradually

while keeping $s_i > 0$ and $\mu_i > 0$:

$$\nabla_Y \mathcal{L} = \nabla_Y C(Y) + J_h^T \lambda + J_g^T \mu \;\; = \;\; 0 \tag{2.55}$$

$$\nabla_\lambda \mathcal{L} = h(Y) \;\; = \;\; 0, \tag{2.56}$$

$$g_i(Y) + s_i \;\; = \;\; 0, \qquad i = 1, \cdots, m \tag{2.57}$$

$$\mu_i s_i - \nu \;\; = \;\; 0, \qquad i = 1, \cdots, m \tag{2.58}$$

Newton's iterative method can be used to solve these equations. Four sets of variables for which these equations are to be solved are Y, λ, μ and s. So let

$$z = \begin{bmatrix} Y \\ \lambda \\ \mu \\ s \end{bmatrix}. \tag{2.59}$$

We will solve the nonlinear equation $\omega(z) = 0$ by the Newton-Raphson method. This algorithm is based on the following Taylor series expansion of $\omega(z)$ about an operating point z^p as:

$$\omega(z) = \omega(z^P) + \left. \frac{\partial \omega(z)}{\partial z} \right|_{z=z^p} \cdot (z - z^p) + \text{higher order terms} = 0, \tag{2.60}$$

Ignoring higher order terms and defining $\Delta z = z - z^p$, the above equation can be rewritten as:

$$\omega(z^p) + \left. \frac{\partial \omega(z)}{\partial z} \right|_{z=z^p} \cdot \Delta z \;\; = \;\; 0 \tag{2.61}$$

$$\Rightarrow \qquad \left. \frac{\partial \omega(z)}{\partial z} \right|_{z=z^p} \cdot \Delta z \;\; = \;\; -\omega(z^p) \tag{2.62}$$

Newton step Δz is the update vector, as mentioned before. The equations needed to find the Newton's step for the equations (2.55 $\sim$ 2.58) can be written, in form similar to (2.62), as follows:

$$
\begin{aligned}
H_Y \Delta Y + J_h^T \Delta \lambda + J_g^T \mu \Delta &= -(\nabla_Y C + J_h^T \lambda + J_g^T \mu) \\
J_h \Delta \lambda &= -h \\
J_g \Delta \mu + I \Delta s &= -(g + s) \\
S \Delta \mu + M \Delta s &= -(MS - \nu I)e
\end{aligned}
$$

These four equations can be written as one matrix equation:

$$
\begin{bmatrix}
H_Y & J_h^T & J_g^T & 0 \\
J_h & 0 & 0 & 0 \\
J_g & 0 & 0 & I \\
0 & 0 & S & M
\end{bmatrix}
\begin{bmatrix}
\Delta Y \\
\Delta \lambda \\
\Delta \mu \\
\Delta s
\end{bmatrix}
= -
\begin{bmatrix}
\nabla_Y C + J_h^T \lambda + J_g^T \mu \\
h \\
g + s \\
(MS - \nu I)e
\end{bmatrix}
\tag{2.63}
$$

or, simply

$$
W \cdot \Delta z = -\omega(z) \tag{2.64}
$$

where

$$
H_Y = \left(\nabla_Y^2 C + \sum_{i=1}^{n} \lambda_i \nabla_Y^2 h_i + \sum_{i=1}^{m} \mu_i \nabla_Y^2 g_i \right)
$$

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and

h : A column vector of size n, containing elements h_i, where $i = 1, \cdots, n$.

g : A column vector of size m, containing elements g_i, where $i = 1, \cdots, m$.

s : A column vector of size m, containing elements s_i, where $i = 1, \cdots, m$.

I : An identity matrix of size $(m \times m)$.

S : A diagonal matrix of size $(m \times m)$ containing elements s_i, where $i = 1, \cdots, m$.

M : A diagonal matrix of size $(m \times m)$ containing elements μ_i, where $i = 1, \cdots, m$.

e : A column vector of size m, with all elements equal to 1.

Central path of the problem

The path defined by the set of solutions $[z^{s_1}, z^{s_2}, z^{s_3} \cdots z^{s_{latest}}]$, whose elements make $\omega(z)$ evaluate to zero for all values of ν $(\nu_1, \nu_2, \nu_3, \cdots, \nu_0)$ respectively, is called the *central path* of the problem. While solving the problem for z for any ν, it is not necessary to force $\omega(z)$ to be zero. Actually we are interested in finding how close we are to the central path. One way to measure the closeness from the central path is by checking how close each product $\mu_i s_i$ is to the barrier parameter ν. $\omega(z)$ will be sufficiently close to zero if each product $\mu_i s_i$ is close to ν and this will happen if following inequality [19, 20] becomes true.

$$\left\| Ms - \frac{1}{m} \sum_{i=1}^{m} \mu_i s_i \right\|_2 = \left\| \begin{array}{c} \mu_1 s_1 - \widehat{\nu} \\ \mu_2 s_2 - \widehat{\nu} \\ \vdots \\ \mu_m s_m - \widehat{\nu} \end{array} \right\|_2 \leq \tau\nu \qquad (0 < \tau < 1) \qquad (2.65)$$

or

$$\mu_i s_i \geq \gamma\widehat{\nu} \qquad \forall\, i = 1, \cdots, m, \qquad (\gamma = 10^{-3}) \qquad (2.66)$$

24
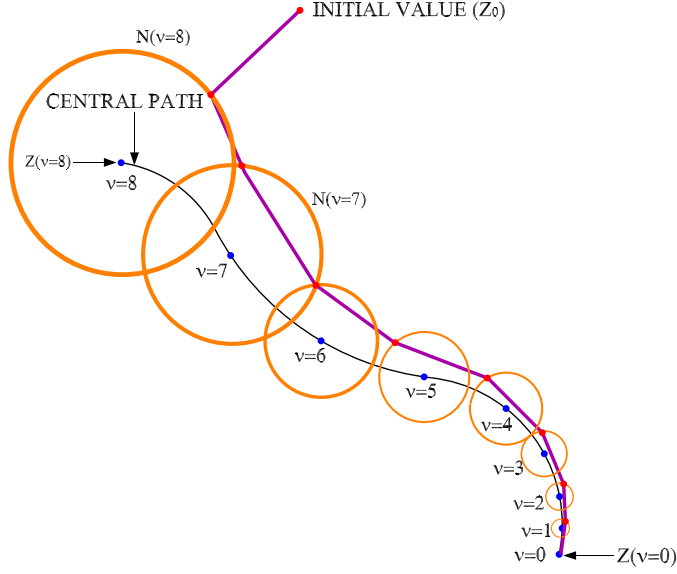
Figure 2.5: Graphical representation of central path

The *duality factor* $\widehat{\nu}$ [20], which measures the average value of the pairwise products, is given by

$$\widehat{\nu} = \frac{1}{m} \sum_{i=1}^{m} \mu_i s_i. \tag{2.67}$$

Once the inequality (2.65) or (2.66) becomes true, we are close to central path of the problem and then ν may be further reduced as shown in figure 2.5 [12].

Scaling factors involved in (PDIP)

Scaling factor α is multiplied with Δz before updating z. This reduces the Newton step size Δz such that μ_i and s_i are always positive in the new updated guess z_{new}. That is, each μ_i and each s_i is kept strictly greater than zero when $\Delta\mu_i$ and Δs_i is added to it, respectively. If the addition of $\Delta\mu_i$ or Δs_i violates this condition, then all $\Delta\mu_i$ and Δs_i must be scaled by some factor α, such that $(0 < \alpha \leq 1)$, as shown in equation (2.73).

For analysis purposes we consider the possibility of different scaling factors α_μ and α_s for calculating the new μ_i and s_i respectively. So μ_i^{new} and s_i^{new} are

found as:

$$\mu_i^{new} = \mu_i^{old} + \alpha_\mu \Delta\mu_i, \qquad i = 1, \cdots, m \qquad (2.68)$$

$$s_i^{new} = s_i^{old} + \alpha_s \Delta s_i, \qquad i = 1, \cdots, m \qquad (2.69)$$

Since we need $\mu_i > 0$ and $s_i > 0$ so we look for those indices i where $\Delta\mu_i < 0$ and $\Delta s_i < 0$, respectively. The value for α is chosen as [11]:

$$\bar{\alpha}_\mu = \min\{-\frac{\mu_i^{old}}{\Delta\mu_i} \mid \forall i : \Delta\mu_i < 0\} \qquad (2.70)$$

$$\bar{\alpha}_s = \min\{-\frac{s_i^{old}}{\Delta s_i} \mid \forall i : \Delta s_i < 0\} \qquad (2.71)$$

$$\alpha = \min\{\bar{\alpha}_\mu, \bar{\alpha}_s, 1\} \qquad (2.72)$$

Initially we will take $\alpha_\mu = \alpha_s = \alpha$.

Scaling factor κ is slightly less then one. κ is multiplied with α so that μ_i and s_i are prevented from being zero. Studies have suggested that κ be 0.9995 [15, 21]. The scaling factor κ forces the variables to stay inside the boundary without touching it. Therefore, the variables are strictly feasible with respect to the inequality constraints, *i.e.* $\forall\ i$, $\mu_i > 0$ and $s_i > 0$ at every iteration [11]. This property yields the term *interior-point*. Therefore, the primal-dual method becomes the *primal-dual interior-point* method.

Scaling factor τ is selected somewhere between 0 and 1 *i.e.* $(0 < \tau < 1)$. If τ is closer to 1, this means that we go very close to the central path. If τ is closer to zero, this means that we go close to central path, but not very close. Stephen J. Wright [20] suggested $\tau = 0.5$. However, there are two views:

One view is that we should go towards the central path and when we reach very close to it then move along the central path by decreasing ν rapidly. Problem will converge to the solution rapidly on the central path.

Another view is that by going too close to the central path, we may unnecessarily increase number of iterations. So perhaps we should go close enough to the central path and then move along the central path as shown in figure 2.5 [12].

If we stay too far from the central path, the problem may not converge or it may converge to a wrong answer. How close we should stay to the central path, to minimize total iterations, but still converge to the solution, is yet to be fully explored.

Scaling factor ρ is used to get new value of ν by reducing its old value. It controls the speed of following the central path. ρ is selected such that $(0 < \rho < 1)$. [19] suggested ρ be (0.4). Another view is that we should not keep ρ constant, instead it should be adjusted depending upon how much progress we make. How it should be adjusted, is yet to be fully researched.

General description of (PDIP) algorithm

1. An initial operating point z_o is chosen and initially barrier parameter ν is set to a relatively large number.

2. Form matrix W and vector $\omega(z)$.

3. Solve $W\Delta z = -\omega(z)$ for Δz, by using LU factorization and forward & backward substitution.

4. A scaling factor α is chosen and then it is multiplied by Δz before updating z. This reduces the Newton step size Δz so that μ_i and s_i remain positive when they are updated.

5. Improved guess for next iteration is found by:

$$z_{new} = z_{old} + \kappa \alpha \Delta z. \tag{2.73}$$

To make sure that each each μ_i^{new} and s_i^{new} is prevented from being zero, $\alpha \Delta z$ is multiplied by another factor κ, that is slightly less than one.

6. Check how close we are to the *central path*.

   - The closeness from the *central path* is the checked by finding how close each product $\mu_i s_i$ is to the barrier parameter ν using equation (2.65) or (2.66).

   - If we are not sufficiently close to the central path go back to step 2 else go to next step.

7. If the barrier parameter ν is sufficiently close to zero, then the current state defined by z is the solution. Otherwise reduce the barrier parameter by using:

   $\nu_{new} = \rho \cdot \nu_{old}$, $(0 < \rho < 1)$, and go to step 2.

   or by using:

   $\nu_{new} = \rho \cdot \widehat{\nu}$, and go to step 2.

   At each step most recent update of variables is used to calculate ν_{new}.

Comparison of (PDIP) with Newton's method

- The main disadvantage of Newton's method is that we must try to identify a set of binding inequality constraints, which are then treated as equality constraints. These equality constraints are put in a so-called *active set*. This set is subject to change when it appears that an assumed active constraint is not binding or when the limits of a constraint previously thought to be inactive are violated. Sometimes a constraint which is expelled from the active set is added again and then expelled again. This can lead to a cycling behavior which can cause

Newton method to fail. In the PDIP method, explicit identification of inequality constraints as active or inactive is avoided by using a barrier method approach. Since there is no active set in PDIP method, there is no question of cycling behavior in PDIP algorithm.

- Comparing equations (2.42) and (2.63), we notice that W matrix in case of Newton is symmetric, whereas the W matrix in case of PDIP is not symmetric. W matrix of equation (2.63) can be made symmetric by multiplying its last row by S^{-1}, but then the matrix will become less well-conditioned, possibly resulting in inaccurate results or failure to converge.

- In case of PDIP, we can tune the solution method by adjusting the factors: ν, κ, τ, and ρ, whereas Newton's method does not give such options.

- Because of including all the inequality constraints, PDIP has an inherent disadvantage that the size of the problem is much bigger than that in case of Newton's method.

2.4 Security of Power Systems

A *contingency* in a power system means the loss of a major piece of equipment, such as a power transmission line or a generator. If a fault occurs on a transmission line, the line is automatically disconnected from the system by the protection system. Contingencies can also occur due to equipment failure at the generation facility which may result in the loss of an on-line generator.

A secure power system is one which can serve load, without violating the operating limits of the system, after the occurrence of a contingency. After the contingency, operating limits may be increased to their *"emergency limits"*. Operation above the normal limits but below the emergency limits is not acceptable for continuous operation, but may be tolerated for a short time (say 15 min or 1 hour) in an emergency.

The system can then be redispatched/reconfigured to remove any overloads (as long as they don't exceed the emergency limits) in a more leisurely fashion, assuming the system has survived the contingency. We assume this slower redispatch is possible once the contingency is survived.

Two or more contingencies may occur at the same time, although this is unlikely if the contingencies are independent. Each contingency will cause changes in the power system state variables: magnitude and angle of voltages on buses and real and reactive power on the lines. In the post-contingency state, if these variables are not within the permissible limits, further contingencies will occur which may lead to a cascading outage or blackout [22].

In last section we learned how to dispatch power in optimal fashion. However, our statement of the problem did not include any constraint regarding the security of the power system. Therefore, the OPF was not able to provide a secure power system operating point.

Power system security is maintained by having spinning reserves on generators and redundancy in designing the transmission system so that the system can be rescheduled, while being within the allowed operational limits, after the occurrence of a contingency.

2.4.1 Security-Constrained Optimal Power Flow (SCOPF)

Programs which can make adjustments to the pre-contingency operation to prevent violations in the post-contingency conditions at the minimum cost are called "security-constrained optimal power flow" (SCOPF) or contingency-constrained OPF. Such programs take count of many contingencies and calculate adjustments to generators MW, generator voltages, transformer taps etc [23].

To find out which of the credible contingencies has a significant impact on the scenario under study, contingency analysis is performed at the current operating point.

Contingency analysis checks all lines and voltages in the system with respect to their respective limits and identifies the contingency cases with violations or near violations. These selected contingency cases are then included in the OPF problem statement. Therefore in mathematical model of SCOPF, OPF problem is stated subject to both the pre-contingency (base-case) constraints, and to the post-contingency constraints of credible contingencies [3].

To mathematically state the SCOPF problem, let us assume that superscripts $(\cdot)^0$ and $(\cdot)^k$ denote the normal operating state and the post contingency state, respectively. The SCOPF problem can be stated as:

$$\min_{Y^0} C(Y^0) \tag{2.74}$$

subject to:

$$
\begin{aligned}
h^0(Y^0) &= 0, \\
g^0(Y^0) &\leq 0, \\
h^k(Y^k) &= 0, \qquad k = 1, \cdots, K \\
g^k(Y^k) &\leq 0, \qquad k = 1, \cdots, K
\end{aligned}
$$

where

Y^0:Vector of pre-contingency state and control variables,

$C(Y^0)$:Cost of pre-contingency system operation,

Y^k:Vector of post-contingency state and control variables,

h^0:Pre-contingency equality constraint vector functions,

g^0:Pre-contingency inequality constraint vector functions,

h^k:Post-contingency equality constraint vector functions,

g^k:Post-contingency inequality constraint vector functions,

K:Total number of credible contingencies.

2.4.2 SCOPF with post-contingency rescheduling

Studies [1] have shown that the security-constrained dispatch as defined above is conservative, because it does not take into account the system corrective capabilities in the post-contingency state. These corrective capabilities include generation rescheduling, partially dropping the load etc. Costs can be reduced by considering the post-contingency rescheduling.

To consider the post-contingency rescheduling we add some constraints, which couple pre-contingency and post contingency states, to the SCOPF problem. These constraints are called *coupling constraints*. One example of coupling constraint is the amount by which a generator can be ramped up (or down), in a specified amount of time, from pre-contingency state to post-contingency state. Mathematically these constraints can be written as:

$$P_{Gi}^k - P_{Gi}^0 \leq \Delta_{max}^+ P_{Gi} \tag{2.75}$$

$$P_{Gi}^0 - P_{Gi}^k \leq \Delta_{max}^- P_{Gi} \tag{2.76}$$

$\Delta_{max}^+ P_{Gi}$ is the amount by which the real power output of a generator at bus i can be increased, and is called ramp up capacity[2] of the generator. Similarly $\Delta_{max}^- P_{Gi}$ is the amount by which the real power output of a generator at bus i can be decreased, and is called ramp down capacity of the generator. Partially dropping the customer

---

[2]Maximum change of real power generation, of a generator, within a specified period of time is called the *ramping rate* of that generator.

load is another coupling constraint and can be written as:

$$P_{Li}^k \leq P_{Li}^0 \tag{2.77}$$

where P_{Li} is the load being served at bus i. From (2.76) and (2.77) it can be noticed that these constraints contain the pre-contingency variables and also limit the post-contingency adjustments to the variables. Therefore, such constraints are function of both the pre-contingency and the post-contingency state. The general form of the coupling inequality constraints is:

$$d^k(Y^k, Y^0) \leq 0. \tag{2.78}$$

The SCOPF problem (2.74) with post-contingency rescheduling can be stated as:

$$\min_{Y^0} C(Y^0) \tag{2.79}$$

subject to:

$$
\begin{aligned}
h^0(Y^0) \quad &= 0, \\
g^0(Y^0) \quad &\leq 0, \\
h^k(Y^k) \quad &= 0, \qquad k = 1, \cdots, K \\
g^k(Y^k) \quad &\leq 0, \qquad k = 1, \cdots, K \\
d^k(Y^k, Y^0) \quad &\leq 0, \qquad k = 1, \cdots, K.
\end{aligned}
$$

2.5   Expected Security Cost Optimal Power Flow

In security constrained optimal power flow, OPF problem is solved by taking security as a constraint. Reference [24] showed that if the security issue is taken care of by

taking security as an economic cost instead of a constraint, then the solution would have a lower expected cost, or increased social welfare. A OPF model was introduced which handles security constraints, at post-contingency states, in terms of economic costs.

When a credible contingency occurs, the power system must re-dispatch from the current operating point to survive the contingency. As a result the new cost of system operation, in the post-contingency state, is obtained. This new cost under the post-contingency state is called the *security cost*. Since post-contingency state is associated with each contingency, therefore each contingency has a security cost that depends upon the pre-contingency state. In post-contingency state, the system might be operating at the *emergency limits* instead of the normal operating limits. Moreover, the post-contingency rescheduling is taken into account, which could be generator rescheduling or demand side rescheduling *i.e.* customer load interruption. If security costs of all credible contingencies are taken into account, the system would dispatch in the pre-contingency state such that the security costs would be as low as possible when the contingencies occur. However, the information on security costs alone is not sufficient to determine the optimal pre-contingency state because during the normal operating state, one does not know which contingency will occur [11]. Each contingency k has a different probability of occurrence, say π^k. A contingency that results in high security cost may be unlikely, while a contingency with lower security cost may be more likely. The pre-contingency optimal dispatch process should consider the significance of all contingencies by taking into account the respective post-contingency security costs and their probabilities of occurrence. Therefore the security cost associated with each constraint is multiplied with its probability of occurrence. This leads to the *expected security cost optimal power flow* (ESCOPF) problem, which is an OPF problem which takes into account the system security in terms of the expected security costs.

The objective of the ESCOPF problem is to minimize the expected system operating cost, which is the sum of the expected pre-contingency cost and expected post-contingency costs [22]. The pre-contingency cost is multiplied by the probability of operating in the pre-contingency state, and each post-contingency cost is multiplied by the probability of the contingency occurring. The sum of these costs is the *expected cost of secured system operation*. Let us consider following notation:

- $C^0(Y^0)$: Cost of operating the system in normal or pre-contingency state Y^0

- π^0: Probability of operating the system in normal or pre-contingency state,

- $C^k(Y^0, Y^k)$: Cost of operating the system in post-contingency state Y^k, after a contingency k, with a pre-contingency operating state Y^0

- π^k: Probability that contingency k will occur

Contingencies are assumed to be independent. The chances of two contingencies occurring simultaneously is so small that we commonly assume the K contingencies to be mutually exclusive. Therefore the probability of normal operation can be written as:

$$\pi^0 = 1 - \sum_{k=1}^{K} \pi^k \tag{2.80}$$

So $\pi^0 C^0(Y^0)$ and $\pi^k C^k(Y^0, Y^k)$ will be the expected costs of system operation in pre-contingency and post-contingency state of contingency k, respectively. Therefore, the total cost of secured system operation will be $\left(\pi^0 C^0(Y^0) + \sum_{k=1}^{K} \pi^k C^k(Y^0, Y^k)\right)$.

Hence the ESCOPF problem can be stated as:

$$\min_{\{Y^0, Y^1, \ldots, Y^K\}} \pi^0 C^0(Y^0) + \sum_{k=1}^{K} \pi^k C^k(Y^0, Y^K) \tag{2.81}$$

subject to:

$$\pi^k h^k(Y^k) = 0, \qquad\qquad k = 0, 1, \ldots, K$$

$$\pi^k g^k(Y^k) \leq 0, \qquad\qquad k = 0, 1, \ldots, K$$

$$\pi^k d^k(Y^0, Y^k) \leq 0, \qquad\qquad k = 1, \ldots, K,$$

h^0: Pre-contingency equality constraints,

g^0: Pre-contingency inequality constraints,

h^k: Post-contingency equality constraints,

g^k: Post-contingency inequality constraints,

d^k: Post-contingency coupling constraints, which include variables from both the pre-contingency and post contingency states.

Reference [11] attempted to solve the problem (3.23) by decoupling the problem in to main problem and K subproblems. Security cost of operating the system in post-contingency state Y^k, $C^k(Y^0, Y^k)$ is obtained by allowing the system to optimally redispatch from the pre-contingency state (Y^0), when the contingency k occurs. The minimum security cost associated with a contingency k can be obtained by solving the OPF for the post-contingency system[3], given the pre-contingency state. So each $C^k(Y^0, Y^k)$ can be evaluated separately. The evaluation of each $C^k(Y^0, Y^k)$ is called the *contingency subproblem* of the ESCOPF problem. The decoupled ESCOPF

---

[3]The structure of the post-contingency system will be different from that of the pre-contingency system due to the outage of an on-line equipment.

problem can be stated as:

$$\min_{Y^0} \pi^0 C^0(Y^0) + \sum_{k=1}^{K} \pi^k C^{k*}(Y^0) \qquad (2.82)$$

$$\text{subject to:} \qquad \pi^0 h^0(Y^0) = 0, \qquad (2.83)$$

$$\pi^0 g^0(Y^0) \leq 0, \qquad (2.84)$$

where, for each $k = \{1, \ldots, K\}$, $\pi_k C^{k*}(Y^0)$ is the solution of the subproblem:

$$C^{k*}(Y^0) = \min_{Y^k} C^k(Y^0, Y^k) \qquad (2.85)$$

$$\text{subject to:} \qquad h^k(Y^k) = 0, \qquad (2.86)$$

$$g^k(Y^k) \leq 0, \qquad (2.87)$$

$$d^k(Y^0, Y^k) \leq 0, \qquad (2.88)$$

The author of [11] tried to solve the decoupled ESCOPF problem (2.82) by Newton method but found that cycling occurred for some cases, in which a small number of constraints repeatedly entered and left the active set of binding constraints. She then explored the application of the PDIP method to the decoupled problem. The algorithm worked well with the decoupled 3-bus case. However, for 5-bus and 14 bus systems the algorithm did not converge.

Therefore, she applied the PDIP to the integrated ESCOPF problem (3.23), which combines the main and the subproblems together, just to see if it worked. While the PDIP failed to solve the cases bigger then 3-bus systems with the decoupled method, because of the reasons discussed below, the corresponding integrated problems of those systems were successfully solved.

2.6 Applications

Whether loadflow/economic dispatch, OPF, loadflow with contingency analysis, SCOPF or ESCOPF is the preferred tool, there are two major applications of these tools.

The first application is in operations, where one would like to know if the current operating state is both secure and efficient (least costly) and, if not, how to adjust the system to make it so. This is information which systems dispatchers should have at all times, and, since the system state is in constant flux, such analyses must be done in near-real time (every 5–15 minutes). Solving any of these problems, but especially SCOPF or ESCOPF, is computationally very demanding. Such algorithms are useless if they require too much time, so a near-real-time method is required.

In system planning, predictions of load growth are combined with planned modifications in generation, transmission and distribution to determine whether additional capacity or other changes must be made to the system over the planning period (one year, two years, five years or possibly even longer). Although planning studies obviously do not require near-real-time speeds, they are often more detailed or cover larger geographical area than what is expected to be available in an operations context, so reasonable speeds are still required even for large problem sizes.

CHAPTER 3

DECOUPLED ESCOPF WITH PARALLEL COMPUTATION

3.1 Advantages associated with distributed and parallel computation of ESCOPF

Reference [11] solved the ESCOPF problem by combining the main and the subproblems together. However, for larger power systems it will take a lot of time because of the size of the problem. If K is the total number of contingencies, the integrated problem will have $(K+1)$ sets of variables: Y^0 and Y^k, where the *contingency index k* denotes operation in the kth contingency ($k \in \{1, \ldots K\}$) and $k = 0$ denotes operation without any contingency occurring, and each set of variables is itself large.

For large power systems, the decoupled method has an advantage over the integrated method because of its ability to divide the ESCOPF problem into smaller subproblems. Let us suppose the size of the matrix to be factorized in the integrated case is $(n \times n)$. If L is the number of nonzero entries in a typical row, this takes about $\frac{1}{3}n^2L$ *flops,*[1] or floating point operations, for the factorization and about $\frac{1}{2}nL$ operations[2] for a forward or backward substitution (so about nL operations for a linear solve once the LU factors of the matrix are known), so for large n the time it takes to factor a matrix is the main influence on solution time. If the problem is decomposed into a main problem and K subproblems, then each problem will have size $\frac{n}{(K+1)} \times \frac{n}{(K+1)}$ or $\frac{n \times n}{(K+1)^2}$. So each factorization will take about $\frac{\frac{1}{3}(n^2L)}{(K+1)^2}$ flops. Since there are $(K+1)$ subproblems, each $(n \times n)$ factorization is replaced by $(K+1)$ smaller

---

[1]For our purposes flops means one multiplication and one addition.
[2]These numbers would be about $\frac{1}{3}n^3$ and $\frac{1}{2}n^2$ if the matrix were dense; given values for sparse matrices ignore fill-ins during factorization.

factorizations, which together take $\frac{1}{3}\frac{L(n\times n)}{(K+1)^2}(K+1)=\frac{\frac{1}{3}Ln^2}{(K+1)}$ flops. This means that the decoupled solution will have to perform $(K+1)$ less floating point operations per factorization as compared to the integrated solution. Therefore the decoupled solution is as much as $(K+1)$ times faster than the integrated solution. In case we have M machines responsible for working on the decoupled problem then this will further reduce the computation time by as much as M times. So a problem solved using distributed and parallel computation techniques will be as much as $M \times (K+1)$ times faster than the integrated solution technique.

3.2   Problem to be solved

3.2.1   Nonlinear ("AC") ESCOPF Problem

In order to calculate the power operation costs, we need the cost of real power production and the benefit received due to real power consumption. The cost of real power production is obtained from the *generator cost curve*, which gives the cost as a function of the amount of real power production. The generator cost function is assumed to be of the form

$$C(P_{Gi}) = \gamma_{Gi}P_{Gi}^2 + \beta_{Gi}P_{Gi} + \alpha_{Gi}. \tag{3.1}$$

Similarly the benefit received by a consumer is obtained from the *consumer benefit curve*, which gives the benefit (in dollars) as a function of real power consumption. The consumer benefit function is assumed to be of the form

$$B(P_{Li}) = \gamma_{Li}P_{Li}^2 + \beta_{Li}P_{Li} + \alpha_{Li}. \tag{3.2}$$

We would like to maximize the expected *social welfare*, defined as expected total benefit minus total cost, but, to be consistent with other OPF literature, we will

minimize expected cost, treating benefits as a negative cost. The ESCOPF problem statement can be written as follows:

$$\min_{Y^0, Y^k} \left\{ \pi^0 C^0(Y^0) + \sum_{k=1}^{K} \pi^k C^k(Y^0, Y^k) \right\} \tag{3.3}$$

subject to pre-contingency equality constraints:

$$
\begin{aligned}
P_i^0(X^0) &= P_{Gi}^0 - P_{Li}^0 &&: \forall\, i, \\
Q_i^0(X^0) &= Q_{Gi}^0 - Q_{Li}^0(P_{Li}^0) &&: \forall\, i,
\end{aligned}
\tag{3.4}
$$

pre-contingency inequality constraints:

$$
\begin{aligned}
X_{\min}^0 &\leq X^0 \leq X_{\max}^0, \\
P_{Gi_{\min}}^0 &\leq P_{Gi}^0 \leq P_{Gi_{\max}}^0 &&: \forall\, i \in \mathcal{G}, \\
Q_{Gi_{\min}}^0 &\leq Q_{Gi}^0 \leq Q_{Gi_{\max}}^0 &&: \forall\, i \in \mathcal{G}, \\
&\quad \left| I_{ij}(X^0) \right| \leq I_{ij_{\max}}^0 &&: \forall\, ij, \\
0 &\leq P_{Li}^0 &&: \forall\, i \in \{\mathcal{I} \cup \mathcal{U}\},
\end{aligned}
\tag{3.5}
$$

post-contingency equality constraints:

$$
\begin{aligned}
P_i^k(X^k) &= P_{Gi}^k - P_{Li}^k &&: \forall\, i, \\
Q_i^k(X^k) &= Q_{Gi}^k - Q_{Li}^k(P_{Li}^k) &&: \forall\, i,
\end{aligned}
\tag{3.6}
$$

post-contingency inequality constraints:

$$
\begin{aligned}
X_{\min}^k &\leq X^k \leq X_{\max}^k, \\
P_{Gi_{\min}}^k &\leq P_{Gi}^k \leq P_{Gi_{\max}}^k &&: \forall\, i \in \mathcal{G}, \\
Q_{Gi_{\min}}^k &\leq Q_{Gi}^k \leq Q_{Gi_{\max}}^k &&: \forall\, i \in \mathcal{G}, \\
&\quad \left| I_{ij}(X^k) \right| \leq I_{ij_{\max}}^k &&: \forall\, ij, \\
0 &\leq P_{Li}^k &&: \forall\, i \in \{\mathcal{I} \cup \mathcal{U}\},
\end{aligned}
\tag{3.7}
$$

and coupling constraints:

$$
\begin{aligned}
P_{Gi}^0 - \Delta_{\max}^- P_{Gi} &\leq P_{Gi}^k \leq P_{Gi}^0 + \Delta_{\max}^+ P_{Gi} &&: \forall\, i \in \mathcal{G}, \\
P_{Li}^0 - \Delta_{\max}^- P_{Li} &\leq P_{Li}^k \leq P_{Li}^0 &&: \forall\, i \in \mathcal{I}.
\end{aligned}
\tag{3.8}
$$

where

$$
\begin{aligned}
C^0(Y^0) &= \sum_{i \in \mathcal{G}} C_{Gi}(P_{Gi}^0) - \sum_{i \in \mathcal{I}} B_{Ii}(P_{Li}^0) - \sum_{i \in \mathcal{U}} B_{Ui}(P_{Li}^0) \tag{3.9} \\
C^k(Y^k, Y^0) &= \sum_{i \in \mathcal{G}} \left[C_{Gi}(P_{Gi}^k) + C_{Ri} \cdot \left(\left| P_{Gi}^k - P_{Gi}^0 \right| \right) \right] \\
&\quad + \sum_{i \in \mathcal{I}} \left[C_{Ii} \cdot (P_{Li}^0 - P_{Li}^k) - B_{Ii}(P_{Li}^k) \right] - \sum_{i \in \mathcal{U}} B_{Ui}(P_{Li}^0). \tag{3.10}
\end{aligned}
$$

The model, expressed in (3.3)-(3.10), represents the ESCOPF objective function and constraints. This ESCOPF problem consists of two parts. The first part is minimization of the cost of operating in pre-contingency state, $C^0 Y^0$ (3.9), subject to equality constraints (3.4) and inequality constraints (3.5). Equality constraints are pre-contingency power balance equations and inequality constraints are pre-contingency system operating limits, which include limits on voltage, tap ratios, phase shifters, power (real and reactive) generation, line flows, and real power consumption. Y^0 represents variables in pre-contingency state. It consists of vector of control variables X^0, which includes voltage magnitudes, voltage angles, transformer tap rations and phase shifters, and vector of state variables, which include real power generation P_{Gi}^0, reactive power generation Q_{Gi}^0, and real power consumed by the loads P_{Li}^0. Power injections at each bus and line currents are function of the control variables X^0.

The probability of operating in pre-contingency state is π^0 (2.80) and π^k is the probability of occurrence of contingency $k, k \in \{1, \cdots, K\}$. We consider two types of groups of consumers $\mathcal{U}$ and $\mathcal{I}$. Loads for consumers in group $\mathcal{U}$ are uninterruptible when a contingency occurs, whereas loads for consumers in group $\mathcal{I}$ can be interrupted

after a contingency occurs. The benefit functions of $\mathcal{U}$ and $\mathcal{I}$ are B_{Ui} and B_{Ii}, which are included in the objective function with negative sign. Equation 3.4 represents real and reactive power flow or power balance equations for every bus i, in pre-contingency state. P_{Gi}^0 and P_{Li}^0 represent real power generated and real power consumed at each bus i. Similarly Q_{Gi}^0 and Q_{Li}^0 represent reactive power generated and reactive power consumed as each bus i. Due to the power factor of the load, reactive power load at any load bus is a function of the real power load at that bus $i.e$ $Q_{Li}^0(P_{Li}^0)$. In equality constraints (3.5) are power system operating constraints in pre-contingency state. These constraints place limits on the control variables X^0 and state variables, which are adjusted so as to minimize the objective function without violating the power balance equations and without exceeding their limits.

The second component of the ESCOPF problem is the minimization of the cost of operating in post-contingency state $C^k(Y^k, Y^0)$ (3.9), subject to post-contingency power balance equations (3.6), post-contingency operating limits (3.7), and coupling constraints (3.8). Y^k is the vector of control variables X^k and vector of state variables $(P_{Gi}^k, Q_{Gi}^k, \& P_{Li}^k)$. $C_{Gi}^k(P_{Gi}^k)$ is the cost of generation in post-contingency state. Cost of ramping a generator up or down when a contingency occurs is called the ramping cost $C_{Ri}\left|P_{Gi}^k - P_{Gi}^0\right|$, which is a function of the change in the real power generation from pre-contingency to post-contingency state. For convenience, we assume that costs of ramping up are same as costs of ramping down. Group $\mathcal{I}$ of consumers agree that their loads are interruptible but in the event of an interruption they experience an economic loss beyond the foregone benefit of consumption, called load interruption cost C_{Ii}, which is a function of the amount of load interrupted $(P_{Li}^0 - P_{Li}^k)$.

Equation (3.6) represents real and reactive power flow equations for every bus i, in the post-contingency state. In equality constraints (3.7) place limits on the control variables X^k and the state variables of the post contingency subproblem. System operating limits in the post-contingency state are emergency limits. $P_{Li}^0 \geq 0$ and

$P_{Li}^k \geq 0$ means that real power consumed cannot be negative, which means that loads cannot act as generators.

Coupling constraints are represented by (3.8), which take care of the generator ramp up limits $\Delta_{\max}^+ P_{Gi}$, generator ramp down limits $\Delta_{\max}^- P_{Gi}$, and load interruption limits $\Delta_{\max}^- P_{Li}$. These constraints are functions of both pre-contingency and post contingency operating point. Real power generation at each bus i has two upper limits and two lower limits. Due to the upper limits, real power generation cannot rise above the maximum emergency generation capacity P_{Gimax}^k, and it cannot ramp up by more then $\Delta_{\max}^+ P_{Gi}$. Real power generation would be limited by either of the limits, whichever is hit first. Similarly one of the lower limits prevents real power generation level from dropping below some minimum generation level and the other lower limit prevents ramping down by more than $\Delta_{\max}^- P_{Gi}$.

For each load of the group $\mathcal{I}$, no more than $\Delta_{\max}^- P_{Li}$ can be interrupted. $P_{Li}^k \leq P_{Li}^0$ means that real power consumed by a load in the post-contingency state cannot exceed the pre-contingency real power consumption.

3.2.2 Linearized ("DC") ESCOPF Problem

For the linearized approximation, called "DC" approximation of a power system, following assumptions are made:

1. Bus voltage magnitudes are approximately 1 per unit,

2. Bus voltage angle differences are small,

3. Transmission line resistances are ignored.

Due to the first two assumptions, reactive power flows are zero and due to the $3rd$ assumption, real power losses in the transmission lines are absent. Real power flow

on a transmission line ij can be found as:

$$P_{ij} = -b_{ij}(\theta_i - \theta_j) \tag{3.11}$$

where b_{ij} is the series susceptance connecting bus i and bus j (in per unit) and θ_i is the voltage angle at bus i in radians. Therefore, the total real power injected at bus i can be expressed as:

$$P_i(\theta) = \sum_j P_{ij} = \sum_j -b_{ij}(\theta_i - \theta_j). \tag{3.12}$$

The model, expressed in (3.13)-(3.20), represents a DC version of ESCOPF problem.

$$\min_{Y^0, Y^k} \left\{ \pi^0 C^0(Y^0) + \sum_{k=1}^{K} \pi^k C^k(Y^0, Y^k) \right\} \tag{3.13}$$

subject to pre-contingency equality constraints:

$$P_i^0(\theta^0) = P_{Gi}^0 - P_{Li}^0 \quad : \forall\, i, \tag{3.14}$$

pre-contingency inequality constraints:

$$
\begin{aligned}
P_{Gi_{\min}}^0 \;\leq\; & P_{Gi}^0 \;\leq\; P_{Gi_{\max}}^0 && : \forall\, i \in \mathcal{G}, \\
& \left| P_{ij}^0 \right| \;\leq\; P_{ij_{\max}}^0 && : \forall\, ij, \\
0 \;\leq\; & P_{Li}^0 && : \forall\, i \in \{\mathcal{I} \cup \mathcal{U}\},
\end{aligned} \tag{3.15}
$$

post-contingency equality constraints:

$$P_i^k(\theta^k) = P_{Gi}^k - P_{Li}^k \quad : \forall\, i, \tag{3.16}$$

post-contingency inequality constraints:

$$
\begin{aligned}
P^k_{Gi_{\min}} &\leq & P^k_{Gi} &\leq & P^k_{Gi_{\max}} &\quad : \forall\, i \in \mathcal{G}, \\
& & \left| P^k_{ij} \right| &\leq & P^k_{ij_{\max}} &\quad : \forall\, ij, \\
0 &\leq & P^k_{Li} & & &\quad : \forall\, i \in \{\mathcal{I} \cup \mathcal{U}\},
\end{aligned}
\tag{3.17}
$$

and coupling constraints:

$$
\begin{aligned}
P^0_{Gi} - \Delta^-_{\max} P_{Gi} &\leq & P^k_{Gi} &\leq & P^0_{Gi} + \Delta^+_{\max} P_{Gi} &\quad : \forall\, i \in \mathcal{G}, \\
P^0_{Li} - \Delta^-_{\max} P_{Li} &\leq & P^k_{Li} &\leq & P^0_{Li} &\quad : \forall\, i \in \mathcal{I}.
\end{aligned}
\tag{3.18}
$$

where

$$
\begin{aligned}
C^0(Y^0) &= \sum_{i \in \mathcal{G}} C_{Gi}(P^0_{Gi}) - \sum_{i \in \mathcal{I}} B_{Ii}(P^0_{Li}) - \sum_{i \in \mathcal{U}} B_{Ui}(P^0_{Li}) \tag{3.19} \\
C^k(Y^k, Y^0) &= \sum_{i \in \mathcal{G}} \left[C_{Gi}(P^k_{Gi}) + C_{Ri} \cdot \left(\left| P^k_{Gi} - P^0_{Gi} \right| \right) \right] \\
&\quad + \sum_{i \in \mathcal{I}} \left[C_{Ii} \cdot (P^0_{Li} - P^k_{Li}) - B_{Ii}(P^k_{Li}) \right] - \sum_{i \in \mathcal{U}} B_{Ui}(P^0_{Li}). \tag{3.20}
\end{aligned}
$$

The ESCOPF model without the assumptions is referred to as AC-ESCOPF. In DC case, Y^0 is same as in AC case, except that variables regarding reactive power and voltages are not present. Objective function remains the same, however, we get rid of constraints related to reactive power and voltages. Moreover, the nonlinear constraints are now linear.

3.3 Decoupled solution method

The ESCOPF problem is stated in a more compact notation as:

$$\min_{\{Y^0,Y^1,...,Y^K\}} \underbrace{\pi^0 C^0(Y^0)}_{\substack{\text{Pre-contingency} \\ \text{case}}} + \underbrace{\sum_{k=1}^{K} \pi^k C^k(Y^0, Y^K)}_{\text{All contingencies}} \tag{3.21}$$

subject to:

$$\pi^0 h^0(Y^0) = 0,$$

$$\pi^0 g^0(Y^0) = 0,$$

$$\pi^k h^k(Y^k) = 0, \qquad\qquad k = 1, \ldots, K.$$

$$\pi^k g^k(Y^k) \leq 0, \qquad\qquad k = 1, \ldots, K.$$

$$\pi^k d^k(Y^0, Y^k) \leq 0, \qquad\qquad k = 1, \ldots, K.$$

h^0: Pre-contingency equality constraints,

g^0: Pre-contingency inequality constraints,

h^k: Post-contingency equality constraints,

g^k: Post-contingency inequality constraints,

d^k: Post-contingency coupling constraints, which include variables from both the pre-contingency and post contingency states.

To get rid of inequality constraints we use *barrier function* as:

$$\mathcal{B}(Y^0, Y^1, ..., Y^K, \lambda^0, \lambda^1, ..., \lambda^K, \nu) = \pi^0 C^0(Y^0) - \nu \ln(-\pi^0 g^0(Y^0))$$

$$+ \sum_{k=1}^{K} \pi^k C^k(Y^k, Y^0) - \nu \sum_{k=1}^{K} \ln(-\pi^k g^k(Y^k)) - \nu \sum_{k=1}^{K} \ln(-\pi^k d^k(Y^k, Y^0)). \tag{3.22}$$

So the new problem can be stated as:

$$\min_{\{Y^0,Y^1,...,Y^K,\nu\}} \mathcal{B}(Y^0, Y^1, ..., Y^K, \lambda^0, \lambda^1, ..., \lambda^K, \nu) \tag{3.23}$$

subject to:

$$\pi^0 h^0(Y^0) = 0, \tag{3.24}$$

$$\pi^k h^k(Y^k) = 0, \qquad\qquad k = 1, \ldots, K$$

The superscript $k = 0$ denotes the main problem, while $k = 1, \ldots, K$ denotes subproblem 1 to K. The Lagrangian function can be written as:

$$
\begin{aligned}
\mathcal{L} &= \pi^0 C^0(Y^0) + \pi^0 \lambda^{0^T} h^0(Y^0) - \nu \ln(-\pi^0 g^0(Y^0)) + \sum_{k=1}^{K} \pi^k C^k(Y^k, Y^0) + \sum_{k=1}^{K} \pi^k \lambda^{k^T} h^k(Y^k) \\
&\quad - \nu \sum_{k=1}^{K} \ln(-\pi^k g^k(Y^k)) - \nu \sum_{k=1}^{K} \ln(-\pi^k d^k(Y^k, Y^0))
\end{aligned}
\tag{3.25}
$$

The gradient of the Lagrangian function with respect to variables Y^0 and Y^k, and Lagrangian multipliers λ^0 and λ^k, are:

$$
\begin{aligned}
\nabla_{Y^0}\mathcal{L} &= \pi^0 \frac{\partial C^0(Y^0)}{\partial Y^0} + \pi^0 \lambda^{0^T} \frac{\partial h^0(Y^0)}{\partial Y^0} - \nu \left[\frac{1}{-g^0(Y^0)} \frac{-\partial g^0(Y^0)}{\partial Y^0} \right] \\
&\quad + \sum_{k=1}^{K} \pi^k \frac{\partial C^k(Y^k, Y^0)}{\partial Y^0} - \nu \sum_{k=1}^{K} \left[\frac{1}{-d^k(Y^k, Y^0)} \frac{-\partial d^k(Y^k, Y^0)}{\partial Y^0} \right], \tag{3.26} \\
\nabla_{Y^k}\mathcal{L} &= \pi^k \frac{\partial C^k(Y^k, Y^0)}{\partial Y^k} + \pi^k \lambda^{k^T} \frac{\partial h^k(Y^k)}{\partial Y^k} - \nu \frac{1}{-g^k(Y^k)} \frac{-\partial g^k(Y^k)}{\partial Y^k} \\
&\quad - \nu \frac{1}{-d^k(Y^k, Y^0)} \frac{-\partial d^k(Y^k, Y^0)}{\partial Y^k}, \tag{3.27} \\
\nabla_{\lambda^0}\mathcal{L} &= \pi^0 h^0(Y^0), \tag{3.28} \\
\nabla_{\lambda^k}\mathcal{L} &= \pi^k h^k(Y^k). \tag{3.29}
\end{aligned}
$$

The primal and dual slacks are defined as follows:

$$s^0 = -g^0(Y^0) \tag{3.30}$$

$$s^k = -g^k(Y^k); \qquad k = 1, \ldots, K \tag{3.31}$$

$$s_d^k = -d^k(Y^k, Y^0); \qquad k = 1, \ldots, K \tag{3.32}$$

$$\mu^0 = \frac{\nu}{-g^0(Y^0)} = \frac{\nu}{s^0} \tag{3.33}$$

$$\mu_d^k = \frac{\nu}{-d^k(Y^k, Y^0)} = \frac{\nu}{s_d^k}; \qquad k = 1, \ldots, K \tag{3.34}$$

After using the primal and dual slacks in the gradients of Lagrangian function, we get the following equations which have to be solved to reach the solution. For the base case we have to solve:

$$\pi^0 \frac{\partial C^0(Y^0)}{\partial Y^0} + \pi^0 \lambda^{0^T} \frac{\partial h^0(Y^0)}{\partial Y^0} + \mu^{0^T} \frac{\partial g^0(Y^0)}{\partial Y^0} + \sum_{k=1}^{K} \pi^k \frac{\partial C^k(Y^k, Y^0)}{\partial Y^0}$$

$$+ \sum_{k=1}^{K} \mu_d^{k^T} \frac{\partial d^k(Y^k, Y^0)}{\partial Y^0} = 0, \tag{3.35}$$

$$\pi^0 h^0(Y^0) = 0, \tag{3.36}$$

$$g^0(Y^0) + s^0 = 0, \tag{3.37}$$

$$(M^0 S^0 - \nu I)e = 0 \tag{3.38}$$

and for each contingency we have to solve:

$$\pi^k \frac{\partial C^k(Y^k, Y^0)}{\partial Y^k} + \pi^k \lambda^{k^T} \frac{\partial h^k(Y^k)}{\partial Y^k} + \mu^{k^T} \frac{\partial g^k(Y^k)}{\partial Y^k} + \mu_d^{k^T} \frac{\partial d^k(Y^k, Y^0)}{\partial Y^k} = 0, \tag{3.39}$$

$$\pi^k h^k(Y^k) \;=\; 0; \qquad k=1,\ldots,K, \tag{3.40}$$

$$g^k(Y^k) + s^k \;=\; 0; \qquad k=1,\ldots,K, \tag{3.41}$$

$$(M^k S^k - \nu I)e^k \;=\; 0; \qquad k=1,\ldots,K, \tag{3.42}$$

$$d^k(Y^k, Y^0) + s_d^k \;=\; 0; \qquad k=1,\ldots,K, \tag{3.43}$$

$$(M_d^k S_d^k - \nu I)e_d^k \;=\; 0; \qquad k=1,\ldots,K, \tag{3.44}$$

and

$$\mu^0, \mu^k, \mu_d^k \;\geq\; 0; \qquad k=1,\ldots,K, \tag{3.45}$$

$$s^0, s^k, s_d^k \;\geq\; 0; \qquad k=1,\ldots,K, \tag{3.46}$$

where

S^0 : A diagonal matrix containing elements s^0,

M^0 : A diagonal matrix containing elements μ^0,

I : An identity matrix,

e : A vector with all elements equal to one,

S^k : A diagonal matrix containing elements s^k,

M^k : A diagonal matrix containing elements μ^k,

$S_d{}^k$: A diagonal matrix containing elements $s_d{}^k$,

$M_d{}^k$: A diagonal matrix containing elements $\mu_d{}^k$.

In order to solve the equations (3.35) to (3.44), we need to find the gradients of these equations with respect to all variables, Lagrangian multipliers and primal and dual

slacks. The update equation would have the following structure:

$$
\left[
\begin{array}{c|cccc}
W_0 & W_{01} & W_{02} & \cdots & W_{0k} \\
\hline
W_{10} & W_1 & 0 & 0 & 0 \\
W_{20} & 0 & W_2 & 0 & 0 \\
\vdots & 0 & 0 & \ddots & 0 \\
W_{k0} & 0 & 0 & 0 & W_K
\end{array}
\right]
\left[
\begin{array}{c}
\Delta z_0 \\
\Delta z_1 \\
\Delta z_2 \\
\vdots \\
\Delta z_K
\end{array}
\right]
= -
\left[
\begin{array}{c}
\omega_0(z) \\
\omega_1(z) \\
\omega_2(z) \\
\vdots \\
\omega_K(z)
\end{array}
\right]
\tag{3.47}
$$

where

W_0: Hessian-like matrix of the main problem,

W_k: Hessian-like matrix of subproblem k, $(k = 1, \cdots, K)$,

W_{0k}: The effects on the main problem due to the changes in subproblem k,

W_{k0}: The effects on the subproblem due to the changes in main problem.

We notice a bordered-block-diagonal structure of the W matrix. The matrices in the diagonal blocks are the Hessian-like matrices of the main and all the subproblems.

$$
W_0 =
\left[
\begin{array}{cccc}
H^0 & \pi^0 J_h^{0\mathrm{T}} & J_g^{0\mathrm{T}} & 0 \\
\pi^0 J_h^0 & 0 & 0 & 0 \\
J_g^0 & 0 & 0 & I \\
0 & 0 & S^0 & M^0
\end{array}
\right]
\tag{3.48}
$$

$$
H^0 = \pi^0 \nabla_{Y^0}^2 C^0(Y^0) + \pi^0 \sum_{i=1}^{n} \lambda_i^0 \nabla_{Y^0}^2 h_i^0 + \sum_{i=1}^{m} \mu_i \nabla_{Y^0}^2 g_i^0
$$

J_h^0 : Jacobian of equality constraint functions $h^0(Y^0)$,

J_g^0 : Jacobian of inequality constraint functions $g^0(Y^0)$,

$$W_k = \begin{bmatrix} H^k & \pi^k J_h^{kT} & J_g^{k\mathrm{T}} & 0 & D_{Y^k}^{\mathrm{T}} & 0 \\ \pi^k J_h^k & 0 & 0 & 0 & 0 & 0 \\ J_g^k & 0 & 0 & I & 0 & 0 \\ 0 & 0 & S^k & M^k & 0 & 0 \\ D_{Y^k} & 0 & 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 & S_d^k & M_d^k \end{bmatrix} \tag{3.49}$$

where

$$H^k = \pi^k \nabla_{Y^k}^2 C^k(Y^k, Y^0) + \pi^k \sum_{i=1}^{n} \lambda_i^k \nabla_{Y^k}^2 h_i^k + \sum_{i=1}^{m} \mu_i \nabla_{Y^k}^2 g_i^k,$$

J_h^k : Jacobian of equality constraint functions $h^k(Y^k)$,

J_g^k : Jacobian of inequality constraint functions $g^k(Y^k)$,

D_{Y^k} : Jacobian of coupling constraint functions $d^k(Y^k, Y^0)$ w.r.t. Y^k.

The effects of the coupling constraints appear only in the blocks, on the top and left borders of the matrix W. These blocks have a very sparse structure due to very few coupling constraints in the optimality conditions. The other blocks are zero because there are no coupling constraints among the subproblems.

The matrices $W_{01}, \cdots, W_{0k}$ can be interpreted as a measure of how much the changes in the variables of subproblem k affect the variables of the main problem. On other hand matrices $W_{10}, \cdots, W_{k0}$ can be interpreted as how the changes in the variables of the main problem affect the variables of the subproblem k. Note that

$W_{ik} = W_{ki}^T$ if $i \neq k$.

$$W_{k0} = W_{0k}^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ D_{Y^0} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.50}$$

D_{Y^0} : Jacobian of coupling constraint functions $d^k(Y^k, Y^0)$ w.r.t. Y^0.

Since in our case the coupling constraints are linear, therefore, these matrices are not only very sparse but also constant. Hence, the integrated problem is almost decoupled. Very little communication is required between the main and the subproblems during their iterative solution. First equation in (3.47) is:

$$W_0 \Delta z_0 + \sum_{i=1}^{K} W_{0i} \Delta z_i = -\omega_0(z) \tag{3.51}$$

$$\text{or} \quad \Delta z_0 = -W_0^{-1}(\omega_0(z) + \sum_{i=1}^{K} W_{0i} \Delta z_i) \tag{3.52}$$

Considering any ith row in (3.47), except the first, one we can write:

$$W_{i0} \Delta z_0 + W_i \Delta z_i = -\omega_i(z) \tag{3.53}$$

$$\text{or} \quad W_i \Delta z_i = -\underbrace{\{\omega_i(z) + W_{i0} \Delta z_0\}}_{\tilde{\omega}_i(z)} \tag{3.54}$$

$$\text{or} \quad \Delta z_i = -W_i^{-1}\{\omega_i(z) + W_{i0} \Delta z_0\} \tag{3.55}$$

Substituting (3.55) in equation (3.51) we obtain:

$$W_0 \Delta z_0 + \sum_{i=1}^{K} W_{0i} [-W_i^{-1} \{\omega_i(z) + W_{i0} \Delta z_0\}] = -\omega_0(z) \qquad (3.56)$$

$$\Rightarrow \quad \underbrace{\{W_0 - \sum_{i=1}^{K} W_{0i} W_i^{-1} W_{i0}\}}_{\tilde{W}_0} \Delta z_0 = - \underbrace{\{\omega_0(z) - \sum_{i=1}^{K} W_{0i} W_i^{-1} \omega_i(z)\}}_{\tilde{\omega}_0(z)} \qquad (3.57)$$

$$\text{or} \quad \tilde{W}_0 \Delta z_0 = -\tilde{\omega}_0(z)$$

The approximation (3.57) can be used as the update equation for the pre-contingency case. It can be seen that the update Δz_0 is affected by each subproblem's Δz_i. Using equations (3.52) and (3.53) we get:

$$W_{i0} \{-W_0^{-1} (\omega_0(z) + \sum_{i=1}^{K} W_{0i} \Delta z_i)\} + W_i \Delta z_i = -\omega_i(z)$$

$$\Rightarrow \quad -W_{i0} W_0^{-1} \omega_0(z) - \sum_{i=1}^{K} W_{i0} W_0^{-1} W_{0i} \Delta z_i + W_i \Delta z_i = -\omega_i(z)$$

$$-W_{i0} W_0^{-1} \omega_0(z) - W_{i0} W_0^{-1} W_{0i} \Delta z_i - \underbrace{\sum_{\substack{j=1 \\ j \neq i}}^{k} W_{j0} W_0^{-1} W_{0j} \Delta z_j}_{\text{ignore}} + W_i \Delta z_i = -\omega_i(z) \ (3.58)$$

$$\Rightarrow \quad \underbrace{\{W_i - W_{i0} W_0^{-1} W_{0i}\}}_{\tilde{W}_i} \Delta z_i = - \underbrace{\{\omega_i(z) - W_{i0} W_0^{-1} \omega_0(z)\}}_{\tilde{\omega}_i(z)} \qquad (3.59)$$

$$\text{or} \quad \tilde{W}_i \Delta z_i = -\tilde{\omega}_i(z) \qquad (3.60)$$

Equation (3.60) is the update equation for the $ith, i \in \{1, \cdots, K\}$ post-contingency subproblem. Each subproblem calculation result Δz_i will have an impact on the main problem's update Δz_0, which will in turn cause the subproblem to adjust its update to take the feedback into account. We must ignore Δz_j in distributed setting. The information that how Δz_0 is affected by Δz_i is used to improve the increment Δz_0, for that iteration. It can be noticed that Δz_i can be calculated by two ways. One way is to use equation (3.60) and the other way is to use equation (3.55). Therefore,

we have two algorithms to solve the decoupled ESCOPF problem.

3.3.1 Algorithm I

First W_0 and $\omega_0(z)$ can be calculated in master machine of the computer cluster. This information can be passed to each of the slave machines for the calculation of $W_i, \omega_i(z)$, W_{i0} and Δz_i, where $i \in \{1, \cdots, K\}$. Computation for Δz_i is performed using equation (3.60). $\tilde{W}_i$ and $\tilde{\omega}_i(z)$ are derived from W_i and $\omega_i(z)$ by incorporating an estimate of the direction of the main problem. The information from all slave machines responsible for all contingencies is collected and sent back to the master machine which can perform the calculations for Δz_0, using equation (3.57). $\tilde{W}_0$ and $\tilde{\omega}_0(z)$ are derived from W_0 and $\omega_0(z)$ by considering the effect of all the subproblems on the main problem. This procedure, shown in Figure 3.1, can be repeated in each iteration until convergence of the problem.

3.3.2 Algorithm II

First W_0 and $\omega_0(z)$ can be calculated in master machine and $W_i, \omega_i(z)$ and, W_{i0} can be computed in slave machines in parallel fashion. The information from all slave machines is collected and sent back to the master machine. Master machine performs the computation for Δz_0, using equation (3.57), which is sent to all the slave machines for the calculation for Δz_i for each contingency. Computation for Δz_i is performed by using equation (3.54). $\tilde{\omega}_i(z)$ appearing in equation (3.54) is different from that appearing in equation (3.60). Both incorporate the effect of the direction of main problem. However, in case of equation (3.60), we consider an estimate of the direction of main problem, while in the case of equation (3.54), we use the exact step taken by the main problem. After the calculation of Δz_i, all the results from all the slaves are gathered in the master machine. This procedure, shown in Figure 3.2, can be repeated in each iteration until the problem is converged.
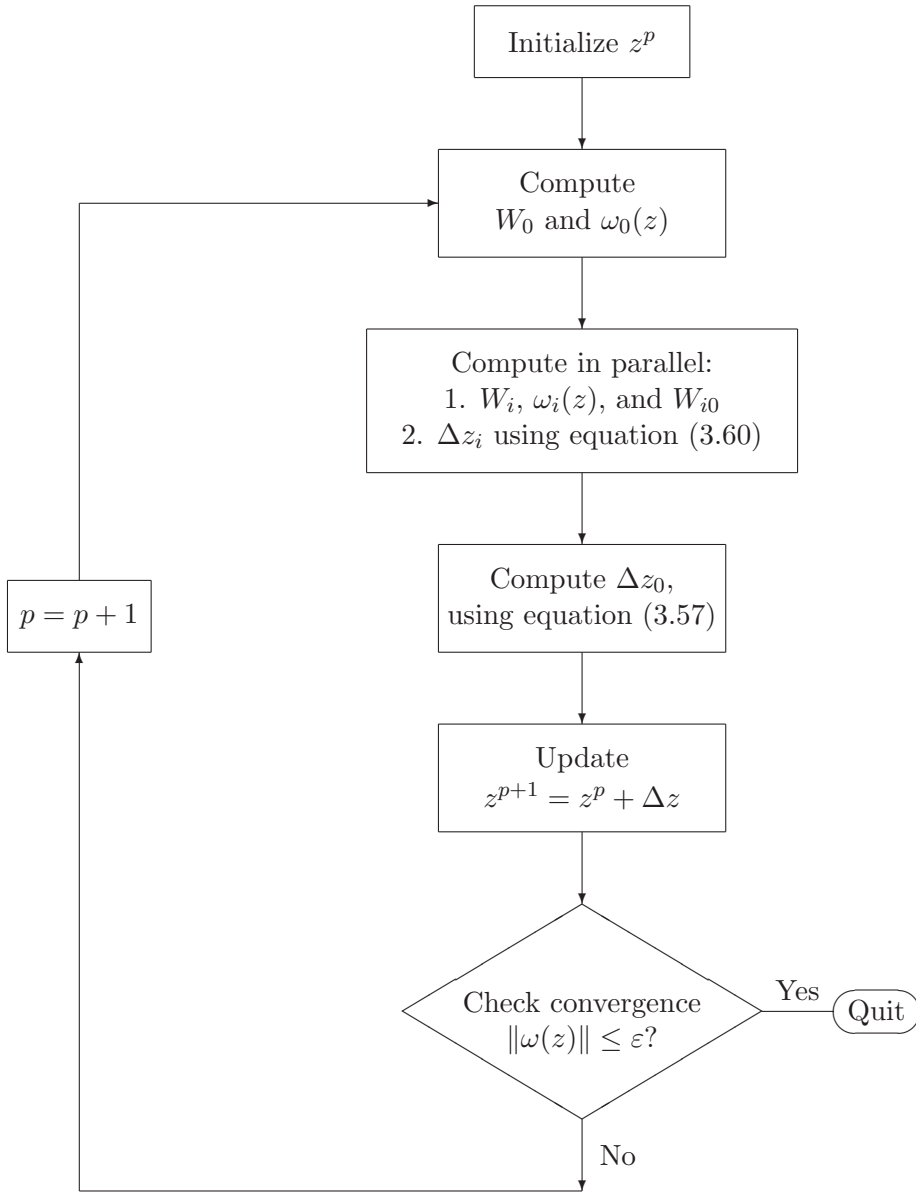
Figure 3.1: Flow chart for algorithm I
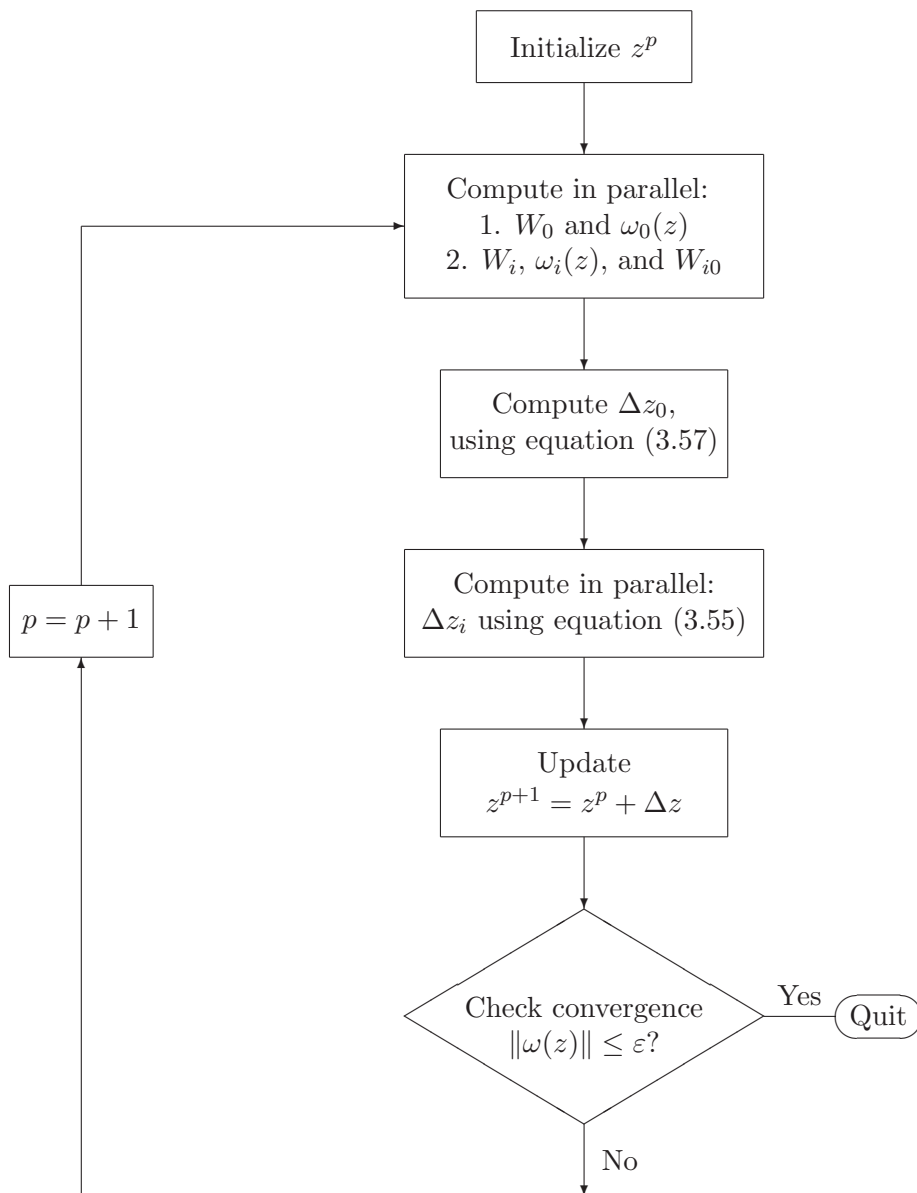
Figure 3.2: Flow chart for algorithm II

3.3.3 Comparison of the two algorithms

The advantage associated with algorithm I is that it requires less communication among the machines of the cluster. However, ignoring terms as shown in equation (3.58), it requires more iterations to reach the solution for smaller test cases. For larger test cases it get gets close to the correct solution but does not quite converge. In comparison to algorithm I, algorithm II needs more communication per iteration but it does converge in relatively small number of iterations.

The idea of the decoupled ESCOPF was successfully implemented for both nonlinear and linear cases by using a Linux-based computer cluster. For the small problems it was noticed that the cluster took more time than the integrated method on a single machine, but as the problem size grew, the time taken by the cluster to solve the problem was significantly less than the time taken by the integrated method.

3.4 Speed up the computation of ESCOPF

3.4.1 Parallel processing

Flynn's Taxonomy

Flynn's Taxonomy, proposed by Michael J. Flynn, is the best-known classification of parallel computer architectures [25, 26]. In this scheme, based upon the number of instruction streams and the number of data streams, there are four classifications of computer systems:

- Single-instruction, single-data (SISD): This category refers to computers which can execute one instruction at a time on one piece of data.

- Single-instruction, multiple-data (SIMD): This category refers to computers which have a central controller that broadcasts the instruction stream to all the subordinate processing elements. Subordinate processors perform the same operation on different data.

- Multiple-instruction, single-data (MISD): This category refers to multiple processors applying different instructions to single data. This class is generally deemed impractical [26].

- Multiple-instruction, multiple-data (MIMD): This category refers to multiple processors simultaneously executing different instructions on different data stream. MIMD systems support parallel solutions that require processors to operate in an autonomous manner. Most modern parallel computers fall into this category. MIMD systems are further divided into shared-memory and distributed-memory systems.
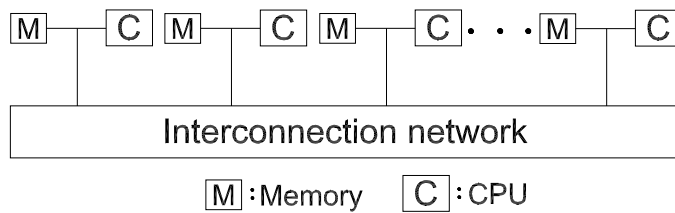
Figure 3.3: Distributed-memory MIMD architecture

Shared-Memory MIMD

Shared memory computer architectures accomplish interprocess communication by using one memory accessible to all processors. There are two major shared memory MIMD interconnection schemes:

- Bus-Based Architectures: All processors are connected to a common memory through a bus[3]. The main disadvantage of these architectures is that large number of processors cannot be used because of the limited bandwidth of the bus.

- Switch-Based Architectures: Such architectures rely on switch-based interconnection network. Unfortunately, this scheme tends to be very expensive because of the large number of hardware switches.

Distributed-Memory MIMD

A distributed-memory system has processors with their own private memory as shown in Figure 3.3. Each node has an autonomous processor and its local memory. All nodes are connected to a processor-to-processor interconnection network. Nodes share data by passing messages through the interconnection network, since there is no shared memory [26]. The main disadvantage of distributed-memory MIMD systems

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[3]In computer terminology bus means the route along which data and instructions travel. In power systems, a bus is simply a node.

is that they are very difficult to program. Software must have the ability to define how and when data is communicated among the processors. The common approach to programming MIMD systems is single-program, multiple-data (SPMD). In this approach the effect of executing different programs is obtained by using conditional branches in the code.

Message Passing

The most commonly used method of programming distributed-memory MIMD systems is message passing, in which processes[4] coordinate their activities by sending and receiving messages [27]. The MPI (Message Passing Interface) standard is the most popular message-passing specification for parallel programming [25]. It is a computer communications protocol. It is a standard for communication among the nodes running a parallel program on a distributed memory system. The Parallel Virtual Machine (PVM)[5] is also a software tool for parallel networking of computers. It is designed to allow a network of heterogeneous machines to be used as a single distributed parallel processor. PVM was a significant step towards modern trends in distributed processing. However, it is not being actively developed.

Matlab is very popular because of its high productivity. One line of Matlab code can typically replace 10 lines of C language [28]. The most common mechanism used for parallel programming is to translate the Matlab code into C, parallelize the resulting C code using MPI and then execute it on a high performance computer. This is an expensive, error-prone and time-consuming activity [29].

MatlabMPI is one of the leading technologies for writing parallel Matlab codes. It was developed by Dr. Jeremy Kepner at Lincoln Laboratory of Massachusetts

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[4]*Process* is a fundamental unit of a parallel software. It is an instance of a program that is executed on a physical processor. A program is parallel if, at any time during its execution, it can executes more than one process [27].

[5]PVM was developed by the University of Tennessee, The Oak Ridge National Laboratory and Emory University.
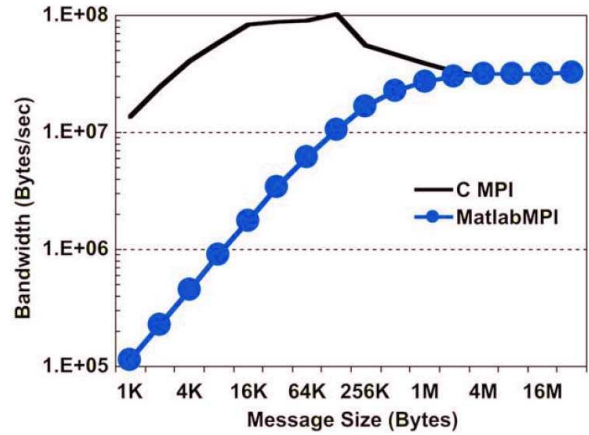
Figure 3.4: MatlabMPI vs. MPI Bandwidth (Courtesy of Dr. Jeremy Kepner)

Institute of Technology. MatlabMPI is a Matlab implementation of the MPI standard and allows any Matlab program to use multiple processors [28]. Using MatlabMPI, MPI communication calls can be embedded in Matlab code. MatlabMPI can run on any combination of computers that Matlab supports. According to reference [29], MatlabMPI has grown quickly in popularity among Department of Defense scientists and engineers.

One issue associated with MatlabMPI is the number of Matlab licenses required to run the parallel program. To overcome this difficulty, Dr. John Nehrbass[6] developed a version of MatlabMPI that would work with Matlab Compiler toolbox and generate executable code, which can be launched for parallel operation and would require zero licenses [29]. This technology is referred to as MatlabMPI compiler technology and it was incorporated in the latest version of MatlabMPI.

Reference [28] compared the performance of C MPI and MatlabMPI. It was found that for large messages ($\sim$ 1 M Byte) MatlabMPI is able to match the performance of C MPI as shown in Figure 3.4.
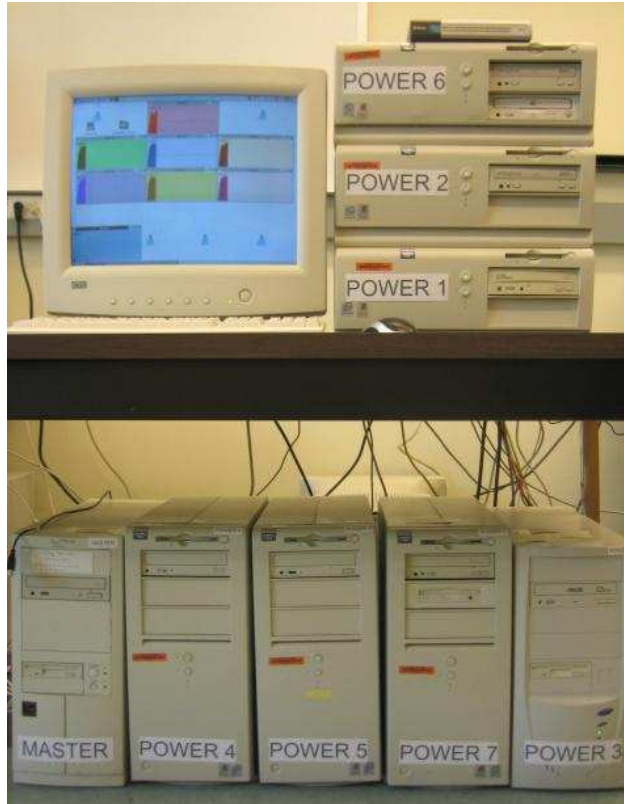
Figure 3.5: OSU Power Beowulf Cluster

Linux cluster

A Linux cluster, a group of computers which work together toward a final goal, has been constructed. We have used Beowulf cluster, originally developed at NASA [30], which is a design for high-performance parallel computing clusters on personal computer hardware. Beowulf systems are now being used worldwide, mainly for the purposes of scientific computing.

Currently our computer cluster is composed of eight machines. One is the master machine and other seven are slave machines as shown in Figure 3.5. The slave machines are given the names `power1` through `power7`. The specifications of the machines in the cluster are given in Table 3.1. The main machine is assigned the main problem and machines `power2`—`power7` are given the task of handling the post-contingency problems. Each machine is assigned a post-contingency problem to solve,

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[6]Dr. John Nehrbass is a Senior Systems Developer/Engineer at Ohio supercomputer center.

Name	Processor Speed GHz	Memory (RAM) MHz
Master	1.20	368
Power 1	0.45	384
Power 2	0.45	384
Power 3	1.20	248
Power 4	0.50	384
Power 5	0.50	384
Power 6	0.45	448
Power 7	0.50	384

Table 3.1: Specifications of the machines in the cluster.

except `power3`, which is given two post-contingency problems to solve because of its faster processor, and `power1,` which monitors the CPU load on each machine in the cluster and displays it using `xload`[7]. All the slave machines can work on subproblems simultaneously before the exchange of the information. The main machine is not only responsible for the calculation of pre-contingency case but also for distributing work among other machines in the cluster.

Figure 3.6 shows the screen shot of `power1` just after the integrated AC ESCOPF has been solved for the IEEE 30 bus test case, using only one of the machines (master) in the cluster. Figure 3.7 shows the screen shot of `power1` after the decoupled AC ESCOPF has been solved, for IEEE 30 bus case, by using seven machines of the cluster. Here, `power1` only monitors the CPU load on each machine of the cluster, so the `xload` display corresponding to `power1` shows that it is not very busy. Comparing Figure 3.6 and Figure 3.7 it can be noticed that the time taken to solve the ESCOPF using the decoupled solution method is much less than the time taken to solve the ESCOPF using the integrated solution method.

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[7]xload is a CPU load monitoring tool, commonly used in Unix. It displays the load of a CPU along y-axis versus time along x-axis.
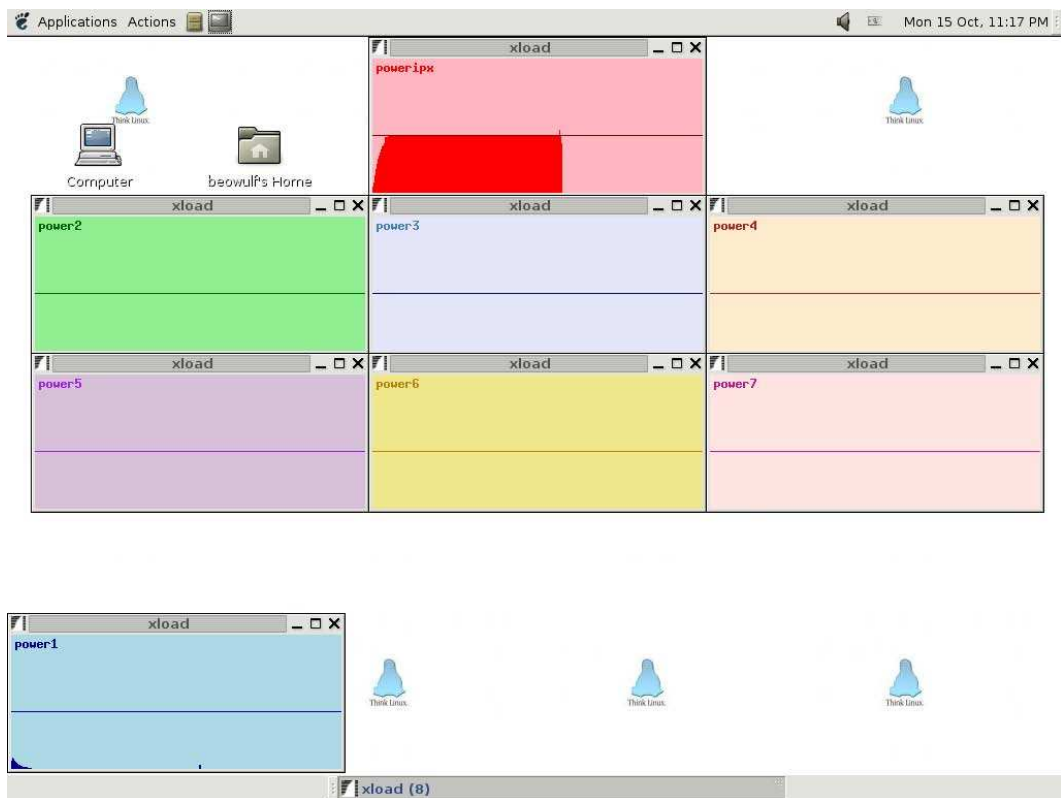
Figure 3.6: Screen shot after completion of the integrated AC ESCOPF for IEEE30 BUS case
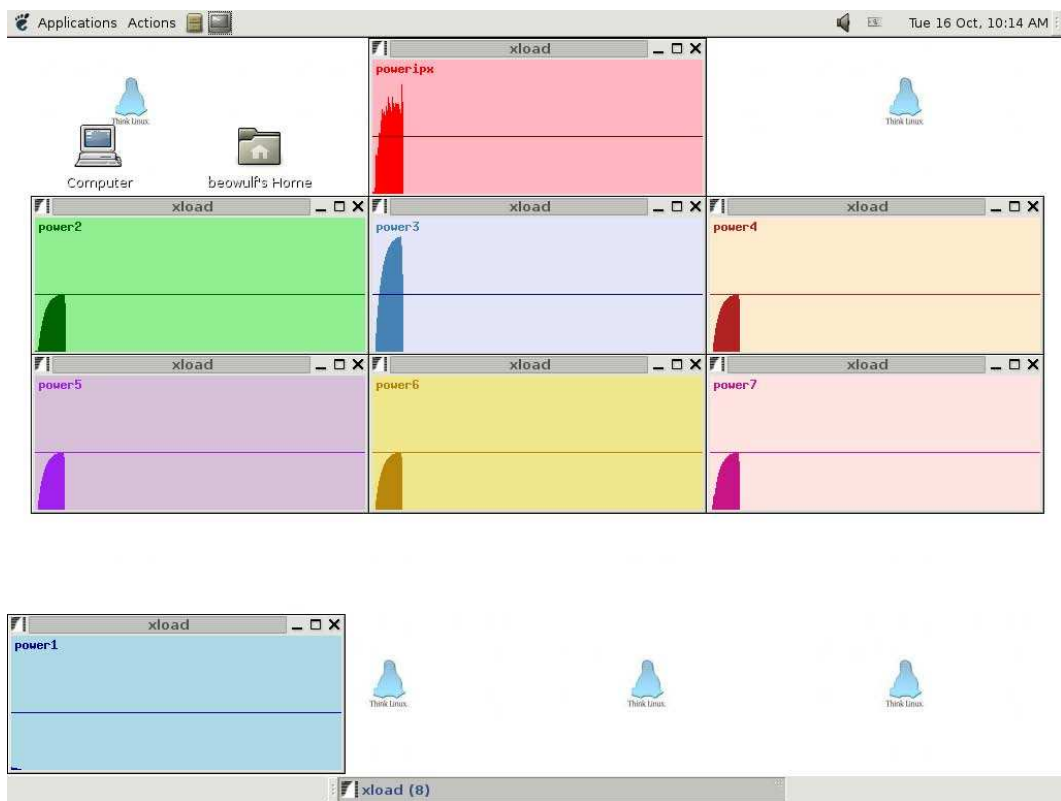
Figure 3.7: Screen shot after completion of the decoupled AC ESCOPF for IEEE30 BUS case (same time scale as that of Figure3.6)

3.4.2 Time Comparison between Integrated and Decoupled ESCOPF methods

Parallel computing can reduce the time to solve a problem but it adds overhead due to the communication involved among the machines. Therefore, for very small problems, which are less computationally intensive, the parallel implementation takes longer than the integrated version of the algorithm, since the computational savings are wiped out by the communication overhead required by the use of multiple processors. But as the size of the computational intensity increases, the communication overhead becomes a small part of the solutions time, so the significant speedup due to parallelism becomes apparent. Integrated (AC & DC) ESCOPF and decoupled (AC & DC) ESCOPF were solved for IEEE 5, 14 and 30 bus cases. Table 3.2 shows the average time taken to solve different test cases. For each test case, Figure 3.8 shows the average solution time versus the size of Zo, which is a vector of variables to be found for the base-case. It can be noticed that in case of 5 and 14 bus DC ESCOPF cases, the integrated solution time is less than the decoupled solution time. Moreover, for 5 bus AC ESCOPF we see the same situation. But for rest of the three cases the cluster takes less time to solve the problem. Therefore, as the computational intensity increases, the time saved by the cluster increases.

It may seem that the 30 bus DC case is more complex than the 14 bus AC case, but actually it is not. The AC versions of the problem not only has non-linear constraints

Case Type	Using Integrated Method	Using Decoupled Method
5 Bus DC-ESCOPF	6.01	29.23
5 Bus AC-ESCOPF	18.95	35.24
14 Bus DC-ESCOPF	15.91	33.53
14 Bus AC-ESCOPF	125.35	52.99
30 Bus DC-ESCOPF	47.49	41.71
30 Bus AC-ESCOPF	1904.72	252.84

Table 3.2: Average time (Seconds) taken to solve different test cases.
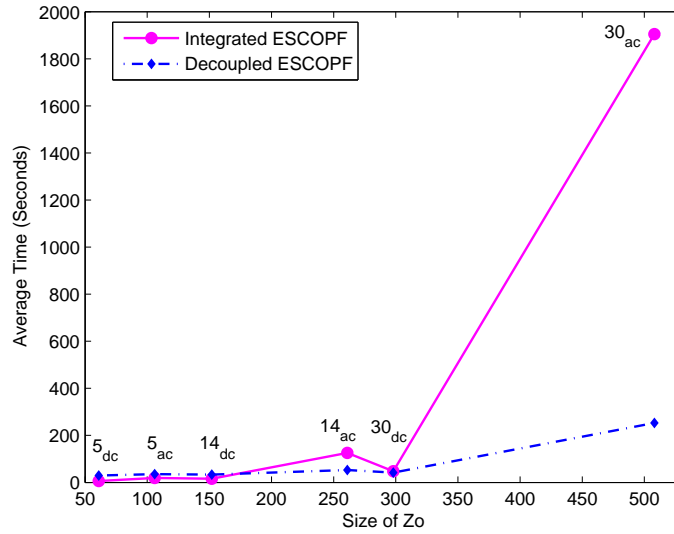
Figure 3.8: Time Comparison (Time versus size of Zo)

but also more constraints than in the DC case, since in the DC case we ignore the voltage and reactive power limits. Due to the non-linear constraints, the Jacobian of equality and inequality constraints are complex to compute in AC case as compared to that in the DC case. In the DC case the elements of the Jacobians are zeros or ones or imaginary part of Y-bus matrix. Moreover, in case of DC there are no Hessians but in case of AC version, we must compute the Hessian matrices of the non-linear constraints. Therefore, the 14 bus AC case is computationally intensive as compared to the 30 bus case.

Another reason that the cluster takes more time than a single machine for smaller cases is the excessive overhead for communication in MatlabMPI for small message

Case Type	Largest Message Size (MB)
5 Bus DC-ESCOPF	0.035
5 Bus AC-ESCOPF	0.057
14 Bus DC-ESCOPF	0.085
14 Bus AC-ESCOPF	0.137
30 Bus DC-ESCOPF	0.163
30 Bus AC-ESCOPF	0.264

Table 3.3: Message sizes for different test cases.

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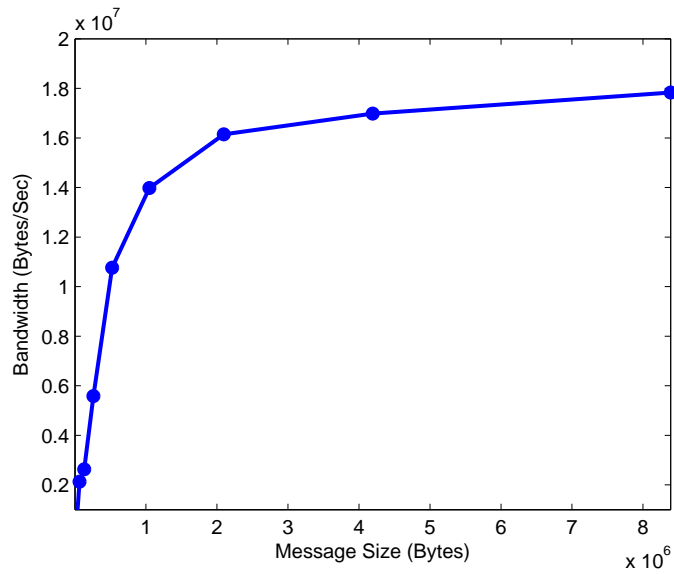
Figure 3.9: Communication Performance of OSU Power Beowulf Cluster

sizes as shown by Figure 3.4. As the size of the problem to be solved grows, the size of the messages communicated between the machines of cluster also increases. Table 3.3 shows the approximate size of messages for different cases. The communication performance of the OSU power cluster is shown in Figure 3.9.

CHAPTER 4

DESCRIPTION OF RESULTS

4.1 Implementation

To solve the decoupled ESCOPF problem several Matlab functions were written. These functions performed the following major tasks:

- Read data from the ASCII text files

- Assign Matlab variable names to data

- Create indices for decision variables, Lagrange multipliers, primal and dual variables

- Initialize the Matlab variables

- Execute the primal-dual interior-point algorithm to solve ESCOPF problem

- Communicate data among different processes for parallel processing

Power system data is present in ASCII text files .cdf, .gdf, and .ldf (see Appendix A.4). In this chapter the results for 5 bus case (AC & DC) have been described. These results have been obtained by using the decoupled ESCOPF solution method. Dr. Damrongkulkamjorn [11] obtained the same results using integrated ESCOPF solution method.

	$P_{Gi_{\min}}$	$P_{Gi_{\max}}$	$\Delta_{\max}^{+}P_{Gi}$	$\Delta_{\max}^{-}P_{Gi}$
Gen 1	45 MW	250 MW	50 MW	50 MW
Gen 2	15 MW	150 MW	35 MW	35 MW

Table 4.1: Characteristics of generators in 5-bus system in pre-contingency state.

4.2 Description of the results of DC ESCOPF for 5-bus system

The DC ESCOPF model formulated in Chapter 3 was tested on a 5-bus power system, which is modified from the IEEE 14-bus system by removing the low voltage buses 6 to 14 and associated transmission lines. Please see Appendix A.4 and A.5 for the data. The characteristics of the 5-bus system are shown by figure 4.1.

This power system consists of two generators connected to bus 1 and bus 2. A synchronous condenser, which supplies only reactive power, is connected to bus 3. Buses 2, 3, 4, and, 5 are load buses. C_{Gi} is the cost curve of generator i, B_{Li} is the benefit curve of consumer load at bus i, and P_{ijmax} is the maximum capacity of line ij. $P_{Gi_{max}}$ and $P_{Gi_{min}}$ are the maximum and minimum capacity limits of generator i, respectively. These capacity limits are given in Table 4.1. When a contingency occurs, flow of power in each transmission line is allowed to slightly exceed its capacity limit for a short time. The flow limit after the occurrence contingency, called *emergency limit*, is shown in parentheses next to $P_{Gi_{max}}$ in Figure 4.1. Maximum ramp up capacity $\Delta_{max}^{+}P_{Gi}$ and maximum ramp down capacity $\Delta_{max}^{-}P_{Gi}$ is given in Table 4.1. C_{Ii} is the interruption cost of load i. Probability of line outage contingency for line ij is represented by π_{ij}. The seven contingencies and the corresponding probabilities of occurrence are given in Table 4.2.

4.2.1 Optimized pre-contingency state

Figure 4.2 shows the solution, for the normal operation, for the IEEE 5 bus system using decoupled ESCOPF method. This case was solved using the "DC" approxi-

Contingency	Lines	π^k
1	1 - 2	0.01
2	1 - 5	0.01
3	2 - 3	0.01
4	2 - 4	0.01
5	2 - 5	0.01
6	3 - 4	0.01
7	4 - 5	0.01

Table 4.2: Contingencies of 5-bus system and their probabilities.

mation. Marginal costs[1] for generators and marginal benefits[2] for loads are shown. At each bus, λ_i, *bus price*, is shown, which is also the Lagrange multiplier for the real power balance equation for the ith bus. Detailed solution can be seen in Appendix B.1. For brevity, the constraints which pertain only to the case subproblem (*i.e.* not coupling) will be called *local constraints*. If the local constraints are binding, the text will be shown as blue and if the *coupling constraints* are binding, the text will be shown as red. The text is shown as purple if both types of constraints are binding.

Generator at bus 1 provides 146.44 MW and generator at bus 2 provides 115.0 MW. The generation capacity limits for generator at bus 1 are: $P_{G1_{max}} = 250$ MW and $P_{G1_{min}} = 45$ MW (see Table 4.1). After the occurrence of a contingency, generator-1 may not be able to enjoy these capacity limits because of the ramp up and ramp down constraints. The capacity limits for generator-1 in the post-contingency state are: $P_{G1_{max}} = 146.44 + 50 = 196.44$ MW and $P_{G1_{min}} = 146.44 - 50 = 96.44$ MW. Similarly the capacity limits for generator-2 in post-contingency state are: $P_{G2_{max}} = 115 + 35 = 150$ MW and $P_{G2_{min}} = 115.44 - 35 = 80.44$ MW, while the nameplate capacity limits for generator-2 are : $P_{G2_{max}} = 150$ MW and $P_{G2_{min}} = 15$ MW. So in the post-contingency state generator-2 is able to use its maximum generation capacity

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[1]Marginal cost is the change in total cost when the power generator is changed by one unit *i.e* $MC(P_{Gi}) = \frac{dC(P_{Gi})}{dP_{Gi}}$.

[2]Marginal benefit is the change in total benefit when the power consumed is changed by one unit *i.e* $MB(P_{Li}) = \frac{dB(P_{Li})}{dP_{Gi}}$.

of 150 MW, but cannot go as low as 15 MW.

4.2.2 Post-contingency state with line 1-2 out

In the normal state, Figure 4.2, transmission line connecting bus-1 and bus-2 was the busiest line with $P_{ik} = 85.12$ MW. Loss of such a line usually forces adjacent lines to reach their maximum capacity limits. This is what happens in this case. The generator at bus-1 has to ramp down by $146.44 - 110 = 36.44$ MW because the emergency power carrying capacity of the only line (between bus-1 and bus-5) connected to generator is 110 MW. So generator-1 will provide 110 MW. Note that the total load being served in the normal state was 261.44 MW. Since 110 MW is provided by generator-1 so generator-2 is now responsible for the rest of the load which is 151.44 MW. However, generator-2 cannot provide more then 150 MW, as we discussed in section 4.2.1. So we have to interrupt 1.44 MW of the load. The Load at bus-2 is interrupted by 0.29 MW and the load at bus-5 is interrupted by 1.15 MW.

There are five constraints which bind the optimal solution for this problem. These constraints are listed in the Matlab-generated solution (Appendix B.1), with their corresponding multipliers[3]. The first constraint is the capacity limit for the line connecting bus-1 and bus-5, as discussed above. The multiplier for this constraint is 101.22, which shows that if one more MW is allowed to flow in the transmission line, the objective function of the post-contingency constraints would improve by 101.22 \$/h. The second constraint is generator maximum power generation limit. The corresponding multiplier for this constraint is 10.42, which shows that the optimal objective function would improve by 10.42 \$/h, if the generator-2 were to generate one more MW in the post contingency state. Third constraint is due to the ramp up capacity of the generator-2. The corresponding multiplier for this constraint is (89.74). This shows that if the ramp up limit of generator is increased by one MW, then the objective

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[3]A constraint multiplier shows the improvement in objective function, if constraint were to be relaxed by one unit

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function would improve by (89.74) \$/h. A comparison of these multipliers shows that the increase in transmission capacity is worth more than increasing the generator's maximum capacity and its ramp up capacity by the same amount. Likewise, increasing the generator's ramp up capacity is worth more that increasing its maximum generation capacity.

4.2.3   Post-contingency state with line 1-5 out

After losing transmission line connecting bus-1 and bus-5, we are left with only one transmission line, connected with generator-1, which is line 1-2 as shown in Figure 4.4. The emergency limit of line 1-2 is 155 MW. So generator-1 is ramped up to 155 MW. Generator-1 can be ramped up to 196.44 MW and has maximum capacity limit of 250 MW. Therefore, generator-1 is neither hit by ramp up limit nor hit by maximum capacity limit. Pre-contingency load was 261.44 MW. Generator-2 is ramped down from 115 MW to 106.44 MW. Neither generator-1 nor generator-2 is hit by any binding constraint. Therefore, the marginal costs of these generators are equal to their respective bus prices.
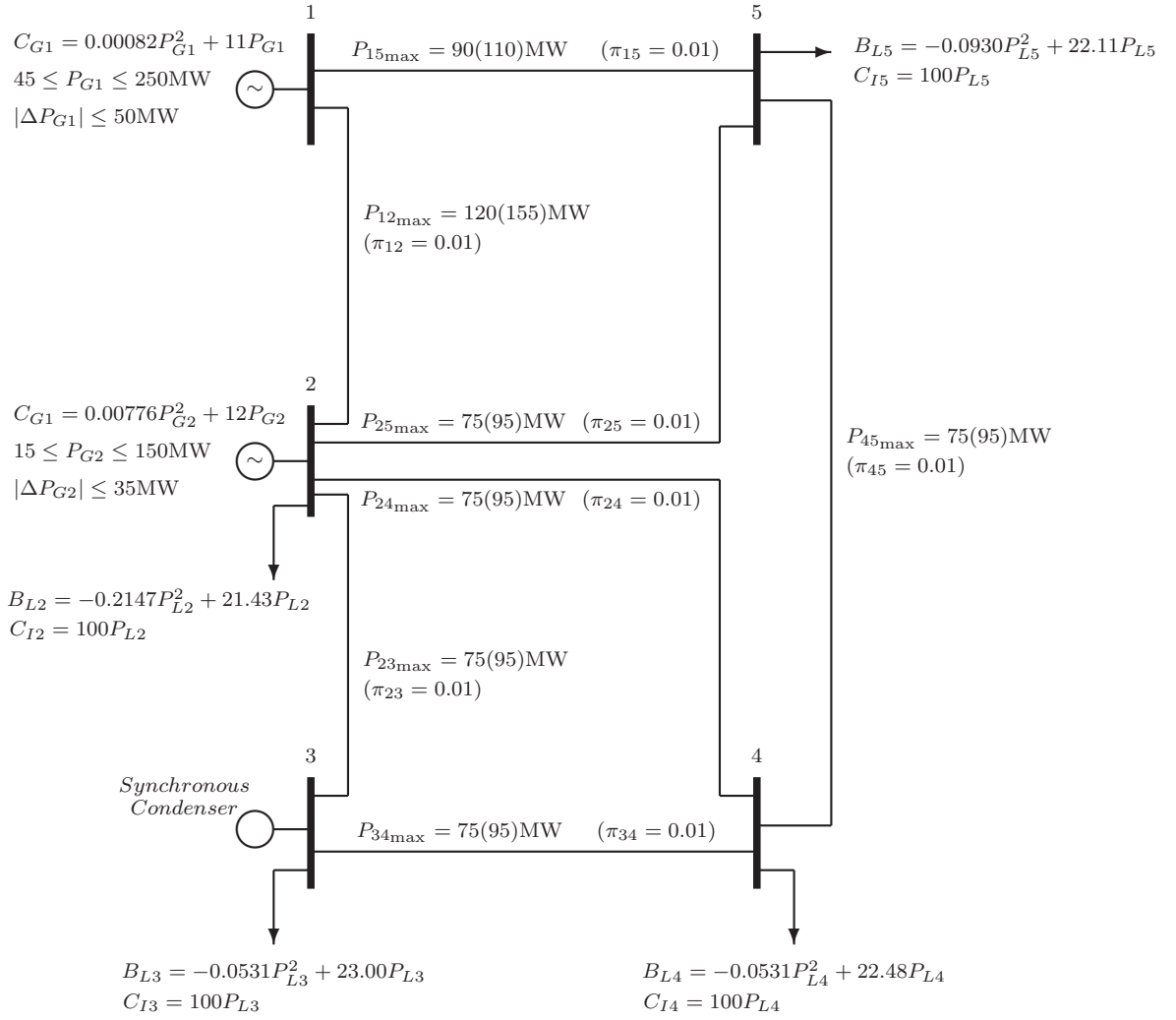
Figure 4.1: Characteristics of 5-bus system.

Total cost = -1577.07 $/hr
Total power generation = 261.44 MW/hr
Total load consumption = 261.44 MW/hr

1

5

$P_{15} = 61.31$ MW

$\theta_5 = -8.295°$
$\lambda_5 = 11.240$ $/MWh
$P_{L5} = 53.32$ MW
$MB_{L5} = 12.191$ $/MWh

$\theta_1 = 0°$
$P_{G1} = 146.44$ MW
$MC_{G1} = 11.240$ $/MWh
$\lambda_1 = 11.240$ $/MWh

$P_{12} = 85.12$ MW

2

$P_{25} = 46.23$ MW

$P_{54} = 54.21$ MW

$\theta_2 = -3.196°$
$P_{G2} = 115.0$ MW
$MC_{G2} = 12.178$ $/MWh
$\lambda_2 = 11.240$ $/MWh

$P_{24} = 58.38$ MW

$P_{L2} = 21.308$ MW
$MB_{L2} = 12.281$ $/MWh

$P_{23} = 74.20$ MW

Synch.
Cond.

3

4

$\theta_3 = -12.086°$
$\lambda_3 = 11.240$$/MWh

$P_{43} = 20.79$ MW

$\theta_4 = -9.735°$
$\lambda_4 = 11.240$ $/MWh
$P_{L4} = 91.809$ MW
$MB_{L4} = 12.730$ $/MWh

$P_{L3} = 95$ MW
$MB_{L3} = 12.911$$/MWh

Figure 4.2: Solution for pre-contingency state ("DC" case)

Total cost = -1396.40 $/hr
Total power generation = 260.00 MW/hr
Total load consumption = 260.00 MW/hr

1   5

$P_{15} = 110.00$ MW $\theta_5 = -14.882°$
                                                    $\lambda_5 = 112.405$ $/MWh
$\theta_1 = 0°$ $P_{L5} = 52.175$ MW
$P_{G1} = 110.00$ MW $MB_{L5} = 12.405$ $/MWh
$MC_{G1} = 11.18$ $/MWh
$\lambda_1 = 11.18$ $/MWh

2

$P_{25} = 22.20$ MW

$\theta_2 = -12.426°$
$P_{G2} = 150.00$ MW
$MC_{G2} = 12.233$ $/MWh $P_{24} = 40.91$ MW $P_{54} = 80.09$ MW
$\lambda_2 = 112.405$ $/MWh

$P_{L2} = 21.016$ MW
$MB_{L2} = 12.406$ $/MWh

$P_{23} = 65.79$ MW

Synch.   3   4
Cond.

$\theta_3 = -20.310°$ $P_{43} = 29.19$ MW
$\lambda_3 = 112.405$$/MWh

                                                    $\theta_4 = -17.00°$
                                                    $\lambda_4 = 112.405$ $/MWh
                                                    $P_{L4} = 91.809$ MW
$P_{L3} = 95.00$ MW $MB_{L4} = 12.732$ $/MWh
$MB_{L3} = 12.911$$/MWh

Figure 4.3: Solution for post-contingency state with line 1-2 gone

Total cost = -1584.98 $/hr
Total power generation = 261.44 MW/hr
Total load consumption = 261.44 MW/hr

1

5

$\theta_1 = 0°$
$P_{G1} = 155.00$ MW
$MC_{G1} = 11.25$ $/MWh
$\lambda_1 = 11.25$ $/MWh

$\theta_5 = -14.171°$
$\lambda_5 = 112.165$ $/MWh
$P_{L5} = 53.326$ MW
$MB_{L5} = 12.191$ $/MWh

$P_{12} = 155.00$MW

2

$P_{25} = 75.71$ MW

$\theta_2 = -5.819°$
$P_{G2} = 106.44$ MW
$MC_{G2} = 12.165$ $/MWh
$\lambda_2 = 12.165$ $/MWh

$P_{24} = 79.88$ MW

$P_{54} = 22.38$ MW

$P_{L2} = 21.308$ MW
$MB_{L2} = 12.28$ $/MWh

$P_{23} = 84.53$ MW

Synch.
Cond.

3

4

$\theta_3 = -15.948°$
$\lambda_3 = 12.165$$/MWh

$P_{43} = 10.46$ MW

$\theta_4 = -14.765°$
$\lambda_4 = 12.165$ $/MWh
$P_{L4} = 91.809$ MW
$MB_{L4} = 12.730$ $/MWh

$P_{L3} = 95.00$ MW
$MB_{L3} = 12.911$$/MWh

Figure 4.4: Solution for post-contingency state with line 1-5 gone
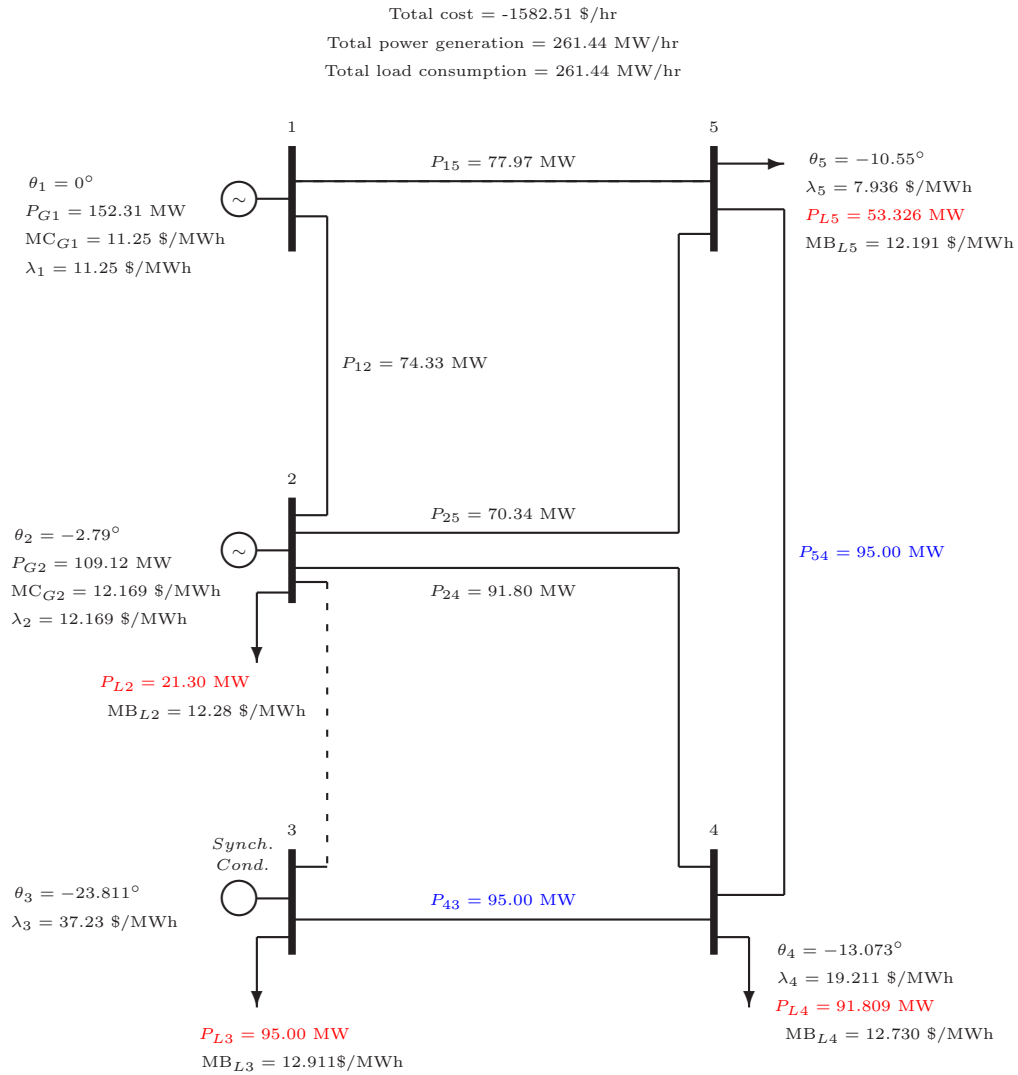
Figure 4.5: Solution for post-contingency state with line 2-3 gone

4.2.4 Post-contingency state with line 2-3 out

Figure 4.5 shows the optimal solution when transmission line 2-3 is lost. As a result of this contingency two transmission lines, 4-3 and 5-4 hit the maximum capacity limit. We do not have to interrupt any load in this case.
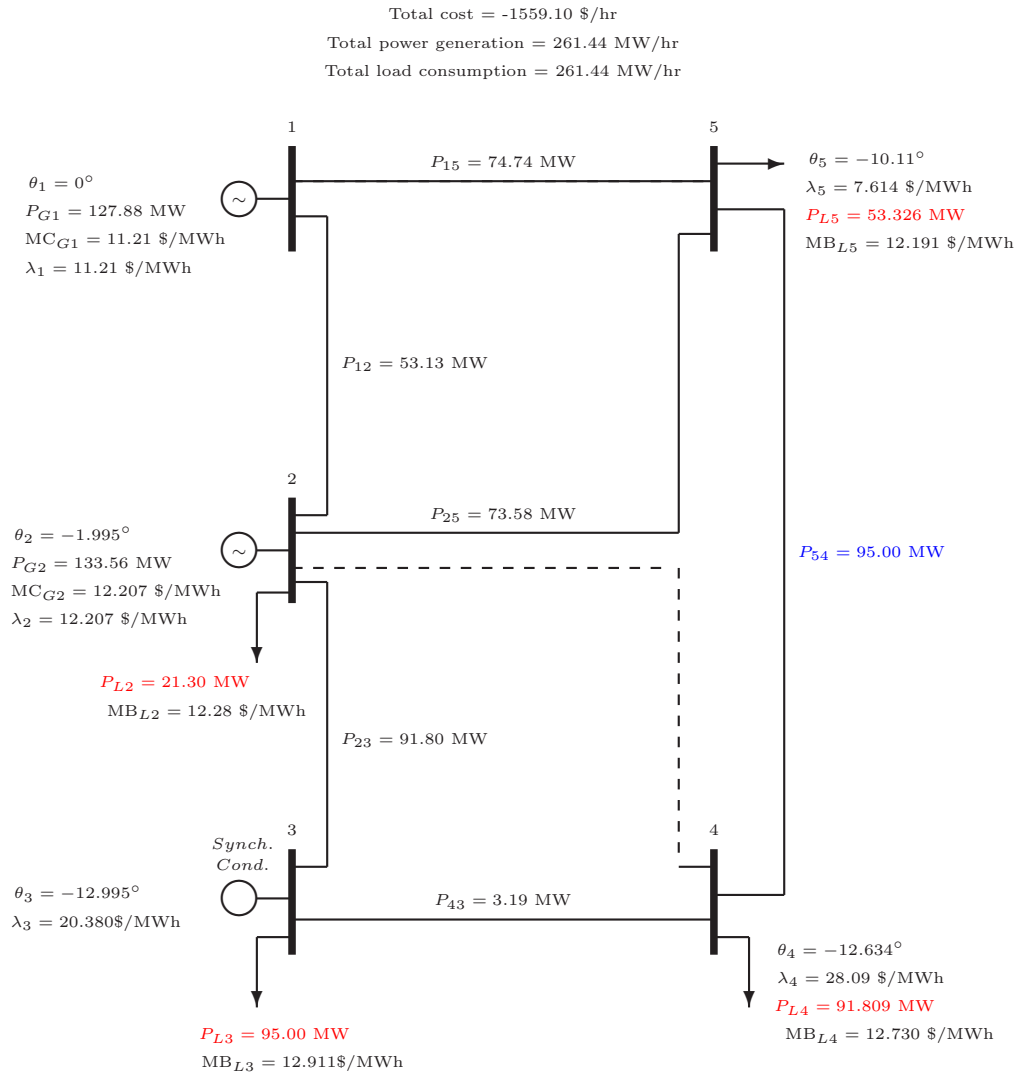
Figure 4.6: Solution for post-contingency state with line 2-4 gone

4.2.5 Post-contingency state with line 2-4 out

Figure 4.6 shows the optimal solution when transmission line 2-4 is lost. As a result of this contingency, transmission line 5-4 hits the maximum capacity limit. No load interruption occurs as a result of this contingency.
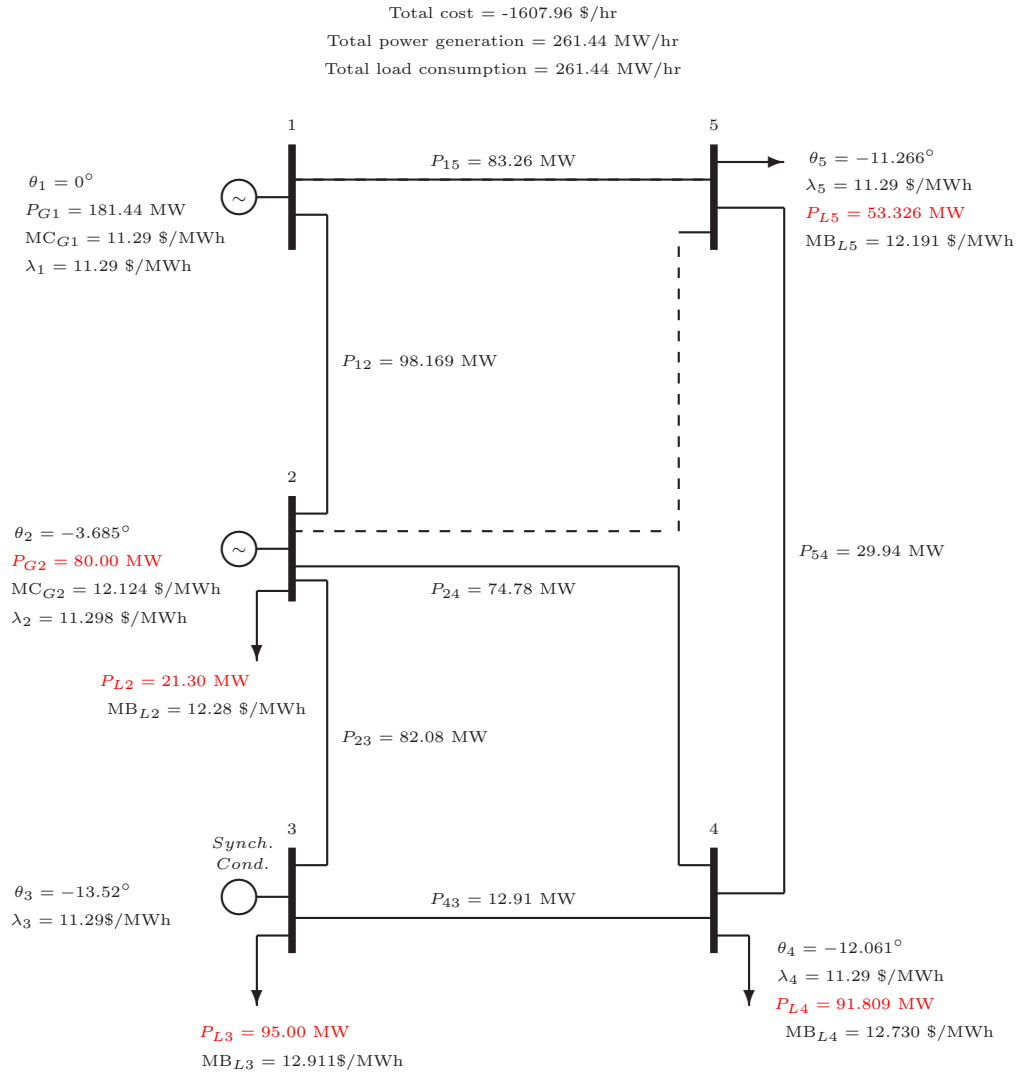
Figure 4.7: Solution for post-contingency state with line 2-5 gone

4.2.6 Post-contingency state with line 2-5 out

Figure 4.7 shows the optimal solution when transmission line 2-5, which is directly connected to generator-2, is lost. Generator-2 is ramped down and generator-1 is ramped up. Since ramp down limit of generator-2 hit, its marginal cost is different from the bus price at bus-2. The difference is the Lagrange multiplier.

In this case the power generated by generator-2 is 80 MW. We also noticed that when line 1-2 is lost, we had to ramp up generator-2 to 150 MW. This means that the 1-2 line outage contingency drives the post-contingency P_{G2} to its maximum, which in turn drives the pre-contingency P_{G2} to 115 MW via the ramp up constraint. At the same time, the 2-5 line loss case has P_{G2} pulled up to 80 MW by the ramp down constraint. These two contingencies influence P_{G2}^0 in opposite directions, with the influence of the 1-2 line outage being dominant.
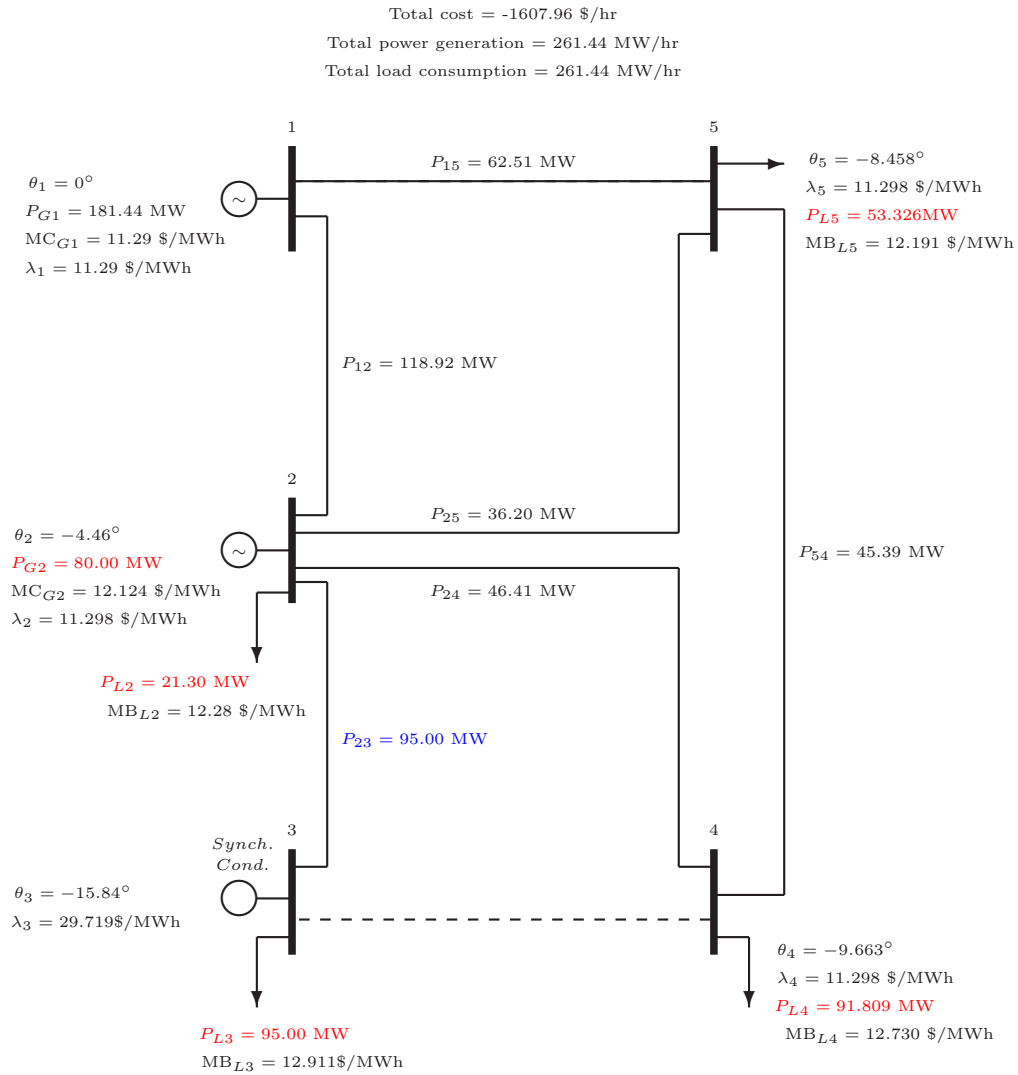
Figure 4.8: Solution for post-contingency state with line 3-4 gone

4.2.7 Post-contingency state with line 3-4 out

After losing transmission line 3-4, the only line supplying power to the load at bus-3 is the line 2-3, which has a maximum capacity of 95 MW. To transmit 95 MW of power through the line 2-3, generator-2 is ramped down and generator-1 is ramped up. The maximum load that can be served at this bus is 95 MW. To avoid the high cost of interruption, the base case load at bus is also set to 95 MW, which then forces all contingencies to have $P_{L3} = 95$ MW, through the coupling constraint $P_{Li}^3 \leq P_{Li}^0$.
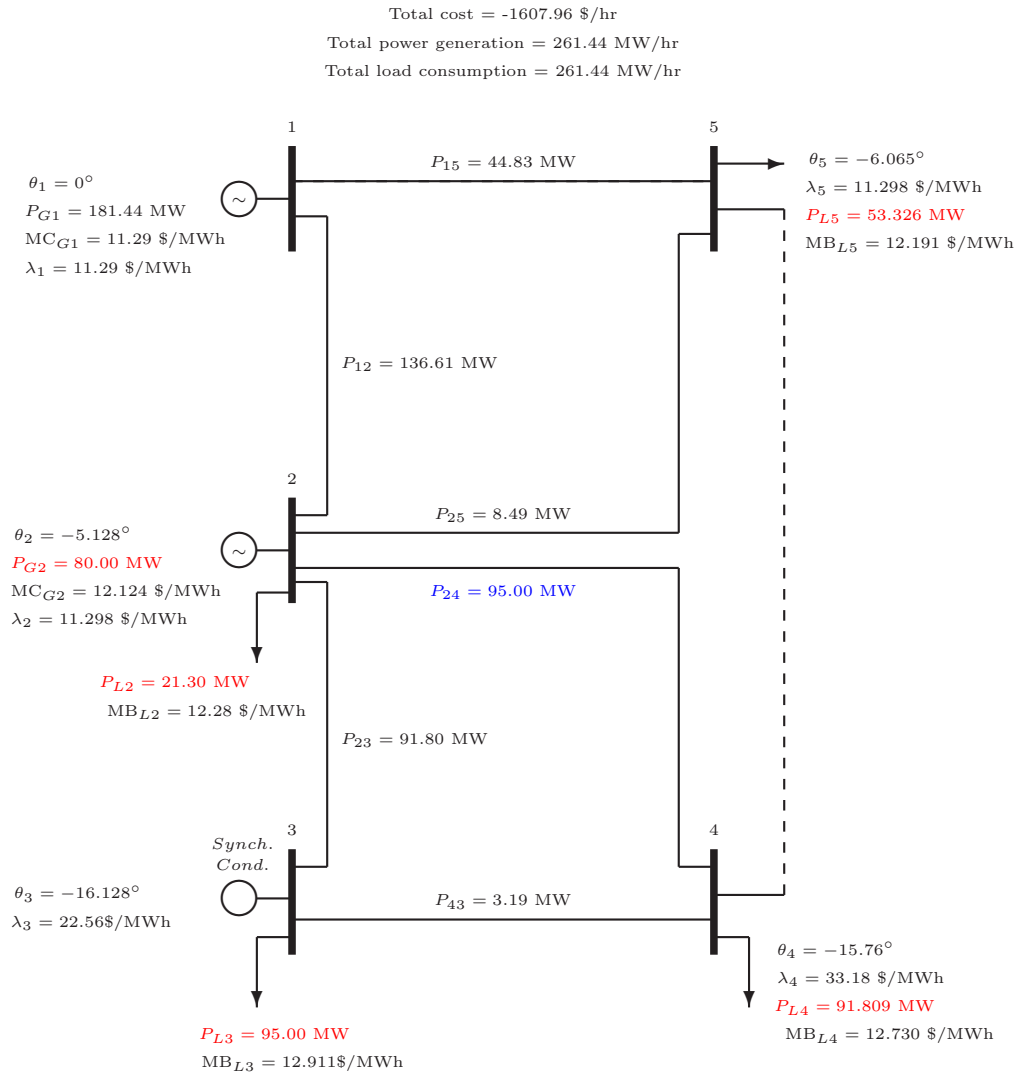
Figure 4.9: Solution for post-contingency state with line 4-5 gone

4.2.8 Post-contingency state with line 4-5 out

Figure 4.9 shows that the transmission line 2-4 is fully loaded as a result of this contingency.

4.3 Description of the results of AC ESCOPF for 5-bus system

The AC ESCOPF model formulated in Chapter 3 was tested on a 5-bus power system. The reactive power is not ignored as was done in the "DC" case. The real and reactive power capacity limits are shown in Table 4.3. In the "AC" case, the power flow equations are nonlinear. Voltage limits are also incorporated. Line resistances are not ignored, therefore, we have real power losses in the transmission lines. The flow from bus i to bus j is different from the flow from bus j to bus i because of the line charging susceptance of the transmission lines.

	$P_{Gi_{\min}}$	$P_{Gi_{\max}}$	$Q_{Gi_{\min}}$	$Q_{Gi_{\max}}$	$\Delta_{\max}^{+}P_{Gi}$	$\Delta_{\max}^{-}P_{Gi}$
Gen 1	45 MW	250 MW	-100 MVar	150 MVar	50 MW	50 MW
Gen 2	15 MW	150 MW	-40 MVar	50 MVar	35 MW	35 MW
SynCon 3	0 MW	0 MW	-40 MVar	40 MVar	0 MW	0 MW

Table 4.3: Characteristics of generators in 5-bus system at pre-contingency state.

4.3.1 Optimized pre-contingency state and post-contingency states

Figure 4.10 shows the solution, for the normal operation, for the IEEE 5 bus system using decoupled ESCOPF method. This case was solved using the "AC" approximation. Figure 4.11 — Figure 4.17 show the post-contingency solutions.

The results of the AC ESCOPF and DC ESCOPF are slightly different. For example in the case of contingency-1, the solution of DC ESCOPF interrupts some of the loads, but the solution of AC ESCOPF does not interrupt any load. This happens because the load consumption in the AC case is less, compared to that in the DC case, because of the inclusion of the real power losses.

In the DC case, when the transmission line 1-2 is lost, both local and coupling constraints are binding on bus 2 real generation, whereas only the coupling constraint is binding in the AC ESCOPF solution (although P_{G2} is very close to its local limit). In DC analysis there are no voltage constraints. In AC analysis we find at least one binding voltage constraint in each contingency, preventing system voltages from becoming unreasonably high as is typical in ACOPF.

Total cost = -1466.9 $/hr
Total power generation = 259.8 MW/hr
Total load consumption = 251.0 MW/hr

$I_{51} = 0.567$ pu
$I_{15} = 0.563$ pu

$\theta_1 = 0°$
$P_{G1} = 151.72$ MW
$MC_{G1} = 11.249$ $/MWh
$\lambda_1 = 11.249$ $/MWh
$Q_{G1} = -15.238$MVar
$V_1 = 1.080$ pu

$\theta_5 = -6.694°$
$\lambda_5 = 12.065$ $/MWh
$V_5 = 1.041$ pu
$P_{L5} = 50.561$ MW
$MB_{L5} = 12.706$ $/MWh

$I_{12} = 0.864$ pu
$I_{21} = 0.852$ pu

$I_{52} = 0.433$ pu
$I_{25} = 0.429$ pu

$\theta_2 = -2.827°$
$P_{G2} = 108.124$ MW
$MC_{G2} = 12.168$ $/MWh
$\lambda_2 = 11.589$ $/MWh
$Q_{G2} = 35.825$MVar
$V_2 = 1.074$ pu

$I_{54} = 0.510$ pu
$I_{45} = 0.509$ pu

$I_{24} = 0.543$ pu
$I_{42} = 0.547$ pu

$P_{L2} = 21.565$ MW
$MB_{L2} = 12.170$ $/MWh

$I_{23} = 0.673$ pu
$I_{32} = 0.672$ pu

Synch. Cond.

$\theta_3 = -10.058°$
$P_{G3} = 0$ MW
$MC_{G3} = 0$$/MWh
$\lambda_3 = 12.434$$/MWh
$Q_{G3} = 31.683$MVar
$V_3 = 1.043$ pu

$I_{34} = 0.209$ pu
$I_{43} = 0.230$ pu

$\theta_4 = -7.886°$
$\lambda_4 = 12.231$ $/MWh
$V_4 = 1.034$ pu
$P_{L4} = 90.11$ MW
$MB_{L4} = 12.91$ $/MWh

$P_{L3} = 88.84$ MW
$MB_{L3} = 13.564$$/MWh

Figure 4.10: Solution for pre-contingency state ("AC" case)

87

Total cost = -1409.58 \$/hr
Total power generation = 261.9 MW/hr
Total load consumption = 251.0 MW/hr

1

$I_{51} = 1.095$ pu
$I_{15} = 1.100$ pu

5

$\theta_1 = 0°$
$P_{G1} = 118.798$ MW
$MC_{G1} = 11.195$ \$/MWh
$\lambda_1 = 11.195$ \$/MWh
$Q_{G1} = 0.634$MVar
$V_1 = 1.080$ pu

$\theta_5 = -13.516°$
$\lambda_5 = 68.471$ \$/MWh
$V_5 = 1.042$ pu
$P_{L5} = 50.561$ MW
$MB_{L5} = 12.706$ \$/MWh

2

$I_{52} = 0.252$ pu
$I_{25} = 0.228$ pu

$\theta_2 = -12.306°$
$P_{G2} = 143.124$ MW
$MC_{G2} = 12.222$ \$/MWh
$\lambda_2 = 67.409$ \$/MWh
$Q_{G2} = 34.413$MVar
$V_2 = 1.080$ pu

$I_{54} = 0.784$ pu
$I_{45} = 0.782$ pu

$I_{24} = 0.377$ pu
$I_{42} = 0.390$ pu

$P_{L2} = 21.565$ MW
$MB_{L2} = 12.170$ \$/MWh

$I_{23} = 0.585$ pu
$I_{32} = 0.587$ pu

3

Synch.
Cond.

4

$I_{34} = 0.305$ pu
$I_{43} = 0.325$ pu

$\theta_3 = -18.522°$
$P_{G3} = 0$ MW
$MC_{G3} = 0$\$/MWh
$\lambda_3 = 71.774$\$/MWh
$Q_{G3} = 33.991$MVar
$V_3 = 1.049$ pu

$\theta_4 = -15.405°$
$\lambda_4 = 69.877$ \$/MWh
$V_4 = 1.037$ pu
$P_{L4} = 90.11$ MW
$MB_{L4} = 12.91$ \$/MWh

$P_{L3} = 88.84$ MW
$MB_{L3} = 13.564$\$/MWh

Figure 4.11: Solution for post-contingency state with line 1-2 gone

88

Total cost = -1414.22 $/hr

Total power generation = 265.2 MW/hr

Total load consumption = 251.0 MW/hr

1   5

$\theta_1 = 0°$

$P_{G1} = 166.65$ MW

$MC_{G1} = 11.273$ $/MWh

$\lambda_1 = 11.273$ $/MWh

$Q_{G1} = -15.772$MVar

$V_1 = 1.080$ pu

$\theta_5 = -11.723°$

$\lambda_5 = 13.17$ $/MWh

$V_5 = 1.007$ pu

$P_{L5} = 50.561$ MW

$MB_{L5} = 12.706$ $/MWh

$I_{12} = 1.550$ pu

$I_{21} = 1.543$ pu

2

$I_{52} = 0.722$ pu

$I_{25} = 0.718$ pu

$\theta_2 = -5.059°$

$P_{G2} = 98.635$ MW

$MC_{G2} = 12.168$ $/MWh

$\lambda_2 = 12.153$ $/MWh

$Q_{G2} = 49.99$MVar

$V_2 = 1.061$ pu

$I_{24} = 0.759$ pu

$I_{42} = 0.762$ pu

$I_{54} = 0.223$ pu

$I_{45} = 0.220$ pu

$P_{L2} = 21.565$ MW

$MB_{L2} = 12.170$ $/MWh

$I_{23} = 0.788$ pu

$I_{32} = 0.787$ pu

Synch. Cond.   3   4

$I_{34} = 0.170$ pu

$I_{43} = 0.202$ pu

$\theta_3 = -13.693°$

$P_{G3} = 0$ MW

$MC_{G3} = 0$$/MWh

$\lambda_3 = 13.255$$/MWh

$Q_{G3} = 39.99$MVar

$V_3 = 1.029$ pu

$\theta_4 = -12.278°$

$\lambda_4 = 13.238$ $/MWh

$V_4 = 1.006$ pu

$P_{L4} = 90.11$ MW

$MB_{L4} = 12.91$ $/MWh

$P_{L3} = 88.84$ MW

$MB_{L3} = 13.564$$/MWh

Figure 4.12: Solution for post-contingency state with line 1-5 gone
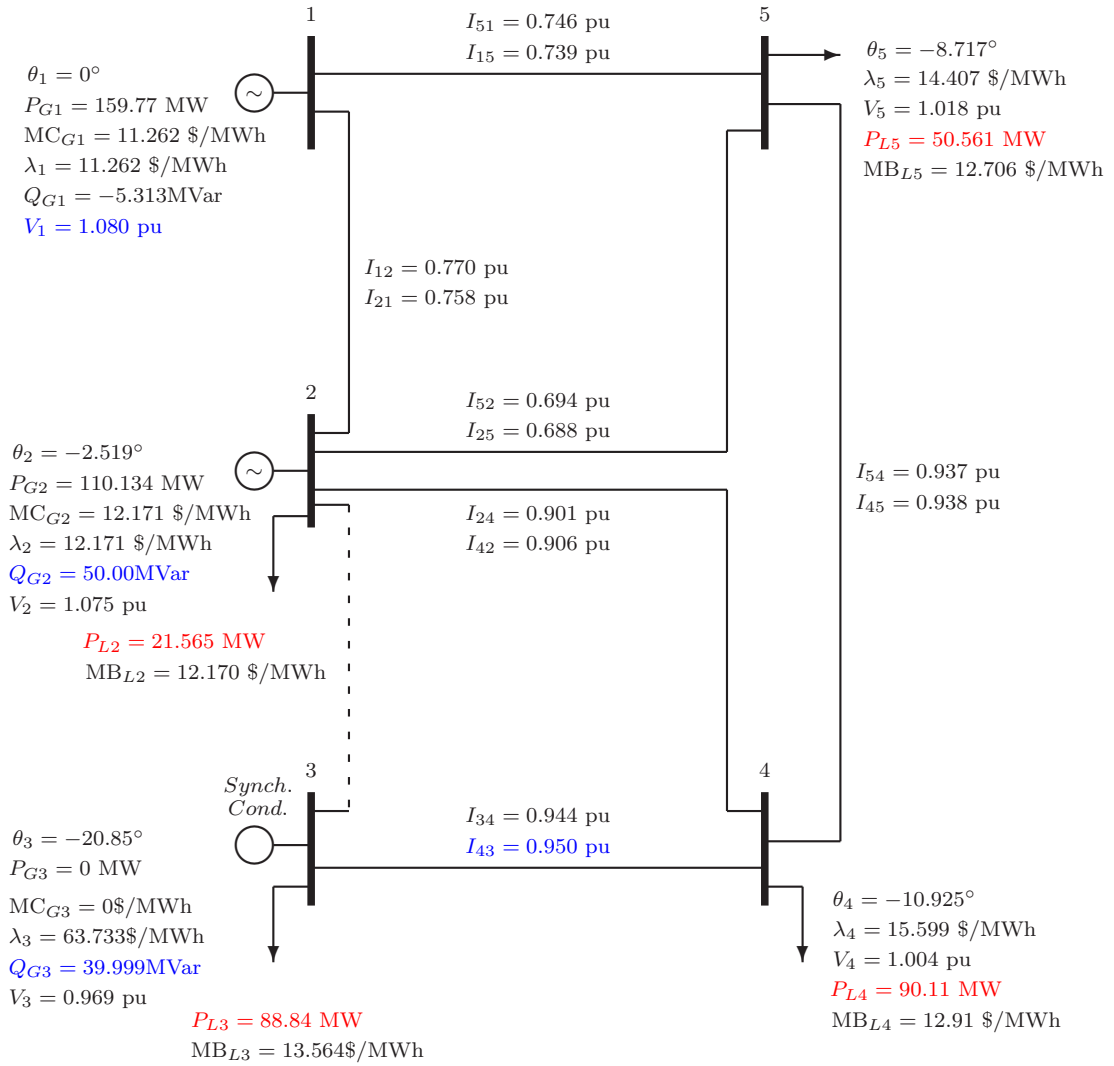
Figure 4.13: Solution for post-contingency state with line 2-3 gone

Total cost = -1457.85 \$/hr
Total power generation = 263.32 MW/hr
Total load consumption = 251.08 MW/hr

$\theta_1 = 0°$
$P_{G1} = 190.196$ MW
$MC_{G1} = 11.312$ \$/MWh
$\lambda_1 = 11.312$ \$/MWh
$Q_{G1} = -12.343$MVar
$V_1 = 1.080$ pu

$I_{51} = 0.775$ pu
$I_{15} = 0.770$ pu

$\theta_5 = -9.172°$
$\lambda_5 = 12.453$ \$/MWh
$V_5 = 1.022$ pu
$P_{L5} = 50.561$ MW
$MB_{L5} = 12.706$ \$/MWh

$I_{12} = 1.022$ pu
$I_{21} = 1.010$ pu

$\theta_2 = -3.354°$
$P_{G2} = 73.124$ MW
$MC_{G2} = 12.113$ \$/MWh
$\lambda_2 = 11.724$ \$/MWh
$Q_{G2} = 42.84$MVar
$V_2 = 1.074$ pu

$I_{52} = 0.649$ pu
$I_{25} = 0.644$ pu

$I_{54} = 0.925$ pu
$I_{45} = 0.925$ pu

$P_{L2} = 21.565$ MW
$MB_{L2} = 12.170$ \$/MWh

$I_{23} = 0.8297$ pu
$I_{32} = 0.8291$ pu

$\theta_3 = -12.321°$
$P_{G3} = 0$ MW
$MC_{G3} = 0$\$/MWh
$\lambda_3 = 12.745$\$/MWh
$Q_{G3} = 39.972$MVar
$V_3 = 1.037$ pu

Synch.
Cond.

$I_{34} = 0.160$ pu
$I_{43} = 0.195$ pu

$\theta_4 = -11.370°$
$\lambda_4 = 12.763$ \$/MWh
$V_4 = 1.009$ pu
$P_{L4} = 90.11$ MW
$MB_{L4} = 12.91$ \$/MWh

$P_{L3} = 88.84$ MW
$MB_{L3} = 13.564$\$/MWh

Figure 4.14: Solution for post-contingency state with line 2-4 gone

Total cost = -1469.79 \$/hr

Total power generation = 262.2 MW/hr

Total load consumption = 251.0 MW/hr

1

$I_{51} = 0.777$ pu

$I_{15} = 0.772$ pu

5

$\theta_5 = -9.183°$

$\lambda_5 = 12.434$ \$/MWh

$V_5 = 1.020$ pu

$P_{L5} = 50.561$ MW

$MB_{L5} = 12.706$ \$/MWh

$\theta_1 = 0°$

$P_{G1} = 189.14$ MW

$MC_{G1} = 11.310$ \$/MWh

$\lambda_1 = 11.310$ \$/MWh

$Q_{G1} = -9.11$MVar

$V_1 = 1.080$ pu

$I_{12} = 1.006$ pu

$I_{21} = 0.995$ pu

2

$\theta_2 = -3.297°$

$P_{G2} = 73.124$ MW

$MC_{G2} = 12.113$ \$/MWh

$\lambda_2 = 11.718$ \$/MWh

$Q_{G2} = 37.12$MVar

$V_2 = 1.073$ pu

$P_{L2} = 21.565$ MW

$MB_{L2} = 12.170$ \$/MWh

$I_{24} = 0.706$ pu

$I_{42} = 0.710$ pu

$I_{54} = 0.287$ pu

$I_{45} = 0.284$ pu

$I_{23} = 0.756$ pu

$I_{32} = 0.755$ pu

Synch.
Cond.

3

$I_{34} = 0.177$ pu

$I_{43} = 0.207$ pu

4

$\theta_3 = -11.472°$

$P_{G3} = 0$ MW

$MC_{G3} = 0$\$/MWh

$\lambda_3 = 12.650$\$/MWh

$Q_{G3} = 38.04$MVar

$V_3 = 1.040$ pu

$\theta_4 = -9.888°$

$\lambda_4 = 12.552$ \$/MWh

$V_4 = 1.019$ pu

$P_{L4} = 90.11$ MW

$MB_{L4} = 12.91$ \$/MWh

$P_{L3} = 88.84$ MW

$MB_{L3} = 13.564$\$/MWh

Figure 4.15: Solution for post-contingency state with line 2-5 gone

Total cost = -1480.95 $/hr
Total power generation = 261.2 MW/hr
Total load consumption = 251.0 MW/hr

1

$I_{51} = 0.584$ pu
$I_{15} = 0.578$ pu

5

$\theta_1 = 0°$
$P_{G1} = 188.15$ MW
$MC_{G1} = 11.309$ $/MWh
$\lambda_1 = 11.309$ $/MWh
$Q_{G1} = -13.368$MVar
$V_1 = 1.080$ pu

$\theta_5 = -6.807°$
$\lambda_5 = 12.182$ $/MWh
$V_5 = 1.034$ pu
$P_{L5} = 50.561$ MW
$MB_{L5} = 12.706$ $/MWh

$I_{12} = 1.185$ pu
$I_{21} = 1.175$ pu

2

$I_{52} = 0.357$ pu
$I_{25} = 0.348$ pu

$\theta_2 = -3.881°$
$P_{G2} = 73.124$ MW
$MC_{G2} = 12.113$ $/MWh
$\lambda_2 = 11.785$ $/MWh
$Q_{G2} = 45.377$MVar
$V_2 = 1.070$ pu

$I_{54} = 0.439$ pu
$I_{45} = 0.441$ pu

$I_{24} = 0.447$ pu
$I_{42} = 0.456$ pu

$P_{L2} = 21.565$ MW
$MB_{L2} = 12.170$ $/MWh

$I_{23} = 0.863$ pu
$I_{32} = 0.859$ pu

Synch.
Cond.

3

4

$\theta_3 = -13.309°$
$P_{G3} = 0$ MW
$MC_{G3} = 0$$/MWh
$\lambda_3 = 12.752$$/MWh
$Q_{G3} = 30.415$MVar
$V_3 = 1.043$ pu

$\theta_4 = -7.780°$
$\lambda_4 = 12.34$ $/MWh
$V_4 = 1.025$ pu
$P_{L4} = 90.11$ MW
$MB_{L4} = 12.91$ $/MWh

$P_{L3} = 88.84$ MW
$MB_{L3} = 13.564$$/MWh

Figure 4.16: Solution for post-contingency state with line 3-4 gone

Total cost = -1448.34 $/hr
Total power generation = 264.1 MW/hr
Total load consumption = 251.0 MW/hr

1

$I_{51} = 0.418$ pu
$I_{15} = 0.414$ pu

5

$\theta_5 = -4.893°$
$\lambda_5 = 11.799$ $/MWh
$V_5 = 1.05$ pu
$P_{L5} = 50.561$ MW
$MB_{L5} = 12.706$ $/MWh

$\theta_1 = 0°$
$P_{G1} = 191.03$ MW
$MC_{G1} = 11.313$ $/MWh
$\lambda_1 = 11.313$ $/MWh
$Q_{G1} = -7.26$MVar
$V_1 = 1.080$ pu

$I_{12} = 1.357$ pu
$I_{21} = 1.353$ pu

2

$I_{52} = 0.857$ pu
$I_{25} = 0.067$ pu

$\theta_2 = -4.403°$
$P_{G2} = 73.124$ MW
$MC_{G2} = 12.113$ $/MWh
$\lambda_2 = 11.876$ $/MWh
$Q_{G2} = 35.432$MVar
$V_2 = 1.06$ pu

$I_{24} = 0.911$ pu
$I_{42} = 0.913$ pu

$P_{L2} = 21.565$MW
$MB_{L2} = 12.170$ $/MWh

$I_{23} = 0.863$ pu
$I_{32} = 0.862$ pu

Synch.
Cond.

3

4

$I_{34} = 0.143$ pu
$I_{43} = 0.178$ pu

$\theta_3 = -13.878°$
$P_{G3} = 0$ MW
$MC_{G3} = 0$$/MWh
$\lambda_3 = 13.021$$/MWh
$Q_{G3} = 39.976$MVar
$V_3 = 1.024$ pu

$\theta_4 = -13.181°$
$\lambda_4 = 13.123$ $/MWh
$V_4 = 0.997$ pu
$P_{L4} = 90.11$ MW
$MB_{L4} = 12.91$ $/MWh

$P_{L3} = 88.84$ MW
$MB_{L3} = 13.564$$/MWh

Figure 4.17: Solution for post-contingency state with line 4-5 gone

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CHAPTER 5

CONCLUSIONS AND FUTURE WORK

The security-constrained optimal power with post-contingency rescheduling minimizes the base-case operating cost, while providing the security but it ignores the actual cost of a contingency as a function of the pre-contingency operating state. There is no upper limit on the cost of surviving the contingency. The expected security cost optimal power flow (ESCOPF) replaces the hard constraints with cost functions, therefore, it is more economical way of securely operating the power system. ESCOPF problem size is very large because the loadflow equations and operating limit constraints for each post-contingency state are combined into one problem. Therefore, a decomposed solution method was needed so that the computational time could be reduced.

The objective of this dissertation is to develop and implement a decoupled solution method by which the ESCOPF problem can be decomposed and distributed to a set of processors which can run together to solve the problem in parallel fashion. Decoupled formulation/solution takes advantage of the nearly decoupled relationship between the pre-contingency and post-contingency variables. MatlabMPI has been used for the parallel implementation of the decoupled ESCOPF on a Beowulf cluster. Steps leading to the successful implementation of the decoupled solution method for ESCOPF have been discussed.

While parallelization can significantly reduce computation time, it adds overhead due to communication among processors in the cluster. Therefore, for very small problems parallel solution using a cluster is not very efficient. For larger problems

which are more computationally intensive, the communication overhead is small compared to the compute time, so the time required for parallel solution is far less than the time to solve by using the integrated ESCOPF method. Significant speed improvements promise the ability to compute results in a reasonable amount of time which would otherwise take far too long to be seriously considered.

In PDIP method, how close we should go to the central path (to minimize the number of iterations without sacrificing convergence) has yet to be fully explored. How should the barrier parameter ν be decreased to zero? Should the scaling factor ρ be kept constant or should it be adjusted depending upon the progress we make while solving the problem? If ρ should be adjusted dynamically, then how? These factors may help us obtain good convergence properties and speed-up the computation process for ESCOPF.

Future studies should focus on the implementation of the decoupled ESCOPF for combined natural gas & electric networks. Initial phase of combining the gas and electric networks has already been completed by our research group at OSU.

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APPENDIX A

DESCRIPTION OF DATA FILE FORMATS

A.1 Common data format file

The IEEE common data format file [42] has *.cdf extension. It contains data about the power system transmission lines and buses. In data file, lines are grouped into section with section headers. Each line in the data file has 128 characters. Data items are stored in specific columns. Following is the description of IEEE *.cdf file.

- **Title Data:**

Columns	2-9	Date, in format DD/MM/YY with leading zeros
	11-30	Originator's name
	32-37	MVA base
	39-42	Year
	44	Season (S - Summer, W - Winter)
	46-73	Case identification

- **Bus Data:**

Columns   1-4   Bus number

Columns	1-4	Bus number
	7-17	Bus name
	25-26	Type:
		0=Unregulated (load, PQ)
		1=Hold MVAR generation within voltage limits (PQ)
		2=Hold voltage within VAR limits (PV)
		3=Hold voltage and angle (Swing)
	28-33	Bus voltage (loadflow result)
	34-40	Bus angle (loadflow result)
	41-49	Load MW (loadflow result)
	50-59	Load MVAR (loadflow result)
	60-67	Generation MW (loadflow result)
	68-75	Generation MVAR (loadflow result)
	91-98	Maximum voltage: for only bus type 0
	99-106	Minimum voltage: for only bus type 0
	107-114	Shunt conductance G (per unit)
	115-122	Shunt susceptance B (per unit)

- **Line Data:**

Columns	1-4	From bus
	6-9	To bus
	19	Line type:
		1=Fixed voltage ratio and/or fixed phase shifter
		2=Fixed phase angle and variable voltage ratio with voltage control
		3=Fixed phase angle and variable voltage ratio with MVAR control
		4=Fixed voltage ratio and variable phase shifter with MW control
	20-29	Branch resistance R, per unit
	30-40	Branch reactance X, per unit
	41-49	Line charging B, per unit
	57-61	Maximum line flow (MVA): for emergency state
	63-67	Maximum line flow (MVA): for normal operating state
	77-82	Transformer tap ratio
	84-90	Phase angle
	91-97	Minimum limit of variable tap ratio or phase shifter angle
	98-104	Maximum limit of variable tap ratio or phase shifter angle
	134-139	Probability of line outage in percent (added for ESCOPF)

A.2 Generator Data format file

The generator data file has a *.gdf extension. It was created to provide generator cost curve data, generator operating voltage and power output operating limits. Following is the description of *.gdf file.

Columns		
	1-5	Generator bus number
	7-15	Generator name
	17-23	Maximum real power generation
	25-29	Minimum real power generation
	31-38	Maximum reactive power generation
	40-45	Minimum reactive power generation
	40-45	Minimum reactive power generation
	47	Adjustable real power generation:
		0=real power generation is not adjustable
		1=real power generation is adjustable
	49	Adjustable reactive power generation:
		0=reactive power generation is not adjustable
		1=reactive power generation is adjustable
	51	Availability:
		0=the generator is not available
		1=the generator is available
	53-75	Generator cost function: $\alpha_{Gi} + \beta_{Gi}P_{Gi} + \gamma_{Gi}P_{Gi}^2$
	53-59	Generator α_{Gi} cost curve coefficient
	61-66	Generator β_{Gi} cost curve coefficient
	68-75	Generator γ_{Gi} cost curve coefficient
	77-81	Maximum voltage magnitude at generator bus
	83-87	Minimum voltage magnitude at generator bus
	89-93	Maximum MW upward ramping
	95-99	Minimum MW downward ramping
	101-106	Maximum MVar upward ramping
	108-113	Minimum MVar downward ramping
	115-120	Generator β_R ramping cost curve coefficient
	122-129	Generator γ_R ramping cost curve coefficient

A.3 Load Data format file

The generator data file has a *.ldf extension. It was created to provide consumer benefit curve data, interruption cost coefficients, and load power factor data. Following is the description of *.ldf file.

Columns	1-5	Load bus number
	7-13	Load bus name
	15-18	Load type:
		1=residential
		2=industrial
		3=commercial
	20-25	Maximum MW load
	27-32	Minimum MW load
	34-40	Maximum MVar load
	42-48	Minimum MVar load
	50	Interruptible load:
		0=not interruptible
		1=interruptable
	52-59	Maximum MW interruptible
	61-87	Consumer benefit function: $\alpha_{Li} + \beta_{Li}P_{Li} + \gamma_{Li}P_{Li}^2$
	61-68	Consumer benefit curve coefficient α_{Li}
	70-77	Consumer benefit curve coefficient β_{Li}
	79-87	Consumer benefit curve coefficient γ_{Li}
	89-96	Interruption cost curve α_{Ii}
	98-104	Interruption cost curve β_{Ii}
	106-113	Interruption cost curve γ_{Ii}
	115-120	Load power factor

A.4 Bus and transmission line data for IEEE 5-bus case

```
09/25/93 UW ARCHIVE          100.0 1962 W IEEE 14 Bus Test Case
BUS DATA FOLLOWS                 5  ITEMS
 1 Bus 1     HV  1  1  3 1.060    0.0     0.0   232.4 -16.9   0.0  1.060  150  -100   0.0  0.0  0.0  0.0   0   1  1.00
 2 Bus 2     HV  1  1  2 1.045   -4.98   21.7   12.7   42.4   0.0  1.045   50.0  -40.0 0.0  0.0  0.0  0.0   0   2  1.00
 3 Bus 3     HV  1  1  2 1.010  -12.72   94.2   19.0   23.4   0.0  1.010   40.0    0.0 0.0  0.0  0.0  0.0   0   3  1.00
 4 Bus 4     HV  1  1  0 1.019  -10.33   91.98 -13.64   0.0   0.0  0.0     1.05   0.95 0.0  0.0  0.0  0.0   0   4  1.00
 5 Bus 5     HV  1  1  0 1.020   -8.78   51.66  14.42   0.0   0.0  0.0     1.05   0.95 0.0  0.0  0.0  0.0   0   5  1.00
-999
BRANCH DATA FOLLOWS              7  ITEMS
 1  2  1  1  1  0 0.01938 0.05917    155  155 120  0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  1
 1  5  1  1  1  0 0.05403 0.22304    110  110  90  0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  2
 2  3  1  1  1  0 0.04699 0.19797     95   95  75  0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  3
 2  4  1  1  1  0 0.05811 0.17632     95   95  75  0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  4
 2  5  1  1  1  0 0.05695 0.17388     95   95  75  0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  5
 3  4  1  1  1  0 0.06701 0.17103     95   95  75  0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  6
 4  5  1  1  1  0 0.01335 0.04211     95   95  75  0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  7
-999
LOSS ZONES FOLLOWS               1 ITEMS
 1 IEEE 14 BUS
-99
INTERCHANGE DATA FOLLOWS         1 ITEMS
 1  2 Bus 2    HV   0.0  999.99  IEEE14  IEEE 14 Bus Test Case
-9
TIE LINES FOLLOWS                0 ITEMS
-999
END OF DATA
```

IEEE 5-bus system common data format

A.5 Generator and Load data

for IEEE 5-bus case

```
12345678901234567890123456789012345678901234567890123456789012345678901234567890
BUNUM GENERNAME MX---MW MN-MW MX--MVR MN-MVR P Q A AL__PHA B-E-TA GAM-M--A V-MAX V-MIN MW-UP MW-DN MVARUP MVARDN BETA_R GAMMA__R
    1 GEN1    250.00 45.0  150.00  -100  1 1 1 0.0    11.0   .000820  1.08  0.950  50.0  50.0  inf    inf    0.0   0.000
    2 GEN2    150.00 15.0   50.00   -40  1 1 1 0.0    12.0   .000776  1.08  0.950  35.0  35.0  inf    inf    0.0   0.000
    3 SynCon 1  0.00  0.0   40.00   -40  0 1 1 0.0     0.0   .000000  1.08  0.950  inf   inf   inf    inf    0.0   0.000
```

IEEE 5-bus system generator data format

```
123456789012345678901234567890123456789012345678901234567890123456789012345
BUNUM BUSNAME TYPE MX--MW MN--MW MX-MVAR MN-MVAR R MX-MW-IN ALPHA-LD BETA--LD GAMMA--LD ALPHA-IN BETA-IN GAMMA-IN P----F
    2 Residen  1   inf    0.0    inf    -inf   1   100     0.00    21.43   -0.2147    0.0    100.0    0.0   0.98
    3 Indust1  2   inf    0.0    inf    -inf   1   100     0.00    23.00   -0.0531    0.0    100.0    0.0   0.98
    4 Indust2  2   inf    0.0    inf    -inf   1   100     0.00    22.48   -0.0531    0.0    100.0    0.0   0.98
    5 Commerc  3   inf    0.0    inf    -inf   1   100     0.00    22.11   -0.0930    0.0    100.0    0.0   0.98
```

IEEE 5-bus system load data format

A.6 Bus and transmission line data

for IEEE 14-bus case

```
07/20/94 OSU mods to UW case  100.0 1962 W IEEE14 W/LINE LIMITS, BUS KV
BUS DATA FOLLOWS                            14 ITEMS
   1 Bus 1     HV  1  1  3 1.0600    .00    .00    .00  232.39 -16.90 230.00 1.0600    .00 150.00 -100.00  .0000  .0000   1  1.00
   2 Bus 2     HV  1  1  2 1.0450  -4.98  21.70  12.70   40.00  42.41 230.00 1.0450    .00  50.00  -40.00  .0000  .0000   2  1.00
   3 Bus 3     HV  1  1  2 1.0100 -12.72  94.20  19.00     .00  23.39 230.00 1.0100    .00  40.00     .00  .0000  .0000   3  1.00
   4 Bus 4     HV  1  1  0 1.0186 -10.32  47.80  -3.90     .00    .00 230.00  .0000    .00    .00     .00  .0000  .0000   4  1.00
   5 Bus 5     LV  1  1  0 1.0203  -8.78   7.60   1.60     .00    .00 230.00  .0000    .00    .00     .00  .0000  .0000   5  1.00
   6 Bus 6     LV  1  1  2 1.0700 -14.22  11.20   7.50     .00  12.24  69.00 1.0700    .00  24.00   -6.00  .0000  .0000   6  1.00
   7 Bus 7     ZV  1  1  0 1.0620 -13.37    .00    .00     .00    .00 161.00  .0000    .00    .00     .00  .0000  .0000   7  0.00
   8 Bus 8     TV  1  1  0 1.0900 -13.37    .00    .00     .00  17.36 138.00 1.0900    .00  24.00   -6.00  .0000  .0000   8  0.00
   9 Bus 9     LV  1  1  0 1.0563 -14.95  29.50  16.60     .00    .00  69.00  .0000    .00    .00     .00  .0000  .1900   9  0.00
  10 Bus 10    LV  1  1  0 1.0513 -15.10   9.00   5.80     .00    .00  69.00  .0000    .00    .00     .00  .0000  .0000  10  0.00
  11 Bus 11    LV  1  1  0 1.0571 -14.80   3.50   1.80     .00    .00  69.00  .0000    .00    .00     .00  .0000  .0000  11  0.00
  12 Bus 12    LV  1  1  0 1.0552 -15.08   6.10   1.60     .00    .00  69.00  .0000    .00    .00     .00  .0000  .0000  12  0.00
  13 Bus 13    LV  1  1  0 1.0504 -15.16  13.50   5.80     .00    .00  69.00  .0000    .00    .00     .00  .0000  .0000  13  0.00
  14 Bus 14    LV  1  1  0 1.0358 -16.04  14.90   5.00     .00    .00  69.00  .0000    .00    .00     .00  .0000  .0000  14  0.00
 -999
BRANCH DATA FOLLOWS                          20 ITEMS
   1   2  1  1  1  0 .019380 .059170  .05280 120. 120. 155.  0  0  .0000   .00  .0000  .0000  .0000  0
   1   5  1  1  1  0 .054030 .223040  .04920  90.  90. 110.  0  0  .0000   .00  .0000  .0000  .0000  0
   2   3  1  1  1  0 .046990 .197970  .04380  75.  75.  95.  0  0  .0000   .00  .0000  .0000  .0000  0
   2   4  1  1  1  0 .058110 .176320  .03740  75.  75.  95.  0  0  .0000   .00  .0000  .0000  .0000  0
   2   5  1  1  1  0 .056950 .173880  .03460  50.  50.  95.  0  0  .0000   .00  .0000  .0000  .0000  0
   3   4  1  1  1  0 .067010 .171030  .03460  50.  50. 110.  0  0  .0000   .00  .0000  .0000  .0000  0
   4   5  1  1  1  0 .013350 .042110  .01280  90.  90.  90.  0  0  .0000   .00  .0000  .0000  .0000  0
   6  11  1  1  1  0 .094980 .198900  .00000  10.  10.  20.  0  0  .0000   .00  .0000  .0000  .0000  0
   6  12  1  1  1  0 .122910 .255810  .00000  20.  20.  30.  0  0  .0000   .00  .0000  .0000  .0000  0
   6  13  1  1  1  0 .066150 .130270  .00000  15.  15.  25.  0  0  .0000   .00  .0000  .0000  .0000  0
   9  10  1  1  1  0 .031810 .084500  .00000  10.  10.  20.  0  0  .0000   .00  .0000  .0000  .0000  0
   9  14  1  1  1  0 .127110 .270380  .00000  10.  10.  20.  0  0  .0000   .00  .0000  .0000  .0000  0
  10  11  1  1  1  0 .082050 .192070  .00000  35.  35.  45.  0  0  .0000   .00  .0000  .0000  .0000  0
  12  13  1  1  1  0 .220920 .199880  .00000  20.  20.  30.  0  0  .0000   .00  .0000  .0000  .0000  0
  13  14  1  1  1  0 .170930 .348020  .00000  50.  50.  60.  0  0  .0000   .00  .0000  .0000  .0000  0
   4   7  1  1  1  0 .000000 .209120  .00000  20.  20.  40.  0  0  .9780   .00  .0000  .0000  .0000  0
   4   9  1  1  1  0 .000000 .556180  .00000  35.  35.  45.  0  0  .9690   .00  .0000  .0000  .0000  0
   5   6  1  1  1  0 .000000 .252020  .00000  35.  35.  45.  0  0  .9320   .00  .0000  .0000  .0000  0
   7   8  1  1  1  0 .000000 .176150  .00000  50.  50.  60. 1.0000      .0000  .0000 0
   7   9  1  1  1  0 .000000 .110010  .00000  35.  35.  45. 1.0000      .0000  .0000 0
 -999
LOSS ZONES FOLLOWS                            1 ITEMS
  1 IEEE 14 BUS
 -99
INTERCHANGE DATA FOLLOWS                       1 ITEMS
  1  2 Bus 2     HV    .0 999.99 IEEE14  IEEE 14 Bus Test Case
 -9
TIE LINES FOLLOW                              0 ITEMS
 -999
```

IEEE 14-bus system common data format

A.7 Generator and Load data for IEEE 14-bus case

```
123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890
```

BUNUM	GENERNAME	MX--GEN	MNGEN	MX---MVR	MN-MVR	P	Q	A	AL--PHA	B-E-TA	GAM---MA	V-MAX	V-MIN	MW-UP	MW-DN	MVARUP	MVARDN	BETA-R	GAMMA__R
1	GEN 1	250.00	45.0	150.00	-100	1	1	1	0.00	11.0	.000820	1.080	0.950	50	50	inf	inf	0.0	.00000
2	GEN 2	150.00	15.0	50.00	-40	1	1	1	0.00	12.0	.000776	1.080	0.950	35	35	inf	inf	0.0	.00000
3	SynCon 1	0.00	0.00	40.00	-40	0	1	1	0.00	0.0	.000000	1.080	0.950	inf	inf	inf	inf	0.0	.00000
6	SynCon 2	0.00	0.00	104.00	-100	0	1	1	0.00	0.0	.000000	1.080	0.950	inf	inf	inf	inf	0.0	.00000
8	SynCon 3	0.00	0.00	104.00	-100	0	1	1	0.00	0.0	.000000	1.080	0.950	inf	inf	inf	inf	0.0	.00000

IEEE 14-bus system generator data format

```
123456789012345678901234567890123456789012345678901234567890123456789012345
```

BUNUM	BUSNAME	TYPE	MX--MW	MN--MW	MX-MVAR	MN-MVAR	R	MX-MW-IN	MN-MVAR	ALPHA-LD	BETA--LD	GAMMA--LD	ALPHA-IN	BETA-IN	GAMMA-IN	P---F
2	Residen	1	inf	0.0	inf	-inf	1	100	-inf	100	21.43	-0.2146	0.00	100.0	0.0	0.98
3	Indust1	2	inf	0.0	inf	-inf	1	100	-inf	100	22.98	-0.0530	0.00	100.0	0.0	0.98
4	Indust2	2	inf	0.0	inf	-inf	1	100	-inf	100	22.45	-0.1021	0.00	100.0	0.0	0.98
5	Commerc	3	inf	0.0	inf	-inf	1	100	-inf	100	22.07	-0.6314	0.00	100.0	0.0	0.98
6	Residen	1	inf	0.0	inf	-inf	1	100	-inf	100	22.11	-0.4292	0.00	100.0	0.0	0.98
9	Residen	1	inf	0.0	inf	-inf	1	100	-inf	100	22.41	-0.1652	0.00	100.0	0.0	0.98
10	Residen	1	inf	0.0	inf	-inf	1	100	-inf	100	22.50	-0.5434	0.00	100.0	0.0	0.98
11	Residen	1	inf	0.0	inf	-inf	1	100	-inf	100	22.39	-1.3904	0.00	100.0	0.0	0.98
12	Residen	1	inf	0.0	inf	-inf	1	100	-inf	100	22.51	-0.8022	0.00	100.0	0.0	0.98
13	Residen	1	inf	0.0	inf	-inf	1	100	-inf	100	22.63	-0.3644	0.00	100.0	0.0	0.98
14	Residen	1	inf	0.0	inf	-inf	1	100	-inf	100	23.02	-0.3359	0.00	100.0	0.0	0.98

IEEE 14-bus system load data format

```
09/25/93 UW ARCHIVE       100.0  1961 W IEEE 30 Bus Test Case
BUS DATA FOLLOWS                   30 ITEMS
  1 Glen Lyn  132 1 1 3  1.060   0.0    0.0   0.0  260.2 -16.1  132.0 1.060  1.05  0.950  0.0 0.0    1
  2 Claytor   132 1 1 2  1.043  -5.48  21.7  12.7   40.0  50.0  132.0 1.045 50.0 -40.0   0.0 0.0    2
  3 Kumis     132 1 1 0  1.021  -7.96   2.4   1.2    0.0   0.0  132.0 0.0    1.05  0.950  0.0 0.0    3
  4 Hancock   132 1 1 0  1.012  -9.62   7.6   1.6    0.0   0.0  132.0 1.010 40.0 -40.0   0.0 0.0    4
  5 Fieldale  132 1 1 2  1.010 -14.37  94.2  19.0    0.0  37.0  132.0 1.010 40.0 -40.0   0.0 0.0    5
  6 Roanoke   132 1 1 0  1.010 -11.34   0.0   0.0    0.0   0.0  132.0 0.0    1.05  0.950  0.0 0.0    6
  7 Blaine    132 1 1 0  1.002 -13.12  22.8  10.9    0.0   0.0  132.0 0.0    1.05  0.950  0.0 0.0    7
  8 Reusens   132 1 1 2  1.010 -12.10  30.0  30.0    0.0  37.3  132.0 1.010 40.0 -10.0   0.0 0.0    8
  9 Roanoke   1.0 1 1 0  1.051 -14.38   0.0   0.0    0.0   0.0    1.0 0.0    1.05  0.950  0.0 0.0    9
 10 Roanoke    33 1 1 0  1.045 -15.97   5.8   2.0    0.0   0.0   33.0 0.0    1.05  0.950  0.0 0.19  10
 11 Roanoke    11 1 1 2  1.082 -14.39   0.0   0.0    0.0  16.2   11.0 1.082 24.0 -6.0    0.0 0.0   11
 12 Hancock    33 1 1 0  1.057 -15.24  11.2   7.5    0.0   0.0   33.0 0.0    1.05  0.950  0.0 0.0   12
 13 Hancock    11 1 1 2  1.071 -15.24   0.0   0.0    0.0  10.6   11.0 1.071 24.0 -6.0    0.0 0.0   13
 14 Bus 14     33 1 1 0  1.042 -16.13   6.2   1.6    0.0   0.0   33.0 0.0    1.05  0.950  0.0 0.0   14
 15 Bus 15     33 1 1 0  1.038 -16.22   8.2   2.5    0.0   0.0   33.0 0.0    1.05  0.950  0.0 0.0   15
 16 Bus 16     33 1 1 0  1.045 -15.83   3.5   1.8    0.0   0.0   33.0 0.0    1.05  0.950  0.0 0.0   16
 17 Bus 17     33 1 1 0  1.040 -16.14   9.0   5.8    0.0   0.0   33.0 0.0    1.05  0.950  0.0 0.0   17
 18 Bus 18     33 1 1 0  1.028 -16.82   3.2   0.9    0.0   0.0   33.0 0.0    1.05  0.950  0.0 0.0   18
 19 Bus 19     33 1 1 0  1.026 -17.00   9.5   3.4    0.0   0.0   33.0 0.0    1.05  0.950  0.0 0.0   19
 20 Bus 20     33 1 1 0  1.030 -16.80   2.2   0.7    0.0   0.0   33.0 0.0    1.05  0.950  0.0 0.0   20
 21 Bus 21     33 1 1 0  1.033 -16.42  17.5  11.2    0.0   0.0   33.0 0.0    1.05  0.950  0.0 0.0   21
 22 Bus 22     33 1 1 0  1.033 -16.41   0.0   0.0    0.0   0.0   33.0 0.0    1.05  0.950  0.0 0.0   22
 23 Bus 23     33 1 1 0  1.027 -16.61   3.2   1.6    0.0   0.0   33.0 0.0    1.05  0.950  0.0 0.0   23
 24 Bus 24     33 1 1 0  1.021 -16.78   8.7   6.7    0.0   0.0   33.0 0.0    1.05  0.950  0.0 0.043 24
 25 Bus 25     33 1 1 0  1.017 -16.35   0.0   0.0    0.0   0.0   33.0 0.0    1.05  0.950  0.0 0.0   25
 26 Bus 26     33 1 1 0  1.000 -16.77   3.5   2.3    0.0   0.0   33.0 0.0    1.05  0.950  0.0 0.0   26
 27 Cloverdle  33 1 1 0  1.023 -15.82   0.0   0.0    0.0   0.0   33.0 0.0    1.05  0.950  0.0 0.0   27
 28 Cloverdle 132 1 1 0  1.007 -11.97   0.0   0.0    0.0   0.0  132.0 0.0    1.05  0.950  0.0 0.0   28
 29 Bus 29     33 1 1 0  1.003 -17.06   2.4   0.9    0.0   0.0   33.0 0.0    1.05  0.950  0.0 0.0   29
 30 Bus 30     33 1 1 0  0.992 -17.94  10.6   1.9    0.0   0.0   33.0 0.0    1.05  0.950  0.0 0.0   30
-999
BRANCH DATA (Continued on next page)
```

IEEE 30-bus system common data format

```
BRANCH DATA FOLLOWS                                 41 ITEMS
```

Tap	Z	A	Zn	C	T	R	X	B	Rate1	Rate2	Tap ratio
1	2	1	1	1	0	0.0192	0.0575	0.0528	155	120	0.0
1	3	1	1	1	0	0.0452	0.1852	0.0408	110	90	0.0
2	4	1	1	1	0	0.0570	0.1737	0.0368	95	75	0.0
3	4	1	1	1	0	0.0132	0.0379	0.0084	95	100	0.0
2	5	1	1	1	0	0.0472	0.1983	0.0418	110	90	0.0
2	6	1	1	1	0	0.0581	0.1763	0.0374	90	90	0.0
4	6	1	1	1	0	0.0119	0.0414	0.0090	35	35	0.0
5	7	1	1	1	0	0.0460	0.1160	0.0204	30	30	0.0
6	7	1	1	1	0	0.0267	0.0820	0.0170	20	35	0.0
6	8	1	1	1	0	0.0120	0.0420	0.0090	25	30	0.0
6	9	1	1	1	1	0.0	0.2080	0.0	20	35	0.978
6	10	1	1	1	1	0.0	0.5560	0.0	20	35	0.969
9	11	1	1	1	0	0.0	0.2080	0.0	20	30	0.0
9	10	1	1	1	0	0.0	0.1100	0.0	45	35	0.0
4	12	1	1	1	1	0.0	0.2560	0.0	60	35	0.932
12	13	1	1	1	0	0.0	0.1400	0.0	45	30	0.0
12	14	1	1	1	0	0.1231	0.2559	0.0	20	35	0.0
12	15	1	1	1	0	0.0662	0.1304	0.0	20	20	0.0
12	16	1	1	1	0	0.0945	0.1987	0.0	20	20	0.0
14	15	1	1	1	0	0.2210	0.1997	0.0	20	20	0.0
16	17	1	1	1	0	0.0824	0.1923	0.0	45	20	0.0
15	18	1	1	1	0	0.1073	0.2185	0.0	45	20	0.0
18	19	1	1	1	0	0.0639	0.1292	0.0	45	45	0.0
19	20	1	1	1	0	0.0340	0.0680	0.0	45	45	0.0
10	20	1	1	1	0	0.0936	0.2090	0.0	35	45	0.0
10	17	1	1	1	0	0.0324	0.0845	0.0	35	45	0.0
10	21	1	1	1	0	0.0348	0.0749	0.0	35	35	0.0
10	22	1	1	1	0	0.0727	0.1499	0.0	35	35	0.0
21	22	1	1	1	0	0.0116	0.0236	0.0	35	35	0.0
15	23	1	1	1	0	0.1000	0.2020	0.0	35	35	0.0
22	24	1	1	1	0	0.1150	0.1790	0.0	35	35	0.0
23	24	1	1	1	0	0.1320	0.2700	0.0	35	35	0.0
24	25	1	1	1	0	0.1885	0.3292	0.0	35	35	0.0
25	26	1	1	1	0	0.2544	0.3800	0.0	35	35	0.0
25	27	1	1	1	0	0.1093	0.2087	0.0	35	35	0.0
28	27	1	1	1	1	0.0	0.3960	0.0	35	35	0.968
27	29	1	1	1	0	0.2198	0.4153	0.0	35	35	0.0
27	30	1	1	1	0	0.3202	0.6027	0.0	35	35	0.0
29	30	1	1	1	0	0.2399	0.4533	0.0	35	35	0.0
8	28	1	1	1	0	0.0636	0.2000	0.0428	35	35	0.0
6	28	1	1	1	0	0.0169	0.0599	0.0130	35	35	0.0

```
-999
LOSS ZONES FOLLOWS                                   1 ITEMS
 1 IEEE 30 BUS
-99
INTERCHANGE DATA FOLLOWS                             1 ITEMS
 1  2 Claytor    132  0.0 1999.99  IEEE30  IEEE 30 Bus Test Case
-9
TIE LINES FOLLOWS                                    0 ITEMS
-999
```

IEEE 30-bus system common data format

A.10 Generator and Load data
for IEEE 30-bus case

12345678901234567890123456789012345678901234567890123456789012345678901234567890

BUNUM	GENERNAME	MX--GEN	MNGEN	MX---MVR	MN-MVR	P	Q	A	AL--PHA	B-E-TA	GAM---MA	V-MAX	V-MIN	MW-UP	MW-DN	MVARUP	MVARDN	BETA-R	GAMMAR
1	GEN 1	280.0	45.0	150.00	-100	1	1	1	0.00	11.00	.000820	1.080	0.950	50	50	inf	inf	0.0	.00000
2	GEN 2	150.0	15.0	50.00	-40	1	1	1	0.00	12.00	.000776	1.080	0.950	35	35	inf	inf	0.0	.00000
5	GEN 3	0.0	0.0	40.00	-40	0	1	1	0.00	0.000	.00	1.080	0.950	inf	inf	inf	inf	0.0	.00000
8	GEN 4	0.0	0.0	40.00	-10	0	1	1	0.00	0.000	.00	1.080	0.950	inf	inf	inf	inf	0.0	.00000
11	GEN 5	0.0	0.0	24.00	-6	0	1	1	0.00	0.000	.00	1.080	0.950	inf	inf	inf	inf	0.0	.00000
13	GEN 6	0.0	0.0	24.00	-6	0	1	1	0.00	0.000	.00	1.080	0.950	inf	inf	inf	inf	0.0	.00000

IEEE 30-bus system generator data format

12345678901234567890123456789012345678901234567890123456789012345

BUNUM	BUSNAME	TYPE	MX--MW	MN--MW	MX-MVAR	MN-MVAR	R	MX-MW-IN	MN-MW-IN	ALPHA-LD	BETA-LD	GAMMA--LD	ALPHA-IN	BETA-IN	GAMMA-IN	P---F
2	Residen	1	inf	0.0	inf	-inf	1	inf	-inf	0.00	21.43	-0.2146	0.0	100.0	0.0	0.98
3	Indust1	2	inf	0.0	inf	-inf	1	inf	-inf	0.00	22.40	-1.5030	0.0	100.0	0.0	0.98
4	Indust2	2	inf	0.0	inf	-inf	1	inf	-inf	0.00	22.07	-0.6314	0.0	100.0	0.0	0.98
5	Commerc	3	inf	0.0	inf	-inf	1	inf	-inf	0.00	22.98	-0.0530	0.0	100.0	0.0	0.98
7	Residen	1	inf	0.0	inf	-inf	1	inf	-inf	0.00	21.61	-0.2492	0.0	100.0	0.0	0.98
8	Residen	1	inf	0.0	inf	-inf	1	inf	-inf	0.00	22.41	-0.1652	0.0	100.0	0.0	0.98
10	Residen	1	inf	0.0	inf	-inf	1	inf	-inf	0.00	22.63	-0.3644	0.0	100.0	0.0	0.98
12	Residen	1	inf	0.0	inf	-inf	1	inf	-inf	0.00	22.11	-0.4292	0.0	100.0	0.0	0.98
14	Residen	1	inf	0.0	inf	-inf	1	inf	-inf	0.00	22.52	-0.8022	0.0	100.0	0.0	0.98
15	Residen	1	inf	0.0	inf	-inf	1	inf	-inf	0.00	22.25	-0.5746	0.0	100.0	0.0	0.98
16	Indust1	2	inf	0.0	inf	-inf	1	inf	-inf	0.00	22.39	-1.3930	0.0	100.0	0.0	0.98
17	Indust2	2	inf	0.0	inf	-inf	1	inf	-inf	0.00	22.50	-0.5434	0.0	100.0	0.0	0.98
18	Commerc	3	inf	0.0	inf	-inf	1	inf	-inf	0.00	22.27	-1.4314	0.0	100.0	0.0	0.98
19	Residen	1	inf	0.0	inf	-inf	1	inf	-inf	0.00	22.55	-0.5092	0.0	100.0	0.0	0.98
20	Residen	1	inf	0.0	inf	-inf	1	inf	-inf	0.00	22.41	-1.6652	0.0	100.0	0.0	0.98
21	Residen	1	inf	0.0	inf	-inf	1	inf	-inf	0.00	22.50	-0.2834	0.0	100.0	0.0	0.98
23	Residen	1	inf	0.0	inf	-inf	1	inf	-inf	0.00	22.30	-1.4322	0.0	100.0	0.0	0.98
24	Residen	1	inf	0.0	inf	-inf	1	inf	-inf	0.00	22.43	-0.5844	0.0	100.0	0.0	0.98
26	Residen	1	inf	0.0	inf	-inf	1	inf	-inf	0.00	22.42	-1.3946	0.0	100.0	0.0	0.98
29	Commerc	3	inf	0.0	inf	-inf	1	inf	-inf	0.00	22.47	-1.6314	0.0	100.0	0.0	0.98
30	Residen	1	inf	0.0	inf	-inf	1	inf	-inf	0.00	22.11	-0.5292	0.0	100.0	0.0	0.98

IEEE 30-bus system load data format

B.1   MATLAB RESULTS FOR 5 BUS DC ESCOPF

```
NORMAL STATE (Pre-contingency state)
--------------------------------------------------------------------------------
               Power        Power                  Generator      Consumer
         Angle  Generated    Consumed    Lambda    Marg. Cost    Marg. Benefit
         (Deg)   (MW)         (MW)       $/MWh      $/MWh         $/MWh
--------------------------------------------------------------------------------
Bus(1)   0.000   146.442      0.000      11.240     11.240        0.000
Bus(2)  -3.196   115.000     21.308      11.240     12.178       12.281
Bus(3) -12.086     0.000     95.000      11.240      0.000       12.911
Bus(4)  -9.735     0.000     91.809      11.240      0.000       12.730
Bus(5)  -8.295     0.000     53.326      11.240      0.000       12.191
--------------------------------------------------------------------------------
Following are the power flows on transmission lines:
Line1--2:  85.1276 MW
Line1--5:  61.3149 MW
Line2--3:  74.2001 MW
Line2--4:  58.3895 MW
Line2--5:  46.2303 MW
Line4--3:  20.7999 MW
Line5--4:  54.2191 MW
--------------------------------------------------------------------------------
Total Power Generation = 261.4425 MW/hour
Total Load Consumption = 261.4425 MW/hour
Total Loss =  0 MW/hour
Total Cost = -1577.07 $/hour
--------------------------------------------------------------------------------
Following are the coupling constraints, at a particular bus, affecting the normal
state in post-contingency states:
At bus (2), Load Interruption limit in postcontingency state-2. (mu=100.1153)
 At bus (2), Load Interruption limit in postcontingency state-3. (mu=100.1111)
 At bus (2), Load Interruption limit in postcontingency state-4. (mu=100.0732)
 At bus (2), Load Interruption limit in postcontingency state-5. (mu=100.9829)
 At bus (2), Load Interruption limit in postcontingency state-6. (mu=100.9829)
 At bus (2), Load Interruption limit in postcontingency state-7. (mu=100.9829)
 At bus (2), Generator ramp up limit in postcontingency state-1. (mu=89.7430)
 At bus (2), Generator ramp down limit in postcontingency state-5. (mu=0.8266)
 At bus (2), Generator ramp down limit in postcontingency state-6. (mu=0.8266)
 At bus (2), Generator ramp down limit in postcontingency state-7. (mu=0.8266)
 At bus (3), Load Interruption limit in postcontingency state-1. (mu=0.5056)
 At bus (3), Load Interruption limit in postcontingency state-2. (mu=100.7458)
 At bus (3), Load Interruption limit in postcontingency state-3. (mu=75.6752)
 At bus (3), Load Interruption limit in postcontingency state-4. (mu=92.5310)
 At bus (3), Load Interruption limit in postcontingency state-5. (mu=101.6134)
 At bus (3), Load Interruption limit in postcontingency state-6. (mu=83.1924)
 At bus (3), Load Interruption limit in postcontingency state-7. (mu=90.3490)
 At bus (4), Load Interruption limit in postcontingency state-1. (mu=0.3245)
 At bus (4), Load Interruption limit in postcontingency state-2. (mu=100.5647)
 At bus (4), Load Interruption limit in postcontingency state-3. (mu=93.5188)
 At bus (4), Load Interruption limit in postcontingency state-4. (mu=84.6398)
 At bus (4), Load Interruption limit in postcontingency state-5. (mu=101.4323)
 At bus (4), Load Interruption limit in postcontingency state-6. (mu=101.4323)
```

```
At bus (4), Load Interruption limit in postcontingency state-7. (mu=79.5412)
At bus (5), Load Interruption limit in postcontingency state-2. (mu=100.0261)
At bus (5), Load Interruption limit in postcontingency state-3. (mu=104.2557)
At bus (5), Load Interruption limit in postcontingency state-4. (mu=104.5769)
At bus (5), Load Interruption limit in postcontingency state-5. (mu=100.8938)
At bus (5), Load Interruption limit in postcontingency state-6. (mu=100.8938)
At bus (5), Load Interruption limit in postcontingency state-7. (mu=100.8938)
==============================================================================
 CONTINGENCY NO:1  Fault on Transmission line connecting Bus(1) & Bus(2)

-------------------------------------------------------------------------------
                    Power       Power                  Generator     Consumer
          Angle   Generated   Consumed    Lambda      Marg. Cost  Marg. Benefit
          (Deg)     (MW)        (MW)      $/MWh         $/MWh        $/MWh
-------------------------------------------------------------------------------
Bus(1)    0.000   110.000       0.000    11.180       11.180        0.000
Bus(2)  -12.426   150.000      21.016   112.405       12.233       12.406
Bus(3)  -20.310     0.000      95.000   112.405        0.000       12.911
Bus(4)  -17.009     0.000      91.809   112.405        0.000       12.730
Bus(5)  -14.882     0.000      52.175   112.405        0.000       12.405
-------------------------------------------------------------------------------
Following are the power flows on transmission lines:
Line1--5: 110.0000 MW (Power flow was hit by binding Constraint with mu =101.2250)
Line2--3:  65.7975 MW
Line2--4:  40.9197 MW
Line2--5:  22.2664 MW
Line4--3:  29.2024 MW
Line5--4:  80.0914 MW
-------------------------------------------------------------------------------
Total Power Generation = 260.0000 MW/hour
Total Load Consumption = 260.0000 MW/hour
Total Loss =   0 MW/hour
Total Cost = -1396.40 $/hour
-------------------------------------------------------------------------------
ALL BINDING CONSTRATINS:
Line capacity limit was hit with mu:101.2250
Generator Pmax limit was hit at bus(2) with mu: 10.4296
Generator ramp up limit (coupling constraint) was hit at bus(2) with mu: 89.7430
Load interruption limit (coupling constraint) was hit at bus(3) with mu:  0.5056
Load interruption limit (coupling constraint) was hit at bus(4) with mu:  0.3245
-------------------------------------------------------------------------------
==============================================================================
 CONTINGENCY NO:2  Fault on Transmission line connecting Bus(1) & Bus(5)

-------------------------------------------------------------------------------
                    Power       Power                  Generator     Consumer
          Angle   Generated   Consumed    Lambda      Marg. Cost  Marg. Benefit
          (Deg)     (MW)        (MW)      $/MWh         $/MWh        $/MWh
-------------------------------------------------------------------------------
Bus(1)    0.000   155.000       0.000    11.254       11.254        0.000
Bus(2)   -5.819   106.443      21.308    12.165       12.165       12.281
Bus(3)  -15.948     0.000      95.000    12.165        0.000       12.911
Bus(4)  -14.765     0.000      91.809    12.165        0.000       12.730
Bus(5)  -14.171     0.000      53.326    12.165        0.000       12.191
-------------------------------------------------------------------------------
Following are the power flows on transmission lines:
Line1--2: 155.0000 MW (Power flow was hit by binding Constraint with mu =  0.9110)
Line2--3:  84.5379 MW
Line2--4:  79.8831 MW
Line2--5:  75.7138 MW
Line4--3:  10.4621 MW
Line5--4:  22.3877 MW
-------------------------------------------------------------------------------
Total Power Generation = 261.4425 MW/hour
Total Load Consumption = 261.4425 MW/hour
Total Loss =   0 MW/hour
Total Cost = -1584.98 $/hour
-------------------------------------------------------------------------------
ALL BINDING CONSTRATINS:
Line capacity limit was hit with mu:  0.9110
```

```
Load interruption limit (coupling constraint) was hit at bus(2) with mu:100.1153
Load interruption limit (coupling constraint) was hit at bus(3) with mu:100.7458
Load interruption limit (coupling constraint) was hit at bus(4) with mu:100.5647
Load interruption limit (coupling constraint) was hit at bus(5) with mu:100.0261

--------------------------------------------------------------------------------
================================================================================
 CONTINGENCY NO:3  Fault on Transmission line connecting Bus(2) & Bus(3)
--------------------------------------------------------------------------------
                      Power      Power                Generator     Consumer
            Angle   Generated  Consumed    Lambda    Marg. Cost   Marg. Benefit
            (Deg)      (MW)      (MW)      $/MWh       $/MWh         $/MWh
--------------------------------------------------------------------------------
Bus(1)    0.000    152.319      0.000     11.250     11.250         0.000
Bus(2)   -2.791    109.123     21.308     12.169     12.169        12.281
Bus(3)  -23.811      0.000     95.000     37.236      0.000        12.911
Bus(4)  -13.073      0.000     91.809     19.211      0.000        12.730
Bus(5)  -10.550      0.000     53.326      7.936      0.000        12.191
--------------------------------------------------------------------------------
Following are the power flows on transmission lines:
Line1--2:  74.3370 MW
Line1--5:  77.9823 MW
Line2--4:  91.8087 MW
Line2--5:  70.3438 MW
Line4--3:  95.0000 MW (Power flow was hit by binding Constraint with mu = 18.0246)
Line5--4:  95.0000 MW (Power flow was hit by binding Constraint with mu = 12.9449)
--------------------------------------------------------------------------------
Total Power Generation = 261.4425 MW/hour
Total Load Consumption = 261.4425 MW/hour
Total Loss =  0 MW/hour
Total Cost = -1582.53 $/hour
--------------------------------------------------------------------------------
ALL BINDING CONSTRATINS:
Line capacity limit was hit with mu: 18.0246
Line capacity limit was hit with mu: 12.9449
Load interruption limit (coupling constraint) was hit at bus(2) with mu:100.1111
Load interruption limit (coupling constraint) was hit at bus(3) with mu: 75.6752
Load interruption limit (coupling constraint) was hit at bus(4) with mu: 93.5188
Load interruption limit (coupling constraint) was hit at bus(5) with mu:104.2557

--------------------------------------------------------------------------------
================================================================================
 CONTINGENCY NO:4  Fault on Transmission line connecting Bus(2) & Bus(4)
--------------------------------------------------------------------------------
                      Power      Power                Generator     Consumer
            Angle   Generated  Consumed    Lambda    Marg. Cost   Marg. Benefit
            (Deg)      (MW)      (MW)      $/MWh       $/MWh         $/MWh
--------------------------------------------------------------------------------
Bus(1)    0.000    127.881      0.000     11.210     11.210         0.000
Bus(2)   -1.995    133.561     21.308     12.207     12.207        12.281
Bus(3)  -12.995      0.000     95.000     20.380      0.000        12.911
Bus(4)  -12.634      0.000     91.809     28.090      0.000        12.730
Bus(5)  -10.112      0.000     53.326      7.614      0.000        12.191
--------------------------------------------------------------------------------
Following are the power flows on transmission lines:
Line1--2:  53.1389 MW
Line1--5:  74.7424 MW
Line2--3:  91.8087 MW
Line2--5:  73.5837 MW
Line4--3:   3.1913 MW
Line5--4:  95.0000 MW (Power flow was hit by binding Constraint with mu = 22.2867)
--------------------------------------------------------------------------------
Total Power Generation = 261.4425 MW/hour
Total Load Consumption = 261.4425 MW/hour
Total Loss =  0 MW/hour
Total Cost = -1559.10 $/hour
--------------------------------------------------------------------------------
ALL BINDING CONSTRATINS:
Line capacity limit was hit with mu: 22.2867
Load interruption limit (coupling constraint) was hit at bus(2) with mu:100.0732
```

```
Load interruption limit (coupling constraint) was hit at bus(3) with mu: 92.5310
Load interruption limit (coupling constraint) was hit at bus(4) with mu: 84.6398
Load interruption limit (coupling constraint) was hit at bus(5) with mu:104.5769

--------------------------------------------------------------------------------
================================================================================
 CONTINGENCY NO:5  Fault on Transmission line connecting Bus(2) & Bus(5)

--------------------------------------------------------------------------------
                     Power       Power                Generator     Consumer
           Angle    Generated   Consumed    Lambda    Marg. Cost  Marg. Benefit
           (Deg)      (MW)        (MW)       $/MWh      $/MWh        $/MWh
--------------------------------------------------------------------------------
Bus(1)    0.000     181.442      0.000      11.298     11.298       0.000
Bus(2)   -3.685      80.000     21.308      11.298     12.124      12.281
Bus(3)  -13.521       0.000     95.000      11.298      0.000      12.911
Bus(4)  -12.061       0.000     91.809      11.298      0.000      12.730
Bus(5)  -11.266       0.000     53.326      11.298      0.000      12.191
--------------------------------------------------------------------------------
Following are the power flows on transmission lines:
Line1--2:  98.1735 MW
Line1--5:  83.2689 MW
Line2--3:  82.0843 MW
Line2--4:  74.7817 MW
Line4--3:  12.9157 MW
Line5--4:  29.9428 MW
--------------------------------------------------------------------------------
Total Power Generation = 261.4425 MW/hour
Total Load Consumption = 261.4425 MW/hour
Total Loss =  0 MW/hour
Total Cost = -1607.96 $/hour
Generator ramp down limit (coupling constraint) was hit at bus(2) with mu:  0.8266
Load interruption limit (coupling constraint) was hit at bus(2) with mu:100.9829
Load interruption limit (coupling constraint) was hit at bus(3) with mu:101.6134
Load interruption limit (coupling constraint) was hit at bus(4) with mu:101.4323
Load interruption limit (coupling constraint) was hit at bus(5) with mu:100.8938

--------------------------------------------------------------------------------
================================================================================
 CONTINGENCY NO:6  Fault on Transmission line connecting Bus(3) & Bus(4)

--------------------------------------------------------------------------------
                     Power       Power                Generator     Consumer
           Angle    Generated   Consumed    Lambda    Marg. Cost  Marg. Benefit
           (Deg)      (MW)        (MW)       $/MWh      $/MWh        $/MWh
--------------------------------------------------------------------------------
Bus(1)    0.000     181.442      0.000      11.298     11.298       0.000
Bus(2)   -4.464      80.000     21.308      11.298     12.124      12.281
Bus(3)  -15.847       0.000     95.000      29.719      0.000      12.911
Bus(4)   -9.663       0.000     91.809      11.298      0.000      12.730
Bus(5)   -8.458       0.000     53.326      11.298      0.000      12.191
--------------------------------------------------------------------------------
Following are the power flows on transmission lines:
Line1--2: 118.9269 MW
Line1--5:  62.5156 MW
Line2--3:  95.0000 MW (Power flow was hit by binding Constraint with mu = 18.4211)
Line2--4:  46.4181 MW
Line2--5:  36.2012 MW
Line5--4:  45.3906 MW
--------------------------------------------------------------------------------
Total Power Generation = 261.4425 MW/hour
Total Load Consumption = 261.4425 MW/hour
Total Loss =  0 MW/hour
Total Cost = -1607.96 $/hour
--------------------------------------------------------------------------------
ALL BINDING CONSTRATINS:
Line capacity limit was hit with mu: 18.4211
Generator ramp down limit (coupling constraint) was hit at bus(2) with mu:  0.8266
Load interruption limit (coupling constraint) was hit at bus(2) with mu:100.9829
Load interruption limit (coupling constraint) was hit at bus(3) with mu: 83.1924
Load interruption limit (coupling constraint) was hit at bus(4) with mu:101.4323
Load interruption limit (coupling constraint) was hit at bus(5) with mu:100.8938
```

```
================================================================================
================================================================================
 CONTINGENCY NO:7  Fault on Transmission line connecting Bus(4) & Bus(5)
--------------------------------------------------------------------------------
                     Power       Power                 Generator    Consumer
           Angle   Generated   Consumed    Lambda     Marg. Cost  Marg. Benefit
           (Deg)     (MW)        (MW)       $/MWh       $/MWh        $/MWh
--------------------------------------------------------------------------------
Bus(1)    0.000    181.442      0.000      11.298      11.298       0.000
Bus(2)   -5.128     80.000     21.308      11.298      12.124      12.281
Bus(3)  -16.129      0.000     95.000      22.562       0.000      12.911
Bus(4)  -15.768      0.000     91.809      33.189       0.000      12.730
Bus(5)   -6.065      0.000     53.326      11.298       0.000      12.191
--------------------------------------------------------------------------------
Following are the power flows on transmission lines:
Line1--2: 136.6111 MW
Line1--5:  44.8313 MW
Line2--3:  91.8087 MW
Line2--4:  95.0000 MW (Power flow was hit by binding Constraint with mu = 32.4202)
Line2--5:   8.4948 MW
Line4--3:   3.1913 MW
--------------------------------------------------------------------------------
Total Power Generation = 261.4425 MW/hour
Total Load Consumption = 261.4425 MW/hour
Total Loss =  0 MW/hour
Total Cost = -1607.96 $/hour
--------------------------------------------------------------------------------
ALL BINDING CONSTRATINS:
Line capacity limit was hit with mu: 32.4202
Generator ramp down limit (coupling constraint) was hit at bus(2) with mu:  0.8266
Load interruption limit (coupling constraint) was hit at bus(2) with mu:100.9829
Load interruption limit (coupling constraint) was hit at bus(3) with mu: 90.3490
Load interruption limit (coupling constraint) was hit at bus(4) with mu: 79.5412
Load interruption limit (coupling constraint) was hit at bus(5) with mu:100.8938
--------------------------------------------------------------------------------
```

B.2   MATLAB RESULTS FOR 5 BUS AC ESCOPF

```
 NORMAL STATE (Pre-contingency state)
--------------------------------------------------------------------------------
                      Power      Power      Power            Generator  Consumer
          Voltage  Angle Generated Generated Consumed Lambda Mrg.Cost Mrg.Benefit
          (p.u.)  (Deg)   (MW)      (MVar)    (MW)   ($/MWh)  ($/MWh)   ($/MWh)
--------------------------------------------------------------------------------
Bus(1) 1.080    0.000   151.722   -15.239     0.000   11.249   11.249     0.000
Bus(2) 1.074   -2.827   108.124    35.826    21.565   11.589   12.168    12.170
Bus(3) 1.043  -10.058     0.000    31.683    88.849   12.434    0.000    13.564
Bus(4) 1.034   -7.886     0.000     0.000    90.111   12.231    0.000    12.910
Bus(5) 1.041   -6.694     0.000     0.000    50.561   12.065    0.000    12.706
--------------------------------------------------------------------------------
Following are the current flows on transmission lines:
Line1--2:  0.8640 pu
Line1--5:  0.5634 pu
Line2--3:  0.6730 pu
Line2--4:  0.5439 pu
Line2--5:  0.4291 pu
Line3--4:  0.2099 pu
Line4--5:  0.5099 pu
Line2--1:  0.8520 pu
Line5--1:  0.5670 pu
Line3--2:  0.6729 pu
Line4--2:  0.5475 pu
Line5--2:  0.4336 pu
Line4--3:  0.2301 pu
Line5--4:  0.5101 pu
```

```
--------------------------------------------------------------------------------
Total Power Generation = 259.8467 MW/hour
Total Load Consumption = 251.0867 MW/hour
Total Loss =  9 MW/hour
Total Cost = -1466.94 $/hour
--------------------------------------------------------------------------------
ALL BINDING CONSTRATINS:
Maximum Voltage limit was hit at bus(1) with mu:  1.9069
Following are the coupling constraints, at a particular bus, affecting the normal
state in post-contingency states:
At bus (2), Load Interruption limit in postcontingency state-1. (mu=44.7607)
 At bus (2), Load Interruption limit in postcontingency state-2. (mu=100.0014)
 At bus (2), Load Interruption limit in postcontingency state-3. (mu=99.6177)
 At bus (2), Load Interruption limit in postcontingency state-4. (mu=100.4458)
 At bus (2), Load Interruption limit in postcontingency state-5. (mu=100.4522)
 At bus (2), Load Interruption limit in postcontingency state-6. (mu=100.3856)
 At bus (2), Load Interruption limit in postcontingency state-7. (mu=100.2946)
 At bus (2), Generator ramp up limit in postcontingency state-1. (mu=55.1872)
 At bus (2), Generator ramp down limit in postcontingency state-4. (mu=0.3892)
 At bus (2), Generator ramp down limit in postcontingency state-5. (mu=0.3956)
 At bus (2), Generator ramp down limit in postcontingency state-6. (mu=0.3290)
 At bus (2), Generator ramp down limit in postcontingency state-7. (mu=0.2381)
 At bus (3), Load Interruption limit in postcontingency state-1. (mu=41.7895)
 At bus (3), Load Interruption limit in postcontingency state-2. (mu=100.2950)
 At bus (3), Load Interruption limit in postcontingency state-3. (mu=49.6985)
 At bus (3), Load Interruption limit in postcontingency state-4. (mu=100.8149)
 At bus (3), Load Interruption limit in postcontingency state-5. (mu=100.9141)
 At bus (3), Load Interruption limit in postcontingency state-6. (mu=100.8128)
 At bus (3), Load Interruption limit in postcontingency state-7. (mu=100.5388)
 At bus (4), Load Interruption limit in postcontingency state-1. (mu=42.8069)
 At bus (4), Load Interruption limit in postcontingency state-2. (mu=99.6147)
 At bus (4), Load Interruption limit in postcontingency state-3. (mu=96.1872)
 At bus (4), Load Interruption limit in postcontingency state-4. (mu=100.0949)
 At bus (4), Load Interruption limit in postcontingency state-5. (mu=100.3161)
 At bus (4), Load Interruption limit in postcontingency state-6. (mu=100.5351)
 At bus (4), Load Interruption limit in postcontingency state-7. (mu=99.7386)
 At bus (5), Load Interruption limit in postcontingency state-1. (mu=43.9830)
 At bus (5), Load Interruption limit in postcontingency state-2. (mu=99.4767)
 At bus (5), Load Interruption limit in postcontingency state-3. (mu=97.4233)
 At bus (5), Load Interruption limit in postcontingency state-4. (mu=100.2045)
 At bus (5), Load Interruption limit in postcontingency state-5. (mu=100.2264)
 At bus (5), Load Interruption limit in postcontingency state-6. (mu=100.4928)
 At bus (5), Load Interruption limit in postcontingency state-7. (mu=100.9564)
================================================================================
 CONTINGENCY NO:1  Fault on Transmission line connecting Bus(1) & Bus(2)

--------------------------------------------------------------------------------
                       Power      Power      Power              Generator  Consumer
          Voltage Angle Generated  Generated  Consumed  Lambda  Mrg.Cost   Mrg.Benefit
          (p.u.)  (Deg)   (MW)       (MVar)     (MW)    ($/MWh)  ($/MWh)    ($/MWh)
--------------------------------------------------------------------------------
Bus(1) 1.080    0.000  118.798     0.633      0.000   11.195   11.195     0.000
Bus(2) 1.080  -12.306  143.124    34.412     21.565   67.409   12.222    12.170
Bus(3) 1.049  -18.522    0.000    33.992     88.849   71.775    0.000    13.564
Bus(4) 1.037  -15.405    0.000     0.000     90.111   69.878    0.000    12.910
Bus(5) 1.042  -13.516    0.000     0.000     50.561   68.472    0.000    12.706
--------------------------------------------------------------------------------
Following are the current flows on transmission lines:
Line1--5:   1.1000 pu (Current flow was hit by binding constraint with mu = 24.1422)
Line2--3:   0.5858 pu
Line2--4:   0.3779 pu
Line2--5:   0.2283 pu
Line3--4:   0.3054 pu
Line4--5:   0.7820 pu
Line5--1:   1.0955 pu
Line3--2:   0.5872 pu
Line4--2:   0.3904 pu
Line5--2:   0.2521 pu
Line4--3:   0.3250 pu
Line5--4:   0.7843 pu
```

```
--------------------------------------------------------------------------------
Total Power Generation = 261.9226 MW/hour
Total Load Consumption = 251.0867 MW/hour
Total Loss = 11 MW/hour
Total Cost = -1409.59 $/hour
--------------------------------------------------------------------------------
ALL BINDING CONSTRATINS:
Maximum Voltage limit was hit at bus(1) with mu: 66.4149
Maximum Voltage limit was hit at bus(2) with mu:  2.5397
Maximum Current flow limit was hit with mu: 24.1422
Generator ramp up limit (coupling constraint) was hit at bus(2) with mu: 55.1872
Following constraints prevent loads in post-contingency constraint to
exceed loads in normal state.
Load interruption limit (coupling constraint) was hit at bus(2) with mu: 44.7607
Load interruption limit (coupling constraint) was hit at bus(3) with mu: 41.7895
Load interruption limit (coupling constraint) was hit at bus(4) with mu: 42.8069
Load interruption limit (coupling constraint) was hit at bus(5) with mu: 43.9830
--------------------------------------------------------------------------------
 CONTINGENCY NO:2  Fault on Transmission line connecting Bus(1) & Bus(5)
--------------------------------------------------------------------------------
                      Power      Power      Power            Generator  Consumer
       Voltage Angle  Generated  Generated  Consumed  Lambda Mrg.Cost  Mrg.Benefit
       (p.u.)  (Deg)  (MW)       (MVar)     (MW)      ($/MWh)($/MWh)    ($/MWh)
--------------------------------------------------------------------------------
Bus(1) 1.080   0.000  166.654   -15.789      0.000   11.273  11.273     0.000
Bus(2) 1.061  -5.059   98.633    50.000     21.565   12.153  12.153    12.170
Bus(3) 1.029 -13.693    0.000    40.000     88.849   13.255   0.000    13.564
Bus(4) 1.006 -12.278    0.000     0.000     90.111   13.237   0.000    12.910
Bus(5) 1.007 -11.723    0.000     0.000     50.561   13.170   0.000    12.706
--------------------------------------------------------------------------------
Following are the current flows on transmission lines:
Line1--2:   1.5500 pu (Current flow was hit by binding constraint with mu =  0.0561)
Line2--3:   0.7885 pu
Line2--4:   0.7592 pu
Line2--5:   0.7182 pu
Line3--4:   0.1708 pu
Line4--5:   0.2207 pu
Line2--1:   1.5432 pu
Line3--2:   0.7870 pu
Line4--2:   0.7625 pu
Line5--2:   0.7220 pu
Line4--3:   0.2025 pu
Line5--4:   0.2232 pu
--------------------------------------------------------------------------------
Total Power Generation = 265.2866 MW/hour
Total Load Consumption = 251.0867 MW/hour
Total Loss = 14 MW/hour
Total Cost = -1414.22 $/hour
--------------------------------------------------------------------------------
ALL BINDING CONSTRATINS:
Maximum Voltage limit was hit at bus(1) with mu:  3.9033
Maximum Current flow limit was hit with mu:  0.0561
Generator Qmax limit was hit at bus(2) with mu:  0.0765
Generator Qmax limit was hit at bus(3) with mu:  0.0701
Following constraints prevent loads in post-contingency constraint to
exceed loads in normal state.
Load interruption limit (coupling constraint) was hit at bus(2) with mu:100.0014
Load interruption limit (coupling constraint) was hit at bus(3) with mu:100.2950
Load interruption limit (coupling constraint) was hit at bus(4) with mu: 99.6147
Load interruption limit (coupling constraint) was hit at bus(5) with mu: 99.4767
--------------------------------------------------------------------------------
 CONTINGENCY NO:3  Fault on Transmission line connecting Bus(2) & Bus(3)
--------------------------------------------------------------------------------
                      Power      Power      Power            Generator  Consumer
       Voltage Angle  Generated  Generated  Consumed  Lambda Mrg.Cost  Mrg.Benefit
       (p.u.)  (Deg)  (MW)       (MVar)     (MW)      ($/MWh)($/MWh)    ($/MWh)
--------------------------------------------------------------------------------
Bus(1) 1.080   0.000  159.779    -5.314      0.000   11.262  11.262     0.000
Bus(2) 1.075  -2.519  110.132    50.000     21.565   12.171  12.171    12.170
```

```
Bus(3) 0.969  -20.850    0.000    40.000    88.849   63.735    0.000    13.564
Bus(4) 1.004  -10.926    0.000     0.000    90.111   15.599    0.000    12.910
Bus(5) 1.018   -8.717    0.000     0.000    50.561   14.407    0.000    12.706
--------------------------------------------------------------------------------
Following are the current flows on transmission lines:
Line1--2:  0.7708 pu
Line1--5:  0.7399 pu
Line2--4:  0.9017 pu
Line2--5:  0.6888 pu
Line3--4:  0.9442 pu
Line4--5:  0.9381 pu
Line2--1:  0.7586 pu
Line5--1:  0.7467 pu
Line4--2:  0.9063 pu
Line5--2:  0.6942 pu
Line4--3:  0.9500 pu (Current flow was hit by binding constraint with mu = 21.1979)
Line5--4:  0.9376 pu
--------------------------------------------------------------------------------
Total Power Generation = 269.9110 MW/hour
Total Load Consumption = 251.0867 MW/hour
Total Loss = 19 MW/hour
Total Cost = -1351.83 $/hour
--------------------------------------------------------------------------------
ALL BINDING CONSTRATINS:
Maximum Voltage limit was hit at bus(1) with mu: 56.5190
Maximum Current flow limit was hit with mu: 21.1979
Generator Qmax limit was hit at bus(2) with mu:  1.8781
Generator Qmax limit was hit at bus(3) with mu:  0.6458
Following constraints prevent loads in post-contingency constraint to
exceed loads in normal state.
Load interruption limit (coupling constraint) was hit at bus(2) with mu: 99.6177
Load interruption limit (coupling constraint) was hit at bus(3) with mu: 49.6985
Load interruption limit (coupling constraint) was hit at bus(4) with mu: 96.1872
Load interruption limit (coupling constraint) was hit at bus(5) with mu: 97.4233

--------------------------------------------------------------------------------
 CONTINGENCY NO:4  Fault on Transmission line connecting Bus(2) & Bus(4)
--------------------------------------------------------------------------------
                      Power     Power     Power          Generator  Consumer
        Voltage Angle Generated Generated Consumed Lambda Mrg.Cost  Mrg.Benefit
        (p.u.)  (Deg) (MW)      (MVar)    (MW)     ($/MWh) ($/MWh)  ($/MWh)
--------------------------------------------------------------------------------
Bus(1) 1.080    0.000 190.197   -12.382    0.000   11.312  11.312    0.000
Bus(2) 1.074   -3.354  73.124    42.858   21.565   11.724  12.113   12.170
Bus(3) 1.037  -12.321   0.000    40.000   88.849   12.745   0.000   13.564
Bus(4) 1.009  -11.370   0.000     0.000   90.111   12.762   0.000   12.910
Bus(5) 1.022   -9.172   0.000     0.000   50.561   12.452   0.000   12.706
--------------------------------------------------------------------------------
Following are the current flows on transmission lines:
Line1--2:  1.0228 pu
Line1--5:  0.7706 pu
Line2--3:  0.8297 pu
Line2--5:  0.6444 pu
Line3--4:  0.1601 pu
Line4--5:  0.9250 pu
Line2--1:  1.0103 pu
Line5--1:  0.7756 pu
Line3--2:  0.8291 pu
Line5--2:  0.6494 pu
Line4--3:  0.1950 pu
Line5--4:  0.9250 pu
--------------------------------------------------------------------------------
Total Power Generation = 263.3217 MW/hour
Total Load Consumption = 251.0867 MW/hour
Total Loss = 12 MW/hour
Total Cost = -1457.85 $/hour
--------------------------------------------------------------------------------
ALL BINDING CONSTRATINS:
Maximum Voltage limit was hit at bus(1) with mu:  3.0142
```

```
Generator Qmax limit was hit at bus(3) with mu:  0.0218
Generator ramp down limit (coupling constraint) was hit at bus(2) with mu:  0.3892
Following constraints prevent loads in post-contingency constraint to
exceed loads in normal state.
Load interruption limit (coupling constraint) was hit at bus(2) with mu:100.4458
Load interruption limit (coupling constraint) was hit at bus(3) with mu:100.8149
Load interruption limit (coupling constraint) was hit at bus(4) with mu:100.0949
Load interruption limit (coupling constraint) was hit at bus(5) with mu:100.2045
--------------------------------------------------------------------------------
 CONTINGENCY NO:5  Fault on Transmission line connecting Bus(2) & Bus(5)
--------------------------------------------------------------------------------
                      Power     Power     Power            Generator Consumer
           Voltage Angle Generated Generated Consumed  Lambda  Mrg.Cost Mrg.Benefit
           (p.u.) (Deg)    (MW)     (MVar)    (MW)   ($/MWh)  ($/MWh)   ($/MWh)
--------------------------------------------------------------------------------
Bus(1) 1.080    0.000  189.142   -9.151    0.000  11.310   11.310    0.000
Bus(2) 1.073   -3.298   73.124   37.113   21.565  11.718   12.113   12.170
Bus(3) 1.040  -11.472    0.000   38.088   88.849  12.650    0.000   13.564
Bus(4) 1.019   -9.888    0.000    0.000   90.111  12.552    0.000   12.910
Bus(5) 1.021   -9.183    0.000    0.000   50.561  12.434    0.000   12.706
--------------------------------------------------------------------------------
Following are the current flows on transmission lines:
Line1--2:   1.0069 pu
Line1--5:   0.7726 pu
Line2--3:   0.7564 pu
Line2--4:   0.7062 pu
Line3--4:   0.1774 pu
Line4--5:   0.2849 pu
Line2--1:   0.9957 pu
Line5--1:   0.7779 pu
Line3--2:   0.7555 pu
Line4--2:   0.7101 pu
Line4--3:   0.2076 pu
Line5--4:   0.2874 pu
--------------------------------------------------------------------------------
Total Power Generation = 262.2660 MW/hour
Total Load Consumption = 251.0867 MW/hour
Total Loss = 11 MW/hour
Total Cost = -1469.79 $/hour
--------------------------------------------------------------------------------
ALL BINDING CONSTRATINS:
Maximum Voltage limit was hit at bus(1) with mu:  2.7109
Generator ramp down limit (coupling constraint) was hit at bus(2) with mu:  0.3956
Following constraints prevent loads in post-contingency constraint to
exceed loads in normal state.
Load interruption limit (coupling constraint) was hit at bus(2) with mu:100.4522
Load interruption limit (coupling constraint) was hit at bus(3) with mu:100.9141
Load interruption limit (coupling constraint) was hit at bus(4) with mu:100.3161
Load interruption limit (coupling constraint) was hit at bus(5) with mu:100.2264
--------------------------------------------------------------------------------
 CONTINGENCY NO:6  Fault on Transmission line connecting Bus(3) & Bus(4)
--------------------------------------------------------------------------------
                      Power     Power     Power            Generator Consumer
           Voltage Angle Generated Generated Consumed  Lambda  Mrg.Cost Mrg.Benefit
           (p.u.) (Deg)    (MW)     (MVar)    (MW)   ($/MWh)  ($/MWh)   ($/MWh)
--------------------------------------------------------------------------------
Bus(1) 1.080    0.000  188.155  -13.421    0.000  11.309   11.309    0.000
Bus(2) 1.070   -3.881   73.124   45.435   21.565  11.784   12.113   12.170
Bus(3) 1.043  -13.308    0.000   30.409   88.849  12.751    0.000   13.564
Bus(4) 1.025   -7.780    0.000    0.000   90.111  12.340    0.000   12.910
Bus(5) 1.034   -6.807    0.000    0.000   50.561  12.182    0.000   12.706
--------------------------------------------------------------------------------
Following are the current flows on transmission lines:
Line1--2:   1.1851 pu
Line1--5:   0.5784 pu
Line2--3:   0.8635 pu
Line2--4:   0.4473 pu
Line2--5:   0.3481 pu
```

```
Line4--5:   0.4411 pu
Line2--1:   1.1750 pu
Line5--1:   0.5844 pu
Line3--2:   0.8598 pu
Line4--2:   0.4565 pu
Line5--2:   0.3577 pu
Line5--4:   0.4392 pu
```

```
Total Power Generation = 261.2795 MW/hour
Total Load Consumption = 251.0867 MW/hour
Total Loss = 10 MW/hour
Total Cost = -1480.95 $/hour
```

ALL BINDING CONSTRATINS:
Maximum Voltage limit was hit at bus(1) with mu: 2.4115
Generator ramp down limit (coupling constraint) was hit at bus(2) with mu: 0.3290
Following constraints prevent loads in post-contingency constraint to
exceed loads in normal state.
Load interruption limit (coupling constraint) was hit at bus(2) with mu:100.3856
Load interruption limit (coupling constraint) was hit at bus(3) with mu:100.8128
Load interruption limit (coupling constraint) was hit at bus(4) with mu:100.5351
Load interruption limit (coupling constraint) was hit at bus(5) with mu:100.4928

 CONTINGENCY NO:7 Fault on Transmission line connecting Bus(4) & Bus(5)

| | Voltage | Angle | Power Generated | Power Generated | Power Consumed | Lambda | Generator Mrg.Cost | Consumer Mrg.Benefit |
	(p.u.)	(Deg)	(MW)	(MVar)	(MW)	($/MWh)	($/MWh)	($/MWh)
Bus(1)	1.080	0.000	191.038	-6.826	0.000	11.313	11.313	0.000
Bus(2)	1.060	-4.399	73.124	34.970	21.565	11.875	12.113	12.170
Bus(3)	1.024	-13.876	0.000	40.000	88.849	13.020	0.000	13.564
Bus(4)	0.997	-13.178	0.000	0.000	90.111	13.123	0.000	12.910
Bus(5)	1.050	-4.891	0.000	0.000	50.561	11.797	0.000	12.706

Following are the current flows on transmission lines:
```
Line1--2:   1.3579 pu
Line1--5:   0.4146 pu
Line2--3:   0.8637 pu
Line2--4:   0.9111 pu
Line2--5:   0.0678 pu
Line3--4:   0.1433 pu
Line2--1:   1.3533 pu
Line5--1:   0.4188 pu
Line3--2:   0.8626 pu
Line4--2:   0.9136 pu
Line5--2:   0.0857 pu
Line4--3:   0.1782 pu
```

```
Total Power Generation = 264.1625 MW/hour
Total Load Consumption = 251.0867 MW/hour
Total Loss = 13 MW/hour
Total Cost = -1448.34 $/hour
```

ALL BINDING CONSTRATINS:
Maximum Voltage limit was hit at bus(5) with mu: 3.1169
Generator Qmax limit was hit at bus(3) with mu: 0.0260
Generator ramp down limit (coupling constraint) was hit at bus(2) with mu: 0.2381
Following constraints prevent loads in post-contingency constraint to
exceed loads in normal state.
Load interruption limit (coupling constraint) was hit at bus(2) with mu:100.2946
Load interruption limit (coupling constraint) was hit at bus(3) with mu:100.5388
Load interruption limit (coupling constraint) was hit at bus(4) with mu: 99.7386
Load interruption limit (coupling constraint) was hit at bus(5) with mu:100.9564

VITA

Qamar H. Arsalan

Candidate for the Degree of

Doctor of Philosophy

Dissertation: EXPECTED SECURITY COST OPTIMAL POWER FLOW USING PARALLEL AND DISTRIBUTED COMPUTATION

Major Field: Electrical Engineering

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Expected security cost optimal power flow (ESCOPF), a form of optimal power flow (OPF), addresses security as an economic cost rather than as a constraint. The objective of this study is to solve the expected security-cost optimal power flow (ESCOPF) problem using a parallel and distributed computation technique. Previous work by our research group at OSU successfully solved the integrated version of the ESCOPF problem on a single machine but pointed out that, due to combining loadflow equations and operating limit constraints for each post-contingency state, the size of the problem becomes very large. Therefore, a decomposed solution method was needed so that the computational time could be reduced. As a result we would be able to use this approach for solving the practical power system problems.

This work presents a decoupled solution method by which the ESCOPF problem can be decomposed into a main problem and many subproblems (one for each contingency). The decoupled ESCOPF has been solved using a Beowulf Linux cluster. MatlabMPI has been used for inter-processor communication. A decoupled ESCOPF method, with reduced computational complexity, has been developed and implemented. This method helps to reduce the computation time and therefore can solve problems associated with larger power systems. As a result of the improved computational efficiency, we are now one step closer to the real-time implementation of OPF and its variants.

ADVISOR'S APPROVAL: _____