DYNAMIC ANALYSIS OF SPATIAL MECHANISMS USING THE METHOD OF DUAL SUCCESSIVE SCREWS

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CHAPTER I

INTRODUCTION

Since the industrial revolution began, work done by animals is being gradually replaced by that of machinery. High-speed mass production forces us to depend heavily on machinery's performance. The design of a new machine for a particular industrial application requires both kinematic and dynamic analysis. Theories for the kinematic analysis of mechanisms have been well developed. Because of the complexity of the nature of the spatial rigid body dynamics, the existing theories for the analysis of spatial mechanism are either too difficult to comprehend or too tedious to apply. The successive screws method, developed recently by Kohli (83) for the analysis and synthesis of spatial two loop mechanisms appeals for popular application. This dissertation attempts to extend the application of the theory of successive screws to perform the dynamic analysis of spatial mechanisms.

Soni and Harrisberger (127) surveyed the state of the art of mechanism science and indicated the existence of nearly 12,000 publications of scholarly level. Many linkages and mechanisms have been installed in mechanical systems including packaging, printing, agricultural, construction, textile, food and chemical processing machinery, typewriters, industrial robots, pneumatic control linkages and prosthetic devices. A detailed study of Soni's work (127) reveals that there is a considerable interest in the development of new theories and applications of spatial mechanisms. Mayer Zur Capellen (94), Beggs (10), Hartenberg and Denavit (69) have provided an extensive list of applications of spatial mechanisms.

In the design of mechanisms for a particular assignment, a kinematic synthesis of types, numbers and dimensions forms an initial design judgement. At Oklahoma State University, Huang (75), has developed a method using graph theory to generate systematically all alternate linkages for the type and number syntheses. An extensive literature survey on type and number syntheses is given by Huang. To make the kinematic synthesis complete, Kohli and Soni (82, 83) apply successive screws to conduct the dimensional synthesis and analysis of spatial two-loop mechanisms. Azeez (3) developed a computer algorithm using FORMAC, an IBM scientific subroutine for algebraic manipulation, to generate dimensional design equations for the planar linkages under the restriction of preassumed input conditions. A complete kinematic analysis (4, 10, 29, 41, 42, 83, 87, 105, 134, 148, 151) of the mechanisms is required before the dynamic analysis can be done. However, prior to adoption of spatial mechanisms in mechanical systems, one must have a better knowledge of their dynamic characteristics.

1.1. Motivations

If a machine is to perform its prescribed function in a satisfactory manner, its reaction forces and torques must be examined to select the proper sizes of bearings, joints and link members so as to withstand the imposed load safely and without deformation and buckling. The dynamic transmission angles are calculated to evaluate the efficiency of power transmission. The input force or torque history is needed to determine the size of the motor. Properly balanced link-members are essential to reduce the serious vibrations.

In general, dual force transmitted by link members may be regarded as dual dynamic force. However, if a mechanism is not operating at high speeds, dual inertia force due to its own inertia can be neglected and dual forces are dual static forces.

Generally speaking, machinery such as cams, gears, fly wheels and rotary epicyclic engines rotate around axes which are fixed to the ground and are of the inertia invariant type because their distance of center of mass are invariant. Machinery with floating links, such as printing and knitting machinery, Hook-joints, reciprocating engines, oil drilling rigs and harvesting machinery are characterized by the varying inertia coefficients in their floating links as the machine system is operating and hence come under inertial variant types.

The two general categories in the dynamics of mechanisms and linkages are kinetostatics and dynamic time response. The kinetostatics is a study of determination of dual input forces required to produce specified output performance and dual reaction forces at pair joints.

The dynamic time response is a study of the time motion history of mechanisms and linkages under the specified dual input forces. Kinematic properties are assumed in both cases but their input and output assumptions are different.

1.2. Review of Literature on Mechanism Dynamics

Because of a widespread application of mechanisms and linkages, it becomes necessary to develop efficient theories to perform dynamic analysis of mechanisms.

In the past, a number of theories to perform dynamic analysis of mechanisms have been developed. This chapter examines the literature on the dynamics of mechanisms and linkages. Erdman (51) summarized fifteen representive contributions on dynamics of mechanisms in his paper "A Guide to Mechanism Dynamics." Paul (101) investigated several existing computer programs to perform dynamics of mechanisms in an attempt to show their advantages and disadvantages.

Graphical methods for dynamic force analysis of planar mechanisms are well known and are subjects of standard tests (71, 121). These methods offer a quick way of analysis of mechanisms. However, the accuracy of the graphical methods is limited and depends heavily on individual analysis in carrying out graphical construction.

The equivalent mass and equivalent force method based on energy principles (power input equals time rate of change of kinetic energy) given by Wittenbauer (144) leads to determination of the input acceleration.

Quinn (102, 103) found that the fraction of the total energy of a mechanism contained in any one link remains the same for a given input position, regardless of the actual velocity. Quinn's energy method leads to the determination of the input velocity but this method only applies to constrained mechanisms.

Given and Wolford (60) extended Quinn's energy distribution method to carry out the study of spatial mechanisms by using a matrix approach in their formulation of equations.

By extending Wittenbauer's and Goodman's contributions (64), Hirschhorn (71, 72) derived the rate of change of energy method. The rate of change of energy method based on the instantaneous energy balance leads to the determination of the input acceleration.

The methods, proposed by Wittenbauer (144), Quinn (102, 103) and Hirschhorn (71, 72), all used graphical-analytical procedures to reach the desired solutions.

Other notable contributions to the dynamics of mechanisms are Van Sickle and Goodman (139), Goodman (64), and Barkan (9). They formulated the equations of motion based on the concept of the equivalent moment of inertia.

Chace and his associates (29-34) utilized a vector approach and generalized the D'Alembert force method for direct inclusion of inertial effect in the equation of motion. Chace and Bayazitogki (33) showed the classical Lagrange method applied to the constrained dynamic machine systems through the use of Lagrangian multipliers by treating all coordinates of a constrained mechanical system as independent coordinates. The reaction forces are found by using a Lagrange multiplier approach (33, 34, 124, 125). A generalized computer program, DAMN, was based on this approach to solve dynamics of constrained mechanisms by Chace and his associates (33, 124, 125). They applied graph theory to generate the Lagrange equation for any link member in their generalized computer program, DAMN.

A 4×4 matrix method, based on a fact that the product of all the transformation matrices taken around the loop equals the unit matrix, has been developed by Uicker, etc. (134), for the displacement analysis. This method has also been developed for the static force analysis of spatial mechanisms by Denavit, etc. (44). The method has been extended to include the dynamic force analysis of spatial linkages by Uicker (135-136). A 4×4 inertial matrix is defined to describe the mass distributions of the links. Kinetic energy is shown in a matrix form. The equations of motion are written by using Lagrange's method without multipliers. Because the reaction forces and torques are not considered in the equation of motion, they are found by using the principle of virtual work after the completion of dynamic response analysis. Uicker and Sheth used the Hamiltonian canonical equations to formulate the equations of motion. Later, Sheth and Uicker (118-120), based on their approaches, developed a generalized computer program, IMP. Later, Carson and Trummel (28) extended Uicker's approach to include external dual force in their formulation of equation of motion.

Soni (126) studied the dynamics of spatial mechanisms based on Uicker's approach using a 3×3 matrix with dual numbers.

Yang (148) applied dual number quaternions to the study of the analysis of spatial mechanisms. Later, he applied the 3×3 matrix with dual numbers to conduct the study of the displacement analysis of spatial mechanisms (147), and the gyrodynamics (149) and inertial forces analysis of spatial mechanisms (150).

Paul (99) showed a method to identify the independent loop of the linkages. The degrees of freedom are determined by the rank of the loop closure equations. Paul and Krajcinovic (100) used the virtual work given by Hartog in their formulation of the equations of motion. The reaction forces and torques are determined by using Newton's law after the completion of the dynamic response analysis.

Modrey (96) is believed to be the first researcher to apply influence coefficients to the linkages. Inspired by Modrey's work, Benedict and Tesar (15-19) formulated equations of motion by using influence coefficients in their studies of mechanism dynamics.

Bonham (22) used Newton's law to formulate the equations of motion for the planar four-bar mechanism. Inspired by Goodman, Wittenbauer and Hirschhorn, he treated input acceleration as one of unknowns and solved the equation of motion in a linear form.

Bagci (5, 6) applied a 3×3 matrix with dual vector to conduct another study of spatial mechanisms dynamics. Bagci wrote the equations of motion in explicit form based on D'Alembert's principle. Dual forces were transformed to a reference system on the link. Later, Bagci, Bransfield and Mehta (23, 24, 27, 95) applied the matrix displacement method (141) to conduct the study of dynamics of the determinate and indeterminate mechanisms.

Woo (145) utilized the screw coordinate method developed by Wittenbauer (144) and demonstrated its application by using an iterative technique to study the dynamic analysis of a RSRC spatial mechanism.

Keler (81) used a 3×3 dual matrix to conduct the static analysis including frictional effects at pair joints.

Langrana and Bartel (86) applied Lagrange's equation based on Goldstein's work (63) to conduct the dynamic study of the human arm.

Artrobolevsky and Loshchinin (2) investigated the distribution of inertial forces and its associated stability criteria.

Recently, in performing dynamic analysis, elastic properties of the materials of linkage are taken into consideration by several researchers. Burns and Crossley (25, 26) showed the linkages with large deformation. Erdman, etc. (49, 50), Iman (78), Kosar (84), Mehta (95), Brasfield and Bagci (23, 24), Newbauer, etc. (97), Mayer Zur Capellen (93), and Winfrey (142, 143) conducted the study of dynamics of elastic link mechanisms.

A linkage generally transmits vibrations to its frame through its ground bearings. Harmonic analysis and balancing of a mechanism enables the designer to prevent the transmission of serious vibrations to the machine body.

Freudenstein (55) gave an approximate series of coefficients of the real system functions and minimized the higher harmonic source. Other notable contributions were Root (104), Flory (54), Gilbert (61), Bogden and his associates (21), Denavit and Hasson (45), Mayer Zur Capellen (92) and Sadler (106).

Machinery without proper balancing always introduces vibration, noise and wear. Balancing is one of the critical design steps toward the application of mechanisms and linkages. Crossley (39) outlined the procedures for the balancing of six-bar linkages. Han (68) applied the least square technique to reduce the vibration. Bessonov (11) minimized the center of mass by treating the relocation of the mass center. Lowen and his associates and others (12-14, 16, 18, 62, 70, 74, 77, 79, 80, 88-91, 98, 111, 112) studied the balancing of mechanisms. Lowen's efforts set the goal of complete force balancing by requiring the mass center to be stationary. However, Lowen's approach may induce more vibration because of torque unbalancing. These unfavorable effects can be reduced by applying optimization techniques (13, 52, 53, 109).

Studies related to dynamic impacts at pair joints due to clearance were carried out by Austin, Denavit and Hartenberg (1), Garrett and Hall (59), and Dubowsky and Freudenstein (48).

1.3. Scope of the Dissertation

Formulation of the equations of motion for a rigid body is essential to the study of mechanisms dynamics. Newton's law, d'Alembert's principle, Lagrange's equation with and without multiplier, Hamiltonian cannonical equations, energy method, equivalent mass and equivalent force method, the rate of change energy method and virtual work are methods which all lead to formulation of the equation of motion.

A comparison of the existing methods is summarized in Table I. Theoretically all these methods are equivalent, but it should be noted that their practical merits, i.e., their complexity to follow and the time taken in solving the equation of motion, may vary.

The approach of Kohli and Soni (83) has been recognized as one of the most efficient tools in kinematic analysis of spatial mechanisms.

The present study proposes to undertake research in dynamic analysis of determinate spatial mechanisms. It is proposed to use the dual successive screw method along with d'Alembert's principle to formulate the equations of motion for any link member involving revolute, prism, helical, cylinder and spherical pairs.

Chapter I presents an intensive survey of literature on dynamics of mechanisms. This literature survey gives us what is known to be possible and points to the new trend and dimension which leads to current study.

Chapter II discusses a kinematic tool - successive screws and dual screws concept. The mathematical procedures of successive screws are discussed and a screw's direction is defined in a new way.

TABLE I

METHODS OF KINEMATIC AND DYNAMIC ANALYSES

Methods of Kinematics	Associated with Methods of Dynamics	Merits of Kinematics
Vectors approach	Newton's law (47, 71) d'Alembert's principle (100) Lagrange equation with multipliers (33) Quinn's energy method (102, 103) Equivalent mass and force method (144) Rate of change of energy method (71, 72)	The mathematics operation is straightforward.
2×2	No application	The procedures to formulate the Cayley-Klein parameter are very tedious and impractical.
3×3	Lagrange equation (126) d'Alembert's Principle (4)	The procedures to formulate the 3×3 dual matrix are simple, however operation is complicated.
4×4	Lagrange equation (135, 136) Hamiltonian Cannonical equation (120)	It is excellent for computer operation.
Screw Coordinate	Newton law (145)	It is based on motor algebra and good for computer operation.
Successive screw	No application	The procedures and operation are simple and straightforward.

Dualization of successive screws are given. Kinematic constraints at pair axis are given. Displacement, velocity and acceleration at any position of the link are defined. Complete kinematic analysis and numerical examples of spatial RCSR, spatial two-loop RCSR-CSR and spatial RCHCH mechanisms shown.

Chapter III presents the studies on dynamic constraints, reaction forces and torques at pair joints, dual inertial forces, and equations of motion. Methods to solve the kinetostatic and dynamic time response are given by demonstrating numerical examples of spatial RCSR, RCSR-CSR and RCHCH mechanisms. A numerical example of kinetostatic analysis of the spatial RCSR mechanism is given. Reaction forces and torques at pair joints are found. The second numerical example is a dynamic response analysis of spatial RCSR mechanism. Equation of motion is linearized by treating input angular acceleration as one of the unknowns. Their reaction force and torques are calculated at the same time when dynamic response is found. The third example is a study of kinetostatic analysis of a two-loop spatial RCSR-CSR mechanism. The fourth example is a study of kinetostatic analysis of RCHCH spatial mechanism. Equations of motion and constraint equations of spatial 7P, RPRHRRR, PRPRRH, RPRRHC, PPHCHP, RCRCR, PHCHC mechanisms are also given.

Chapter IV draws findings and conclusions and potential applications of the new theories in mechanisms analysis.

CHAPTER II

KINEMATIC ANALYSIS OF SPATIAL MECHANISMS USING THE METHOD OF SUCCESSIVE SCREWS AND DUAL VECTORS

The science of kinematics may be regarded as a geometrical study of motion. It centers on the time history of displacements, velocities, and accelerations. The results of the kinematic analyses are the basic ingredients in the dynamic analysis of any mechanical system. In the following sections, the fundamental concepts of successive screws for a rigid body being constrained by a set of kinematic pairs which permits a mechanism move with respect to a reference frame are discussed.

2.1. Kinematic Pairs

A kinematic pair is a pair of elements which permits relative motion and have some form of contact between each other. A rigid body that moves freely in space without constraints has six degrees of freedom. The degrees of freedom are defined on the basis that the rigid body can experience rotation about three independent axes and translation along the same axes. Hence, a kinematic pair can have a maximum of five degrees of freedom and a minimum of one degree of freedom. The number of degrees of freedom a pair may have is related to its class. The kinematic pairs are listed in Table II.

Reuleaux classified the kinematic pairs into two groups; lower kinematic pairs and higher kinematic pairs. The classification has

TABLE II

CLASSIFICATION OF KINEMATIC PAIRS

Class of Kinematic Pairs	Degrees of Freedom	Kinematic Pairs		
I	1			
		Revolute	Prism	Helical
II	2			
		Slotted Spheric	Cylinder	Cam
III	3			
		Spheric	Sphere Slotted C.	Plane
IV	4			
		Sphere Grove		Cylinder Plane
V	5			
			Sphere Plane	

been drawn on the basis of the form of contacts. The lower kinematic pairs permit surface contact between two elements. The higher kinematic pairs make either point or line contact between two elements. It was pointed out by Soni (135) that lower kinematic pairs are employed in a linkage when large forces are transmitted. Therefore, for practical purposes, he confines his study on lower kinematic pairs such as revolute, prism, helical, cylinder, and spherical pairs.

2.1.1. Cylinder Pair (Figure 1)

It is the most general pair with variable pitches. It has two degrees of freedom. It permits rotation about its axis $\hat{A}$ and translation along the same screw axis with variable pitch.

2.1.2. Revolute Pair (Figure 2)

It is a degenerated case of a cylinder pair with zero pitch. It has one degree of freedom namely rotation about screw axis $\hat{A}$. Translation along $\hat{A}$ is not permitted.

2.1.3. Prism Pair (Figure 3)

It is a degenerated case of a cylinder pair with infinite pitch. It has one degree of freedom, namely translation along screw axis $\hat{A}$. Rotation about $\hat{A}$ is not permitted.

2.1.4. Helical Pair (Figure 4)

It is a degenerated case of a cylinder pair with constant pitch. It has one degree of freedom, namely translation corresponding to a rotation is permitted along screw axis $\hat{A}$. The ratio of translation and

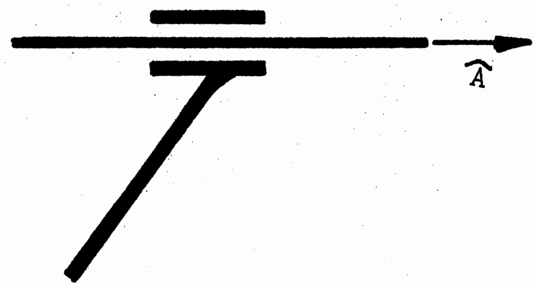

Figure 1. A Cylinder Pair

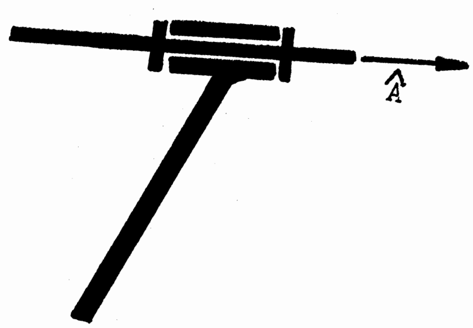

Figure 2. A Revolute Pair

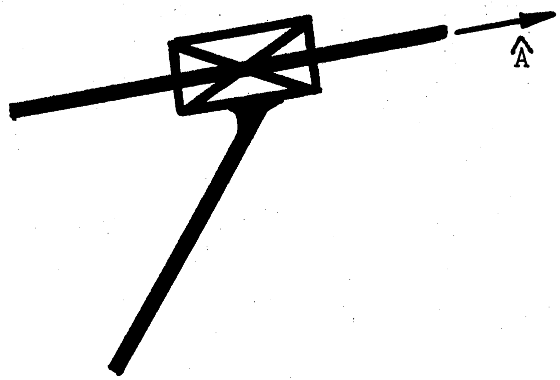

Figure 3. A Prism Pair

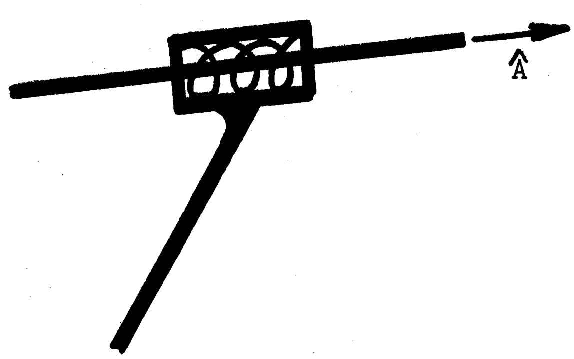

Figure 4. A Helical Pair

rotation is called the pitch of the screw pair.

2.1.5. Spherical Pair (Figure 5)

It has three degrees of freedom. It permits rotational motion about three independent screw axes $\hat{A}$, $\hat{B}$ and $\hat{C}$, where

$$\hat{A} \times \hat{B} = \hat{C} \qquad (2.1)$$

$$\hat{B} \times \hat{C} = \hat{A} .$$

2.2. Successive Screws

The science of kinematics deals with the study of motion of a rigid body. When two positions of a rigid body are given, there are infinite ways by which a rigid body can be displaced from one position to the other. Chasle's theorem states that any displacement of a rigid body can be specified by a movement consisting of a rotation around a straight line accompanied by a translation parallel to the straight line. The theorem implies that the motion of a rigid body can be separated into two parts: rotation and translation.

Generally, six coordinates are necessary to locate a rigid body in space. Because of the constrained characteristics of pair geometry, six coordinates are reduced to five. Hence, a point and a line vector attached to a rigid body are necessary and sufficient to specify a rigid body being constrained by a kinematic pair in spatial mechanisms without redundant freedom.

Figure 6 shows a rigid body R being attached to a cylindrical pair at O_1 in a carteasian coordinate system OXYZ. The path or a trajectory of the point P_2 with respect to the OXYZ reference system is indicated

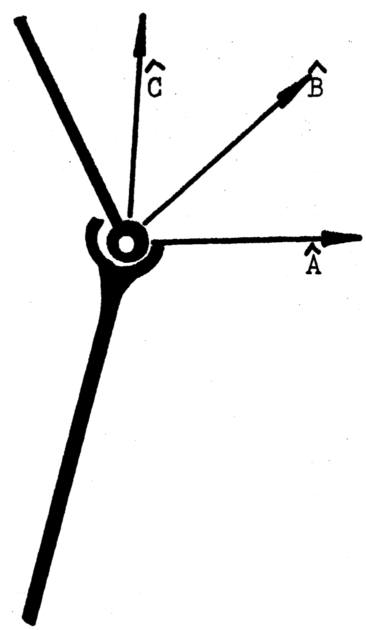

Figure 5. A Spherical Pair

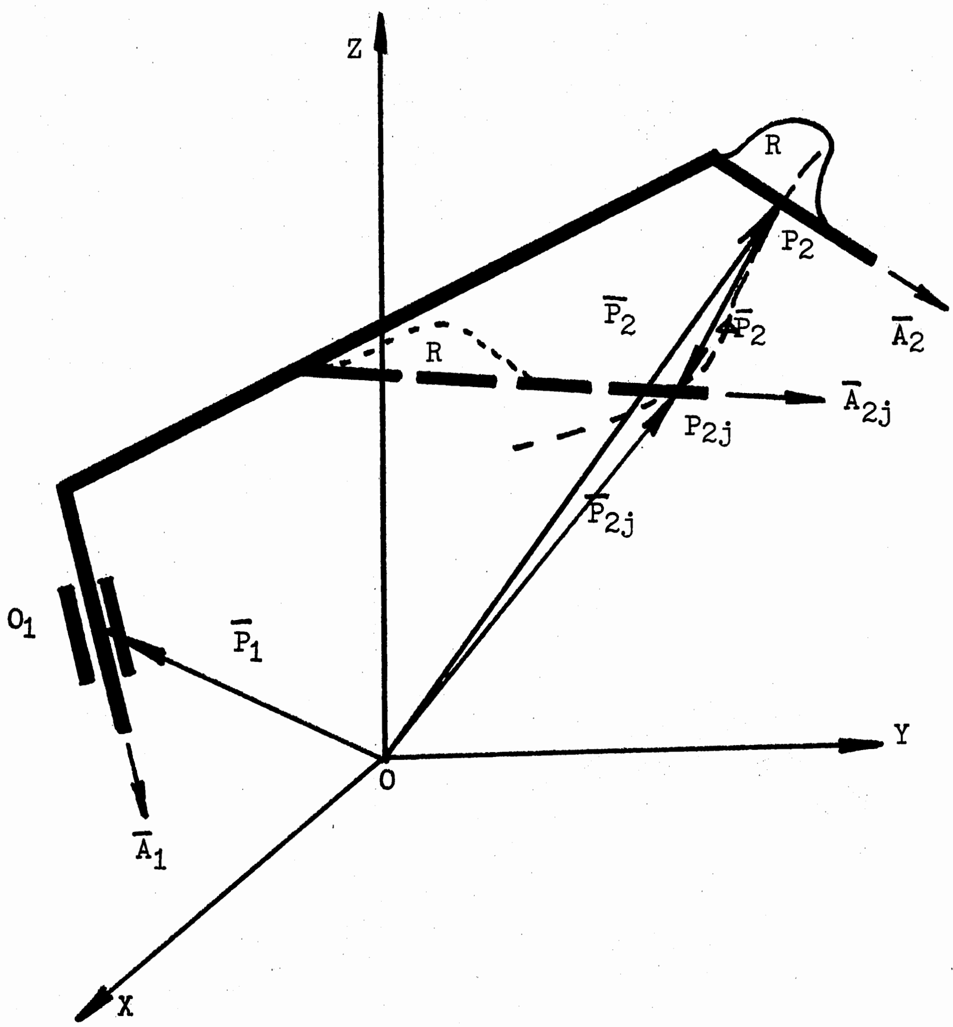

Figure 6. A Rigid Body R Attached to a Cylinder Pair

by the dotted curve C. This pair can experience dual motion, i.e., rotations about and translations along the axis $\overline{A}_1$ of the cylinderical pair.

The following vectors are defined:

$\overline{A}_1$ is a unit vector defining the direction of the pair axis.

$\overline{A}_2$ is a unit vector defining the direction of a line in R, or a orientation of a rigid body.

$\overline{P}_1$ is a vector from the origin to a point O_1 on the axis.

$\overline{P}_2$ is a vector from the origin to a point P_2 on the line.

The rigid body R is displaced to a new position R_j through dual motion at pair O_1 consisting of a rotation about $\overline{A}_1$ by θ_1 and translation S_1 along the axis $\overline{A}_1$.

The direction vector $\overline{A}_{2j}$ in the displaced R_j position is given in (82) as

$$\overline{A}_{2j} = (1 - \mathrm{COS}\theta_1)\ (\overline{A}_2 - \overline{A}_1) \cdot \overline{A}_1 + \mathrm{COS}\theta_1\overline{A}_2 + \mathrm{SIN}\theta_1\ (\overline{A}_1 \times \overline{A}_2) \quad (2.3)$$

The new position vector for $\overline{P}_{2j}$ is given as

$$\overline{P}_{2j} = \mathrm{COS}\theta_1\ ((\overline{P}_2 - \overline{P}_1) - ((\overline{P}_2 - \overline{P}_1)\ \overline{A}_1)\ \overline{A}_1) + \overline{P}_1 + \overline{A}_1 \cdot S_1 +$$
$$\mathrm{SIN}\theta_1\ (\overline{A}_1 \times (\overline{P}_2 - \overline{P}_1)) + ((\overline{P}_2 - \overline{P}_1)\ \overline{A}_1)\ \overline{A}_1 \quad (2.4)$$

Equations (2.3) and (2.4) constitute a set of equations which allow one sufficiently to specify a rigid body motion in space.

Figure 7 shows a rigid body being attached to two links where both joints can experience dual motion. Two dual rotations, $\theta_1 + \varepsilon\ S_1$ and $\theta_2 + \varepsilon\ S_2$ are about $\overline{A}_1$ and $\overline{A}_2$ axes respectively.

The direction vector $\overline{A}_{3j}$ corresponding to position R_j is given in

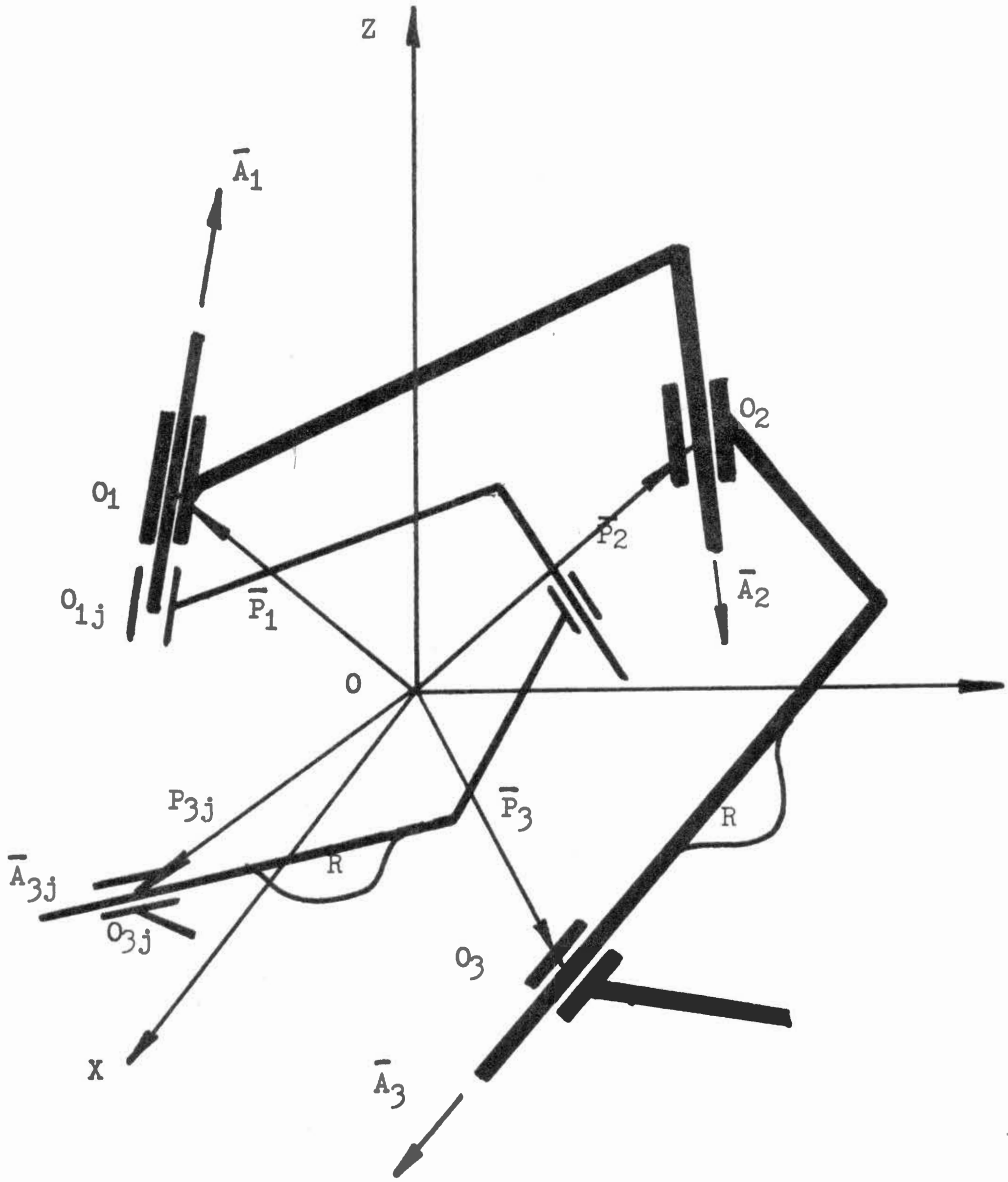

Figure 7. A Rigid Body R Attached to Two Cylinder Pairs

(82) as

$$\overline{A}_{3j} = (1 - \text{COS}\theta_1)\ (\overline{A}_{32} - \overline{A}_1)\ \overline{A}_1 + \text{COS}\theta_1\overline{A}_{32} + \text{SIN}\theta_1(\overline{A}_1 \times \overline{A}_{32}) \quad (2.5)$$

where

$$\overline{A}_{32} = (1 - \text{COS}\theta_2)\ (\overline{A}_3 - \overline{A}_1)\ \overline{A}_2 + \text{COS}\theta_2\overline{A}_2 + \text{SIN}\theta_2(\overline{A}_2 \times \overline{A}_3)$$

The new position vector is defined in (82) as

$$\overline{P}_{3j} = \text{COS}\theta_1((\overline{P}_{32} - \overline{P}_1) - ((\overline{P}_{32} - \overline{P}_1))\ \overline{A}_1) + \overline{P}_1 + \overline{A}_1\overline{S}_1 +$$

$$\text{SIN}\theta_1(\overline{A}_1 \times (\overline{P}_{32} - \overline{P}_1)) + ((\overline{P}_{32} - \overline{P}_1)\ \overline{A}_1)\ \overline{A}_1 \quad (2.6)$$

where

$$\overline{P}_{32} = \text{COS}\theta_2((\overline{P}_3 - \overline{P}_2) - ((\overline{P}_3 - \overline{P}_2)\ \overline{A}_2)\ \overline{A}_2) + \overline{P}_2 + \overline{A}_2\overline{S}_2 +$$

$$\text{SIN}\theta_2(\overline{A}_2 \times (\overline{P}_3 - \overline{P}_2))\ .$$

2.3. Velocity and Acceleration Analysis

One begins by considering cartesian coordinates OXYZ and a point P_2 on the rigid body R which moves with respect to reference frame as indicated in Figure 6. Let $\overline{P}_2$ denote the position vector $\overline{OP}_2$. At some instant of time, the point P_2 traces to point P_2 with trajectory C. The corresponding displacement vector is $\Delta P_2 = \overline{P_2P_{2i}}$. The velocity, V of the point P_2 with respect to the reference frame is the time rate of change of position. Thus,

$$V = \lim_{\Delta t \to 0} \frac{\Delta P_2}{\Delta t}$$

$$= \frac{dP_2}{dt}$$

$$= \bar{A}_1 \times (P_2 - P_1) \frac{d\theta_1}{dt} + \bar{A}_1 \frac{dS_1}{dt} . \qquad (2.7)$$

In general, the velocity is a function of time. The time rate of change of V gives the acceleration of point P_2 with respect to the reference frame.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$$= \frac{dv}{dt}$$

$$= \bar{A}_1 \times (\bar{P}_2 - \bar{P}_1) \frac{d^2\theta_1}{dt^2} + \frac{d\theta_1}{dt} \bar{A} \times \left(\frac{d\bar{P}_2}{dt} - \frac{d\bar{P}_1}{dt} \right) + \bar{A}_1 \frac{d^2S_1}{dt^2} . \qquad (2.8)$$

In Figure 6 direction vector $\bar{A}_2$ defines the orientation of a rigid body at P_2 while $\bar{A}_{2j}$ defines that of the same rigid body at position P. The angular velocity of a rigid body is the time rate of change of angular position with respect to the reference frame OXYZ.

$$\omega = \frac{dA_{2j}}{dt}$$

$$= (\bar{A}_1 \times \bar{A}_2) \times \frac{d\theta_1}{dt} . \qquad (2.9)$$

The angular velocity usually is also a function of time. The time rate of change of angular velocity is the angular acceleration with respect to reference frame OXYZ.

$$\alpha = \frac{d\omega}{dt}$$

$$= \frac{d^2\theta_1}{dt^2} (\bar{A}_1 \times \bar{A}_2) + (\bar{A}_1 + \frac{d\bar{A}_2}{dt}) \frac{d\theta_1}{dt} . \qquad (2.10)$$

2.4. Kinematic Analysis Utilizing Intermediate Frame

In order to describe the kinematic properties of any point located in the rigid body, viz., a center point, or a coupler point or a point where external load is acting, we introduce the concept of intermediate frames.

In Figure 8 the basic reference frame is OXYZ and the intermediate frame is o'x'y'z'. Our problem is to determine velocity and acceleration of point P with respect to reference frame OXYZ.

The position vector for point P is

$$\bar{R} = \bar{R}_0 + \bar{r} .$$

The velocity of point P with respect to the basic reference coordinate system is then,

$$\begin{aligned} V &= \frac{d\bar{R}}{dt} \\ &= \frac{d\bar{R}_0}{dt} + \frac{d\bar{r}}{dt} \\ &= \frac{dR_0}{dt} + \left(\left(\frac{\partial}{\partial t} \right)_{rel} + \omega X \right) \bar{r} \\ &= \frac{dR_0}{dt} + \left(\frac{\partial \bar{r}}{\partial t} \right)_{rel} + \omega X \bar{r} . \end{aligned} \tag{2.11}$$

The acceleration point P with respect to the reference frame OXYZ is

$$\begin{aligned} a &= \frac{dv}{dt} \\ &= \frac{d^2R_0}{dt^2} + \left(\left(\frac{\partial}{\partial t} \right)_{rel} + \omega X \right) \left(\left(\frac{\partial \bar{r}}{\partial t} \right)_{rel} + \omega X \bar{r} \right) \\ &= \frac{d^2R_0}{dt^2} + a_{rel} + \dot{\omega} X \bar{r} + 2\omega X \bar{V}_{rel} + \omega X (\omega X \bar{r}) \end{aligned} \tag{2.12}$$

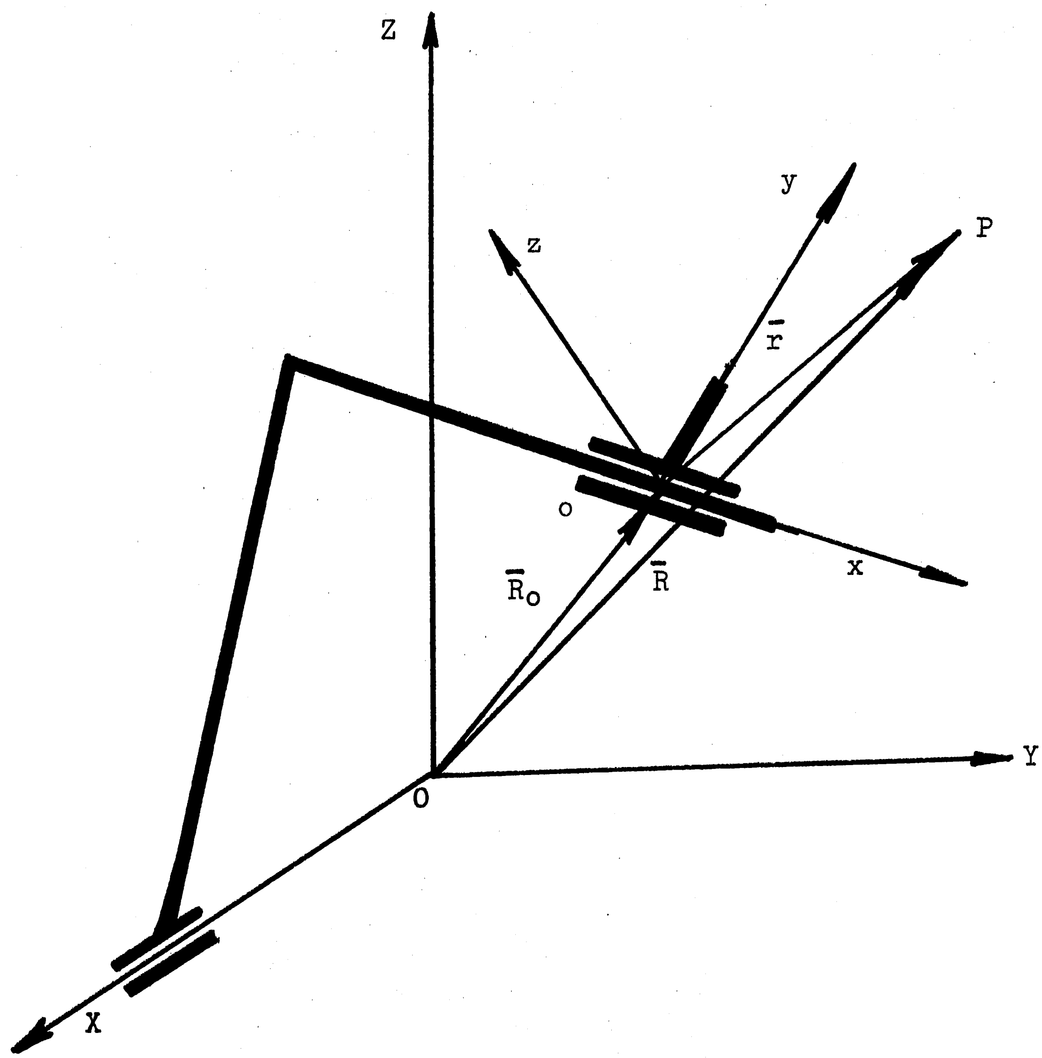

Figure 8. An Intermediate Coordinate Frame O'X'Y'Z' Respects to a Reference Frame OXYZ

where a_{rel} and ω are the relative linear acceleration and angular velocity of intermediate frame o'x'y'z'. The resultant acceleration of p with respect to frame OXYZ can be separated into five terms:

1. The first term indicates the acceleration of the origin of the intermediate coordinate O'X'Y'Z' with respect to coordinate OXYZ.
2. The second term is the relative acceleration of point P with respect to coordinate system O'X'Y'Z'.
3. The third term indicates the contribution due to angular acceleration.
4. The fourth term is called coriolis acceleration.
5. The last term is called centripetal acceleration.

The expressions of Equations (2.11) and (2.12) are corresponding to those of Equations (2.8) and (2.10). This is to prove the generality of the Equations (2.8) and (2.10).

2.5. Dual Vectors

There are three kinds of vectors: free, sliding and fixed vectors. Vectors having the same magnitude and direction but different properties cannot be mathematically operated together. Our experience shows that the combination of force and moment vectors becomes a mathematical problem. This problem could be resolved by introducing the concept of dual-complex numbers. A dual number is defined as a pair of real numbers expressed as

$$\hat{X} = X_0 + \varepsilon X_1 \qquad (2.13)$$

where X_0 is a real part and X_1 a dual part of dual $\hat{X}$. ε is a

Clifford's operator, where by definition $\varepsilon^2 = 0$. $\hat{V} = V + \varepsilon\omega$, $\hat{P} = P + \varepsilon H$ and $\hat{F} = F_0 + \varepsilon F_1$ represent dual velocity with linear velocity V and angular velocity ω, dual momentum $\hat{P}$ with linear momentum, H, and angular momentum H and dual force $\hat{F}$ with force F_0 and moment F_1.

The screw $\hat{a}$ can be drawn as shown in Figure 9 where

$$\bar{a}_0 = a_{0x}\bar{i} + a_{0y}\bar{j} + a_{0z}\bar{k}$$

and

$$\bar{a}_1 = a_{1x}\bar{i} + a_{1y}\bar{j} + a_{1z}\bar{k} .$$

Vector r is the vector product of a_0 and a_1 and indicates the position of the line vector a_0. $\hat{a}$ is parallel to a_0.

The location of a rigid body can be written in a dual form.

$$\hat{a} = \bar{a}_0 + \varepsilon\, \bar{a}_0 \times \bar{a}_1 \qquad (2.14)$$

where

a_0 represents the direction of the line vector on the rigid body.

a_1 represents the position vector of a point on the rigid body.

To convert a screw into a dual vector, multiply the dual vector by the screw. The multiplication of a dual vector by a dual scalar changes the dual magnitude of the screw but does not change its direction. Figure 10 shows a dual vector $\hat{A} = \bar{A}_0 + \varepsilon\, \bar{A}_1 \times \bar{A}_1$ where $\hat{A}$ is parallel to vector A_0 and position vector $\bar{A}_1 = \bar{A}_0 \times (\bar{A}_0 \times \bar{A}_1)$.

An ordinary function F (X) with dual argument $\hat{X} = X_0 + \varepsilon X_1$ is decomposed into real and dual parts by the formula:

$$\hat{F}\,(X) = F\,(X_0) + \varepsilon\, X_1 \frac{dF(X_0)}{dX_0} \qquad (2.15)$$

For example:

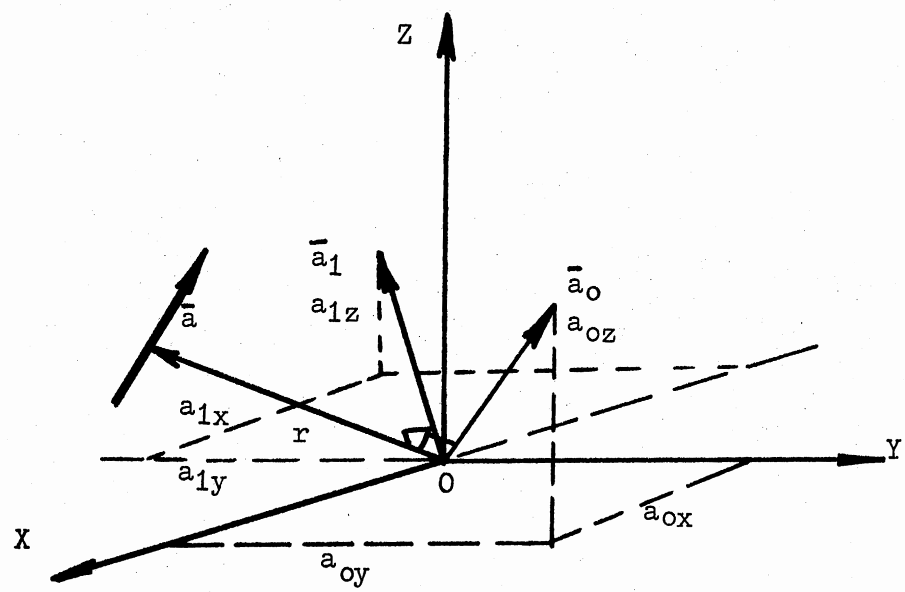

Figure 9. A Dual Vector $\hat{a}$

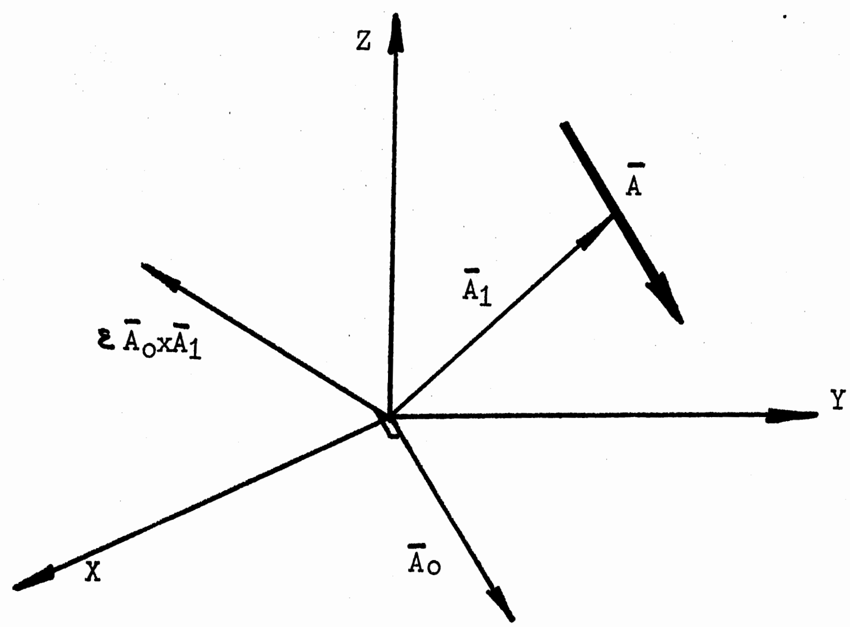

Figure 10. A Dual Vector $\hat{A}$

$$SIN(X) = SIN(X_0) + \varepsilon\ X_1 COS(X_0)$$

$$COS(X) = COS(X_0) - \varepsilon\ X_1 SIN(X_0)$$

2.6. Kinematic Analysis of the RCSR Spatial Mechanism

The geometry and the parameters of the RCSR mechanism are shown in Figure 11. θ_1 is an input angle. θ_4 is an output angle. P_1, P_2, P_3, and P_4 are the link lengths of link 1, 2 and 3 respectively. a_1, a_4 are link twist angles of links 1 and 4 respectively. S_1 and S_4 are link lengths of link 1 and 4 respectively. θ_2, S_2, θ_4 are variables.

2.6.1. Freudenstein Displacement Equation of the RCSR Spatial Mechanism

Since the spherical pair is the highest kinematic pair in this case, one breaks the mechanism at this pair. Therefore, the open loop chain is unfolded as shown in Figure 12. Sign conversions are defined such that screws that locate left of origin are negative screws, others are positive screws.

The position vectors and direction vectors at the unfolded position are defined from Figure 12.

$$\overline{A}_1 = \overline{i}$$

$$\overline{A}_2 = COSa_1\overline{i} + SINa_1\overline{k}$$

$$\overline{A}_4 = COSa_4\overline{i} - SINa_4\overline{k}$$

$$\overline{0} = 0\ .$$

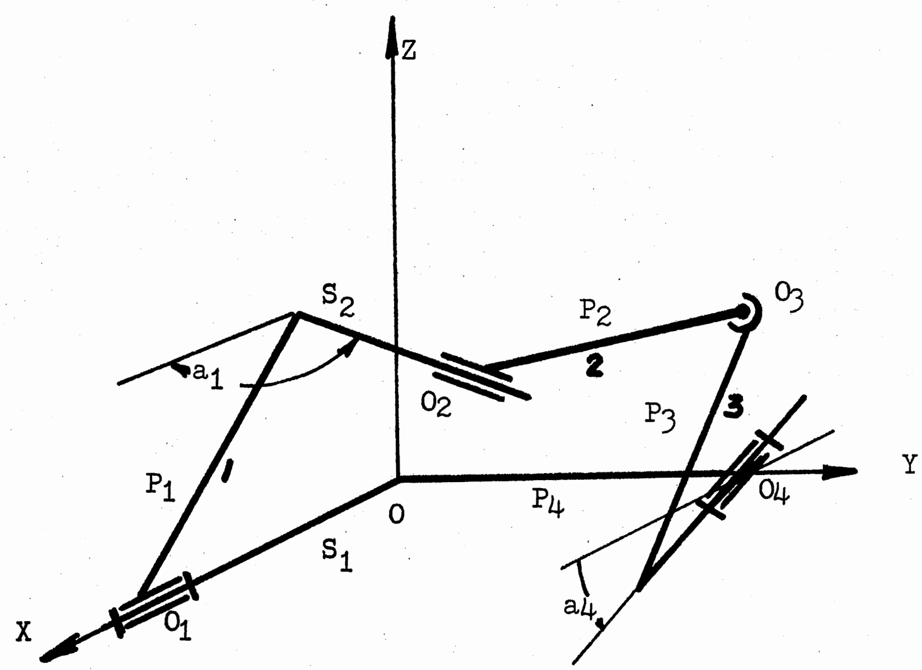

Figure 11. The RCSR Spatial Mechanism

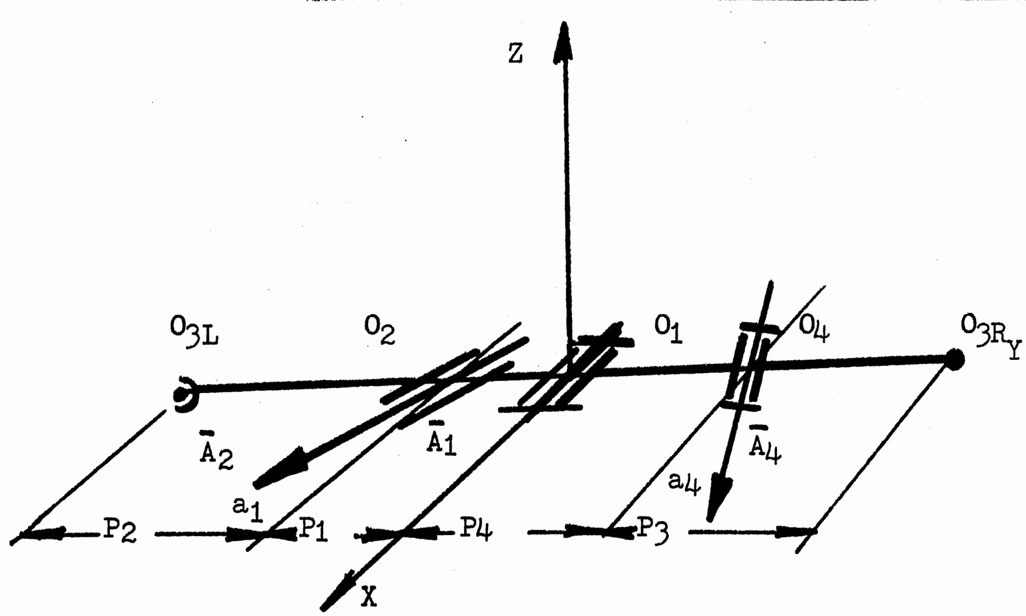

Figure 12. The Open Loop Chain SCRRS

$$\overline{O}_2 = -P_1\overline{j}$$

$$\overline{O}_{3L} = -(P_1 + P_2)\overline{j}$$

$$\overline{O}_4 = P_4\overline{j}$$

$$\overline{O}_{3R} = (P_4 + P_3)\overline{j}$$

The loop-closure-equation is written by transforming O_{3L} through screw $\overline{A}_2$ with rotation θ_2 and translation S_2. This screw is a negative screw. Let subscript j denote the new position j.

$$\overline{O}_{32j} = \text{COS}\theta_2((\overline{O}_3 - \overline{O}_2) - ((\overline{O}_3 - \overline{O}_2)\overline{A}_2)\overline{A}_2) -$$

$$\text{SIN}\theta_2(\overline{A}_2 \times (\overline{O}_3 - \overline{O}_2)) - S_2\overline{A}_2 + \overline{O}_2$$

$$= \begin{Bmatrix} -P_2Sa_1S\theta_2 - S_2Ca_1 \\ -P_2C\theta_2 - P_1 \\ P_2Ca_1S\theta_2 - S_2Sa_1 \end{Bmatrix} \qquad (2.16)$$

where sin and cosin are abbreviated by S and C respectively.

Transforming O_{32j} through negative screw A_1 with rotation θ_1 and translation S_1, we have

$$\overline{O}_{31j} = \text{COS}\theta_1((\overline{O}_{32j} - \overline{O}_1) - ((\overline{O}_{32j} - \overline{O}_1)\overline{A}_1)\overline{A}_1) - \text{SIN}\theta_1(\overline{A}_1 \times$$

$$(\overline{O}_{32j} - \overline{O}_1)) - S_1\overline{A}_1 + \overline{O}_1$$

$$= \begin{Bmatrix} -S_1 - P_2Sa_1S\theta_2 - S_2Ca_1 \\ C\theta_1(-P_2C\theta_2 - P_1) + S\theta_1(P_2Ca_1S\theta_2 - S_2Sa_1) \\ C\theta_1(P_2Ca_1S\theta_2 - S_2Sa_1) + S\theta_1(P_2C\theta_2 + P_1) \end{Bmatrix}. \qquad (2.17)$$

$\overline{O}_{3Lj}$ is the new position of $\overline{O}_{3L}$ after applying successive screws transformation.

The second-loop-closure equation is obtained by transforming the O_{3R} through positive screw $\overline{A}_4$ with rotation θ_4 and translation S_4. Thus,

$$\overline{O}_{3Rj} = C\theta_4((\overline{O}_{3R} - \overline{O}_4) - ((\overline{O}_{3R} - \overline{O}_4)\ \overline{A}_4)\ \overline{A}_4) + S\theta_4(\overline{A}_4 \times (\overline{O}_{3R} - \overline{O}_4)) + S_4\overline{A}_4 + \overline{O}_4$$

$$= \begin{Bmatrix} P_3Sa_4S\theta_4 + S_4Ca_4 \\ P_3C\theta_4 + P_4 \\ P_3Ca_4S\theta_4 - Sa_4S_4 \end{Bmatrix} . \qquad (2.18)$$

Equating the two final positions of $\overline{O}_{3Rj}$ and $\overline{O}_{3Lj}$, one has

$$\overline{O}_{3rj} = \overline{O}_{3Lj} . \qquad (2.19)$$

After rearranging Equation (2.19) to have

$$S\theta_2\overline{D}_8 + C\theta_2\overline{D}_7 + S_2\overline{D}_9 + \overline{D}_{10} = S\theta_4\overline{D}_1 + C\theta_4\overline{D}_2 + \overline{D}_3 \qquad (2.20)$$

where

$$\overline{D}_1 = P_3Sa_4\overline{i} + P_3Ca_4\overline{k}$$

$$\overline{D}_2 = P\ \overline{j}$$

$$\overline{D}_3 = S_4Ca_4\overline{i} + P_4\overline{j} - Sa_4S_4\overline{k}$$

$$\overline{D}_7 = -\ P_2C\theta_1\overline{j} + P_2S\theta_1\overline{k}$$

$$\overline{D}_8 = -\ P_2Sa_1\overline{i} + P_2Ca_1S\theta_1\overline{j} + P_2Ca_1C\theta_1\overline{k}$$

$$\overline{D}_9 = - Ca_1\overline{i} - Sa_1S\theta_1\overline{j} - Sa_1C\theta_1\overline{k}$$

$$D_{10} = - S_1\overline{i} - P_1C\theta_1\overline{j} + P_1S\theta_1\overline{k}$$

Eliminating $S\theta_2$ and $C\theta_2$ from Equation (2.20) by dotting $(\overline{D}_8 \times \overline{D}_9)$ and $(\overline{D}_7 \times \overline{D}_9)$, one has

$$C\theta_2 = S\theta_4E_4 + C\theta_4E_5 + E_6 \tag{2.21}$$

and

$$S\theta_2 = S\theta_4E_1 + C\theta_4E_2 + E_3 \tag{2.22}$$

where

$$E_1 = \overline{D}_1 (\overline{D}_7 \times \overline{D}_9) / \overline{D}_8 (\overline{D}_7 \times \overline{D}_9)$$

$$E_2 = \overline{D}_2 (\overline{D}_7 \times \overline{D}_9) / \overline{D}_8 (\overline{D}_7 \times \overline{D}_9)$$

$$E_3 = (\overline{D}_3 - \overline{D}_{10}) (\overline{D}_7 \times \overline{D}_9) / \overline{D}_8 (\overline{D}_7 \times \overline{D}_9)$$

$$E_4 = \overline{D}_1 (\overline{D}_8 \times \overline{D}_9) / \overline{D}_7 (\overline{D}_8 \times \overline{D}_9)$$

$$E_5 = \overline{D}_2 (\overline{D}_8 \times \overline{D}_9) / \overline{D}_7 (\overline{D}_8 \times \overline{D}_9)$$

$$E_6 = (\overline{D}_3 - \overline{D}_{10}) (\overline{D}_8 \times \overline{D}_9) / \overline{D}_7 (\overline{D}_8 \times \overline{D}_9)$$

Squaring Equations (2.21) and (2.22), using the identity, $C\theta_2^2 + S\theta_2^2 = 1$, and introducing the half angle relationship, $S\theta_4 = 2\Phi_4 / (1 + \Phi_4^2)$ and $C\theta_4 = (1 - \Phi_4^2) / (1 + \Phi_4^2)$ where $\Phi = \tan\left(\frac{\theta_4}{2}\right)$, one has

$$\Phi_4^4C(1) + \Phi_4^3C(2) + \Phi_4^2C(3) + \Phi_4C(4) + C(5) = 0 \tag{2.23}$$

where

$$C(1) = F_1 + F_3 - F_5$$

$$C(2) = -2F_4 + 2F_6$$

$$C(3) = -2F_1 + 4F_2 + 2F_3$$

$$C(4) = 2F_4 + 2F_6$$

$$C(5) = F_1 + F_3 + F_5$$

$$F_1 = E_2^2 + E_5^2$$

$$F_2 = E_1^2 + E_4^2$$

$$F_3 = E_3^2 + E_6^2 - 1$$

$$F_4 = 2E_1E_2 + 2E_4E_5$$

$$F_5 = 2E_2E_3 + 2E_5E_6$$

$$F_6 = 2E_1E_3 + 2E_4E_6$$

After solving Equation (2.23), one has

$$\theta_4 = 2 * \tan^{-1} (\Phi_4) .$$

2.6.2. Expressions for θ_2 and S_2 and Specified Positions P_{m1}, P_{m2}, P_{m3}, O_1, O_{2j}, O_{3j} and O_4

Substituting θ_4 from Equation (2.23), into Equations (2.21), and (2.22), one has

$$\theta_2 = \tan^{-1} \left(\frac{S\theta_2}{C\theta_2} \right) .$$

Care must be taken in order to decide the correct quadrant of θ_2.

Dotting Equation (2.20) by $(\overline{D}_8 \times \overline{D}_7)$, one gets

$$S_2 = \frac{(S\theta_4\overline{D}_1 + C\theta_4\overline{D}_2 + \overline{D}_3 - \overline{D}_{10})\ (\overline{D}_8 \times \overline{D}_7)}{\overline{D}_9 \cdot (\overline{D}_8 \times \overline{D}_7)} . \qquad (2.24)$$

After calculations of θ_4, θ_2 and S_2 are completed, his next task is to locate specified points being investigated on the rigid body. Figure 13 shows mass centers of link 1, 2, and 3 are P_{m1}, P_{m2}, and P_{m3} whose positions are preferably measured parallel to the pair axes reference coordinates.

The position of P_{m3} at initial unfolded position is (X_3, Y_3, Z_3) which is measured to the reference coordinates. Thus,

$$\overline{P}_{m3} = \left\{\begin{matrix} Ca_4 & 0 & Sa_4 \\ 0 & 1 & 0 \\ -Sa_4 & 0 & Ca_4 \end{matrix}\right\} \begin{bmatrix} X_3 \\ Y_3 \\ Z_3 \end{bmatrix} + \overline{P}_4$$

$$= \left\{\begin{matrix} X_3Ca_4 + Z_3Sa_4 \\ Y_3 \\ -X_3Sa_4 + Z_3Ca_4 \end{matrix}\right\} + \overline{P}_4 . \qquad (2.25)$$

The new displaced position of P_{m3} after rotating about A_4 axis with θ_4 and translation S_4 along the same axis becomes

$$\overline{P}_{m3j} = COS\theta_4((\overline{P}_{m3} - \overline{P}_4) - (\overline{P}_{m3} - \overline{P}_4)\ \overline{A}_4)\ \overline{A}_4) +$$

$$SIN\theta_4(\overline{A}_4 \times (\overline{P}_{m3} - \overline{P}_4)) + S_4\overline{A}_4 +$$

$$((\overline{P}_{m3} - \overline{P}_4)\ \overline{A}_4)\ \overline{A}_4 + \overline{P}_4$$

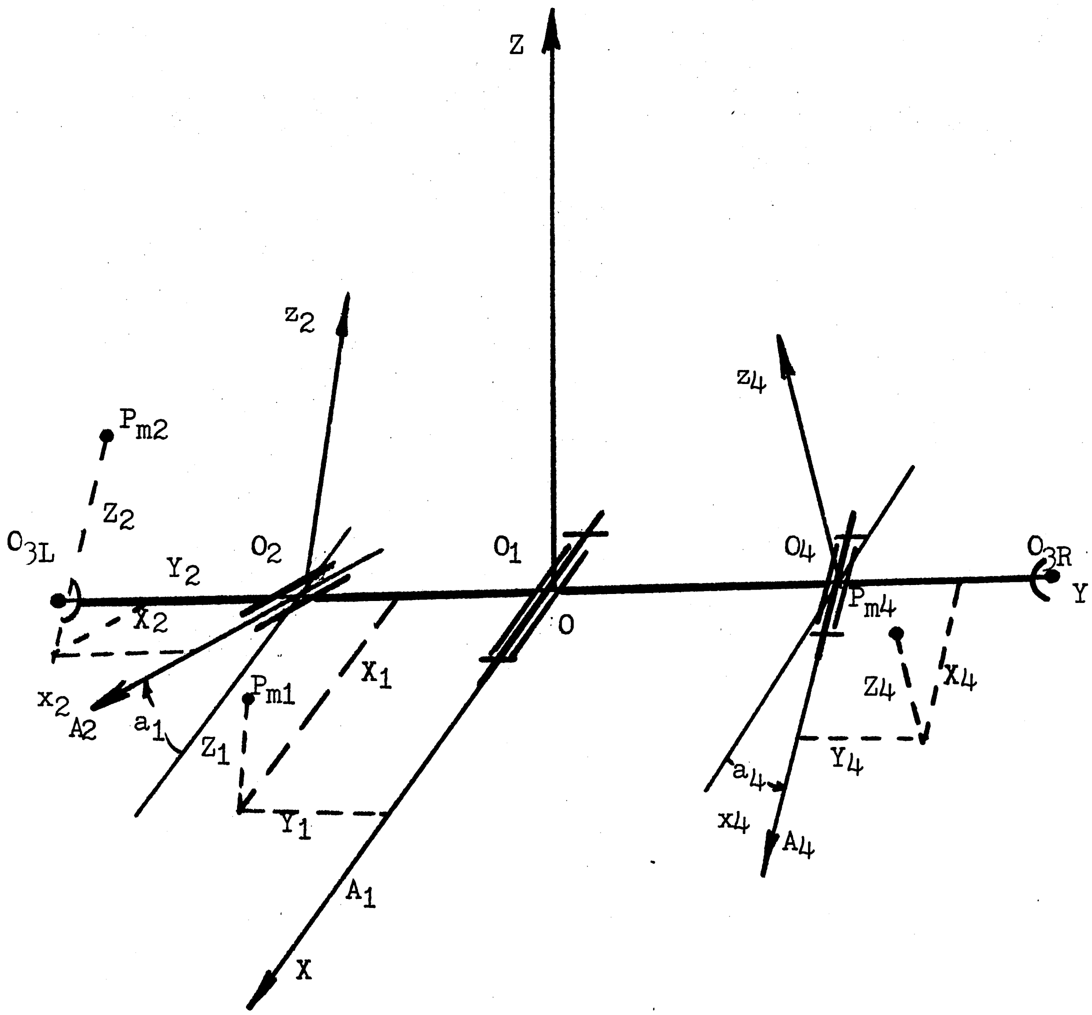

Figure 13. Location of a Point on Each Link

$$= \begin{Bmatrix} Z_3Sa_4C\theta_4 + Y_3S\theta_4Sa_4 + S_4Ca_4 + X_3Ca_4 \\ Y_3C\theta_4 - Z_3S\theta_4 + P_4 \\ Z_3C\theta_4Ca_4 + Y_3S\theta_4Ca_4 - S_4Sa_4 - X_3Sa_4 \end{Bmatrix} . \qquad (2.26)$$

Using the same procedure for P_{m1} and P_{m2}, one has

$$\overline{P}_{m1j} = C\theta_1 \; ((\overline{P}_{m1} - \overline{P}_1) - ((\overline{P}_{m1} - \overline{P}_1) \; \overline{A}_1) \; \overline{A}_1) -$$

$$S\theta_1 \; (\overline{A}_1 \times (\overline{P}_{m1} - \overline{P}_1)) - S_1\overline{A}_1 +$$

$$((\overline{P}_{m1} - \overline{P}_1) \; \overline{A}_1) \; \overline{A}_1 + \overline{P}_1$$

$$= \begin{Bmatrix} X_1 - S_1 \\ Y_1C\theta_1 + Z_1S\theta_1 \\ -Y_1S\theta_1 + Z_1C\theta_1 \end{Bmatrix} \qquad (2.27)$$

$$\overline{P}_{m2} = \begin{Bmatrix} X_2Ca_1 - Z_2Sa_1 \\ Y_2 \\ X_2Sa_1 + Z_2Ca_1 \end{Bmatrix} + \overline{P}_2$$

$$\overline{P}_{m2j} = \begin{Bmatrix} -S_1 - Z_2Sa_1C\theta_2 + S\theta_2Sa_1Y_2 - S_2Ca_1 + X_2Ca_1 \\ C\theta_1(Y_2C\theta_2 + S\theta_2Z_2 - P_2) - S\theta_1(-Ca_1Z_2C\theta_2 + Ca_1Y_2S\theta_2 + \\ S_2Sa_1 - X_2Sa_1) \\ C\theta_1(Ca_1Z_2C\theta_2 - Ca_1Y_2S\theta_2 - S_2Sa_1 + X_2Sa_1) - S\theta_1(Y_2C\theta_2 + \\ S\theta_2Z_2 - P_2) \end{Bmatrix} \qquad (2.28)$$

The new displaced position O_2 of cylinder pair is

$$\overline{O}_{2j} = \overline{O}_{2j} + \overline{A}_{2j}S_2$$

$$= \begin{Bmatrix} -S_1 - S_2Ca_1 \\ -C\theta_1P_1 - S_2S\theta_1Sa_1 \\ -C\theta_1S_2Sa_1 + S\theta_1P_1 \end{Bmatrix} . \tag{2.29}$$

The new displaced position O_3 of spherical pair is

$$\overline{O}_{3j} = \overline{O}_{3Lj} . \tag{2.30}$$

The new displaced position O_1 of revolute pair is

$$\overline{O}_1 = \begin{Bmatrix} -S_1 \\ 0 \\ 0 \end{Bmatrix} . \tag{2.31}$$

The new displaced position O_4 of revolute pair is

$$\overline{O}_4 = \begin{Bmatrix} S_4Ca_4 \\ P_4 \\ -Sa_4S_4 \end{Bmatrix} . \tag{2.32}$$

2.6.3. Dual Screws

Figure 14 shows direction vectors $\overline{A}_1$, $\overline{A}_2$, $\overline{A}_4$, $\overline{C}$ and $\overline{B}$ at the unfolded position. The dual vector $\hat{A}_1$, $\hat{A}_2$, $\hat{A}_4$, $\hat{C}$, $\hat{B}$, $\hat{D}$ at the displaced position are shown in Figure 15. Let us define $\overline{B}$ and $\overline{C}$ at the unfolded

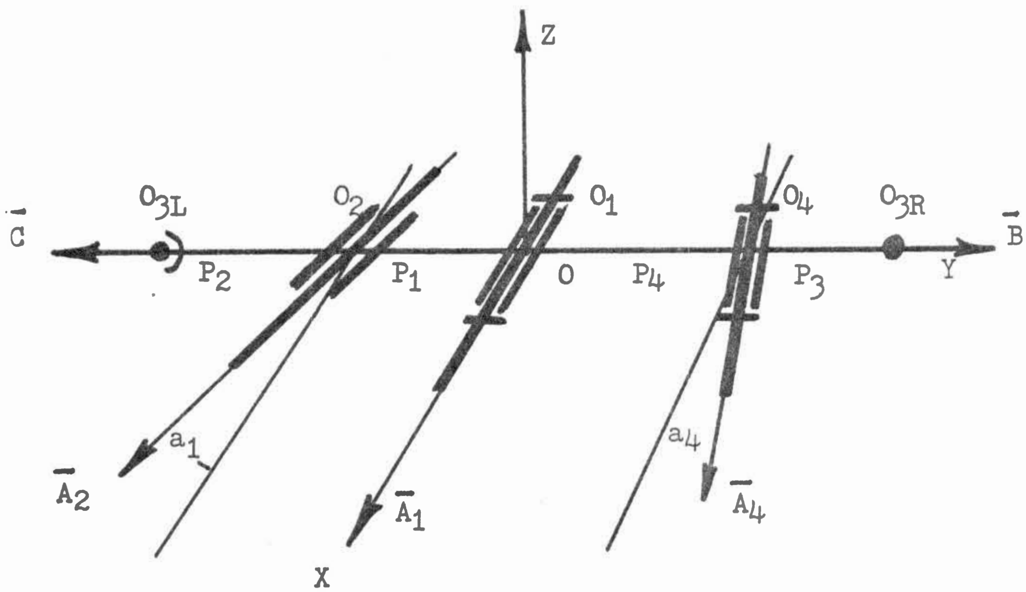

Figure 14. The Direction Vectors $\bar{B}$, $\bar{C}$, $\bar{A}_1$, $\bar{A}_2$ and $\bar{A}_3$ at the Unfolded Position

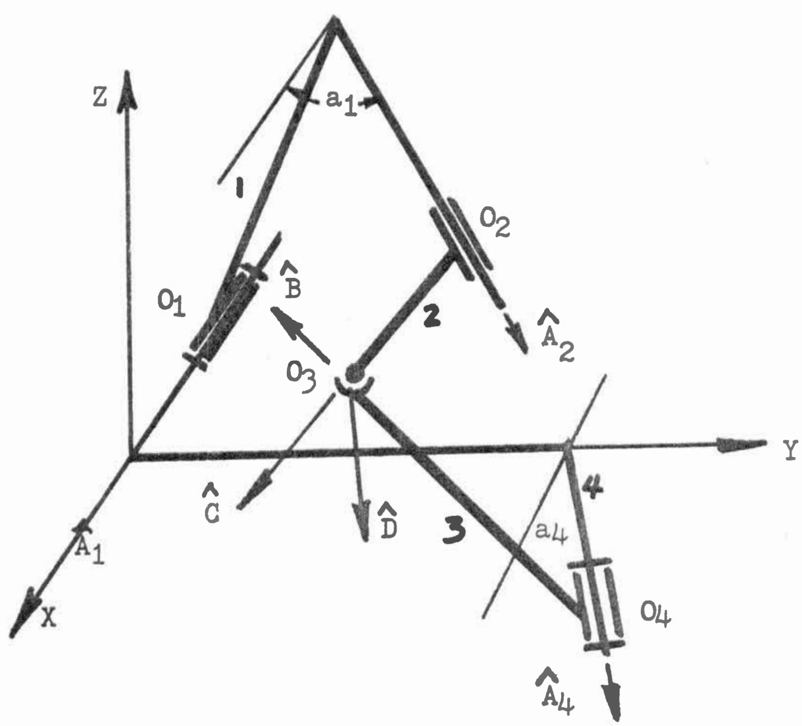

Figure 15. The Dual Vectors $\hat{A}_1$, $\hat{A}_2$, $\hat{A}_4$, $\hat{C}$, $\hat{B}$, $\hat{D}$ at the Unfolded Position

position as in the following manner:

$$\bar{B} = j$$

$$\bar{C} = -j$$

At the displaced position, $\bar{B}$ and $\bar{C}$ become

$$\bar{B}_j = \begin{Bmatrix} S\theta_4 Sa_4 \\ C\theta_4 \\ S\theta_4 Ca_4 \end{Bmatrix} \tag{2.33}$$

$$\bar{C}_j = \begin{Bmatrix} -S\theta_2 Sa_1 \\ -C\theta_1 C\theta_2 + S\theta_1 S\theta_2 Ca_1 \\ C\theta_1 S\theta_2 Ca_1 + S\theta_1 C\theta_2 \end{Bmatrix} \tag{2.34}$$

Thus, $\bar{D}$ can be found from Equation (2.35)

$$\bar{D}_j = \bar{B}_j \times \bar{C}_j \tag{2.35}$$

The displaced direction vector A_2 becomes

$$\bar{A}_{2j} = \begin{Bmatrix} Ca_1 \\ S\theta_1 Sa_1 \\ C\theta_1 Sa_1 \end{Bmatrix} \tag{2.36}$$

Applying the procedures defined in Section 2.5, we define the following

$$\hat{A}_1 = \overline{A}_{10} + \varepsilon \overline{A}_{11} \times \overline{A}_{10} \quad (2.37)$$

$$\hat{A}_2 = \overline{A}_{20} + \varepsilon \overline{A}_{21} \times \overline{A}_{20} \quad (2.38)$$

$$\hat{A}_4 = \overline{A}_{40} + \varepsilon \overline{A}_{41} \times \overline{A}_{40} \quad (2.39)$$

$$\hat{B} = \overline{B}_0 + \varepsilon \overline{B}_1 \times \overline{B}_0 \quad (2.40)$$

$$\hat{C} = \overline{C}_0 + \varepsilon \overline{C}_1 \times \overline{C}_0 \quad (2.41)$$

$$\overline{D}_0 = \overline{B}_0 \times \overline{C}_0 \quad (2.42)$$

$$\hat{D} = \overline{D}_0 + \varepsilon \overline{D}_1 \times \overline{D} \quad (2.43)$$

where

$$\overline{A}_{11} = \overline{O}_1$$

$$\overline{A}_{10} = \overline{A}_1$$

$$\overline{A}_{20} = \overline{A}_{2j}$$

$$\overline{A}_{21} = \overline{O}_{2j}$$

$$\overline{A}_{40} = \overline{A}_4$$

$$\overline{A}_{41} = \overline{O}_4$$

$$\overline{B}_1 = \overline{O}_{3j}$$

$$\overline{C}_1 = \overline{O}_{3j}$$

$$\overline{D}_1 = \overline{O}_{3j}$$

$$\overline{B}_0 = \overline{B}_j$$

$$\overline{C}_0 = \overline{C}_j$$

$$\overline{D}_0 = \overline{D}_j$$

2.6.4. Velocity Analysis

Differentiating Equations (2.17) and (2.18), one has

$$\frac{d\overline{O}_{3Rj}}{dt} = (\overline{A}_{2j} \times (\overline{P}_{31j} - \overline{P}_{2j}))\ \omega_2 + \overline{A}_{2j}\dot{S}_2 + (\overline{A}_1 \times (\overline{P}_{3Lj} - \overline{P}_{1j}))\ \omega_1$$

$$= \overline{N}_1\omega_2 + \overline{N}_2\dot{S}_2 - \overline{N}_4\omega_1 \qquad (2.44)$$

and

$$\frac{d\overline{O}_{3Rj}}{dt} = \overline{A}_4 \times (\overline{P}_{3Rj} - \overline{P}_{4j})\ \omega_4$$

$$= \overline{N}_3\omega_4 \qquad (2.45)$$

where

$$\overline{N}_1 = \begin{Bmatrix} P_2Sa_1C\theta_2 \\ -P_2S\theta_2C\theta_1 - P_2Ca_1C\theta_2S\theta_1 \\ S\theta_1P_2S\theta_2 - P_2Ca_1C\theta_1C\theta_2 \end{Bmatrix}$$

$$\overline{N}_2 = \begin{Bmatrix} Ca_1 \\ S\theta_1Sa_1 \\ C\theta_1Sa_1 \end{Bmatrix}$$

$$\overline{N}_4 = \begin{Bmatrix} 0 \\ C\theta_1(P_2Ca_1S\theta_2 - S_2Sa_1) + S\theta_1(P_2C\theta_2 + P_1) \\ C\theta_1(-P_2C\theta_2 - P_1) + S\theta_1(P_2Ca_1S\theta_2 - S_2Sa_1) \end{Bmatrix}$$

$$\overline{N}_3 = \begin{Bmatrix} +P_3 Sa_4 C\theta_4 \\ -P_3 S\theta_4 \\ +P_3 Ca_4 C\theta_4 \end{Bmatrix} \omega_4$$

Equating Equations (2.44) and (2.45), one gets

$$\frac{d\overline{O}_{3Lj}}{dt} = -\frac{d\overline{O}_{3Rj}}{dt} \tag{2.46}$$

or

$$\overline{N}_1\omega_2 + \overline{N}_2\dot{S}_2 + \overline{N}_3\omega_4 = \overline{N}_4\omega_1 \tag{2.47}$$

Solving for ω_2, $\mathring{S}_2$ and ω_4, one has

$$\omega_2 = \frac{\overline{N}_4\ (\overline{N}_2 \times \overline{N}_3)}{N_1\ (N_2 \times N_3)}\ \omega_1 \tag{2.48}$$

$$\mathring{S}_2 = \frac{\overline{N}_4\ (\overline{N}_1 \times \overline{N}_3)}{\overline{N}_2\ (\overline{N}_1 \times \overline{N}_3)}\ \omega_1 \tag{2.49}$$

$$\omega_4 = \frac{\overline{N}_4\ (\overline{N}_1 \times \overline{N}_2)}{\overline{N}_3\ (\overline{N}_1 \times \overline{N}_2)}\ \omega_1 \tag{2.50}$$

Differentiating Equation (2.36), one gets

$$\frac{d\overline{A}_{2j}}{dt} = \begin{Bmatrix} 0 \\ -C\theta_1 Sa_1 \\ S\theta_1 Sa_1 \end{Bmatrix} \omega_1 \tag{2.51}$$

The velocity of point O_3 is

$$\frac{d\overline{O}_{3j}}{dt} = \overline{N}_1\omega_2 + \overline{N}_2\dot{S}_2 - \overline{N}_4\omega_1 \tag{2.52}$$

The velocity of point O_2 is

$$\frac{d\overline{O}_{2j}}{dt} = \overline{N}_2\dot{S}_2 + \begin{Bmatrix} 0 \\ C\theta_1S_2Sa_1 - P_1S\theta_1 \\ -C\theta_1P_1 - S_2S\theta_1Sa_1 \end{Bmatrix} \omega_1 \qquad (2.53)$$

The velocity of point P_{m3} is

$$\frac{d\overline{P}_{m3j}}{dt} = \overline{A}_4 \times (\overline{P}_{m3j} - \overline{P}_{4j})\ \omega_4$$

$$= \begin{Bmatrix} V_{pm3x} \\ V_{pm3y} \\ V_{pm3z} \end{Bmatrix} \omega_4 \qquad (2.54)$$

where

$$V_{pm3x} = Sa_4\ (Y_3C\theta_4 - Z_3S\theta_4)$$

$$V_{pm3y} = -Z_3C\theta_4 - Y_3S\theta_4$$

$$V_{pm3z} = Ca_4(Y_3C\theta_4 - Z_3S\theta_4)$$

The velocity of point P_{m1} is

$$\frac{d\overline{P}_{m1j}}{dt} = \overline{A}_1 \times (\overline{P}_{m1j} - \overline{P}_{1j})\ \omega_1$$

$$= \begin{Bmatrix} 0 \\ V_{pm1y} \\ V_{pm1z} \end{Bmatrix} \omega_1 \qquad (2.55)$$

where

$$V_{pm1y} = -(Y_1 S\theta_1 + Z_1 C\theta_1)$$

$$V_{pm1z} = Y_1 C\theta_1 + Z_1 S\theta_1$$

The velocity of point P_{m2} is

$$\frac{d\bar{P}_{m2j}}{dt} = \bar{A}_{2j} \times (\bar{P}_{m2j} - \bar{P}_{2j})\ \omega_2 + \bar{A}_2\ S_{2j} + \bar{A}_1 \times (\bar{P}_{m2j} - \bar{P}_{1j})\ \omega_1$$

$$= \begin{Bmatrix} V_{pm2x} \\ V_{pm2y} \\ V_{pm2z} \end{Bmatrix} \qquad (2.56)$$

where

$$V_{pm2x} = (S\theta_1 Sa_1 (C\theta_1 (Ca_1 Z_2 C\theta_2 - Ca_1 Y_2 S\theta_2 + X_2 Sa_1) - S\theta_1 (Y_2 C\theta_2 + S\theta_2 Z_2)) - C\theta_1 Sa_1 (C\theta_1 (Y_2 C\theta_2 + S\theta_2 Z_2) - S\theta_1 (-Ca_1 Z_2 C\theta_2 + Ca_1 Y_2 S\theta_2 - X_2 Sa_1))\omega_2 + Ca_1 \dot{S}_2$$

$$V_{pm2y} = (-Ca_1 (C\theta_1 (Ca_1 Z_2 C\theta_2 - Ca_1 Y_2 S\theta_2 + X_2 Sa_1) - S\theta_1 (Y_2 C\theta_2 + S\theta_2 Z_2)) + C\theta_1 Sa_1 (-Z_2 Sa_1 C\theta_2 + S\theta_2 Sa_1 Y_2 + X_2 Ca_1))\omega_2 + S\theta_1 Sa_1 \mathring{S}_2 + (-C\theta_1 (Ca_1 Z_2 C\theta_2 - Ca_1 Y_2 S\theta_2 - S_2 Sa_1 + X_2 Sa_1) + S\theta_1 (Y_2 C\theta_2 + S\theta_2 Z_2 - P_2))\omega_1$$

$$V_{pm2z} = (Ca_1 (C\theta_1 (Y_2 C\theta_2 + S\theta_2 Z_2) - S\theta_1 (-Ca_1 Z_2 C\theta_2 + Ca_1 Y_2 S\theta_2 - X_2 Sa_1)) - S\theta_1 Sa_1 (-Z_2 Sa_1 C\theta_2 + S\theta_2 Sa_1 Y_2 + X_2 Ca1))\omega 2 + C\theta_1 Sa_1 \mathring{S}_2 + (C\theta_1 (Y_2 C\theta_2 + S\theta_2 Z_2 - P_2) - S\theta_1 (-Ca_1 Z_2 C\theta_2 + Ca_1 Y_2 S\theta_2 + S_2 Sa_1 - X_2 Sa1))\omega 1$$

2.6.5. Acceleration Analysis

Differentiating Equations (2.44) and (2.45), we get

$$\frac{d^2\overline{P}_{3Lj}}{dt} = (\overline{A}_{2j} \times (\frac{d\overline{P}_{3j}}{dt} - \frac{d\overline{P}_{2j}}{dt}) + \frac{d\overline{A}_{2j}}{dt} \quad (\overline{P}_{3j} - \overline{P}_{2j}))\omega_2 +$$

$$(\overline{A}_{2j} \times (\overline{P}_{3j} - \overline{P}_{2j}))\alpha_2 + \overline{A}_{2j}\ddot{S}_2 + \frac{d\overline{A}_{2j}}{dt}\dot{S}_2 +$$

$$(A_{1j} \times (\frac{d\overline{P}_{3j}}{dt})\omega_1 + (\overline{A}_{1j} \times (\overline{P}_{3j} - \overline{P}_{1j}))\alpha_1 \tag{2.57}$$

$$\frac{d^2P_{3Rj}}{dt} = \overline{D}_6\alpha_4 + \overline{A}_4 \times \overline{D}_6\omega_4^2 \tag{2.58}$$

Equating Equations (2.57) and (2.58), one gets

$$\frac{d^2\overline{P}_{3Lj}}{dt^2} = -\frac{d^2\overline{P}_{3Rj}}{dt^2} \tag{2.59}$$

or

$$\overline{N}_1\alpha_2 + \overline{N}_2\ddot{S}_2 + \overline{N}_3\alpha_4 = \overline{N}_4\alpha_1 + \overline{N}_5 \tag{2.60}$$

where

$$N5_x = Sa_4P_3S\theta_4\omega_4^2 - (P_2S\theta_2Sa_1\omega_2 - Sa_1P_2Ca_1S\theta_2\omega_1)\omega_1 +$$
$$Sa_1P_2Ca_1S\theta_2\omega_2$$

$$N5_y = P_3C\theta_4\omega_4^2 + (Ca_1((S\theta_1P_2S\theta_2 - P_2Ca_1C\theta_1C\theta_2)\omega_2 + (-C\theta_1P_2C\theta_2 +$$
$$S\theta_1P_2Ca_1S\theta_2)\omega_1) - C\theta_1Sa_1P_2Sa_1C\theta_2\omega_2)\omega_2 + S\theta_1Sa_1P_2Sa_1S\theta_2\omega_2 +$$
$$C\theta_1Sa_1\mathring{S}_2 + ((S\theta_1P_2S\theta_2 - P_2Ca_1C\theta_1C\theta_2)\omega_2 + C\theta_1Sa_1\dot{S}_2 +$$
$$(C\theta_1(-P_2C\theta_2 - P_1) + S\theta_1(P_2Ca_1S\theta_2 - S_2Sa_1))\omega_1)\omega_1$$

$$N5_z = P_3S\theta_4Ca_4\omega_4^2 + (-Ca_1((-P_2S\theta_2C\theta_1 - P_2Ca_1C\theta_2S\theta_1)\omega_2 -$$
$$(C\theta_1P_2Ca_1S\theta_2 + S\theta_1P_2C\theta_2)\omega_1 + S\theta_1Sa_1P_2Sa_1C\theta_2\omega_2)\omega_2 +$$

$C\theta_1 Sa_1 P_2 Sa_1 S\theta_2 \omega_2 - S\theta_1 Sa_1 \dot{S}_2 + (P_2 S\theta_2 C\theta_1 +$

$P_2 Ca_1 C\theta_2 S\theta_1)\omega_2 - S\theta_1 Sa_1 \mathring{S}_2 + ((C\theta_1 P_2 Ca_1 S\theta_2 - S_2 Sa_1) +$

$S\theta_1 (P_2 C\theta_1 + P_1))\omega_1)\omega_1$

After solving Equation (2.60), one gets

$$\alpha_2 = \frac{\overline{N}_4 \cdot (\overline{N}_2 \times \overline{N}_3)}{\overline{N}_1 \cdot (\overline{N}_2 \times \overline{N}_3)} \alpha_1 + \frac{\overline{N}_5 \cdot (\overline{N}_2 \times \overline{N}_3)}{\overline{N}_1 \cdot (\overline{N}_2 \times \overline{N}_3)} \tag{2.61}$$

$$\ddot{S}_2 = \frac{\overline{N}_4 \cdot (\overline{N}_1 \times \overline{N}_3)}{\overline{N}_2 \cdot \overline{N}_1 \times \overline{N}_3)} \alpha_1 + \frac{\overline{N}_5 \cdot (\overline{N}_1 \times \overline{N}_3)}{\overline{N}_2 \cdot (\overline{N}_1 \times \overline{N}_3)} \tag{2.62}$$

$$\alpha_4 = \frac{\overline{N}_4 \cdot (\overline{N}_1 \times \overline{N}_3)}{\overline{N}_2 \cdot \overline{N}_1 \times \overline{N}_2)} \alpha_1 + \frac{\overline{N}_5 \cdot (\overline{N}_1 \times \overline{N}_2)}{\overline{N}_3 \cdot (\overline{N}_1 \times \overline{N}_2)} \tag{2.63}$$

The acceleration of pair O_2 is

$$\frac{d^2\overline{O}_{2j}}{dt^2} = \begin{Bmatrix} 0 \\ -C\theta_1 Sa_1 \omega_1 \\ S\theta_1 Sa_1 \omega_1 \end{Bmatrix} \mathring{S}_2 + \begin{Bmatrix} Ca_1 \\ S\theta_1 Sa_1 \\ C\theta_1 Sa_1 \end{Bmatrix} \ddot{S}_2 +$$

$$\begin{Bmatrix} 0 \\ -C\theta_1 Sa_1 \mathring{S}_2 + (C\theta_1 P_1 + S\theta_1 S_2 Sa_1)\omega_1 \\ S\theta_1 Sa_1 \mathring{S}_2 + (C\theta_1 S_2 Sa_1 - S\theta_1 P_1)\omega_1 \end{Bmatrix} \omega_1 +$$

$$\begin{Bmatrix} 0 \\ C\theta_1 S_2 Sa_1 - S\theta_1 P_1 \\ -C\theta_1 P_1 - S\theta_1 S_2 Sa_1 \end{Bmatrix} \alpha_1 \tag{2.64}$$

The acceleration of pair O_3 is

$$\frac{d^2\overline{O}_{3j}}{dt^2} = \frac{d^2\overline{O}_{3Lj}}{dt^2} \tag{2.65}$$

The acceleration of the point P_{m2} is

$$\frac{d^2P_{m1j}}{dt^2} = \begin{Bmatrix} 0 \\ Y_1S\theta_1 - Z_1C\theta_1 \\ Y_1C\theta_1 + Z_1S\theta_1 \end{Bmatrix} \alpha_1 + \begin{Bmatrix} 0 \\ -Y_1C\theta_1 - Z_1S\theta_1 \\ Y_1S\theta_1 + Z_1C\theta_1 \end{Bmatrix} \tag{2.66}$$

The acceleration of point P_{m2} is

$$\frac{d^2\overline{P}_{m2}}{dt^2} = (\overline{A}_{2j} \times (\frac{d\overline{P}_{m2j}}{dt} - \frac{d\overline{P}_{2j}}{dt}) + \frac{d\overline{A}_{2j}}{dt} \times (\overline{P}_{m2j} - \overline{P}_{2j}))\omega_2 +$$

$$(\overline{A}_{2j} \times (\overline{P}_{m2j} - \overline{P}_{2j}))\alpha_2 + \overline{A}_{2j}\ddot{S}_2 + \frac{d\overline{A}_{2j}}{dt}\dot{S} +$$

$$(\overline{A}_1 \times (\overline{P}_{m2j} - \overline{P}_1))\alpha_1 + \overline{A}_1 \times (\frac{d\overline{P}_{m2j}}{dt})\omega_1 \tag{2.67}$$

The acceleration of point P_{m3} is

$$\frac{d^2\overline{P}_{m3j}}{dt^2} = \overline{A}_4 \times (\overline{P}_{m3j} - \overline{P}_4)\alpha_4 + \overline{A}_4 \times \overline{A}_4 \times (\overline{P}_{m3j} - \overline{P}_4)\omega_4^2$$

$$= \begin{Bmatrix} Sa_4(Y_3C\theta_4 - Z_3S\theta_4) \\ -Z_3C\theta_4 - Y_3S\theta_4 \\ Ca_4(Y_3C\theta_4 - Z_3S\theta_4) \end{Bmatrix} \alpha_4 + \begin{Bmatrix} Sa_4(-Z_3C\theta_4 - Y_3S\theta_4) \\ -(Y_3C\theta_4 - Z_3S\theta_4) \\ Ca_4(-Z_3C\theta_4 - Y_3S\theta_4 \end{Bmatrix} \omega_4^2$$

(2.68)

2.6.6. Numerical Example of Kinematic Analysis of the RCSR Mechanism

The kinematic analysis of the RCSR spatial mechanism is performed with the dimensions: $a_1 = 12°$, $a_2 = 20°$, $4_4 = 15°$, $P_1 = .5$ in., $P_2 = .8$ in., $P_3 = .4$ in., $P_4 = .5$ in., $\omega_1 = .1$ rps, $\alpha_1 = .2$ revolute per second^2, $X_1 = .5$ in., $Y_1 = 1.$ in., $Z_1 = 1.$ in., $X_2 = .5$ in., $Y_2 = 1.$ in., $Z_2 = 1.$ in., $X_3 = .5$ in., $Y_3 = 1.$ in., $Z_3 = 1.$ in. Numerical results are listed in Tables III through IX.

2.7. Kinematic Analysis of the RCSR-CSR Spatial Mechanism

The geometry and the parameters of the RCSR-CSR spatial mechanism are shown in Figure 16. θ_1 is the input angle. θ_4 and θ_7 are output angles. P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8 are the link lengths; a_1, a_4, a_5, a_8 are link twist angles. S_1, S_4 and S_6 are the link lengths of link 1, 4 and 6. θ_2, S_2, θ_4, θ_5, S_5 and θ_7 are variables.

2.7.1. Freudenstein Displacement Equation of the RCSR-CSR Mechanism

Figure 17 shows the first loop SCRRS which is the unfolded chain breaking at the highest kinematic pair S. The closed-form displacement equation is developed in Section 2.6.1.

Figure 18 shows the second loop SCCRRS which is the unfolded chain breaking at the highest kinematic pair S. The position vectors and direction vectors at unfolded position are defined from Figure 18.

$$\overline{A}_1 = i$$

TABLE III

ANALYSIS OF RCSR MECHANISM

θ_1	θ_2	S_2	θ_4	ω_2	$\dot{S}_2$	ω_4	α_2	$\ddot{S}_2$	α_4
150.	-103.882	-8.961	37.232	0.143	0.006	-0.007	0.282	0.015	-0.083
152.	-101.038	-8.958	36.853	0.142	0.007	-0.031	0.284	0.017	-0.130
154.	-98.194	-8.956	35.997	0.142	0.008	-0.055	0.287	0.019	-0.178
156.	-95.334	-8.953	34.661	0.144	0.009	-0.079	0.291	0.021	-0.228
158.	-92.445	-8.949	32.837	0.145	0.010	-0.104	0.296	0.024	-0.279
160.	-89.522	-8.945	30.514	0.147	0.012	-0.129	0.299	0.027	-0.330
162.	-86.564	-8.941	27.683	0.149	0.013	-0.154	0.301	0.030	-0.382
164.	-83.577	-8.936	24.341	0.150	0.014	-0.180	0.302	0.034	-0.433
166.	-80.577	-8.931	20.490	0.150	0.016	-0.205	0.299	0.037	-0.481
168.	-77.585	-8.925	16.145	0.149	0.018	-0.229	0.293	0.041	-0.526
170.	-74.629	-8.919	11.328	0.146	0.020	-0.252	0.283	0.044	-0.567
172.	-71.743	-8.911	6.074	0.142	0.021	-0.273	0.268	0.047	-0.602
174.	-68.966	-8.904	0.426	0.135	0.023	-0.291	0.250	0.049	-0.632
176.	-66.337	-8.896	-5.567	0.127	0.024	-0.307	0.227	0.050	-0.657
178.	-63.894	-8.887	-11.854	0.117	0.024	-0.321	0.202	0.051	-0.677
180.	-61.675	-8.879	-18.386	0.105	0.025	-0.332	0.173	0.050	-0.693
182.	-59.710	-8.870	-25.118	0.091	0.025	-0.341	0.142	0.049	-0.706
184.	-58.027	-8.861	-32.017	0.077	0.024	-0.349	0.110	0.047	-0.717
186.	-56.648	-8.853	-39.057	0.061	0.023	-0.355	0.076	0.044	-0.729
188.	-55.591	-8.845	-46.226	0.044	0.022	-0.362	0.040	0.040	-0.742
190.	-54.875	-8.838	-53.525	0.027	0.021	-0.369	0.003	0.036	-0.758
192.	-54.516	-8.831	-60.976	0.009	0.019	-0.377	-0.037	0.031	-0.781
194.	-54.538	-8.825	-68.620	-0.011	0.016	-0.388	-0.082	0.025	-0.814
196.	-54.977	-8.820	-76.529	-0.033	0.013	-0.404	-0.134	0.017	-0.863
198.	-55.892	-8.816	-84.833	-0.059	0.010	-0.428	-0.201	0.008	-0.944
200.	-57.393	-8.813	-93.766	-0.093	0.005	-0.469	-0.301	-0.004	-1.092
202.	-59.734	-8.812	-103.842	-0.146	-0.001	-0.549	-0.503	-0.028	-1.443

TABLE IV

ANALYSIS OF RCSR MECHANISM-P_{m1}, P_{m2}, P_{m3}

θ_1	P_{m1x}	P_{m1y}	P_{m1z}	P_{m2x}	P_{m2y}	P_{m2z}	P_{m3x}	P_{m3y}	P_{m3z}
150.	-4.5	-0.366	-1.366	4.102	2.822	-1.467	4.709	0.691	0.189
152.	-4.5	-0.413	-1.352	4.087	2.763	-1.633	4.709	0.700	0.188
154.	-4.5	-0.460	-1.337	4.073	2.692	-1.796	4.708	0.721	0.185
156.	-4.5	-0.507	-1.320	4.058	2.610	-1.956	4.707	0.754	0.179
158.	-4.5	-0.553	-1.302	4.044	2.516	-2.111	4.704	0.798	0.171
160.	-4.5	-0.598	-1.282	4.029	2.411	-2.262	4.701	0.854	0.158
162.	-4.5	-0.642	-1.260	4.015	2.294	-2.407	4.696	0.921	0.139
164.	-4.5	-0.686	-1.237	4.000	2.167	-2.545	4.689	0.999	0.113
166.	-4.5	-0.728	-1.212	3.986	2.030	-2.674	4.680	1.087	0.078
168.	-4.5	-0.770	-1.186	3.971	1.884	-2.795	4.667	1.182	0.032
170.	-4.5	-0.811	-1.158	3.957	1.730	-2.904	4.651	1.284	-0.028
172.	-4.5	-0.851	-1.129	3.943	1.570	-3.003	4.631	1.389	-0.102
174.	-4.5	-0.890	-1.099	3.930	1.405	-3.089	4.607	1.493	-0.192
176.	-4.5	-0.928	-1.067	3.917	1.238	-3.163	4.579	1.592	-0.297
178.	-4.5	-0.964	-1.034	3.904	1.071	-3.225	4.547	1.684	-0.418
180.	-4.5	-1.000	-1.000	3.892	0.906	-3.275	4.511	1.764	-0.553
182.	-4.5	-1.034	-0.964	3.881	0.744	-3.314	4.471	1.830	-0.700
184.	-4.5	-1.067	-0.928	3.870	0.587	-3.343	4.429	1.878	-0.858
186.	-4.5	-1.099	-0.890	3.861	0.436	-3.363	4.385	1.907	-1.023
188.	-4.5	-1.129	-0.851	3.852	0.293	-3.376	4.339	1.914	-1.194
190.	-4.5	-1.158	-0.811	3.844	0.157	-3.383	4.292	1.899	-1.367
192.	-4.5	-1.186	-0.770	3.837	0.031	-3.385	4.246	1.860	-1.541
194.	-4.5	-1.212	-0.728	3.831	-0.086	-3.382	4.200	1.796	-1.712
196.	-4.5	-1.237	-0.686	3.826	-0.193	-3.378	4.155	1.705	-1.879
198.	-4.5	-1.260	-0.642	3.823	-0.289	-3.371	4.112	1.586	-2.040
200.	-4.5	-1.282	-0.598	3.822	-0.369	-3.364	4.071	1.432	-2.192
202.	-4.5	-1.302	-0.553	3.824	-0.429	-3.358	4.033	1.232	-2.334

TABLE V

ANALYSIS OF RCSR MECHANISM-O_2, O_3

θ_1	O_{2x}	O_{2y}	O_{2z}	O_{3x}	O_{3y}	O_{3z}
150.	3.765	1.365	-1.363	3.926	0.818	-0.802
152.	3.763	1.316	-1.410	3.926	0.820	-0.804
154.	3.760	1.266	-1.454	3.925	0.824	-0.808
156.	3.757	1.214	-1.497	3.923	0.829	-0.816
158.	3.754	1.161	-1.538	3.920	0.836	-0.826
160.	3.750	1.106	-1.577	3.916	0.845	-0.839
162.	3.746	1.050	-1.613	3.912	0.854	-0.856
164.	3.741	0.993	-1.648	3.906	0.864	-0.876
166.	3.736	0.934	-1.681	3.900	0.875	-0.900
168.	3.730	0.875	-1.711	3.892	0.884	-0.928
170.	3.724	0.814	-1.739	3.884	0.892	-0.959
172.	3.717	0.753	-1.765	3.875	0.898	-0.994
174.	3.709	0.691	-1.789	3.864	0.900	-1.032
176.	3.701	0.628	-1.810	3.854	0.898	-1.073
178.	3.693	0.564	-1.829	3.842	0.891	-1.115
180.	3.685	0.500	-1.846	3.831	0.880	-1.157
182.	3.676	0.435	-1.860	3.820	0.862	-1.199
184.	3.668	0.370	-1.873	3.809	0.839	-1.240
186.	3.660	0.305	-1.883	3.798	0.811	-1.279
188.	3.652	0.239	-1.891	3.789	0.777	-1.314
190.	3.644	0.173	-1.896	3.780	0.738	-1.346
192.	3.638	0.107	-1.900	3.773	0.694	-1.373
194.	3.632	0.041	-1.901	3.767	0.646	-1.395
196.	3.627	-0.025	-1.900	3.763	0.593	-1.411
198.	3.623	-0.091	-1.898	3.761	0.536	-1.420
200.	3.620	-0.157	-1.893	3.760	0.474	-1.421
202.	3.620	-0.223	-1.886	3.763	0.404	-1.410

TABLE VI

ANALYSIS OF RCSR MECHANISM-$\dot{P}_{m1}$, $\dot{P}_{m2}$, $\dot{P}_{m3}$

θ_1	$\dot{P}_{m1x}$	$\dot{P}_{m1y}$	$\dot{P}_{m1z}$	$\dot{P}_{m2x}$	$\dot{P}_{m2y}$	$\dot{P}_{m2z}$	$\dot{P}_{m3x}$	$\dot{P}_{m3y}$	$\dot{P}_{m3z}$
150.	0.	-0.137	0.037	-0.042	-0.153	-0.479	-0.000	0.010	-0.001
152.	0.	-0.135	0.041	-0.042	-0.187	-0.472	-0.002	0.043	-0.006
154.	0.	-0.134	0.046	-0.042	-0.220	-0.462	-0.003	0.076	-0.012
156.	0.	-0.132	0.051	-0.041	-0.253	-0.452	-0.005	0.110	-0.019
158.	0.	-0.130	0.055	-0.042	-0.285	-0.439	-0.008	0.143	-0.030
160.	0.	-0.128	0.060	-0.042	-0.318	-0.424	-0.012	0.176	-0.044
162.	0.	-0.126	0.064	-0.042	-0.349	-0.405	-0.017	0.208	-0.063
164.	0.	-0.124	0.069	-0.042	-0.379	-0.384	-0.023	0.238	-0.087
166.	0.	-0.121	0.073	-0.041	-0.406	-0.359	-0.031	0.264	-0.116
168.	0.	-0.119	0.077	-0.041	-0.431	-0.330	-0.041	0.284	-0.151
170.	0.	-0.116	0.081	-0.040	-0.451	-0.299	-0.051	0.297	-0.191
172.	0.	-0.113	0.085	-0.039	-0.466	-0.265	-0.063	0.300	-0.234
174.	0.	-0.110	0.089	-0.038	-0.476	-0.230	-0.075	0.294	-0.279
176.	0.	-0.107	0.093	-0.037	-0.479	-0.194	-0.087	0.276	-0.324
178.	0.	-0.103	0.096	-0.035	-0.477	-0.160	-0.098	0.248	-0.367
180.	0.	-0.100	0.100	-0.033	-0.470	-0.127	-0.109	0.210	-0.405
182.	0.	-0.096	0.103	-0.031	-0.457	-0.097	-0.117	0.164	-0.438
184.	0.	-0.093	0.107	-0.029	-0.441	-0.070	-0.124	0.111	-0.464
186.	0.	-0.089	0.110	-0.027	-0.422	-0.047	-0.129	0.052	-0.483
188.	0.	-0.085	0.113	-0.024	-0.400	-0.027	-0.132	-0.011	-0.494
190.	0.	-0.081	0.116	-0.021	-0.376	-0.011	-0.133	-0.077	-0.498
192.	0.	-0.077	0.119	-0.019	-0.350	0.001	-0.133	-0.147	-0.495
194.	0.	-0.073	0.121	-0.015	-0.322	0.011	-0.130	-0.220	-0.486
196.	0.	-0.069	0.124	-0.011	-0.290	0.017	-0.126	-0.299	-0.470
198.	0.	-0.064	0.126	-0.006	-0.254	0.020	-0.120	-0.388	-0.449
200.	0.	-0.060	0.128	0.001	-0.206	0.020	-0.113	-0.499	-0.422
202.	0.	-0.055	0.130	0.012	-0.131	0.015	-0.104	-0.665	-0.388

TABLE VII

ANALYSIS OF RCSR MECHANISM-$\dot{O}_2$, $\dot{O}_3$

θ_1	$\dot{O}_{2x}$	$\dot{O}_{2y}$	$\dot{O}_{2z}$	$\dot{O}_{3x}$	$\dot{O}_{3y}$	$\dot{O}_{3z}$
150.	-0.006	-0.137	-0.135	-0.001	0.002	-0.002
152.	-0.007	-0.142	-0.130	-0.003	0.007	-0.010
154.	-0.008	-0.146	-0.125	-0.005	0.013	-0.017
156.	-0.009	-0.150	-0.120	-0.007	0.018	-0.025
158.	-0.010	-0.155	-0.114	-0.009	0.022	-0.034
160.	-0.011	-0.158	-0.108	-0.011	0.026	-0.043
162.	-0.013	-0.162	-0.102	-0.014	0.029	-0.053
164.	-0.014	-0.166	-0.096	-0.017	0.030	-0.063
166.	-0.016	-0.169	-0.090	-0.020	0.029	-0.074
168.	-0.017	-0.172	-0.084	-0.023	0.026	-0.085
170.	-0.019	-0.175	-0.077	-0.026	0.020	-0.096
172.	-0.021	-0.177	-0.071	-0.028	0.012	-0.105
174.	-0.022	-0.179	-0.064	-0.030	0.001	-0.113
176.	-0.023	-0.181	-0.058	-0.032	-0.012	-0.118
178.	-0.024	-0.183	-0.051	-0.033	-0.026	-0.121
180.	-0.024	-0.185	-0.045	-0.033	-0.042	-0.122
182.	-0.024	-0.186	-0.038	-0.032	-0.058	-0.119
184.	-0.024	-0.187	-0.032	-0.031	-0.074	-0.114
186.	-0.023	-0.188	-0.026	-0.029	-0.090	-0.107
188.	-0.022	-0.188	-0.019	-0.026	-0.104	-0.097
190.	-0.020	-0.189	-0.013	-0.023	-0.119	-0.085
192.	-0.018	-0.189	-0.007	-0.019	-0.132	-0.071
194.	-0.016	-0.189	-0.001	-0.015	-0.145	-0.055
196.	-0.013	-0.189	0.005	-0.010	-0.157	-0.036
198.	-0.010	-0.189	0.011	-0.004	-0.171	-0.015
200.	-0.005	-0.189	0.017	0.003	-0.187	0.012
202.	0.001	-0.189	0.022	0.014	-0.213	0.051

TABLE VIII

ANALYSIS OF RCSR MECHANISM-$\ddot{P}_{m1}$, $\ddot{P}_{m2}$, $\ddot{P}_{m3}$

θ_1	$\ddot{P}_{m1x}$	$\ddot{P}_{m1y}$	$\ddot{P}_{m1z}$	$\ddot{P}_{m2x}$	$\ddot{P}_{m2y}$	$\ddot{P}_{m2z}$	$\ddot{P}_{m3x}$	$\ddot{P}_{m3y}$	$\ddot{P}_{m3z}$
150.	0.	-0.270	0.087	-0.083	-0.403	-0.937	-0.004	0.116	-0.015
152.	0.	-0.266	0.096	-0.083	-0.468	-0.919	-0.007	0.182	-0.026
154.	0.	-0.263	0.105	-0.083	-0.534	-0.897	-0.011	0.248	-0.042
156.	0.	-0.259	0.115	-0.083	-0.599	-0.870	-0.017	0.315	-0.064
158.	0.	-0.255	0.124	-0.083	-0.664	-0.838	-0.025	0.382	-0.095
160.	0.	-0.250	0.132	-0.083	-0.727	-0.799	-0.036	0.446	-0.135
162.	0.	-0.246	0.141	-0.083	-0.786	-0.754	-0.050	0.506	-0.186
164.	0.	-0.241	0.149	-0.083	-0.840	-0.701	-0.067	0.556	-0.250
166.	0.	-0.235	0.158	-0.082	-0.888	-0.641	-0.087	0.594	-0.325
168.	0.	-0.230	0.166	-0.081	-0.926	-0.574	-0.110	0.616	-0.410
170.	0.	-0.224	0.174	-0.079	-0.953	-0.504	-0.134	0.617	-0.502
172.	0.	-0.217	0.182	-0.076	-0.968	-0.431	-0.160	0.597	-0.596
174.	0.	-0.211	0.189	-0.073	-0.970	-0.358	-0.185	0.553	-0.689
176.	0.	-0.204	0.196	-0.069	-0.960	-0.288	-0.208	0.487	-0.775
178.	0.	-0.197	0.203	-0.065	-0.940	-0.223	-0.228	0.401	-0.851
180.	0.	-0.190	0.210	-0.061	-0.910	-0.164	-0.245	0.299	-0.913
182.	0.	-0.183	0.217	-0.056	-0.873	-0.112	-0.257	0.185	-0.961
184.	0.	-0.175	0.223	-0.051	-0.831	-0.068	-0.266	0.060	-0.992
186.	0.	-0.167	0.229	-0.046	-0.784	-0.032	-0.270	-0.071	-1.008
188.	0.	-0.159	0.234	-0.041	-0.734	-0.004	-0.270	-0.207	-1.009
190.	0.	-0.151	0.240	-0.035	-0.680	0.018	-0.267	-0.349	-0.997
192.	0.	-0.142	0.245	-0.028	-0.622	0.034	-0.261	-0.497	-0.972
194.	0.	-0.134	0.250	-0.020	-0.559	0.044	-0.251	-0.656	-0.936
196.	0.	-0.125	0.254	-0.010	-0.485	0.047	-0.238	-0.835	-0.889
198.	0.	-0.116	0.258	0.004	-0.390	0.045	-0.222	-1.054	-0.830
200.	0.	-0.107	0.262	0.026	-0.249	0.034	-0.203	-1.367	-0.758
202.	0.	-0.097	0.266	0.071	0.033	0.009	-0.179	-1.967	-0.667

TABLE IX

ANALYSIS OF RCSR MECHANISM-$\ddot{O}_2$, $\ddot{O}_3$

θ_1	$\ddot{O}_{2x}$	$\ddot{O}_{2y}$	$\ddot{O}_{2z}$	$\ddot{O}_{3x}$	$\ddot{O}_{3y}$	$\ddot{O}_{3z}$
150.	-0.015	-0.288	-0.256	-0.007	0.020	-0.026
152.	-0.017	-0.297	-0.246	-0.011	0.031	-0.040
154.	-0.019	-0.305	-0.235	-0.015	0.041	-0.056
156.	-0.021	-0.313	-0.224	-0.020	0.050	-0.074
158.	-0.023	-0.321	-0.212	-0.025	0.057	-0.093
160.	-0.026	-0.328	-0.200	-0.030	0.061	-0.113
162.	-0.029	-0.335	-0.188	-0.036	0.063	-0.135
164.	-0.033	-0.341	-0.175	-0.042	0.060	-0.157
166.	-0.036	-0.347	-0.162	-0.048	0.052	-0.180
168.	-0.040	-0.352	-0.149	-0.054	0.038	-0.201
170.	-0.043	-0.357	-0.136	-0.059	0.020	-0.220
172.	-0.046	-0.361	-0.123	-0.063	-0.004	-0.234
174.	-0.048	-0.365	-0.110	-0.066	-0.032	-0.245
176.	-0.049	-0.368	-0.097	-0.067	-0.063	-0.249
178.	-0.050	-0.371	-0.084	-0.066	-0.096	-0.248
180.	-0.049	-0.373	-0.071	-0.064	-0.129	-0.241
182.	-0.048	-0.375	-0.058	-0.061	-0.162	-0.228
184.	-0.046	-0.377	-0.046	-0.056	-0.193	-0.210
186.	-0.043	-0.378	-0.033	-0.050	-0.223	-0.188
188.	-0.039	-0.378	-0.021	-0.043	-0.250	-0.162
190.	-0.035	-0.379	-0.009	-0.035	-0.276	-0.132
192.	-0.030	-0.379	0.004	-0.026	-0.301	-0.098
194.	-0.024	-0.379	0.016	-0.016	-0.325	-0.060
196.	-0.017	-0.378	0.027	-0.004	-0.351	-0.016
198.	-0.008	-0.378	0.039	0.010	-0.383	0.038
200.	0.004	-0.377	0.049	0.030	-0.430	0.112
202.	0.027	-0.377	0.058	0.066	-0.532	0.247

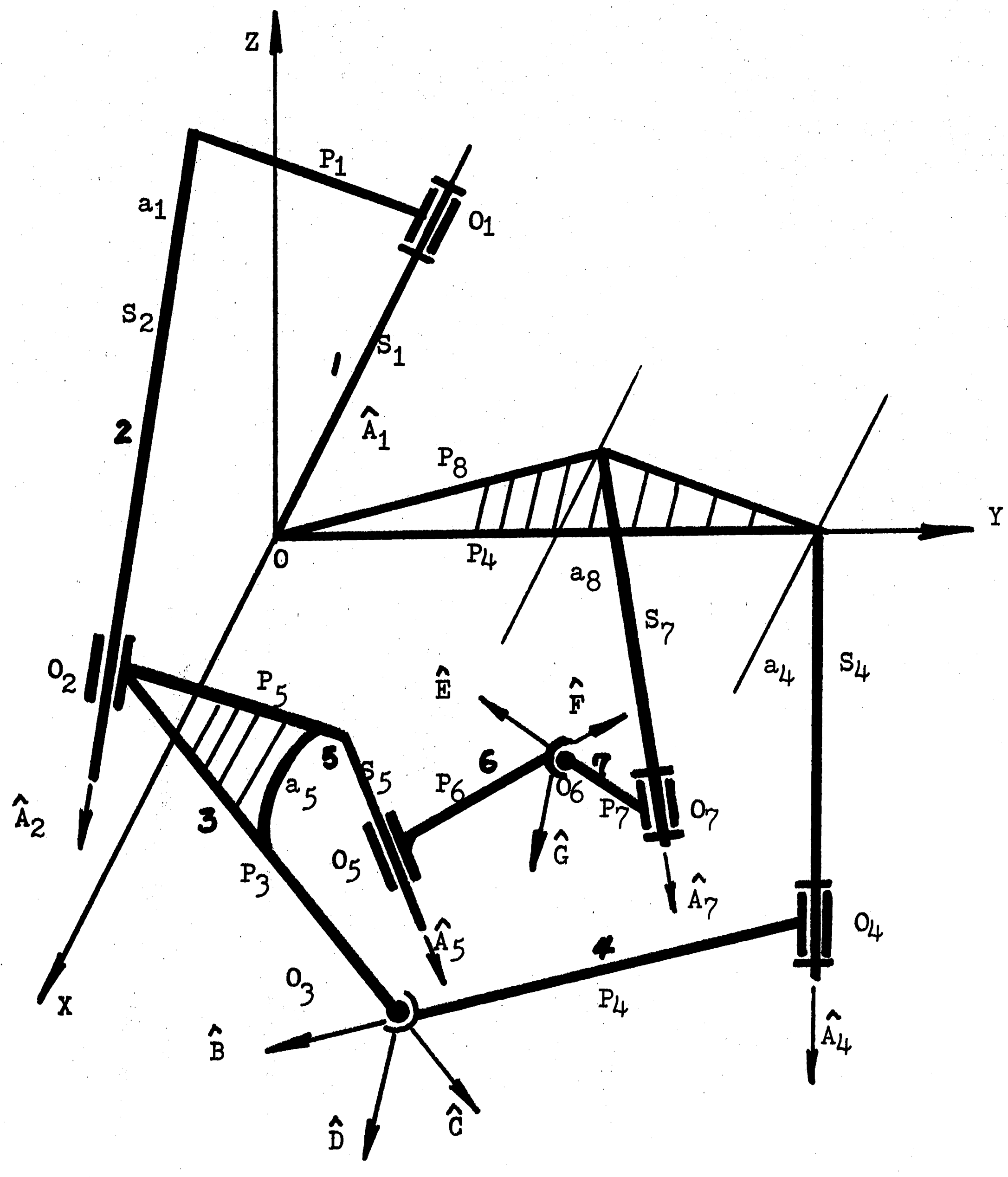

Figure 16. The RCSR-CSR Spatial Mechanism

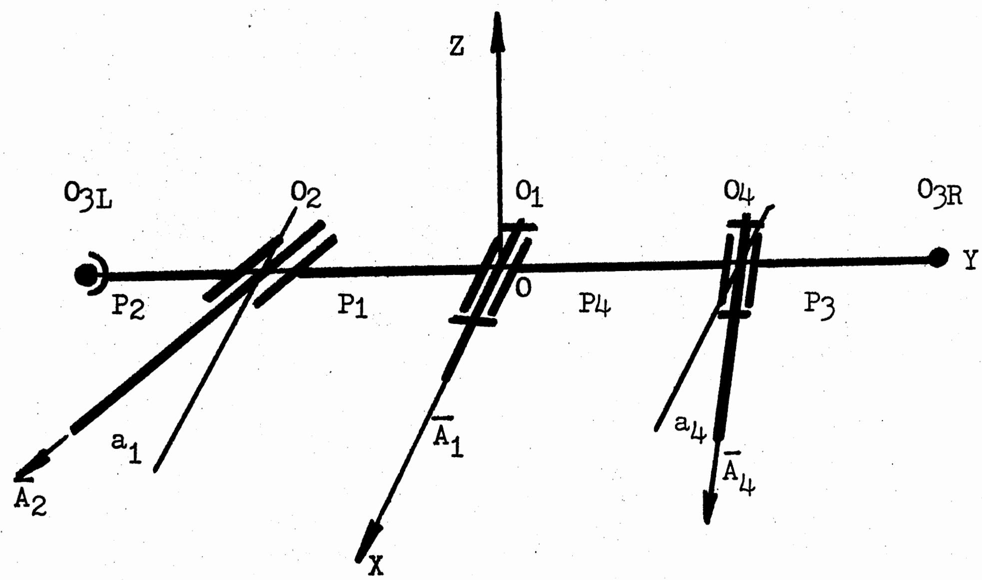

Figure 17. The First Loop Chain SCRRS

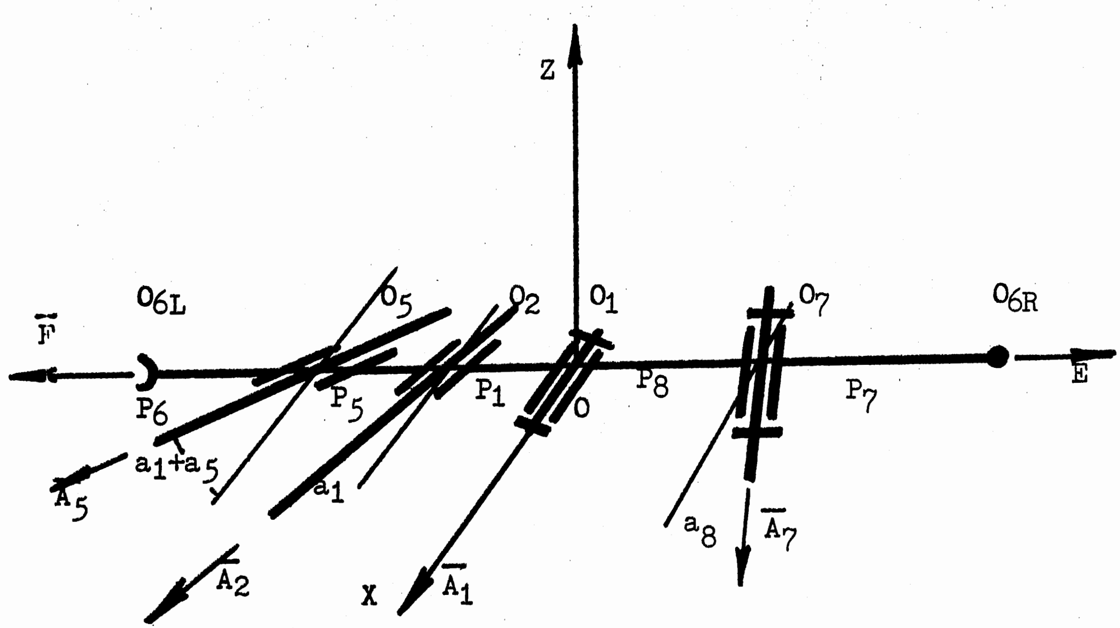

Figure 18. The Second Loop Chain SCRRS

$$\overline{A}_2 = COSa_1\overline{i} + SINa_1\overline{k}$$

$$\overline{A}_5 = COS\ (a_1 + a_5)\ \overline{i} + SIN\ (a_1 + a_5)\ \overline{k}$$

$$\overline{A}_7 = COSa_8\overline{i} - SINa_8\overline{k}$$

$$\overline{O}_1 = 0$$

$$\overline{O}_2 = -P_2j$$

$$\overline{O}_5 = -(P_1 + P_5)\ j$$

$$\overline{O}_{6L} = -(P_1 + P_5 + P_6)\ j$$

$$\overline{O}_7 = P_8j$$

$$\overline{O}_{6R} = (P_8 + P_7)\ j$$

The position $\overline{O}_{6L}$ is displaced through negative screw motion, $\overline{A}_5$ with rotation θ_5, translation S_5, $\overline{A}_2$ with rotation θ'_2, translation S'_2 and $\overline{A}_1$ with rotation θ_1, translation S_1. One has

$$\overline{O}_{6Lj} = C\theta_5\ ((\overline{O}'_{6L} - \overline{O}'_5) - ((\overline{O}'_{6L} - \overline{O}'_5)\ \overline{A}_{5j})\ \overline{A}_{5j}) - S\theta_5\ (\overline{A}_{5j} \times (\overline{O}'_{6L} - \overline{O}'_5)) + ((\overline{O}'_{6L} - \overline{O}'_5)\ \overline{A}_{5j})\ \overline{A}_{5j} + \overline{O}'_5 - \overline{A}_{5j}S_5 \tag{2.69}$$

where

$$\overline{O}'_{6L} = C\theta_1\ ((\overline{O}''_{6L} - O_1) - ((\overline{O}''_{6L} - \overline{O}_1)\ \overline{A}_1)\ \overline{A}_1) - S\theta_1\ (\overline{A}_1 \times (\overline{O}''_{6L} - \overline{O}_1)) + \overline{O}_1 - \overline{A}_1\overline{S}_1 + ((\overline{O}''_{6L} - \overline{O}_1)\ \overline{A}_1)\ \overline{A}_1$$

$$= \begin{Bmatrix} -S\theta'_2(P_6 + P_5)\ Sa_1 - S'_2Ca_1 - S_1 \\ C\theta_1(-C\theta'_2(P_6 + P_5) + P_1) - S\theta_1(-S\theta'_2(P_5 + P_6)\ Ca_1 + S'_2Sa_1) \\ C\theta_1(S\theta'_2(P_5 + P_6)\ Ca_1 - S'_2Sa_1) - S\theta_1(-C\theta'_2(P_5 + P_6) + P_1) \end{Bmatrix}$$

(2.70)

$$\overline{O}''_{6L} = \begin{Bmatrix} -S\theta'_2(P_6 + P_5)\ Sa_1 - S_2Ca_1 \\ -C\theta'_2(P_6 + P_5) + P_1 \\ S\theta'_2(P_5 + P_6)\ Ca_1 - S_2Sa_1 \end{Bmatrix} \quad (2.71)$$

$$\overline{O}'_5 = C\theta_1((\overline{O}''_5 - \overline{O}_1) - ((\overline{O}''_5 - O_1)\ \overline{A}_1)\ \overline{A}_1) - S\theta_1(\overline{A}_1 \times (\overline{O}''_5 - \overline{O}_1)) - \overline{A}_1S_1 + ((\overline{O}''_5 - \overline{O}_1)\ \overline{A}_1)\ \overline{A}_1) + \overline{O}_1 \quad (2.72)$$

$$\overline{O}''_5 = C\theta'_2((\overline{O}_5 - \overline{O}_2) - ((\overline{O}_5 - \overline{O}_2)\ \overline{A}_2)\ \overline{A}_2) - S\theta'_2(\overline{A}_2 \times (\overline{O}_5 - \overline{O}_2)) + \overline{O}_2 - \overline{A}_2S'_2 + ((\overline{O}_5 - \overline{O}_2)\ \overline{A}_2)\ \overline{A}_2$$

$$= \begin{Bmatrix} -S\theta'_2P_5Sa_1 - S'_2Ca_1 \\ -C\theta'_2P_5 + P_1 \\ -S\theta'_2P_5Ca_1 - S'_2Sa_1 \end{Bmatrix} \quad (2.73)$$

$$\overline{A}_{5j} = C\theta_1(\overline{A}'_{5j} - (\overline{A}'_{5j} \cdot \overline{A}_1)\ \overline{A}_1) - S\theta_1(\overline{A}_1 \times \overline{A}'_{5j}) + (\overline{A}'_{5j} \cdot \overline{A}_1)\ \overline{A}_1 \quad (2.74)$$

$$\overline{A}_{5j} = C\theta'_2(\overline{A}_5 - (\overline{A}_5 \cdot \overline{A}_2)\ A_2) - S\theta'_2(\overline{A}_2 \times \overline{A}_5) + (\overline{A}_5 \cdot \overline{A}_2)\ \overline{A}_2$$

(2.75)

$$\theta'_2 = \theta_2 + O_{c2}$$

and

$$S'_2 = S_2 + S_{c2}$$

The position $\overline{O}_{6R}$ is displaced through successive screw motions $\overline{A}_7$ with rotation θ_7, translation S_7 and $\overline{A}_1$ with rotation θ_{c1} and translation S_{c1} one obtains

$$\overline{O}_{6Rj} = C\theta_7((\overline{O}'_{6R} - \overline{O}'_7) - ((\overline{O}'_{6R} - \overline{O}'_7)\ \overline{A}_{7j})\ \overline{A}_{7j}) + S\theta_7(\overline{A}_{7j} \times (\overline{O}'_{6R} - \overline{O}'_7)) + \overline{O}'_7 + S_7\overline{A}_{7j} + ((\overline{O}'_{6R} - \overline{O}'_7)\ \overline{A}_{7j})\ \overline{A}_7 \qquad (2.76)$$

where

$$\overline{O}'_{6R} = C\theta_{c1}((\overline{O}_{6L} - \overline{O}_1) - ((\overline{O}_{6R} - \overline{O}_1)\ \overline{A}_1)\ \overline{A}_1) + S\theta_{c1}(\overline{A}_1 \times (\overline{O}_{6L} - \overline{O}_1)) + \overline{A}_1 \cdot S_{c1} + ((\overline{O}_{6R} - \overline{O}_1)\ \overline{A}_1)\ \overline{A}_1$$

$$= \begin{Bmatrix} S_{c1} \\ C\theta_{c1}(P_8 + P_7) \\ S\theta_{c1}(P_8 + P_7) \end{Bmatrix} \qquad (2.77)$$

$$\overline{O}_7' = C\theta_{c1}((\overline{O}_7 - \overline{O}_1) - ((\overline{O}_7 - \overline{O}_1)\ \overline{A}_1)\ \overline{A}_1) + S\theta_{c1}(\overline{A}_1 \times (\overline{O}_7 - \overline{O}_1)) + \overline{A}_1 \cdot S_{c1}$$

$$= \begin{Bmatrix} S_{c1} \\ P_8 C\theta_{c1} \\ P_8 S\theta_{c1} \end{Bmatrix} \tag{2.78}$$

$$A_{7j} = \begin{Bmatrix} Ca_8 \\ S\theta_{c1} Sa_8 \\ -C\theta_{c1} Sa_8 \end{Bmatrix} \tag{2.79}$$

Equating Equations (2.69) and (2.76) and rearranging the equations, one has

$$C\theta_5 \overline{H}_7 + S\theta_5 \overline{H}_8 + S_5 \overline{H}_9 + \overline{H}_{10} = S\theta_7 \overline{H}_1 + C\theta_7 \overline{H}_2 + \overline{H}_3 \tag{2.80}$$

where

$$\overline{H}_7 = ((\overline{O}_{6L}' - \overline{O}_5') - ((\overline{O}_{6L}' - \overline{O}_5')\,\overline{A}_{5j})\,\overline{A}_{5j})$$

$$\overline{H}_8 = -(\overline{A}_{5j} \times (\overline{O}_{6L}' - \overline{O}_5')$$

$$\overline{H}_9 = -\overline{A}_{5j}$$

$$\overline{H}_{10} = ((\overline{O}_{6L}' - \overline{O}_5')\,\overline{A}_{5j})\,\overline{A}_{5j} + \overline{O}_5'$$

$$H_1 = (\overline{A}_{7j} \times (\overline{O}_6\ ' - \overline{O}_7'))$$

$$H_2 = ((\overline{O}_{6R}' - \overline{O}_7') - ((\overline{O}_{6R}' - \overline{O}_7')\,\overline{A}_{7j})\,\overline{A}_{7j})$$

$$H_3 = \overline{O}_7' + S_7 \overline{A}_{7j} + ((\overline{O}_{6R}' - \overline{O}_7')\,\overline{A}_{7j})\,\overline{A}_{7j}$$

Dotting Equation (2.80) by $(\overline{H}_8 \times \overline{H}_9)$ and $(\overline{H}_7 \times \overline{H}_9)$, we have

$$C\theta_5 = S\theta_7 E_1 + C\theta_7 E_2 + E_3 \tag{2.81}$$

$$S\theta_5 = S\theta_7 E_4 + C\theta_7 E_5 + E_6 \tag{2.82}$$

where

$$E_1 = \frac{\overline{H}_1 \cdot (\overline{H}_7 \times \overline{H}_9)}{\overline{H}_8 \cdot (\overline{H}_7 \times \overline{H}_9)}$$

$$E_2 = \frac{\overline{H}_2 \cdot (\overline{H}_7 \times \overline{H}_9)}{\overline{H}_8 \cdot (\overline{H}_7 \times \overline{H}_9)}$$

$$E_3 = \frac{(\overline{H}_3 - \overline{H}_{10})\ (\overline{H}_7 \times \overline{H}_9)}{\overline{H}_8\ (\overline{H}_7 \times \overline{H}_9)}$$

$$E_4 = \frac{\overline{H}_1 \cdot (\overline{H}_8 \times \overline{H}_9)}{\overline{H}_7 \cdot (\overline{H}_8 \times \overline{H}_9)}$$

$$E_5 = \frac{\overline{H}_2 \cdot (\overline{H}_8 \times \overline{H}_9)}{\overline{H}_7 \cdot (\overline{H}_8 \times \overline{H}_9)}$$

$$E_6 = \frac{(\overline{H}_3 - \overline{H}_{10})\ (\overline{H}_8 \times \overline{H}_9)}{\overline{H}_7 \cdot (\overline{H}_8 \times \overline{H}_9)}$$

Squaring Equations (2.81) and (2.82), using the identity $C^2\theta_5 + S^2\theta_5 = 1$, and introducing the half angles relationships, $S\theta_7 = 2\Phi_7 / (1 + \Phi_7^2)$ and $C\theta_7 = (1 - \Phi_7^2) / (1 + \Phi_7^2)$, where $\Phi_7 = \tan\left(\frac{\theta_7}{2}\right)$, we have

$$\Phi_7^4 C(1) + \Phi_7^3 C(2) + \Phi_7^2 C(3) + \Phi_7 C(4) + C(5) = 0 \tag{2.83}$$

where C(1), C(2), C(3), C(4), C(5) are defined in Equation (2.23).

Solving Equation (2.83), one has

$$\theta_7 = 2 * \tan^{-1}(\Phi_7) \tag{2.84}$$

After solving θ_7, one substitutes into Equations (2.81) and (2.82). Thus,

$$\theta_5 = \tan^{-1} \left(\frac{S\theta_5}{C\theta_5} \right) \tag{2.85}$$

Dotting Equation (2.80) by $(H_7 \times H_8)$, one has

$$S_5 = \frac{(S\theta_2\overline{H}_1 + C\theta_2\overline{H}_2 + (\overline{H}_3 - \overline{H}_{10})\ (\overline{H}_7 \times \overline{H}_8)}{\overline{H}_9\ (\overline{H}_7 \times \overline{H}_8)} \tag{2.86}$$

2.7.2. Dual Screws of the RCSR-CSR Spatial Mechanism

Figure 16 shows dual screws $\hat{A}_1$, $\hat{A}_2$, $\hat{A}_4$, $\hat{A}_5$, $\hat{A}_7$, $\hat{E}$, $\hat{F}$, $\hat{G}$, $\hat{B}$, $\hat{C}$, and $\hat{D}$ at the displaced position. $\overline{E}$ and $\overline{F}$ are defined from Figure 18.

$$\overline{E} = j \tag{2.87}$$

$$\overline{F} = -j \tag{2.88}$$

At the displaced position, $\overline{E}$ and $\overline{F}$ becomes

$$\overline{E}_j = \begin{Bmatrix} S\theta_7 Sa_8 \\ C\theta_{c1}C\theta_7 - Sa_1 S\theta_7 Ca_8 \\ C\theta_{c1}S\theta_7 Ca_8 + S\theta_{c1}C\theta_7 \end{Bmatrix} \tag{2.89}$$

$$\overline{F}_j = \begin{Bmatrix} F_{ix} \\ C\theta_1 F_{iy} + S\theta_1 F_{iz} \\ C\theta_1 F_{iz} - S\theta_1 F_{iy} \end{Bmatrix} \tag{2.90}$$

where

$$F_{ix} = C\theta'_2(S(a_1 + a_5)\ S\theta_5 - (S(a_1 + a_5)\ S\theta_5 Ca_1 - S\theta_5 C(a_1 + a_5)\ Sa_1)\ Ca_1) - S\theta'_2 C\theta_5 Sa_1 + (S(a_1 + a_5)\ S\theta_5 Ca_1 - S\theta_5 C(a_1 + a_5)\ Sa_1)\ Ca_1$$

$$F_{iy} = -C\theta'_2 C\theta_5 - S\theta'_2(Ca_1 S\theta_5 C(a_1 + a_5) + Sa_1 S(a_1 + a_5)\ S\theta_5)$$

$$F_{iz} = C\theta'_2(-S\theta_5 C(a_1 + a_5) - (S(a_1 + a_5)\ S\theta_5 Ca_1 - S\theta_5 C(a_1 + a_5)\ Sa_1)\ Sa_1) + S\theta'_2 Ca_1 C\theta_5 + (S(a_1 + a_5)\ S\theta_5 Ca_1 - S\theta_5 C(a_1 + a_5)\ Sa_1)\ Sa_1$$

Thus,

$$\overline{G}_j = \overline{E}_j \times \overline{F}_j \qquad (2.91)$$

Applying the procedures defined in Section 2.5, we obtain

$$E = \overline{E}_0 + \varepsilon\ \overline{E}_1 \times \overline{E}_0 \qquad (2.92)$$

$$F = \overline{F}_0 + \varepsilon\ \overline{F}_1 \times \overline{F}_0 \qquad (2.93)$$

$$G = \overline{G}_0 + \varepsilon\ \overline{G}_1 \times \overline{G}_0 \qquad (2.94)$$

where

$$\overline{E}_0 = \overline{E}_j$$

$$\overline{F}_0 = \overline{F}_j$$

$$\overline{G}_0 = \overline{G}_j$$

$$\overline{E}_1 = \overline{O}_{6Rj}$$

$$\overline{F}_1 = \overline{O}_{6Rj}$$

$$\overline{G}_1 = \overline{O}_{6Rj}$$

2.7.3. Numerical Example of Kinematic Analysis of RCSR-CSR Mechanism

Numerical example of kinematic analysis of RCSR-CSR spatial mechanism is performed with the dimensions: $a_1 = 10°$, $a_2 = 30°$, $a_4 = 10°$, $a_5 = 20°$, $a_6 = 30°$, $a_8 = 20°$, $\theta_1 = 20°$, $\theta_2 = 20°$, $P_1 =$ .5 in., $P_2 =$ 1. in., $P_3 =$ .4 in., $P_4 =$ .6 in., $S_1 =$ .5 in., $S_2 =$ .6 in., and $S_7 =$ .8 in. Numerical results are listed in Table X.

2.8. Kinematic Analysis of the RCHCH Spatial Mechanism

The geometry and the parameters of the RCHCH spatial mechanism are shown in Figure 19. θ_1 is an input angle, θ_5 is an output angle. P_1, P_2, P_3, P_4 and P_5 are the link lengths of link 1, 2, 3, 4 and 5 respectively. a_1, a_2, a_3, a_4 and a_5 are the twist angle of links 1 through 5 respectively.

2.8.1. Freudenstein Displacement Equation of the RCHCH Spatial Mechanism

For the RCHCH spatial mechanisms, we break the mechanism at one of these C pairs. The open loop chain is unfolded as shown in Figure 20. The position vectors and the direction vectors at the unfolded position are defined from Figure 20.

TABLE X

ANALYSIS OF RCSR-CSR MECHANISM

θ_1	θ_2	S_2	θ_4	S_5	θ_7
160.	-84.007	-8.966	65.565	2.150	163.256
162.	-81.292	-8.964	60.048	2.153	160.014
164.	-78.539	-8.961	54.103	2.157	156.968
166.	-75.793	-8.958	47.806	2.161	154.148
168.	-73.101	-8.955	41.235	2.166	151.577
170.	-70.503	-8.951	34.466	2.171	149.272
172.	-68.038	-8.946	27.568	2.176	147.236
174.	-65.734	-8.941	20.597	2.179	145.464
176.	-63.616	-8.936	13.604	2.182	143.941
178.	-61.701	-8.930	6.624	2.183	142.643
180.	-60.001	-8.924	-0.316	2.183	141.539
182.	-58.520	-8.918	-7.198	2.181	140.597
184.	-57.262	-8.912	-14.012	2.178	139.780
186.	-56.227	-8.906	-20.756	2.174	139.055
188.	-55.413	-8.899	-27.429	2.169	138.390
190.	-54.816	-8.894	-34.040	2.163	137.758
192.	-54.432	-8.888	-40.600	2.156	137.136
194.	-54.258	-8.882	-47.123	2.148	136.507
196.	-54.292	-8.877	-53.630	2.140	135.857
198.	-54.534	-8.872	-60.144	2.131	135.180
200.	-54.984	-8.868	-66.698	2.122	134.472
202.	-55.649	-8.864	-73.329	2.112	133.734
204.	-56.541	-8.861	-80.092	2.103	132.972
206.	-57.680	-8.858	-87.062	2.092	132.196
208.	-59.105	-8.855	-94.350	2.081	131.426
210.	-60.884	-8.854	-102.146	2.069	130.695
212.	-63.165	-8.854	-110.824	2.055	130.070
214.	-66.351	-8.855	-121.392	2.037	129.737

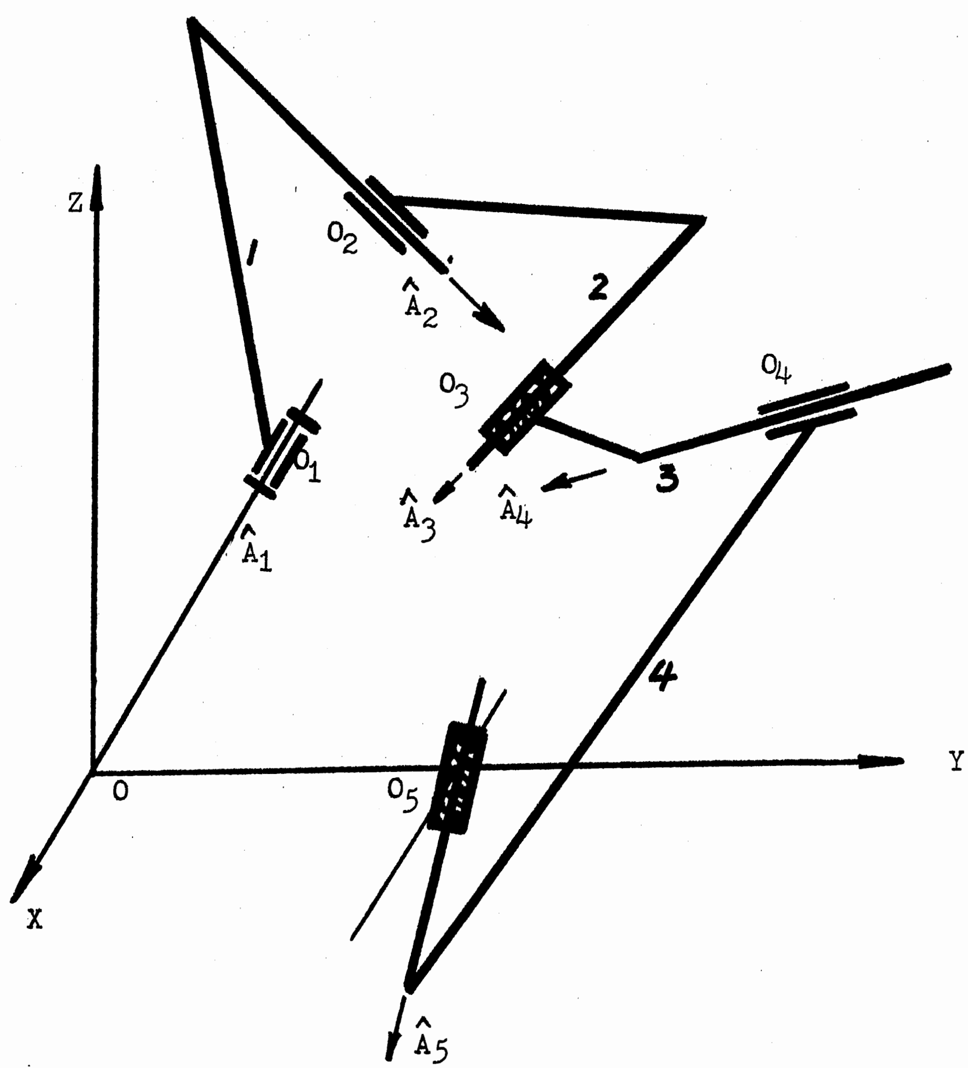

Figure 19. The RCHCH Spatial Mechanism

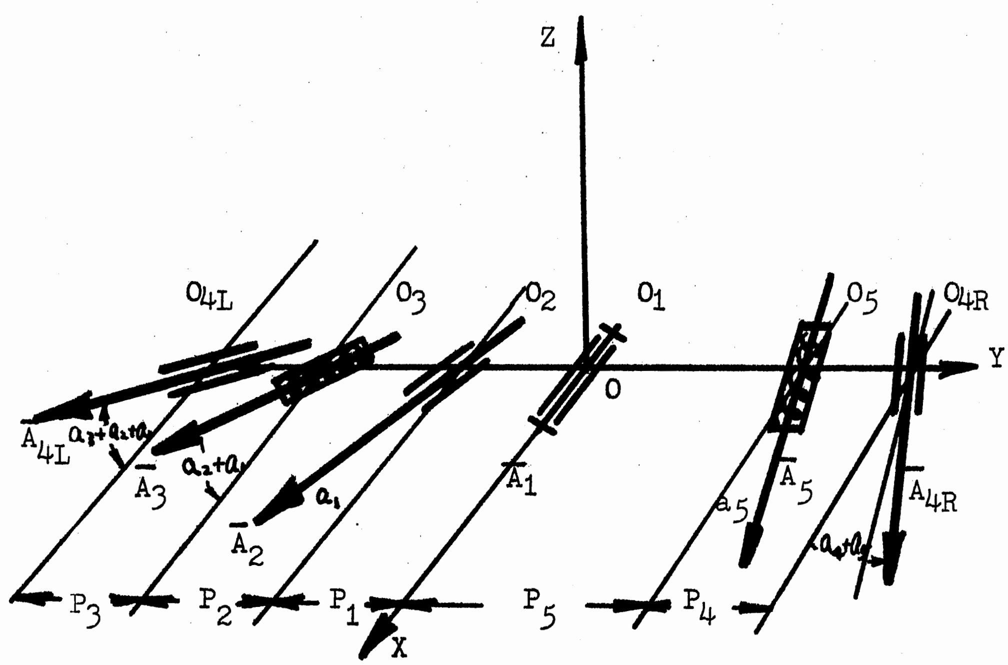

Figure 20. The Open Loop Chain CHRHCH

$$\overline{O}_1 = 0$$

$$\overline{O}_2 = -P_1\overline{j}$$

$$\overline{O}_3 = -(P_1 + P_2)\ \overline{j}$$

$$\overline{O}_{41} = -(P_1 + P_2 + P_3)\ \overline{j}$$

$$\overline{O}_5 = P_5\overline{j}$$

$$\overline{O}_{4R} = (P_5 + P_4)\ \overline{j}$$

$$\overline{A}_1 = \overline{i}$$

$$\overline{A}_2 = COSa_1\overline{i} + SINa_1\overline{k}$$

$$\overline{A}_3 = COS(a_1 + a_2)\ \overline{i} + SIN(a_1 + a_2)\ \overline{k}$$

$$= C_{12}\ \overline{i} + S_{12}\ \overline{k}$$

$$\overline{A}_{41} = COS(a_1 + a_2 + a_3)\ \overline{i} + SIN(a_1 + a_2 + a_3)\ \overline{k}$$

$$= C_{123}\ \overline{i} + S_{123}\ \overline{k}$$

$$\overline{A}_5 = COSa_5\ \overline{i} - SINa_5\ \overline{k}$$

$$\overline{A}_{4R} = COS(a_5 + a_4)\ \overline{i} - SIN(a_5 + a_4)\ \overline{k}$$

$$= C_{54}\ \overline{i} = S_{54}\ \overline{k}$$

Direction vector, $\overline{A}_{4L}$, is displaced to new position $\overline{A}_{4Lj}$ about axes $\overline{A}_3$, $\overline{A}_2$ and $\overline{A}_1$ by angles θ_3, θ_2 and θ_1 respectively. Thus, one has

$$\overline{A}_{4Lj} = C\theta_3\overline{N}_1 + S\theta_3\overline{N}_2 + \overline{N}_3 \tag{2.95}$$

where

$$\overline{N}_1 = \begin{Bmatrix} C\theta_2 C_{5x} + C_{7x} \\ C\theta_1 S\theta_2 C_{6y} + S\theta_1 C\theta_2 C_{5z} + S\theta_1 C_{7z} \\ C\theta_1 C\theta_2 C_{5z} + C\theta_1 C_{7z} - S\theta_1 S\theta_2 C_{6y} \end{Bmatrix}$$

$$\overline{N}_2 = \begin{Bmatrix} -S\theta_2 C_{9x} \\ -C\theta_1 C\theta_2 C_{15y} - S\theta_1 S\theta_2 C_{9z} \\ -S\theta_2 C\theta_1 C_{9z} + S\theta_1 C\theta_2 C_{15y} \end{Bmatrix}$$

$$\overline{N}_3 = \begin{Bmatrix} C\theta_2 C_{11x} + C_{13x} \\ -C\theta_1 S\theta_2 C_{12y} + S\theta_1 (C\theta_2 C_{11z} + C_{13z}) \\ C\theta_1 (C\theta_2 C_{11z} + C_{13z}) + S\theta_1 S\theta_2 C_{12y} \end{Bmatrix}$$

$$\overline{C}_{14} = \begin{Bmatrix} C_{123} - (C_{123} \cdot C_{12} + S_{123} \cdot S_{12}) \cdot C_{12} \\ 0 \\ S_{123} - (C_{123} \cdot C_{12} + S_{123} \cdot S_{12}) \cdot S_{12} \end{Bmatrix}$$

$$\overline{C}_{15} = \begin{Bmatrix} 0 \\ -C_{12} \cdot S_{123} + S_{12} \cdot C_{123} \\ 0 \end{Bmatrix}$$

$$\overline{C}_{16} = \begin{Bmatrix} (C_{123} \cdot C_{12} + S_{123} \cdot S_{12})\ C_{12} \\ 0 \\ (C_{123} \cdot C_{12} + S_{123} \cdot S_{12})\ S_{12} \end{Bmatrix}$$

$$\overline{C}_{7} = \begin{Bmatrix} (C_{14x} \cdot Ca_1 + C_{14z} \cdot Sa_1)\ Ca_1 \\ 0 \\ (C_{14x} \cdot Ca_1 + C_{14z} \cdot Sa_1)\ Sa_1 \end{Bmatrix}$$

$$\overline{C}_{5} = \begin{Bmatrix} C_{14x} - C_{7x} \\ 0 \\ C_{14z} - C_{7z} \end{Bmatrix}$$

$$\overline{C}_{6} = \begin{Bmatrix} 0 \\ -Ca_1 \cdot C_{14z} + Sa_1 \cdot C_{14x} \\ 0 \end{Bmatrix}$$

$$\overline{C}_{8} = \begin{Bmatrix} 0 \\ C_{15y} \\ 0 \end{Bmatrix}$$

$$\overline{C}_9 = \begin{Bmatrix} -Sa_1 \cdot C_{15y} \\ 0 \\ Ca_1 \cdot C_{15y} \end{Bmatrix}$$

$$\overline{C}_{10} = 0$$

$$\overline{C}_{11} = \begin{Bmatrix} C_{16x} - C_{13x} \\ 0 \\ C_{16z} - C_{13z} \end{Bmatrix}$$

$$\overline{C}_{12} = \begin{Bmatrix} 0 \\ -Ca_1 C_{16z} + Sa_1 C_{16x} \\ 0 \end{Bmatrix}$$

$$\overline{C}_{13} = \begin{Bmatrix} (C_{16x} \cdot Ca_1 + C_{16z} \cdot Sa_1)\ Ca_1 \\ 0 \\ (C_{16x} \cdot Ca_1 + C_{16z} \cdot Sa_1)\ Sa_1 \end{Bmatrix}$$

Direction vector, $\overline{A}_{4R}$, is displaced to new position $\overline{A}_{4Rj}$ about axis $\overline{A}_5$ by angle θ_5. One obtains

$$A_{4Rj} = C\theta_5 \overline{N}_4 + S\theta_5 \overline{N}_5 + \overline{N}_6 \tag{2.96}$$

where

$$\overline{N}_4 = \begin{Bmatrix} C_{45} - (Ca_5 \cdot C_{45} + Sa_5 \cdot S_{45})\ Ca_5 \\ 0 \\ -S_{45} + (Ca_5 \cdot C_{45} + Sa_5 \cdot S_{45})\ Sa_5 \end{Bmatrix}$$

$$\overline{N}_5 = \begin{Bmatrix} 0 \\ Ca_5 S45 - Sa_5 C45 \\ 0 \end{Bmatrix}$$

$$\overline{N}_6 = \begin{Bmatrix} (Ca_5 C45 + Sa_5 S45)\ Ca_5 \\ 0 \\ -(Ca_5 C45 + Sa_5 S45)\ Sa_5 \end{Bmatrix}$$

Equating the two displaced direction vectors, Equations (2.95) and (2.96) obtained from both sides, one gets

$$\overline{A}_{4Rj} = \overline{A}_{4Lj} \tag{2.97}$$

Equation (2.97) is a loop-closure equation which is a function of θ_1, θ_2, θ_3 and θ_5. By using the vector identities, one has

$$C\theta_3 = \frac{\overline{A}_{4Rj} \cdot (\overline{N}_2 \times \overline{N}_3)}{\overline{N}_1 \cdot (\overline{N}_2 \times \overline{N}_3)} \tag{2.98}$$

and

$$S\theta_3 = \frac{\overline{A}_{4Rj} \cdot (\overline{N}_1 \times \overline{N}_3)}{\overline{N}_2 \cdot (\overline{N}_1 \times \overline{N}_3)} \tag{2.99}$$

Using the identity, $C^2\theta_3 + S^2\theta_3 = 1$, one has

$$\left(\frac{\overline{A}_{4Rj} \cdot (\overline{N}_2 \times \overline{N}_3)}{\overline{N}_1 \cdot \overline{N}_2 \times \overline{N}_3}\right)^2 + \left(\frac{\overline{A}_{4Rj} \cdot (\overline{N}_1 \times \overline{N}_3)}{\overline{N}_2 \cdot \overline{N}_1 \times \overline{N}_3}\right)^2 = 1 \qquad (2.100)$$

Equation (2.100) is a function of θ_1, θ_2 and θ_5 by using the half angle relationship $\Phi_2 = \tan\frac{\theta_2}{2}$, where $C\theta_2 = \frac{1 - \Phi_2^2}{1 + \Phi_2^2}$, and $S\theta_2 = \frac{2\Phi_2}{1 + \Phi_2^2}$, to have

$$\Phi_2^8 J1 + \Phi_2^7 J2 + \Phi_2^6 J3 + \Phi_2^5 J4 + \Phi_2^4 J5 + \Phi_2^3 J6 + \Phi_2^2 J7 + \Phi_2 J8 + J9 = 0 \qquad (2.101)$$

where

$$J1 = I1 - I5 + I10 - I13 + I15$$

$$J2 = -2I3 + 2I8 - 2I12 + 2I14$$

$$J3 = -4I1 + 2I5 + 4I7 - 4I9 + 4I11 - 2I13 + 4I15$$

$$J4 = 6I3 + 8I6 - 2I8 - 2I12 + 6I14$$

$$J5 = 6I1 + 16I2 - 8I7 - 2I10 + 8I11 + 6I15$$

$$J6 = -6I3 - 2I4 + 8I6 - 2I8 + 2I12 + 6I14$$

$$J7 = -4I1 - 2I5 + 4I7 + 4I9 + 4I11 + 2I13 + 4I15$$

$$J8 = 2I3 + 2I4 + 2I8 + 2I12 + 2I14$$

$$J9 = I1 + I5 + I10 + I13 + I15$$

$$I1 = gi^2 + hi^2 - fi^2$$

$$I2 = f3^2$$

$$I3 = 2h1h2 + 2g1g3 - 2f1f2$$

$$I4 = 2f2f3 + 2g2g3$$

$$I5 = 2g1g4 + 2h1h3 - 2f1f4$$

$$I6 = -2f3f5 + 2g2g5$$

$$I7 = h2^2 + g3^2 + 2g1g2 - f2^2 + 2f1f3$$

$$I8 = 2g5g1 + 2h2h3 - 2f2f4 + 2g3g4 + 2f1f5 + 2h1h4$$

$$I9 = 2g3g5 + 2g2g4 + 2h2h4 + 2f2f5 + 2f3f4$$

$$I10 = g4^2 + h3^2 + 2h1h5 - f4^2$$

$$I11 = g2^2 + g5^2 + h4^2 - f5^2$$

$$I12 = 2g4g5 + 2h2h5 + 2h3h4 + 2f4f5$$

$$I13 = 2h3h5$$

$$I14 = 2h4h5$$

$$I15 = h5^2$$

$$g1 = C\theta_2^2 C_{11x} C\theta_1 C_{15y}$$

$$g2 = C\theta_1 C_{12y} C_{9x}$$

$$g3 = C_{11x} S\theta_1 C_{9z} - S\theta_1 C_{11z} C_{9x}$$

$$g4 = C\theta_1 C_{13x} C_{15y} - C\theta_5 N_{4x} C\theta_1 C_{15y} - N_{6x} C\theta_1 C_{15y}$$

$$g5 = C_{13x} S\theta_1 C_{9z} - C\theta_5 N_{4x} S\theta_1 C_{9z} - N_{6x} S\theta_1 C_{9z} - S\theta_1 C_{13z} C_{9x} - S\theta_5 N_{5y} C_{9x}$$

$$h1 = -S\theta_1 C_{11x} C_{5z} + S\theta_1 C_{11z} C_{5x}$$

$$h2 = -C_{11x} C\theta_1 C_{6y} - C\theta_1 C_{12y} C_{5x}$$

$$h3 = -C_{11x} S\theta_1 C_{7z} + S\theta_1 C_{5z}(C\theta_5 N_{4x} + N_{6x}) - S\theta_5 N_{5y} C_{5x} +$$

$$C_{11z} S\theta_1 C_{7x} + S\theta_1 C_{13z} C_{5x}$$

$$h4 = -C_{13x} C\theta_1 C_{6y} + C\theta_1 C_{6y}(C\theta_5 N_{4x} + N_{6x}) + C\theta_1 C_{12y} C_{7x}$$

$$h5 = -C_{13x} S\theta_1 C_{7z} + (C\theta_5 N_{4x} + N_{6x})\ S\theta_1 C_{7z} + S\theta_1 C_{13z} C_{7x} -$$

$$S\theta_5 N_{5y} C_{7x}$$

Equation (2.101) is a function of θ_1, θ_2 and θ_5. To eliminate θ_2 from Equation (2.101), one needs to derive another loop-closure equation. Care must be exercised in breaking the loop in order to prevent introducing higher order loop-closure equation. The open-loop chain is unfolded as shown in Figure 21.

The newly defined vectors are

$$\overline{A}_{3L} = \cos(a_1 + a_2)\ \overline{i} + \sin(a_1 + a_2)\ \overline{k}$$

$$= C_{12} i + S_{12} \overline{k}$$

$$\overline{A}_{3R} = \cos(a_5 + a_4 + a_3)\ \overline{i} - \sin(a_5 + a_4 + a_3)\ \overline{k}$$

$$= C_{543} i - S_{543} \overline{k}$$

Direction vector, A_{3L}, is displaced to new position A_{3Lj} about axes $\overline{A}_2$ and $\overline{A}_1$ by angles θ_2 and θ_1 respectively. One has

$$\overline{A}_{3Lj} = C\theta_2 \overline{F}_4 - S\theta_2 \overline{F}_5 + \overline{F}_6 \qquad (2.102)$$

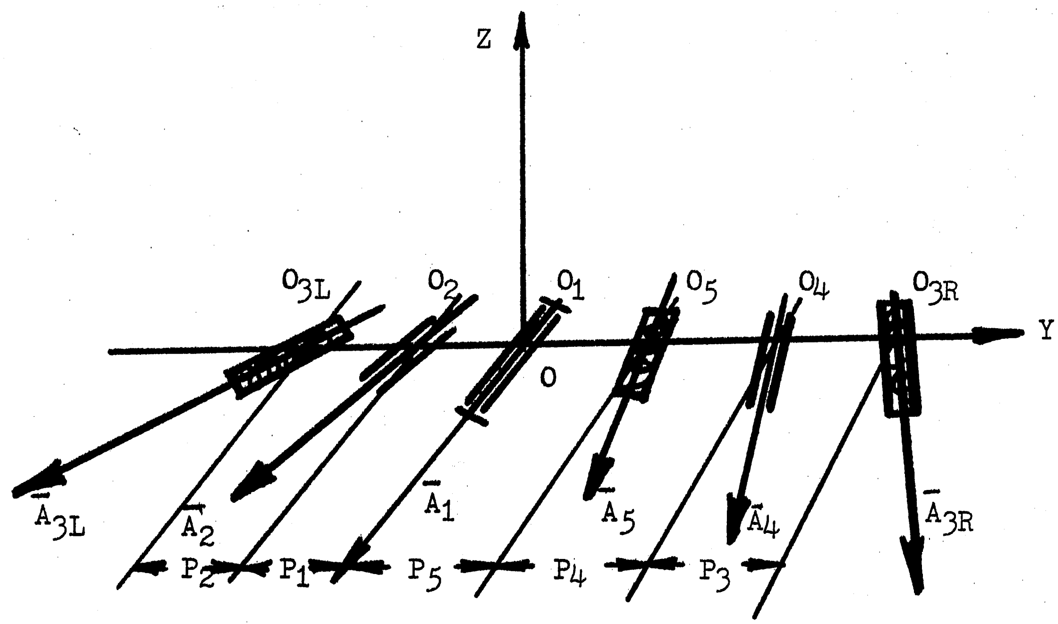

Figure 21. The Open Loop Chain HCRHCH

where

$$\overline{D}16 = \begin{Bmatrix} (C12Ca_1 + S12Sa_1)\ Ca_1 \\ 0 \\ (C12Ca_1 + S12Sa_1)\ Sa_1 \end{Bmatrix}$$

$$D12 = \begin{Bmatrix} 0 \\ (C12Ca_1 + S12Sa_1)\ Sa_1 \\ 0 \end{Bmatrix}$$

$$\overline{D}13 = \begin{Bmatrix} (C12Ca_1 + S12Sa_1)\ Ca_1 \\ 0 \\ 0 \end{Bmatrix}$$

$$\overline{D}14 = \begin{Bmatrix} c12 - D16x \\ 0 \\ S12 - D16z \end{Bmatrix}$$

$$\overline{D}15 = \begin{Bmatrix} 0 \\ -Ca_1S12 + Sa_1C12 \\ 0 \end{Bmatrix}$$

$$\overline{D5} = \left\{ \begin{array}{c} 0 \\ 0 \\ D14z \end{array} \right\}$$

$$D6 = \left\{ \begin{array}{c} 0 \\ -D14z \\ 0 \end{array} \right\}$$

$$\overline{D7} = \left\{ \begin{array}{c} D14x \\ 0 \\ 0 \end{array} \right\}$$

$$\overline{D8} = \left\{ \begin{array}{c} 0 \\ D15y \\ 0 \end{array} \right\}$$

$$\overline{D9} = \left\{ \begin{array}{c} 0 \\ 0 \\ D15y \end{array} \right\}$$

$$\overline{D10} = \left\{ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right\}$$

$$\overline{F4} = \begin{Bmatrix} D14x \\ -S\theta_1 D14z \\ C\theta_1 D14z \end{Bmatrix}$$

$$\overline{F5} = \begin{Bmatrix} 0 \\ C\theta_1 D15y \\ S\theta_1 D15y \end{Bmatrix}$$

$$\overline{F6} = \begin{Bmatrix} D16x \\ S\theta_1 D16z \\ C\theta_1 D16z \end{Bmatrix}$$

The direction vector, $\overline{A}_{3R}$, is displaced to new position $\overline{A}_{3Rj}$ about axes $\overline{A}_4$ and $\overline{A}_5$ by angles θ_4 and θ_5 respectively to yield

$$\overline{A3Rj} = C\theta 4\overline{F1} + S\theta 4\overline{F2} + \overline{F3} \qquad (2.103)$$

where

$$\overline{E15} = \begin{Bmatrix} 0 \\ C45S345 - S45C345 \\ 0 \end{Bmatrix}$$

$$\overline{E16} = \begin{Bmatrix} (C345C45 + S345S45)\ C45 \\ 0 \\ -(C345C45 + S345S45)\ S45 \end{Bmatrix}$$

$$\overline{E14} = \begin{Bmatrix} C345 - E16x \\ 0 \\ -S345 - E16z \end{Bmatrix}$$

$$\overline{E12} = \begin{Bmatrix} 0 \\ -Ca_5E16z - Sa_5E16x \\ 0 \end{Bmatrix}$$

$$\overline{E13} = \begin{Bmatrix} (E16xCa_1 - E16zSa_5)\ Ca_5 \\ 0 \\ -(E16xCa_1 - E16zSa_5)\ Sa_5 \end{Bmatrix}$$

$$\overline{E11} = \begin{Bmatrix} E16x - (E16xCa_5 - E16zSa_5)\ Ca_5 \\ 0 \\ E16z + (E16xCa_5 - E16zSa_5)\ Sa_5 \end{Bmatrix}$$

$$\overline{E5} = \begin{Bmatrix} E14x - E7x \\ 0 \\ E14z - E7z \end{Bmatrix}$$

$$\overline{E6} = \begin{Bmatrix} 0 \\ -Ca_5E14z - Sa_5E14x \\ 0 \end{Bmatrix}$$

$$\overline{E7} = \begin{Bmatrix} (E14xCa_5 - E14zSa_5)\ Ca_5 \\ 0 \\ -(E14xCa_5 - E14zSa_5)\ Sa_5 \end{Bmatrix}$$

$$\overline{E8} = \begin{Bmatrix} 0 \\ E15y \\ 0 \end{Bmatrix}$$

$$\overline{E9} = \begin{Bmatrix} Sa_5E15y \\ 0 \\ Ca_5E15y \end{Bmatrix}$$

$$\overline{E10} = 0$$

$$\overline{F1} = \begin{Bmatrix} C\theta_5E5x + E7x \\ S\theta_5E6y \\ C\theta_5E5z + E7z \end{Bmatrix}$$

$$\overline{F2} = \begin{Bmatrix} S\theta_5E9x \\ C\theta_5E15y \\ S\theta_5E9z \end{Bmatrix}$$

and

$$\overline{F3} = \begin{Bmatrix} C\theta_5 E11x + E13x \\ S\theta_5 E12y \\ C\theta_5 E11z + E13z \end{Bmatrix}$$

To equate the two direction vectors obtained from both sides to yield

$$\overline{A}_{3Lj} = \overline{A}_{3Rj} \tag{2.104}$$

By using the vector identities, we obtain

$$C\theta_4 = \frac{\overline{A}_{3Lj} \cdot (\overline{F}_2 \times \overline{F}_3)}{\overline{F}_1 \cdot (\overline{F}_2 \times \overline{F}_3)} \tag{2.105}$$

and

$$S\theta_4 = \frac{\overline{A}_{3Lj} \cdot (\overline{F}_1 \times \overline{F}_3)}{\overline{F}_2 \cdot (\overline{F}_1 \times \overline{F}_3)} \tag{2.106}$$

To eliminate θ_4 from Equations (2.105) and (2.106) to have

$$\left(\frac{\overline{A}_{31j} \cdot (\overline{F}_2 \times \overline{F}_3)}{\overline{F}_1 \cdot (\overline{F}_2 \times \overline{F}_3)}\right)^2 + \left(\frac{\overline{A}_{31j} \cdot (\overline{F}_1 \times \overline{F}_3)}{\overline{F}_2 \cdot (\overline{F}_1 \times \overline{F}_3)}\right)^2 = 1 \tag{2.107}$$

Equation (2.107) is a function of θ_1, θ_2 and θ_5. By substituting the half angle relationship, $\Phi_2 = \tan\frac{\theta_2}{2}$ where $SIN\theta_2 = \frac{2\Phi}{1 + \Phi_2^2}$, and $COS\theta_2 = \frac{1 - \Phi_2^2}{1 + \Phi_2^2}$ into Equation (2.107), one obtains

$$\Phi_2^4 C1 + \Phi_2^3 C2 + \Phi_2^2 C3 + \Phi_2 C4 + C5 = 0 \tag{2.108}$$

where

$$C1 = H1 + H3 - H5$$

$$C2 = -2H4 + 2H6$$

$$C3 = -2H1 + 4H2 + 2H3$$

$$C4 = 2H4 + 2H6$$

$$C5 = H1 + H3 + H5$$

$$H1 = G2^2 + G5^2$$

$$H2 = G1^2 + G4^2$$

$$H3 = G3^2 + G6^2 - 1$$

$$H4 = 2G1G2 + 2G4G5$$

$$H5 = 2G2G3 + 2G5G6$$

$$H6 = 2G1G3 + 2G4G6$$

Using Sylvester's dyalitic eliminant technique, one eliminates the common root Φ_2 from Equations (2.101) and (2.108), to obtain Equation (2.109)

$$\begin{bmatrix} J1 & J2 & J3 & J4 & J5 & J6 & J7 & J8 & J9 & 0 & 0 & 0 \\ 0 & J1 & J2 & J3 & J4 & J5 & J6 & J7 & J8 & J9 & 0 & 0 \\ 0 & 0 & J1 & J2 & J3 & J4 & J5 & J6 & J7 & J8 & J9 & 0 \\ 0 & 0 & 0 & J1 & J2 & J3 & J4 & J5 & J6 & J7 & J8 & J9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & C1 & C2 & C3 & C4 & C5 \\ 0 & 0 & 0 & 0 & 0 & 0 & C1 & C2 & C3 & C4 & C5 & 0 \\ 0 & 0 & 0 & 0 & 0 & C1 & C2 & C3 & C4 & C5 & 0 & 0 \\ 0 & 0 & 0 & 0 & C1 & C2 & C3 & C4 & C5 & 0 & 0 & 0 \\ 0 & 0 & 0 & C1 & C2 & C3 & C4 & C5 & 0 & 0 & 0 & 0 \\ 0 & 0 & C1 & C2 & C3 & C4 & C5 & 0 & 0 & 0 & 0 & 0 \\ 0 & C1 & C2 & C3 & C4 & C5 & 0 & 0 & 0 & 0 & 0 & 0 \\ C1 & C2 & C3 & C4 & C5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = 0 \qquad (2.109)$$

Equation (2.109) is a function of input and output rotation angles. It may be regarded as a Freudenstein type displacement equation in a matrix form. Because of its complex nature, it is wise not to expand the matrix in a polynomial form. Instead, one solves the displacement equation by using a suitable iteration technique to find θ_5.

2.8.2. Expressions for O_2, O_3, O_4, S_2, S_3, S_4 and S_5

Substituting θ_5 from Equation (2.109) into Equation (2.108),

$$\theta_2 = 2 \tan^{-1} \Phi_2 \qquad (2.110)$$

Substituting θ_2 and θ_5 into Equations (2.105) and (2.106), one obtains

$$\theta_4 = \tan^{-1} \left(\frac{S\theta_4}{C\theta_4} \right) \qquad (2.111)$$

Substituting θ_4 and θ_5 into Equation (2.98) and (2.99), one obtains

$$\theta_3 = \tan^{-1} \left(\frac{S\theta_3}{C\theta_3} \right) \qquad (2.112)$$

The linear displacements at the helical pairs are constrained by their pitch values.

$$S_3 = \rho_3 \theta_3 \qquad (2.113)$$

$$S_5 = \rho_5 \theta_5 \qquad (2.114)$$

From Figure 20, $\overline{O}_{3L}$ is displaced to new position $\overline{O}_{3Lj}$ by the screw motions through $\overline{A}_2$ then $\overline{A}_1$.

$$\overline{O}_{3Lj} = C\theta_2(C\theta_1(\overline{D}_1 - (\overline{D}_1 \cdot \overline{A}_1)\,\overline{A}_1) - S\theta_1(\overline{A}_1 \times \overline{D}_1) + (\overline{D}_1 \cdot \overline{A}_1)\,\overline{A}_1)) - S\theta_2(C\theta_1(\overline{D}_2 - (\overline{D}_2 \cdot \overline{A}_1)\,\overline{A}_1 - S\theta_1(\overline{A}_1 \times \overline{D}_2) + (\overline{D}_2 \cdot \overline{A}_1)\,\overline{A}_1) + C\theta_1((\overline{D}_3 - \overline{P}_1) - (\overline{D}_3 - \overline{P}_1)\,\overline{A}_1\,\overline{A}_1) - S\theta_1(\overline{A}_1 \times (\overline{P}_3 - \overline{P}_1)) - S_1\overline{A}_1 + \overline{P}_1 \qquad (2.115)$$

$\overline{O}_{3R}$ is displaced to new position O_{3Rj} by the screw motions through $\overline{A}_4$ then $\overline{A}_5$ to yield

$$\overline{O}_{3Rj} = C\theta_4(C\theta_5(\overline{E}_1 - (\overline{E}_1 \cdot \overline{A}_5)\,\overline{A}_5) + S\theta_5(\overline{A}_5 \times \overline{E}_1) + (\overline{E}_1 \cdot \overline{A}_5)\,\overline{A}_5) + S\theta_4(C\theta_5(\overline{E}_2 - (\overline{E}_2 \cdot \overline{A}_5)\,\overline{A}_5) + S\theta_5(\overline{A}_5 \times \overline{E}_2) + (\overline{E}_2 \cdot \overline{A}_5)\,\overline{A}_5) + S_4(C\theta_5(\overline{A}_4 - (\overline{A}_4 \cdot \overline{A}_5)\,\overline{A}_5) + S\theta_5(\overline{A}_5 \times \overline{A}_4) + (\overline{A}_4 \cdot \overline{A}_5)\,\overline{A}_5) + C\theta_5((\overline{E}_3 - \overline{P}_5) - ((\overline{E}_3 - \overline{P}_5)\,\overline{A}_5)\,\overline{A}_5) + S\theta_5(\overline{A}_5 \times (\overline{E}_3 - \overline{P}_5)) + \overline{S}_2\overline{A}_5 + \overline{P}_5 + ((\overline{E}_3 - \overline{P}_5)\,\overline{A}_5)\,\overline{A}_5 + \overline{A}_{3Rj} \cdot S_3 \qquad (2.116)$$

where

$$\overline{D}_1 = \begin{Bmatrix} 0 \\ -P_2 \\ 0 \end{Bmatrix}$$

$$\overline{D}_2 = \begin{Bmatrix} Sa_1P_2 \\ 0 \\ -Ca_1P_2 \end{Bmatrix}$$

$$\overline{D}_3 = \begin{Bmatrix} 0 \\ -P_1 \\ 0 \end{Bmatrix}$$

$$\overline{E}_1 = \begin{Bmatrix} 0 \\ P_3 \\ 0 \end{Bmatrix}$$

$$E_2 = \begin{Bmatrix} S_{45}P_3 \\ 0 \\ C_{45}P_3 \end{Bmatrix}$$

$$E_3 = \begin{Bmatrix} 0 \\ P_5 + P_4 \\ 0 \end{Bmatrix}$$

Equating the results of Equation (2.115) with those of Equation (2.116), one obtains three linear equations which, in turn, can be solved for the unknown S_2 and S_4.

2.8.3. Velocity Analysis of the RCHCH Spatial Mechanism

Taking the time derivative of Equation (2.97), one gets

$$\begin{aligned} -\dot{\overline{A}}_{4Lj} &= (\overline{A}_1\omega_1 + \overline{A}_{2j}\omega_2 + \overline{A}_{3j}\omega_3) \times \overline{A}_{4Lj} \\ &= -(\overline{A}_5\omega_5) \times \overline{A}_{4Rj} \end{aligned} \qquad (2.117)$$

Note that ω_2, ω_3 and ω_5 can be solved simultaneously from Equation (2.117). $\dot{S}_2$ and $\dot{S}_4$ can be obtained from the following equation.

$$-\dot{\overline{O}}_{4j} = \overline{A}_1 \times (\overline{O}_{41j} - \overline{O}_1)\ \omega_1 + \overline{A}_{2j} \times (\overline{O}_{41j} - \overline{O}_{2j})\ \omega_2 + \overline{A}_{2j} \cdot \dot{S}_2 + \overline{A}_{3j} \times (\overline{O}_{41j} - \overline{O}_{3j})\ \omega_3 + \overline{A}_{3j} \cdot \dot{S}_3 = \overline{A}_5 \times (\overline{O}_{4Rj} - \overline{O}_{5j})\ \omega_5 - \overline{A}_5 \cdot \dot{S}_5 - \overline{A}_{4Rj} \cdot \dot{S}_4 \qquad (2.118)$$

The linear velocities at helical pairs are

$$\dot{S}_3 = \rho_3\omega_3 \qquad (2.119)$$

and

$$\dot{S}_5 = \rho_5\omega_5 \qquad (2.120)$$

The linear velocity at point O_2 is

$$-\dot{\overline{O}}_{2j} = \overline{A}_{2j} \times (\overline{O}_{2j} - \overline{O}_1)\ \omega_1 + \overline{A}_1\dot{S}_2 \qquad (2.121)$$

The linear velocity at point O_3 is

$$-\dot{\overline{O}}_{3j} = \overline{A}_{2j} \times (\overline{O}_{3j} - \overline{O}_{2j})\ \omega_2 + \overline{A}_{3j} \times (\overline{O}_{3j} - \overline{O}_1)\ \omega_1 + \overline{A}_{3j}\ \dot{S}_3 + \overline{A}_{2j} \cdot \dot{S}_2 \qquad (2.122)$$

The linear velocity at point C_4 is

$$-\dot{\overline{O}}_4 = \overline{A}_{3j} \times (\overline{O}_{4j} - \overline{O}_{3j})\ \omega_3 + \overline{A}_{2j} \times (\overline{O}_{4j} - \overline{O}_{2j})\ \omega_2 + \overline{A}_1 \times (\overline{O}_{3j} - \overline{O}_1)\ \omega_1 + \overline{A}_{3j} \cdot \dot{S}_3 + \overline{A}_{2j} \cdot \dot{S}_2 \qquad (2.123)$$

2.8.4. Acceleration Analysis of the RCHCH Spatial Mechanism

Taking the time derivative of Equation (2.117), one gets

$$-\ddot{\overline{A}}_{41} = (\overline{A}_1\omega_1 + \overline{A}_{2j}\omega_2 + \overline{A}_{3j}\omega_3) \times \dot{\overline{A}}_4 + (\overline{A}_{2j}\omega_2 + \dot{\overline{A}}_{3j}\omega_3 +$$

$$\overline{A}_1\alpha_1 + \overline{A}_{2j}\alpha_2 + \overline{A}_{3j}\alpha_3) \times \overline{A}_{4j}$$

$$= -(\overline{A}_5\alpha_5) \times \overline{A}_{4Rj} - (\overline{A}_5\omega_5) \times \overline{A}_{4Rj} \qquad (2.124)$$

Equation (2.124) can be solved simultaneously for α_2, α_3 and α_5. Taking the time derivate of Equation (2.118), one obtains

$$-\ddot{\overline{O}}_{4j} = \overline{A}_1 \times (\overline{O}_{41j})\ \omega_1 + \overline{A}_1 \times (\overline{O}_{41j} - \overline{O}_1)\ \alpha_1 + \dot{\overline{A}}_{2j} \times (\overline{O}_{41j} -$$

$$\overline{O}_{2j})\ \omega_2 + \overline{A}_{2j} \times (\dot{\overline{O}}_{4j} - \dot{O}_{2j})\ \omega_2 + \overline{A}_{2j} \times (O_{41j} -$$

$$O_1)\ \alpha_2 + \dot{\overline{A}}_{2j}\dot{S}_2 + \overline{A}_{2j}\ddot{S}_2 + (\dot{\overline{A}}_3 + (\overline{O}_{41j} - \overline{O}_{3j}) +$$

$$\overline{A}_{3j} \times (\dot{\overline{O}}_{41j} - \dot{\overline{O}}_{3j}))\ \omega_3 + \overline{A}_{3j} \times (\overline{O}_{41j} - \overline{O}_{3j})\ \alpha_3 +$$

$$\dot{\overline{A}}_{3j}\dot{S}_3 + \overline{A}_{3j}\ddot{S}_3$$

$$= -(\overline{A}_{5j} \times (\dot{\overline{O}}_{4Rj} - \dot{\overline{O}}_{5j}))\ \omega_5 - \overline{A}_{5j} \times (\overline{O}_{4Rj} - \overline{O}_{5j})\ \alpha_5 -$$

$$\overline{A}_{5j}\ddot{S}_5 - \dot{\overline{A}}_{4Rj}\dot{S}_4 - \overline{A}_{4Rj}\ddot{S}_4 \qquad (2.125)$$

$\ddot{S}_2$ and $\ddot{S}_4$ can be solved simultaneously from Equation (2.125). The linear acceleration at helical pairs are:

$$\ddot{S}_3 = \rho_3\alpha_3 \qquad (2.126)$$

$$\ddot{S}_3 = \rho_5\alpha_5 \qquad (2.127)$$

2.8.5. Kinematic Analysis of the Mass Center Point of Each Link of RCHCH Spatial Mechanism

As shown in Figure 20, we define the coordinates of each mass center at unfolded position (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) and (x_4, y_4, z_4) for links 1, 2, 3 and 4 respectively.

The position P of mass center of link 4 is

$$\overline{P}_{m4j} = \begin{Bmatrix} z_4Sa_5C\theta_5 + S\theta_5Sa_5y_4 + s_5Ca_5 + x_4Ca_5 \\ C\theta_5y_4 - S\theta_5z_4 + P_5 \\ C\theta_5Ca_5z_4 + S\theta_5Ca_5y_4 - s_5Sa_5 - x_4Sa_5 \end{Bmatrix} \quad (2.128)$$

The velocity of mass center $\frac{dP_{m4j}}{dt}$ of link 4 is

$$\frac{dP_{m4j}}{dt} = \begin{Bmatrix} Sa_5(y_4C\theta_5 - z_4S\theta_5) \\ -z_4C\theta_5 - Y_4S\theta_4 \\ Ca_5(y_4C\theta_5 - z_4S\theta_5) \end{Bmatrix} \omega_5 + \begin{Bmatrix} Ca_5\dot{S}_5 \\ 0 \\ -Sa_5\dot{S}_5 \end{Bmatrix} \quad (2.129)$$

The acceleration of mass center $\frac{d^2P_{m4j}}{dt^2}$ of link 4 is

$$\frac{d^2P_{m4j}}{dt^2} = \begin{Bmatrix} Sa_5(y_4CO_5 - z_4S\theta_5) \\ -z_4C\theta_5 - y_4S\theta_5 \\ Ca_5(y_4C\theta_5 - z_4S\theta_5) \end{Bmatrix} \alpha_5 + \begin{Bmatrix} Sa_5(-z_4C\theta_5 - y_4S\theta_5) \\ -(y_4C\theta_5 - z_4S\theta_5) \\ Ca_5(-z_4C\theta_5 - y_4S\theta_5) \end{Bmatrix} \omega_5^2 +$$

$$\begin{Bmatrix} Ca_5\ddot{S}_5 \\ 0 \\ -Sa_5\ddot{S}_5 \end{Bmatrix} \quad (2.130)$$

The position P_{m1j} of mass center of link 1 is

$$\overline{P}_{m1j} = \begin{Bmatrix} x_1 - S_1 \\ y_1 C\theta_1 - z_1 S\theta_1 \\ -y_1 S\theta_1 + z_1 C\theta_1 \end{Bmatrix} \tag{2.131}$$

The velocity $\frac{d\overline{P}_{m1j}}{dt}$ of mass center of link 1 is

$$\frac{-d\overline{P}_{m1j}}{dt} = \begin{Bmatrix} 0 \\ -y_1 S\theta_1 + z_1 C\theta_1 \\ -y_1 C\theta_1 + z_1 S\theta_1 \end{Bmatrix} \omega_1 \tag{2.132}$$

The acceleration $\frac{d^2\overline{P}_{m1j}}{dt^2}$ of mass center of link 1 is

$$\frac{-d^2\overline{P}_{m1j}}{dt^2} = \frac{d\overline{p}_{m1j}}{dt}_{1} + \begin{Bmatrix} 0 \\ -(y_1 C\theta_1 + z_1 S\theta_1) \\ (-y_1 S\theta_1 + z_1 C\theta_1) \end{Bmatrix} \omega_1^{\;2}{}_{1^2} \tag{2.133}$$

The position P_{m2j} of mass center of link 2 is

$$\overline{P}_{m2j} = \begin{Bmatrix} -S_1 - z_2 Sa_1 C\theta_2 + S\theta_2 Sa_1 y_2 - S_2 Ca_1 + x_2 Ca_1 \\ C\theta_1 (y_2 C\theta_2 + S\theta_2 z_2 - p_1) - S\theta_1 (-Ca_1 z_2 C\theta_2 + Ca_1 y_2 S\theta_2 + \\ S_2 Sa_1 - x_2 Sa_1) \\ C\theta_1 (Ca_1 z_2 C\theta_2 - Ca_1 y_2 S\theta_2 - S_2 Sa_1 + x_2 Sa_1) - S\theta_1 (y_2 C\theta_2 + \\ S\theta_2 z_2 - p_1) \end{Bmatrix}$$

(2.134)

The velocity $\frac{d\overline{P}_{m2j}}{dt}$ of mass center of link 2 is

$$\frac{-d\overline{P}_{m2j}}{dt} = \overline{A}_{2j} \times (\overline{P}_{m2j} - \overline{O}_{2j})\ \omega_2 + \overline{A}_{2j} \cdot\ \dot{S}_2 + \overline{A}_1 \times (\overline{P}_{m2j} - \overline{O}_1)\ \omega_1 \tag{2.135}$$

The acceleration $\frac{d^2\overline{P}_{m2j}}{dt^2}$ of mass center of link 2 is

$$\frac{-d^2\overline{P}_{m2j}}{dt^2} = (\overline{A}_{2j} \times (\frac{d\overline{P}_{m2j}}{dt} - \frac{d\overline{O}_{2j}}{dt}) + \frac{d\overline{A}_{2j}}{dt} \times (\overline{P}_{m2j} - \overline{O}_{2j}))\ \omega_2 + (\overline{A}_{2j} \times (\overline{P}_{m2j} - \overline{O}_{2j}))\ \alpha_2 + \overline{A}_{2j}\ddot{S}_2 + \frac{dA_{2j}}{dt}\ \dot{S}_2 + (\overline{A}_1 \times (\overline{P}_{m2j} - \overline{O}_1))\ \alpha_1 + \overline{A}_1 \times (\frac{d\overline{P}_{m2j}}{dt})\ \omega_1 \tag{2.136}$$

The position $\overline{P}_{m3j}$ of the mass center of link 3 is

$$\overline{P}_{m3j} = C\theta_1((\overline{P}'_{m3} - \overline{O}_1) - ((\overline{P}'_{m3} - \overline{O}_1)\ \overline{A}_1)\ \overline{A}_1) - S\theta_1(\overline{A}_1 \times (\overline{P}'_{m3} - \overline{O}_1)) - S_1\overline{A}_1 + \overline{O}_1 + ((\overline{P}'_{m3} - \overline{O}_1)\ \overline{A}_1)\ \overline{A}_1) \tag{2.137}$$

where

$$\overline{P}_{m3} = \left\{ \begin{array}{l} -S_2Ca_1 - z_3S_{12}C\theta_3 + S\theta_3S_{12}y_3 - S_3C_{12} + x_3C_{12} \\ C\theta_2(y_3C\theta_3 + S\theta_3z_3 - p_2) - S\theta_2(-C_{12}z_3C\theta_3 + C_{12}y_3S\theta_3 + S_3S_{12} - x_3S_{12}) \\ C\theta_2(C_{12}z_3CO_3 - C_{12}y_3S\theta_3 - S_3S_{12} + x_3S_{12}) - S\theta_2(y_3C\theta_3 + S\theta_3z_3 - p_2) \end{array} \right\}$$

The velocity $\frac{d\overline{P}_{m3j}}{dt}$ of the mass center of link 3 is

$$\frac{-d\overline{P}_{m3j}}{dt} = \overline{A}_{3j} \times (\overline{P}_{m3j} - \overline{O}_{3j})\ \omega_3 + \overline{A}_{3j}\dot{S}_3 + \overline{A}_{2j} \times (\overline{P}_{m3j} - \overline{O}_{2j})\ \omega_2 +$$

$$\overline{A}_{2j}\dot{S}_2 + \overline{A}_1 \times (\overline{P}_{m3j})\ \omega_1 \qquad (2.138)$$

The acceleration $\frac{d^2P_{m3j}}{dt^2}$ of the mass center of link 3 is

$$\frac{-d^2\overline{P}_{m3j}}{dt^2} = \overline{A}_{3j} \times (\overline{P}_{m3j} - \overline{O}_{3j})\ \alpha_3 + \left(\frac{dA_{3j}}{dt} \times (\overline{P}_{m3j} - \overline{O}_{3j}) +\right.$$

$$\overline{A}_{3j} \times \left(\frac{d\overline{P}_{m3j}}{dt} - \frac{d\overline{O}_{3j}}{dt} \right)\Big)\ \omega_3 + A_{3j}\ddot{S}_3 + \frac{d\overline{A}_{3j}}{dt}\dot{S}_3 +$$

$$\overline{A}_{2j} \times (\overline{P}_{m3j} - \overline{O}_{2j})\ \alpha_2 + \left(\frac{dA_{2j}}{dt} \times (\overline{P}_{m3j} - \overline{O}_{2j}) +\right.$$

$$\overline{A}_{2j} \times \left(\frac{d\overline{P}_{m3j}}{dt} - \frac{d\overline{O}_{2j}}{dt} \right)\Big)\ \omega_2 + \overline{A}_{2j}\ddot{S}_2 + \frac{d\overline{A}_{2j}}{dt}\dot{S}_2 +$$

$$A_1 \times \overline{P}_{m3j}\ \alpha_1 + \left(\overline{A}_1 \times \frac{d\overline{P}_{m3j}}{dt}\right)\ \omega_1 \qquad (2.139)$$

2.8.6. Numerical Example of the Kinematic Analysis of the RCHCH Spatial Mechanism

Numerical example of kinematic analysis of RCHCH spatial mechanism is performed with the dimensions: $a_1 = 10°$, $a_2 = 80°$, $a_3 = 40°$, $a_4 = 50°$, $a_5 = 60°$, $P_1 = 1$ in., $P_2 = .5$ in., $P_3 = .8$ in., $P_4 = .6$ in., $P_5 = .4$ in., $\omega_1 = .1$ rps, $\alpha_1 = .1$ revolute per second^2, $\rho_3 = 2$, $\rho_5 = 5$.

The numerical results are listed in Table XI through XIII.

TABLE XI

ANALYSIS OF RCHCH MECHANISM

θ_1	θ_2	θ_3	θ_5	S_2	S_3	S_4	S_5
-65.	-15.10	63.55	-29.61	-0.17	2.22	-3.18	-2.58
-60.	-18.55	60.64	-23.62	-0.29	2.12	-2.26	-2.06
-55.	-22.05	58.67	-18.08	-0.36	2.05	-1.50	-1.58
-50.	-25.45	57.54	-12.81	-0.39	2.01	-0.82	-1.12
-45.	-28.48	57.24	-7.62	-0.36	2.00	-0.18	-0.66
-40.	-30.68	57.77	-2.26	-0.23	2.02	0.46	-0.20
-35.	-40.74	54.62	-5.00	0.21	1.91	0.16	-0.44
-30.	-45.55	55.20	-2.24	0.44	1.93	0.54	-0.20
-25.	-50.87	56.22	-0.12	0.53	1.96	0.90	-0.01
-20.	-51.38	58.90	5.39	0.53	2.06	1.63	0.47
-15.	-51.60	61.38	10.21	0.51	2.14	2.31	0.89
-10.	-51.63	63.77	14.71	0.48	2.23	2.98	1.28
-5.	-51.60	66.11	19.05	0.43	2.31	3.65	1.66
0.	-51.49	68.43	23.37	0.37	2.39	4.36	2.04
5.	-51.31	70.73	27.74	0.28	2.47	5.12	2.42
10.	-51.07	72.99	32.23	0.15	2.55	5.97	2.81
15.	-57.57	75.34	31.82	0.15	2.63	5.97	2.78
20.	-50.23	77.24	41.94	-0.26	2.70	8.11	3.66
25.	-49.51	79.14	47.44	-0.60	2.76	9.62	4.14
30.	-48.33	80.85	53.82	-1.15	2.82	11.86	4.70
35.	-45.66	82.57	62.64	-2.29	2.88	16.50	5.47
40.	-62.78	-96.58	50.30	-1.06	-3.37	3.35	4.39
45.	43.20	-48.48	54.93	-1.47	-1.69	7.42	4.79
50.	-55.73	-98.93	69.84	-2.95	-3.45	12.36	6.09
55.	-50.82	-102.73	87.18	-13.28	-3.59	76.31	7.61

TABLE XII

VELOCITY ANALYSIS OF RCHCH MECHANISM

θ_1	ω_2	$\dot{S}_2$	ω_3	S_3	ω_5	S_5
-65.	-0.55	1.00	-0.26	-0.51	-0.50	-2.50
-60.	-0.37	1.46	-0.32	-0.64	-0.67	-3.33
-55.	-1.11	-0.03	0.18	0.35	0.00	0.00
-50.	0.03	2.47	-0.39	-0.77	-1.00	-5.00
-45.	-1.14	0.02	0.18	0.37	-0.00	-0.00
-40.	0.12	1.13	-0.12	-0.25	-0.94	-4.69
-35.	0.23	0.55	-0.32	-0.63	-1.13	-5.63
-30.	-1.18	0.94	0.21	0.42	0.00	0.00
-25.	0.25	-10.83	-0.07	-0.15	-1.00	-5.00
-20.	-2.64	-13.34	-0.63	-1.25	0.56	2.82
-15.	-1.42	-5.62	2.11	4.22	1.50	7.50
-10.	-1.31	-2.02	0.57	1.14	0.44	2.19
-5.	-0.47	2.18	-0.68	-1.35	-0.92	-4.58
0.	1.17	8.02	-2.66	-5.32	-3.28	-16.41
5.	-1.00	-0.28	0.16	0.33	-0.00	-0.00
10.	-0.53	1.19	-0.44	-0.89	-0.73	-3.65
15.	-0.43	1.41	-0.50	-1.00	-0.81	-4.06
20.	-0.52	0.89	-0.48	-0.96	-0.75	-3.75
25.	-0.60	0.62	-0.41	-0.82	-0.65	-3.23
30.	-0.52	0.65	-0.60	-1.20	-0.81	-4.06
35.	-17.83	-20.30	32.42	64.84	30.75	153.75
40.	-0.43	0.56	-0.65	-1.29	-0.81	-4.06
45.	0.95	5.56	-2.98	-5.96	-3.27	-16.33
50.	-1.61	-0.09	1.91	3.83	1.25	6.25
55.	-1.07	0.18	0.22	0.44	0.00	0.00

TABLE XIII

ACCELERATION ANALYSIS OF RCHCH MECHANISM

θ_1	α_2	$\ddot{S}_2$	α_3	$\ddot{S}_3$	α_5	$\ddot{S}_5$
-65.	0.23	0.34	-0.37	-0.75	-0.20	-0.98
-60.	0.35	0.02	-0.49	-0.98	-0.31	-1.56
-55.	-0.33	-1.15	0.32	0.63	0.38	1.89
-50.	-0.09	-3.05	-0.25	-0.50	-0.26	-1.31
-45.	-1.10	-2.72	0.64	1.28	1.12	5.61
-40.	-0.16	9.07	-0.06	-0.12	-0.15	-0.73
-35.	-0.18	14.46	-0.13	-0.26	-0.21	-1.04
-30.	-4.30	-3.73	1.76	3.52	3.99	19.97
-25.	-0.28	400.41	-0.02	-0.04	-0.09	-0.46
-20.	-6.04	-30.88	-3.08	-6.17	1.38	6.89
-15.	1.59	17.92	-1.06	-2.12	-0.75	-3.75
-10.	0.26	-1.82	-0.09	-0.19	-0.06	-0.32
-5.	0.19	4.02	-0.14	-0.28	-0.29	-1.44
0.	1.47	33.19	-1.52	-3.04	-1.83	-9.15
5.	-0.08	-1.44	0.16	0.32	0.18	0.88
10.	-0.04	1.68	0.08	0.16	-0.12	-0.60
15.	-0.10	2.27	0.13	0.26	-0.11	-0.53
20.	-0.11	2.05	0.22	0.44	-0.03	-0.16
25.	-0.09	1.92	0.22	0.45	-0.00	-0.02
30.	-0.17	3.22	0.41	0.82	0.08	0.42
35.	-567.97	-284.47	495.23	990.46	288.38	1441.90
40.	-0.27	1.15	0.49	0.98	0.12	0.59
45.	-4.46	31.10	4.96	9.93	2.39	11.95
50.	-2.53	29.23	1.57	3.13	0.82	4.10
55.	-0.13	1.47	0.11	0.21	0.02	0.12

CHAPTER III

DYNAMIC ANALYSIS OF SPATIAL MECHANISMS

Time response and kinetostatics are the two general problems in the dynamic analysis of mechanism. Both problems require complete kinematic analysis of the mechanism before the dynamic analysis can be done. When the motion of the system is specified, that is, the input velocity and acceleration are assumed at any position of the system, input torque or input force or both becomes an unknown at that position. This type of problem is known as a kinetostatics problem.

In contrast to this, time response is the dynamic analysis of mechanisms which are moved from one position to another as consequence of a specified input dual force, such as one provided by a spring or a weight. The motions of the mechanism are unknowns in the time response analysis while the driving forces are the knowns.

Essentially, formulation of the equations of motion leads to the important step of dynamic analysis of the system. Newton's law, d'Alembert's principle, Lagrange's equation with and without multiplier, methods, Hamilton's canonical equations, energy methods, influence coefficients method, virtual work method, etc., all are seeking the common goal of formulating the equations of motion. Each has its significance and is tied to the concept of power or energy or dual force balance.

3.1. Dual Forces Acting on Kinematic Pairs

Reuleaux classified the kinematic pairs into two groups: lower kinematic pairs and higher kinematic pairs. The male and female elements of lower kinematic pairs make surface contacts, while the higher kinematic pairs make either point or line contact. However, another classification should also be noted. From the dynamics point of view, the magnitude and the direction of the internal reaction dual force of the lower kinematic pairs are both unknown quantities, while higher kinematic pair direction of the dual forces is known because it is always perpendicular to the tangent of the acting surface.

Applying dual forces at the structure joint, one can predict the dual forces at a joint as

$$\hat{F} = (F_x + \varepsilon\, M_x)\,\bar{i} + (F_y + \varepsilon\, M_y)\,\bar{j} + (F_z + \varepsilon\, M_z)\,\bar{K}$$

where F_x, F_y, F_z are the force components of $\hat{F}$ and M_x, M_y, M_z are the moment components of $\hat{F}$.

3.1.1. Revolute Pair

Revolute pair permits the rotation freedom about its pair axis $\bar{A}$ as shown in Figure 2. It doesn't permit translation along its pair axis $\bar{A}$. Thus, the velocity screw is

$$V = \omega\hat{A} \tag{3.1}$$

The dual force screw is

$$\hat{R}_{ij} = \hat{F}_{ij} + F^A\,\hat{A} \tag{3.2}$$

which is acting from link i to link j and F^A is the reaction force acting along the screw axis $\hat{A}$.

By observing this fact, there can be no torque exerted on link i to link j along the pair axis $\hat{A}$. Thus, the constraint equation is

$$\hat{F}_{ij}\ \hat{A} = 0 \tag{3.3}$$

where

$$\hat{F}_{ij} = (F_{ijx} + \varepsilon\ M_{ijx})\ \overline{i} + (F_{ijy} + \varepsilon\ M_{ijy})\ \overline{j} + (F_{ijz} + \varepsilon\ M_{ijz})\ \overline{K}$$

3.1.2. Prism Pair

Prism pair permits the translation along its pair axis $\hat{A}$ as shown in Figure 3. No force can be transmitted from link i to link j along its pair axis $\hat{A}$. Thus, the velocity screw is

$$V = \varepsilon V\hat{A} \tag{3.4}$$

The dual force screw is

$$\hat{R}_{ij} = \hat{F}_{ij} + \varepsilon\ M^a \cdot \hat{A} \tag{3.5}$$

The constraint equation is

$$\hat{F}_{ij}\hat{A} = 0 \tag{3.6}$$

3.1.3. Helical Pair

Helical pair permits rotation about and translation along its axis $\hat{A}$ as shown in Figure 4. The ratio of translation and rotation is called pitch, ρ.

The velocity screw is

$$\hat{V} = (\omega + \varepsilon\, v) \cdot \hat{A} \tag{3.7}$$

The dual force screw at pair joint is

$$\hat{R}_{ij} = \hat{F}_{ij} + (F^a + \varepsilon\, M^a) \cdot \hat{A} \tag{3.8}$$

The constraint equation is

$$\hat{R}_{ij} \cdot \hat{A} = 0 \tag{3.9}$$

and

$$\rho\, F^a + M^a = 0 \tag{3.10}$$

3.1.4. Cylinder Pair

Cylinder pair permits dual screw motion along the pair axis $\hat{A}$ as shown in Figure 1.

The velocity screw is

$$V = (\omega + \varepsilon\, v) \cdot \hat{A} \tag{3.11}$$

The dual force screw at pair joint is

$$\hat{R}_{ij} = \hat{F}_{ij} \tag{3.12}$$

No forces and torques can be transmitted from link i to link j along the pair axis. Thus, the constraint equation is

$$\hat{F}_{ij}\, \hat{A} = 0 \tag{3.13}$$

3.1.5. Spherical Pair

Spherical pairs permit three independent rotational motions at a pair joint as shown in Figure 5.

The velocity screw at pair joint is

$$V = \omega_1\hat{A} + \omega_2\hat{B} + \omega_3\hat{C} \tag{3.14}$$

$$\hat{C} = (\hat{A} \times \hat{B})$$

where A, B and C are three independent screws.

The dual force at pair joint is

$$\hat{R}_{ij} = F^A \cdot \hat{A} + F^B \cdot \hat{B} + F^C \cdot \hat{C} \tag{3.15}$$

where F^A, F^B and F^C are the reaction forces acting along screw axes $\hat{A}$, $\hat{B}$ and $\hat{C}$ respectively. No torques can be transmitted from link i to link j. Since R_{ij} truly represents reaction force from link i to link k, there is no constraint equation for a spherical pair.

3.2. Dual Inertial Forces Acting on the i-th Moving Link

A rigid body as shown in Figure 22 can be considered to be a distribution of mass fixed to a rigid frame with no elastic properties. If M denotes the total mass of a rigid body,

$$M = \sum_{i=1}^{n} m_i = \int m_i \tag{3.16}$$

where m_i is the mass of particle i.

Because the motion of a rigid body can experience linear as well as rotational motion, the dynamic properties of a rigid body are more complex than those of a particle. In this study, theory is based on

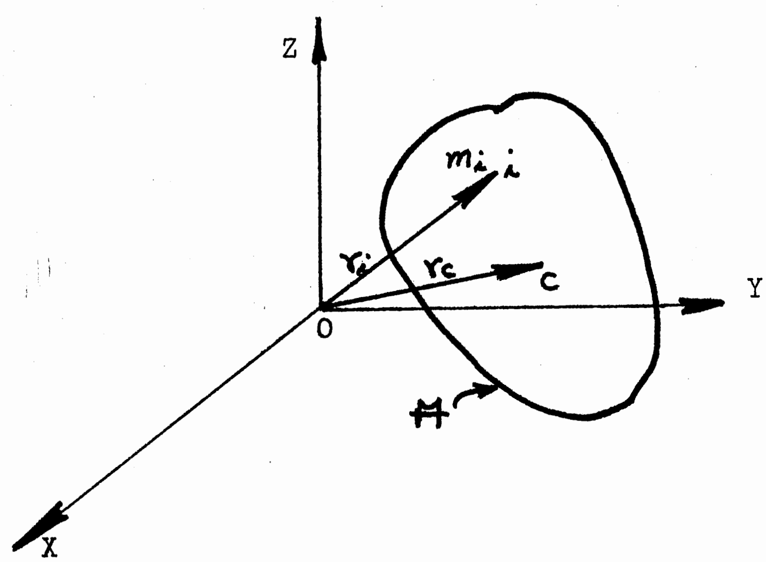

Figure 22. A Rigid Body

the classical Newtonian dynamics.

The dual inertial forces are dual forces which resist the change of the state of motion. The dual inertial forces acting on the floating link of a mechanism vary continually as the mechanism performs its function. The dual inertial force acting at the center of mass of link i is $\hat{F}^{Ii} = F_{Ii} + \varepsilon M_{Ii}$. The dual inertial forces are obtained as the negative of the time rate of change in the dual momentum M.

Varignon's theorem in statics states that, if C is taken as center

$$M\, r_c = \sum_{i=1}^{n} m_i r_i \tag{3.17}$$

where M is the total mass of a rigid body.

Differentiate Equation (3.17) with respect to time and introduce the linear momentum to have

$$M\, \dot{\bar{r}}_c = M\, V_c \tag{3.18}$$

$$= P_c \tag{3.19}$$

$$= \sum m_i \cdot \dot{r}_i$$

$$= \sum m_i \cdot v_i \tag{3.20}$$

$$= \sum P_i \tag{3.21}$$

where

P_c is the total linear momentum with respect to mass center c.

P_i is the linear momentum of mass particle i.

r_c is the position of the mass center c.

r_i is the position of the mass particle i.

$\dot{\bar{r}}_i$ and v_i are the velocities of the mass particle i.

$\dot{r}_c$ and v_c are the velocities of the mass center c.

Equations (3.19) and (3.21) state that the total linear momentum is equal to the summation of linear momentum of particle i. Equation (3.18) states that in Newtonian mechanics, the total linear momentum of a system is the product of the total mass M and absolute velocity of the centroid and is invariant with the choice of the reference frame.

Applying the Newton second law, one obtains

$$F_{Ii} = \frac{dP}{dt}$$

$$= \sum_{j=1}^{n} m_j \cdot v_j$$

$$= M \cdot \dot{v}_{ci}$$

$$= M\, a_{ci} \tag{3.22}$$

where a_{ci} is the acceleration of the center of link i. Index j is referred to mass particle j.

The angular momentum of a rigid body about a fixed point O is defined as the vector sum of the moments of the linear momentum about O of all mass particles of the rigid body. Thus,

$$H_0 = \sum r_i \times m_i v_i \tag{3.23}$$

Differentiate Equation (3.23) to have

$$\dot{H}_0 = \sum \dot{r}_i \times m_i v_i + \sum r_i \times m_i \dot{v}_i \tag{3.24}$$

Substituting Equation (3.22) into (3.24), one has

$$\dot{H}_0 = \sum \mathring{r}_i \times m_i v_i + \sum r_i \times F_i \tag{3.25}$$

$$= \sum \dot{r}_i \times m_i v_i + M_o \tag{3.26}$$

where

F_i is the inertial force acting at particle i.

M_0 is the moment about o.

Let us consider that the position o of the rigid body is moving with velocity V_o and the rigid body is experiencing angular rotation ω. Thus, Equation (3.23) becomes

$$H = \sum r_i \times m_i \, (V_o + \omega x r_i)$$

$$= M \, r_c \times V_o + I_o \tag{3.27}$$

where

I_o is a moment of inertial about o.

Equation (3.26) becomes

$$M_o = M \, (V_o \quad V_c) + \dot{H}_o \tag{3.28}$$

where

$$\dot{H} = \left(\frac{d}{dt} + \omega x \right) H_o$$

Parallel axis theorem states

$$I_o = I_c + M \begin{bmatrix} y^2 + z^2 & -xy & -xz \\ -xy & z^2 + x^2 & -yz \\ -xz & -yz & x^2 + y^2 \end{bmatrix} \tag{3.29}$$

where x, y and z are defined in (37).

Substituting Equation (3.29) into Equation (3.28), one gets

$$M_{Ii} = M\ r_c \times a_o + \frac{dI_o}{dt}\ \omega + I_o\ \alpha + \omega \times M\ r_c \times V_o + \omega \times I_o \tag{3.30}$$

$$= M \begin{Bmatrix} ya_{oz} - za_{oy} \\ za_{ox} - xa_{oz} \\ xa_{oz} - ya_{ox} \end{Bmatrix} + M \begin{Bmatrix} \omega_y(xV_{oy} - yV_{ox}) - \omega_z(zV_{ox} - xV_{oz}) \\ \omega_z(yV_{oz} - zV_{oy}) - \omega_x(xV_{oy} - yV_{ox}) \\ \omega_x(zV_{ox} - xV_{oz}) - \omega_y(yV_{oz} - zV_{oy}) \end{Bmatrix} +$$

$$M \begin{Bmatrix} -y\omega_y - z\omega_z \\ -y\omega_x + 2x\omega_y \\ -z\omega_x + 2x\omega_z \end{Bmatrix} + \begin{Bmatrix} I_{xx}\alpha_x + I_{xy}\alpha_y + I_{xz}\alpha_z \\ I_{xy}\alpha_x + I_{yy}\alpha_y + I_{xz}\alpha_z \\ I_{xz}\alpha_x + I_{yz}\alpha_y + I_{zz}\alpha_z \end{Bmatrix} +$$

$$\begin{Bmatrix} \omega_y(I_{xz}\omega_x + I_{yz}\omega_y + I_{zz}\omega_z) - \omega_z(I_{xy}\omega_x + I_{yy}\omega_y + I_{xz}\omega_z) \\ \omega_z(I_{xx}\omega_x + I_{xy}\omega_y + I_{xz}\omega_z) - \omega_x(I_{xz}\omega_x + I_{yz}\omega_y + I_{zz}\omega_z) \\ \omega_x(I_{xy}\omega_x + I_{yy}\omega_y + I_{xz}\omega_z) - \omega_y(I_{xx}\omega_x + I_{xy}\omega_y + I_{xz}\omega_z) \end{Bmatrix} \tag{3.31}$$

The dual inertial force is obtained from Equations (3.22) and (3.30)

$$\hat{F}_{Ii} = -F_{Ii} - \varepsilon\ M_{Ii} \tag{3.32}$$

3.3. Equation of Motion for Spatial Mechanism

If a linkage, as a whole, is in equilibrium, then each of its links is in equilibrium also. Hence, any link may be considered to be in equilibrium under the combined action of all dual internal and external forces. This principle is known as d'Alembert's principle. The dual equations of motion, one for each link, are written in dual vector form resulting in siz equations of motion for each link, three equations from real part of x, y and z components and three equations from dual part of x, y and z components.

The i-th moving link of a mechanism is shown in Figure 23. The OXYZ system is the fixed inertial system. The $O_iX_iY_iZ_i$ system is the reference system for the i-th moving link which is preferably fixed to the i-th moving link.

When all the dual forces, including both external and internal dual forces, acting on a link of a mechanism are added, the resultant dual forces-vector must be a null vector if the link is to be in equilibrium with the entire mechanism.

Let us define $\hat{R}^{ij}$ as dual reaction force acting from link i to link j. Thus, Newton's action and reaction law states that $\hat{R}^{ij} = -\hat{R}^{ij}$. $\hat{R}^{ij}$ are representing dual reaction forces acting from link i to link j, where j = 1 to n if there are n links connected with link i. The power is needed to drive the mechanism whether it is in a form of force or torque. We define this input power in a dual form as $\hat{F}^{In}$. The mechanism are used to generate a useful form of work to perform the prescribed tasks. This useful form of work is designated as $\hat{F}^{out}$. The dual inertial of link i is taken as $\hat{F}_{Ii}$. The dual external applying force at link i is designated $\hat{F}_{Ai}$. Thus, for the i-th link

$$\sum_{j=1}^{n} \hat{R}^{ji} + \hat{F}_{Ii} + \hat{F}^{out} + \hat{F}^{In} + \hat{F}_{Ai} = 0 \qquad (3.33)$$

3.4. Kinetostatic Analysis of the RCSR Spatial Mechanism

The geometry and the parameters of the RCSR spatial mechanism is

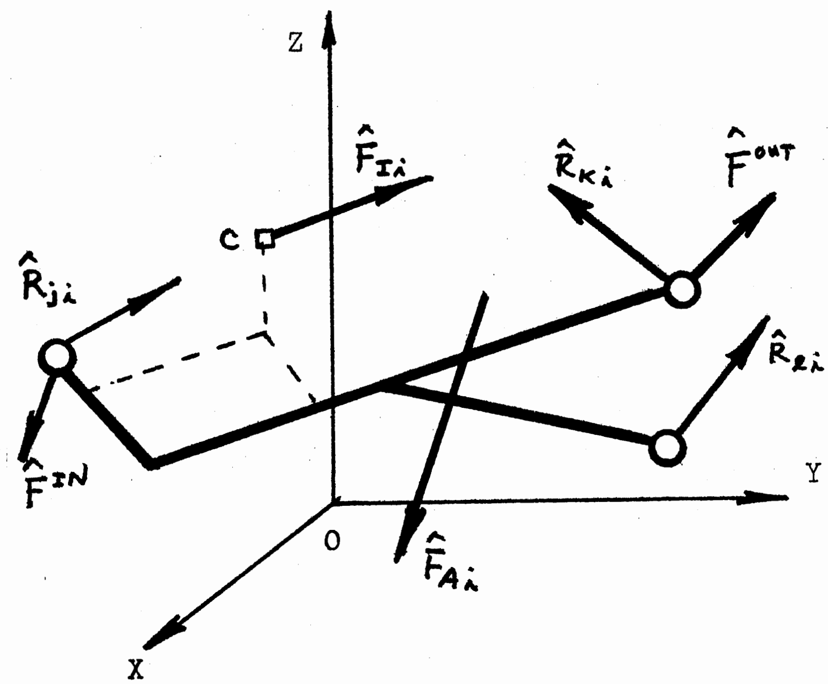

Figure 23. The i-th Link

shown in Figure 11. A complete kinematic analysis of this mechanism has been carried out in Chapter II. The following procedures are used to explain the method which the author develops in this dissertation.

From Chapter 3.2, one defines the dual reaction forces at pair joints as:

$$\hat{R}_{12} = \hat{F}_{12} \tag{3.34}$$

$$\hat{R}_{23} = F^B\hat{B} + F^C\hat{C} + F^D\hat{D} \tag{3.35}$$

$$\hat{R}_{41} = \hat{F}_{41} + F^{A1}\hat{A}_1 \tag{3.36}$$

$$\hat{R}_{43} = \hat{F}_{43} + F^{A1}\hat{A}_4 \tag{3.37}$$

where

$\hat{R}_{12}$ is a dual reaction force acting from link 1 to link 2 at cylinder pair O.

$\hat{R}_{23}$ is a dual reaction force acting from link 2 to link 3 at spherical pair O.

$\hat{R}_{41}$ is a dual reaction force acting from link 4 to link 1 at revolute pair O.

$\hat{R}_{43}$ is a dual reaction force acting from link 4 to link 3 at revolute pair O.

$\hat{A}_1$, $\hat{A}_4$, $\hat{B}$, $\hat{C}$ and $\hat{D}$ are dual screws which are defined in Chapter II Section 7.

The constraint equation for revolute pair O_1 is

$$\hat{F}_{41} \cdot \hat{A}_1 = 0 \tag{3.38}$$

The constraint equation for cylinder pair O_2 is

$$\hat{F}_{12} \cdot \hat{A}_2 = 0 \tag{3.39}$$

The constraint equation for revolute pair 0_4 is

$$\hat{F}_{41} \cdot \hat{A}_4 = 0 \tag{3.40}$$

The equation of motion for link 1 is

$$\hat{R}^{41} + \hat{R}^{21} + \hat{F}^{IN} + \hat{F}_{I1} = 0 \tag{3.41}$$

where

$$\hat{F}^{IN} = \varepsilon \, M^{IN}\hat{A}_1 \tag{3.42}$$

M^{IN} is an input torque.

The equation of motion for link 2 is

$$\hat{R}^{12} + \hat{R}^{32} + \hat{F}_{I2} = 0 \tag{3.43}$$

The equation of motion for link 3 is

$$\hat{R}^{43} + \hat{R}^{23} + \hat{F}^{out} + \hat{F}_{I3} = 0 \tag{3.44}$$

where

$$\hat{F}^{out} = \varepsilon \, M^{out}\hat{A}_4 \tag{3.45}$$

M^{out} is an output torque.

Equation (3.41) becomes

$$\begin{Bmatrix} F_{41}^x + \varepsilon \, M_{41}^x \\ F_{41}^y + \varepsilon \, M_{41}^y \\ F_{41}^z + \varepsilon \, M_{41}^z \end{Bmatrix} + F^{A1} \begin{Bmatrix} A_{10}^x + \varepsilon \, A_{11}^x \\ A_{10}^y + \varepsilon \, A_{11}^y \\ A_{10}^z + \varepsilon \, A_{11}^z \end{Bmatrix} - \begin{Bmatrix} F_{12}^x + \varepsilon \, M_{12}^x \\ F_{12}^y + \varepsilon \, M_{12}^y \\ F_{12}^z + \varepsilon \, M_{12}^z \end{Bmatrix}$$

$$+ \varepsilon M^{IN} \begin{Bmatrix} A_{10}^{x} + \varepsilon A_{11}^{x} \\ A_{10}^{y} + \varepsilon A_{11}^{y} \\ A_{10}^{z} + \varepsilon A_{11}^{z} \end{Bmatrix} + \begin{Bmatrix} F_{I1}^{x} + M_{I1}^{x} \\ F_{I1}^{y} + M_{I1}^{y} \\ F_{I1}^{z} + M_{I1}^{z} \end{Bmatrix} = 0 \qquad (3.46)$$

Equation (3.43) becomes

$$\begin{Bmatrix} F_{12}^{x} + \varepsilon M_{12}^{x} \\ F_{12}^{y} + \varepsilon M_{12}^{y} \\ F_{12}^{z} + \varepsilon M_{12}^{z} \end{Bmatrix} - F^{B} \begin{Bmatrix} B_{0}^{x} + \varepsilon B_{1}^{x} \\ B_{0}^{y} + \varepsilon B_{1}^{y} \\ B_{0}^{z} + \varepsilon B_{1}^{z} \end{Bmatrix} - F^{C} \begin{Bmatrix} C_{0}^{x} + \varepsilon C_{1}^{x} \\ C_{0}^{y} + \varepsilon C_{1}^{y} \\ C_{0}^{z} + \varepsilon C_{1}^{z} \end{Bmatrix}$$

$$- F^{D} \begin{Bmatrix} D_{0}^{x} + \varepsilon D_{1}^{x} \\ D_{0}^{y} + \varepsilon D_{1}^{y} \\ D_{0}^{z} + \varepsilon D_{1}^{z} \end{Bmatrix} + \begin{Bmatrix} F_{I2}^{x} + \varepsilon M_{I2}^{x} \\ F_{I2}^{y} + \varepsilon M_{I2}^{y} \\ F_{I2}^{z} + \varepsilon M_{I2}^{z} \end{Bmatrix} = 0 \qquad (3.47)$$

Equation (3.44) becomes

$$\begin{Bmatrix} F_{43}^{x} + \varepsilon M_{43}^{y} \\ F_{43}^{y} + \varepsilon M_{43}^{y} \\ F_{43}^{z} + \varepsilon M_{43}^{z} \end{Bmatrix} + F^{B} \begin{Bmatrix} B_{0}^{x} + \varepsilon B_{1}^{x} \\ B_{0}^{y} + \varepsilon B_{1}^{y} \\ B_{0}^{z} + \varepsilon B_{1}^{z} \end{Bmatrix} + F^{C} \begin{Bmatrix} C_{0}^{x} + \varepsilon C_{1}^{x} \\ C_{0}^{y} + \varepsilon C_{1}^{y} \\ C_{0}^{z} + \varepsilon C_{1}^{z} \end{Bmatrix}$$

$$+ F^{D} \begin{Bmatrix} D_{0}^{x} + \varepsilon D_{1}^{x} \\ D_{0}^{y} + \varepsilon D_{1}^{y} \\ D_{0}^{z} + \varepsilon D_{1}^{z} \end{Bmatrix} + F^{A4} \begin{Bmatrix} A_{40}^{x} + \varepsilon A_{41}^{x} \\ A_{40}^{y} + \varepsilon A_{41}^{y} \\ Z_{40}^{z} + \varepsilon A_{41}^{z} \end{Bmatrix}$$

$$+ \varepsilon M^{out} \begin{Bmatrix} A_{40}^{x} + A_{41}^{x} \\ A_{40}^{y} + A_{41}^{y} \\ A_{40}^{z} + A_{41}^{z} \end{Bmatrix} + \begin{Bmatrix} F_{I3}^{x} + M_{I3}^{x} \\ F_{I3}^{y} + M_{I3}^{y} \\ F_{I3}^{z} + M_{I3}^{z} \end{Bmatrix} = 0 \tag{3.48}$$

Equation (3.38) becomes

$$\begin{Bmatrix} F_{12}^{x} + \varepsilon M_{12}^{x} \\ F_{12}^{y} + \varepsilon M_{12}^{y} \\ F_{12}^{z} + \varepsilon M_{12}^{z} \end{Bmatrix} \cdot \begin{Bmatrix} A_{20}^{x} + \varepsilon A_{21}^{x} \\ A_{20}^{y} + \varepsilon A_{21}^{y} \\ A_{20}^{z} + \varepsilon A_{21}^{z} \end{Bmatrix} = 0 \tag{3.49}$$

Equation (3.39) becomes

$$\begin{Bmatrix} F_{43}^{x} + \varepsilon M_{43}^{x} \\ F_{43}^{y} + \varepsilon M_{43}^{y} \\ F_{43}^{z} + \varepsilon M_{43}^{z} \end{Bmatrix} \cdot \begin{Bmatrix} A_{40}^{x} + \varepsilon A_{41}^{x} \\ A_{40}^{y} + \varepsilon A_{41}^{y} \\ A_{40}^{z} + \varepsilon A_{41}^{z} \end{Bmatrix} = 0 \tag{3.50}$$

Expand Equation (3.40) becomes

$$\begin{Bmatrix} F_{41}^{x} + \varepsilon M_{41}^{x} \\ F_{41}^{y} + \varepsilon M_{41}^{y} \\ F_{41}^{z} + \varepsilon M_{41}^{z} \end{Bmatrix} \cdot \begin{Bmatrix} A_{10}^{x} + \varepsilon A_{11}^{x} \\ A_{10}^{y} + \varepsilon A_{11}^{y} \\ A_{10}^{z} + \varepsilon A_{11}^{z} \end{Bmatrix} = 0 \tag{3.51}$$

The total numbers of equations are twenty-four of which eighteen equations are equations of motion and the others are constraint equations. The twenty-four unknowns are:

$$\{U\}^T = \{F_{41}^x, F_{41}^y, F_{41}^z, F_{12}^x, F_{12}^y, F_{12}^z, F_{43}^x, F_{43}^y, F_{43}^z,$$
$$M_{41}^x, M_{41}^y, M_{41}^z, M_{12}^x, M_{12}^y, M_{12}^z, M_{43}^x, M_{43}^y, M_{43}^z,$$
$$F^{A1}, F^{A4}, F^B, F^C, F^D, M^{IN}\} \quad (3.52)$$

In this investigation the 24-th unknown is an input torque. Having 24 equations of static equilibriums and 24 unknowns, the RCSR spatial mechanism is a statically determinate mechanism.

Equations (3.46) through (3.51) can be solved for the unknown forces and torques by matrix algebra. Thus,

$$\{Q\}_{24 \times 24} \times \{U\}_{24 \times 1} = \{C\}_{24 \times 1} \quad (3.53)$$

where nonzero coefficients of {Q} and {C} are listed in Table XIV.

The kinetostatic analysis of RCSR mechanism is performed with the dimensions listed in Chapter II Section 6.6 and $I_{xx1} = 0.2$, $I_{xy1} = .05$, $I_{xz1} = .1$, $I_{xx2} = .01$, $I_{yy2} = .2$, $I_{zz2} = .2$, $I_{xz2} = .05$, $I_{yz2} = .4$, $I_{xx3} = .02$, $I_{yy3} = .3$, $I_{zz3} = .4$, $I_{xy3} = .5$, $I_{yz3} = .2$. Numerical results are listed in Tables XV through XVIII.

3.5. Dynamic Response Analysis of RCSR Spatial Mechanism

In Chapter 3.4, our goal is to define input torque for motor sizing. For the dynamic response problem, input torque becomes known while motion state becomes unknown. Unlike the traditional method in engaging to solve a set of differential equations, one treats input acceleration as one of unknowns. Using the information of input position, velocity and acceleration to define the motion state of

TABLE XIV

NONZERO COEFFICIENTS OF {Q} AND {C} OF THE RCSR MECHANISM

$C(1) = -F_{I1}^{x}$	$C(17) = -M_{I3}^{y} - M^{out}A_{40}^{y}$	$Q(4, 24) = A_{11x}$
$C(2) = -F_{I1}^{y}$	$C(18) = -M_{I3}^{z} - M^{out}A_{40}$	$Q(5, 11) = 1$
$C(3) = -F_{I1}^{z}$	$Q(1, 1) = 1$	$Q(5, 14) = -1$
$C(4) = -M_{I1}^{x}$	$Q(1, 4) = -1$	$Q(5, 19) = A_{11y}$
$C(5) = -M_{I1}^{y}$	$Q(1, 19) = A_{10x}$	$Q(5, 24) = A_{11y}$
$C(6) = -M_{I1}^{z}$	$Q(1, 24) = A_{10x}$	$Q(6, 12) = 1$
$C(7) = -F_{I2}^{x}$	$Q(2, 2) = 1$	$Q(6, 15) = -1$
$C(8) = -F_{I2}^{y}$	$Q(2, 19) = A_{10y}$	$Q(6, 19) = A_{11z}$
$C(9) = -F_{I2}^{z}$	$Q(2, 24) = A_{10y}$	$Q(6, 24) = A_{11z}$
$C(10) = -M_{I2}^{x}$	$Q(3, 3) = 1$	$Q(7, 4) = 1$
$C(11) = -M_{I2}^{y}$	$Q(3, 6) = -1$	$Q(7, 21) = -B_{ox}$
$C(12) = -M_{I2}^{z}$	$Q(3, 19) = A_{10z}$	$Q(7, 22) = -C_{ox}$
$C(13) = -F_{I3}^{x}$	$Q(3, 24) = A_{10z}$	$Q(7, 23) = -D_{ox}$
$C(14) = -F_{I3}^{y}$	$Q(4, 10) = 1$	$Q(8, 5) = 1$
$C(15) = -F_{I3}^{z}$	$Q(4, 13) = -1$	$Q(8, 21) = -B_{oy}$
$C(16) = -M_{I3}^{x} - M^{out}A_{40}^{x}$	$Q(4, 19) = A_{11x}$	$Q(8, 22) = -C_{oy}$

TABLE XIV (Continued)

$Q(8, 23) = -D_{oy}$	$Q(13, 7) = 1$	$Q(16, 21) = B_{1x}$
$Q(9, 6) = 1$	$Q(13, 20) = A_{40x}$	$Q(16, 22) = C_{1x}$
$Q(9, 21) = -B_{oz}$	$Q(13, 21) = B_{ox}$	$Q(16, 23) = D_{1x}$
$Q(9, 22) = -C_{oz}$	$Q(13, 22) = C_{ox}$	$Q(17, 17) = 1$
$Q(9, 23) = -D_{oz}$	$Q(13, 23) = D_{ox}$	$Q(17, 20) = A_{41y}$
$Q(10, 13) = 1$	$Q(14, 8) = 1$	$Q(17, 21) = B_{1y}$
$Q(10, 21) = -B_{1x}$	$Q(14, 20) = A_{40y}$	$Q(17, 22) = C_{1y}$
$Q(10, 22) = -C_{1x}$	$Q(14, 21) = B_{oy}$	$Q(17, 23) = D_{1y}$
$Q(10, 23) = -D_{1x}$	$Q(14, 22) = C_{oy}$	$Q(18, 18) = 1$
$Q(11, 14) = 1$	$Q(14, 23) = D_{oy}$	$Q(18, 20) = A_{41z}$
$Q(11, 21) = -B_{1y}$	$Q(15, 9) = 1$	$Q(18, 21) = B_{1z}$
$Q(11, 22) = -C_{1y}$	$Q(15, 20) = A_{40z}$	$Q(18, 22) = C_{1z}$
$Q(11, 23) = -D_{1y}$	$Q(15, 21) = B_{oz}$	$Q(18, 23) = D_{1z}$
$Q(12, 15) = 1$	$Q(15, 22) = C_{oz}$	$Q(19, 4) = A_{20x}$
$Q(12, 21) = -B_{1z}$	$Q(15, 23) = D_{oz}$	$Q(19, 5) = A_{20y}$
$Q(12, 22) = -C_{1z}$	$Q(16, 16) = 1$	$Q(19, 6) = A_{20z}$
$Q(12, 23) = -D_{1z}$	$Q(16, 20) = A_{41x}$	$Q(20, 4) = A_{21x}$

TABLE XIV (Concluded)

$Q(20, 5) = A_{21y}$	$Q(19, 4) = A_{20x}$	$Q(22, 9) = A_{41z}$
$Q(20, 6) = A_{21z}$	$Q(19, 5) = A_{20y}$	$Q(22, 16) = A_{40x}$
$Q(20, 13) = A_{20x}$	$Q(19, 6) = A_{20z}$	$Q(22, 17) = A_{40y}$
$Q(20, 14) = A_{20y}$	$Q(20, 4) = A_{21x}$	$Q(22, 18) = A_{40z}$
$Q(17, 17) = 1$	$Q(20, 5) = A_{21y}$	$Q(23, 1) = A_{10x}$
$Q(17, 20) = A_{41y}$	$Q(20, 6) = A_{21z}$	$Q(23, 2) = A_{10y}$
$Q(17, 21) = B_{1y}$	$Q(20, 13) = A_{20x}$	$Q(23, 3) = A_{10z}$
$Q(17, 22) = C_{1y}$	$Q(20, 14) = A_{20y}$	$Q(24, 1) = A_{11x}$
$Q(17, 23) = D_{1y}$	$Q(20, 15) = A_{20z}$	$Q(24, 2) = A_{11y}$
$Q(18, 18) = 1$	$Q(21, 7) = A_{40x}$	$Q(24, 3) = A_{11z}$
$Q(18, 20) = A_{41z}$	$Q(21, 8) = A_{40y}$	$Q(24,10) = A_{10x}$
$Q(18, 21) = B_{1z}$	$Q(21, 9) = A_{40z}$	$Q(24, 11) = A_{10y}$
$Q(18, 22) = C_{1z}$	$Q(22, 7) = A_{41x}$	$Q(24, 12) = A_{10z}$
$Q(18, 23) = D_{1z}$	$Q(22, 8) = A_{41y}$	

TABLE XV

DUAL REACTION FORCE AT REVOLUTE PAIR O_1

θ_1	F_{41x}	F_{41y}	F_{41z}	M_{41x}	M_{41y}	M_{41z}	F^{A1}
150.	0.	20.67	-17.26	-0.	53.87	93.79	-5.46
152.	0.	18.86	-18.30	-0.	58.33	87.89	-5.41
154.	0.	17.10	-19.29	-0.	62.64	82.21	-5.37
156.	0.	15.36	-20.20	-0.	66.73	76.55	-5.34
158.	0.	13.58	-21.00	-0.	70.50	70.77	-5.31
160.	0.	11.77	-21.67	0.	73.86	64.78	-5.27
162.	0.	9.91	-22.17	0.	76.69	58.54	-5.22
164.	0.	8.01	-22.49	0.	78.92	52.06	-5.15
166.	0.	6.10	-22.61	0.	80.49	45.38	-5.07
168.	0.	4.21	-22.54	0.	81.40	38.59	-4.96
170.	0.	2.36	-22.30	-0.	81.70	31.77	-4.84
172.	0.	0.58	-21.91	0.	81.50	24.99	-4.72
174.	0.	-1.11	-21.43	0.	80.92	18.33	-4.59
176.	0.	-2.71	-20.89	0.	80.09	11.82	-4.48
178.	0.	-4.23	-20.33	0.	79.14	5.46	-4.38
180.	0.	-5.67	-19.80	0.	78.18	-0.76	-4.30
182.	0.	-7.06	-19.30	0.	77.27	-6.86	-4.24
184.	0.	-8.42	-18.87	0.	76.50	-12.89	-4.22
186.	0.	-9.76	-18.51	0.	75.93	-18.92	-4.22
188.	0.	-11.11	-18.25	0.	75.63	-25.03	-4.26
190.	0.	-12.52	-18.09	0.	75.69	-31.36	-4.34
192.	0.	-14.01	-18.08	0.	76.24	-38.08	-4.47
194.	0.	-15.66	-18.24	0.	77.47	-45.48	-4.66
196.	0.	-17.56	-18.64	0.	79.66	-54.02	-4.93
198.	0.	-19.88	-19.40	0.	83.30	-64.50	-5.32
200.	0.	-22.91	-20.70	0.	89.18	-78.53	-5.89
202.	0.	-27.11	-22.66	0.	97.94	-99.63	-6.71

TABLE XVI

DUAL REACTION AT CYLINDER PAIR O_2

θ_1	F_{21x}	F_{21y}	F_{21z}	M_{21x}	M_{21y}	M_{21z}
150.	-5.46	21.21	-17.43	8.38	54.06	93.97
152.	-5.41	19.39	-18.49	6.37	58.52	88.07
154.	-5.37	17.63	-19.50	4.44	62.83	82.39
156.	-5.34	15.88	-20.43	2.57	66.92	76.73
158.	-5.31	14.09	-21.25	0.73	70.69	70.95
160.	-5.27	12.27	-21.93	-1.09	74.05	64.96
162.	-5.22	10.40	-22.45	-2.89	76.88	58.72
164.	-5.15	8.49	-22.79	-4.64	79.11	52.24
166.	-5.07	6.57	-22.93	-6.33	80.68	45.56
168.	-4.96	4.67	-22.87	-7.95	81.59	38.77
170.	-4.84	2.81	-22.64	-9.48	81.89	31.95
172.	-4.72	1.02	-22.27	-10.94	81.69	25.17
174.	-4.59	-0.69	-21.80	-12.33	81.11	18.51
176.	-4.48	-2.30	-21.28	-13.68	80.28	12.00
178.	-4.38	-3.83	-20.74	-14.99	79.34	5.64
180.	-4.30	-5.29	-20.22	-16.31	78.37	-0.58
182.	-4.24	-6.70	-19.74	-17.66	77.46	-6.68
184.	-4.22	-8.07	-19.32	-19.04	76.69	-12.71
186.	-4.22	-9.42	-18.97	-20.50	76.12	-18.74
188.	-4.26	-10.80	-18.72	-22.05	75.82	-24.85
190.	-4.34	-12.21	-18.57	-23.75	75.88	-31.18
192.	-4.47	-13.72	-18.57	-25.65	76.43	-37.90
194.	-4.66	-15.39	-18.74	-27.85	77.66	-45.30
196.	-4.93	-17.31	-19.15	-30.51	79.85	-53.84
198.	-5.32	-19.65	-19.92	-33.91	83.49	-64.32
200.	-5.89	-22.70	-21.22	-38.59	89.37	-78.35
202.	-6.71	-26.91	-23.19	-45.57	98.13	-99.45

TABLE XVII

DUAL REACTION FORCE AT REVOLUTE PAIR O_4

θ_1	F_{43x}	F_{43y}	F_{43z}	M_{43x}	M_{43y}	M_{43z}	FA^4
150.	3.50	-22.64	13.07	-17.63	-54.11	-91.96	1.58
152.	3.77	-20.82	14.08	-15.56	-58.63	-86.24	1.23
154.	4.03	-19.06	15.04	-13.58	-63.01	-80.74	0.90
156.	4.26	-17.30	15.91	-11.64	-67.17	-75.25	0.60
158.	4.47	-15.50	16.67	-9.73	-71.02	-69.63	0.31
160.	4.63	-13.67	17.27	-7.83	-74.46	-63.79	0.05
162.	4.74	-11.80	17.70	-5.97	-77.38	-57.68	-0.20
164.	4.80	-9.91	17.93	-4.14	-79.69	-51.32	-0.41
166.	4.81	-8.04	17.94	-2.37	-81.34	-44.74	-0.61
168.	4.76	-6.22	17.75	-0.67	-82.32	-38.02	-0.77
170.	4.66	-4.48	17.38	0.94	-82.68	-31.25	-0.91
172.	4.52	-2.87	16.87	2.48	-82.53	-24.51	-1.02
174.	4.36	-1.40	16.29	3.94	-81.99	-17.86	-1.09
176.	4.20	-0.06	15.67	5.36	-81.18	-11.33	-1.14
178.	4.04	1.14	15.07	6.75	-80.23	-4.93	-1.17
180.	3.89	2.24	14.53	8.15	-79.24	1.34	-1.16
182.	3.77	3.25	14.08	9.56	-78.31	7.51	-1.14
184.	3.68	4.22	13.73	11.01	-77.50	13.63	-1.09
186.	3.62	5.15	13.51	12.53	-76.87	19.76	-1.02
188.	3.59	6.09	13.41	14.15	-76.51	25.98	-0.92
190.	3.61	7.07	13.47	15.91	-76.50	32.43	-0.81
192.	3.67	8.13	13.70	17.87	-76.98	39.29	-0.67
194.	3.79	9.32	14.14	20.13	-78.12	46.84	-0.51
196.	3.98	10.71	14.86	22.85	-80.22	55.56	-0.31
198.	4.28	12.43	15.98	26.31	-83.74	66.25	-0.06
200.	4.74	14.62	17.68	31.05	-89.49	80.54	0.28
202.	5.39	17.24	20.11	38.11	-98.04	102.02	0.81

TABLE XVIII

REACTION FORCES AT SPHERICAL PAIR O_3

θ	F^B	F^C	F^D	M^{IN}
150.	7.71	-25.04	-1.26	8.38
152.	7.94	-24.82	-1.20	6.37
154.	8.20	-24.72	-1.15	4.45
156.	8.50	-24.67	-1.11	2.58
158.	8.82	-24.63	-1.06	0.73
160.	9.13	-24.55	-1.00	-1.09
162.	9.42	-24.40	-0.92	-2.88
164.	9.66	-24.15	-0.83	-4.63
166.	9.85	-23.77	-0.72	-6.33
168.	9.95	-23.27	-0.59	-7.94
170.	9.97	-22.67	-0.44	-9.48
172.	9.89	-21.99	-0.29	-10.94
174.	9.72	-21.27	-0.13	-12.33
176.	9.49	-20.55	0.02	-13.67
178.	9.20	-19.89	0.16	-14.99
180.	8.87	-19.32	0.28	-16.31
182.	8.54	-18.87	0.38	-17.65
184.	8.20	-18.55	0.45	-19.04
186.	7.88	-18.40	0.51	-20.50
188.	7.58	-18.43	0.54	-22.05
190.	7.32	-18.67	0.56	-23.75
192.	7.08	-19.16	0.55	-25.64
194.	6.87	-19.98	0.53	-27.85
196.	6.66	-21.28	0.49	-30.51
198.	6.38	-23.31	0.40	-33.91
200.	5.79	-26.73	0.22	-38.58
202.	3.61	-33.45	-0.26	-45.57

points located in any link in the graphical kinematic analysis of planar linkages has been developed and recognized for at least a half century. The mathematical expressions of acceleration of any point in any link can be written easily in a linear function of input acceleration in planar linkages. However, due to the complexity of the operation of 4×4 displacement matrix and 3×3 dual matrix, it is very tedious, even impossible, to find such expression if one takes the time factor into account. This difficulty has been overcome by introducing successive displacement screws. The motion state of any point can be written in a linear function of the input function. This is due to the fact that displacement, velocity and acceleration equations are in linear forms in terms of unknown components and input state. Therefore, the equations of motion can be linearized in terms of unknown forces and torques and input acceleration.

Equations (2.61), (2.62) and (2.63) give the angular accelerations α_2 and α_4 and a linear acceleration $\ddot{S}_2$ as

$$\alpha_2 = DR_1 \cdot \alpha_1 + DR_2 \qquad (3.54)$$

$$\ddot{S}_2 = DR_3 \cdot \alpha_1 + DR_4 \qquad (3.55)$$

and

$$\alpha_4 = DR_5 \cdot \alpha_1 + DR_6 \qquad (3.56)$$

where

$$DR_1 = \frac{\bar{N}_4 \cdot (\bar{N}_2 \times \bar{N}_3)}{\bar{N}_1 \cdot (\bar{N}_2 \times \bar{N}_3)}$$

$$DR_2 = \frac{\overline{N}_5 \cdot (\overline{N}_2 \times \overline{N}_3)}{\overline{N}_1 \cdot (\overline{N}_2 \times \overline{N}_3)}$$

$$DR_3 = \frac{\overline{N}_4 \cdot (\overline{N}_1 \times \overline{N}_3)}{\overline{N}_2 \cdot (\overline{N}_1 \times \overline{N}_3)}$$

$$DR_4 = \frac{\overline{N}_5 \cdot (\overline{N}_1 \times \overline{N}_3)}{\overline{N}_2 \cdot \overline{N}_1 \times \overline{N}_3}$$

$$DR_5 = \frac{\overline{N}_4 \cdot (\overline{N}_1 \times \overline{N}_3)}{\overline{N}_2 \cdot (\overline{N}_1 \times \overline{N}_2)}$$

$$DR_6 = \frac{\overline{N}_5 \cdot (\overline{N}_1 \times \overline{N}_2)}{\overline{N}_2 \cdot (\overline{N}_1 \times \overline{N}_2)}$$

Regrouping Equations (2.65), (2.66), and (2.67) in linear functions of α_1, one obtains

$$\frac{d^2\underline{P}_{m1j}}{dt^2} = \overline{K}_1\alpha_1 + \overline{K}_2 \tag{3.57}$$

$$\frac{d^2\underline{P}_{m2j}}{dt^2} = \overline{K}_3\alpha_1 + \overline{K}_4 \tag{3.58}$$

and

$$\frac{d^2\underline{P}_{m3j}}{dt^2} = \overline{K}_5\alpha_1 + \overline{K}_6 \tag{3.59}$$

where

$$\overline{K}_1 = \begin{Bmatrix} 0 \\ y_1 s\theta_1 - z_1 c\theta_1 \\ y_1 c\theta_1 + z_1 s\theta_1 \end{Bmatrix}$$

$$\overline{K}_2 = \begin{Bmatrix} 0 \\ -y_1 c\theta_1 - z_1 s\theta_1 \\ y\ s\theta_1 - z\ c\theta \end{Bmatrix} \omega_1^2$$

$$\overline{K}_3 = (\overline{A}_{2j} \times (\overline{P}_{m2j} - \overline{P}_{2j}))\ DR_1 + \overline{A}_{2j} DR_3 + (\overline{A}_1 \times (\overline{P}_{m2j} - \overline{P}_{1j}))$$

$$\overline{K}_4 = (\overline{A}_{2j} \times (\overline{P}_{m2j} - \overline{P}_{2j}))\ DR_2 + \overline{A}_{2j} DR_4 + (\overline{A}_{2j} \times (\frac{d\overline{P}_{m2j}}{dt} - \frac{d\overline{P}_{2j}}{dt}) + \frac{d\overline{A}_{2j}}{dt} \times (P_{m2j} - P_{2j}))\ \omega_2 + \frac{d\overline{A}_{2j}}{dt} \mathring{S} + \overline{A} \times (\frac{d\overline{P}_{m2j}}{dt})\ \omega_1$$

$$\overline{K}_5 = \begin{Bmatrix} sa_4(y_3 c\theta_4 - z_3 s\theta_4) \\ -z_3 c\theta_4 - y_3 s\theta_4 \\ ca_4(y_3 c\theta_4 - z_3 s\theta_4 \end{Bmatrix} DR_5$$

$$\overline{K}_6 = \begin{Bmatrix} sa_4(y_3 c\theta_4 - z_3 s\theta_4) \\ -z_3 c\theta_4 - y_3 s\theta_4 \\ ca_4(y_3 c\theta_4 - z_3 s\theta_4) \end{Bmatrix} DR_6 +$$

$$\begin{Bmatrix} sa_4(-z_3 c\theta_4 - y_3 s\theta_4) \\ -y_3 c\theta_4 + z_3 s\theta_4 \\ ca_4(-z_3 c\theta_4 - y_3 s\theta_4) \end{Bmatrix} \omega_4^2$$

Regrouping Equations (2.64) and (2.65) in linear functions of α_1, one gets

$$\frac{d^2P_{02j}}{dt^2} = \overline{K}_7\alpha_1 + \overline{K}_8 \tag{3.60}$$

and

$$\frac{d^2P_{03j}}{dt^2} = \overline{K}_9\alpha_1 + \overline{K}_{10} \tag{3.61}$$

where

$$\overline{K}_7 = \begin{Bmatrix} ca_1 \\ s\theta_1 sa_1 \\ c\theta_1 sa_1 \end{Bmatrix} DR_3 + \begin{Bmatrix} 0 \\ c\theta_1 s_2 sa_1 - s\theta_1 p_1 \\ -c\theta_1 p_1 - s\theta_1 s_2 sa_1 \end{Bmatrix}$$

$$\overline{K}_8 = \begin{Bmatrix} 0 \\ -c\theta_1 sa_1\omega_1 \\ s\theta_1 sa_1\omega_1 \end{Bmatrix} \dot{S}_2 + \begin{Bmatrix} ca_1 \\ s\theta_1 sa_1 \\ c\theta_1 sa_1 \end{Bmatrix} DR_4 +$$

$$\begin{Bmatrix} 0 \\ -c\theta_1 sa_1\mathring{S}_2 + (c\theta_1 P_1 + s\theta_1 S_2 sa_1)\omega_1 \\ s\theta_1 sa_1\mathring{S}_2 + (c\theta_1 S_2 sa_1 - s\theta_1 P_1)\omega_1 \end{Bmatrix} \omega_1$$

$$\overline{K}_9 = \overline{D}_6 \cdot DR_5$$

$$\overline{K}_{10} = \overline{A}_4 \times \overline{D}_6\omega_4^2 + \overline{D}_6 \cdot DR_6$$

Substituting Equations (3.57), (3.58) and (3.59) into Equation (3.22), one has inertial forces for link 1, 2 and 3.

The inertial force of link 1 is

$$F_{I1} = -M_1 \frac{d^2P_{m1j}}{dt^2}$$

$$= -M_1\overline{K}_1\alpha_1 - M_1\overline{K}_2 \qquad (3.62)$$

The inertial force of link 2 is

$$F_{I2} = -M \frac{d^2P_{m2j}}{dt^2}$$

$$= -M_2\overline{K}_3\alpha_1 - M_2\overline{K}_4 \qquad (3.63)$$

The inertial force of link 3 is

$$F_{I3} = -M \frac{d^2P_{m3j}}{dt^2}$$

$$= -M_3\overline{K}_5\alpha_1 - M_3\overline{K}_6 \qquad (3.64)$$

The inertial torque of link 1 is

$$M_1 = -(I_{01})_1\alpha_1 - \omega_1 \times I_0\omega_1$$

$$= -\begin{Bmatrix} I_{xx1} \\ I_{xy1} \\ I_{xy1} \end{Bmatrix} \alpha_1 - \begin{Bmatrix} 0 \\ -\omega_1^2 I_{xz1} \\ \omega_1^2 I_{xy1} \end{Bmatrix} \qquad (3.65)$$

The inertial torque of link 2 is

$$M_2 = -M_2 r_{c2} \times a_{02} + \frac{dI_{02}}{dt}\omega_2 + I_{02}\omega_2 + \omega \times M_2 r_{c2} \times V_{02} + \omega_2 \times I_{02}\omega_2$$

$$= -M_2 \begin{Bmatrix} y_2K_7 - z_2K_7 \\ z_2K_7 - X_2K_7 \\ X_2K_{7y} - y_2K_{7x} \end{Bmatrix} \alpha_1 - M_2 \begin{Bmatrix} y_2K_8 - z_2K_8 \\ z_2K_8 - X_2K_8 \\ X_2K_{8y} - y\ K_{8x} \end{Bmatrix}$$

$$- M_2 \begin{Bmatrix} \omega_{2y}(X_2V_{02y} - y_2V_{02x}) - \omega_{2z}(z_2V_{02x} - X_2V_{02z}) \\ \omega_{2z}(y_2V_{02z} - z_2V_{02y}) - \omega_{2x}(X_2V_{02y} - y_2V_{02x}) \\ \omega_{2x}(z_2V_{02x} - X_2V_{02z}) - \omega_{2y}(y_2V_{02z} - z_2V_{02y}) \end{Bmatrix}$$

$$- M_2 \begin{Bmatrix} -y_2\omega_{2y} - z_2\omega_{2z} \\ -y_2\omega_{2x} + 2X_2\omega_{2y} \\ -z_2\omega_{2x} + 2X_2\omega_{2z} \end{Bmatrix}$$

$$- \begin{Bmatrix} I_{xx2}A_{2jx} + I_{xy2}A_{2jy} + I_{xx2}A_{2jz} \\ I_{xy2}A_{2jx} + I_{yy2}A_{2jy} + I_{xz2}A_{2jz} \\ I_{xz2}A_{2jx} + I_{yz2}A_{2jy} + I_{zz2}A_{2jz} \end{Bmatrix} (DR_1 \cdot \alpha_1 + DR_2)$$

$$- \begin{Bmatrix} \omega_{2y}(I_{xz2}\omega_{2x} + I_{yz2}\omega_{2y} + I_{zz2}\omega_{2z}) - \omega_{2z}(I_{xy2}\omega_{2x} + I_{yy2}\omega_{2y} + I_{xz2}\omega_{2z}) \\ \omega_{2z}(I_{xx2}\omega_{2x} + I_{xy2}\omega_{2y} + I_{xz2}\omega_{2z}) - \omega_{2x}(I_{xz2}\omega_{2x} + I_{yz2}\omega_{2y} + I_{zz2}\omega_{2z}) \\ \omega_{2x}(I_{xy2}\omega_{2x} + I_{yy2}\omega_{2y} + I_{xz2}\omega_{2z}) - \omega_{2y}(I_{xx2}\omega_{2x} + I_{xy2}\omega_{2y} + I_{xz2}\omega_{2z}) \end{Bmatrix}$$

The inertial torque of link 3 is

$$M = -I_{03}\bar{\alpha}_4 - \bar{\omega}_4 \times I_{03}\bar{\omega}_4$$

$$= - \begin{Bmatrix} I_{xx3}A_{4x} + I_{xy3}A_{4y} + I_{xz3}A_{4z} \\ I_{xy3}A_{4x} + I_{yy3}A_{4y} + I_{xz3}A_{4z} \\ I_{xz3}A_{4x} + I_{yz3}A_{4y} + I_{zz3}A_{4z} \end{Bmatrix} (DR_5 \cdot \alpha_1 + DR_6)$$

$$- \begin{Bmatrix} \omega_{4y}(I_{xz3}\omega_{4x} + I_{yz3}\omega_{4y} + I_{zz3}\omega_{4z}) - \omega_{4z}(I_{xy3}\omega_{4x} + I_{yy3}\omega_{4y} + I_{xz3}\omega_{4z}) \\ \omega_{4z}(I_{xx3}\omega_{4x} + I_{xy3}\omega_{4y} + I_{xz3}\omega_{4z}) - \omega_{4x}(I_{xz3}\omega_{4x} + I_{yz3}\omega_{4y} + I_{zz3}\omega_{4z}) \\ \omega_{4x}(I_{xy3}\omega_{4x} + I_{yy3}\omega_{4y} + I_{xz3}\omega_{4z}) - \omega_{4y}(I_{xx3}\omega_{4x} + I_{xy3}\omega_{4y} + I_{xz3}\omega_{4z}) \end{Bmatrix}$$

(3.67)

The constraint equations and equations of motion for the dynamic response analysis are the same as Equations (3.38) to (3.45). However, input torque becomes known while input angular acceleration becomes unknown. Thus, we have twenty-four unknowns. They are

$$\{U\}^T = \{F_{41}^x, F_{41}^y, F_{41}^z, F_{12}^x, F_{12}^x, F_{12}^y, F_{12}^z, F_{43}^x, F_{43}^y, F_{43}^z, M_{41}^x, M_{41}^y,$$

$$M_{41}^z, M_{12}^x, M_{12}^y, M_{12}^z, M_{43}^x, M_{43}^y, M_{43}^z, F^{A1}, F^{A4}, F^B, F^C, F^D, \alpha_1\}$$

The twenty-fourth unknown is the input angular acceleration α_1.

The twenty-four equations can be written in matrix form as

$$\{Q\}_{24 \times 24} \times \{U\}_{24 \times 1} = \{C\}_{24 \times 1} \tag{3.68}$$

where

$C(1) = M_1K_{2x}$

$C(2) = M_1K_{2y}$

$C(3) = M_1K_{2z}$

$C(4) = 0$

$C(5) = -\omega_1^2 I_{xz1}$

$C(6) = \omega_1^2 I_{xy1}$

$$C(7) = M_2K_{4x}$$

$$C(8) = M_2K_{4y}$$

$$C(9) = M_2K_{4z}$$

$$C(10) = M_2(y_2K_{8z} - z_2K_{8y}) + M_2(\omega_{2y}\ (X_2V_{02y} - y_2V_{02x}) -$$

$$\omega_{2z}(z_2V_{02x} - x_2V_{02z})) + M_2(-y_2\omega_{2y} - z_2\omega_{2z}) +$$

$$(I_{xx2}A_{2jx} + I_{xy2}A_{2jy} + I_{xz2}A_{2jz})\ DR_2 +$$

$$\omega_{2z}(I_{xx2}\omega_{2x} + I_{xy2}\omega_{2y} + I_{xz2}\omega_{2z}) - \omega_{2x}(I_{xz2}\omega_{2x} +$$

$$I_{yz2}\omega_{2y} + I_{zz2}\omega_{2z})$$

$$C(12) = M_2(x_2K_{8y} - y_2k_{8x}) + M_2(\omega_2 \times (z_2V_{02x} - x_2V_{02z}) -$$

$$\omega_{2y}(y_2V_{02z} - z_2V_{02y})) + M_2(-z_2\omega_{2x} + 2x_2\omega_{2z}) +$$

$$(I_{xz2}A_{2jx} + I_{yz2}A_{2jy} + I_{zz2}A_{2jz})\ DR_2 +$$

$$\omega_{2x}(I_{xy2}\omega_{2x} + I_{yy2}\omega_{2y} + I_{xz2}\omega_{2z}) - \omega_{2y}(I_{xx2}\omega_{2x} +$$

$$I_{xy2}\omega_{2y} + I_{xz2}\omega_{2z})$$

$$C(13) = M_3K_{6x}$$

$$C(14) = M_3K_{6y}$$

$$C(15) = M_3K_{6z}$$

$$C(16) = (I_{xx3}A_{4x} + I_{xy3}A_{4y} + I_{xz3}A_{4z})\ DR_6 + \omega_{4y}(I_{xz3}\omega_{4x} +$$

$$I_{yz3}\omega_{4y} + I_{zz3}\omega_{4z}) - \omega_{4z}(I_{xy3}\omega_{4x} + I_{yy3}\omega_{4y} + I_{xx3}\omega_{4z})$$

$$C(17) = (I_{xy3}A_{4x} + I_{yy3}A_{4y} + I_{xz3}A_{4z})\ DR_6 + \omega_{4z}(I_{xx3}\omega_{4x} + I_{xy3}\omega_{4y} + I_{xz3}\omega_{4z}) - \omega_{4x}(I_{xz3}\omega_{4x} + I_{yz3}\omega_{4y} + I_{zz3}\omega_{4z})$$

$$C(18) = (I_{xz3}A_{4x} + I_{xz3}A_{4y} + I_{zz3}A_{4z})\ DR_6 + \omega_{4x}(I_{xy3}\omega_{4x} + I_{yy3}\omega_{4y} + I_{xz3}\omega_{4z}) - \omega_{4y}(I_{xx3}\omega_{4x} + I_{xy3}\omega_{4y} + I_{xz3}\omega_{4z})$$

$$Q(1.24) = -\ M_1K_{1x}$$

$$Q(2.24) = -\ M_1K_{1y}$$

$$Q(3.24) = -\ M_1K_{1z}$$

$$Q(4.24) = -I_{xx1}$$

$$Q(5.24) = -I_{xy1}$$

$$Q(6.24) = -I_{xz1}$$

$$Q(7.24) = -\ M_2K_{3x}$$

$$Q(8.24) = -\ M_2K_{3y}$$

$$Q(9.24) = -\ M_2K_{3z}$$

$$Q(10.24) = -\ M_2(y_2k_{7j} - z_2K_{7y}) - (I_{xx2}A_{2jx} + I_{xy2}A_{2jy} + I_{xz2}A_{2jz})\ DR_1$$

$$Q(11.24) = -M_2(z_2K_{7x} - X_2K_{7z}) - I_{xy2}A_{2jx} + I_{yy2}A_{2jy} + I_{xz2}A_{2jz})\ DR_1$$

$$Q(12.24) = -M_2(X_2K_{7y} - y_2K_{7x}) - I_{xy2}A_{2jx} + I_{yz2}A_{2jy} + I_{zz2}A_{2jz})\ DR_1$$

$$Q(13.24) = -M_3K_{5x}$$

$$Q(14.24) = -M_3K_{5y}$$

$$Q(15.24) = -M_3K_{5z}$$

$$Q(16.24) = -(I_{xx3}A_{4x} + I_{xy3}A_{xy} + I_{xz3}A_{4z})\ DR_5$$

$$Q(17.24) = -(I_{xy3}A_{4x} + I_{yy3}A_{4y} + I_{xz3}A_{4z})\ DR_5$$

$$Q(18.24) = -(I_{xz3}A_{4x} + I_{yz3}A_{4y} + I_{zz3}A_{4z})\ DR_5$$

The rest of the coefficients remain the same as shown in Equation (3.53).

Solving Equation (3.68) by matrix algebra, one has components of forces and torques at pair joint and input acceleration α_1. Thus, input angular velocity and displacement can be determined by the following finite difference equation.

$$V_{i+1} = V_i + \alpha_i \cdot \Delta t \quad (3.69)$$

$$\theta_{i+1} = \theta_i + \left(\frac{V_i + V_{i+1}}{2} \right) \cdot \Delta t \quad (3.70)$$

where

index i denotes time step at t_i (3.70) index $i + 1$ denotes time step at t_{i+1}

Δt is a time increment and

$\Delta t = t_{i+1} - t_i$

Dynamic response analysis of the RCSR spatial mechanism is performed with the dimensions: $a_1 = 10°$, $a_2 = 15°$, $a_4 = 20°$, $P_1 = 5$ in., $P_2 = 10$ in., $P_3 = 5$ in., $P_4 = 10$ in., $I_{xx1} = .02$, $I_{xy1} = .05$, $I_{xz1} = .1$, $I_{xx2} = .01$, $I_{xy2} = .2$, $I_{zz2} = .2$, $I_{xz2} = .05$, $I_{yz2} = .4$, $I_{xx3} = .02$, $I_{xy3} = .3$, $I_{zz3} = .4$, $I_{xy3} = .5$, $I_{yz3} = .2$ (unit of components of inertial is lbn/in^2). $M^{In} = 10$ in.-lb., $X_1 = .5$ in., $Y_1 = 1.$ in., $Z_1 = 1.$ in., $X_2 = .5$ in., $Y_2 = 1.$ in., $Z_2 = 1.$ in. $X_3 = .5$ in., $Y_3 = 1.$ in., $Z_1 = 1.$ in. Numerical results are listed in Tables XIX through XXIII.

3.6. Kinetostatic Analysis of RCSR-CSR Spatial Mechanism

The geometry and parameters of the RCSR-CSR mechanism are shown in Figure 16. A complete kinematic analysis of this mechanism is given in Chapter 2.7. In this investigation we neglect the dual inertial forces due to its own mass.

Define the following dual reacting forces at pair joints in the following manner:

$$\hat{R}_{12} = \hat{F}_{12} \tag{3.71}$$

$$\hat{R}_{23} = F^B\hat{B} + F^C\hat{C} + F^D\hat{D} \tag{3.72}$$

$$\hat{R}_{41} = \hat{F}_{41} + F^{A1}\hat{A}_1 \tag{3.73}$$

TABLE XIX

DYNAMIC RESPONSE OF THE RCSR MECHANISM

θ	ω_1	ω_2	ω_4	α_1	α_2	α_4	t
20.000	0.009	0.000	0.000	0.175	0.000	0.000	0.0
20.025	0.026	-0.012	-0.016	0.175	-0.230	-0.328	0.1
20.125	0.044	-0.034	-0.049	0.178	-0.226	-0.319	0.2
20.326	0.062	-0.057	-0.080	0.182	-0.222	-0.308	0.3
20.629	0.080	-0.079	-0.110	0.186	-0.219	-0.294	0.4
21.036	0.099	-0.100	-0.139	0.190	-0.214	-0.279	0.5
21.549	0.118	-0.122	-0.166	0.192	-0.209	-0.263	0.6
22.171	0.137	-0.142	-0.192	0.194	-0.203	-0.246	0.7
22.904	0.157	-0.162	-0.215	0.194	-0.198	-0.231	0.8
23.747	0.176	-0.182	-0.238	0.194	-0.192	-0.218	0.9
24.701	0.196	-0.201	-0.259	0.194	-0.188	-0.207	1.0
25.766	0.215	-0.219	-0.279	0.194	-0.184	-0.198	1.1
26.943	0.234	-0.238	-0.299	0.194	-0.183	-0.193	1.2
28.230	0.254	-0.256	-0.318	0.196	-0.182	-0.189	1.3
29.629	0.274	-0.274	-0.337	0.198	-0.183	-0.188	1.4
31.140	0.294	-0.293	-0.356	0.201	-0.185	-0.189	1.5
32.765	0.314	-0.311	-0.375	0.205	-0.189	-0.192	1.6
34.505	0.335	-0.330	-0.394	0.211	-0.194	-0.196	1.7
36.363	0.356	-0.350	-0.414	0.217	-0.200	-0.201	1.8
38.342	0.378	-0.370	-0.434	0.224	-0.207	-0.207	1.9
40.446	0.401	-0.391	-0.455	0.231	-0.215	-0.214	2.0

TABLE XX

DUAL REACTION FORCE AT REVOLUTE PAIR O_1

θ_1	F_{41x}	F_{41y}	F_{41z}	M_{41x}	M_{41y}	M_{41z}	F^{A1}
20.00	-0.00	0.15	-2.21	-0.00	-53.85	17.43	0.36
20.03	-0.00	0.15	-2.20	0.00	-53.83	17.51	0.36
20.13	-0.00	0.19	-2.25	-0.00	-54.51	18.63	0.36
20.33	-0.00	0.23	-2.32	-0.00	-55.50	20.58	0.37
20.63	-0.00	0.24	-2.38	-0.00	-56.44	22.94	0.38
21.04	-0.00	0.20	-2.42	-0.00	-57.05	25.39	0.39
21.55	-0.00	0.09	-2.42	-0.00	-57.23	27.70	0.40
22.17	-0.00	-0.07	-2.38	0.00	-57.01	29.81	0.41
22.90	-0.00	-0.27	-2.30	0.00	-56.46	31.74	0.41
23.75	-0.00	-0.51	-2.19	0.00	-55.73	33.59	0.41
24.70	-0.00	-0.77	-2.06	-0.00	-54.91	35.47	0.41
25.77	-0.00	-1.04	-1.90	0.00	-54.09	37.49	0.40
26.94	-0.00	-1.32	-1.73	-0.00	-53.34	39.75	0.40
28.23	-0.00	-1.61	-1.54	-0.00	-52.69	42.34	0.40
29.63	-0.00	-1.90	-1.32	-0.00	-52.14	45.36	0.40
31.14	-0.00	-2.20	-1.09	-0.00	-51.70	48.87	0.40
32.76	-0.00	-2.50	-0.82	-0.00	-51.36	52.94	0.40
34.51	-0.00	-2.81	-0.53	0.00	-51.08	57.63	0.40
36.36	-0.00	-3.14	-0.19	0.00	-50.81	63.01	0.40
38.34	-0.00	-3.47	0.20	-0.00	-50.51	69.12	0.40
40.45	-0.00	-3.80	0.65	-0.00	-50.10	75.96	0.40

TABLE XXI

DUAL REACTION FORCE AT CYLINDER PAIR O_2

θ_1	F_{12x}	F_{12y}	F_{12z}	M_{12x}	M_{12y}	M_{12z}
20.00	0.36	0.15	-2.21	10.00	-53.86	17.41
20.03	0.36	0.15	-2.21	10.00	-53.82	17.51
20.13	0.36	0.18	-2.26	10.00	-54.46	18.66
20.33	0.37	0.22	-2.34	10.00	-55.42	20.65
20.63	0.38	0.23	-2.41	10.00	-56.32	23.05
21.04	0.39	0.18	-2.46	10.00	-56.90	25.53
21.55	0.40	0.07	-2.47	10.00	-57.05	27.88
22.17	0.41	-0.10	-2.44	10.00	-56.78	30.03
22.90	0.41	-0.31	-2.37	10.00	-56.19	32.00
23.75	0.41	-0.55	-2.28	10.00	-55.42	33.89
24.70	0.41	-0.82	-2.15	10.00	-54.56	35.80
25.77	0.40	-1.10	-2.01	10.00	-53.71	37.86
26.94	0.40	-1.38	-1.85	10.00	-52.92	40.15
28.23	0.40	-1.68	-1.67	10.00	-52.22	42.79
29.63	0.40	-1.98	-1.47	10.00	-51.63	45.84
31.14	0.40	-2.29	-1.25	10.00	-51.16	49.39
32.76	0.40	-2.61	-1.00	10.00	-50.77	53.50
34.51	0.40	-2.93	-0.73	10.00	-50.45	58.24
36.36	0.40	-3.27	-0.40	10.00	-50.14	63.66
38.34	0.40	-3.62	-0.03	10.00	-49.80	69.80
40.45	0.40	-3.98	0.40	10.00	-49.34	76.68

TABLE XXII

DUAL REACTION FORCE AT SPHERICAL PAIR O_3

θ_1	F^B	F^C	F^D	α_1
20.000	21.653	22.414	-10.207	0.175
20.025	21.505	22.262	-10.145	0.175
20.125	21.159	21.976	-9.937	0.178
20.326	20.420	21.324	-9.516	0.182
20.629	19.299	20.280	-8.900	0.186
21.036	17.882	18.906	-8.141	0.190
21.549	16.302	17.328	-7.311	0.192
22.171	14.695	15.691	-6.479	0.194
22.904	13.164	14.112	-5.696	0.194
23.747	11.777	12.670	-4.989	0.194
24.701	10.553	11.396	-4.366	0.194
25.766	9.494	10.300	-3.821	0.194
26.943	8.585	9.371	-3.341	0.194
28.230	7.808	8.592	-2.914	0.196
29.629	7.142	7.944	-2.524	0.198
31.140	6.568	7.406	-2.159	0.201
32.765	6.067	6.960	-1.808	0.205
34.505	5.624	6.591	-1.459	0.211
36.363	5.225	6.283	-1.104	0.217
38.342	4.856	6.022	-0.731	0.224
40.446	4.505	5.795	-0.331	0.231

TABLE XXIII

DUAL REACTION FORCE AT REVOLUTE PAIR O_4

θ_1	F_{43x}	F_{43y}	F_{43z}	M_{43x}	M_{43y}	M_{43z}	F^{A4}
20.00	-0.57	-2.84	-1.57	-24.63	6.39	-18.73	-3.94
20.03	-0.57	-2.83	-1.57	-24.56	6.39	-18.64	-3.92
20.13	-0.56	-2.90	-1.55	-24.41	6.42	-18.76	-3.87
20.33	-0.55	-3.00	-1.52	-24.05	6.48	-18.80	-3.77
20.63	-0.54	-3.07	-1.48	-23.46	6.54	-18.57	-3.61
21.04	-0.52	-3.09	-1.42	-22.67	6.59	-17.98	-3.39
21.55	-0.50	-3.05	-1.37	-21.73	6.63	-17.08	-3.14
22.17	-0.48	-2.97	-1.32	-20.73	6.64	-15.99	-2.88
22.90	-0.46	-2.87	-1.26	-19.72	6.62	-14.84	-2.62
23.75	-0.44	-2.76	-1.21	-18.75	6.59	-13.74	-2.38
24.70	-0.42	-2.65	-1.16	-17.83	6.55	-12.76	-2.17
25.77	-0.41	-2.57	-1.12	-16.98	6.51	-11.95	-1.97
26.94	-0.39	-2.51	-1.06	-16.16	6.48	-11.30	-1.79
28.23	-0.37	-2.47	-1.01	-15.38	6.47	-10.82	-1.62
29.63	-0.35	-2.46	-0.95	-14.59	6.50	-10.49	-1.46
31.14	-0.32	-2.46	-0.89	-13.78	6.58	-10.30	-1.29
32.76	-0.30	-2.49	-0.81	-12.91	6.72	-10.22	-1.12
34.51	-0.26	-2.54	-0.73	-11.96	6.96	-10.24	-0.94
36.36	-0.23	-2.60	-0.63	-10.90	7.29	-10.34	-0.73
38.34	-0.19	-2.66	-0.52	-9.68	7.76	-10.52	-0.49
40.45	-0.14	-2.72	-0.39	-8.29	8.39	-10.76	-0.22

$$\hat{R}_{43} = \hat{F}_{43} + F^{A4}\hat{A} \tag{3.74}$$

$$\hat{R}_{25} = \hat{F}_{25} \tag{3.75}$$

$$\hat{R}_{56} = F^{E}\hat{E} + F^{F}\hat{F} + F^{G}\hat{G} \tag{3.76}$$

$$\hat{R}_{46} = \hat{F}_{46} + F^{A7}\hat{A}_7 \tag{3.77}$$

where

$\hat{R}_{12}$ is a dual reaction force acting from link 1 to link 2 at cylinder pair O_2.

$\hat{R}_{23}$ is a dual reaction force acting from link 2 to link 3 at spherical pair O_3.

$\hat{R}_{41}$ is a dual reaction force acting from link 4 to link 1 at revolute pair O_1.

$\hat{R}_{43}$ is a dual reaction force acting from link 4 to link 3 at revolute pair O_4.

$\hat{R}_{25}$ is a dual reaction force acting from link 2 to link 5 at cylinder pair O_5.

$\hat{R}_{56}$ is a dual reaction force acting from link 5 to link 6 at spherical pair O_6.

$\hat{R}_{47}$ is a dual reaction force acting from link 4 to link 7 at revolute pair O_7.

$\hat{B}$, $\hat{C}$, $\hat{D}$, $\hat{A}_1$, $\hat{A}_4$, $\hat{E}$, $\hat{F}$, $\hat{G}$ and $\hat{A}_7$ are dual screws which are defined in Chapter II Section 7.

The constraint equation for revolute O_1 is

$$\hat{F}_{41} \cdot \hat{A}_1 = 0 \tag{3.78}$$

The constraint equation for cylinder pair O_2 is

$$\hat{F}_{12} \cdot \hat{A}_2 = 0 \tag{3.79}$$

The constraint equation for revolute pair O_4 is

$$\hat{F}_{43} \cdot \hat{A}_4 = 0 \tag{3.80}$$

The constraint equation for cylinder pair O_5 is

$$\hat{F}_{25} \cdot \hat{A}_5 = 0 \tag{3.81}$$

The constraint equation for revolute pair O_7 is

$$\hat{F}_{46} \cdot \hat{A}_7 = 0 \tag{3.82}$$

The equation of motion for link 1 is

$$\hat{R}^{41} + \hat{R}^{21} + \hat{F}^{IN} = 0 \tag{3.83}$$

where

$$\hat{F}^{IN} = \varepsilon\, M^{IN}\hat{A}_1 \tag{3.84}$$

The equation of motion for link 2 is

$$\hat{R}^{12} + \hat{R}^{52} + \hat{R}^{32} = 0 \tag{3.85}$$

The equation of motion for link 3 is

$$\hat{R}^{23} + \hat{R}^{43} + \hat{F}^{out} = 0 \tag{3.86}$$

where

$$\hat{F}^{out} = \varepsilon\, M^{out}\, \hat{A}_4$$

The equation of motion for link 5 is

$$\hat{R}^{25} + \hat{R}^{6R} = 0 \tag{3.87}$$

The equation of motion for link 6 is

$$\hat{R}^{56} + \hat{R}^{46} = 0 \tag{3.88}$$

Equations (3.87) through (3.88) give thirty-nine unknown forces and torques and unknown input torque.

$$\{U\}^T = \{F_{41x}, F_{41y}, F_{41z}, F_{12x}, F_{12y}, F_{12z}, F_{43x}, F_{43y}, F_{43z}, M_{41x}, M_{41y}, M_{41z}, M_{12x}, M_{12y}, M_{12z}, M_{43x}, M_{43y}, M_{43z}, F^{A1}, F^{A4}, F^B, F^C, F^D, M^{IN}, F_{25x}, F_{25y}, F_{25z}, M_{25x}, M_{25y}, M_{25z}, F^E, F^F, F^G, F_{46x}, F_{46y}, F_{46z}, M_{46x}, M_{46y}, M_{46z}, F^{A7}\}$$

Equations (3.78) through (3.88) can be written in a linear form

$$\{Q\}_{40 \times 40} \times \{U\}_{40 \times 1} = \{C\}_{40 \times 1} \tag{3.89}$$

where nonzero coefficients of {O} and {C} are listed in Table XXIV.

A numerical analysis of kinetostatic analysis of the RCSR-CSR is performed with the dimensions: $a_1 = 10°$, $a_2 = 15°$, $\theta_{c1} = 20°$, $S_{c1} =$ .5 in., $a_4 = 10°$, $S_4 = 4$ in., $S_1 = 5$ in., $P_1 = .5$ in., $P_2 = 1$ in., $P_3 = .4$ in., $P_4 = .6$ in., $P_5 = .5$ in., $P_6 = .4$ in., $P_7 = 1$ in., $P_8 =$ .5 in., $S_7 = .8$ in., $\theta_{c2} = 20°$, $S_{c2} = .6$ in., $a_5 = 20°$, $a_6 = 30°$, $a_8 = 20°$, $\omega_1 = .1$, $\alpha_1 = .2$, $M^{out1} = 1$ in.-lb., $M^{out2} = 4$ in.-lb. The results are shown in Table XXV through XXXI.

3.7. Dynamic Analysis of RCHCH Spatial Mechanism

The geometry and dimensions of the RCHCH spatial mechanism are

TABLE XXIV

NONZERO COEFFICIENTS OF {Q} AND {C} OF THE RCSR-CSR MECHANISM

Q(1, 1) = 1	Q(5, 14) = -1	Q(9, 21) = $-B_{oz}$
Q(1, 4) = -1	Q(5, 19) = A_{11y}	Q(9, 22) = $-C_{oz}$
Q(1, 19) = A_{10x}	Q(5, 24) = A_{11y}	Q(9, 23) = $-D_{oz}$
Q(1, 24) = A_{10x}	Q(6, 12) = 1	Q(10, 13) = 1
Q(2, 2) = 1	Q(6, 15) = -1	Q(10, 21) = $-B_{1x}$
Q(2, 19) = A_{10y}	Q(6, 19) = A_{11z}	Q(10, 22) = $-C_{1x}$
Q(2, 24) = A_{10y}	Q(6, 24) = A_{11z}	Q(10, 23) = $-D_{1x}$
Q(3, 3) = 1	Q(7, 4) = 1	Q(11, 14) = 1
Q(3, 6) = -1	Q(7, 21) = $-B_{ox}$	Q(11, 21) = $-B_{1y}$
Q(3, 19) = A_{10z}	Q(7, 22) = $-C_{ox}$	Q(11, 22) = $-C_{1y}$
Q(3, 24) = A_{10z}	Q(7, 23) = $-D_{ox}$	Q(11, 23) = $-D_{1y}$
Q(4, 10) = 1	Q(8, 5) = 1	Q(12, 15) = 1
Q(4, 13) = -1	Q(8, 21) = $-B_{oy}$	Q(12, 21) = $-B_{1z}$
Q(4, 19) = A_{11x}	Q(8, 22) = $-C_{oy}$	Q(12, 22) = $-C_{1z}$
Q(4, 24) = A_{11x}	Q(8, 23) = $-D_{oy}$	Q(12, 23) = $-D_{1z}$
Q(5, 11) = 1	Q(9, 6) = 1	Q(13, 7) = 1

TABLE XXIV (Continued)

$Q(13, 20) = A_{40x}$	$Q(16, 22) = C_{1x}$	$Q(20, 6) = A_{21z}$
$Q(13, 21) = B_{ox}$	$Q(16, 23) = D_{1x}$	$Q(2, 13) = A_{20x}$
$Q(13, 22) = C_{ox}$	$Q(17, 17) = 1$	$Q(20, 14) = A_{20y}$
$Q(13, 23) = D_{ox}$	$Q(17, 20) = A_{41y}$	$Q(20, 15) = A_{20z}$
$Q(14, 8) = 1$	$Q(17, 21) = B_{1y}$	$Q(21, 7) = A_{40x}$
$Q(14, 20) = A_{40y}$	$Q(17, 22) = C_{1y}$	$Q(21, 8) = A_{40y}$
$Q(14, 21) = B_{oy}$	$Q(17, 23) = D_{1y}$	$Q(21, 9) = A_{40z}$
$Q(14, 22) = C_{oy}$	$Q(18, 18) = 1$	$Q(22, 7) = A_{40x}$
$Q(14, 23) = D_{oy}$	$Q(18, 20) = A_{41z}$	$Q(22. 8) = A_{40y}$
$Q(15, 9) = 1$	$Q(18, 21) = B_{1z}$	$Q(22, 9) = A_{40z}$
$Q(15, 20) = A_{40z}$	$Q(18, 22) = C_{1z}$	$Q(22, 16) = A_{41x}$
$Q(15, 21) = B_{oz}$	$Q(18, 23) = D_{1z}$	$Q(22, 17) = A_{41y}$
$Q(15, 22) = C_{oz}$	$Q(19, 4) = A_{20x}$	$Q(22, 18) = A_{41z}$
$Q(15, 23) = D_{oz}$	$Q(19, 5) = A_{20y}$	$Q(23, 1) = A_{40x}$
$Q(16, 16) = 1$	$Q(19, 6) = A_{20z}$	$Q(23, 2) = A_{40y}$
$Q(16, 20) = A_{41x}$	$Q(20, 4) = A_{21x}$	$Q(23, 3) = A_{40z}$
$Q(16, 21) = B_{1x}$	$Q(20, 5) = A_{21y}$	$Q(24, 1) = A_{10x}$

TABLE XXIV (Continued)

$Q(24, 2) = A_{10y}$	$Q(27, 27) = 1$	$Q(33, 31) = E_{oz}$
$2(24, 3) = A_{10z}$	$Q(37, 27) = A_{50z}$	$Q(34, 31) = E_{1x}$
$Q(24, 10) = A_{11x}$	$Q(38, 27) = A_{51z}$	$Q(35, 31) = E_{1y}$
$Q(24, 11) = A_{11y}$	$Q(28, 28) = 1$	$Q(36, 31) = E_{1z}$
$Q(24, 12) = A_{11z}$	$Q(38, 28) = A_{50x}$	$Q(25, 32) = -F_{ox}$
$Q(25, 25) = 1$	$Q(29, 29) = 1$	$Q(26, 32) = -F_{oy}$
$Q(37, 25) = A_{50x}$	$Q(38, 29) = A_{50y}$	$Q(27, 32) = -F_{oz}$
$Q(38, 25) = A_{51x}$	$Q(30, 30) = 1$	$Q(28, 32) = -F_{1x}$
$Q(7, 25) = -1$	$Q(38, 30) = A_{50z}$	$Q(29, 32) = -F_{1y}$
$Q(8, 26) = -1$	$Q(25, 31) = -E_{ox}$	$Q(30, 32) = -F_{1z}$
$Q(9, 27) = -1$	$Q(26, 31) = -E_{oy}$	$Q(31, 32) = F_{ox}$
$Q(10, 28) = -1$	$Q(27, 31) = -E_{oz}$	$Q(32, 32) = F_{oy}$
$Q(11, 29) = -1$	$Q(28, 31) = -E_{1x}$	$Q(33, 32) = F_{oz}$
$Q(12, 30) = -1$	$Q(29, 31) = -E_{1y}$	$Q(34, 32) = F_{1x}$
$Q(26, 26) = 1$	$Q(30, 31) = -E_{1z}$	$Q(35, 32) = F_{1y}$
$Q(37, 26) = A_{50y}$	$Q(31, 31) = E_{ox}$	$Q(36, 32) = F_{1z}$
$Q(38, 26) = A_{51y}$	$Q(32, 31) = E_{oy}$	$Q(25, 33) = -G_{ox}$

TABLE XXIV (Concluded)

$Q(25, 33) = -G_{oy}$	$Q(31, 34) = 1$	$Q(36, 40) = A_{71z}$
$Q(27, 33) = -G_{oz}$	$Q(31, 40) = A_{70x}$	$Q(39, 34) = A_{70x}$
$Q(28, 33) = -G_{1x}$	$Q(32, 35) = 1$	$Q(39, 35) = A_{70y}$
$Q(29, 33) = -G_{1y}$	$Q(33, 36) = 1$	$Q(39, 36) = A_{70z}$
$Q(30, 33) = -G_{1z}$	$Q(32, 40) = A_{70y}$	$Q(40, 34) = A_{71x}$
$Q(31, 33) = G_{ox}$	$Q(33, 40) = A_{70z}$	$Q(40, 35) = A_{71y}$
$Q(32, 33) = G_{oy}$	$Q(34, 37) = 1$	$Q(40, 36) = A_{71z}$
$Q(33, 33) = G_{oz}$	$Q(34, 40) = A_{71x}$	$Q(40, 37) = A_{70x}$
$Q(34, 33) = G_{1x}$	$Q(35, 38) = 1$	$Q(40, 38) = A_{70y}$
$Q(35, 33) = G_{1y}$	$Q(35, 40) = A_{71y}$	$Q(40, 39) = A_{70z}$
$Q(36, 33) = G_{1z}$	$Q(36, 39) = 1$	

TABLE XXV

DUAL REACTION FORCE AT REVOLUTE PAIR O_1

θ_1	F_{41x}	F_{41y}	F_{41z}	M_{41x}	M_{41y}	M_{41z}	F^{A1}
160.	0.	0.257	-6.051	0.000	28.364	1.676	-1.018
162.	0.	-0.644	-5.514	0.000	25.979	-2.378	-0.890
164.	0.	-1.460	-4.827	0.000	22.956	-6.150	-0.747
166.	0.	-2.164	-4.021	0.000	19.418	-9.478	-0.596
168.	0.	-2.734	-3.147	0.000	15.561	-12.244	-0.443
170.	0.	-3.161	-2.257	0.000	11.609	-14.388	-0.295
172.	0.	-3.449	-1.398	0.000	7.769	-15.912	-0.160
174.	0.	-3.612	-0.607	0.000	4.204	-16.864	-0.040
176.	0.	-3.669	0.096	0.0	1.018	-17.322	0.062
178.	0.	-3.642	0.702	0.000	-1.744	-17.376	0.146
180.	0.	-3.550	1.215	0.000	-4.084	-17.115	0.214
182.	0.	-3.412	1.641	0.000	-6.034	-16.616	0.268
184.	0.	-3.239	1.994	0.000	-7.642	-15.944	0.311
186.	0.	-3.042	2.286	0.000	-8.963	-15.149	0.345
188.	0.	-2.828	2.528	0.0	-10.046	-14.267	0.372
190.	0.	-2.601	2.729	0.000	-10.937	-13.326	0.394
192.	0.	-2.366	2.897	0.000	-11.673	-12.346	0.413
194.	0.	-2.124	3.039	0.000	-12.285	-11.339	0.429
196.	0.	-1.876	3.161	0.000	-12.801	-10.312	0.445
198.	0.	-1.623	3.266	0.000	-13.242	-9.270	0.459
200.	0.	-1.364	3.360	0.000	-13.629	-8.211	0.474
202.	0.	-1.097	3.447	0.000	-13.981	-7.127	0.491
204.	0.	-0.817	3.531	0.000	-14.319	-6.006	0.510
206.	0.	-0.518	3.619	0.000	-14.670	-4.820	0.534
208.	0.	-0.187	3.723	0.000	-15.077	-3.520	0.564
210.	0.	0.204	3.862	-0.000	-15.618	-2.002	0.608
212.	0.	0.727	4.088	0.000	-16.496	0.004	0.679
214.	0.	1.671	4.612	-0.000	-18.559	3.603	0.839

TABLE XXVI

DUAL REACTION FORCE AT CYLINDER PAIR O_2

θ_1	F_{21x}	F_{21y}	F_{21z}	M_{21x}	M_{21y}	M_{21z}
160.	-1.018	0.257	-6.051	-6.022	28.364	1.676
162.	-0.890	-0.644	-5.514	-6.458	25.979	-2.378
164.	-0.747	-1.460	-4.827	-6.753	22.956	-6.150
166.	-0.596	-2.164	-4.021	-6.901	19.418	-9.478
168.	-0.443	-2.734	-3.147	-6.911	15.561	-12.244
170.	-0.295	-3.161	-2.257	-6.807	11.609	-14.388
172.	-0.160	-3.449	-1.398	-6.618	7.769	-15.912
174.	-0.040	-3.612	-0.607	-6.374	4.204	-16.864
176.	0.062	-3.669	0.096	-6.099	1.018	-17.322
178.	0.146	-3.642	0.702	-5.811	-1.744	-17.376
180.	0.214	-3.550	1.215	-5.522	-4.084	-17.115
182.	0.268	-3.412	1.641	-5.238	-6.034	-16.616
184.	0.311	-3.239	1.994	-4.961	-7.642	-15.944
186.	0.345	-3.042	2.286	-4.691	-8.963	-15.149
188.	0.372	-2.828	2.528	-4.429	-10.046	-14.267
190.	0.394	-2.601	2.729	-4.173	-10.937	-13.326
192.	0.413	-2.366	2.897	-3.921	-11.673	-12.346
194.	0.429	-2.124	3.039	-3.674	-12.285	-11.339
196.	0.445	-1.876	3.161	-3.428	-12.801	-10.312
198.	0.459	-1.623	3.266	-3.184	-13.242	-9.270
200.	0.474	-1.364	3.360	-2.938	-13.629	-8.211
202.	0.491	-1.097	3.447	-2.688	-13.981	-7.127
204.	0.510	-0.817	3.531	-2.427	-14.319	-6.006
206.	0.534	-0.518	3.619	-2.147	-14.670	-4.820
208.	0.564	-0.187	3.723	-1.833	-15.077	-3.520
210.	0.608	0.204	3.862	-1.452	-15.618	-2.002
212.	0.679	0.727	4.088	-0.921	-16.496	0.004
214.	0.839	1.671	4.612	0.098	-18.559	3.603

TABLE XXVII

DUAL REACTION FORCE AT REVOLUTE PAIR O_4

θ_1	F_{43x}	F_{43y}	F_{43z}	M_{43x}	M_{43y}	M_{43z}	FA^4
160.	1.918	2.273	10.878	8.119	-44.350	7.877	-0.461
162.	1.823	3.164	10.338	8.412	-42.107	11.435	-0.335
164.	1.695	3.995	9.635	8.564	-39.200	14.765	-0.205
166.	1.550	4.716	8.789	8.552	-35.718	17.666	-0.070
168.	1.383	5.292	7.842	8.377	-31.833	19.995	0.070
170.	1.207	5.705	6.843	8.056	-27.746	21.681	0.208
172.	1.030	5.956	5.840	7.620	-23.656	22.726	0.339
174.	0.859	6.060	4.873	7.103	-19.729	23.188	0.459
176.	0.701	6.041	3.973	6.542	-16.083	23.162	0.562
178.	0.557	5.926	3.156	5.965	-12.790	22.757	0.648
180.	0.429	5.743	2.431	5.397	-9.875	22.081	0.718
182.	0.317	5.513	1.796	4.854	-7.333	21.227	0.771
184.	0.220	5.257	1.245	4.344	-5.137	20.267	0.811
186.	0.136	4.988	0.771	3.873	-3.252	19.257	0.841
188.	0.064	4.715	0.364	3.440	-1.637	18.233	0.861
190.	0.002	4.445	0.014	3.045	-0.253	17.222	0.876
192.	-0.051	4.182	-0.287	2.686	0.937	16.240	0.885
194.	-0.097	3.928	-0.548	2.358	1.966	15.294	0.891
196.	-0.137	3.685	-0.776	2.057	2.865	14.390	0.893
198.	-0.172	3.452	-0.977	1.781	3.659	13.527	0.894
200.	-0.204	3.228	-1.158	1.523	4.375	12.701	0.892
202.	-0.233	3.012	-1.324	1.280	5.037	11.906	0.889
204.	-0.261	2.801	-1.483	1.045	5.674	11.130	0.884
206.	-0.289	2.589	-1.642	0.810	6.318	10.354	0.876
208.	-0.320	2.367	-1.813	0.561	7.019	9.542	0.864
210.	-0.356	2.116	-2.019	0.271	7.868	8.620	0.846
212.	-0.408	1.782	-2.312	-0.129	9.084	7.378	0.816
214.	-0.513	1.126	-2.909	-0.936	11.561	4.897	0.748

TABLE XXVIII

REACTION FORCE AT SPHERICAL PAIR O_3

θ	F^B	F^C	F^D	M^{IN}
160.	-7.816	-4.043	0.772	-6.022
162.	-7.585	-3.975	0.533	-6.458
164.	-7.308	-3.875	0.300	-6.753
166.	-6.981	-3.753	0.074	-6.901
168.	-6.612	-3.618	-0.137	-6.911
170.	-6.215	-3.478	-0.326	-6.807
172.	-5.804	-3.341	-0.488	-6.618
174.	-5.396	-3.209	-0.617	-6.374
176.	-5.004	-3.088	-0.714	-6.099
178.	-4.637	-2.977	-0.783	-5.811
180.	-4.304	-2.878	-0.826	-5.522
182.	-4.007	-2.791	-0.850	-5.238
184.	-3.748	-2.716	-0.859	-4.961
186.	-3.526	-2.653	-0.858	-4.691
188.	-3.340	-2.602	-0.851	-4.429
190.	-3.187	-2.564	-0.841	-4.173
192.	-3.066	-2.538	-0.830	-3.921
194.	-2.976	-2.527	-0.819	-3.674
196.	-2.917	-2.530	-0.810	-3.428
198.	-2.889	-2.551	-0.803	-3.184
200.	-2.897	-2.593	-0.800	-2.938
202.	-2.946	-2.662	-0.801	-2.688
204.	-3.047	-2.766	-0.807	-2.427
206.	-3.221	-2.921	-0.819	-2.147
208.	-3.506	-3.155	-0.839	-1.833
210.	-3.989	-3.533	-0.866	-1.452
212.	-4.904	-4.219	-0.896	-0.921
214.	-7.262	-5.920	-0.832	0.098

TABLE XXIX

DUAL REACTION FORCE AT CYLINDER PAIR O_5

θ_1	F_{25x}	F_{25y}	F_{25z}	M_{25x}	M_{25y}	M_{25z}
160.	0.446	2.529	4.906	3.130	-15.986	9.651
162.	0.603	2.520	4.882	2.974	-16.128	9.082
164.	0.749	2.535	4.844	2.817	-16.243	8.563
166.	0.885	2.552	4.780	2.644	-16.300	8.056
168.	1.009	2.557	4.682	2.443	-16.271	7.537
170.	1.116	2.543	4.549	2.212	-16.137	6.997
172.	1.204	2.507	4.382	1.951	-15.887	6.440
174.	1.271	2.448	4.187	1.666	-15.524	5.879
176.	1.316	2.372	3.971	1.369	-15.065	5.334
178.	1.341	2.284	3.746	1.072	-14.534	4.824
180.	1.349	2.192	3.521	0.786	-13.959	4.369
182.	1.344	2.101	3.303	0.521	-13.366	3.982
184.	1.329	2.018	3.099	0.284	-12.779	3.670
186.	1.309	1.946	2.912	0.078	-12.215	3.438
188.	1.284	1.887	2.742	-0.094	-11.683	3.284
190.	1.259	1.844	2.591	-0.234	-11.190	3.205
192.	1.234	1.816	2.456	-0.343	-10.735	3.197
194.	1.210	1.805	2.336	-0.424	-10.319	3.256
196.	1.187	1.809	2.230	-0.479	-9.936	3.376
198.	1.167	1.829	2.134	-0.512	-9.583	3.555
200.	1.149	1.864	2.047	-0.523	-9.254	3.789
202.	1.133	1.916	1.968	-0.515	-8.944	4.079
204.	1.119	1.984	1.895	-0.489	-8.645	4.428
206.	1.106	2.071	1.826	-0.444	-8.352	4.843
208.	1.095	2.180	1.760	-0.377	-8.057	5.339
210.	1.085	2.321	1.697	-0.284	-7.750	5.944
212.	1.075	2.509	1.634	-0.150	-7.412	6.726
214.	1.063	2.798	1.573	0.068	-6.998	7.885

TABLE XXX

REACTION FORCES AT SPHERICAL PAIR O_6

θ_1	F^E	F^F	F^G
160.	0.878	5.981	-1.012
162.	0.640	5.809	-0.798
164.	0.387	5.661	-0.626
166.	0.133	5.522	-0.486
168.	-0.111	5.383	-0.372
170.	-0.340	5.234	-0.278
172.	-0.545	5.071	-0.202
174.	-0.724	4.892	-0.142
176.	-0.874	4.701	-0.095
178.	-0.992	4.503	-0.061
180.	-1.082	4.303	-0.039
182.	-1.145	4.108	-0.025
184.	-1.186	3.923	-0.019
186.	-1.207	3.751	-0.020
188.	-1.214	3.594	-0.025
190.	-1.208	3.453	-0.032
192.	-1.193	3.328	-0.042
194.	-1.172	3.220	-0.053
196.	-1.145	3.127	-0.064
198.	-1.115	3.048	-0.075
200.	-1.081	2.984	-0.086
202.	-1.045	2.935	-0.097
204.	-1.006	2.902	-0.107
206.	-0.963	2.885	-0.118
208.	-0.915	2.889	-0.129
210.	-0.859	2.920	-0.143
212.	-0.787	2.995	-0.160
214.	-0.680	3.162	-0.187

TABLE XXXI

DUAL REACTION FORCE AT REVOLUTE PAIR O_7

θ_1	F_{46x}	F_{46y}	F_{46z}	M_{46x}	M_{46y}	M_{46z}	F^{A7}
160.	-1.256	0.0	-0.644	-5.514	0.000	25.979	-2.378
162.	-1.268	0.0	-1.460	-4.827	0.000	22.956	-6.150
164.	-1.272	0.0	-2.164	-4.021	0.000	19.418	-9.478
166.	-1.267	0.0	-2.734	-3.147	0.000	15.561	-12.244
168.	-1.251	0.0	-3.161	-2.257	0.000	11.609	-14.388
170.	-1.225	0.0	-3.449	-1.398	0.000	7.769	-15.912
172.	-1.189	0.0	-3.612	-0.607	0.000	4.204	-16.864
174.	-1.144	0.0	-3.669	0.096	0.0	1.018	-17.322
176.	-1.093	0.0	-3.642	0.702	0.000	-1.744	-17.376
178.	-1.037	0.0	-3.550	1.215	0.000	-4.084	-17.115
180.	-0.980	0.0	-3.412	1.641	0.000	-6.034	-16.616
182.	-0.924	0.0	-3.239	1.994	0.000	-7.642	-15.944
184.	-0.870	0.0	-3.042	2.286	0.000	-8.963	-15.149
186.	-0.819	0.0	-2.828	2.528	0.0	-10.046	-14.267
188.	-0.771	0.0	-2.601	2.729	0.000	-10.937	-13.326
190.	-0.727	0.0	-2.366	2.897	0.000	-11.673	-12.346
192.	-0.686	0.0	-2.124	3.039	0.000	-12.285	-11.339
194.	-0.649	0.0	-1.876	3.161	0.000	-12.801	-10.312
196.	-0.613	0.0	-1.623	3.266	0.000	-13.242	-9.270
198.	-0.580	0.0	-1.364	3.360	0.000	-13.629	-8.211
200.	-0.548	0.0	-1.097	3.447	0.000	-13.981	-7.127
202.	-0.516	0.0	-0.817	3.531	0.000	-14.319	-6.006
204.	-0.485	0.0	-0.518	3.619	0.000	-14.670	-4.820
206.	-0.453	0.0	-0.187	3.723	0.000	-15.077	-3.520
208.	-0.420	0.0	0.204	3.862	-0.000	-15.618	-2.002
210.	-0.384	0.0	0.727	4.088	0.000	-16.496	0.004
212.	-0.344	0.0	1.671	4.612	-0.000	-18.559	3.603
214.	-0.292	-2.702	-1.837	-3.967	6.794	-9.484	-0.820

shown in Figure 19. A complete kinematic analysis of RCHCH spatial mechanism is given in Chapter II Section 8.

Let us define the following unknown dual forces acting at pair joints.

$$\hat{R}_{51} = \hat{F}_{51} + F^{A1}\hat{A}_1 \qquad (3.90)$$

$$\hat{R}_{12} = \hat{F}_{12} \qquad (3.91)$$

$$\hat{F}^{IN} = \varepsilon\ M\ \hat{A}_1 \qquad (3.92)$$

$$\hat{R}_{32} = \hat{F}_{32} + (F^{A3} + \varepsilon\ M^{A3})\ \hat{A}_3 \qquad (3.93)$$

$$\hat{R}_{43} = \hat{F}_{43} \qquad (3.94)$$

$$\hat{R}_{54} = \hat{F}_{54} + (F^{A5} + \varepsilon\ M^{A5})\ \hat{A}_5 \qquad (3.95)$$

where

$\hat{R}_{51}$ is a dual reaction force acting from link 5 to link 1 at revolute pair 0.

$\hat{R}_{32}$ is a dual reaction force acting from link 3 to link 2 at cylinder pair O_2.

$\hat{R}_{43}$ is a dual reaction force acting from link 4 to link 3.

$\hat{R}_{54}$ is a dual reaction force acting from link 5 to link 4.

$\hat{F}^{IN}$ is a dual input force.

$\hat{A}_1$, $\hat{A}_3$ and $\hat{A}_5$ are dual screw vectors which are defined in Chapter II Section 8.

The constraint equation for revolute pair O_1 is

$$\hat{F}_{51} \cdot \hat{A}_1 = 0 \qquad (3.96)$$

The constraint equation for cylinder pair O_2 is

$$\hat{F}_{21} \cdot \hat{A}_2 = 0 \tag{3.97}$$

The constraint equations for helical pair O_3 are

$$\hat{F}_{32} \cdot \hat{A} = O_3 \tag{3.98}$$

and

$$\rho_3 \dot{F}^{A3} + M^{A3} = 0$$

The constraint equation for cylinder pair O_4 is

$$\hat{F}_{43} \cdot \hat{A}_4 = 0 \tag{3.99}$$

The constraint equations for helical pair O_5 are

$$\hat{F}_{54} \cdot \hat{A}_5 = 0 \tag{3.100}$$

and

$$\rho_5 F^{A5} + M^{A5} = 0$$

The equation of motion for link 1 is

$$\hat{R}_{51} + \hat{R}_{21} + \hat{F}_{I1} + \hat{F}^{IN} = 0 \tag{3.101}$$

The equation of motion for link 2 is

$$\hat{R}_{12} + \hat{R}_{32} + \hat{F}_{I2} = 0 \tag{3.102}$$

The equation of motion for link 3 is

$$\hat{R}_{23} + \hat{R}_{43} + \hat{F}_{I3} = 0 \tag{3.103}$$

The equation of motion for link 4 is

$$\hat{R}_{34} + \hat{R}_{54} + \hat{F}_{I4} + \hat{F}^{OUT} = 0 \tag{3.104}$$

In the above equations (3.101) through (3.104) there are thirty-three unknown forces and torques and an input torque. They are

$$\{U\}^T = \{F_{51x}, F_{51y}, F_{51z}, M_{51x}, M_{51y}, M_{51z}, F^{A1}, F_{21x}, F_{21y}, F_{21z}, M_{21x}, M_{21y}, M_{21z}, M^{IN}, F_{32x}, F_{32y}, F_{32z}, M_{32x}, M_{32y}, M_{32z}, F^{A3}, F_{43x}, F_{43y}, F_{43z}, M_{43x}, M_{43y}, M_{43z}, F_{54x}, F_{54y}, F_{54z}, M_{54x}, M_{54y}, M_{54z}, F^{A5}\}$$

Equation (3.96) through (3.104) can be arranged in a linear matrix form.

$$\{Q\}_{34 \times 34} \times \{U\}_{34 \times 1} = \{C\}_{34 \times 1} \tag{3.105}$$

where nonzero coefficients of {Q} and {C} are listed in Table XXXII.

A numerical example of the dynamic analysis of the RCHCH spatial mechanism is performed with the dimensions: $a_1 = 10°$, $a_2 = 80°$, $a_3 = 40°$, $a_4 = 50°$, $a_5 = 60°$, $P_1 = 1$ in., $P_2 = .5$ in., $P_3 = .8$ in., $P_4 = .6$ in., $P_5 = .4$ in., $S_1 = .2$ in., $\rho_3 = 2$, $\rho_5 = 5$, $\omega_1 = .1$ rps, $\alpha_1 = .1$ rps^2, $x_1 = .1$ in., $y_1 = .1$ in., $z_1 = .01$ in., $x_2 = .2$ in., $y_2 = .1$ in., $z_2 = .1$ in., $x_3 = .01$ in., $y_3 = .2$ in., $z_3 = 0.01$ in., $x_4 = .1$ in., $y_4 = .1$ in., $z_4 = .01$ in., $M_1 = .1$ lb., $M_2 = .5$ lb., $M_3 = .4$ lb., $M_4 = .5$ lb., $I_{xx1} = .01$, $I_{xy1} = .01$, $I_{xz1} = .001$, $I_{yy1} = .002$, $I_{yz1} = .01$, $I_{zz1} = .02$, $I_{xx2} = .01$, $I_{xy2} = .02$, $I_{xz2} = .001$, $I_{yy2} = .002$, $I_{yz2} = .01$, $I_{zz2} = .02$, $I_{xx3} = .01$, $I_{xy3} = .02$, $I_{xz3} = .001$, $I_{yy3} = .002$, $I_{yz3} = .01$, $I_{zz3} = .02$, $I_{xx4} = .01$, $I_{xy4} = .02$, $I_{yy4} = .001$, $I_{yz4} = .01$, $I_{zz4} = .02$ and $M^{out} = 10$ in.-lbs.

Kinematic numerical results are shown in Table XI. Numerical results of dynamic analysis are listed in Tables XXXIII through XXXVII.

TABLE XXXII

NONZERO COEFFICIENTS OF {Q} AND {C} OF THE RCHCH SPATIAL MECHANISM

$Q(1, 1) = 1$	$Q(5, 14) = A_{1y}$	$Q(7, 1) = A_{1x}$
$Q(2, 2) = 1$	$Q(6, 14) = A_{1z}$	$Q(7, 2) = A_{1y}$
$Q(3, 3) = 1$	$Q(1, 8) = 1$	$Q(7, 3) = A_{1z}$
$Q(4, 4) = 1$	$Q(2, 9) = 1$	$Q(8, 1) = A_{11x}$
$Q(5, 5) = 1$	$Q(3, 10) = 1$	$Q(8, 2) = A_{11y}$
$Q(6, 6) = 1$	$Q(4, 11) = 1$	$Q(8, 3) = A_{11z}$
$Q(15, 21) = -A_{3x}$	$Q(5, 12) = 1$	$Q(8, 4) = A_{1x}$
$Q(16, 21) = -A_{3y}$	$Q(1, 7) = A_{1x}$	$Q(8, 5) = A_{1y}$
$Q(17, 21) = -A_{3z}$	$Q(2, 7) = A_{1y}$	$Q(8, 6) = A_{1z}$
$Q(18, 21) = -A_{33x} + \rho_3 * A_{3x}$	$Q(3, 7) = A_{1z}$	$Q(9, 8) = -1$
	$Q(4, 7) = A_{11x}$	$Q(10, 9) = -1$
$Q(19, 21) = -A_{33x} + \rho_3 \times A_{3y}$	$Q(5, 7) = A_{11y}$	$Q(11, 10) = -1$
$Q(20, 21) = -A_{33z} + \rho_3 \cdot A_{3z}$	$Q(6, 7) = A_{11z}$	$Q(12, 11) = -1$
	$Q(4, 14) = A_{1x}$	$Q(13, 12) = -1$
$Q(6, 13) = 1$	$Q(5, 14) = A_{1y}$	$Q(14, 13) = -1$
$Q(4, 14) = A_{1x}$	$Q(6, 14) = A_{1z}$	$Q(9, 15) = 1$

TABLE XXXII (Continued)

Q(10, 16) = 1	Q(27, 8) = A_{2x}	Q(34, 20) = A_{3z}
Q(11, 17) = 1	Q(27, 9) = A_{2y}	Q(15, 22) = 1
Q(12, 18) = 1	Q(27, 10) = A_{2z}	Q(16, 23) = 1
Q(13, 19) = 1	Q(28, 8) = A_{22x}	Q(17, 24) = 1
Q(14, 20) = 1	Q(28, 9) = A_{22y}	Q(18, 25) = 1
Q(9, 21) = A_{3x}	Q(28, 10) = A_{22z}	Q(19, 26) = 1
Q(10, 21) = A_{3y}	Q(28, 11) = A_{2x}	Q(20, 27) = 1
Q(11, 21) = A_{3z}	Q(28, 12) = A_{2y}	Q(21, 22) = -1
Q(12, 21) = $A_{31x} - \rho_3 * A_{3x}$	Q(28, 13) = A_{2z}	Q(22, 23) = -1
Q(13, 21) = $A_{31y} - \rho_3 * A_{3y}$	Q(33, 15) = A_{3x}	Q(23, 24) = -1
Q(14, 21) = $A_{31z} - \rho_3 * A_{3z}$	Q(33, 16) = A_{3y}	Q(24, 25) = -1
Q(15, 15) = -1	Q(33, 17) = A_{3z}	Q(25, 26) = -1
Q(16, 16) = -1	Q(34, 15) = A_{31x}	Q(26, 27) = -1
Q(17, 17) = -1	Q(34, 16) = A_{31y}	Q(21, 28) = 1
Q(18, 18) = -1	Q(34, 17) = A_{31z}	Q(22, 29) = 1
Q(19, 19) = -1	Q(34, 18) = A_{3x}	Q(23, 30) = 1
Q(20, 20) = -1	Q(34, 19) = A_{3y}	Q(24, 31) = 1

TABLE XXXII (Concluded)

Q(25, 32) = 1	Q(31, 28) = A_{5x}	C(11) = $-F_{I2z}$
Q(26, 33) = 1	Q(31, 29) = A_{5y}	C(12) = $-M_{I2x}$
Q(21, 34) = A_{5x}	Q(31, 30) = A_{5z}	C(13) = $-M_{I2y}$
Q(22, 34) = A_{5y}	Q(32, 28) = A_{51x}	C(14) = $-M_{I2z}$
Q(23, 34) = A_{5z}	Q(32, 29) = A_{51y}	C(15) = $-F_{I3x}$
Q(24, 34) = $A_{51x} - \rho_5 * A_{5x}$	Q(32, 30) = A_{51z}	C(16) = $-F_{I3y}$
Q(25, 34) = $A_{51y} - \rho_5 * A_{5y}$	Q(32, 31) = A_{5x}	C(17) = $-F_{I3z}$
Q(26, 34) = $A_{51z} - \rho_5 * A_{5z}$	Q(32, 32) = A_{5y}	C(18) = $-M_{I3x}$
Q(29, 22) = A_{4x}	Q(32, 33) = A_{5z}	C(19) = $-M_{I3y}$
Q(29, 23) = A_{4y}	C(1) = $-F_{I1x}$	C(20) = $-M_{I3z}$
Q(29, 24) = A_{4z}	C(2) = $-F_{I1y}$	C(21) = $-F_{I4x}+A_{5x} \cdot M^{out}/\rho_5$
Q(30, 22) = A_{41x}	C(3) = $-F_{I1z}$	C(22) = $-F_{I4y}+A_{5x} \cdot M^{out}/\rho_5$
Q(30, 23) = A_{41y}	C(4) = $-M_{I1x}$	C(23) = $-F_{I4z}+A_{5x} \cdot M^{out}/\rho_5$
Q(30, 24) = A_{41z}	C(5) = $-M_{I1y}$	C(24) = $-M_{I4x} - A_{5x} * M^{out}$
Q(30, 25) = A_{4x}	C(6) = $-M_{I1z}$	C(25) = $-M_{I4y} - A_{5y} * M^{out}$
Q(30, 26) = A_{4y}	C(9) = $-F_{I2x}$	C(26) = $-M_{I4z} - A_{5z} * M^{out}$
Q(30, 27) = A_{4z}	C(10) = $-F_{I2y}$	

TABLE XXXIII

DUAL REACTION FORCE AT REVOLUTE PAIR O_1

θ_1	F_{51x}	F_{51y}	F_{51z}	M_{51x}	M_{51y}	M_{51z}	F^{A1}	M^{IN}
-65.	0.	-1.38	0.53	0.	2.37	-2.35	-0.26	-0.96
-60.	0.	-1.88	0.61	0.	3.48	-2.75	-0.34	-1.20
-55.	0.	2.54	-1.21	0.	-3.97	3.81	0.49	1.89
-50.	0.	3.66	-0.00	0.	-3.06	3.72	0.49	1.98
-45.	0.	18.02	-2.54	0.	-32.76	13.81	2.56	8.80
-40.	0.	-59.46	-0.48	0.	97.45	-24.36	-6.67	-23.08
-35.	0.	-59.50	-2.60	0.	50.56	-12.32	-5.64	-24.33
-30.	0.	67.62	0.11	0.	-73.32	-4.63	5.94	26.94
-25.	0.	-3816.94	-308.59	0.	-1575.24	102.48	-235.12	-1382.06
-20.	0.	-344.29	0.01	0.	-473.79	-80.98	-20.77	-125.17
-15.	0.	2375.32	867.07	0.	-16950.38	-4803.25	-39.28	-273.98
-10.	0.	-10.10	-0.87	0.	16.28	6.05	-0.16	-1.12
-5.	0.	10.30	2.37	0.	-23.97	-8.43	-0.26	-0.76
0.	0.	41.00	10.28	0.	-101.94	-25.75	-1.81	-7.50
5.	0.	-3.11	0.12	0.	3.34	0.95	0.03	0.05
10.	0.	1.32	1.09	0.	-4.37	-1.37	-0.23	-1.01
15.	0.	1.49	1.01	0.	-5.13	-0.66	-0.24	-1.09
20.	0.	0.23	0.90	0.	-3.11	0.30	-0.16	-0.81
25.	0.	-0.24	0.91	0.	-2.74	0.56	-0.13	-0.61
30.	0.	-1.08	1.07	0.	-3.94	2.09	-0.07	-0.32
35.	0.	-398.55	2042.39	0.	-1956.69	-6963.89	-254.69	-232.75
40.	0.	0.30	-0.41	0.	-1.92	3.12	0.02	-0.14
45.	0.	-16.23	12.92	0.	3.64	-14.78	0.41	4.54
50.	0.	-19.97	-32.69	0.	64.76	152.87	6.40	10.94
55.	0.	-1.03	-15.72	0.	184.91	63.05	1.74	-23.36

TABLE XXXIV

DUAL REACTION FORCE AT CYLINDER PAIR O_2

θ_1	F_{21x}	F_{21y}	F_{21z}	M_{21x}	M_{21y}	M_{21z}
-65.	0.26	1.38	-0.53	0.97	-2.37	2.35
-60.	0.34	1.88	-0.61	1.20	-3.48	2.75
-55.	-0.49	-2.54	1.21	-1.89	3.97	-3.81
-50.	-0.49	-3.66	0.00	-1.98	3.06	-3.72
-45.	-2.56	-18.02	2.54	-8.80	32.76	-13.80
-40.	6.67	59.46	0.48	23.08	-97.45	24.36
-35.	5.64	59.50	2.60	24.33	-50.56	12.32
-30.	-5.94	-67.62	-0.11	-26.94	73.32	4.63
-25.	235.12	3816.94	308.59	1382.06	1575.24	-102.48
-20.	20.77	344.29	-0.01	125.18	473.79	80.98
-15.	39.28	-2375.32	-867.07	273.98	16950.38	4803.25
-10.	0.16	10.10	0.87	1.12	-16.28	-6.05
-5.	0.26	-10.30	-2.37	0.76	23.97	8.43
0.	1.81	-41.00	-10.28	7.50	101.94	25.75
5.	-0.03	3.11	-0.12	-0.05	-3.34	-0.95
10.	0.23	-1.32	-1.09	1.01	4.37	1.37
15.	0.24	-1.49	-1.01	1.09	5.13	0.66
20.	0.16	-0.23	-0.90	0.81	3.11	-0.30
25.	0.13	0.24	-0.91	0.61	2.74	-0.56
30.	0.07	1.08	-1.07	0.32	3.94	-2.09
35.	254.69	398.55	-2042.39	232.75	1956.69	6963.89
40.	-0.02	-0.30	0.41	0.14	1.52	-3.12
45.	-0.41	16.23	-12.92	-4.54	-3.64	14.78
50.	-6.40	19.97	32.69	-10.94	-64.76	-152.87
55.	-1.74	1.03	15.72	23.36	-184.90	-63.05

TABLE XXXV

DUAL REACTION FORCE AT HELICAL PAIR O_3

θ_1	F_{32x}	F_{32y}	F_{32z}	M_{32x}	M_{32y}	M_{32z}	F^{A3}	M^{A3}
-65.	0.11	0.12	0.23	-1.28	0.35	1.03	-1.51	3.03
-60.	0.35	0.34	0.51	-1.62	-0.37	0.52	-1.91	3.82
-55.	0.05	-0.40	-0.50	2.21	0.12	0.04	2.77	-5.55
-50.	0.95	-1.78	-1.85	2.31	-0.47	0.60	2.91	-5.83
-45.	-1.50	-9.30	-7.83	11.26	17.60	8.56	13.73	-27.46
-40.	3.03	40.29	28.04	-26.83	-61.55	-32.16	-34.35	68.70
-35.	-0.34	47.16	25.61	-14.29	-18.15	-29.50	-27.37	54.75
-30.	-5.43	-57.72	-24.04	9.31	40.98	45.62	26.15	-52.30
-25.	92.65	3575.34	1103.58	200.15	2557.32	-1512.37	-867.01	1734.03
-20.	40.47	326.04	67.75	32.02	542.14	-41.68	-68.53	137.06
-15.	66.93	-2445.33	-311.66	-491.63	17413.64	3775.56	-562.74	1125.48
-10.	1.04	10.00	0.31	1.95	-16.63	-5.10	0.59	-1.18
-5.	-1.51	-9.99	0.59	-3.73	25.22	2.30	-3.30	6.59
0.	-13.26	-39.02	6.23	-19.18	103.47	-11.50	-19.62	39.24
5.	0.65	2.99	-0.72	0.90	-3.20	0.34	0.68	-1.37
10.	-0.49	-0.79	0.29	-1.20	3.76	-1.56	-1.61	3.23
15.	-0.74	-0.78	0.40	-1.28	4.08	-2.41	-1.77	3.53
20.	-0.78	0.27	-0.09	-0.76	1.87	-1.99	-1.13	2.27
25.	-0.76	0.67	-0.37	-0.60	1.42	-1.59	-0.87	1.73
30.	-1.48	1.31	-0.90	-0.50	2.67	-2.41	-0.58	1.15
35.	460.85	1164.28	-1085.65	-1543.13	-1573.02	6927.23	-1226.20	2452.40
40.	-0.57	-0.26	0.38	-0.04	1.22	-3.09	-0.13	0.26
45.	-15.64	16.04	-11.68	-8.83	-12.14	13.52	-3.12	6.23
50.	-22.99	-7.48	15.37	31.64	25.36	-191.88	29.80	-59.61
55.	-2.91	-5.97	11.99	36.21	-127.30	-143.17	7.63	-15.26

TABLE XXXVI

DUAL REACTION FORCE AT CYLINDER PAIR O_4

θ_1	F_{43x}	F_{43y}	F_{43z}	M_{43x}	M_{43y}	M_{43z}
-65.	-0.60	1.27	-0.99	1.00	-2.34	2.50
-60.	-0.31	1.66	-1.21	1.23	-3.44	2.88
-55.	0.84	-4.14	1.99	-1.72	4.09	-3.86
-50.	1.80	-5.30	0.84	-1.89	3.19	-3.94
-45.	0.39	-20.02	5.01	-8.59	32.95	-14.05
-40.	-1.59	62.32	-3.50	23.00	-97.96	25.61
-35.	-7.71	63.34	-4.22	24.20	-51.38	14.21
-30.	-1.34	-68.74	6.58	-26.83	73.53	4.40
-25.	-129.99	3891.38	99.77	1377.07	1554.01	-51.52
-20.	43.21	331.99	24.23	127.43	473.42	74.45
-15.	22.56	-2386.06	-879.33	274.42	16948.61	4804.77
-10.	1.88	7.32	2.22	1.51	-16.31	-6.46
-5.	-4.01	-11.63	-4.64	0.73	23.78	9.19
0.	-32.51	-61.58	-26.63	7.38	98.71	30.33
5.	1.46	2.42	1.40	0.12	-3.27	-1.08
10.	-1.54	-2.20	-1.64	0.99	4.31	1.74
15.	-2.02	-2.62	-1.74	1.03	5.04	1.12
20.	-1.83	-1.37	-1.36	0.76	3.03	0.09
25.	-1.71	-0.92	-1.17	0.59	2.68	-0.20
30.	-2.95	-0.88	-1.49	0.24	3.80	-1.57
35.	796.97	-54.53	-136.05	92.97	1794.58	7081.52
40.	-1.66	-0.50	-0.51	0.13	1.43	-2.83
45.	-25.62	-6.83	-12.51	-5.75	-6.05	19.26
50.	-29.55	6.90	16.84	-11.16	-66.15	-149.42
55.	-2.58	1.30	13.42	23.23	-184.89	-62.87

TABLE XXXVII

DUAL REACTION FORCE AT HELICAL PAIR O_5

θ_1	F_{54x}	F_{54y}	F_{54z}	M_{54x}	M_{54y}	M_{54z}	F^{A5}	M^{A5}
-65.	-0.88	1.26	-0.51	1.69	-1.26	1.45	2.07	-10.34
-60.	-0.76	1.64	-0.44	1.98	-2.08	1.55	2.11	-10.55
-55.	1.51	-4.14	0.87	-2.13	2.89	-2.04	1.64	-8.21
-50.	1.71	-5.35	0.99	-2.91	3.84	-2.59	1.51	-7.57
-45.	2.51	-20.02	1.45	-11.97	31.60	-8.25	0.65	-3.27
-40.	-2.72	62.28	-1.57	29.02	-97.86	18.21	3.87	-19.36
-35.	-7.62	63.28	-4.40	22.74	-51.13	17.19	1.28	-6.41
-30.	2.01	-68.75	1.16	-16.69	72.80	-10.71	5.62	-28.09
-25.	-54.30	3891.33	-31.35	946.21	1554.07	575.26	-149.63	748.16
-20.	42.95	331.96	24.80	139.47	475.16	57.62	6.07	-30.33
-15.	-363.90	-2386.16	-210.10	2470.53	16947.08	1620.45	772.93	-3864.66
-10.	2.37	7.31	1.37	-0.93	-16.51	-1.80	0.86	-4.29
-5.	-5.05	-11.67	-2.91	5.73	22.74	6.00	3.30	-16.48
0.	-36.21	-62.01	-20.91	23.55	90.87	32.91	4.23	-21.16
5.	1.71	2.41	0.99	0.65	-2.30	-0.53	1.96	-9.79
10.	-1.88	-2.22	-1.09	3.28	3.60	2.90	2.35	-11.74
15.	-2.29	-2.64	-1.32	3.12	4.43	3.02	2.23	-11.16
20.	-1.98	-1.39	-1.14	2.77	2.81	2.66	2.18	-10.91
25.	-1.80	-0.94	-1.04	2.38	2.69	2.33	2.15	-10.75
30.	-2.88	-0.90	-1.66	2.17	4.74	2.79	2.02	-10.12
35.	505.20	-85.53	291.68	5537.59	5327.30	2927.70	1239.27	-6196.32
40.	-1.49	-0.52	-0.86	1.65	2.65	1.74	1.91	-9.53
45.	-24.98	-7.20	-14.42	25.48	19.99	28.04	6.00	-29.98
50.	-14.93	6.84	-8.62	-84.72	-55.08	-40.95	-25.31	126.54
55.	3.88	1.30	2.24	-12.67	-184.49	-9.38	-10.85	54.27

3.8. Dynamic Analysis of the 7R Spatial Mechanism

The dimensions and dual vectors of the 7R spatial mechanism are defined in Figure 24. The unknown forces and torques at pair joints are written as follows:

At revolute pair O_1,

$$\hat{R}_{71} = \hat{F}_{71} + F^{A1}\hat{A}_1 \tag{3.106}$$

and

$$\hat{F}^{IN} = \varepsilon M^{A1}\hat{A}_1$$

At revolute pair O_2, dual reaction force is

$$\hat{R}_{12} = \hat{F}_{12} + F^{A2}\hat{A}_2 \tag{3.107}$$

At revolute pair O_3, dual reaction force is

$$\hat{R}_{23} = \hat{F}_{23} + F^{A3}\hat{A}_3 \tag{3.108}$$

At revolute pair O_4, dual reaction force is

$$\hat{R}_{34} = \hat{F}_{34} + F^{A4}\hat{A}_4 \tag{3.109}$$

At revolute pair O_5, dual reaction force is

$$\hat{R}_{45} = \hat{F}_{45} + F^{45}\hat{A}_5 \tag{3.110}$$

At revolute pair O_6, dual reaction force is

$$\hat{R}_{56} = \hat{F}_{56} + F^{A6}\hat{A}_6 \tag{3.111}$$

At revolute pair O_7, dual reaction force is

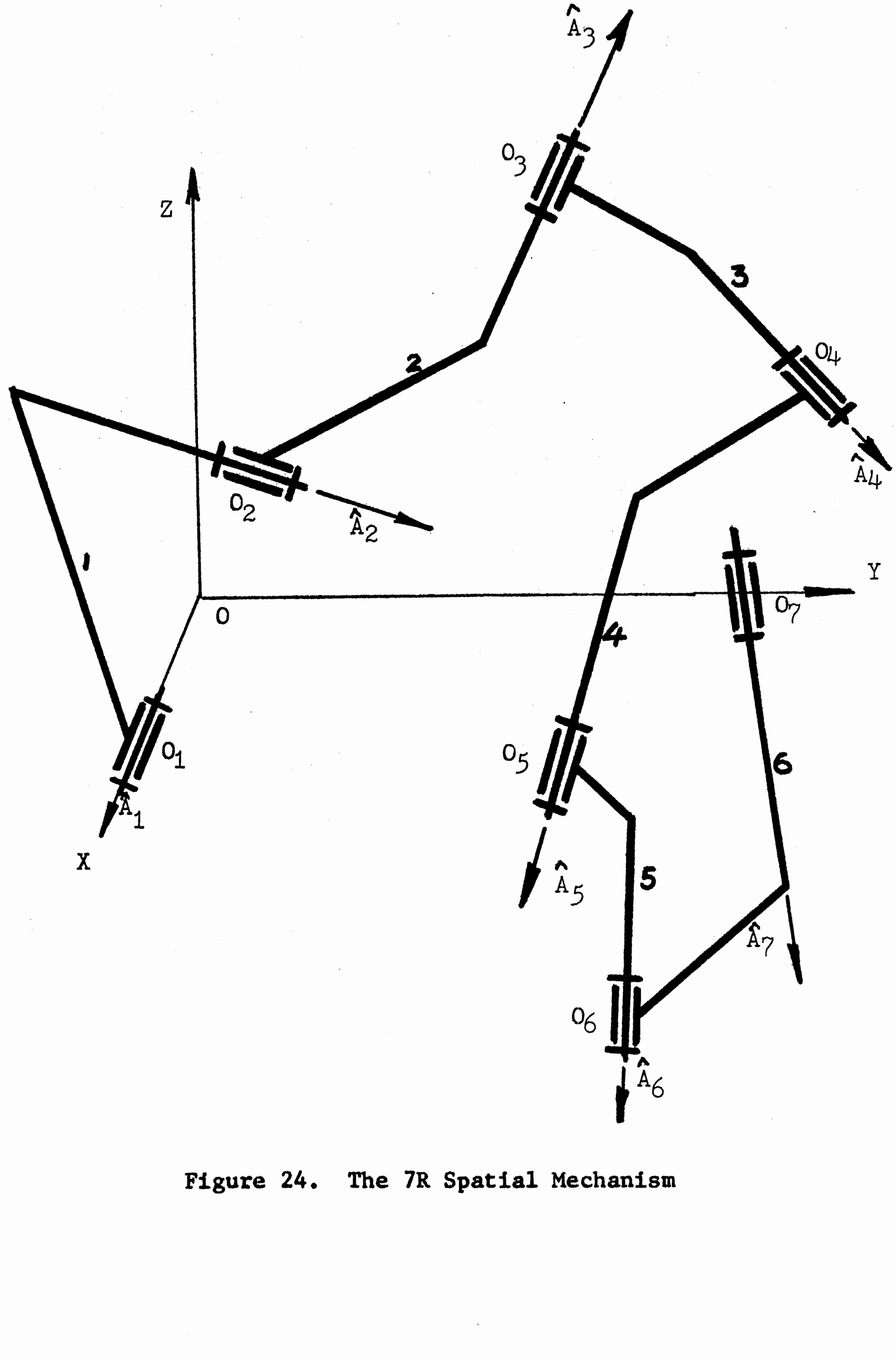

Figure 24. The 7R Spatial Mechanism

$$\hat{R}_{67} = \hat{F}_{67} + F^{A7}\hat{A}_7 \qquad (3.112)$$

The equations of motion of link 1 through 6 are

$$R_{71} + R_{21} + F_{I1} + F^{IN} = 0 \qquad (3.113)$$

$$\hat{R}_{12} + \hat{R}_{32} + \hat{F}_{I2} = 0 \qquad (3.114)$$

$$\hat{R}_{23} + \hat{R}_{43} + \hat{F}_{I3} = 0 \qquad (3.115)$$

$$\hat{R}_{34} + \hat{R}_{54} + \hat{F}_{I4} = 0 \qquad (3.116)$$

$$\hat{R}_{45} + \hat{R}_{65} + \hat{F}_{I5} = 0 \qquad (3.117)$$

and

$$\hat{R}_{56} + \hat{R}_{76} + \hat{F}_{I6} + \hat{F}^{out} = 0 \qquad (3.118)$$

The equations of constraint at kinematic pairs are

$$\hat{F}_{71} \cdot \hat{A}_1 = 0 \qquad (3.119)$$

$$\hat{F}_{12} \cdot \hat{A}_2 = 0 \qquad (3.120)$$

$$\hat{F}_{23} \cdot \hat{A}_3 = 0 \qquad (3.121)$$

$$\hat{F}_{34} \cdot \hat{A}_4 = 0 \qquad (3.122)$$

$$\hat{F}_{45} \cdot \hat{A}_5 = 0 \qquad (3.123)$$

$$\hat{F}_{56} \cdot \hat{A}_6 = 0 \qquad (3.124)$$

and

$$\hat{F}_{67} \cdot \hat{A}_7 = 0 \qquad (3.125)$$

Dynamic forces and torques analysis of the 7R spatial mechanism is a determinate system with fifty unknowns and fifty equations.

3.9. Dynamic Analysis of the RPRHRRR Spatial Mechanism

The dimensions and dual vectors of the RPRHRRR spatial mechanism are defined in Figure 25. The unknown forces and torques at pair joints are written as follows:

At revolute pair O_1,

$$\hat{R}_{71} = \hat{F}_{71} + F^{A1}\hat{A}_1 \tag{3.126}$$

$$\hat{F}^{IN} = \varepsilon\, M^{A1}\hat{A}_1$$

At prism pair O_2,

$$\hat{R}_{12} = \hat{F}_{12} + \varepsilon\, M^{A2}\hat{A}_2 \tag{3.127}$$

At revolute pair O_3, dual reaction force is

$$\hat{R}_{23} = \hat{F}_{23} + F^{A3}\hat{A}_3 \tag{3.128}$$

At helical pair O_4, dual reaction force is

$$\hat{R}_{34} = \hat{F}_{34} + (F^{A4} + \varepsilon\, M^{A4})\,\hat{A}_4 \tag{3.129}$$

and constraint equation is

$$\rho F^{A4} + M^{A4} = 0$$

At revolute pair O_4, dual reaction force is

$$\hat{R}_{45} = \hat{F}_{45} + F^{A5}\hat{A}_5 \tag{3.130}$$

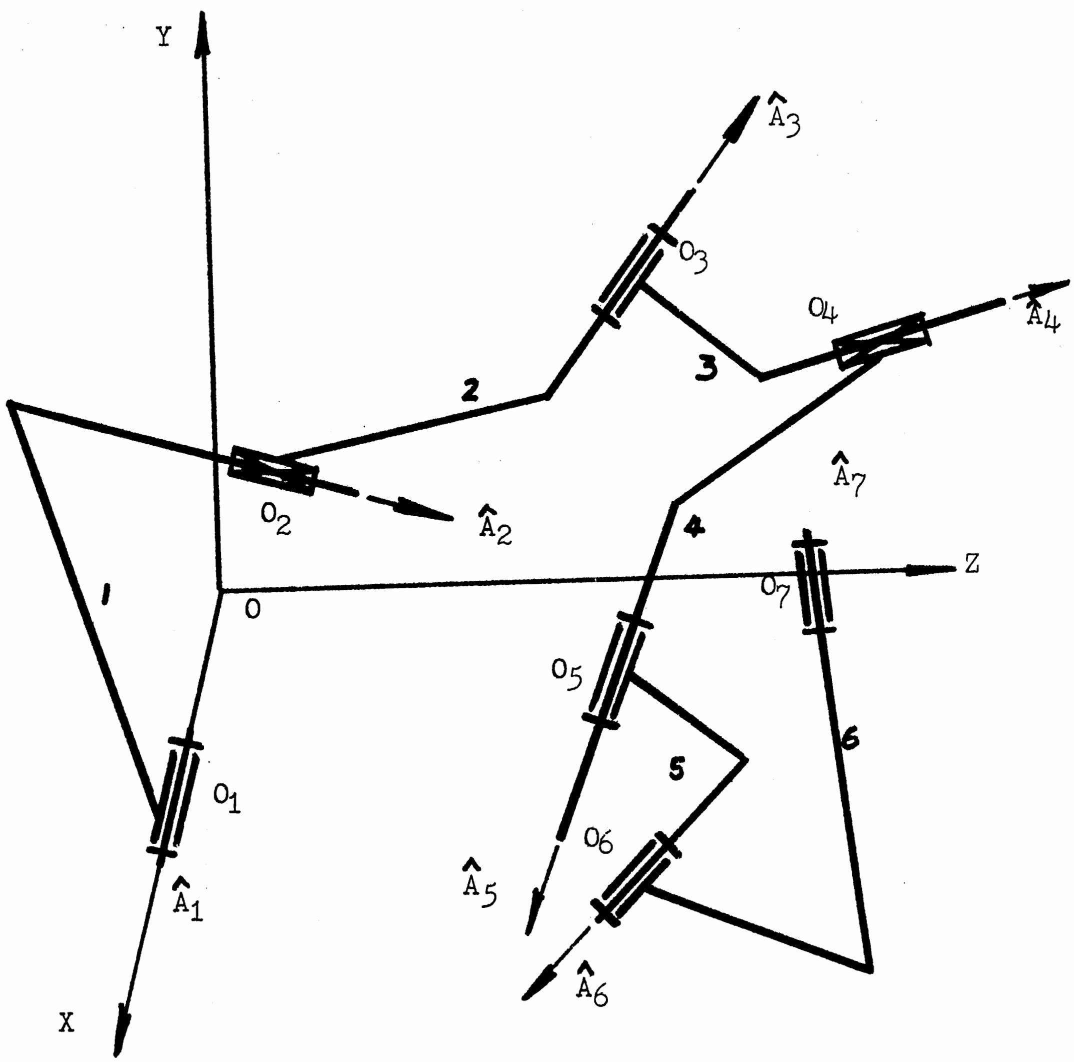

Figure 25. The RPRHRRR Spatial Mechanism

At revolute pair O_5, dual reaction force is

$$\hat{R}_{56} = \hat{F}_{56} + F^{A6}\hat{A}_6 \tag{3.131}$$

At revolute pair O_6, dual reaction force is

$$\hat{R}_{76} = \hat{F}_{76} + \hat{F}^{A7}A_7 \tag{3.132}$$

The equations of motion of link 1 through 6 are

$$\hat{R}_{71} + \hat{R}_{21} + \hat{F}_{I1} + \hat{F}^{IN} = 0 \tag{3.133}$$

$$\hat{R}_{12} + \hat{R}_{32} + \hat{F}_{I2} = 0 \tag{3.134}$$

$$\hat{R}_{23} + \hat{R}_{43} + \hat{F}_{I3} = 0 \tag{3.135}$$

$$\hat{R}_{34} + \hat{R}_{54} + \hat{F}_{I4} = 0 \tag{3.136}$$

$$\hat{R}_{45} + \hat{R}_{65} + \hat{F}_{I5} = 0 \tag{3.137}$$

and

$$\hat{R}_{56} + \hat{R}_{76} + \hat{F}_{I6} + \hat{F}^{out} = 0 \tag{3.138}$$

The equations of constraint at kinematic pairs are

$$\hat{F}_{71} \cdot \hat{A}_1 = 0 \tag{3.139}$$

$$\hat{F}_{12} \cdot \hat{A}_2 = 0 \tag{3.140}$$

$$\hat{F}_{23} \cdot \hat{A}_3 = 0 \tag{3.141}$$

$$\hat{F}_{34} \cdot \hat{A}_4 = 0 \tag{3.142}$$

$$\hat{F}_{45} \cdot \hat{A}_5 = 0 \tag{3.143}$$

$$\hat{F}_{56} \cdot \hat{A}_6 = 0 \tag{3.144}$$

and

$$\hat{F}_{76} \cdot \hat{A}_7 = 0 \tag{3.145}$$

Dynamic forces and torques analysis of the RPRHRRR spatial mechanism is a determinate system with fifty unknowns and fifty equations.

3.10. Dynamic Analysis of the RRRRRH Spatial Mechanism

The dimensions and dual vectors of the RRRRRH spatial mechanism are in Figure 26. The unknown forces and torques at pair joints are written as follows:

At revolute pair O_1, dual reaction force is

$$\hat{R}_{71} = \hat{F}_{71} + F^{A1}\hat{A}_1 \tag{3.146}$$

and

$$\hat{F}^{IN} = \varepsilon M^{IN} \hat{A}_1$$

At revolute O_2, dual reaction force is

$$\hat{R}_{12} = \hat{F}_{12} + F^{A2}\hat{A}_2 \tag{3.147}$$

At revolute pair O_3, dual reaction force is

$$\hat{R}_{23} = \hat{F}_{23} + F^{A3}\hat{A}_3 \tag{3.148}$$

At revolute pair O_4, dual reaction force is

$$\hat{R}_{34} = \hat{F}_{34} + F^{A4}\hat{A}_4 \tag{3.149}$$

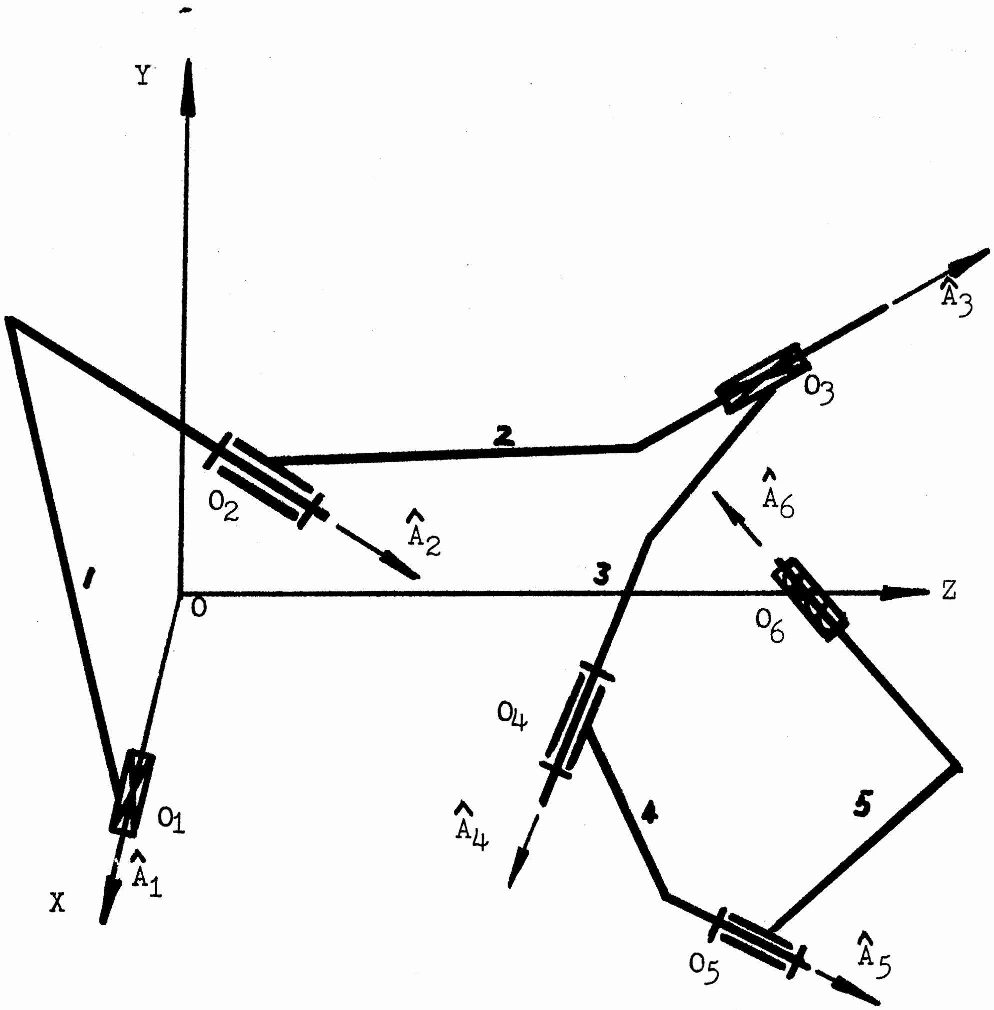

Figure 26. The PRPRRH Spatial Mechanism

At revolute pair O_5, dual reaction force is

$$\hat{R}_{45} = \hat{F}_{45} + F^{A5}\hat{A}_5 \tag{3.150}$$

At helical pair O_6, dual reaction force is

$$\hat{R}_{76} = \hat{F}_{76} + (F^{A7} + \varepsilon\, M^{A7})\, \hat{A}_7 \tag{3.151}$$

and

$$\rho F^{A7} + M^{A7} = 0$$

The equations of motion of link 1, 2, 3, 4, and 5 are

$$\hat{R}_{71} + \hat{R}_{21} + \hat{F}_{I1} + \hat{F}^{IN} = 0 \tag{3.152}$$

$$\hat{R}_{12} + \hat{R}_{32} + \hat{F}_{I2} = 0 \tag{3.153}$$

$$\hat{R}_{23} + \hat{R}_{43} + \hat{F}_{I3} = 0 \tag{3.154}$$

$$\hat{R}_{34} + \hat{R}_{54} + \hat{F}_{I4} = 0 \tag{3.155}$$

$$\hat{R}_{45} + \hat{R}_{65} + \hat{F}_{I5} = 0 \tag{3.156}$$

and

$$\hat{R}_{56} + \hat{R}_{76} + \hat{F}_{I6} + \hat{F}^{out} = 0 \tag{3.157}$$

The equations of constraint at kinematic pairs are

$$\hat{F}_{71} \cdot \hat{A}_1 = 0 \tag{3.158}$$

$$\hat{F}_{12} \cdot \hat{A}_2 = 0 \tag{3.159}$$

$$\hat{F}_{23} \cdot \hat{A}_3 = 0 \tag{3.160}$$

$$\hat{F}_{34} \cdot \hat{A}_4 = 0 \tag{3.161}$$

$$\hat{F}_{45} \cdot \hat{A}_5 = 0 \tag{3.162}$$

$$\hat{F}_{56} \cdot \hat{A}_6 = 0 \tag{3.163}$$

and

$$\hat{F}_{76} \cdot \hat{A}_7 = 0 \tag{3.164}$$

Dynamic force torques analysis of the RRRRRRH spatial mechanism is a determinate system with fifty unknowns and fifty equations.

3.11. Dynamic Analysis of the RPRRHC Spatial Mechanism

The dimensions and dual vectors of the RPRRHC spatial mechanism are defined in Figure 27. The unknown forces and torques at pair joints are written as follows:

At revolute pair O_1, dual reaction force is

$$\hat{R}_{61} = \hat{F}_{61} + F^{A1}\hat{A}_1 \tag{3.165}$$

and

$$\hat{F}^{IN} = \varepsilon\, M^{IN}\, \hat{A}_1$$

At prism pair O_2, dual reaction force is

$$\hat{R}_{12} = \hat{F}_{12} + F^{A2}\hat{A}_2 \tag{3.166}$$

At revolute pair O_3, dual reaction force is

$$\hat{R}_{23} = \hat{F}_{23} + F^{A3}\hat{A}_3 \tag{3.167}$$

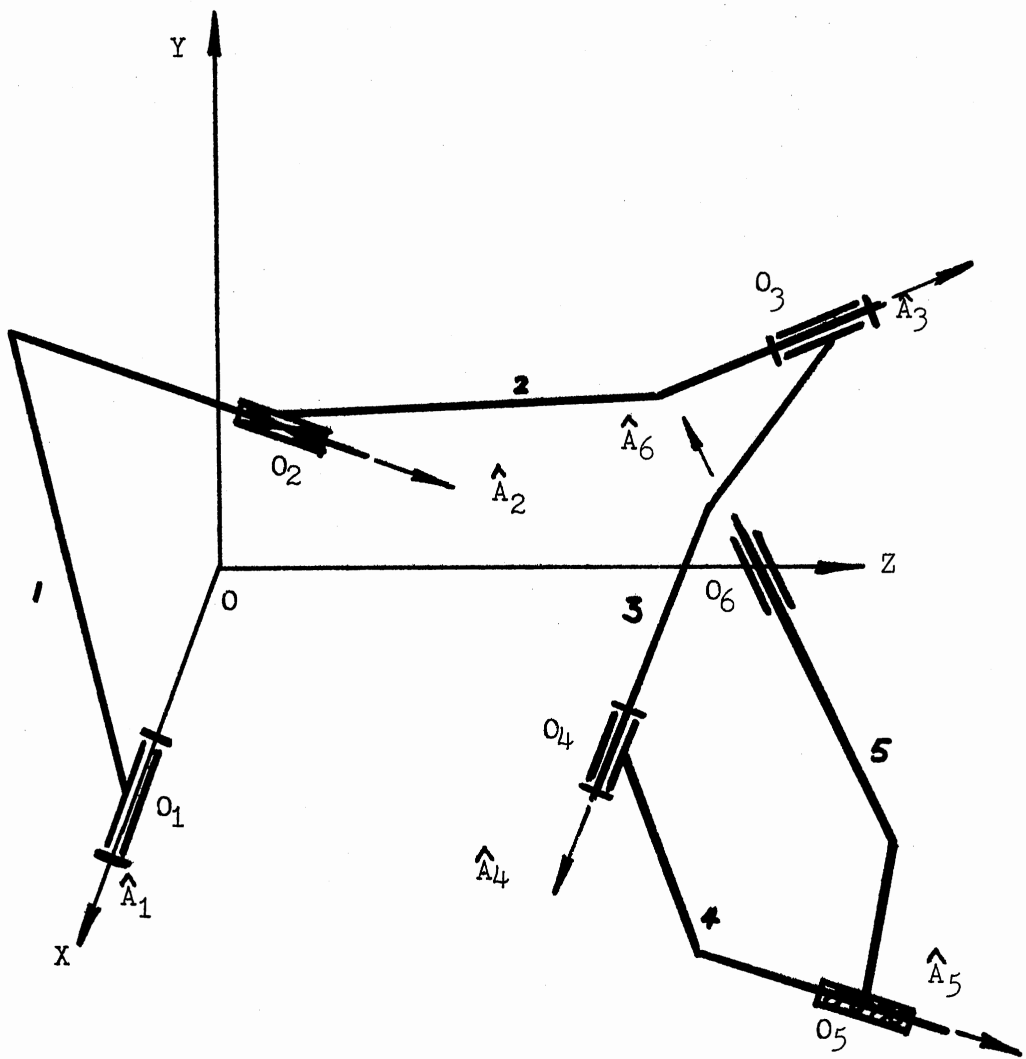

Figure 27. The RPRRHC Spatial Mechanism

At revolute pair O_4, dual reaction force is

$$\hat{F}_{34} = \hat{F}_{34} + F^{A4}\hat{A}_4 \qquad (3.168)$$

At helical pair O_5, dual reaction force is

$$\hat{R}_{45} = \hat{F}_{45} + (F^{A5} + \varepsilon\, M^{A5})\, \hat{A}_5 \qquad (3.169)$$

$$\rho F^{A5} + M^{A5} = 0$$

At cylinder pair O_6, dual reaction force is

$$\hat{R}_{65} = \hat{F}_{65} \qquad (3.170)$$

The equations of motion of link 1 through 5 are

$$\hat{R}_{61} + \hat{R}_{21} + \hat{F}_{I1} + \hat{F}^{IN} = 0 \qquad (3.171)$$

$$\hat{R}_{12} + \hat{R}_{32} + \hat{F}_{I2} = 0 \qquad (3.172)$$

$$\hat{R}_{23} + \hat{R}_{43} + \hat{F}_{I3} = 0 \qquad (3.173)$$

$$\hat{R}_{34} + \hat{R}_{54} + \hat{F}_{I4} = 0 \qquad (3.174)$$

and

$$\hat{R}_{45} + \hat{R}_{65} + \hat{F}_{I5} + \hat{F}^{out} = 0 \qquad (3.175)$$

The constraint equations at kinematic pairs are

$$\hat{F}_{61} \cdot \hat{A}_1 = 0 \qquad (3.176)$$

$$\hat{F}_{12} \cdot \hat{A}_2 = 0 \qquad (3.177)$$

$$\hat{F}_{23} \cdot \hat{A}_3 = 0 \qquad (3.178)$$

$$\hat{F}_{34} \cdot \hat{A}_4 = 0 \tag{3.179}$$

$$\hat{F}_{45} \cdot \hat{A}_5 = 0 \tag{3.180}$$

and

$$\hat{F}_{65} \cdot \hat{A}_6 = 0 \tag{3.181}$$

Dynamic forces and torques analysis of the RPRRHC spatial mechanism is a determinate system with forty-two unknowns and forty-two equations.

3.12. Dynamic Analysis of the PPHCHP Spatial Mechanism

The dimensions and dual vectors of the PPHCHP spatial mechanism are shown in Figure 28. The unknown forces and torques at pair joints are written as follows:

At prism pair O_1, dual reaction force is

$$\hat{R}_{61} = \hat{F}_{61} + \varepsilon M^{A1}\hat{A}_1 \tag{3.182}$$

and

$$\hat{F}^{IN} = F^{IN} \hat{A}_1$$

At prism pair O_2, dual reaction force is

$$\hat{R}_{12} = \hat{F}_{12} + M^{A2}\hat{A}_2 \tag{3.183}$$

At helical pair O_3, dual reaction force is

$$\hat{R}_{23} = \hat{F}_{23} + (F^{A3} + \varepsilon M^{A3}) \hat{A}_3 \tag{3.184}$$

$$\rho_3 F^{A3} + M^{A3} = 0$$

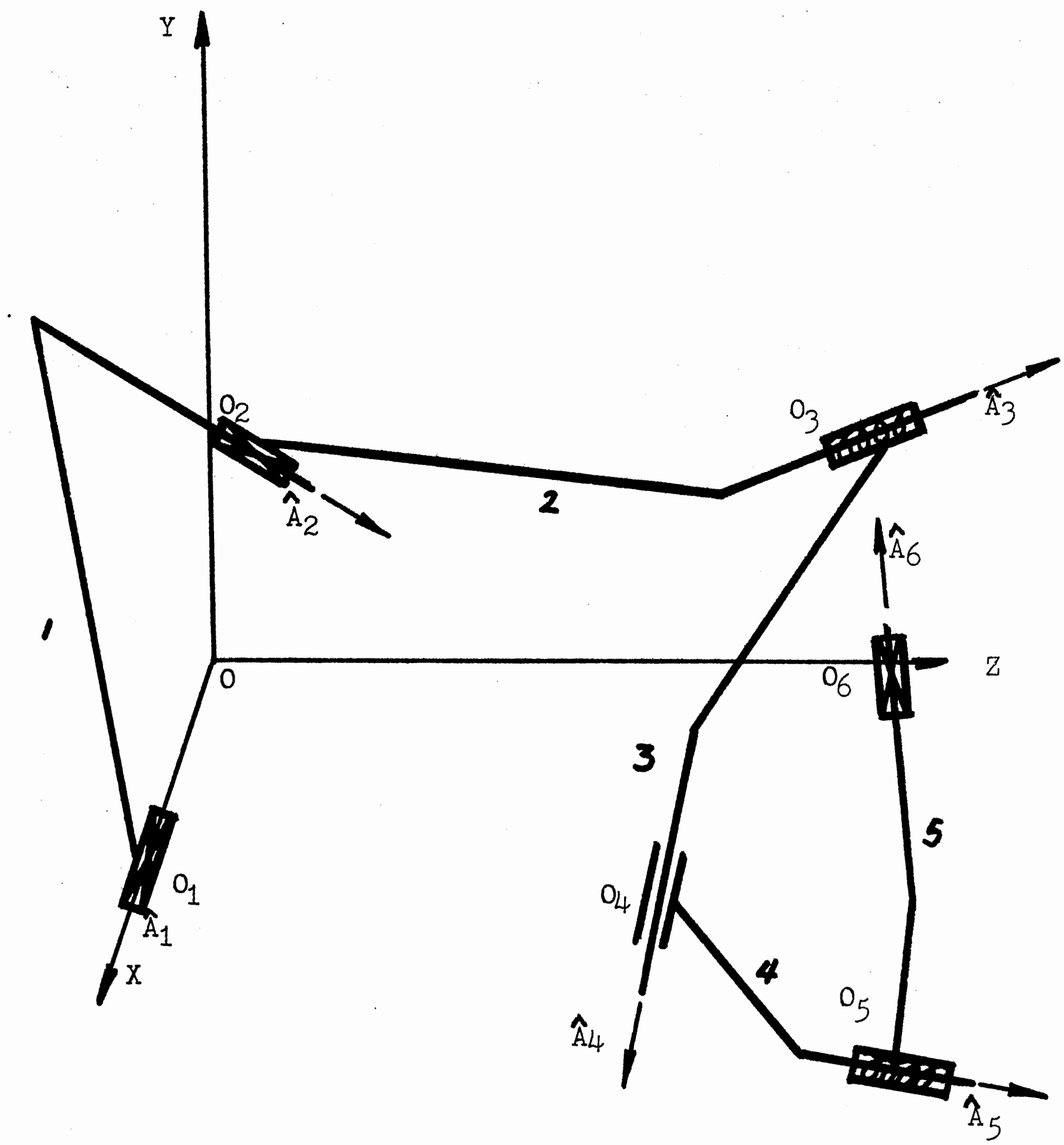

Figure 28. The PPHCHP Spatial Mechanism

At cylinder pair O_4, dual reaction force is

$$\hat{R}_{34} = \hat{F}_{34} \tag{3.185}$$

At helical pair O_5, dual reaction force is

$$\hat{R}_{45} = \hat{F}_{45} + (F^{5} + \varepsilon M^{5}) \hat{A}_5 \tag{3.186}$$

$$\rho_5 F^{A5} + M^{A5} = 0$$

At prism pair O_6, dual reaction force is

$$\hat{R}_{65} = \hat{F}_{65} + M^{A6}\hat{A}_6 \tag{3.187}$$

The equations of motion of link 1 through link 5

$$\hat{R}_{61} + \hat{R}_{21} + \hat{F}_{1} + \hat{F}^{IN} = 0 \tag{3.188}$$

$$\hat{R}_{12} + \hat{R}_{32} + \hat{F}_{I2} = 0 \tag{3.189}$$

$$\hat{R}_{23} + \hat{R}_{43} + \hat{F}_{I3} = 0 \tag{3.190}$$

$$\hat{R}_{34} + \hat{R}_{54} + \hat{F}_{I4} = 0 \tag{3.191}$$

and

$$\hat{R}_{45} + \hat{R}_{65} + \hat{F}_{I5} + \hat{F}^{out} = 0 \tag{3.192}$$

The equations of constraint at kinematic pair are

$$\hat{F}_{61} \cdot \hat{A}_1 = 0 \tag{3.193}$$

$$\hat{F}_{12} \cdot \hat{A}_2 = 0 \tag{3.194}$$

$$\hat{F}_{23} \cdot \hat{A}_3 = 0 \tag{3.195}$$

$$\hat{F}_{34} \cdot \hat{A}_4 = 0 \tag{3.196}$$

$$\hat{F}_{45} \cdot \hat{A}_5 = 0 \tag{3.197}$$

and

$$\hat{F}_{65} \cdot \hat{A}_6 = 0 \tag{3.198}$$

Dynamic forces and torques analysis of PPHCHP spatial mechanism is a determinate system with forty-two unknowns and forty-two equations.

3.13. Dynamic Analysis of the RCRCR Spatial Mechanism

The dimensions and dual vectors of the RCRCR spatial mechanism is shown in Figure 29. The unknown forces and torques at pair joints are written as follows:

At revolute pair O_1, dual reaction force is

$$\hat{R}_{51} = \hat{F}_{51} + \varepsilon\, M^{A1}\hat{A}_1 \tag{3.199}$$

and

$$F^{IN} = \varepsilon\, M^{A1}\hat{A}_1$$

At cylinder pair O_2, dual reaction force is

$$\hat{R}_{12} = \hat{F}_{12} \tag{3.200}$$

At revolute pair O_3, dual reaction force is

$$\hat{R}_{23} = \hat{F}_{23} + \varepsilon\, M^{A3}\hat{A}_3 \tag{3.201}$$

At cylinder pair O_4, dual reaction force is

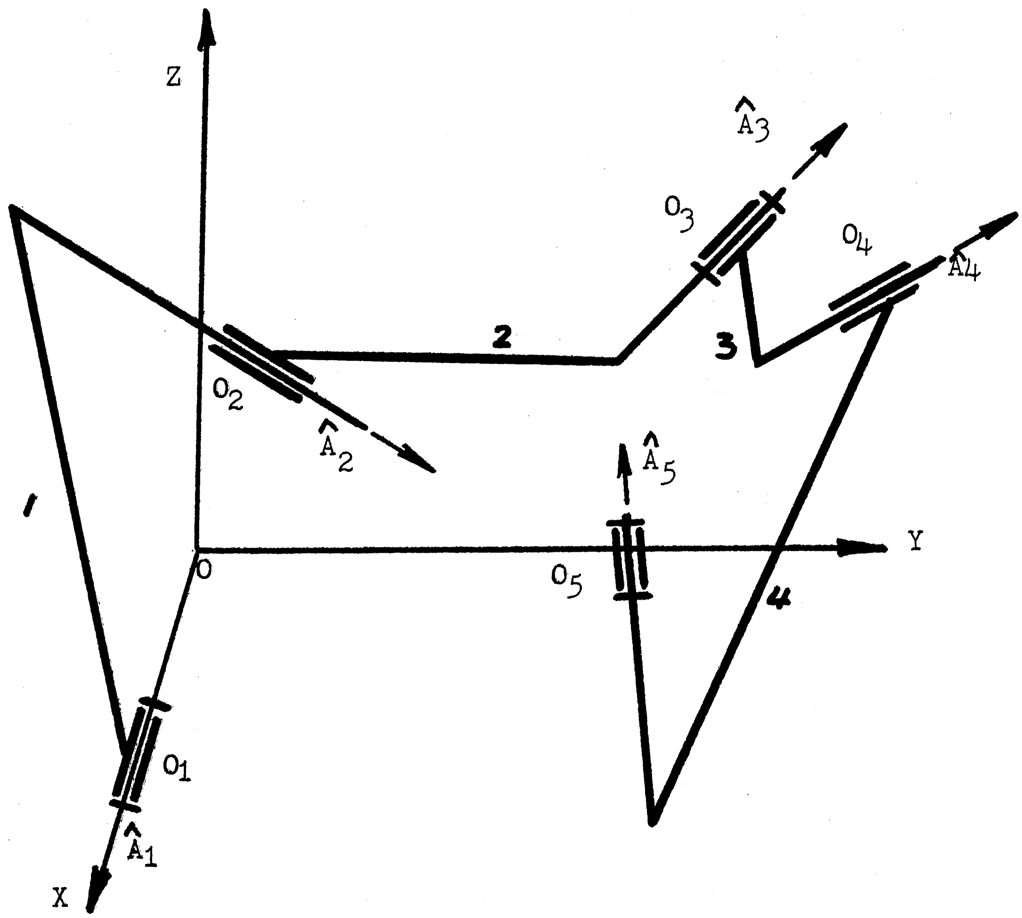

Figure 29. The RCRCR Spatial Mechanism

$$\hat{R}_{34} = \hat{F}_{34} \quad (3.202)$$

At revolute pair O_5, dual reaction force is

$$\hat{R}_{54} = \hat{F}_{54} + \varepsilon\ M^{A5}\hat{A}_5 \quad (3.203)$$

The equations of motion of link 1 through 4 are

$$\hat{R}_{51} + \hat{R}_{21} + \hat{F}_{I1} + \hat{F}^{IN} = 0 \quad (3.204)$$

$$\hat{R}_{12} + \hat{R}_{32} + \hat{F}_{I2} = 0 \quad (3.205)$$

$$\hat{R}_{23} + \hat{R}_{43} + \hat{F}_{I3} = 0 \quad (3.206)$$

and

$$\hat{R}_{34} + \hat{R}_{54} + \hat{F}_{I4} + \hat{F}^{out} = 0 \quad (3.207)$$

The constraint equations of kinematic pair are

$$\hat{F}_{51} \cdot \hat{A}_1 = 0 \quad (3.208)$$

$$\hat{F}_{12} \cdot \hat{A}_2 = 0 \quad (3.209)$$

$$\hat{F}_{23} \cdot \hat{A}_3 = 0 \quad (3.210)$$

$$\hat{F}_{34} \cdot \hat{A}_4 = 0 \quad (3.211)$$

and

$$\hat{F}_{34} \cdot \hat{A}_4 = 0 \quad (3.212)$$

Dynamic forces and torques analysis of the RCRCR spatial mechanism is a determinate system with thirty-four unknowns and thirty-four equations.

3.14. Dynamic Analysis of the PHCHC Spatial Mechanism

The dimensions and dual vectors of the PHCHC spatial mechanism is shown in Figure 30. The unknown forces and torques at pair joints are written as follows:

At prism pair O_1, dual reaction force is

$$\hat{R}_{51} = \hat{F}_{51} + F^{A1}\hat{A}_1 \qquad (3.213)$$

At helical pair O_2, dual reaction force is

$$\hat{R}_{12} = \hat{F}_{12} + (F^{A2} + \varepsilon\ M^{A2})\ \hat{A}_2 \qquad (3.214)$$

and

$$\rho_2 F^{A2} + M^{A2} = 0$$

At cylinder pair O_3, dual reaction force is

$$\hat{R}_{23} = \hat{F}_{23} \qquad (3.215)$$

At helical pair O_4, dual reaction force is

$$\hat{R}_{34} = \hat{F}_{34} + (F^{A4} + \varepsilon\ M^{A4})\ \hat{A}_4 \qquad (3.216)$$

and

$$\rho_4 F^{A4} + M^{A4} = 0$$

At cylinder pair O_5, dual reaction force is

$$\hat{R}_{54} = \hat{F}_{54} \qquad (3.217)$$

The equations of motion of link 1 through 4 are

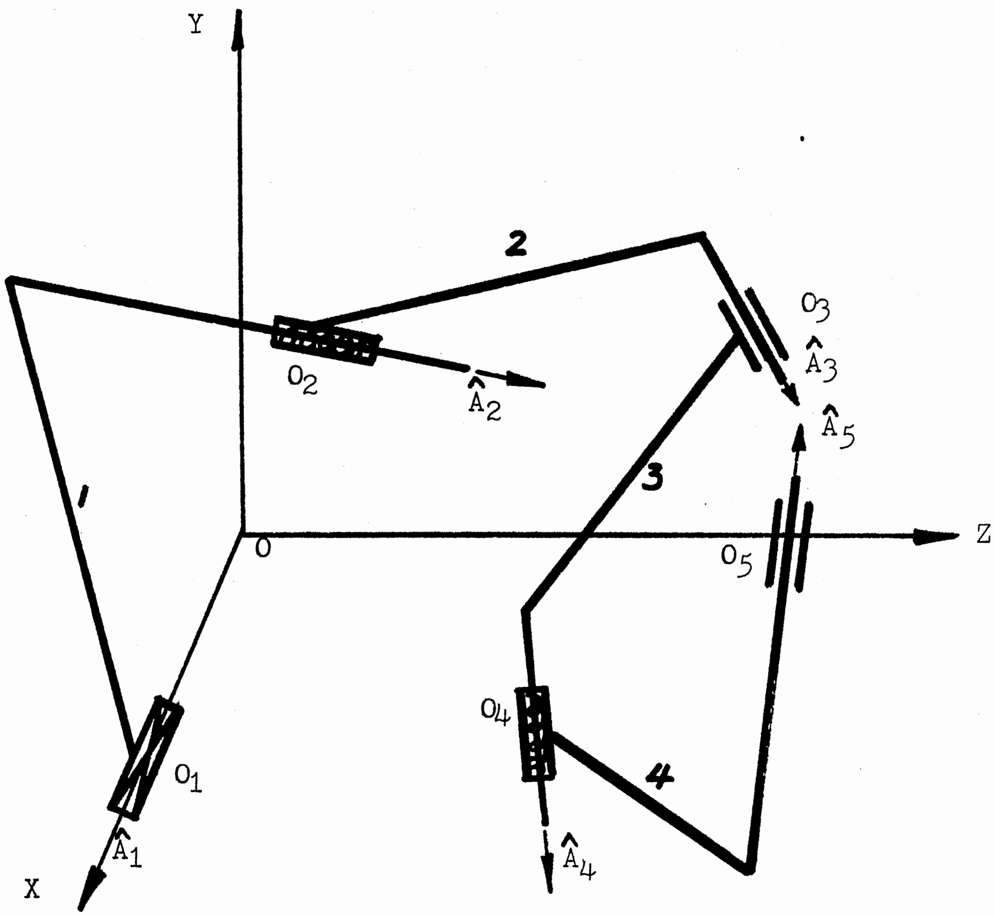

Figure 30. The PHCHC Spatial Mechanism

$$\hat{R}_{51} + \hat{R}_{21} + \hat{F}_{I1} + \hat{F}^{IN} = 0 \tag{3.218}$$

$$\hat{R}_{12} + \hat{R}_{32} + \hat{F}_{I2} = 0 \tag{3.219}$$

$$\hat{R}_{23} + \hat{R}_{43} + \hat{F}_{I3} = 0 \tag{3.220}$$

and

$$\hat{R}_{34} + \hat{R}_{54} + \hat{F}_{I4} = 0 \tag{3.221}$$

The equations of constraint at kinematic pairs are

$$\hat{F}_{51} \cdot \hat{A}_1 = 0 \tag{3.222}$$

$$\hat{F}_{12} \cdot \hat{A}_2 = 0 \tag{3.223}$$

$$\hat{F}_{23} \cdot \hat{A}_3 = 0 \tag{3.224}$$

$$\hat{F}_{34} \cdot \hat{A}_4 = 0 \tag{3.225}$$

and

$$\hat{F}_{54} \cdot \hat{A}_5 = 0 \tag{3.226}$$

Dynamic forces and torques analysis of the PHCHC spatial mechanism is a determinate system with thirty-four unknowns and thirty-four equations.

CHAPTER IV

SUMMARY AND CONCLUSIONS

In this dissertation, an intensive literature search on the dynamics of mechanisms is done. It discusses the existing approaches with their advantages and drawbacks and leads to current study. A successive screw method has been developed for the kinematic analysis of spatial mechanisms. The simplicity of this method appears in its expressions of the geometrical location of the rigid body. The Rodrigues' formulae to express the position and the orientation of the rigid body are in vector forms. The expressions obtained by using a 3×3 dual matrix method are in real and dual vector forms, where the real part represents the orientation of the rigid body and the dual part represents the moment arm of the rigid body. Because the expression to describe the position of the rigid body is not in an explicit form itself, this contributes to its complexity.

In deriving the displacement equation of the input and output relationship, one is trying to eliminate unwanted variables from the loop-closure equation. This becomes a problem when unwanted variables are more than one. It is believed that the process of mathematical elimination could introduce extraneous roots into displacement equations. In recent years, there has been a growing interest in the determination of closed-form displacement equations of spatial mechanisms. The author (87) has proved that a unified theory failed to predict the correct order of displacement equations. The method of successive screws has never been extended to the kinematic analysis of spatial mechanisms in which loop-closure equations have more than one unwanted variable. The author has shown procedures and criteria in dealing with problems having two unwanted variables by an example of RCHCH spatial mechanism. Because of the simplicity of the method of successive screws, extraneous roots can be prevented from entering into the displacement equation. Kinematic analyses of the RCSR, RCSR-CSR and RCHCH spatial mechanisms are used as examples to illustrate this method. Procedures to unfold the mechanism are also discussed in order to reduce the higher order displacement equation.

The procedure to locate a point on the rigid body is given. The expressions obtained for the velocity analysis are in linear in input velocity while for the acceleration analysis are in linear in input acceleration. Numerical examples illustrating the kinematic analysis of RCSR, RCSR-CSR and RCHCH spatial mechanisms are given.

In the past the vector method, the 3×3 dual matrix method, the 4×4 matrix method and the screw coordinates method have been applied to perform the dynamic analysis of mechanisms. Each one of the methods has its significance. The method of successive screws developed in this thesis is extended to conduct dynamic analysis of spatial mechanisms for the first time. Kinetostatic and dynamic response analysis are the two general problems in the dynamic analysis of mechanisms. Using the kinematic pair constraints along with successive screws method in formulation of the equation proves to be an efficient approach, because of the fact that the kinematic pair constraint can be physically visualized. Formulation of the equation of motion are based on the d'Alembert principle.

The method can be employed to conduct the dynamic analysis of spatial mechanisms having binary or ternary links with kinematic pairs such as: revolute, helical, prism, cylinderical and spherical pairs. Equations of motion are general and good for any number of links and loops in which mechanisms constitute a force and torque determinate system.

In the dynamic response analysis of spatial mechanism, all the existing methods treat the equations of motion as a set of differential equations and apply numerical integrating techniques to find the responses. However, for a constrained mechanical system, other accelerations can be written as linear functions of the input acceleration. Therefore, one treats the input acceleration as well as reaction forces and torques as unknown quantities. This procedure enables us to save a tremendous amount of time in computing.

Kinetostatic analyses are illustrated by numerical examples of the single loop 4-link RCSR, 5-link RCHCH and two-loop six-link RCSR-CSR mechanisms. Another RCSR spatial mechanism is used to demonstrate the dynamic response analysis. Equations of motion and kinematic pair constraints are also discussed by presenting examples of 7R, RRRRRH, RRRRHC, PPHCHP, RCRCR, PHCHC spatial mechanisms.

There are points for further study that become apparent at this stage:

1. The methodology developed in this dissertation can be mechanized by using the graph theory to identify variables, links and pairs, and automatically generate equations of motion and constraints for the machine system. The practical

value of such programs are great for the designers in industries.

2. The hydrodynamic pressure distributions in the revolute, prism, helical, cylinder, and spherical pairs should be investigated before direct inclusion of the frictional forces into the equations of motion. The author believes that it should be carried out in another independent study with emphasis on fluid dynamics.
3. The solution of the equations of motion is obtained using the Gaussian elimination method. The algorithms used in the elimination method are good for full, nonsymmetric matrices. An algorithm for solving nonfull matrices can be developed which will save tremendous computing time.
4. An appealing study would be to formulate equations of motion including impact and friction.

It is expected that the present study will provide the basis for these future research opportunities.

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