THE STUDY AND ESTABLISHMENT OF AN ELECTRICAL ANALOGY FOR AN HYDRAULIC PIPE LINE

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H. H. M.

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INTRODUCTION

The Electrical Network Analyzer

Most electrical engineers are familiar with the use of network analyzers in the study of power distribution systems. The calculating board is now regarded as an indispensible instrument for determining the transient currents and voltages resulting from the interconnection of several generating stations, lines, and loads. Difficult problems can be solved in a very short period of time with improved accuracy and at a reduced cost to the utility companies. Transient and steady state currents and voltages can be predetermined for all types of fault conditions after the calculating board has been set up for a particular system. This is not the only type of problem that may be solved by the network analyzer; it is a relative easy operation to evaluate the effect of any additions to the distribution system together with the effect of any changes in loading or input to the existing network.

In addition to the savings in time and money for the initial installations, there results savings in maintenance and loss of equipment due to the absence of sufficient protection for the various electrical units. These economies by themselves are adequate justification for the employment of a network analyzer. Problems solved by using the calculating board may be worked out mathematically by properly trained engineers if a sufficient amount of time is allowed.

The opportunities for error in many cases are numerous, and consequently all calculations made independently of a calculating board must be checked and rechecked.

Civil engineers have an equally difficult problem in the solution of problems involving complex hydraulic pipe line networks. Just as the network analyzer has resulted in a tremendous acceleration of the studies involving networks of power systems, so would an analyzer for hydraulic distribution systems tend to speed up the solution of many complex problems involved in the flow of water, oil, and other fluids through complex networks of pipe lines.

The Problem

A calculating board for hydraulic systems that would permit the determination of the hydraulic pressures at any point in the system has been sought for many years with very little success. It is the purpose of this research project to establish an electrical network that is an analogy of hydraulic pipe line flow and pressures. However such a development, if successful, is by no means a complete solution to the measurement of flows and pressures of a complex pipe line network. In this study no attempt is made to set up an electrical analogy for a multi-pipe system, but only to establish a few definite relationships between certain hydraulic problems and an electrical network. It is believed that these relationships may be of considerable assistance in any further investigation of this field.

Necessity for This Study

From among the many problems that arise in the study of hydraulic systems, there are at least three that assume a role of major importance. The design of a calculation board for hydraulic systems must certainly include devices which will permit the evaluation of the various parameters dictated by the requirements established in these problems.

In the following list are three reasons for the need of such an electric analyzer for hydraulic problems.

(1) Calculations of the pressure changes when an addition is made to an existing complex hydraulic system are now impossible unless numerous long and tedious calculations are made. Such problems should be solved with an analyzer in a few minutes.

(2) The phenomenon of the water hammer has been a major problem and has caused enormous losses of equipment. By extending the research started in this project it may be possible to predetermine the pressure developed by the sudden closing of a valve. If the changes in pressures are known, adequate protection may be instituted to guard against them.

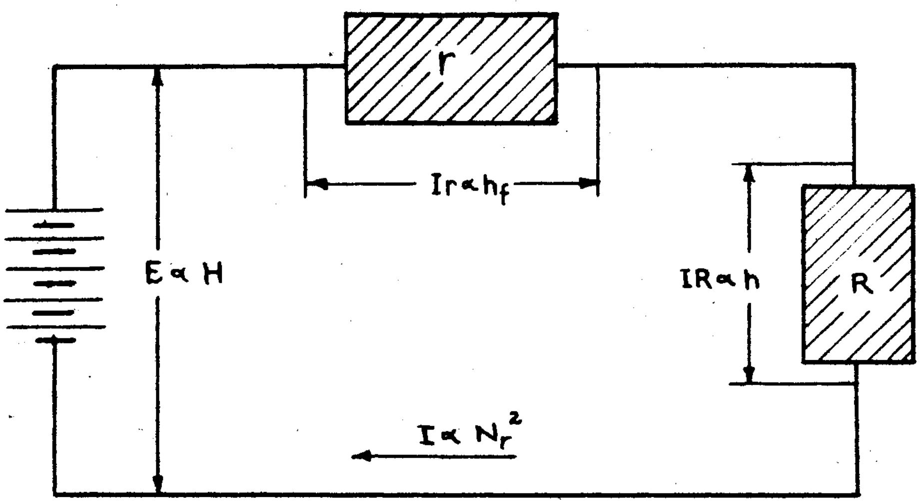
(3) The results of a change in pressure due to the change of elevation of the input head should be readily determined by an adequate calculating board. This involves a simple extension of the established electrical analogy before applying it to a multi-pipe hydraulic system.

Delimitations

The electrical network which is to be the analyzer must be developed so that the voltages in the electrical network will represent either the pressure head at various points in the pipe line or the head loss through the pipe line. To know which voltage drop corresponds to the pressure head and which voltage drop corresponds to the head loss will depend entirely upon which resistor in the circuit the voltage is measured across. Total pressure and elevation head at the pipe input will be represented by the applied voltage of the electrical circuit because the total head of the pipe line system is the sum of the output pressure head and the head loss in the pipe. Direct voltages will be used in the analyzer because the normal flow of the fluid in the pipe lines is a steady unidirectional movement. A simplified block diagram of the analyzer circuit is included in Figure 1.

Rather than to attempt the solution of a complex pipe line network, this project has been confined to the simplest type of mesh. A single straight pipe with a constant diameter and a constant head at the input end presents the type of problem that will be considered. Experimental results with hydraulic systems have shown that two types of flow may exist in a pipe line. These are designated as laminar flow and turbulent flow. The laminar flow is a smooth type of flow that exists if the velocity of the fluid is very low. The turbulent flow is a rough movement of the fluid in the pipe which takes place whenever fluid velocity is normal or higher than normal. Turbulent flow is discussed more fully in the next chapter. In the establishment of the analogy, laminar flow will be given consideration where the roughness of the pipe has no effect. The turbulent region of flow will be analyzed where the various degrees of pipe roughness are important. Laminar flow has very little practical importance, and consequently is not considered in great detail. Fortunately it developes that basic principles involved in the construction and operation of the analyzer are the same for both types of flow.

BLOCK DIAGRAM
OF ELECTRICAL ANALOGY



E – Appied Voltage (volts)
H – Hydraulic head at input to pipe (ft.)
I – Current (amperes)
N_r – Reynolds number
h_f – Head loss in pipe line (ft.)
h – Hydraulic head at pipe outlet (ft.)
r – Resistance (voltage across r represents h_f)
R – Resistance (voltage across R represents h)

FIGURE 1

CHAPTER II

THEORY OF RELATIONSHIPS BETWEEN AN ELECTRICAL NETWORK AND AN HYDRAULIC SYSTEM

The Hydraulic System

The type of flow existing in a pipe line depends upon the value of the Reynolds number which is defined as DF/W, where D is the diameter of the pipe, F is the average velocity of the fluid, and W is the kinematic viscosity. Laminar, or smooth flow, occurs if the Reynolds number is below 2,200 and turbulent flow exists if N_r, the Reynolds number, is above this value.

The loss of head due to friction depends upon the Reynolds number N_r, length of pipe L, Diameter of pipe D, gravitational constant g, and coefficient of friction f, that is,

$$h_f = f \frac{L}{D} \frac{F^2}{2g} = f \frac{LW^2}{2D^3 g} N_r^2 \qquad (1)$$

The coefficient of friction is not a fixed quantity, but varies with N_r as shown in Figure 2[1].

For a fixed value of N_r in the turbulent flow region, f depends on the degree of roughness in the pipe as designated by the r/K values. For laminar flow the curve may be expressed in the form of an equation,

$$f = \frac{64}{N_r} \propto \frac{64}{I} \qquad (2)$$

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1. Hunter Rouse, Elementary Mechanics of Turbulent Flow (New York, New York: John Wiley and Sons, Inc., 1946), p.376.

This relationship is the same for all values of r/K.

Establishing the Analogy

The condition of laminar flow will be considered first. A possible solution can be obtained by setting the current in the electrical network to values that are proportional to the Reynolds number for the pipe line. It remains to be seen what the values of the resistances r and R must be in order that the voltage drop across them will be proportional to h_f and h, respectively, of the hydraulic system. Combining the relationship

$$I \propto N_r \tag{3}$$

and equation (1) an equation relating h_f to I may be obtained,

$$h_f \propto f \frac{LW^2}{2D^3 g} I^2 \tag{4}$$

Substituting equation (2) into (4) gives

$$h_f \propto \frac{32LW^2}{D^3 g} I = Ir \tag{5}$$

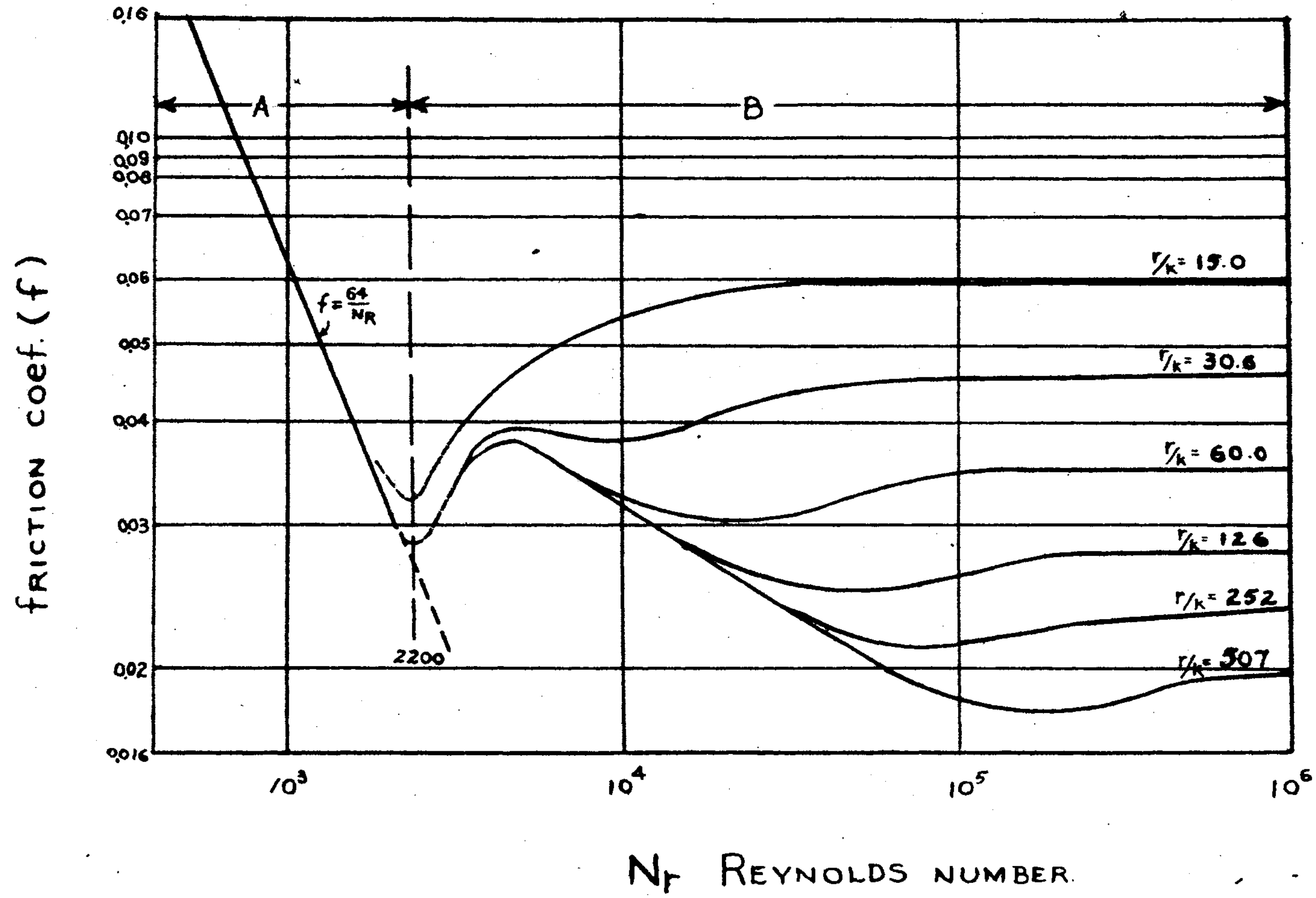
By making r

$$r \propto \frac{32LW^2}{D^3 g} \tag{5a}$$

the head loss will always be proportional to the voltage drop across the resistor r. By examining (5a) it can be seen that the resistance r will not change unless the physical dimensions of the pipe line are altered or the viscosity of the fluid being transmitted is changed, therefore, r will be a fixed resistor which will not vary as the fluid velocity changes.

To evaluate R the relationship between h and F

$$h = \frac{F^2}{2g} \tag{6}$$



A - Laminar Flow.
B - Turbulent Flow.

Figure 2

and the definition of N_r may be combined to give

$$h = \frac{F^2}{2g} = \frac{N_r^{\,2} W^2}{2D^2 g} \tag{6a}$$

and from equation (3) and (6a) it follows that

$$h \propto \frac{W^2 I^2}{2D^2 g} \tag{7}$$

This shows that the voltage across R must be made to vary as the current squared. The only way this can be accomplished is for the resistance R to change as I changes, i.e.;

$$R \propto \frac{W^2 I}{2D^2 g} \tag{8}$$

This solution gives one fixed resistance r and one resistance R that must vary as the first power of I.

Another possible attack on the problem is to set

$$I \propto N_r^{\,2} \tag{9}$$

By substituting (9) into equation (1)

$$h_f \propto f \frac{LW^2}{2D^3} I \tag{10}$$

For this approach (9) can be combined with the relationship between f and N_r and there results

$$f = \frac{64}{N_r} \propto \frac{64}{\sqrt{I}} \tag{11}$$

By substituting (11) into (10) it is permissable to write

$$h_f \propto \frac{64W^2}{2D^3 g} \sqrt{I} = \frac{32W^2}{D^3 g\sqrt{I}}\, I \tag{12}$$

Upon examining equation (12) it becomes obvious that it is necessary to make the resistance

$$r \propto \frac{32W^2}{D^3 g\sqrt{I}} \tag{13}$$

Hence the resistance r must vary inversely as the square root of the current.

The resistance R may be evaluated from (6) and (9) giving,

$$h \propto \frac{W^2}{2D^2 g} I \qquad (14)$$

and,

$$R \propto \frac{W^2}{2D^2 g} \qquad (15)$$

Consequently the voltage drop across R will be proportional to the outlet head h; therefore

$$h \propto IR \qquad (15a)$$

This solution, in common with the previous solution, leads to the necessity of one fixed and one variable resistance for the basic circuit. From the linearity standpoint the parameters r and R are interchanged in these two solutions. This is apparent if reference is made to equations (5a), (8), (13), and (15). It is evident that both methods are equally satisfactory for the laminar flow region. It can be shown, however, that they are not equally satisfactory for turbulent flow. This will now be illustrated.

Using the first approach, that is, making the current I proportional to the Reynolds number, it was found from equation (4) that

$$h_f \propto f \frac{LW^2}{2D^3 g} I^2 \qquad (4)$$

For the turbulent region f cannot be expressed in terms of

I or N_r as was the case for the laminar region. This makes it necessary to let

$$r \propto f \frac{LW^2}{2D^3g} I \qquad (16)$$

It is obvious that r is not a fixed quanity for all values of I. The outlet head and the resistance R across which the resistance drop is measured is the same for turbulent as for laminar flow, hence

$$R \propto \frac{W^2 I}{2D^2 g} \qquad (8)$$

Therefore both resistances R and r must vary with the current if I is to be proportional to N_r.

For the case of I being proportional to N_r^2, the head loss and outlet head are given by equations (10) and (14) respectively. Consequently it follows that the value of the two resistances must be

$$r \propto f \frac{LW^2}{2D^3g} \qquad (17)$$

and

$$R \propto \frac{W^2}{2D^2g} \qquad (15)$$

Values for r vary as I changes due to the fact that f is a variable. The resistance R is constant as it was in the case for the laminar region. For this reason the network will be developed on the bases of I being proportional to N_r^2, and as a result there will be only one non-linear resistance in both the turbulent and laminar regions of flow.

The Camp and Hazen Analyzer

One of the first network analyzers was developed by Camp and Hazen[1], who used resistances which were manually variable. These resistances were adjusted to the proper value by a successive approximation method. Usually three trial adjustments were necessary before the proper pressure readings were obtained. While their method was reasonably accurate, it was not applicable to all types of hydraulic problems. The operator had to be an expert to obtain results in any reasonable length of time. A detailed explanation of the operation of this analyzer may be found in the New England Water Works Association Journal[2].

Non-Linear Resistances

In order to make the analyzer satisfactory for any type of problem a resistance which is non-linear with respect to the current is required for the parameter r. Referring to equation (18) and Figure 2, it may be seen that several different resistance characteristics are necessary if any consideration is to be given to the roughness factor, r/K, of the pipe. Both the triode and the pentode type vacuum tubes have the above mentioned characteristics, but they do not vary in the proper manner. Numerous attempts were made to correlate the plate

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1. T.R. Camp and H.L. Hazen, "Hydraulic Analysis of Water Distribution Systems as an Electric Analyzer." New England Water Works Association Journal, Vol. 48 (December 1934), pp. 383-407.

2. Ibid.

current characteristics of this type tube with curves of the Reynolds number squared versus head loss. In these attempts the grid bias was taken as a constant for a specific value of r/K, and was varied proportionally to r/K to simulate the changes in head loss resulting from the various degrees of pipe roughness.

Since the vacuum tube has many different values of resistance depending upon the operating point selected, it is possible to simulate the Reynolds number squared versus head loss curves providing the control grid bias is varied as the current in the tube varies. The above method is limited by the maximum allowable plate current of the tube used. This difficulty is partially overcome by changing the proportionality constants used in the analogy. This point will be treated in detail later.

A means for obtaining the control grid voltage to result in the proper plate current to plate voltage relationship for the vacuum tube representing the resistance r must be employed. The grid bias voltage used for this vacuum tube must be predetermined and is accomplished by a method to be described later in this chapter. A photoformer circuit developed by D.E. Sustain[1] together with several auxiliary circuits will be used for automatically controlling the grid bias voltage in the prescribed manner as the tube current varies.

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1. D. E. Sustain, "Photoelectric Waveform Generator." Electronics, Vol. 22 (February, 1949), pp. 120-121.

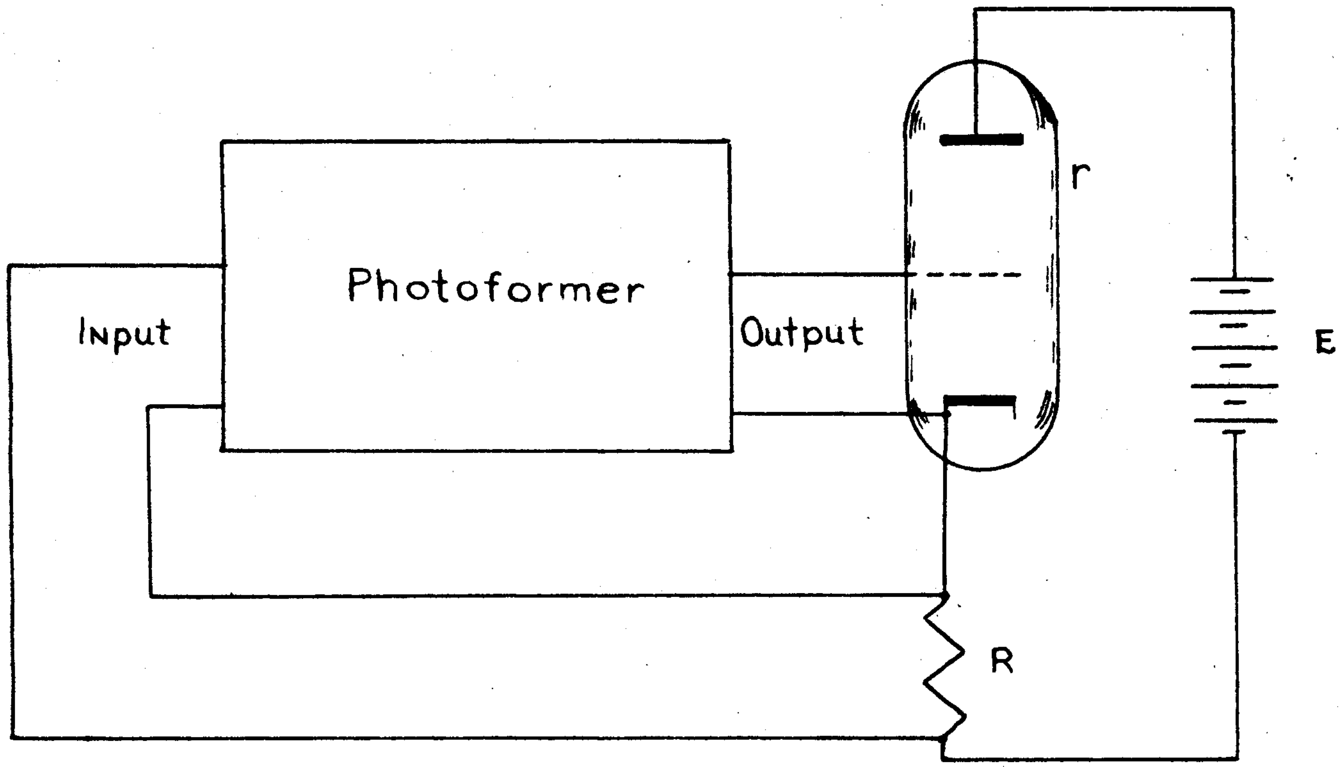


fig. 3

Block diagram of Electrical Analyzer

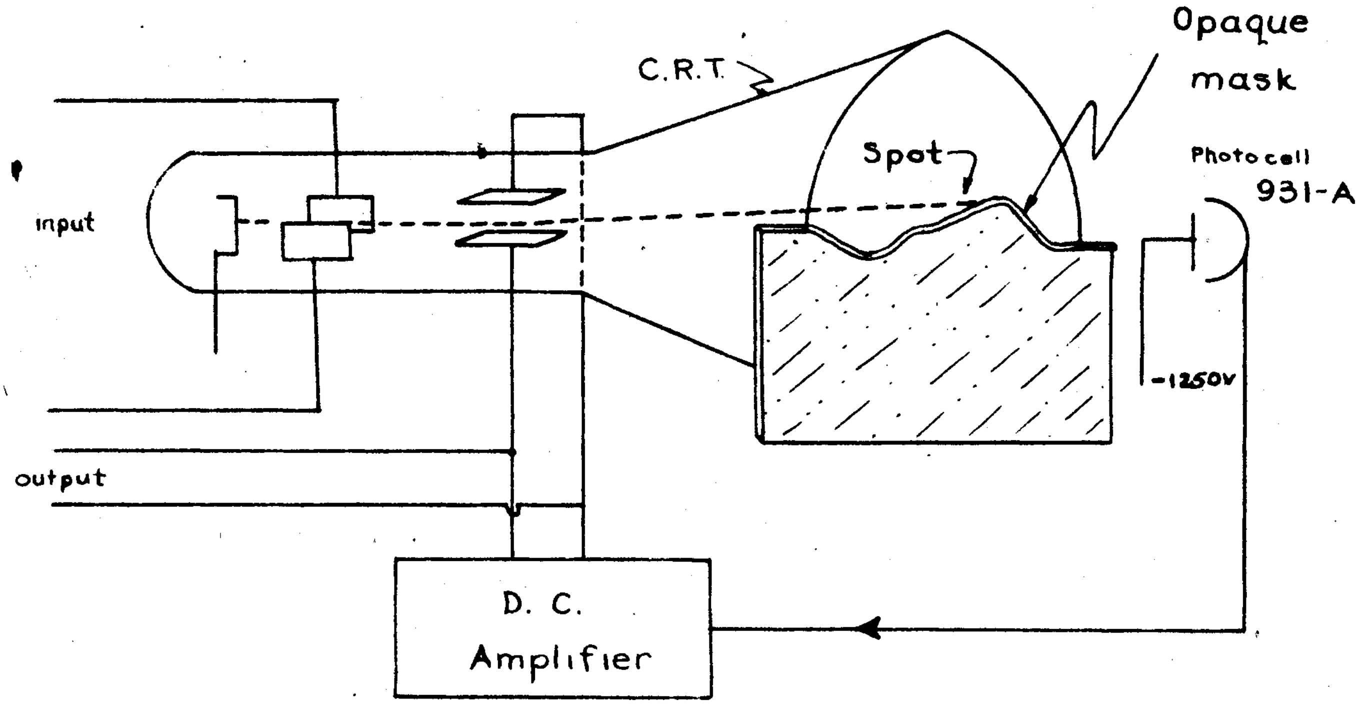


fig. 3a.

Simplified Photoformer Circuit

The Photoformer Circuit

Since the photoformer is an important part of the electric analyzer, it will be discussed briefly before proceeding further with the analogy. There are three major components to the photoformer, they are a cathode ray tube, a 931-A photo-electric tube, and a direct current amplifier. An opaque paper shield is placed over the face of the cathode ray tube, and the output voltage is caused to vary in such a manner that it conforms to the contour of the paper shield. This shield is designed to give the desired values of control grid on the analyzer vacuum tube that represents the head loss resistor r. The beam of the cathode ray tube will be deflected in a horizontal direction by the input on the horizontal deflection plates. For zero current flow in the analyzer circuit the spot is adjusted to the extreme right of the screen of the cathode ray tube by a horizontal positioning control.

As long as the spot is not behind the shield the light in front of the cathode ray tube screen will fall upon the 931-A phototube. The output of the phototube is connected to the vertical deflection plates of the cathode ray tube through a suitable amplifier. With this arrangement any change in light intensity causes a change in the position of the spot. This, in turn, will cause a further change in light intensity on the phototube. The amplifier is phased so that an increase in light will cause the spot to be deflected downward, and conversely, a decrease in light will cause the spot to be deflected upward. By the proper adjustment of the gain of the direct amplifier, the spot will always come to rest at the edge of the opaque shield fixed to the cathode ray tube screen.

Due to the fact that the deflection sensitivity is independent of beam height, the output will be proportional to the spot height at any position in the horizontal direction. A chart showing how the output voltage varies with shield height is given in Figure 4. Therefore, it follows that the input to output relationship is substantially identical to the contour of the shield. By this method any relationship between the input and output may be obtained by constructing an optical shield of the proper shape.

Shield Construction

Input to the photoformer is supplied by the voltage across the resistor R of the analyzer circuit, and consequently the input will be directly proportional to the current I. This is apparent if it is recalled that R does not vary as the current I changes. Figure 3 shows a simplified circuit diagram of the analyzer. Grid bias for the non-linear resistance element is taken from the output of the photoformer, and this output depends upon the proper construction of the shield. In order that the magnitude of these grid voltages may be evaluated, a type 6G6 vacuum tube connected as a triode was selected to be the non-linear resistance r across which the head loss is to be measured. Plate current characteristics for the 6G6 tube are available in the General Electric tube manual, but for convience they are reproduced in Figure 5

PHOTOFORMER OUTPUT VOLTAGE
VERSUS OPAQUE MASK HEIGTH

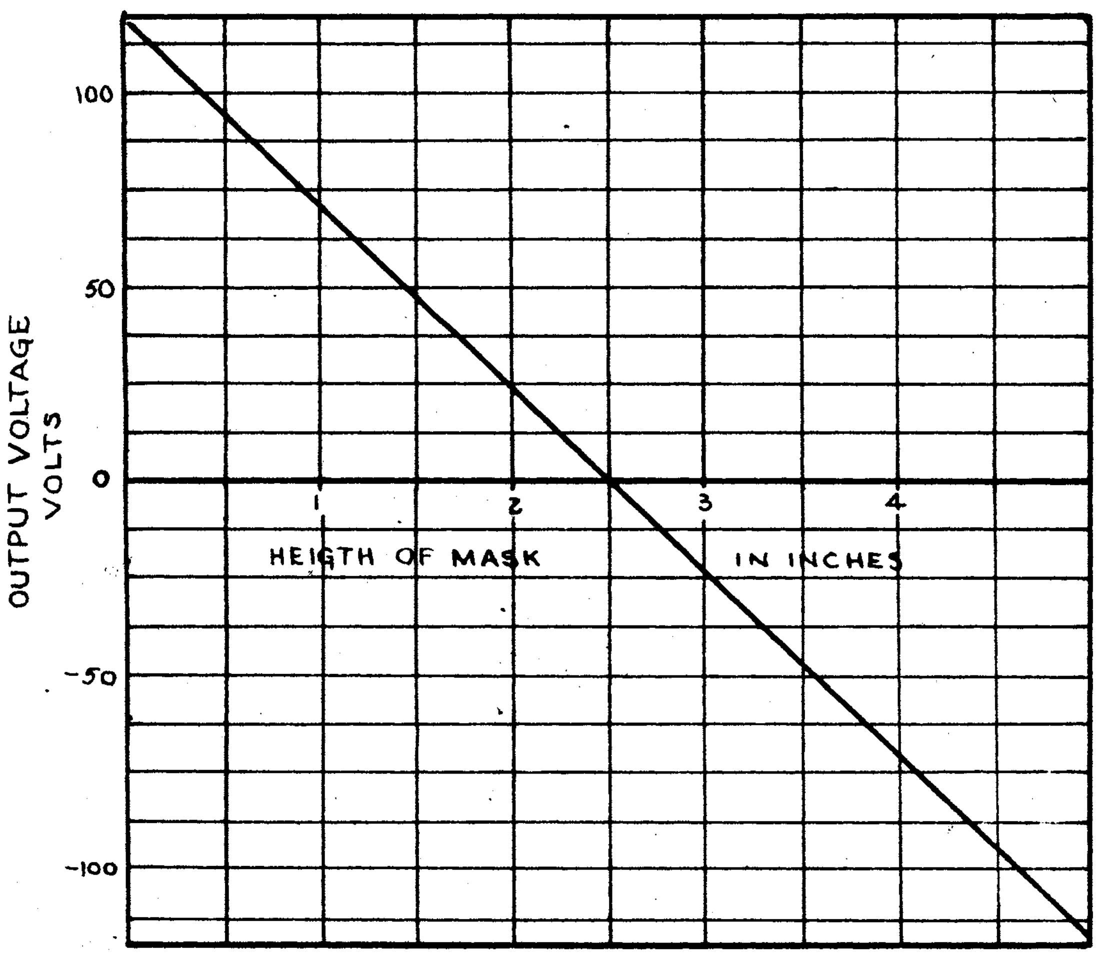


FIGURE 4

of this thesis. Also the sensitivity of the cathode ray tube must be known before the opaque shield can be designed, and consequently it is necessary to calibrate both the horizontal and vertical deflection plates. Figures 9 and 9a show the calibration for the cathode ray tube used in this experiment.

Experimentation in this project was carried out for a pipe line 1,000 feet long, one half foot in diameter, and with water as the fluid passing through the pipe line. Table I indicates the values of both h_f and h for the Reynolds numbers ranging from 2,200 to 10^6 for the pipe under consideration. These calculations of h_f and h were made for two values of the roughness factor, r/K, namely: 507 and 15. These values represent conditions for a smooth and a rough pipe respectively.

Previously it was established that the plate voltage of the 6G6 vacuum tube was to be proportional to the head loss of the pipe line. The plate current that exists simultaneously with the plate voltage of the 6G6 tube must also be proportional to the Reynolds number employed in calculating the corresponding head loss. The current of the tube and the voltage across the tube are therefore fixed by the dimensions of the pipe and the Reynolds number. Control grid bias of the 6G6 tube is fixed if the plate current and plate voltage are specified. It follows that the output of the photoformer must be made equal to this dictated value of the grid voltage for a photoformer input that corresponds to the plate current used in finding this grid bias.

TABLE I

HYDRAULIC PIPE LINE PRESSURE HEADS AND LOSSES FOR REYNOLDS NUMBERS FROM 2,200 to 10^6

N_r	h	h_f @ r/K = 507	h_f @ r/K = 15
	Feet	Feet	Feet
2,200	3.03×10^{-5}	0.001815	0.001815
2,510	3.88×10^{-5}	0.00248	0.00248
3,160	6.21×10^{-5}	0.00475	0.00476
3,980	9.86×10^{-5}	0.0076	0.00857
5,000	15.55×10^{-5}	0.0115	0.01442
6,300	24.60×10^{-5}	0.0177	0.0246
7,910	39.20×10^{-5}	0.0256	0.0408
10,000	62.10×10^{-5}	0.0395	0.0665
12,580	98.00×10^{-5}	0.0585	0.1080
15,800	15.55×10^{-4}	0.0885	0.1750
19,900	24.70×10^{-4}	0.1286	0.283
25,200	39.20×10^{-4}	0.197	0.464
31,600	62.10×10^{-4}	0.293	0.745
39,800	98.00×10^{-4}	0.440	1.192
50,000	15.55×10^{-3}	0.644	1.888
63,000	24.70×10^{-3}	0.984	2.990
79,100	39.20×10^{-3}	1.49	4.710
100,000	62.10×10^{-3}	2.34	7.55
125,800	98.00×10^{-3}	3.58	11.90
158,000	15.50×10^{-2}	5.51	18.9
199,000	24.60×10^{-2}	8.75	30.0
252,000	39.20×10^{-2}	14.26	47.7
316,000	62.10×10^{-2}	23.38	75.5
398,000	98.00×10^{-2}	37.8	119.0
500,000	15.55×10^{-1}	60.4	189.0
630,000	24.70×10^{-1}	97.5	300.0
791,000	39.20×10^{-1}	149.6	477.0
1,000,000	62.10×10^{-1}	236.0	755.0

L = 1,000 feet, D = ½ foot, W = 10^{-5}.

PLATE CURRENT
CHARACTERISTICS

TYPE 6G6-G
(TRIODE CONNECTION)

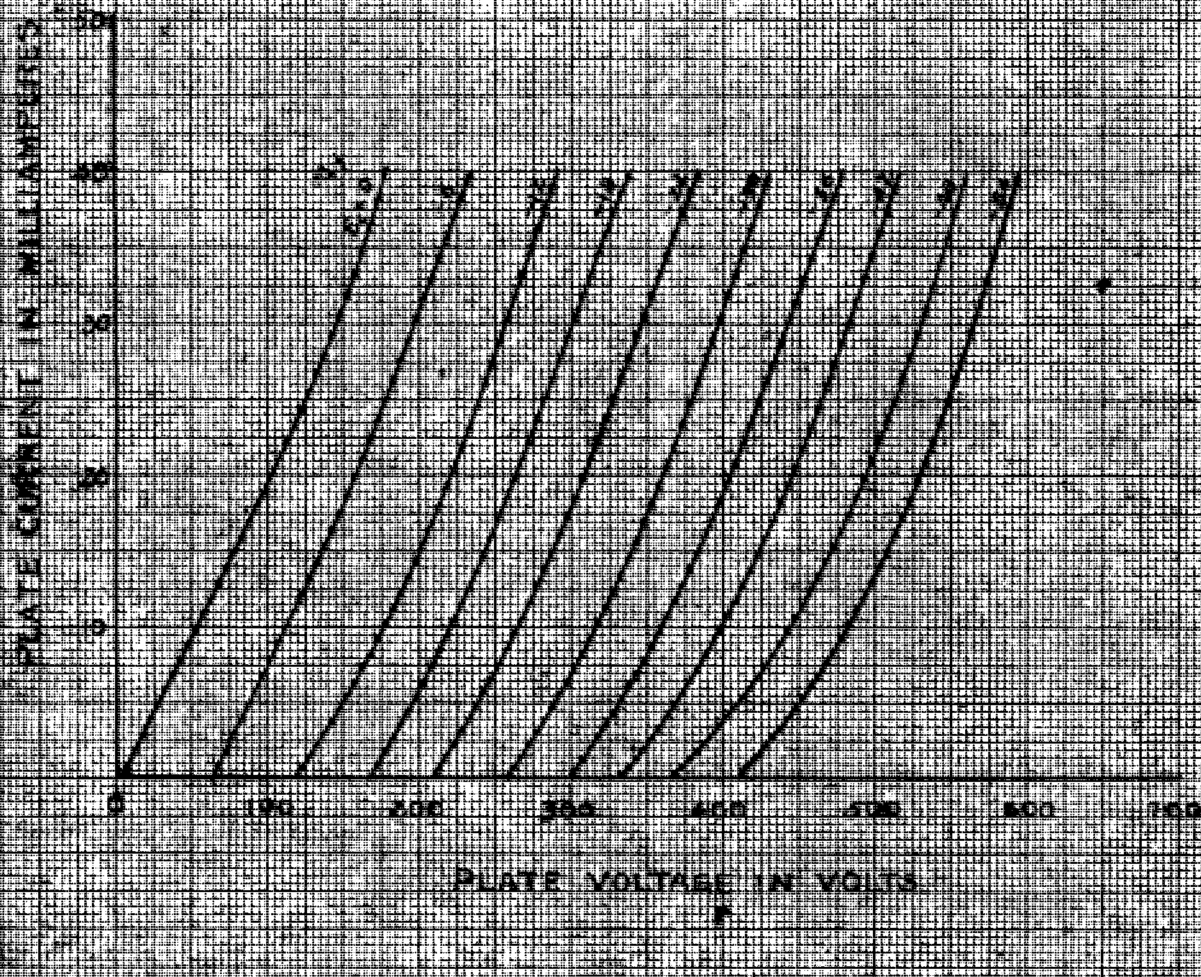


FIGURE 5

Knowing the grid bias and cathode ray tube sensitivity, the shield could be constructed from the data in Table I if it were not for a number of physical limitations. Just how these limitations are overcome will be the next topic of discussion.

Proportionality Constants

The proportionality constants that have been referred to throughout the theory presented up to this point must be evaluated. Before they can be definitely fixed the circuit elements must be selected and the characteristics of the pipe line to be represented by the analyzer must be determined. The necessity for the evaluation of the constants results from the fact that the head loss and Reynolds numbers squared are never numerically equal to the vacuum tube voltage and the vacuum tube current ratings, respectively, for the conventional types of tubes. This point can be clarified by an example.

For the purpose of the discussion a pipe 1,000 feet long, one half foot in diameter, and with water ($W = 10^{-5}$) as the fluid passing through the pipe will be the hydraulic system used for explaining the system established for the evaluation of the proportionality constants. The pipe will be assumed to have a roughness factor r/K of 507. A type 6G6 pentode connected as a triode will be used for the non-linear resistance in the electrical network. The plate current characteristics for this tube are shown in Figure 5.

As a starting point a Reynolds number of 2,200, the lowest value to give turbulent flow, was selected. Referring to Table I, the head loss and outlet head are found to be 1.8×10^{-3} feet and 3.03×10^{-5} feet respectively. From equation (9) there results,

$$K_1 I = N_r^2 \tag{9a}$$

where K_1 is the first proportionality constant that must be employed to bring the current within the range of the 6G6 tube, and hence

$$I = \frac{(2,200)^2}{K_1} \tag{9b}$$

$$= \frac{4.84 \times 10^6}{K_1} \quad \text{amperes}$$

The plate current is arbritrarily selected as 4.84 milliamperes, and so

$$K_1 = \frac{4.84 \times 10^6}{4.84 \times 10^{-3}} = 10^9.$$

From equation (15) the value of K_2, the second proportionality constant, may be evaluated. It follows that

$$R = \frac{W^2}{2D^2gK_2} = \frac{6.21 \times 10^{-11}}{K_2} \quad \text{ohms.} \tag{15a}$$

The constant K_2 must be established so that the voltage IR when multiplied by a third constant K_3 will give the value of the outlet head, hence K_3 must be evaluated before K_2 can be determined. The constant K_3 should be selected so that the values of h_f/K_3 will yield values of tube voltage that are within the range of the 6G6. For a Reynolds number of 2,200, K_3 is arbitrarily taken as 0.5×10^{-4}. The tube voltage is then found to be $\frac{1.8 \times 10^{-3}}{0.5 \times 10^{-4}}$ or 36.50 volts at a current of 4.84 ma. From equation (15a) and the fact that IRK_3 equals h, it is now possible to evaluate K_2. Substitution of the values of I, (15a), and h at a Reynolds number of 2,200 into the equation

$$IRK_3 = h$$

yields

$$(0.00\,484)\left(\frac{6.21 \times 10^{-11}}{K_2}\right)(0.5 \times 10^{-4}) = 3.03 \times 10^{-5}$$

or K_2 equals 0.5×10^{-12}; so that

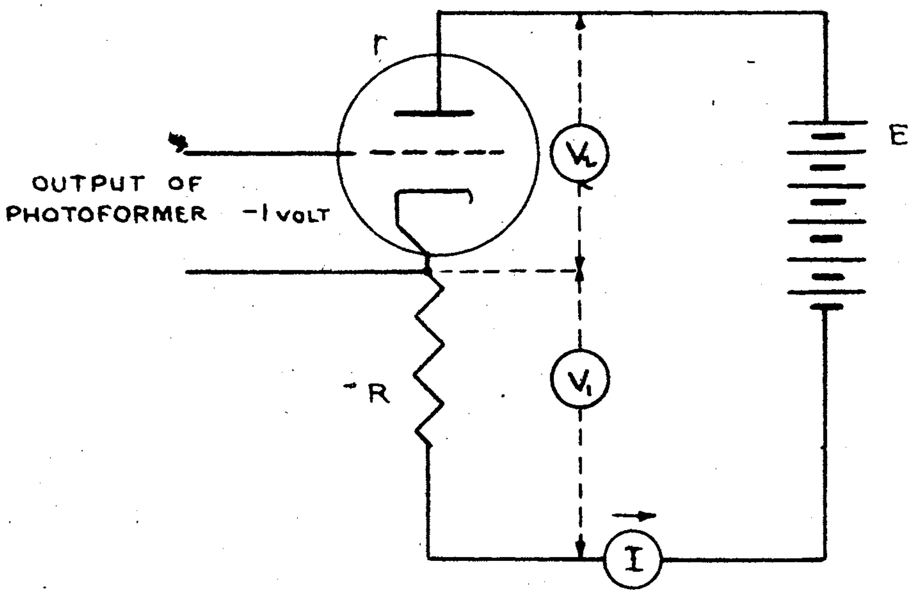
$$R = \frac{W^2}{2D^2gK_2} = \frac{6.21 \times 10^{-11}}{0.5 \times 10^{-12}} = 124.2 \text{ ohms} \qquad (15b)$$

results from (15a).

For a N_r equal to 2,200 the electrical network has the voltage and current as illustrated in Figure 6.

The value of minus one volt used for the control grid bias is determined from the plate current characteristic of the 6G6 tube at a plate current of 4.84 ma. and a plate voltage of 36.5 volts; it is obtained from the photoformer circuit by the proper shield construction. Before the opaque shield can be constructed, the procedure just presented must be carried out for the entire range of Reynolds numbers. In doing this it will be necessary to change K_1 and K_3 several times due to the maximum current rating of the 6G6. On the other hand K_1 and K_3 should not be changed unless it is absolutely necessary to do so, because it is impossible to tell which value of K_1 and K_3 is the proper value to use without some type of indicating device. The voltage applied by the horizontal positioning control must also be changed when K_1 and K_3 are altered. It

VOLTAGE AND CURRENT RELATIONSHIPS FOR THE ANALOGY AT $N_r = 2200$



$E = 37.11$ VOLTS $= H \times K_3$

$V_1 = 0.606$ VOLTS $= h \times K_3$

$V_2 = 36.5$ VOLTS $= h_f \times K_3$

$I = 4.84$ M.A. $= N_r^2 \div K_1$

$R = 124.2$ OHMS $= w^2/2D^2gK_2$

FIGURE 6

TABLE II

ELECTRICAL EQUIVALENTS OF PIPE LINE FLOW AND PRESSURES AT REYNOLDS NUMBERS FROM 2,200 TO 6.3×10^5

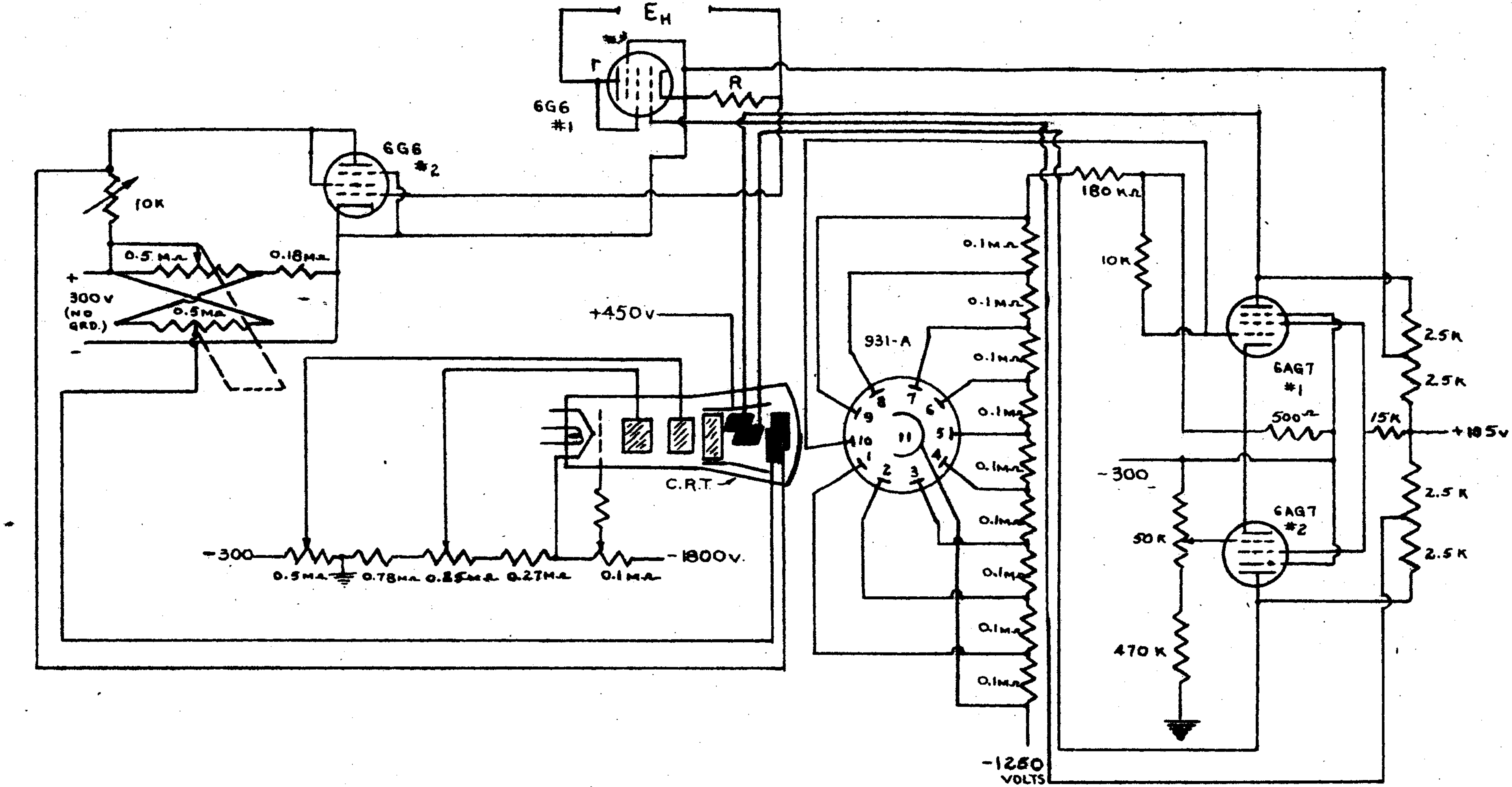
						$r/K = 507$		$r/K = 15$
N_r	K_1	I	IR	K_2	K_3	E_b	E_{cc}	E_b
		ma	volts			volts	volts	volts
2,200	10^9	4.84	0.606	0.5×10^{-12}	0.5×10^{-4}	36.3	-1.0	36.3
2,510	10^9	6.3	0.782	0.5×10^{-12}	0.5×10^{-4}	50.6	-1.9	49.6
3,160	10^9	10.0	1.242	0.5×10^{-12}	0.5×10^{-4}	95.0	-3.9	95.2
3,980	10^9	15.8	1.972	0.5×10^{-12}	0.5×10^{-4}	152.0	-7.2	171.4
5,000	10^9	25.0	3.11	0.5×10^{-12}	0.5×10^{-4}	230.0	-11.0	288.4
6,300	10^9	39.8	4.92	0.5×10^{-12}	0.5×10^{-4}	354.0	-20.0	492.0
7,910	10^{10}	6.3	0.782	0.5×10^{-12}	0.5×10^{-3}	51.2	-2.0	80.4
10,000	10^{10}	10.0	1.242	0.5×10^{-12}	0.5×10^{-3}	79.0	-2.5	133.0
12,580	10^{10}	15.8	1.972	0.5×10^{-12}	0.5×10^{-3}	117.0	-4.0	216.0
15,840	10^{10}	25.0	3.11	0.5×10^{-12}	0.5×10^{-3}	177.0	-5.5	350.0
19,900	10^{10}	39.8	4.92	0.5×10^{-12}	0.5×10^{-3}	257.2	-8.0	566.0
25,200	10^{11}	6.3	0.782	0.5×10^{-12}	0.5×10^{-2}	39.4	-0.1	92.8
31,600	10^{11}	10.0	1.242	0.5×10^{-12}	0.5×10^{-2}	59.6	0	149.0
39,800	10^{11}	15.8	1.972	0.5×10^{-12}	0.5×10^{-2}	88.0	0	238.0
50,000	10^{11}	25.0	3.11	0.5×10^{-12}	0.5×10^{-2}	128.8	-0.1	377.6
63,000	10^{11}	39.8	4.92	0.5×10^{-12}	0.5×10^{-2}	196.8	-1.0	598.0
79,100	10^{12}	6.3	0.782	0.5×10^{-12}	0.5×10^{-1}	29.8	0.4	94.2
100,000	10^{12}	10.0	1.242	0.5×10^{-12}	0.5×10^{-1}	46.8	0.4	151.0
125,800	10^{12}	15.8	1.972	0.5×10^{-12}	0.5×10^{-1}	71.6	1.0	238.0
158,000	10^{12}	25.0	3.11	0.5×10^{-12}	0.5×10^{-1}	110.2	1.6	378.0
199,000	10^{12}	39.8	4.92	0.5×10^{-12}	0.5×10^{-1}	175.0	1.0	600.0
252,000	10^{13}	6.30	0.782	0.5×10^{-12}	0.5	28.52	0.5	95.4
316,000	10^{13}	10.0	1.242	0.5×10^{-12}	0.5	46.76	0.4	151.0
398,000	10^{13}	15.8	1.972	0.5×10^{-12}	0.5	75.6	0.6	238.0
500,000	10^{13}	25.2	3.11	0.5×10^{-12}	0.5	120.8	0.5	378.0
630,000	10^{13}	39.8	4.92	0.5×10^{-12}	0.5	194.0	-1.0	600.0

$h = 1{,}000$ feet, $D = \frac{1}{2}$ foot, $W = 10^{-5}$, $R = 124.2$ ohms is this voltage that is used to operate the indicating device; however no automatic means of changing the voltage on the positioning control has been developed. This is the only operation that must be done manually on the analyzer as it now stands.

A table of K_1, K_2, K_3, I, V_1, V_2, IR, and E_{cc} for all of the Reynolds numbers ranging from 2,200 to 6.3×10^5 are listed in Table II.

By an examination of the constants K_1 and K_3 in Table II, it can be seen that the applied voltage, the voltage drop across R, and the voltage drop across the vacuum tube may represent any one of five different values of head or head loss. The indicating device will establish which value of K_1 and K_3 is to be used. This is determined from the magnitude of the horizontal positioning voltage. A voltmeter may be employed as the indicator, and it will be found that the voltage magnitude that designates which value of K_1 and K_3 is applicable depends upon the construction of the shield used in conjunction with the photoformer. More will be said about this after the design of the shield has been discussed. Assuming that K_1 and K_3 are known the voltages V_1 and V_2 together with the current I are read. These values are then multiplied by the proper proportionality constant as indicated in Figure 6 to obtain the head loss, the outlet head, and the Reynolds number for any applied head at the input to the pipe.

There are a number of details that have been omitted from the previous discussion in order that the over-all picture might be clearer. Now that all the necessary data have been compiled and the general concept is established, it becomes necessary to consider these details. A complete circuit diagram of the analyzer is included in Figure 7. The power supply circuits are not shown for the reason that any regulated supply may be utilized providing its current capacity is sufficient. Likewise a photograph of equipment that was used in obtaining experimental data can be found on page 41. By an examination of the analyzer schematic diagram in Figure 7, it is obvious that several different values of direct voltage are necessary. For example, direct voltage of negative 1,250 volts, negative 300 volts, and positive 105 volts must be supplied to the direct current amplifier of the photoformer. Futhermore a negative 1,800 volts and a positive 450 volts are required to supply the anode and cathode of the cathode ray tube for the photoformer. The power supplies and the inter connections between these units will be discussed in Chapter III. A majority of the power supply units that were utilized consisted of units removed from a war surplus SCR-545-A radar set. Two circuits of the analyzer that have not been discussed are the horizontal position control and a second direct voltage amplifier employed to amplify the voltage drop across the resistance R. These circuit units are necessary in order to obtain the necessary control and amplification of the horizontal deflection voltage indicated on the screen of the cathode ray tube.



Complete Circuit Diagram
Of Electric Analyzer
(Power Supply Unit is
not Shown.)

Note :
6.3 volts to all Filaments

Figure 7

The Horizontal Deflecting Circuits

The horizontal positioning control varies the direct voltage applied to the horizontal deflection plates. This voltage is in series with the output of the direct voltage amplifier that also varies the electron beam horizontal deflection. In this direct current amplifier, which employs a 6G6 pentode tube connected as a triode, the input to output voltage is a linear function. Figure 8 shows a plot of the grid bias versus the voltage drop across the 7,000 ohm resistance, R_x. Throughout the region from zero grid voltage to minus twelve volts the curve is either a straight line or at least so close to being a straight line that very little error is introduced; hence, the horizontal deflection of the spot on the cathode ray tube screen is still proportional to the current flow in R.

One more item must be clarified before the actual shield can be constructed, and that is the matter of the sensitivity of the deflection plates in the cathode ray tube. For the five inch tube used in gathering experimental data the calibration curve for both the horizontal and vertical plates are included in Figures 9 and 9a. From these curves the sensitivity was found to be 56 volts per inch for the horizontal deflection plates and 47 volts per inch for the vertical plates.

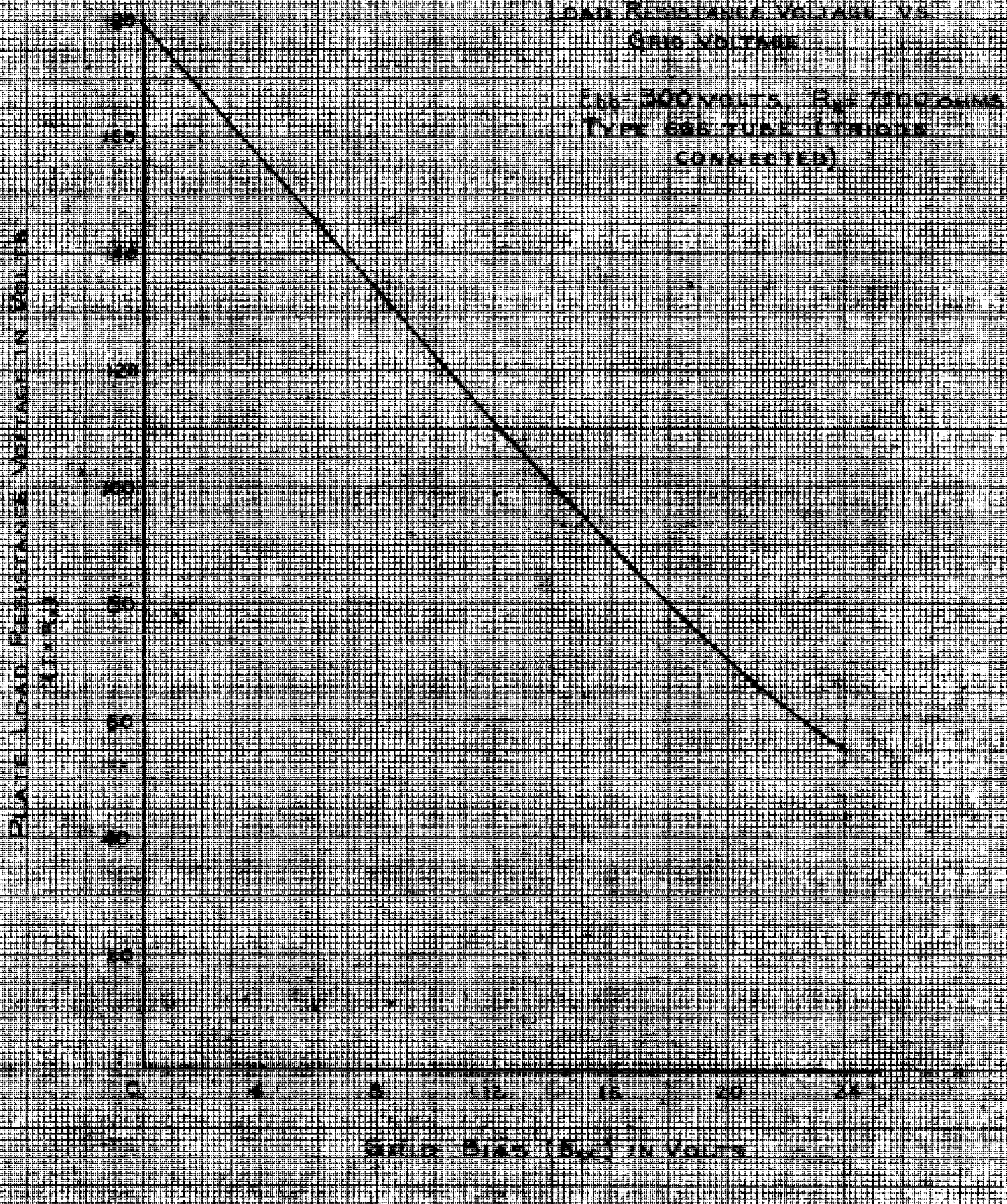
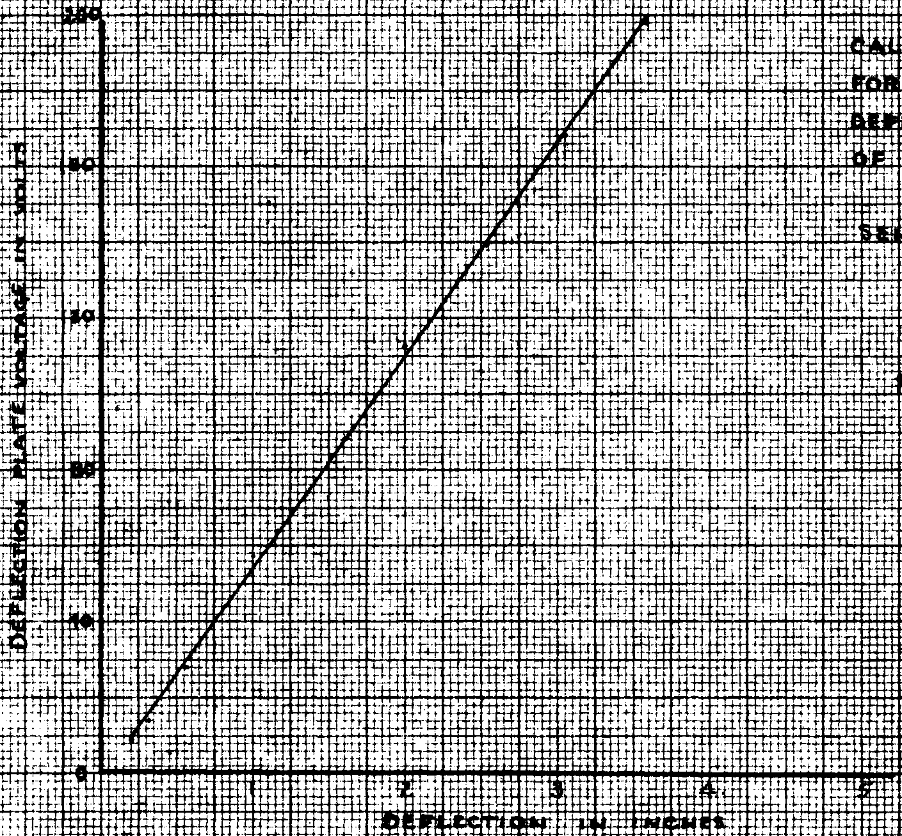

Load Resistance Voltage vs
Grid Voltage
E_{bb} = 300 volts, R_L = 7500 ohms
Type 6G6 tube (triode
connected)
Plate Load Resistance Voltage in Volts
($I_b R_L$)
180
160
140
120
100
80
60
40
20
0
4
8
12
16
20
24
Grid Bias (E_{cc}) in Volts

Figure 8

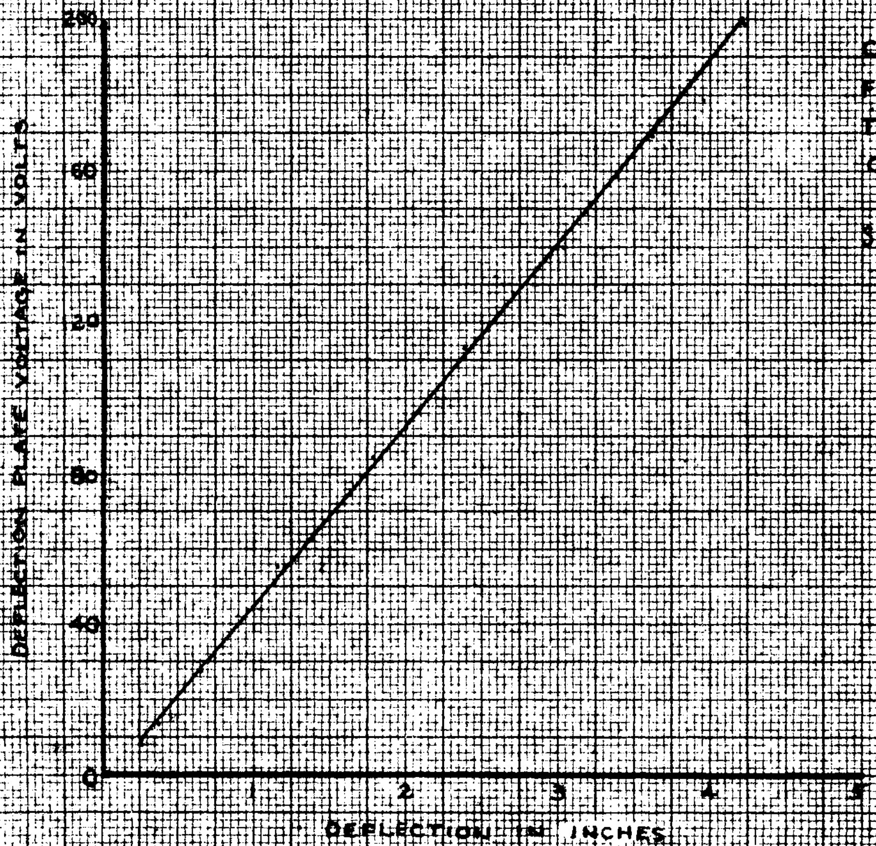
Design of Shield

Extreme care should be employed in the construction of the shield; any discrepancies in either the design or formation of the opaque shield will result in an incorrect value of the grid bias voltage on the 6G6 vacuum tube. Head loss measurements are taken from a meter in the plate circuit of the 6G6 tube, and consequently an error in these readings will result if the grid bias voltage of the tube is not accurately maintained. If the readings of the head loss h_f are incorrect it follows that the readings of the input head will be in error. This is due to the fact that the input head is always equal to the sum of the output head and the head loss. In the example it was necessary to use five different values of K_1 and K_3. This implies that the 6G6 maximum current range must be covered five times in order to accommodate the variations of N_r from 2,200 to 10^6. It follows that the shield must be subdivided into five sections in the horizontal direction; consequently five different settings of horizontal positioning control are required. Five inches are available on the screen of the cathode ray tube, but only the center two and one half inches can be utilized because of the limited operating angle of the phototube. From this it follows that only one half inch can be assigned to each division. The direct voltage amplifier using the 6G6 tube is adjusted by varying its load resistance R_x, so that a 40 ma. current in the resistance R will cause one half inch deflection on the cathode ray tube screen. Due to the fact that R has a resistance of 124.2 ohms,



CALIBRATION CURVE
FOR HORIZONTAL
DEFLECTION PLATES
OF C.R.T. USED IN
PROJECT
SENSITIVITY = 56V/IN

FIGURE 9



CALIBRATION CURVE
FOR VERTICAL DEFLEC-
TION PLATES OF
C.R.T. USED IN
PROJECT
SENSITIVITY = 47 V/IN

FIGURE 9a.

and the current is 40 ma., there will be a voltage of 4.968 volts across R. Consequently the direct voltage amplifier was adjusted so that the combined sensitivity of the amplifier and the deflection plates of the cathode ray tube was 9.936 volts per inch. It follows that the scale used for the X axis of the shield will be 9.936 volts per inch. The IR column of Table II provides the horizontal values to be used in constructing the shield, while the corresponding values in the E_{cc} column are for the vertical or Y axis values. By an examination of the schematic diagram it can be seen that only one half of the total output from the photoformer is applied to the grid of the first 6G6 tube of Figure 7. The sensitivity of the vertical deflection plates is forty seven volts per inch, and therefore the scale for the Y axis is 23.5 volts to the inch. In laying out the shield in the X direction, five different ordinates are used. They will be separated by one half inch, and the one on the extreme left will correspond to the lowest range of Reynolds numbers associated with the lowest values of K_1 and K_3. The second ordinate is used for the next to the lowest range of N_r. This system is followed to the last ordinate which is for the highest value of N_r.

A typical shield is drawn to scale in Figure 10. The data for its construction were taken from Table II. An identical shield was used for the experimental data tabulated in Table III and Table IV of Chapter III.

<u>The Indicator</u>

It is now possible to discuss the voltages utilized in the device designed to indicate the proportionality constant. Since the sensitivity for the horizontal deflection plates was determined to be 56 volts per inch, a total horizontal positioning voltage of 70 volts is required to locate the spot at point e of Figure 10. This value is based upon the voltage required to produce a one and one quarter inch beam deflection to the left of the shield's center line. This voltage of 70 volts will read positive provided the positive terminal of the voltmeter is connected to the left hand deflection plate of the cathode ray tube. In the discussion that follows this connection will be employed. Whenever the value of the indicator voltage is between a positive 70 volts and a positive 42 volts, the photoformer output will be controlled by the curve e-f of the shield shown in Figure 10, and consequently the proportionality constants K_1 and K_2 will be 10^9 and $1/2 \times 10^{-4}$ respectively. The procedure that was followed for the curve e-f must be repeated for the remaining divisions, namely f-g, g-h, h-i, and i-j. The shield constructed for the example under consideration employed the following proportionality constants for the consecutive voltage ranges involved.

Indicator voltage range in volts	K_1	K_2
+70 to +42	10^9	$1/2 \times 10^{-4}$
+41 to +14	10^{10}	$1/2 \times 10^{-3}$
+13 to -13	10^{11}	$1/2 \times 10^{-2}$
-14 to -41	10^{12}	$1/2 \times 10^{-1}$
-42 to -70	10^{13}	$1/2 \times 10^{0}$

TYPICAL PHOTOFORMER
SHIELD

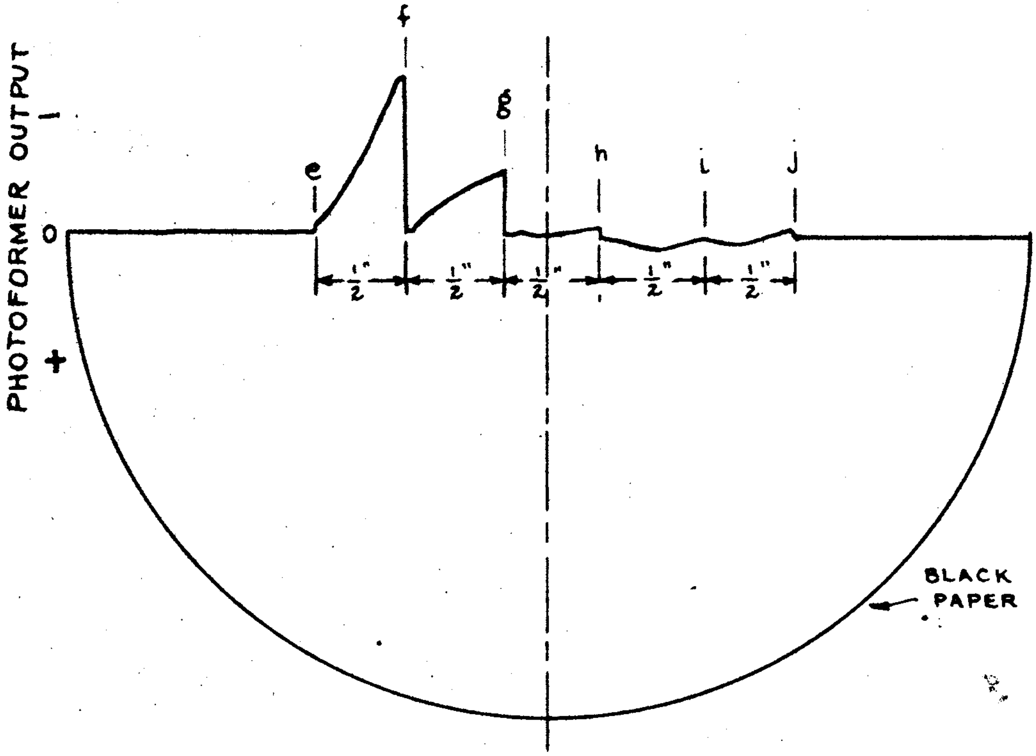


FIGURE 10

CHAPTER III

EQUIPMENT, EXPERIMENTAL DATA, AND INTERPRETATION OF THE DATA

Construction

As a check on the correctness of the theoretical principles set forth in Chapter II for an electrical analyzer for an hydraulic system, an analyzer was constructed from the equipment that was available at Oklahoma A. and M. College. The equipment used was not specifically designed for this project, but it proved to be satisfactory for preliminary checks. Some difficulty was encountered in the adjustments of the analyzer. This was due to the lack of a control panel. Before applying the principles of the analogy to a complex pipe line system, a compact unit should be designed. After the design and construction of a compact unit has been accomplished there remains only the relatively easy task of compiling data for the complex hydraulic network.

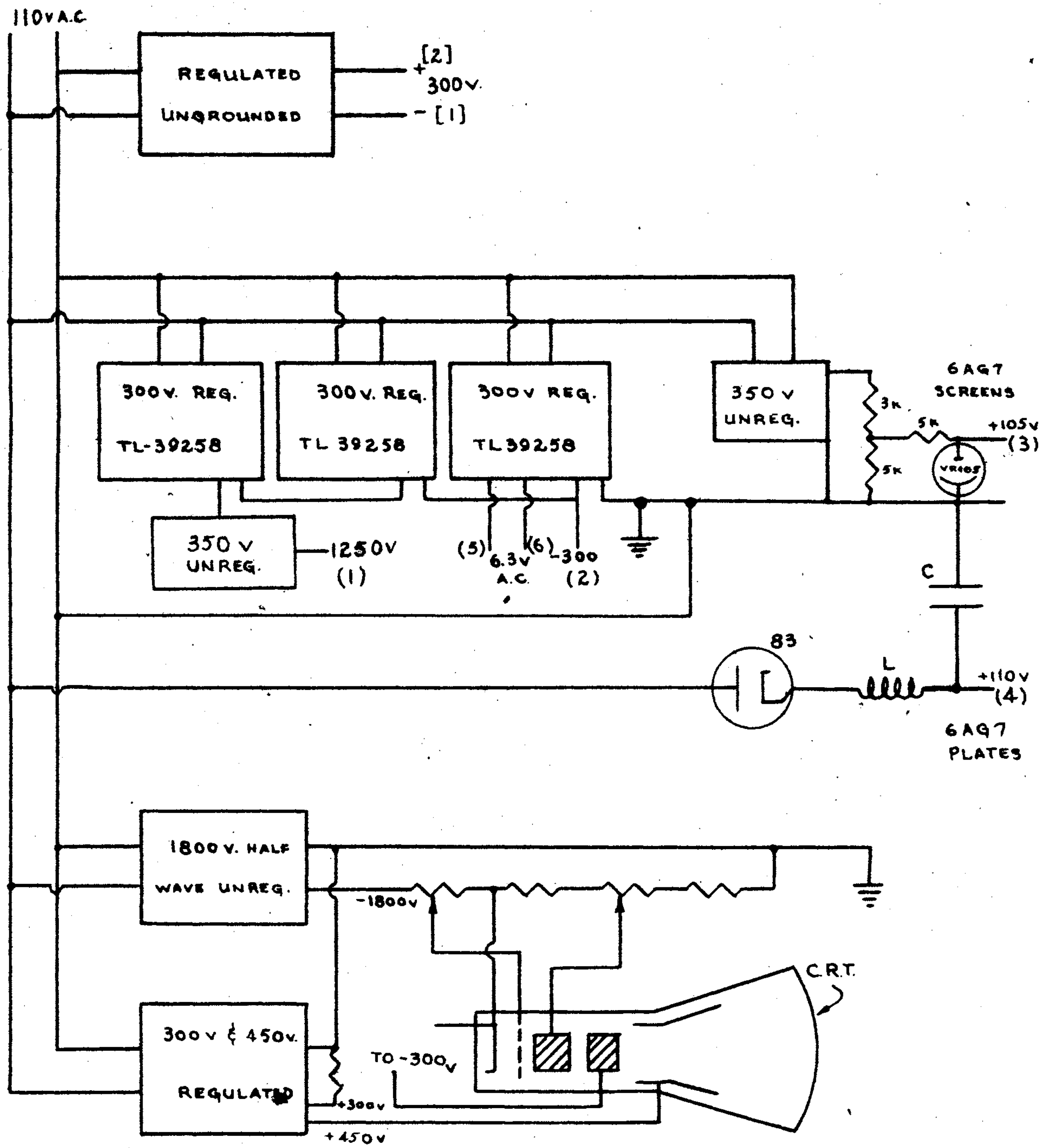
Several power supply units were required in the construction of the analyzer. Voltage ratings on these may be readily determined from Figure 7. All units should be regulated in order to obtain successful operation of the analyzer. In addition to the power supplies mentioned, it is necessary to have a cathode ray tube, two 6AG7 vacuum tubes, two 6G6 vacuum tubes, together with a variety of resistors of conventional types. Reference should be made to Figure 7 for the resistance ratings of the resistors. Since the exact values of voltage output for the above mentioned direct voltage supplies were not available in one unit in all cases, the following arrangement was employed. To obtain the minus 1,250 volts for the phototube circuit, three 300 volt units and one 350 volt unit were connected in series. The negative 300 volt supply was also obtained from one of these units. All three 300 volt units were part of a war surplus radar set SCR-545-A. Their circuit diagrams may be found on drawing number TL-39258 of the service manual for that equipment. No circuit diagram is available for the 350 volt unit, since it was a standard unregulated rectifier power supply unit. However the load was small and very little voltage fluctuation was observed. The cathode ray tubes were also a part of the SCR-545-A radar as were the power supplies that furnished voltages for these tubes. Circuit diagrams for these components are available on drawing TL-39286 in the service manual for SCR-545-A radar. All other circuits in the indicator unit were rendered inactive.

The two 6AG7 tubes required a 105 volt supply with a 200 milliampere current capacity. There were no power supplies with this rating available at the time of the construction, consequently it became necessary to obtain the plate and screen voltages from separate units. For the operating condition assigned to the 6AG7 tubes, the screen current did not exceed 30 milliamperes. This made it possible to design a regulating circuit that utilized a VR-105-30 tube. The supply voltage for this regulator circuit was a standard unregulated 350 volt unit. The plate current demand of the 6AG7 tubes, however, was approximately 170 milliamperes, and therefore a VR-105-30 tube could not be used. This led to the construction of power supply with a high current capacity and a low internal resistance. With this type of rectifier very little fluctuation of voltage was observed.

An ungrounded source of 300 volts was required to operate the horizontal positioning control and the direct voltage amplifier equipped with a 6G6 tube. A standard regulated power supply was used for this purpose. The interconnection of the power supplies previously discussed are indicated schematically in Figure 11.

Connections to the deflection plates of the cathode ray tube were made at points identified by the numbers 252-1F, 252-1D, 252-1B, and 252-1A on drawing TL-39286 found in the SCR-545-A service manual. All connections that were made at these points when the equipment was used as a radar indicator were removed.

The analyzer was built in three separate parts and interconnected by cables as shown in Figure 12. The first of the three units housed the analogy circuit and the horizontal control circuits for positioning the electron beam of the cathode ray tube. The complete photoformer network comprised unit 2, while unit 3 consisted of the several power supplies for the system. Each component is identified in the photograph of the apparatus included on page 41.



( ) INDICATES PIN OF POWER SUPPLY PLUG
[ ] INDICATES PIN OF D.C. AMPLIFIER PLUG

POWER SUPPLY UNIT

FIGURE 11

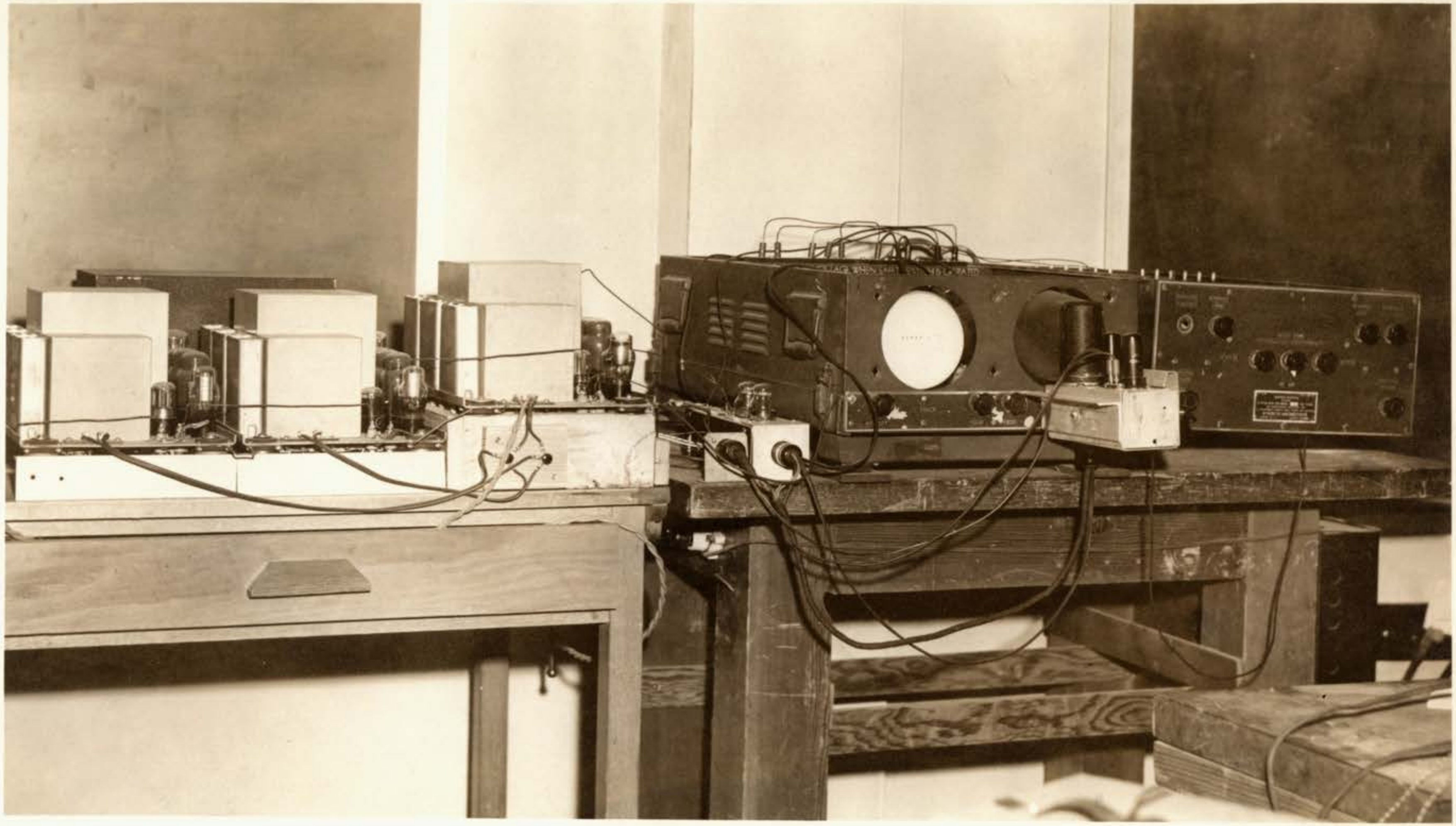
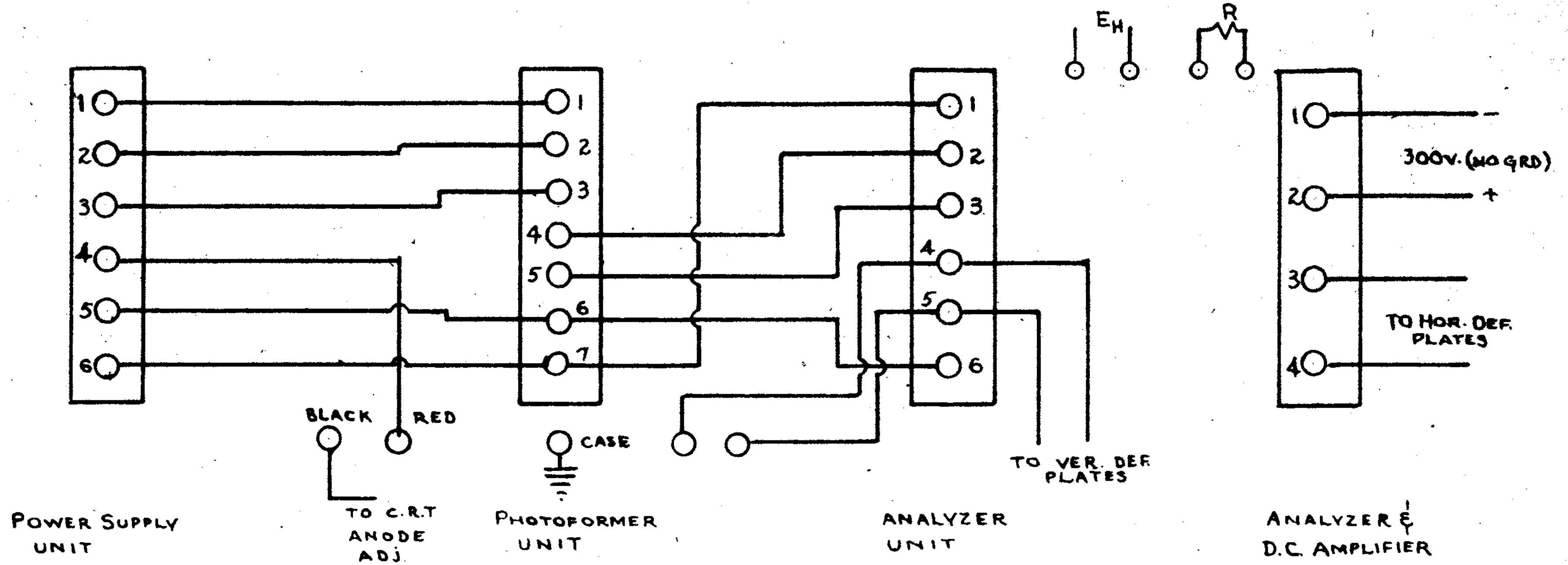


Figure 11a Photograph of the Analyzer

CABLES
MALE CONNECTIONS



PLUGS
FEMALE CONNECTORS

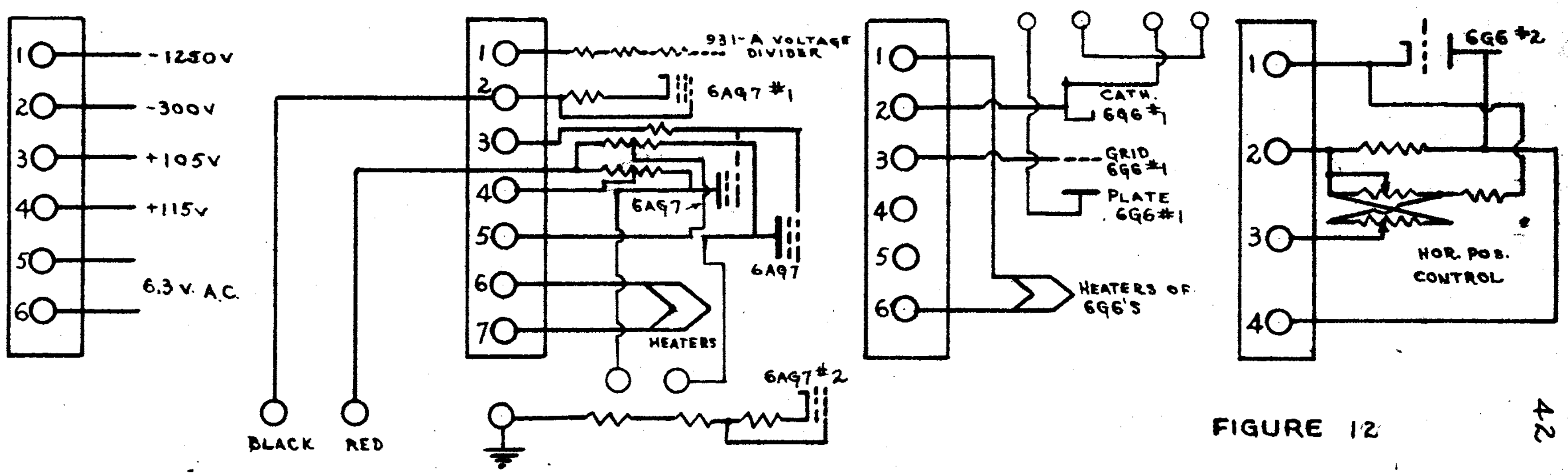


FIGURE 12

Adjustments

In adjusting the circuit to function properly the spot of the cathode ray tube must be made to follow the contours of the shield. This was accomplished by varying both the spot intensity and the control grid voltage on the second 6AG7 tube shown in Figure 7. To prevent damage to the flourescent screen the intensity of the spot should be kept low. However, it must be remembered that the purpose of the cathode ray tube was to supply light to the phototube, and consequently the intensity cannot be decreased below a value required by the photoformer for satisfactory operation. This results in the determination of an optimum light intensity. The sensitivity of the photoformer may also be increased or decreased by increasing or decreasing the spacing between the screen of the cathode ray tube and the phototubes. However this adjustment is limited because the minimum distance that can be employed is determined by the 32 degree horizontal angle required by the phototube. The light from the cathode ray tube must be within the 32 degree angle originating at the phototube. Therefore, ample spacing between the cathode ray tube and the phototube should exist, so that this angle will include the contours of the opaque shield fixed to the cathode ray tube screen. With the spacing that existed in the experimental equipment it was permissable to use the center two and one half inches of the cathode ray tube screen.

In addition to adjusting the spot to the edge of the shield, the position of the shield must be adjusted on the face of the cathode ray tube. This is an important procedure, because the grid bias voltage supplied to the 6G6 tube used as the non-linear resistance depends upon the photoformer output. Points e and j of Figure 10 are selected as the zero voltage reference points. It follows that the adjustment of shield position will be made to result in zero voltage output of the photoformer when the cathode ray tube beam is horizontally located at either e or j. In the experiment this was accomplished by a trial and error procedure. The shield was glued to the screen of the cathode ray tube, and whenever the position of the shield was altered it was necessary to remove the hood between the phototube and the cathode ray tube. After dismantling the hood the position of the shield was shifted. Following this adjustment it was necessary to reassemble the hood. One adjustment did not always result in zero voltage output, and it was usually necessary to repeat this procedure several times. The sensitivity of the vertical deflection plates is 47 volts per inch, consequently the adjustment can vary less than 1/47 of an inch if the photoformer output is to be accurate within plus or minus one volt. This degree of accuracy could not be obtained with the arrangement used in this project. An excellent improvement in the analyzer would result from the design of a mechanism for making these adjustments. In designing such a device it should not be forgotten that the shield must be directly against the screen of the cathode ray tube.

There is another factor which must not be overlooked if accurate experimental results are to be obtained. The plate current characteristics employed in making the calculations outlined in Chapter II should be for the actual 6G6 tube used in the analyzer. The average characteristics that are available in the tube manual are not accurate enough for this purpose. The plate current characteristics should be measured and plotted very accurately in order to permit precise determination of the grid voltage. This is one of the extremely critical factors for obtaining correct data from this phase of the experiment.

The refinement just mentioned is necessary if the results obtained from the analyzer are to be accurate. However it must be kept in mind that at the present there is considerable error in estimating the roughness factor, r/K, for the pipe under consideration. Until more accurate means are developed for determining this factor, the advisability of striving for a high degree of accuracy for the grid bias voltage is somewhat dubious. Consequently it was deemed inadvisable to prepare an elaborate set of plate current characteristics for the 6G6 tube used in this experiment.

The Experiment

Experimental data were taken over the complete turbulent range for a roughness factor of r/K equal to 507 and are compiled in Table III. Data for any other pipe roughness can be secured by an identical procedure, but an opaque shield must be made for the value of r/K in question. Changes in applied head were the only condition analyzed in this study. In taking data the current was set to a value that corresponds to the N_r in question by adjusting E_H, and all corresponding voltages were recorded together with the current readings. Values of E_H, h, H, h_f, and N_r were calculated from these voltages and current by applying the correct proportionality constant as described in Chapter II. A comparison of data in Table II with that in Table IV shows the experimental readings of h_f to be within $\pm$ 15 per cent of the theoritical values set forth in Figure 13. Two factors contributed to this error. An examination of Table II and Table III will show that the experimental value of E_{cc} was not exactly as calculated. This was due to the inaccuracies in the construction of the shield contours and the adjustment of the shield on the cathode ray tube screen. The second source of error was due to the fact the average plate current characteristics of the 6G6 tube were taken as a source of data and considered as sufficiently accurate for the calculation of the grid bias voltage required by the 6G6 tube. This action is justified by the reasoning that was presented in the preceeding paragraph.

TABLE III

EXPERIMENTAL DATA

I ma.	E_H volts	E_{cc} volts	E_{h_f} volts	IR volts
4.84	34.0	- 1.0	33.39	0.606
6.3	48.0	- 1.5	47.23	0.782
10.0	80.0	- 3.1	78.76	1.242
15.8	115.0	- 4.7	113.0	1.972
25.0	229.0	-10.5	225.3	3.11
30.0	285.0	-13.0	281.3	3.75
35.0	300.0	-16.7	295.6	4.37
40.0	350.0	-20.4	345.0	5.00
6.3	52.5	- 2.0	51.72	0.782
10.0	79.0	- 2.85	77.76	1.242
15.8	107.0	- 3.85	105.0	1.972
25.0	175.0	- 6.5	171.9	3.11
30.0	228.0	- 7.9	224.3	3.75
35.0	256.0	- 8.5	251.6	4.37
40.0	268.0	- 8.71	263.0	5.00
6.3	35.0	0.25	34.22	0.782
10.0	50.0	0.38	48.76	1.242
15.8	67.0	0.28	65.0	1.972
25.0	112.0	- 0.125	108.9	3.11
30.0	140.0	- 0.4	136.3	3.75
35.0	170.0	- 0.66	165.6	4.37
40.0	206.0	- 1.0	201.0	5.00
6.3	34.0	0.0	33.22	0.782
10.0	54.0	0.16	52.76	1.242
15.8	67.0	0.31	65.03	1.972
25.0	113.0	0.38	109.9	3.11
30.0	137.0	0.23	133.3	3.75
35.0	176.0	- 0.17	171.6	4.37
40.0	210.0	- 0.60	205.0	5.00
6.3	40.0	- 0.5	39.22	0.782
10.0	59.8	- 0.66	58.6	1.242
15.8	76.0	- 0.81	74.03	1.972
25.0	126.0	- 1.0	122.9	3.11
30.0	148.0	- 1.0	144.3	3.75
35.0	190.0	- 1.1	185.6	4.37
40.0	220.0	- 1.4	215.0	5.00

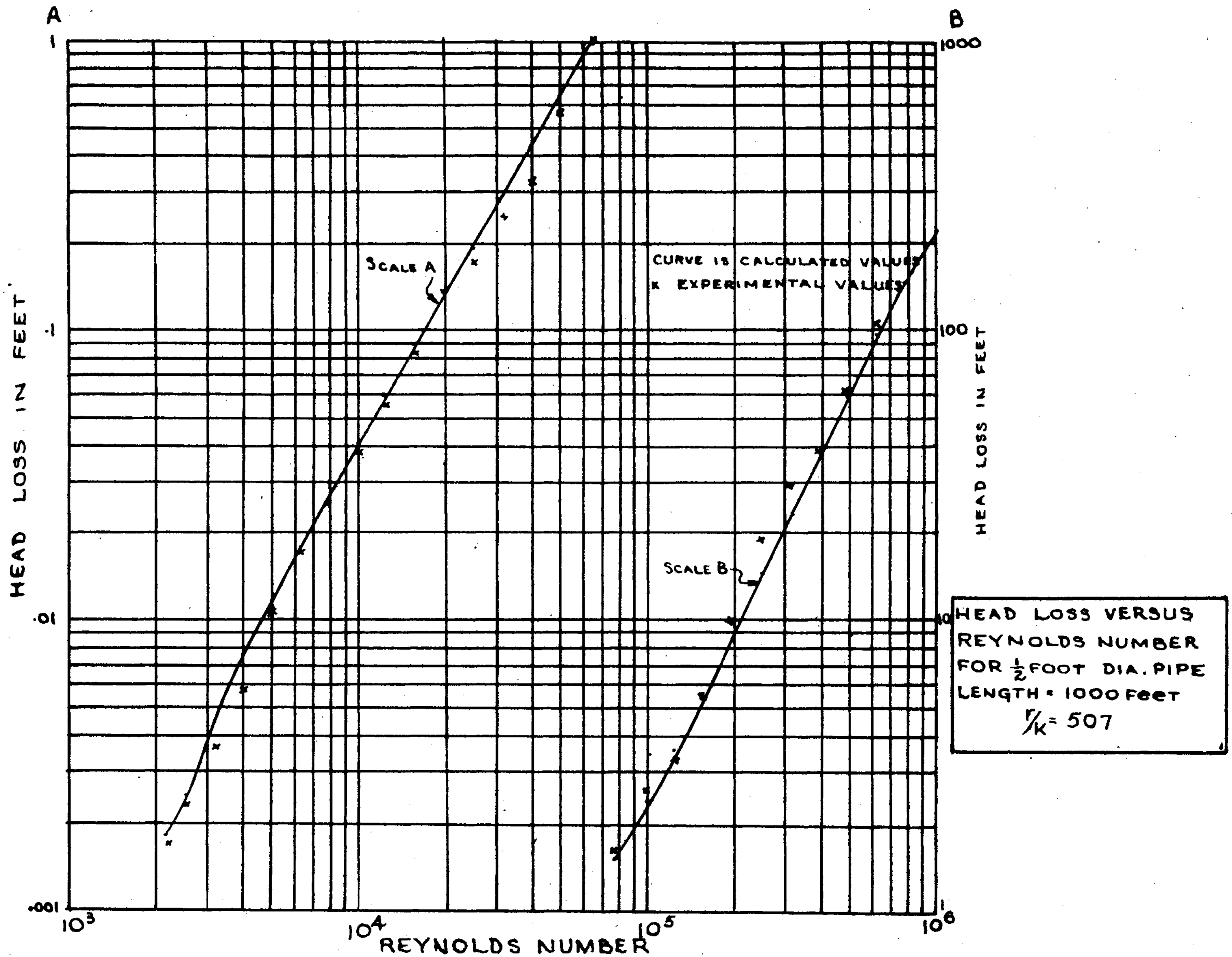
D = 1/2 foot, L = 1,000 feet, and r/K = 507

TABLE IV

CALCULATED EXPERIMENTAL DATA

H Feet	h_f Feet	h Feet	N_r
17.0×10^{-4}	16.9×10^{-4}	3.03×10^{-5}	2.200×10^{3}
24.0×10^{-4}	23.6×10^{-4}	3.88×10^{-5}	2.510×10^{3}
40.0×10^{-4}	39.4×10^{-4}	6.21×10^{-5}	3.160×10^{3}
57.5×10^{-4}	56.5×10^{-4}	9.86×10^{-5}	3.980×10^{3}
114.5×10^{-4}	112.5×10^{-4}	15.55×10^{-5}	5.000×10^{3}
142.5×10^{-4}	140.8×10^{-4}	18.75×10^{-5}	5.460×10^{3}
150.0×10^{-4}	147.9×10^{-4}	21.85×10^{-5}	5.920×10^{3}
175.0×10^{-4}	172.5×10^{-4}	25.00×10^{-5}	6.300×10^{3}
26.3×10^{-3}	25.9×10^{-3}	3.88×10^{-4}	7.910×10^{3}
39.5×10^{-3}	38.9×10^{-3}	6.21×10^{-4}	10.00×10^{3}
53.5×10^{-3}	52.5×10^{-3}	9.86×10^{-4}	12.58×10^{3}
87.5×10^{-3}	85.7×10^{-3}	15.55×10^{-4}	15.84×10^{3}
114.0×10^{-3}	112.2×10^{-3}	18.75×10^{-4}	17.30×10^{3}
128.1×10^{-3}	125.8×10^{-3}	21.85×10^{-4}	18.60×10^{3}
134.1×10^{-3}	131.6×10^{-3}	25.00×10^{-4}	20.00×10^{3}
17.5×10^{-2}	17.16×10^{-2}	3.88×10^{-3}	2.51×10^{4}
25.0×10^{-2}	24.4×10^{-2}	6.21×10^{-3}	3.16×10^{4}
33.54×10^{-2}	32.5×10^{-2}	9.86×10^{-3}	3.98×10^{4}
56.0×10^{-2}	54.4×10^{-2}	15.55×10^{-3}	5.00×10^{4}
70.0×10^{-2}	68.15×10^{-2}	18.75×10^{-3}	5.46×10^{4}
85.0×10^{-2}	82.8×10^{-2}	21.85×10^{-3}	5.92×10^{4}
103.0×10^{-2}	100.5×10^{-2}	25.00×10^{-3}	6.30×10^{4}
17.0×10^{-1}	16.55×10^{-1}	3.88×10^{-2}	7.91×10^{4}
27.0×10^{-1}	26.4×10^{-1}	6.21×10^{-2}	10.0×10^{4}
33.5×10^{-1}	32.6×10^{-1}	9.86×10^{-2}	12.58×10^{4}
55.7×10^{-1}	54.9×10^{-1}	15.55×10^{-2}	15.84×10^{4}
68.5×10^{-1}	66.6×10^{-1}	18.75×10^{-2}	17.30×10^{4}
88.0×10^{-1}	85.8×10^{-1}	21.85×10^{-2}	18.60×10^{4}
105.0×10^{-1}	102.5×10^{-1}	25.00×10^{-2}	20.00×10^{4}
20.0×10^{0}	19.6×10^{0}	3.88×10^{-1}	2.51×10^{5}
29.9×10^{0}	29.3×10^{0}	6.21×10^{-1}	3.16×10^{5}
38.0×10^{0}	37.0×10^{0}	9.86×10^{-1}	3.98×10^{5}
63.0×10^{0}	61.5×10^{0}	15.55×10^{-1}	5.00×10^{5}
74.0×10^{0}	72.2×10^{0}	18.75×10^{-1}	5.46×10^{5}
95.0×10^{0}	92.8×10^{0}	21.85×10^{-1}	5.92×10^{5}
110.0×10^{0}	107.5×10^{0}	25.00×10^{-1}	6.30×10^{5}

$D = 1/2$ foot, $L = 1{,}000$ feet, and $r/K = 507$



A
B
1
.1
.01
.001
1000
100
10
1
HEAD LOSS IN FEET
HEAD LOSS IN FEET
10^3
10^4
10^5
10^6
REYNOLDS NUMBER
SCALE A
SCALE B
CURVE IS CALCULATED VALUES
x EXPERIMENTAL VALUES
HEAD LOSS VERSUS
REYNOLDS NUMBER
FOR $\frac{1}{2}$ FOOT DIA. PIPE
LENGTH = 1000 FEET
$r/k = 507$

FIGURE 13

CHAPTER IV

SUMMARY AND CONCLUSIONS

Summary

The ultimate goal of this project was to establish an electrical analogy between a pipe line system and an electrical network. Due to the complexity of the problem when applied to a complete pipe line network, the research has been confined to a single straight pipe and the equivalent electrical circuit.

First it was necessary to determine theoretically how the various flows and pressures of an hydraulic line could be represented in the electrical system. After several lines of attack had been tried, the analogy was established by making the current in the electrical network proportional to the Reynolds number for the hydraulic network. By setting I proportional to N_r^2 it was possible to make the voltage readings in the electrical circuit analogous to the head loss and pressure readings of the pipe line.

The resistances to be used in the analyzer were evaluated from the known parameters of the pipe line. The resistance R, across which the outlet pressure head is measured, is proportional to $W^2/2D^2g$, while the resistance r, across which the head loss is measured, is proportional to f $LW^2/2D^3g$. Since the coefficient of friction is a variable, the resistance r is non-linear with respect to the analyzer current. The resistance R, however, is constant for all values of current.

Since r is not fixed, a vacuum tube was employed to represent this resistance. It must be remembered, however, that the control grid bias on this vacuum tube must be changed throughout the range of operation to secure the proper values of resistance at all values of plate current.

A 6G6 pentode tube, connected as a triode, was selected to perform the function of the variable resistance r. The required grid bias voltage was calculated for the entire range of Reynolds numbers from the plate current characteristics. A photoformer circuit was employed as a means of varying the grid bias voltage in accordance with these calculations. The ratios of the input voltage to the output voltage for the photoformer are identical to the contours of a shield used in conjunction with this unit. Since the resistance r is nonlinear with respect to the current I of the analyzer, the input to the photoformer was obtained from the fixed resistance R. Consequently the input to the photoformer was directly proportional to the current I. The output from the photoformer controlled the magnitude of the resistance r by supplying the correct grid bias voltage to the 6G6 tube. Therefore it followed that the shield used in conjunction with the photoformer was designed so that the photoformer would furnish the desired values of grid bias voltage for the 6G6 tube.

Before the shield of the photoformer could be constructed, it was necessary to calibrate the equipment employed in terms of the unknown constants of the pipe line. As soon as the required calibrations were made, a shield was constructed for a pipe 1,000 feet long, 1/2 foot in diameter, and a roughness factor of r/K equal to 507. Experimental data for this sample problem are tabulated in Table III and the theoretical values as developed in Chapter II may be found in Table II. In order to facilitate comparison these two values have been plotted in Figure 13.

Experimental results verify the basic principles involved in the analyzer. It must be remembered that the accuracy of all readings could be improved by the use of precision made parts and equipment. However the results of the present experiment were within ± 15 per cent of the calculated values.

Conclusions

The ultimate results to be accomplished from the project undertaken in this thesis is the establishment of an electrical analogy for the pressures and flows that exist in a complex hydraulic system. In this thesis the problem was limited to the establishment of the electrical analogy for a single straight pipe with a constant pressure head at the input. Futhermore, the various problems involved in dealing with the phenomenon of water hammer were not investigated.

Within the limits set forth in this project, it can be safely claimed that the goal has been attained. The analyzer as developed is capable of predetermining the output pressure and head losses for any fluid velocity. Also the input head required to produce this velocity may be predetermined.

Agreement between the values read from the analyzer and the values calculated from empirical formulas verify the principles developed.

The fundamental principles of the analogy can be applied to the complex pipe line network. It must be remembered, however, that the proportionality constants established for the electrical network must be uniform throughout the network. Should it become necessary to deviate from this uniformity of the proportionality constants, the problem of correlating the constants established for the various branches arises. Such procedure may not be an easy obstacle to surmount. And may require considerable additional research.

Necessary Improvements

When a new field of engineering is investigated it is not unusual for shortcomings to exist in the first product. Future research and development in such a new field can be simplified if the original experimenter indicates the existence and nature of these weak points. For this reason the shortcomings of the analyzer will be summarized.

In order to operate the photoformer the intensity of the light from the cathode ray tube must be at approximately normal brilliance. However the spot is not in continuous motion, and this may result in damage to the screen of the cathode ray tube. Therefore a refinement in the photoformer circuit is necessary. This difficulty might be overcome by increasing the sensitivity of the phototube circuit, in which event the brilliance of the light could be decreased. An alternative procedure might involve the selection of a cathode ray tube with a flourescent screen that is not easily damaged by a continuous electron bombardment.

Turbulent flow takes place in a pipe line whenever the Reynolds number exceeds 2,200. For all practical purposes the upper limit of N_r can be taken as 10^6. From this it follows that the largest value of N_r is 4.55×10^3 times larger than the lowest value of N_r for turbulent flow.
Equation (9a) gives the relation

$$K_1 I = N_r^2 \qquad (9a)$$

Therefore it can be seen that the current I for a value of N_r equal to 10^6 must be 2.07×10^5 times larger than the current corresponding to a N_r of 2,200. Due to the fact the maximum current rating of the 6G6 tube is 40 milliamperes it becomes necessary to use a very small value of I to simulate the lower values of N_r. However the sensitivity of the circuits involved in controlling the resistance r prevented the utilization of these low current values. Because of this lack of sensitivity in the control circuits the range of N_r was subdivided into five sections. This process of subdividing the range leads to a unique value of K_1 for each division. For each value of K_1 there must be a corresponding section of the shield. Consequently a manual adjustment of the horizontal positioning voltage was required for each value of K_1.

In order to obtain completely automatic operation of the analyzer it will be necessary to develop a relay system that will make the required adjustments. An alternate remedy might be to develop a detector that would respond to the current values for the complete range of N_r. However such a design is not feasible at the present stage of the electronic art.

The analyzer in its present stage is now capable of solving problems which involve changes in the input head of the simple pipe line. In the future the application of the analyzer may be extended to solve problems involving complex pipe line systems. Again it must be observed that no provisions have been incorporated in the electrical network for obtaining a solution to the water hammer problem.

The initial cost of constructing a calculating board for a complex pipe line system will be high due to the amount of equipment required to simulate a pipe line electrically. Justification for the installation and purchase costs will be determined by the demand for the analyzer.

BIBLIOGRAPHY

Hydraulic

Bokhmeteff, B. A., The Mechanics of Turbulent Flow. 2nd printing. Princeton, New Jersey: Princeton University Press, 1941. Pp. 101.

Rouse, H., Elementary Mechanics of Turbulent Flow. New York, New York: John Wiley and Sons, Inc., 1946. Pp. 376.

Electrical

Camp, Thos. R. and H. L. Hazen, "Hydraulic Analysis of Water Distribution Systems as an Electric Analyzer," Journal of New England Water Works Association, Vol. 48, (December, 1934), pp. 383-407.

Kron, Gabiel, "Electric Circuit Models of Partial Differential Equations," Electrical Engineer, Vol. 67, (July, 1948), pp. 672-684.

McCann, G. D. and E. L. Harder, "Computer-Mathematical Merlin," Westinghouse Engineer, Vol.8, (November, 1948), pp. 178-183.

Sustain, D. E., "Photoelectric Waveform Generator," Electronics, Vol. 22, (Feb. 1949), pp. 100-103.

Travers, H. A., "The Network Calculator Brought up to Date." Westinghouse Engineer, Vol. 4, (July, 1944), pp. 111-119.

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