HARMONIC MITIGATION IN DC-AC CONVERSION BY UTILIZING MULTI-LEVEL INVERTERS AND ITS APPLICATION TO PV SYSTEMS FOR GRID INTERCONNECTION

By

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Abstract: Small (0.2-50 kW) and medium (50-1000 kW) PV systems typically utilize square wave inverters for DC-AC voltage conversion. The output voltage of square wave inverters consists of two voltage levels V_{DC-} and V_{DC+}. The square wave outputs are filtered to obtain a sinusoidal waveform before interconnection to the electric grid. While the simplicity of this approach makes it a popular choice, the voltage harmonics of the inverter output require a significant amount of filtering to obtain an acceptable sinusoidal waveform. Lowering voltage harmonic content can significantly lower filtering requirements and the need to invest in bulky filtering equipment.

Total Harmonic Distortion (THD) for a square wave inverter is 48.34%; use of a multilevel inverter can significantly reduce this value. The first part of this thesis summarizes several harmonic minimization techniques for multilevel inverters. An equal angle approach can be used to decrease THD to as low as 12%. Elimination of selected harmonics can be used to completely eliminate select low order harmonics and their integer multiples, and reduce THD to below 12% for a 7-level inverter. A THD minimization approach can be used to lower harmonics to below 17% for a 5-level inverter. These results can be achieved without using Pulse Width Modulation (PWM), which is unsuitable for high power applications due to high switching frequency requirements.

The second part of this thesis presents the application of multilevel inverters for the interconnection of PV systems to the electric grid. The main principle behind the strategy is designing a separate DC bus for each voltage level to which solar panel modules and a battery backup system are connected. Each DC bus will act as a DC voltage source for the multilevel inverter.

Design recommendations will be limited to 7-level inverters since higher level inverters are impractical due to the cost of additional components. A scheme with 3^{rd} order harmonic elimination with THD reduced to 11.82% is presented. A scheme which simplifies equalizing voltage stress by using the equal angle approach with THD reduced to 25.47% is also presented.

TABLE OF CONTENTS

Chapter Page

I. INTRODUCTION 1

1.1 Background 1
1.2 The Green Shift 2
1.3 PV Systems 3

II. HARMONICS 4

2.1 Theory 4
2.1.1 Fourier Series 4
2.1.2 Properties of Interest 7
2.1.3 Waveforms of Interest 9
2.1.4 Total Harmonic Distortion (THD) 15
2.2 Harmonics in Power Systems 17
2.2.1 Sources 17
2.2.2 Detrimental Effects 18
2.2.3 Limits 19
2.3 Filtering Techniques 20
2.3.1 Passive Filtering 20
2.3.2 Active Filtering 20
2.3.3 Hybrid Filtering 21

III. INVERTERS 22

3.1 Applications in Power Systems 22
3.2 Single Phase Inverters 22
3.2.1 Single Voltage Source Inverters 22
3.2.2 Multilevel Inverters 29
3.3 Three Phase Inverters 32

Chapter Page

IV. PHOTOVOLTAIC SYSTEMS ... 35

4.1 Overview ... 35
4.2 Modeling PV Cells ... 37
4.3 Interconnection to the Electric Grid ... 44
4.4 Harmonics from PV Systems ... 45

V. A MULTILEVE INVERTER SCHEME FOR PV SYSTEMS ... 46

5.1 Overview ... 46
5.2 Practical Considerations ... 50
5.2.1 Cost Limit ... 50
5.2.2 Voltage Stress Balancing of Inverter Switches ... 52
5.2.3 Power Balancing of PV Modules ... 52
5.2.4 Varying Voltage Levels ... 53
5.3 Control Strategy ... 53
5.3.1 Minimization of Harmonics in Output Voltage ... 54
5.3.2 Maximization of Output Power of PV Modules ... 55
5.4 Objective Function ... 58
5.5 Equal Angle Approach ... 59
5.6 Elimination of Select Harmonic Orders ... 61
5.7 THD Minimization ... 63
5.8 Minimization of Selected Harmonic Orders ... 66

VI. PERFORMANCE RESULTS ... 70

6.1 Standard on Harmonics ... 70
6.2 Square Wave Operation Results ... 74
6.3 Equal Angle Operation Results ... 77
6.4 Select Harmonic Order Elimination Results ... 80
6.4.1 3-level Inverters Harmonics Order Elimination ... 80
6.4.1 5-level and 7-level Inverters Harmonic Order Elimination ... 83

VII. CONCLUSION AND RECOMMENDATIONS ... 86

7.1 The Quick and Economic Fix ... 86
7.2 Multilevel Inverter Configuration for THD Minimization ... 87
7.3 Multilevel Inverter Configuration for Voltage Stress Balancing ... 89
7.4 Contributions ... 94

7.5 Further Work 94
REFERENCES... 95

APPENDICES
APPENDIX A MATLAB CODE.. 97

A.1 MATLAB Code: HarmonicAnalysis.m 98
A.2 MATLAB Code: ThreeLevelTHDminimization.m 101
A.3 MATLAB Code: FiveLevelTHDminimization.m 102
A.4 MATLAB Code: FifthSeventhDistortionPercentageMinimization.m 103

LIST OF TABLES

Table Page

2.1 THD of Various Waveforms 17
3.1 H-Bridge Inverter Operation Modes 23
3.2 5-level Inverter Operation Modes 31
6.1 Voltage Harmonics Limits from the IEEE 519 Standard 70
6.2 Voltage Harmonics Limits from the IEC 61000-3 Standard 71
6.3 Output Voltage and Harmonics RMS Values for Square Wave Operation 74
6.4 THD and Harmonics Distortion % for Square Wave Operation 74
6.5 Harmonics Improvement vs. Square Wave for Square Wave Operation 75
6.6 IEEE519 Limits Exceedance for Square Wave Operation 75
6.7 RMS Value of IEEE519 Limits Exceedance for Square Wave Operation 76
6.8 Output Voltage and Harmonics RMS Values for Equal Angle Operation 77
6.9 THD and Harmonics Distortion % for Equal Angle Operation 77
6.10 Harmonics Improvement vs. Square Wave for Equal Angle Operation 78
6.11 IEEE519 Limits Exceedance for Equal Angle Operation 78
6.12 RMS Value of IEEE519 Limits Exceedance for Equal Angle Operation 79
6.13 Output Voltage and Harmonics RMS Values for 3-level Inverter Harmonic Elimination 80
6.14 THD and Harmonics Distortion % for 3-level Inverter Harmonic Elimination 81
6.15 Harmonics Improvement vs. Square Wave for 3-level Inverter Harmonic Elimination 81
6.16 IEEE519 Limits Exceedance for 3-level Inverter Harmonic Elimination 82
6.17 RMS Value of IEEE519 Exceedance for 3-level Inverter Harmonic Elimination 82
6.18 Output Voltage and Harmonics RMS Values for 5/7-level Inverter Harmonic Elimination 83
6.19 THD and Harmonics Distortion % for 5/7-level Inverter Harmonic Elimination 83
6.20 Harmonics Improvement vs. Square Wave for 5/7-level Inverter Harmonic Elimination 84
6.21 IEEE519 Limits Exceedance for 5/7-level Inverter Harmonic Elimination 84
6.22 RMS Value of IEEE519 Exceedance for 5/7-level Inverter Harmonic Elimination 85
7.1 7-level Inverter Equal Angle Operation Firing Angles 90

LIST OF FIGURES

Figure Page

2.1 Plot of a Square Wave Plot ... 12
2.2 Fourier Series Magnitude Spectrum of a Square Wave ... 13
2.3 Plot of a Square Wave with Firing Angle ... 14
2.4 Fourier Series Magnitude Spectrum of a Square Wave with Firing Angle ... 15
2.5 Plot of a 5-level Step Wave ... 16
2.6 Fourier Series Magnitude Spectrum of a 5-level Step Wave ... 17
3.1 H-Bridge Inverter ... 23
3.2 Output Voltage of a Square Wave Inverter with f = 60Hz ... 24
3.3 Uniform PWM ... 25
3.4 Sinusoidal PWM ... 27
3.5 Switching Frequency versus Power for Various Switching Devices ... 28
3.6 N-level H-Bridge Inverter ... 29
3.7 5-level Inverter ... 30
3.8 Output Voltage of a 5-level Inverter ... 31
3.9 3-Phase N-level Inverter ... 33
3.10 3-Phase 5-level Inverter ... 33
3.11 Output Voltages of a 3-Phase, 5-level Inverter with f = 60Hz ... 34
4.1 Typical PV System Schematic ... 36
4.2 Semiconductor Energy Band Diagram ... 38
4.3 P-N Junction Energy Band Diagram ... 39
4.4 Equivalent Circuit of a PV Cell ... 40
4.5 V-I Curve of a Typical PV Cell ... 40
4.6 V-I Curves for Different Irradiance ... 41
4.7 Range of Operating Voltage with Changing Irradiance ... 42
4.8 V-I Curves for Different Series Resistances ... 42
4.9 V-I Curves for Different Fill Factors ... 43
4.10 Multi-section Passive Filter ... 45
5.1 Block Diagram of a Multilevel Inverter PV System ... 60
5.2 Circuit Diagram of N PV Arrays Connected to N DC Buses ... 61
5.3 Circuit Diagram of an N-level 3-Phase Inverter ... 62
5.4 Output Voltage Pattern for Equalizing Voltage Stress ... 65
5.5 V-I Curve for Fixed Voltage Operation ... 67
5.6 Bus Voltage Control Algorithm ... 69
5.7 Bus Voltage Control Algorithm with Voltage Constraints ... 70
5.8 5-level Equal Angle Output Voltage Waveform ... 72
5.9 11-level Equal Angle Output Voltage Waveform ... 73

5.10 21-level Equal Angle Output Voltage Waveform 73
5.11 101-level Equal Angle Output Voltage Waveform 74
5.12 Inverter Output Voltage and Fourier Spectrum with 7th Order Elimination 76
5.13 Firing Angle vs. THD for a 3-level Inverter 77
5.14 Firing Angles vs. THD for a 5-level Inverter 78
5.15 3-level Inverter Output Voltage 5th and 7th Order Minimization..................... 82
7.1 7-level Inverter PV System Schematic 87
7.2 Harmonic Spectrums: Square Wave vs.7-level, THD Minimization 88
7.3 7-level Inverter Output for Equalizing Voltage Stress 90
7.4 Harmonic Magnitudes: Square Wave vs.7-level, Equal Angle 91
7.5 3-Phase DC Bus Output Currents 92
7.6 3-Phase Inverter and Battery Charging Currents 93

LIST OF SYMBOLS

$\boldsymbol{f(t)}$	Periodic function of time t
$\boldsymbol{f(\omega t)}$	Fourier series of periodic function $f(t)$
$\boldsymbol{t}$	Time (s)
$\boldsymbol{f}$	Frequency of periodic function $f(t)$ (Hz)
$\boldsymbol{\omega}$	Angular frequency of periodic function $f(t)$ $(\frac{rad}{s})$
$\boldsymbol{T}$	Period of periodic function $f(t)$ (s) /PN Junction temperature (°K)
$\boldsymbol{n}$	Fourier series component number or harmonic order number
$\boldsymbol{N}$	Number of voltage levels
$\boldsymbol{x}$	Number of DC voltage sources of an N-level inverter (Equation 2.22)
$\boldsymbol{a_0}$	Average (DC) value of a Fourier series (Equation 2.3)
$\boldsymbol{a_n}$	n^{th} Fourier series coefficient (Equation 2.4)
$\boldsymbol{b_n}$	n^{th} Fourier series coefficient (Equation 2.5)
$\boldsymbol{A_n}$	n^{th} Fourier series coefficient vector magnitude of the (Equation 2.7)
$\boldsymbol{A_1}$	Magnitude of the fundamental frequency Fourier series coefficient vector
$\boldsymbol{\varphi_n}$	Phase angle of the Fourier series coefficient vector (Equation 2.7)
$\boldsymbol{V_{rms}}$	RMS value of a voltage waveform (V)
$\boldsymbol{V_n}$	n^{th} voltage harmonic order coefficient magnitude (V)
$\boldsymbol{V_{n,rms}}$	RMS value of n^{th} voltage harmonic order (V)
$\boldsymbol{V_{1,rms}}$	RMS value of the fundamental component of a voltage waveform (V)
$\boldsymbol{V_{DCn}}$	Voltage of DC voltage source n (V)
$\boldsymbol{V_{multiple\ orders,rms}}$	Total RMS value of multiple harmonic orders (V)
$\boldsymbol{V_{DC}}$	Magnitude of a DC voltage (V)
$\boldsymbol{V_{AC}}$	RMS value of an AC voltage (V)

$\boldsymbol{V_{max}}$	Maximum constraint voltage for bus voltage control algorithm (V)
$\boldsymbol{V_{min}}$	Minimum constraint voltage for bus voltage control algorithm (V)
$\boldsymbol{V_{ref}}$	Reference voltage for bus voltage control algorithm (V)
$\boldsymbol{V_{bus}}$	Bus voltage for bus voltage control algorithm (V)
$\boldsymbol{V_{AN}}$	Line to neutral voltage of phase A (V)
$\boldsymbol{V_{BN}}$	Line to neutral voltage of phase B (V)
$\boldsymbol{V_{CN}}$	Line to neutral voltage of phase C (V)
$\boldsymbol{V_{oc}}$	Open circuit voltage of a PV cell/module (V)
$\boldsymbol{V_{m}}$	Maximum power point voltage of a PV cell/module (V)
$\boldsymbol{v_{o}(t)}$	Output voltage of a PWM inverter, function of time t (V)
$\boldsymbol{v_{ref}(t)}$	Reference voltage of a PWM inverter, function of time t (V)
$\boldsymbol{v_{tri}(t)}$	Triangular wave voltage of a PWM inverter, function of time t (V)
$\boldsymbol{P_{max}}$	Maximum power of a PV cell/module (W)
$\boldsymbol{P_{out,n}}$	Output power of PV array n for bus voltage control algorithm (W)
FF	Fill Factor of a PV cell/module V-I curve
$\boldsymbol{\alpha}$	Firing angle ($\frac{rad}{s}$ or degrees)
$\boldsymbol{\Delta\alpha}$	Firing angle increments for the Equal Angle Approach ($\frac{rad}{s}$ or degrees)
$\boldsymbol{\alpha_{n}}$	Firing angle of voltage level n ($\frac{rad}{s}$ or degrees)
$\boldsymbol{\alpha_{+}}$	Firing angle of the positive cycle of a step wave ($\frac{rad}{s}$ or degrees)
$\boldsymbol{\alpha_{-}}$	Firing angle of the negative cycle of a step wave ($\frac{rad}{s}$ or degrees)
$\boldsymbol{I}$	DC bus current (A)
$\boldsymbol{I_{L}}$	Photon current source current (A)
$\boldsymbol{I_{S}}$	Shunt resistor current of a PV cell equivalent circuit (A)
$\boldsymbol{I_{D}}$	Diode current of a PV cell equivalent circuit (A)

$\boldsymbol{I_n}$	RMS value of n^{th} order current harmonic (A)
$\boldsymbol{I_{sc}}$	Short circuit current of a PV cell/module (A)
$\boldsymbol{I_o}$	Reverse saturation current of a PV cell (A)
$\boldsymbol{I_m}$	Maximum power point current of a PV cell/module (A)
$\boldsymbol{R_n}$	Impedance at the n^{th} harmonic order frequency (Ω)
$\boldsymbol{R_s}$	Series resistance of a PV cell equivalent circuit (Ω)
$\boldsymbol{R_{sh}}$	Shunt resistance of a PV cell equivalent circuit (Ω)
Zn	Filter impedance tuned to the n^{th} harmonic order frequency (Ω)
$\boldsymbol{S}n$	Switch number n
$\boldsymbol{q}$	Electron charge: $1.60217657 \times 10^{-19}$ coulombs
$\boldsymbol{k}$	Boltzmann's constant: $1.3806488 \times 10^{-23}$ m^2 kg s^{-2} K^{-1}
$\boldsymbol{m}$	Non-ideality factor of a PV cell

ACRONYMS

AC	Alternating Current
AC-DC	AC to DC
BJT	Bipolar Junction Transistor
CT	Current Transformer
DC	Direct Current
DC-AC	DC to AC
D/C	Disconnect device
GTO	Gate Turn-off Thyristor
IEC	International Electromechanical Commission
IEEE	Institute of Electrical and Electronics Engineers
IGBT	Insulated-gate Bipolar Transistor
MCT	MOS Controlled Thyristor
MOSFET	Metal-Oxide-Semiconductor Field-Effect Transistor
PN Junction	Junction of a P-type and N-type semiconductor
PT	Potential Transformer
PV	Photovoltaic
p.u.	per unit
P&C	Protection and Control
PWM	Pulse Width Modulation
RMS	Root mean square
SCR	Silicon Controlled Rectifier
THD	Total Harmonic Distortion
%THD	Distortion percentage
V-I	Voltage versus current

CHAPTER I

INTRODUCTION

1.1 Background

Voltage and current waveform distortion has existed in the AC electric power system ever since AC generators were first used to generate electricity. The magnitude of the distortion was negligible since all system loads were essentially linear and the currents drawn by the loads were sinusoidal, or identical to the voltage waveform. The main source of distortion was non-ideal AC generators due to the distortion of air gap flux distribution. With the increase in the usage of nonlinear components in electrical systems, system voltage distortion would reach unacceptable levels if not mitigated [1]. Voltage and current distortions are expressed in terms of harmonics. Aside from the lowered quality of power delivered, other undesired effects of harmonics include equipment failures, errors in measuring equipment, neutral currents in three phase systems, and audible noise [8]. Interest in harmonics mitigation has increased in recent history due to the increased usage of significant harmonic sources which include electric power converters, variable frequency drives, electronic ballasts, and various other power electronic systems [2]. These harmonic sources draw or inject non-sinusoidal current waveforms from/to the electric grid. Several standards organizations, IEEE and IEC, recommend harmonic limits for utility companies and generators to ensure safe and reliable operation of electric power systems. Generators of harmonics use various strategies to suppress harmonics to acceptable levels as required by their governing standards. Mitigation of harmonics first requires a careful analysis of its characteristics. Different techniques are then employed to minimize harmonics at the source and the remaining harmonics are passed through filters to reduce distortion to acceptable levels. Filtering techniques may utilize passive filters, active filters or hybrid filters which employ both passive and active filters [2]. Even though harmonics can be completely eliminated by filtering, filtering equipment is costly [1]. Cost minimization is thus one of the main objectives when researching new and innovative solutions for eliminating harmonics. The rapidly growing entry of renewable energy sources is closely tied in with harmonics due to extensive use of power converters, a significant source of voltage and current harmonics. A brief overview of renewable energy technologies employing converters is provided in Section 1.2.

1.2 The Green Shift

With growing awareness of global warming and increasing effort in reducing greenhouse gas emissions caused by electric power generation, investment in ecofriendly generation options such as wind and solar power has increased drastically. Rectifiers, inverters, battery chargers and other power electronics utilized by renewable energy systems are significant sources of harmonics in the electric power system. Their effects on system voltage are greater when connected to distribution or micro-grid systems. As most developed countries are striving to generate a significant amount of their electric power using renewable energy sources, more harmonic sources are being connected to the electric system and the need to mitigate harmonics is thus increasing.

PV generators and wind turbines are the leading renewable energy inputs due to the abundant supply of wind and solar energy for harnessing. PV generators utilize inverters or DC-AC converters for interconnection to the electric grid. Although not as significant a harmonic source as rectifiers and other non-linear loads, if the voltage harmonic content is large enough, significant harmonic currents are injected into the electric grid [9][10]. Minimization of voltage harmonics is thus required to ensure that this ecofriendly source of energy does not become a significant source of lowering the power quality of electrical systems. A brief introduction of PV systems and inverters is provided in Section 1.3.

1.3 PV Systems

The abundance of insolation that is available for harnessing makes PV generation a very attractive ecofriendly option. PV generation produces virtually zero carbon emissions and is a very reliable source of electricity. One drawback of the technology is its high capital cost; minimization of this cost is a main focus of PV research. The major harmonic sources of the system are the inverter and backup battery charger. The component of interest for this thesis is inverters used to convert DC voltage produced by PV panels into AC voltage for interconnection to the electric grid. Square wave inverters are the technology of choice for the purpose of DC-AC conversion in small (0.2-10 kW) and medium (50-1000 kW) PV systems. While the technology is simple, the harmonic profile of the component can be improved significantly if multilevel inverters are utilized. Using multilevel inverters increases the number of components needed but greatly reduces lower order harmonics. The other major advantage is lower ratings for major components such as DC link capacitors and switching devices.

The scheme presented in this thesis will consist of a separate DC bus for each voltage level. Bus voltages will be varied to obtain maximum power output from PV modules. The design and harmonic profile of this scheme will be presented and compared with that of a conventional square wave scheme. Some basic harmonics, inverters and PV system concepts will first be reviewed in Chapters II through Chapter IV. A multilevel inverter scheme will then be presented in Chapter V and its performance results in Chapter VI. The thesis will conclude with a brief discussion of conclusions and recommendations for further work.

CHAPTER II

HARMONICS

2.1 Theory

2.1.1 Fourier Series

Any periodic function can be expressed as a Fourier series, which is a summation of sinusoids with frequencies that are integer multiples of the function's fundamental frequency. The Fourier series of a periodic function $f(t)$with fundamental frequency f can be expressed as follows:

$$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos(n2\pi ft) + b_n \sin(n2\pi ft)) \qquad (2.1)$$

Equation 2.1 can be rewritten in terms of the function's angular frequency ω as:

$$f(\omega t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos(n\omega t) + b_n \sin(n\omega t)) \qquad (2.2)$$

a_0 is the average (DC) value of the function, whereas a_n and b_n are the n^{th} coefficients of the series. The coefficients can be computed as follows:

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\omega t)\, d\omega t \qquad (2.3)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\omega t)\ \cos(n\omega t)\ d\omega t \qquad (2.4)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\omega t)\ \sin(n\omega t)\ d\omega t \qquad (2.5)$$

Each n^{th} combination of sine and cosine term $(a_n \cos(n\omega t) + b_n \sin(n\omega t))$ is the n^{th} Fourier series component of the periodic wave. Each component can be expressed as a vector with magnitude A_n and phase angle φ_n in the following rectangular and polar forms:

$$A_n \angle \varphi_n = a_n + j b_n \qquad (2.6)$$

The magnitude (A_n) and phase angle (φ_n) of the n^{th} component can be calculated as follows:

$$A_n = \sqrt{{a_n}^2 + {b_n}^2} \qquad (2.7)$$

$$\varphi_n = \tan^{-1}(\frac{b_n}{a_n}) \qquad (2.8)$$

In terms of its component vectors, a periodic function can also be written in the following two forms:

$$f(\omega t) = a_0 + \sum_{n=1}^{\infty} A_n \angle \varphi_n \qquad (2.9)$$

$$f(\omega t) = a_0 + \sum_{n=1}^{\infty} A_n \sin(n\omega t + \varphi_n) \qquad (2.10)$$

Since each component of the Fourier series can be expressed as a sinusoid, the average or root mean square (RMS) value of component n can be calculated using the following equation:

$$RMS\ value\ of\ component\ n = \frac{A_n}{\sqrt{2}} \qquad (2.11)$$

2.1.2 Properties of Interest

Several properties of interest for simplifying the calculation of symmetric waveforms are listed below:

Odd Symmetry [1]: A periodic function $f(t)$ with frequency ω has odd symmetry if $f(t) = -f(-t)$. The Fourier series consists of sine terms only and the coefficients are simplified to:

$$a_n = 0, for\ all\ n \qquad (2.12a)$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(\omega t)\ \sin(\mathrm{n}\omega t)\ d\omega t \qquad (2.12b)$$

Even Symmetry [1]: A periodic function $f(t)$ with frequency ω has even symmetry if $f(t) = f(-t)$. The Fourier series consists of cosine terms only and the coefficients are simplified to:

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(\omega t)\ \cos(\mathrm{n}\omega t)\ d\omega t \qquad (2.13a)$$

$$b_n = 0, for\ all\ n \qquad (2.13b)$$

Half Wave Symmetry [1]: A periodic function $f(t)$ with frequency ω has half wave symmetry if $f(t) = -f\left(t + \frac{T}{2}\right)$. The coefficients are simplified to:

If n is EVEN:

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(\omega t)\ \cos(\mathrm{n}\omega t)\ d\omega t \qquad (2.14a)$$

$$b_n = 0 \qquad (2.14b)$$

If n is ODD:

$$a_n = \ 0 \qquad (2.15a)$$

$$b_n = \ \frac{2}{\pi} \int_0^{\pi} f(\omega t)\ \sin(\mathrm{n}\omega t)\ d\omega t \qquad (2.15b)$$

Using these properties, if an analyzed waveform has the same symmetry as a sine wave, it will have odd and half wave symmetries; these properties would simplify its Fourier series to contain only odd order sine terms as per equations 2.12 and 2.15. Likewise, if an analyzed voltage or current waveform has the same symmetry as a cosine wave, it will have even and half wave symmetries; this would simplify its Fourier series to contain only even order cosine terms as per equations 2.13 and 2.14.

Superposition [5]: If a periodic waveform can be expressed as a sum of multiple periodic waveforms, the Fourier series of the waveform is the summation of the multiple waveforms` Fourier series. This property simplifies the computation of the Fourier series of step waveforms of multi-level inverter output voltage which is discussed in Section 2.1.3.

2.1.3 Waveforms of Interest

Two waveforms of interest in the study of harmonic characteristics of inverter output voltages are square and step waves.

Square Wave: A unit square wave with half wave and odd symmetries can be mathematically expressed as follows:

$$f(t) = 1;\ for\ 0 < \omega t < \pi \qquad (2.16a)$$

$$f(t) = -1;\ for\ \pi < \omega t < 2\pi \qquad (2.16b)$$

A plot of a unit square wave is shown in Figure 2.1.

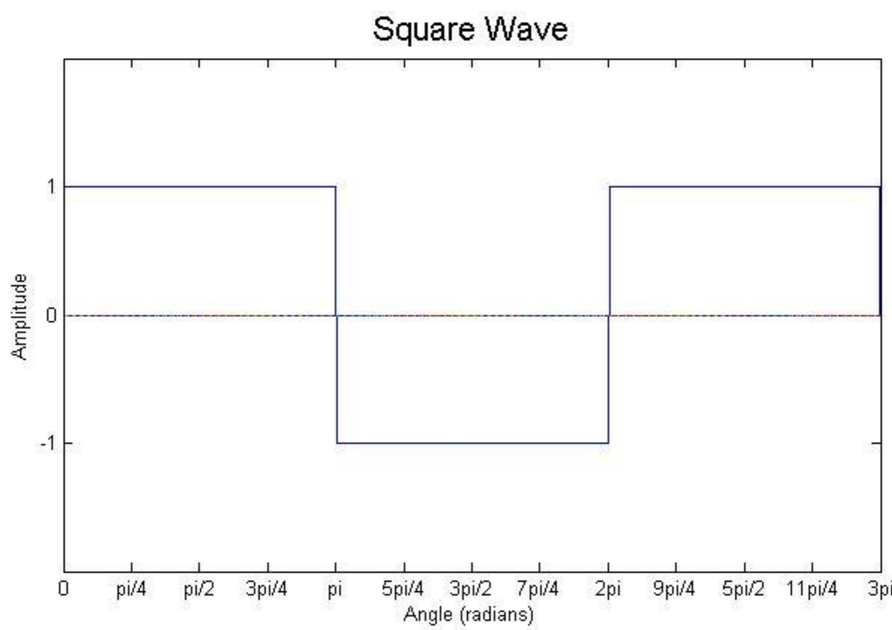

Figure 2.1: Plot of Square Wave

The Fourier series of a unit square wave is expressed as follows [5]:

$$f(\omega t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{4}{n\pi}\sin(\mathrm{n}\omega\mathrm{t}) \qquad (2.17a)$$

The Fourier series of a square wave with amplitude A is expressed as follows:

$$f(\omega t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{4A}{n\pi} \sin(n\omega t) \qquad (2.17b)$$

For a graphical representation of Fourier series, the magnitude of its components A_n are normalized to the magnitude of the fundamental frequency component A_1, and plotted against the order of the component. The Fourier series magnitude plot normalized to the magnitude of the fundamental frequency component for the first 20 terms of a square wave is illustrated in Figure 2.2.

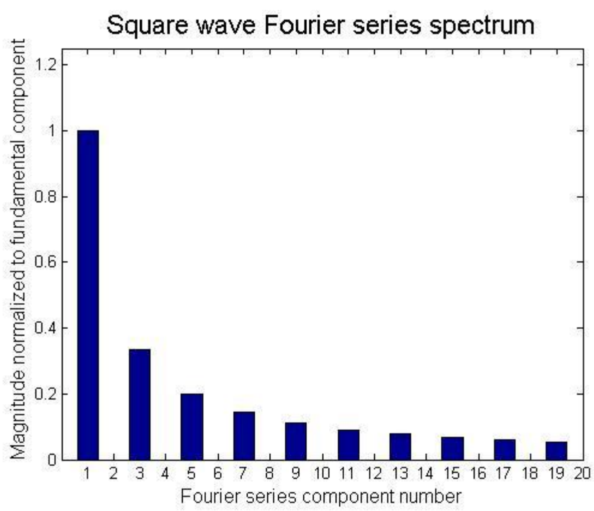

Figure 2.2: Fourier Series Magnitude Spectrum of a Square Wave

The square wave shown in Figure 2.1 has a duty cycle of 50%. The term duty cycle will refer to the percentage of the total period for which the value of the waveform has a positive non-zero value. A square wave can also have a duty cycle less than 50%. The angle at which a square wave begins is called the firing angle and is denoted α. The firing angle and has a range of 0 to $\frac{\pi}{2}$ radians or 0° to 90°. In general, for a square wave with firing angle α, the duty cycle can be calculated as follows:

$$Duty\ Cycle = \frac{(\pi - 2\alpha)}{2\pi} \qquad (2.18)$$

Using Equation 2.18, a square wave with firing angle $\alpha = \frac{\pi}{6}$ corresponds to a duty cycle of 33.33% and can be mathematically expressed as follows:

$$f(t) = 1;\ for\ \frac{\pi}{6} < \omega t < \frac{5\pi}{6} \qquad (2.19a)$$

$$f(t) = -1;\ for\ \frac{7\pi}{6} < \omega t < \frac{11\pi}{6} \qquad (2.19b)$$

A plot of a unit square wave with a firing angle of $\alpha = \frac{\pi}{6}$ is shown in Figure 2.3.

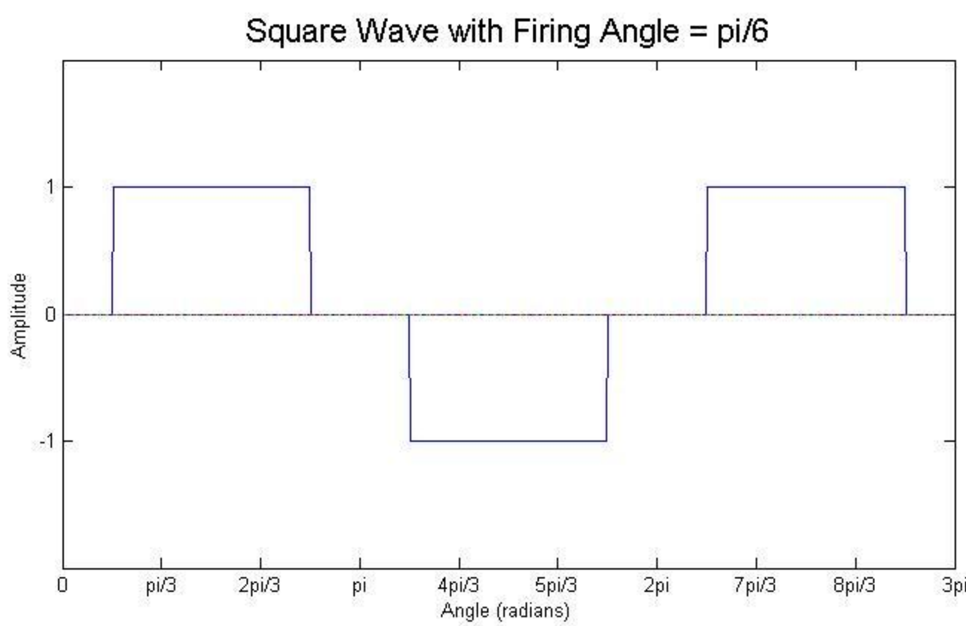

Figure 2.3: Plot of a Square Wave with Firing Angle

The Fourier series of a unit square wave with firing angle α is expressed as follows [5]:

$$f(\omega t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{4}{n\pi} \cos(\alpha n)\ \sin(n\omega t) \qquad (2.20a)$$

The Fourier series of a square wave with amplitude A and firing angle α is expressed as follows:

$$f(\omega t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{4A}{n\pi} \cos(\alpha n) \ \sin(\mathrm{n\omega t}) \qquad (2.20b)$$

The Fourier series magnitude plot normalized to the magnitude of the fundamental frequency component for the first 20 terms of a square wave with firing angle $\alpha = \frac{\pi}{6}$ is shown in Figure 2.4.

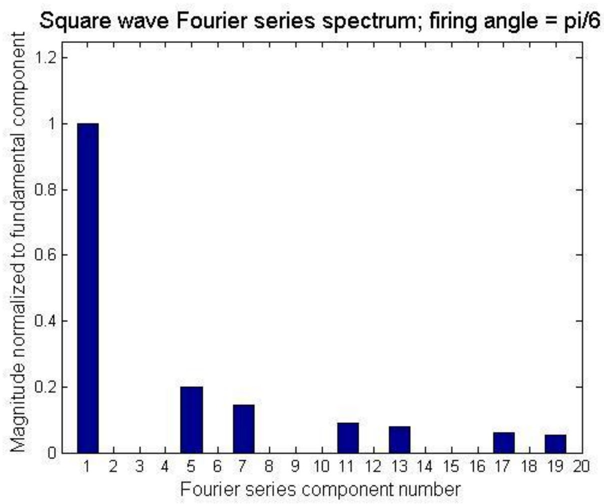

Figure 2.4: Fourier Series Magnitude Spectrum of a Square Wave with Firing Angle

A quick point to note is that by selecting a firing angle of $\alpha = \frac{\pi}{6}$, the 3rd, 9th, 15th, and higher order components which are integer multiples of three (triplens) are eliminated due to the cancellation of the cosine term in Equation 2.20a. This technique is referred to as the elimination of selected harmonic orders and is discussed in Section 5.6.

Step Wave: A step wave with half wave and odd symmetries is one which resembles a sine wave, but increases in discrete steps. Step waves are a summation of multiple square waves with duty cycles equal to or less than 50%. The plot for a 5-level step wave and its individual components is shown in Figure 2.5.

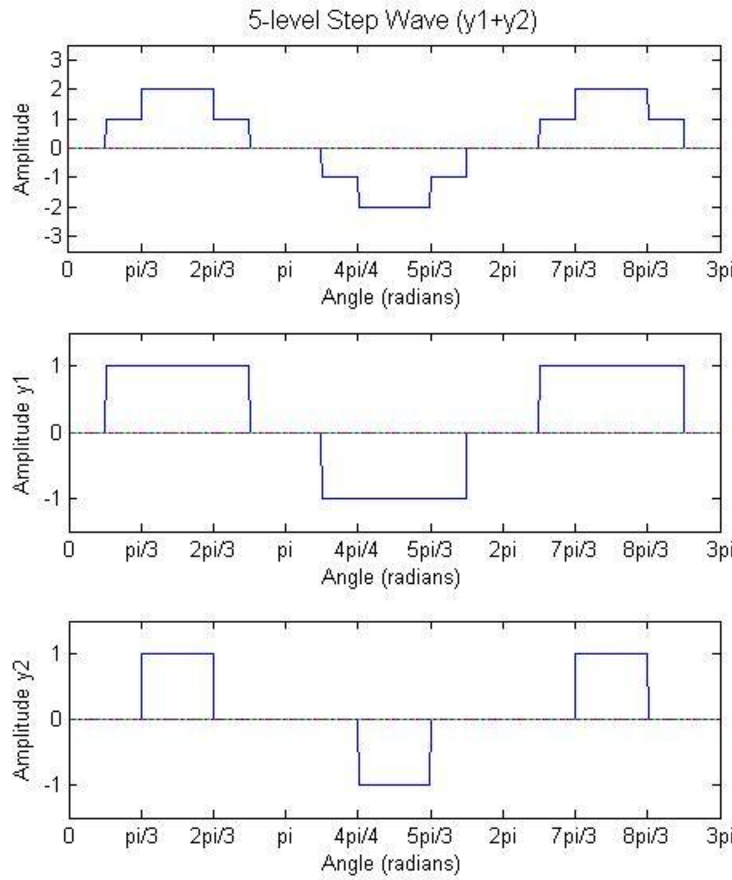

Figure 2.5: Plot of a 5-level Step Wave

Component y1 is a unit step wave with a firing angle of $\alpha = \frac{\pi}{3}$ and y2 a unit step wave with a firing angle of $\alpha = \frac{\pi}{6}$.

The Fourier series of the 5-level step waveform in Figure 2.5 is simply the summation of the Fourier series of the individual components y1 and y2 as per the superposition property discussed in section 2.1.2.

Using Equation 2.17a, the Fourier series of component y1 is expressed as follows:

$$f(\omega t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{4}{n\pi} \cos(\frac{\pi}{3} n) \ \sin(\mathrm{n\omega t}) \qquad (2.20a)$$

Similarly, the Fourier series of component y2 is expressed as follows:

$$f(t) = \sum_{n=1,3,5...}^{\infty} \frac{4}{n\pi} \cos(\frac{\pi}{6} n) \ \sin(\mathrm{n\omega t}) \qquad (2.20b)$$

The Fourier series of the 5-level step wave is the summation of Equations 2.20a and 2.20b and is expressed as follows:

$$f(\omega t) = \sum_{n=1,3,5...}^{\infty} \frac{4}{n\pi} \left(\cos\left(\frac{\pi}{3} n\right) + \cos(\frac{\pi}{6} n) \right) \ \sin(\mathrm{n\omega t}) \qquad (2.20c)$$

The Fourier series magnitude plot normalized to the magnitude of the fundamental frequency component for the first 20 terms of the function in Equation 2.20c is shown in Figure 2.6.

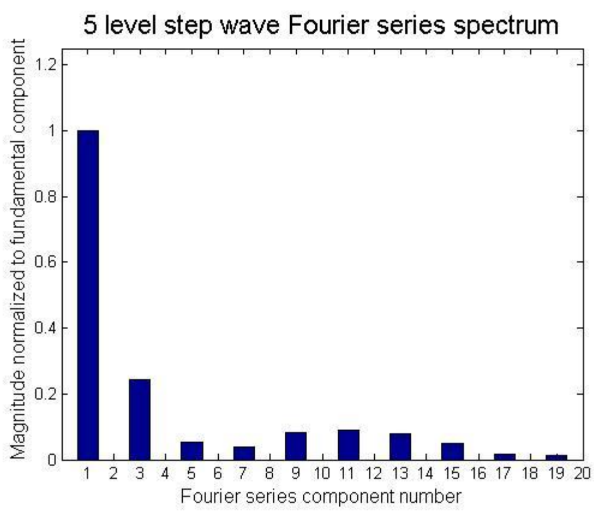

Figure 2.6: Fourier Series Magnitude Spectrum of a 5-level Step Wave

A quick point to note is that the magnitude of the non-fundamental Fourier series components decrease significantly compared to those of a square wave spectrum in Figure 2.2.

In general, for an N-level step wave, using Equation 2.20b, the Fourier series is expressed as follows:

$$f(\omega t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{4}{n\pi} \left(\sum\nolimits_{y=1}^{x} A_y \cos(\alpha_y n)\right) \sin(\mathrm{n\omega t}) \qquad (2.21)$$

In Equation 2.21, x is the number of firing angles required for an N-level step wave and is calculated as follows:

$$x = \frac{(N-1)}{2} \qquad (2.22)$$

For a voltage or current waveform, if the only component of interest is the fundamental frequency component ($n = 1$), then the total magnitude of higher frequency components ($n > 1$) are compared with that of the fundamental frequency component to calculate Total Harmonic Distortion. This concept is discussed in Section 2.1.4.

2.1.4 Total Harmonic Distortion (THD)

If a periodic voltage waveform $v(t)$ is expressed in terms of its Fourier series in the form:

$$v(t) = V_{DC} + \sum_{n=1}^{\infty} V_n \angle \varphi_n \qquad (2.23)$$

The fundamental frequency component $(V_1 \angle \varphi_1)$ is the component of interest. The remainder of the voltage waveform $(\sum_{n\neq 1}^{\infty} V_n \angle \varphi_n)$ is the summation of undesired voltage distorting components referred to as harmonics. The undesired n^{th} order component is referred to as the n^{th} harmonic. Assuming no DC offset, the root mean square (RMS) value of $v(t)$ can be expressed in terms of its Fourier series components as follows:

$$V_{rms}^2 = \sum_{n=1}^{\infty} V_{n,rms}^2 \qquad (2.24)$$

The RMS value of the n^{th} harmonic ($V_{n,rms}$) can be calculated using Equation 2.11 as follows:

$$V_{n,rms} = \frac{V_n}{\sqrt{2}} \qquad (2.25)$$

Rearranging Equation 2.24 and taking the fundamental frequency voltage component out of the summation term ($\sum_{n=1}^{\infty} V_{n,rms}^2$), V_{rms} can be expressed as follows:

$$V_{rms} = \sqrt{V_{1,rms}^2 + \sum_{n=2}^{\infty} V_{n,rms}^2} \qquad (2.26)$$

The magnitude of the voltage distortion($\sum_{n=2}^{\infty} V_{n,rms}$) can be expressed by rearranging Equation 2.26 as follows:

$$|\sum_{n=2}^{\infty} V_{n,rms}^2| = \sqrt{V_{rms}^2 - V_{1,rms}^2} \qquad (2.27)$$

Voltage distortion is measured in terms of Total Harmonic Distortion (THD). THD is the ratio of the magnitude of the distortion components to the magnitude of the fundamental frequency component and is expressed as follows [1]:

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty} V_{n,rms}^2}}{V_{1,rms}} \qquad (2.28a)$$

In terms of the RMS value of the voltage waveform, using 2.27, THD can be rewritten as follows:

$$THD = \sqrt{\left(\frac{V_{rms}}{V_{1,rms}}\right)^2 - 1} \qquad (2.28b)$$

Using Equation 2.28b, THD of the square and step waveforms discussed in Section 2.1.3 are summarized in Table 2.1. The results were obtained using the MATLAB code in Appendix A1.

Waveform	V_{rms}	$V_{1,rms}$	**THD**
Square wave	1	0.9003	0.4834
Square wave with $\alpha = \frac{\pi}{6}$	1	0.9549	0.3108
5-level step wave with $\alpha_1 = \frac{\pi}{6}, \alpha_2 = \frac{\pi}{3}$	1	0.9520	0.3192

Table 2.1: THD of Various Waveforms

The contribution of individual harmonic orders to voltage distortion can be expressed as distortion percentage of a particular order n and is calculated as follows:

$$Distortion\ \%\ of\ harmonic\ order\ n = \frac{V_{n,rms}}{V_{1,rms}} \qquad (2.29)$$

2.2 Harmonics in Power Systems

2.2.1 Sources

Any component connected to the electric power system which distorts system voltage, or draws or injects a non-sinusoidal current waveform is a harmonic source. The most common sources of harmonics are electric power converters and arc furnaces [2]. While inverters (DC to AC converters) are not as significant a harmonics source as rectifiers (AC to DC converters), the potential to inject significant harmonics current exists, especially if a large number of inverters are connected to an electric system [9][10]. The focus of this thesis will be the harmonic characteristics of inverters and utilizing multilevel inverters to reduce output voltage harmonics of PV systems feeding power to the grid.

2.2.2 Detrimental Effects

The detrimental effects of harmonics range from nuisances to equipment failures. Some of the most common effects are listed below:

Power Quality Degradation [2]: It is the utility companies' responsibility to deliver power of reasonably high quality. If significant harmonics are present in electricity delivered by a utility, power quality is degraded. Customers with sensitive loads would have to face the nuisance and cost of purchasing equipment to improve power quality to a reasonable level for sensitive loads such as computers, data servers amongst various other electronic equipment and consumer electronics. This would reflect poorly upon the ability of the utility company to deliver a product of high quality.

Resonance Conditions [8]: The reactive nature of electric systems may create resonance conditions for certain harmonic frequencies. At high frequencies, impedance of capacitors decrease and the impedance of inductors increase. This may create resonance conditions for certain harmonic orders and promote large currents at their frequencies. Large harmonic currents can lead to the following undesirable consequences:

- Equipment failures
- Equipment overheating
- Measurement equipment errors
- Protection and control nuisances (tripping of breakers)
- Protection and control equipment malfunctions

Heating [8]: Heating losses can be approximated as follows:

$$heating\ losses = I^2R \qquad (2.30a)$$

This expression can be expanded to include harmonic current contributions as follows [8]:

$$heating\ losses \cong I_1^2 R_1 + \sum_{n=2}^{\infty} I_n^2 R_n \qquad (2.30b)$$

Aside from the expected heat generated by the fundamental frequency current, additional heat is generated due to the presence of harmonic currents ($\sum_{n=2}^{\infty} I_n^2 R_n$). For equipment rated at the fundamental frequency, to operate it at its specified equipment rating, the equipment may have to be derated to account for additional heat losses due to harmonics.

Neutral Currents: Harmonic components present in a single phase or unbalanced harmonic components present in multiple phases of a three phase system would cause voltage unbalance. Voltage unbalance causes current flow in the neutral wire. Under ideal system operating conditions, neutral wire current should be zero. Current flow due to the presence of harmonics causes heating losses in the neutral wire.

Noise [8]: Magnetic fields created by harmonic currents at high frequencies can induce voltages in nearby circuits. The induced current or voltage may affect communication circuits, cause protection and control relay malfunctions, or simply introduce noise in whichever circuit is affected.

2.2.3 Limits

IEEE Standard 519 and IEC 61000-3 are two widely accepted power quality standards that provide the basis for the electric power industry to establish their harmonic limits. The standards specify the allowable %THD for both current and voltage harmonics at the point of common coupling. The limits are also further divided into harmonic order ranges. IEEE Standard 1547 also provides harmonic limits for connection to distribution systems derived from IEEE Standard 519.

2.3 Filtering Techniques

2.3.1 Passive Filtering

Passive filters use a combination of capacitors and inductors to filter harmonics. One simple passive filter is a line filter, which is a reactor connected in series with the harmonic source [2]. The reactor acts as a low pass filter, providing a low impedance path for the fundamental frequency, and a high impedance path for high frequency harmonics. The most common filter used is a shunt passive filter, which is a filter connected in parallel with the harmonic source [2]. The main principle behind the technique is to provide a path with high impedance for the fundamental frequency, and a low impedance path for high frequency harmonics to ground. Passive filtering is a suitable choice for systems in which the voltage waveform is constant because of its simple design. The components required for this filtering technique include capacitor banks, reactors and disconnects. These components are quite basic, making passive filters a suitable for high power applications as well.

2.3.2 Active Filtering

Active filtering is a strategy which monitors output current and injects an adaptive waveform to eliminate distortion or harmonics. While the obvious benefit of this filtering technique is that it is adaptive to changing harmonic conditions, the heavy usage of semiconductors and high frequency switching makes it unsuitable for high power applications [2]. This is due to switching frequency limitations of switches with high power ratings. Active filtering is an effective filtering option for changing load conditions, but for the purpose of filtering a fairly constant voltage waveform such as a PV system output voltage, passive filtering is a simpler and more economical choice.

2.3.3 Hybrid Filtering

Hybrid filtering utilizes both passive and active filters. Applications which typically utilize this filtering technique consist of fixed harmonic conditions for which passive filters are used, and smaller changing harmonic elements for which adaptive or active filtering is used [2].

CHAPTER III

INVERTERS

3.1 Applications in Power Systems

Inverters are used to convert DC voltage into AC voltage. Typical applications for inverters in power systems include interconnection of DC voltage sources to the electric grid, backup battery voltage conversion, buffer in AC-AC conversion, and AC motor speed control. The focus of this thesis is the use of inverters to convert DC power generated by PV arrays to AC for interconnection to the electric grid. The theory of single phase inverters will first be discussed in Section 3.2 and then extended to three phase inverters in Section 3.3.

3.2 Single Phase Inverters

The basic theory of operation of inverters is to alternate the polarity of an input DC voltage to a positive voltage for half a period and a negative voltage for the second half of the period across output terminals. Two inverter topologies are single-voltage source inverters and multilevel inverters discussed in Sections 3.2.1 and 3.2.2 respectively.

3.2.1 Single-Voltage Source Inverters

A single-voltage source inverter utilizes one DC voltage source, the polarity of which is alternated between positive and negative voltage for half a period each at the output terminals.

The most common circuit used is the H-bridge inverter shown in Figure 3.1.

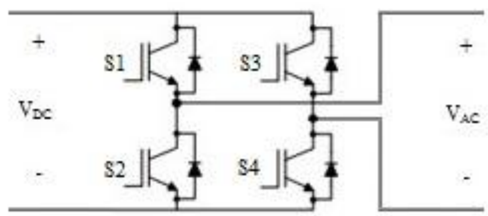

Figure 3.1: H-Bridge Inverter

The inverter consists of four switches S1 to S4, the operation of which is summarized in Table 3.1. GTO thyristors and SCRs are typically used for high power applications.

Switch S1	Switch S2	Switch S3	Switch S4	Output (V_{AC})
ON	OFF	OFF	ON	V_{DC}
OFF	ON	ON	OFF	$-V_{DC}$
OFF	ON	OFF	ON	0V
ON	OFF	ON	OFF	0V

Table 3.1: H-Bridge Inverter Operation Modes

The most common modes of operation are Square Wave Operation and Pulse Width Modulation which are discussed as follows [7]:

Square Wave Operation: Under square wave operation, the output waveform is $+V_{DC}$ for half a cycle and $-V_{DC}$ for the second half. The output voltage waveform resembles a unit square wave described by Equation 2.16 but with a magnitude of V_{DC}. The output voltage waveform of a square wave inverter with magnitude V_{DC} and a frequency of 60Hz is shown in Figure 3.2.

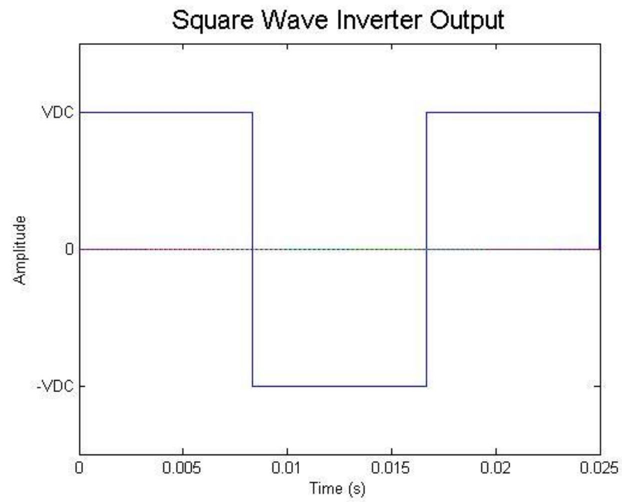

Figure 3.2: Square Wave Inverter Output Voltage with f = 60Hz

The RMS value of the output waveform V_{rms} with a period T and magnitude V_{DC} is computed as follows:

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T {V_{DC}}^2 \, dt} \qquad (3.1)$$

Using Equation 3.1, the RMS value for a square wave is $V_{rms} = V_{DC}$.

While the obvious advantage of this scheme is its simplicity and low switching frequency, the biggest disadvantage is the presence of significant harmonic components.

The Fourier series of this waveform from Equation 2.17b is expressed as follows:

$$v(t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{4V_{DC}}{n\pi} \sin(\mathrm{n\omega t}) \qquad (3.2)$$

The Fourier series magnitude plot is the same as that of a square wave as shown in Figure 2.2.

It is apparent that the large magnitudes of low harmonic orders require heavy filtering. This can also be inferred intuitively by the fact that the output waveform lacks smoothness and bears little resemblance to a sinusoidal waveform except its symmetry.

THD can be calculated using Equation 2.28b, and is equal to 0.438 from Table 2.1.

Pulse Width Modulation: Pulse width modulation (PWM) is the usage of a high frequency switching technique which allows control over the output voltage of an inverter. The basic concept of PWM is the comparison of a reference signal to that of another waveform, typically triangular, to control the gating signals for inverter switches. Two widely used PWM techniques are Uniform PWM and Sinusoidal PWM.

Uniform PWM: A constant reference voltage ($v_{ref}(t)$) to a triangular waveform ($v_{tri}(t)$) to produce pulses with the same width at the inverter output ($v_o(t)$). The pulses have a magnitude of $+V_{DC}$for the first half cycle and $-V_{DC}$ for the second half of the cycle. This principle is illustrated in Figure 3.3 [5].

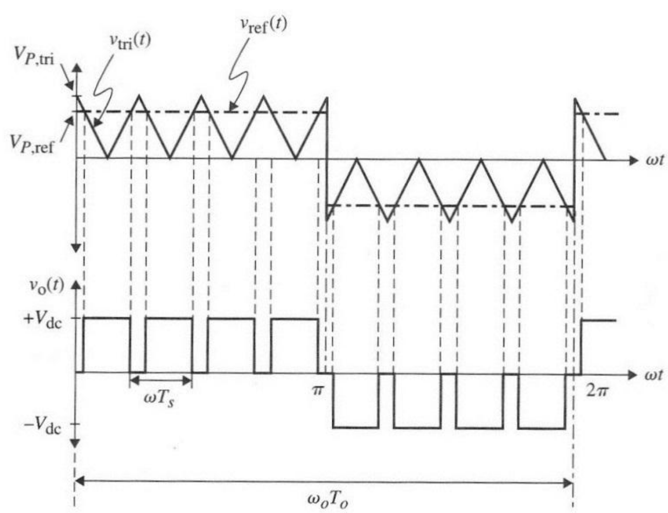

Figure 3.3: Uniform PWM
Source: [5] pp.458, Figure 9.28

The principle of operation is as follows:

For the positive part of the cycle:

$$v_o(t) = V_{DC} \; when \; |v_{ref}(t)| > |v_{tri}(t)| \qquad (3.3a)$$

$$v_o(t) = 0V \; when \; |v_{ref}(t)| < |v_{tri}(t)| \qquad (3.3b)$$

For the negative part of the cycle:

$$v_o(t) = -V_{DC} \; when \; \left|v_{ref}(t)\right| > |v_{tri}(t)| \qquad (3.3c)$$

$$v_o(t) = 0V \; when \; |v_{ref}(t)| < |v_{tri}(t)| \qquad (3.3d)$$

The RMS value of the output voltage ($v_o(t)$) is the summation of the RMS values of all the pulses in a period. The output voltage can be controlled by changing the reference voltage (v_{ref}), which would change the width of the pulses.

Sinusoidal PWM: For a sinusoidal PWM scheme, the reference voltage is a sinusoidal waveform instead of a constant value. The same principle of operation described by Equation 3.3 is used for comparison of the reference waveform to a triangular waveform. This principle is illustrated in Figure 3.4 [5].

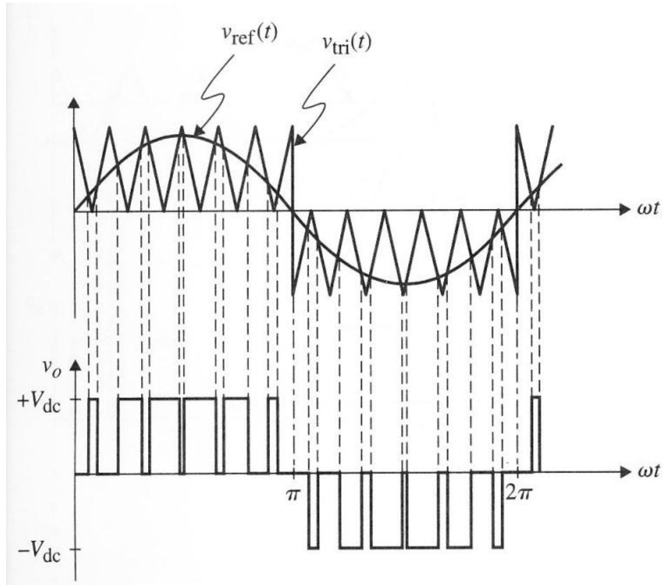

Figure 3.4: Sinusoidal PWM
Source: [5] pp.459, Figure 9.29

PWM Harmonics: The harmonics of PWM voltage pulses is simply the summation of the harmonics of each individual periodic pulse. In terms of Fourier series, the principle of superposition discussed in section 2.1.3 applies. While there is greater control over the magnitude and shape of the output voltage waveform allowing greater harmonic elimination control, the algorithm can get complex. The focus of this paper is non-PWM inverters, the justification for which is provided in the next subsection.

The Case Against PWM: The switching frequency of PWM is much higher than that of square wave operation. In low power operation where high switching devices such as IGBTs can be utilized, PWM would be the choice of operation due to higher versatility in harmonic elimination. Higher power applications such as small and medium PV systems use inverters with high switch power ratings but low frequency switching capabilities [13][14]. GTO thyristors have been the technology of choice for 2kV to 13.8kV inverters [5]. SCR is another technology which can be used for high power applications. Figure 3.5 shows a plot of switching frequency versus power rating for several different switching devices [5].

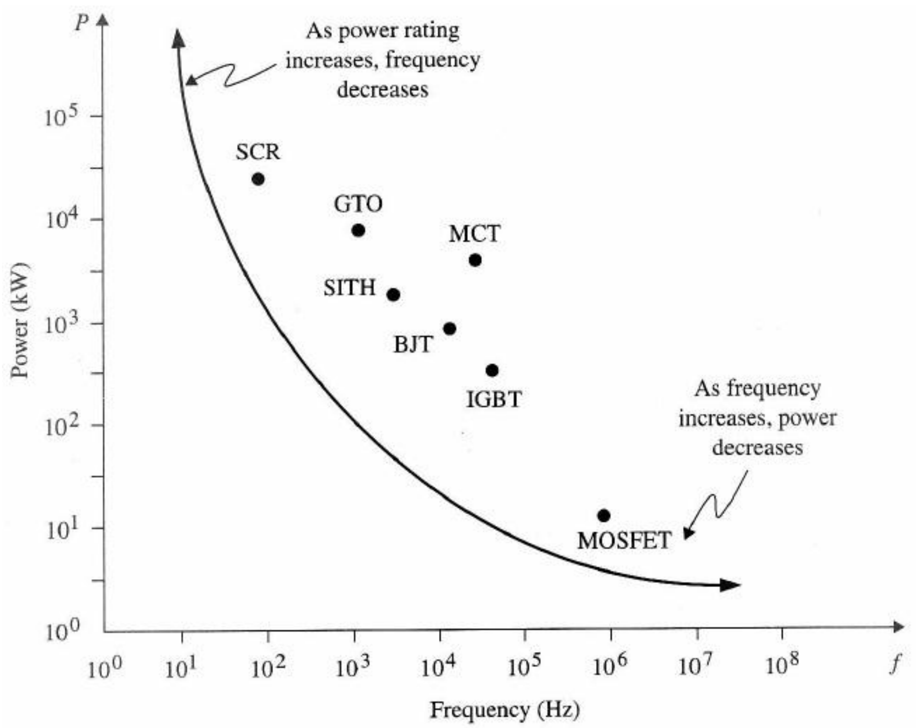

Figure 3.5: Switching Frequency vs. Power for Various Switching Devices
Source: [5] pp.68, Figure 2.40

The generation voltage of PV modules is typically maintained within a small range. There is no need to control and vary the output voltage of a PV system since it has to match the electric grid voltage to which it is connected. The usage of PWM, which allows voltage control would only add complexity to the system. Higher frequency switching also results in greater switching losses and stresses which are more significant when dealing with high power applications. It is thus more preferable to use square wave operation or block switching (discussed in Section 3.2.2) with lower switching frequencies [13][14]. The focus of this paper will therefore be square wave and block switching operation of inverters for high power PV systems. The control algorithm for the proposed system will maintain the PV cell generation voltage within a small range, making inverter input and output voltage fairly constant. This further eliminates the need for output voltage control provided by PWM.

3.2.2 Multilevel Inverters

Multilevel inverters utilize multiple DC voltage sources to create an output voltage with a step waveform as described in section 2.1.3. The obvious benefit of this inverter scheme is that the output voltage resembles a sinusoidal waveform more than a square wave, hence containing lower harmonic content. As the number of voltage levels increases, the harmonic contents decrease greatly, especially low order ones. The most significant drawback of increasing the number of voltage levels is a significant increase in the number of components, gating signal complexity, and cost. A multilevel inverter can be realized by connecting multiple H-Bridge inverters in series [5]. The circuit diagram of a multilevel inverter with N-levels is shown in Figure 3.6.

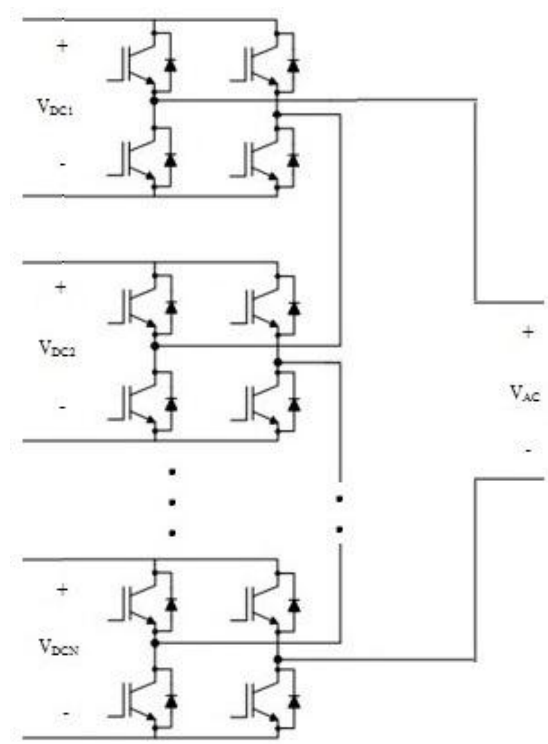

Figure 3.6: N-level H-Bridge Inverter

The most basic multilevel inverter is a 3-level inverter, which uses the same circuit as a square wave inverter shown in Figure 3.1. The only difference between the two is that a 3-level inverter uses $0V$ as a third voltage level in addition to $+V_{DC}$ and $-V_{DC}$. The output voltage waveform resembles that of a square wave with a firing angle greater than zero as shown in Figure 2.3.

In general, to realize an N-level inverter, x DC sources are required. The value of x is calculated as follows:

$$x = \frac{(N-1)}{2} \qquad (3.4)$$

Equation 3.4 is the same as Equation 2.22 since there is one firing angle associated with each DC voltage source. The circuit diagram of a 5-level inverter is shown in Figure 3.7.

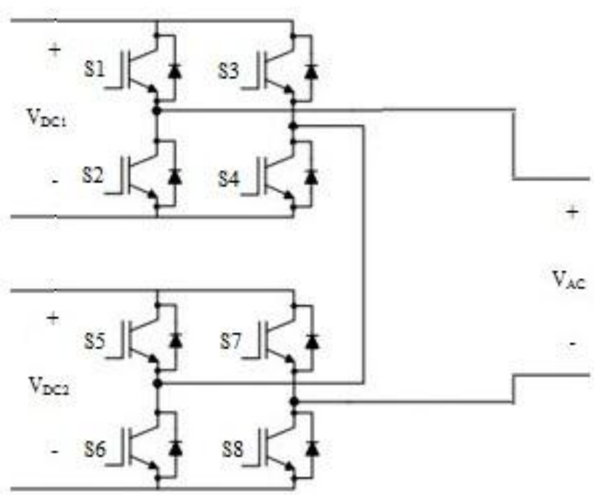

Figure 3.7: 5-level Inverter

This inverter consists of eight switches S1 to S8, the operation of which is summarized in Table 3.2.

Switch S1	Switch S2	Switch S3	Switch S4	Switch S5	Switch S6	Switch S7	Switch S8	Output (V_{AC})
ON	OFF	OFF	ON	OFF	ON	OFF	ON	V_{DC1}
ON	OFF	OFF	ON	ON	OFF	OFF	ON	$V_{DC1}+V_{DC2}$
OFF	ON	ON	OFF	OFF	ON	OFF	ON	$-V_{DC1}$
OFF	ON	ON	OFF	OFF	ON	ON	OFF	$-(V_{DC1}+V_{DC2})$
OFF	ON	OFF	ON	OFF	ON	OFF	ON	0V
ON	OFF	ON	OFF	OFF	ON	OFF	ON	0V

Table 3.2: 5-level Inverter Operation Modes

As is the case with single-voltage source inverters, multilevel inverters can operate with block switching or PWM.

Block Switching Operation [6]: Under block switching operation, the output voltage waveform of a 5-level inverter resembles the 5-level step wave shown in Figure 2.5. The magnitude of each step is the voltage of the DC voltage sources V_{DC1} and V_{DC2} for levels 1 and 2 respectively. The firing angles for each level are $\alpha_1 = \frac{\pi}{8}$ and $\alpha_2 = \frac{\pi}{4}$, corresponding to the DC voltage source number. The output voltage waveform for a 5-level inverter is shown in Figure 3.8.

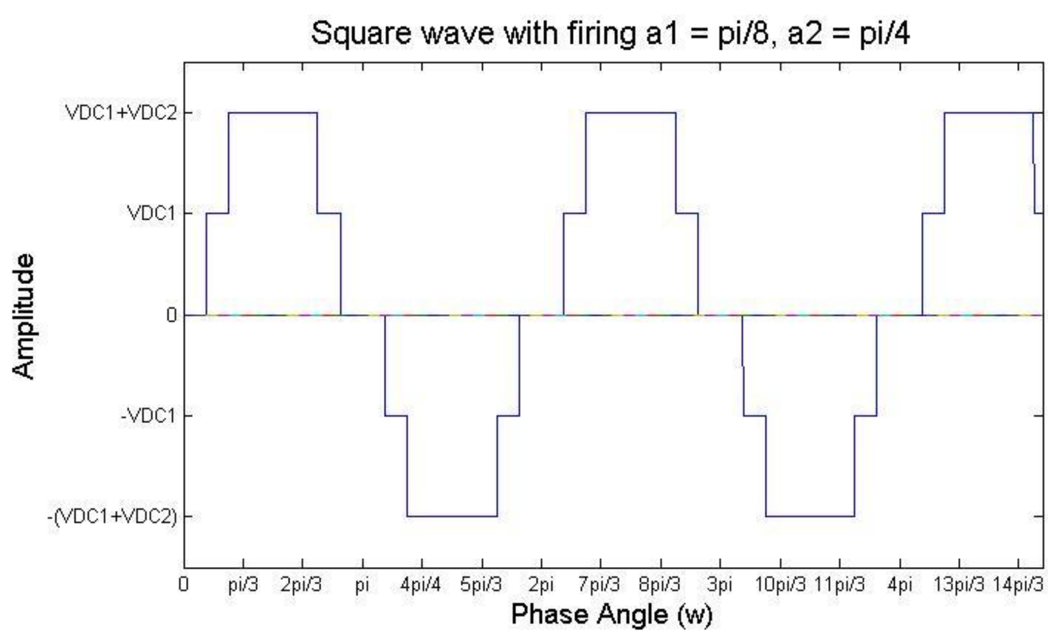

Figure 3.8: 5-level Inverter Output Voltage

Using Equation 2.21, the Fourier series of an N-level inverter output voltage is expressed as follows:

$$v(t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{4}{n\pi}\left[\sum\nolimits_{y=1}^{x} V_{DCy}(\cos(\alpha_y n))\right]\sin(\mathrm{n\omega t}) \qquad (3.5)$$

The output voltage waveforms of multilevel inverters resemble a sine wave more than that of a square wave inverter. Based on greater resemblance to a sine wave, it can be intuitively inferred that their harmonic contents are lower. The performance results of multilevel inverters will be discussed in Chapter VI.

Pulse Width Modulation: There are several PWM techniques for multilevel inverters with the same operating principle as that of a single-voltage source inverter. The complexity of the gating signal algorithm is much higher due to the additional number of levels and thus switches. While the harmonic elimination versatility is much greater, the switching frequency limitation of high power switches and the simplicity of PV system application make it a less favorable choice [13][14]. As mentioned earlier in Section 3.2.1, square wave and block switching will be the only modes of operation discussed in this thesis.

3.3 Three Phase Inverters

Three phase inverters are essentially an extension of single phase inverters. Three single phase inverters are utilized, one per phase, and their gating signals are shifted by 120°. The circuit diagram of a three phase, N-level inverter is shown in Figure 3.9.

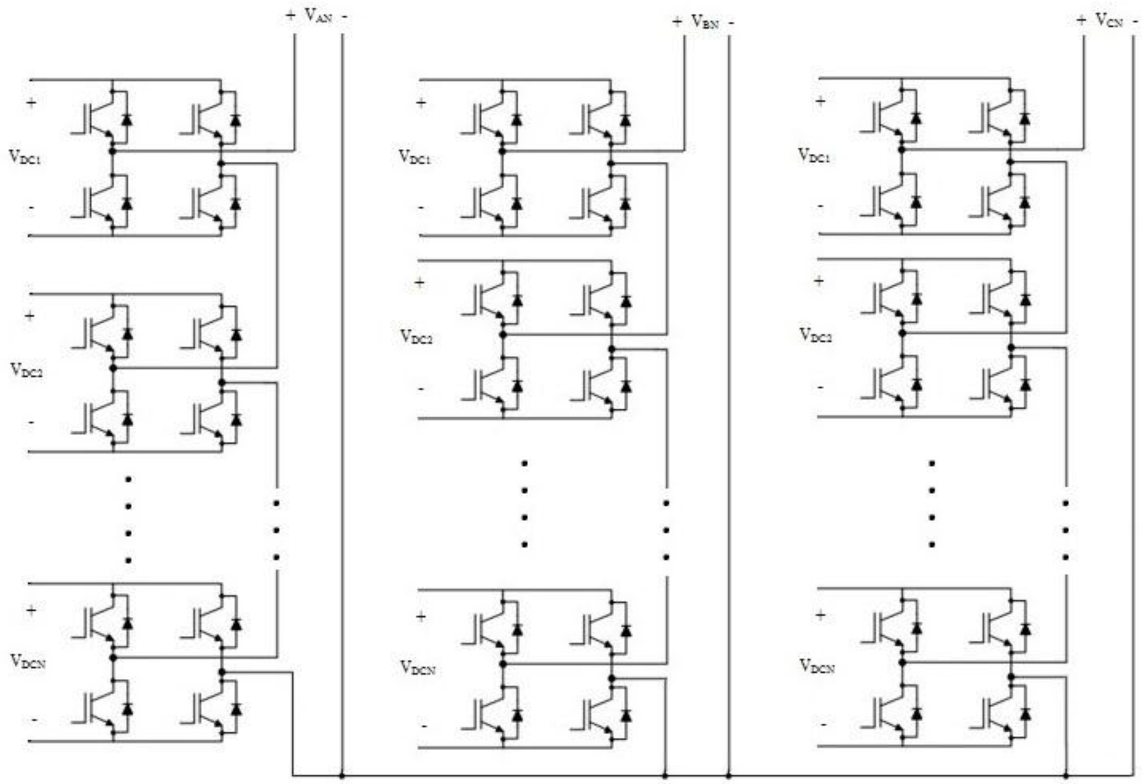

Figure 3.9: 3-Phase, N-level Inverter

The circuit diagram of a 3-phase, 5-level inverter is shown in Figure 3.10.

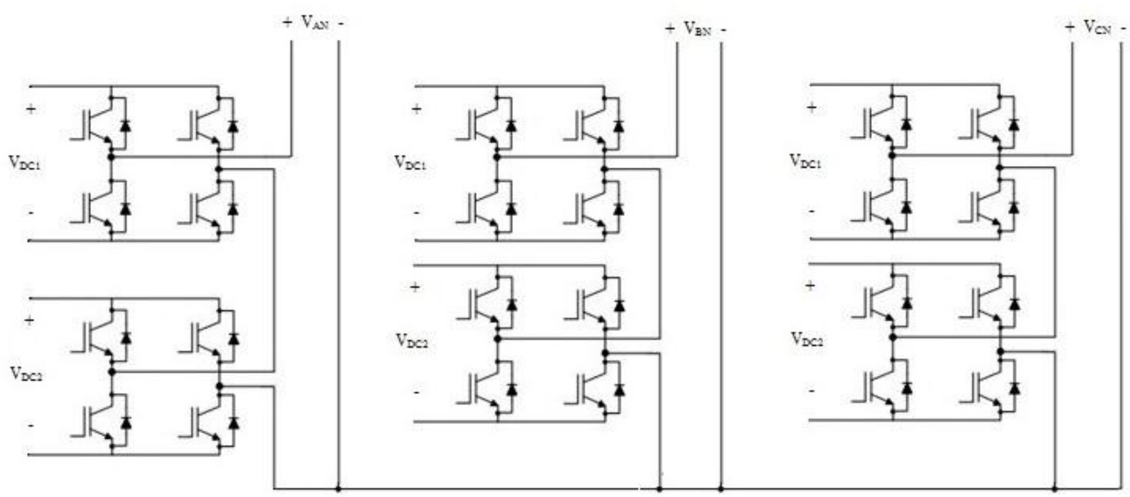

Figure 3.10: 3-Phase, 5-level Inverter

The output voltage waveform of a 3-phase, 5-level inverter with a frequency of 60Hz is shown in Figure 3.11.

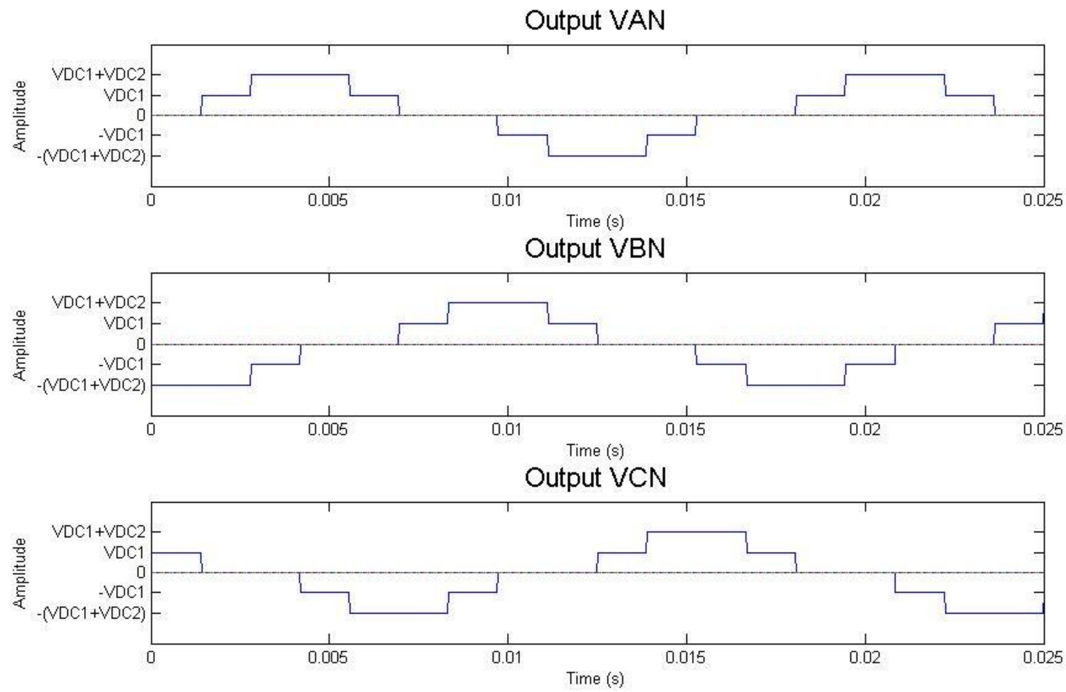

Figure 3.11: Output Voltage of a 3-Phase, 5-level Inverter with f = 60Hz

CHAPTER IV

PHOTOVOLTAIC SYSTEMS

4.1 Overview

Photovoltaic or PV systems convert incident solar energy (insolation) into low-voltage DC electricity. Solar energy is a very attractive ecofriendly generation option due to the fact that it emits virtually zero greenhouse gases and the abundance of insolation available for harnessing. It is estimated that $1.2\ x\ 10^{11}\ MW$ of solar power is received by the Earth's surface on average which is more than enough to generously meet all the electricity needs of the human population for years [4]. Other benefits of PV systems are that they contain very few moving parts, are very reliable and require minimal maintenance [3][4]. The low maintenance requirements make PV generation a very viable option for remote locations such as deserts and offshore sites. Studies have even gone as far as exploring the option of installing solar panels in space. The main drawbacks of PV generation are the high capital cost, intermittence due to inconsistent irradiance (day and night), and low efficiency of energy conversion. Increasing PV system efficiency and lowering generation costs are some of the most researched areas in the field. The research efforts are fueled by the recent increase in investment in the technology.

A typical PV system configuration is shown in Figure 4.1.

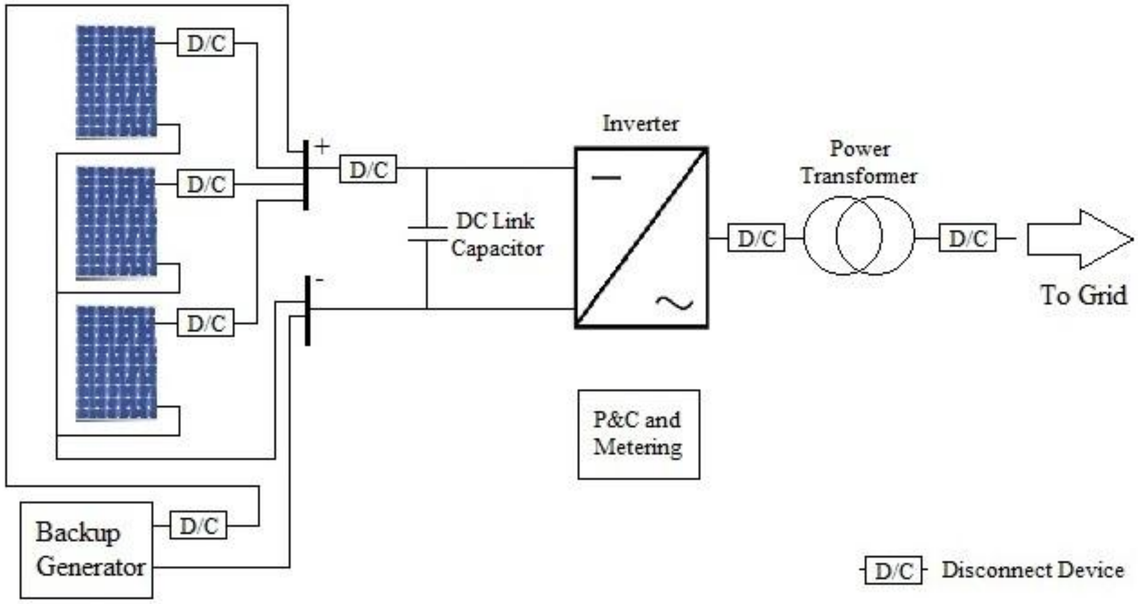

Figure 4.1: Typical PV System Schematic

The major components of PV systems are listed below along with brief descriptions.

PV cells: Convert solar energy into electrical energy using semi-conductor technology. The theory and modeling of PV cells is discussed in the Section 4.2.

PV panels: A collection of PV cells connected in series and parallel. Panels can be treated as one large PV cell for modeling purposes.

DC Bus: Common conductor to which all PV panels, backup generators and energy storage devices are connected.

DC Link Capacitor: Used to stabilize DC voltage at inverter input. Switching transients from either side of the capacitor may cause voltage ripples or even lead to inadvertent tripping of breakers if their magnitude is large enough. DC voltage is thus stabilized by DC link capacitors.

Inverter: Convert DC voltage produced by PV arrays to AC voltage. Small and medium PV generators typically use square wave inverters as described in Section 3.2.1.

Step-up transformer: Steps up inverter output voltage to electric grid voltage. This transformer also provides isolation from the electric grid.

Disconnects: Disconnects labeled D/C are used for isolation of devices and for protection and control purposes. Switches are typically used for isolation purposes, and fuses and breakers for overcurrent protection.

Backup Generator: A backup generator can be used to mitigate the intermittence of PV generation. If a battery backup system is used, it can also be used to store electrical energy.

Protection and Control, and Metering Equipment: Protection and Control equipment is used to monitor any anomalies which can damage equipment and initiate the appropriate response such as opening disconnects as per the protection scheme. Control circuitry is also used to control inverter and bus voltages with changing insolation conditions. Measurement equipment such as Current Transformers (CTs) and Potential Transformers (PTs) are used to measure system parameters such as bus and inverter input and output voltages for protection and control purposes. Metering equipment is used for the financial aspect of PV generation which is to measure total power output of the system for payment, and billing of electricity used from the grid.

4.2 Modeling PV Cells

In simple terms, a PV cell is a large area of a *p*-type and *n*-type semiconductor junction (*pn* junction) with suitable connections. The principles of PV cells can be explained using an energy band diagram of a semiconductor illustrated in Figure 4.2.

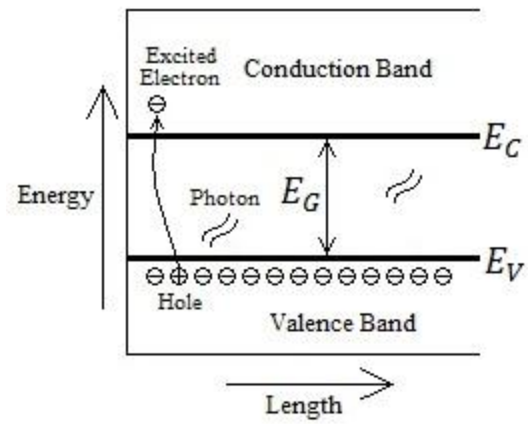

Figure 4.2: Semiconductor Energy Band Diagram

When light is incident on a semiconductor, photons with energy greater than the band gap energy (E_G) are absorbed electrons in the valence band. These electrons initially have less energy than the top of the valence band (E_V). E_G is the minimum amount of energy required for an electron to absorb and move to the conduction band. The excited electrons which gain more energy than the bottom of the conduction band energy (E_C) break free from their covalent bonds in the semiconductor structure and move to the conduction band. When an electron moves to the conduction band, a hole is created where the electron was bound; this pair is referred to as an electron-hole pair. PV cells operate on this principle by using a *pn* junction, which is essentially the junction of a *p*-type and *n*-type semiconductor. E_V and E_C have higher values for a *p*-type semiconductor. PV cell operation can be explained using an energy band diagram of a *pn* junction illustrated in Figure 4.3.

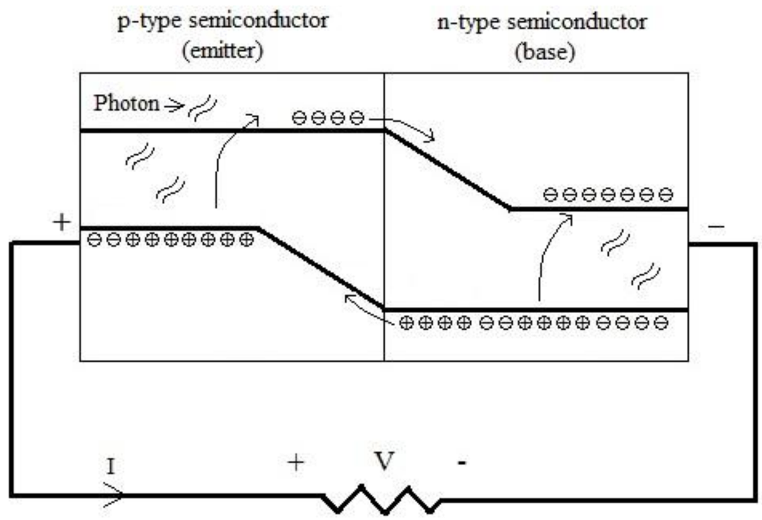

Figure 4.3: Operation of a PV Cell

When electron-hole pairs are created by the absorption of photons from the incident light, electrons in the p-type semiconductor and holes in the n-type semiconductor diffuse towards the junction or the depletion region. The junction field created by the composition of a *pn* junction transfers the electrons from the conduction band of the p-type semiconductor across the junction to the conduction band of the n-type semiconductor. The holes which are part of the electrons' electron-hole pairs are similarly transferred across the junction and collected in the valence band of the p-type semiconductor. This results in excess electrons in the base which gives the base a negative charge. Similarly, the excess holes in the emitter give it a positive charge. Under open circuit conditions, a DC voltage V_{OC} develops across the *pn* junction. If a current path is provided between the p-type and n-type semiconductor, current flows from the p-type semiconductor to the n-side semiconductor due to the voltage generated across the *pn* junction. I_{SC} is the short circuit current between the emitter and base. If a resistance R is connected between the emitter and base, the generation of DC voltage V across the emitter and base can provide useful DC current I as illustrated in Figure 4.3.

PV cells essentially operate as a diode with an externally imposed forward bias by the generated DC voltage. This is modeled by placing a current source in parallel with a diode. The equivalent circuit of a PV cell is shown in Figure 4.4 [3].

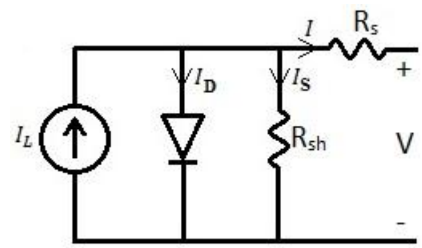

Figure 4.4: Equivalent Circuit of a PV Cell

Ignoring series resistance ($R_s = 0$) and shunt resistance ($R_{sh} = \infty$) the ideal V-I relationship of a PV cell is expressed as follows [3]:

$$I = I_L - I_O[\exp\left(\frac{qV}{kT}\right) - 1] \qquad (4.1)$$

In the Equation 4.1, I_O is the reverse saturation current of the cell, q is electron charge, k is Boltzmann's constant and T is the junction temperature. The V-I relationship is a non-linear one making the computation of either output voltage or current a fairly non-intuitive one. A V-I plot as shown in the Figure 4.5 can be used instead for analysis purposes.

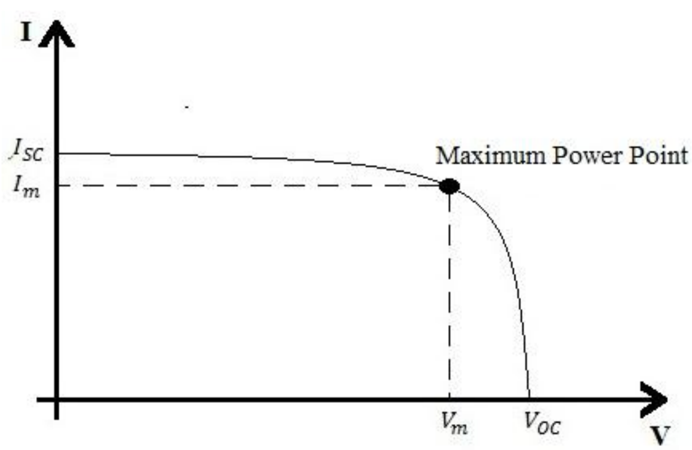

Figure 4.5: V-I Curve of a Typical PV Cell

The points of interest are I_{SC}, V_{OC} and P_{max} which are the short circuit current ($V = 0$), open circuit voltage ($I = 0$), and the maximum power point respectively.

The most important property of a PV cell is the effect on the V-I curve as irradiance changes. Change in irradiance effects all three factors of the V-I curve I_{SC}, V_{OC} and P_{max}. The effect of change in irradiance is illustrated in Figure 4.6.

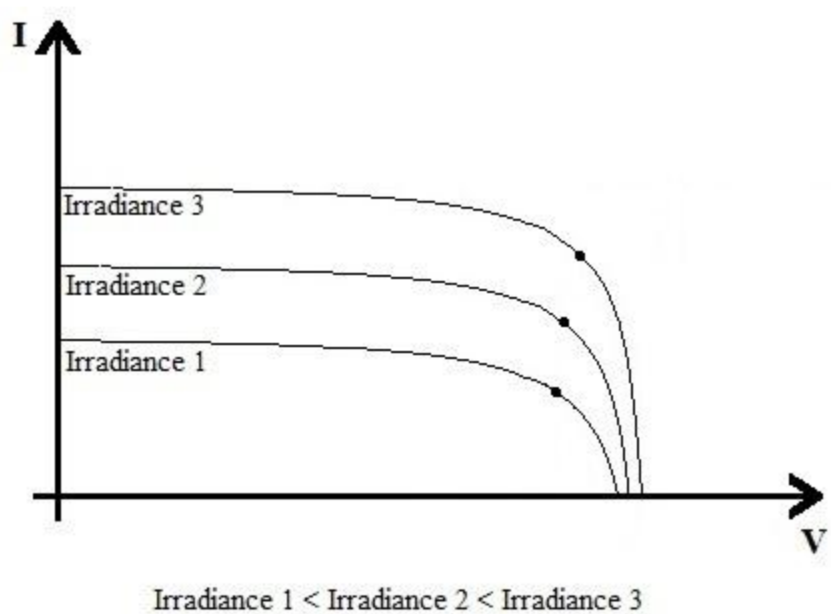

Figure 4.6: V-I Curves for Different Irradiance

It can be seen that V_{OC} fluctuates less than I_{SC} as irradiance changes. The advantage of the small fluctuation of V_{OC} with changing irradiance is that it allows operation of the PV cell close to the maximum power point within a small voltage range. This range is highlighted in Figure 4.7.

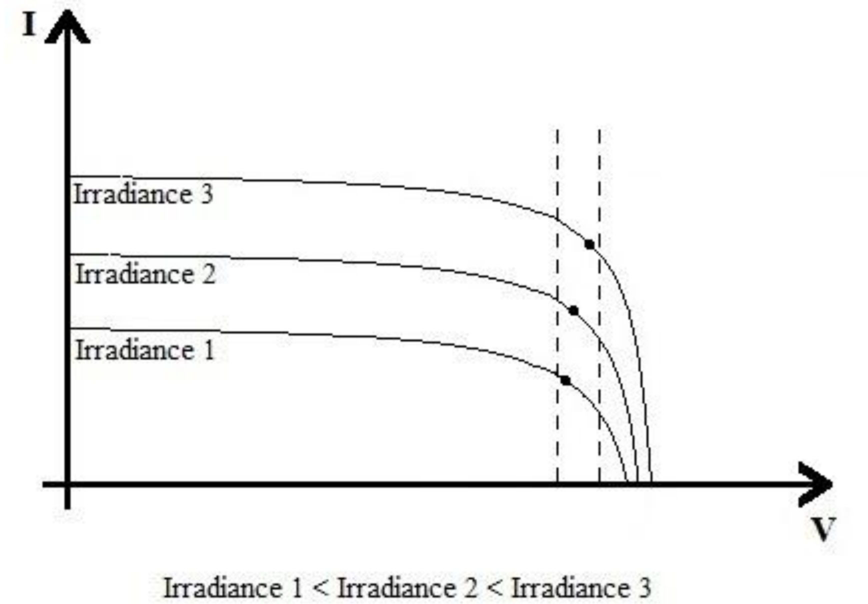

Figure 4.7: Range of Operating Voltage with Changing Irradiance

Losses associated with current flow is grouped together and represented as a series resistance R_S shown in Figure 4.4. The effects of different values of R_S on the V-I curve is illustrated in Figure 4.8.

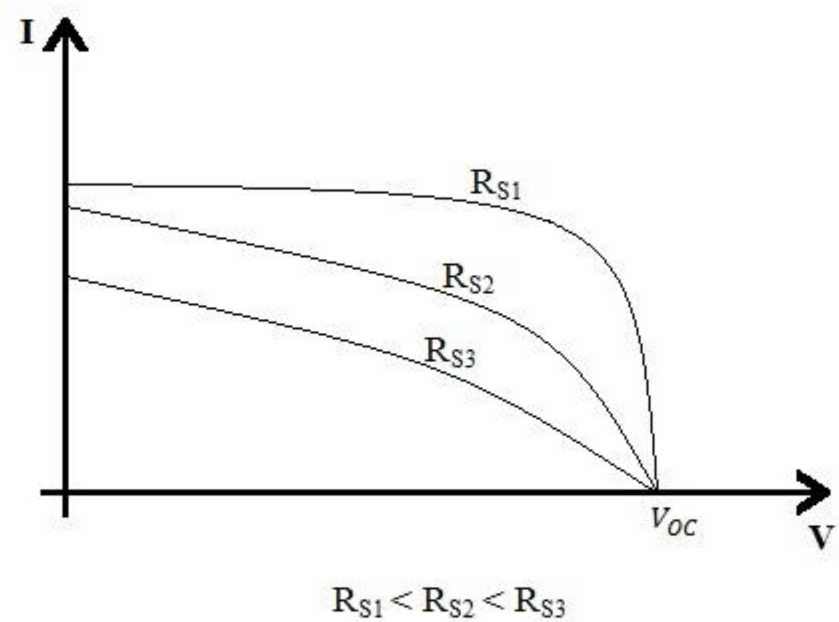

Figure 4.8: V-I Curves for Different Series Resistances

With the inclusion of series resistance R_S, the V-I relationship is expressed as follows [3]:

$$I = I_L - I_O[\exp\left(\frac{qV + IR_S}{mkT}\right) - 1] \qquad (4.2)$$

The variable m is the non-ideality factor with a value close to unity. As R_S changes, the shape of the V-I curve changes but the values of I_{SC} and V_{OC} remain unaffected. An intuitive variable Fill Factor (FF) helps describe the relative shape of the V-I curve and is defined as follows:

$$FF = \frac{P_{max}}{I_{SC} \text{ x } V_{OC}} \qquad (4.3a)$$

$$FF = \frac{I_m \text{ x } V_m}{I_{SC} \text{ x } V_{OC}} \qquad (4.3b)$$

I_m and V_m are the current and voltage at the maximum power point respectively. The larger the FF, the larger the maximum power is. Also, the larger the value of R_S, the smaller the FF is. The effects of different values of FF on the V-I curve is illustrated in Figure 4.9 [3].

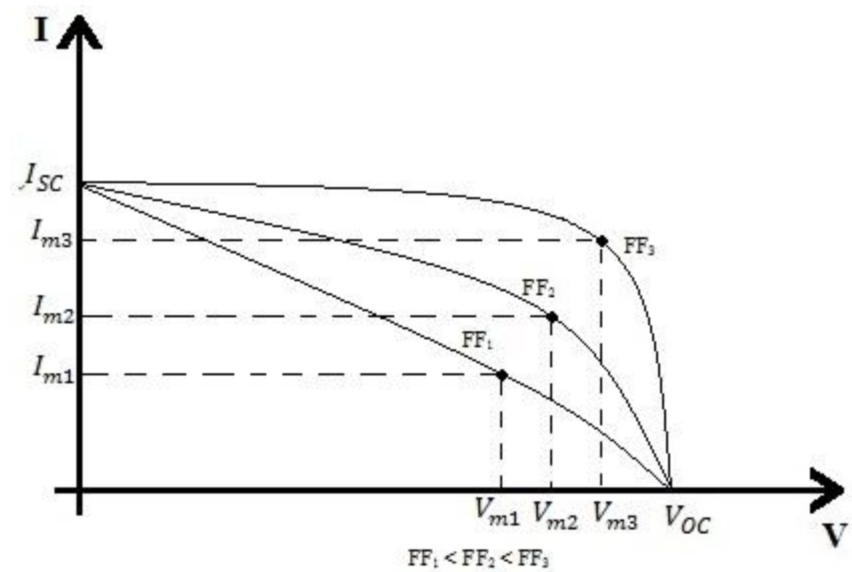

Figure 4.9: V-I Curves for Different Fill Factors

4.3 Interconnection to the Electric Grid

Connecting individual PV cells to the grid would require a large voltage step-up conversion from about 0.6V to the electric grid voltage. PV cells are thus connected in series and parallel to form a PV module with a higher output voltage. Modules are then connected in series and parallel to from PV panels, which are connected together to form arrays. Modules, panels and arrays are essentially larger blocks of PV cells. These arrays are connected to appropriate power electronic systems for power control and conversion to AC voltage for interconnection to either the electric grid or local loads. The equivalent circuit of a PV cell shown in Figure 4.4 is an accurate approximation of a larger number of cells connected together as a module [3]. The values of V_{OC} and I_{SC} change, but the general shape of the V-I curve remains the same for the approximation.

To control the power of a PV array, it is treated as a current source. If the bus voltage of a PV array is set to a certain value, the output current would be the corresponding value along the V-I curve described by equation 4.2 and shown in Figure 4.5. To maximize output power, generation voltage is maintained close to the maximum power point. The generation voltage of PV arrays can be controlled by a voltage source connected in parallel to it. Battery banks connected via DC-DC converters is a simple voltage control mechanism which allows energy storage as well. If the PV system is part of a hybrid system, other generation options such as diesel generators or gas turbines can also be used to control the common bus voltage.

Small (0.2-50 kW) and medium (50-1000 kW) PV systems typically use square wave inverters to convert DC voltage to AC voltage for interconnection to the grid. Using square wave inverters allows operation with a relatively simple control scheme. Inverter output is stepped up by a power transformer for interconnection to the electric grid. The transformer also provides system isolation from the grid.

4.4 Harmonics from PV Systems

Inverters and battery chargers are the most significant harmonic sources of PV systems [9]. Typical small and medium PV systems, as mentioned earlier, utilize square wave inverters for DC-AC voltage conversion. The biggest drawback of this scheme is the high harmonic content in the output voltage requiring significant filtering. Shunt passive filtering is the most common method of harmonic filtering in PV systems [2][11]. The main principle is providing a low impedance diversion path for harmonic currents. Multi-section filters utilize one shunt filter per harmonic order, tuned to the harmonic order frequency. Figure 4.10 shows a multi-section filter for a PV system targeting the 3rd, 5th and 7th harmonics orders. Z3, Z5 and Z7 are the impedances for the 3rd, 5th and 7th harmonic order filters respectively.

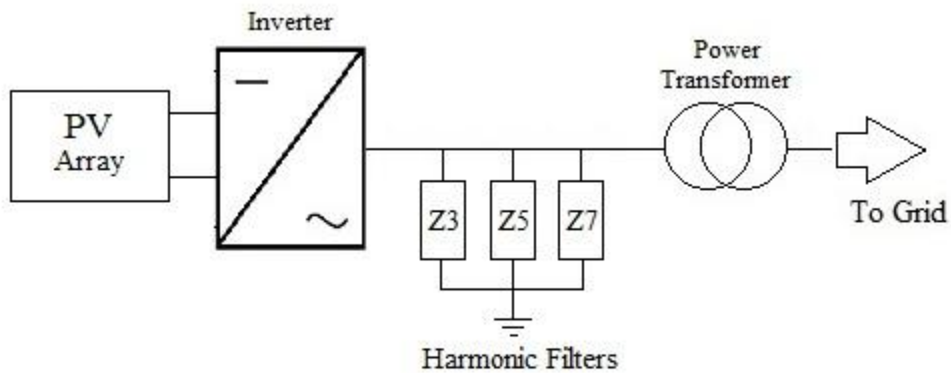

Figure 4.10: Multi-section Passive Filter

The greater the magnitude of harmonics of inverter output voltage is, the higher the filtering equipment has to be rated, resulting in higher filtering costs. The performance results of a PV system utilizing a square wave inverter are discussed in Chapter VI.

CHAPTER V

A MULTILEVEL INVERTER SCHEME FOR PV SYSTEMS

5.1 Overview

Using the concepts discussed in earlier chapters, a PV scheme which replaces the conventional square wave inverter with a multilevel inverter will be discussed. The major benefit of this strategy is lower inverter output voltage harmonics and consequently, lower filtering requirements [13][14][15]. Chapter V discusses the system, components and control strategy. Chapter VI presents the performance results, and compares them to square wave operation and filtering requirements of IEEE 519.

The basic block diagram of this scheme is shown in Figure 5.1.

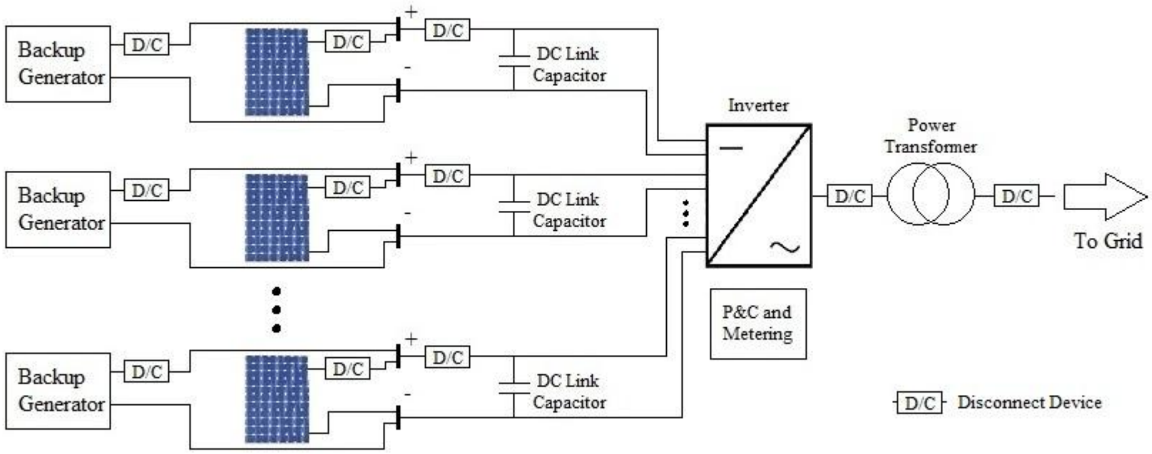

Figure 5.1: Block Diagram of a Multilevel Inverter PV System

The circuit diagram of the PV arrays' connection to DC buses is shown in Figure 5.2, followed by the circuit diagram of an N-level inverter to which the DC buses are connected, shown in Figure 5.3.

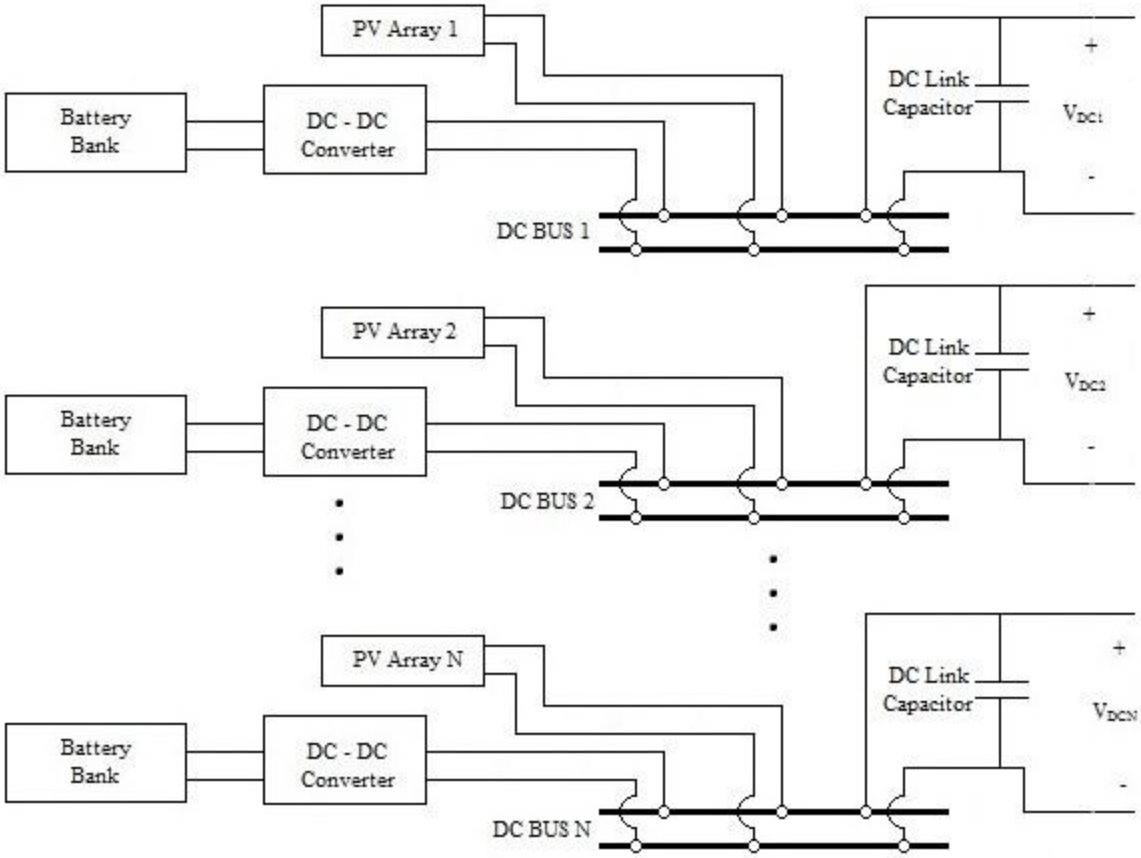

Figure 5.2: Circuit Diagram of N PV Arrays Connected to N DC Buses

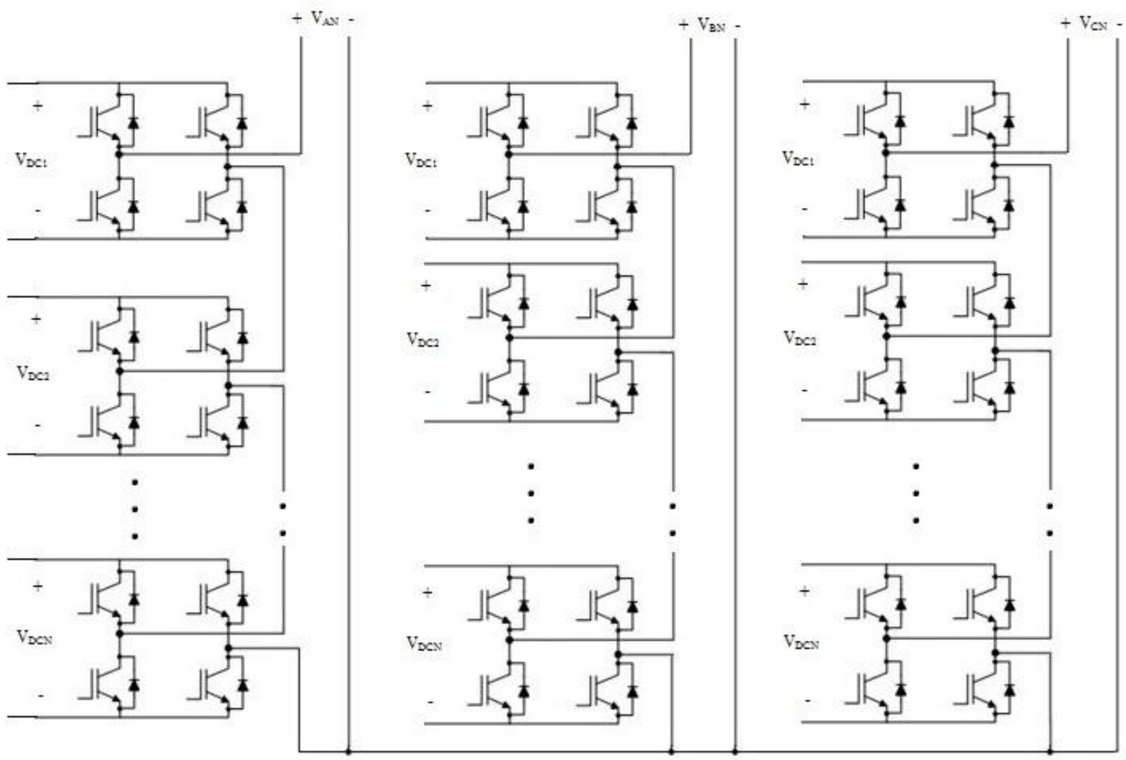

Figure 5.3: Circuit Diagram of an N-level 3-Phase Inverter

The basic characteristics of this scheme are discussed in the following subsections.

Multiple DC Voltage Sources: PV arrays are grouped into smaller modules to realize multiple DC voltage sources. The multiple PV array groups have a lower power rating than being grouped together as one large current source. In general, for an N- level inverter, x groups of PV arrays are required. The variable x is defined by Equation 3.4 as $\left[\frac{(N-1)}{2}\right]$.

Multiple Buses with Individual Bus Voltage Control: Each group of PV arrays is connected to a separate DC bus. The DC bus voltages may be fixed, or varied to operate the PV arrays at their maximum power point. Bus voltages will be controlled by backup power sources as shown in Figure 5.1. In general, x DC buses are required for a N-levels. The variable x is defined by Equation 3.4 as $\left[\frac{(N-1)}{2}\right]$.

Voltage Regulating Backup Power Source: The voltage of each individual DC bus will be controlled by a backup power source. The system shown in Figure 5.2 uses battery banks with DC-DC converters for this purpose.

A hybrid system with other generation sources may also be used to regulate the DC bus voltages. If constant power output is required from such a hybrid system, the alternate source can produce power when there is no insolation available. The alternate source may be an ecofriendly option such as a diesel generator using methane gas as its fuel, or even wind turbines. To integrate the backup generation to DC buses, additional appropriate power converters would be required.

Inverter: A multilevel inverter described in Section 3.2.2 is used for DC-AC voltage conversion. The focus of this thesis is to study the improvement of output voltage harmonics realized by using multilevel inverters instead of square wave inverters. The theory is discussed in the sections that follow and the results are discussed in Chapter VI.

5.2 Practical Considerations

When theory is extended to application, various practical factors have to be taken into consideration due to real life constraints. Several design considerations are discussed in subsections 5.2.1 to 5.2.4.

5.2.1 Cost Limit

Increasing the number of voltage levels leads to a significant increase in the total number of components required and hence cost. For a square wave inverter configuration as shown in Figure

4.1, one DC bus, one DC-link capacitor and 4 inverter switches are required per phase. To realize an N-level inverter, the following numbers of major components are required:

- Inverter Switches: $4 \times \frac{N-1}{2}$
- DC Bus Bars: $2 \times \frac{N-1}{2}$ (one each for + and - terminals)
- DC link Capacitors: $\frac{N-1}{2}$
- Backup Generators: $\frac{N-1}{2}$

Additional components such as cabling, protection/control/metering equipment and wiring, and disconnect devices are also required.

Aside from lowering harmonics, the other main advantage of using multilevel inverters is lower component ratings. Since the power processing is divided into lower power modules, the ratings of major components can be lowered. Major components include inverter switches, DC link capacitors and backup power sources. Instead of handling total power generation from all the PV arrays combined, these components operate at the lower power of smaller PV array groups. If a DC link capacitor were to handle the total generation from a single DC source as shown in Figure 4.1, the same PV arrays if connected via an N-level inverter as shown in Figure 5.1 would require $\left(\frac{N-1}{2}\right)$ DC link capacitors, each with a voltage rating of approximately $[\frac{1}{\left(\frac{N-1}{2}\right)}]^{th}$ of the original one.

Another advantage of increasing the number of components is redundancy. If a DC bus is taken out of service due to failure or maintenance, the remaining buses can still be used to provide power. The control scheme would have to compensate by assigning new control parameters to ensure that output voltage remains at the desired level.

5.2.2 Equalizing Voltage Stress of Inverter Switches

To ensure that all similar components are utilized equally, and hence have the same life cycles and switching losses, switching techniques which utilize all voltage levels for approximately the same duty cycle is crucial [5]. If some voltage levels are utilized more than others, the heavily used switches would be under more voltage stress and would thus have a shorter life cycle. To illustrate the principle of equalizing voltage stress, a 5-level inverter is used. If all DC voltages are the same, to equalize voltage stress on the switches, the switching pattern shown in Figure 5.4 can be utilized.

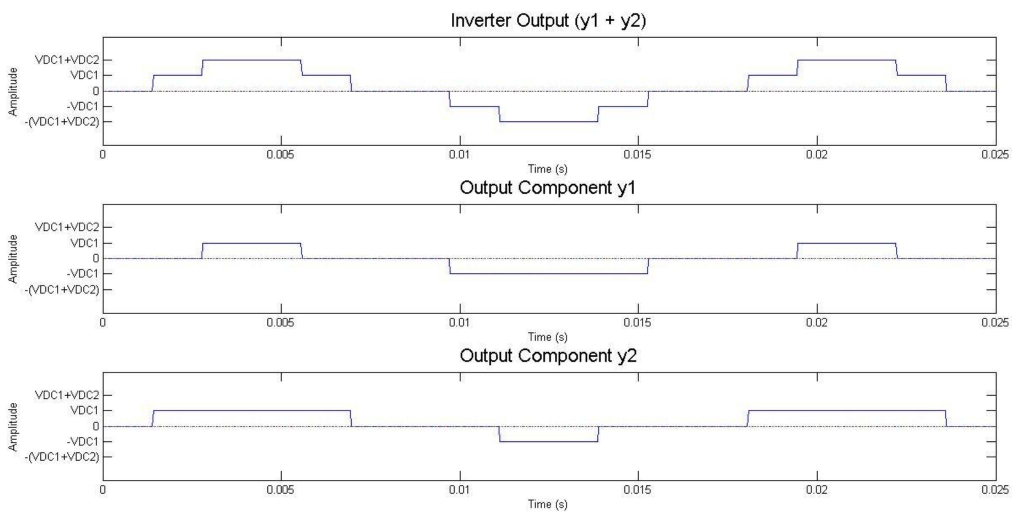

Figure 5.4: Output Voltage Pattern for Equalizing Voltage Stress

The voltage waveforms of both DC sources, V_{DC1} and V_{DC2} in Figure 5.4 have the same duty cycle but opposite polarity.

5.2.3 Power Balancing of PV Modules

Power balancing of PV arrays is crucial to ensure that any DC bus can be used to supply any voltage level. PV panels have to thus be divided into arrays with equal rated power or more importantly equal expected output power. The main challenge is installing PV arrays in a manner which would ensure that all modules produce approximately the same power output under the same insolation conditions.

5.2.4 Varying Voltage Levels

PV module generation voltages may be varied to operate at the maximum power point on the V-I curve. Voltage variations can be restricted to a small range around a design nominal voltage. If the voltage of the buses differ greatly or have a large operating range, the control scheme can become complex. Changing DC bus voltages would require adjusting the firing angles to keep the RMS value of the desired output voltage component, the fundamental frequency component ($V_{1,rms}$) constant. Defining an operating range of bus voltages would define a range for firing angles. Changing the firing angles to keep the output voltage constant would result in changing harmonic conditions. The ranges of harmonic magnitudes have to be taken into account when defining a voltage range for DC buses to ensure that harmonic magnitudes stay within limits. For the purpose of simulating the harmonic performance results in Chapter VI, all DC bus voltages will be assumed to be equal.

5.3 Control Strategy

The control strategy will be based on controlling the following two sets of parameters:

- $\boldsymbol{V_{DCN}}$**:** This is the magnitude of the voltage of DC bus N
- $\boldsymbol{\alpha_N}$**:** This is the firing angle of level N of the inverter

Two approaches can be considered for the control strategy of a PV system with a multilevel inverter:

1- Minimize harmonics in output voltage
2- Maximize output power of the PV arrays

5.3.1 Minimization of Harmonics in Output Voltage

The basis of this approach is to keep DC bus voltages and firing angles constant at the values chosen to minimize harmonics. By predefining PV array voltages, maximum power will not always be generated under changing irradiance conditions. As the amount of insolation incident on PV cells changes over the course of the day, the operating voltage would sway away from the maximum power point.

Under fixed voltage operation, if irradiance increases, the PV arrays would operate at a lower voltage than the maximum power point voltage. Likewise if irradiance decreases, the PV arrays would operate at a voltage higher than the maximum power point voltage; this is illustrated in Figure 5.5.

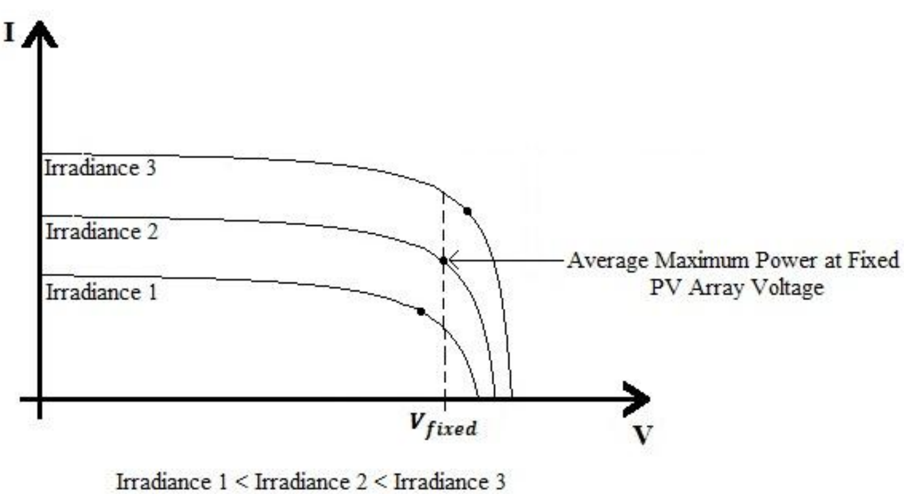

Figure 5.5: V-I Curve for Fixed Voltage Operation

When irradiance increases from Irradiance 2 to Irradiance 3, the PV module output voltage is lower than the maximum power point voltage. Similarly, when irradiance decreases from Irradiance 2 to Irradiance 1, the PV module output is higher than the maximum power point voltage. To ensure that the operating voltage does not sway too far away from the maximum power point under changing insolation conditions, choosing the right fixed DC bus voltage is crucial. The fixed DC bus voltage can be the average value of its maximum power voltage operating range as solar conditions vary from the minimum to maximum irradiance.

Maintaining constant DC bus voltages and firing angles results in a constant output voltage waveform. The harmonics of the output voltage remain unchanged with changing irradiance and are hence predictable. Filter design is greatly simplified by maintaining constant harmonics conditions.

5.3.2 Maximization of Output Power of PV Modules

If a power maximization approach is taken, bus voltages and firing angles would be controlled to obtain the maximum power output from PV arrays. As mentioned earlier in section 5.2.4, changing DC bus voltages would require adjusting the firing angles to keep system output voltage constant. The component of interest to regulate or use as reference is the fundamental frequency component V_{rms1}.

The principle behind the strategy is to vary DC bus voltages V_{DCN} by predefined steps ΔV to check if output power P_{outN} increases until the maximum power point is reached. If a DC bus voltage is varied, the firing angle of that voltage level is readjusted to ensure that $V_{1,rms}$ remains at the desired value.

The flow chart in Figure 5.6 demonstrates this algorithm for the N^{th} DC bus.

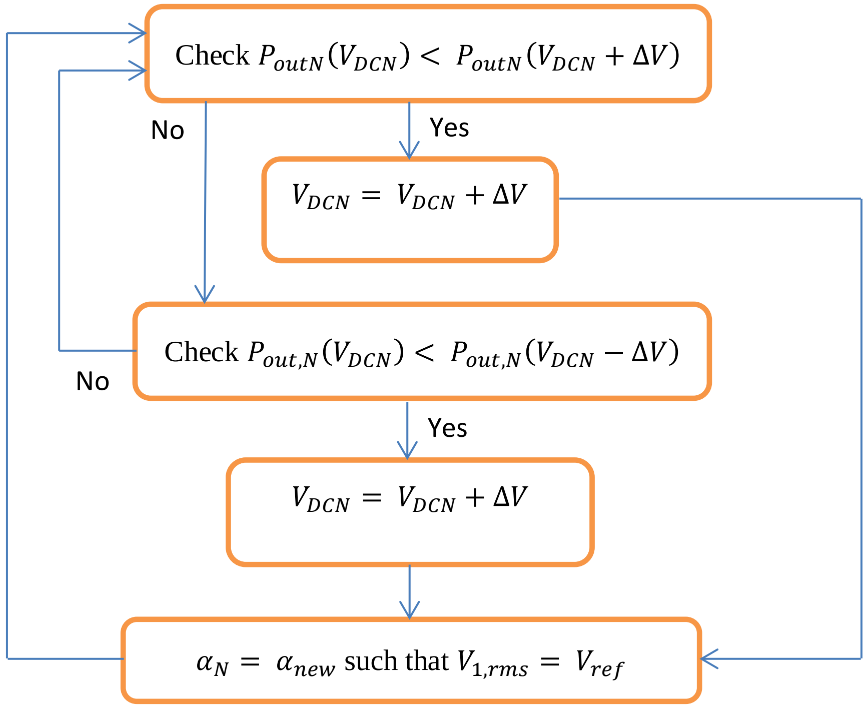

Figure 5.6: Bus Voltage Control Algorithm

The algorithm in Figure 5.6 ensures that the operating voltage of the bus is close to the maximum power point without any DC bus voltage constraints. To ensure that the operating voltage does not sway out of the design operating range, constraints on maximum and minimum operating voltages are required. If the desired operating range for a DC bus voltage is $\pm 5\%$ of the nominal voltage, then maximum and minimum voltages can be defined as follows:

$$V_{max} = V_{bus} * 1.05 \qquad (5.1a)$$

$$V_{max} = V_{bus} * 0.95 \qquad (5.1b)$$

The flow chart in Figure 5.7 demonstrates this algorithm with voltage constraints for the N^{th} DC bus.

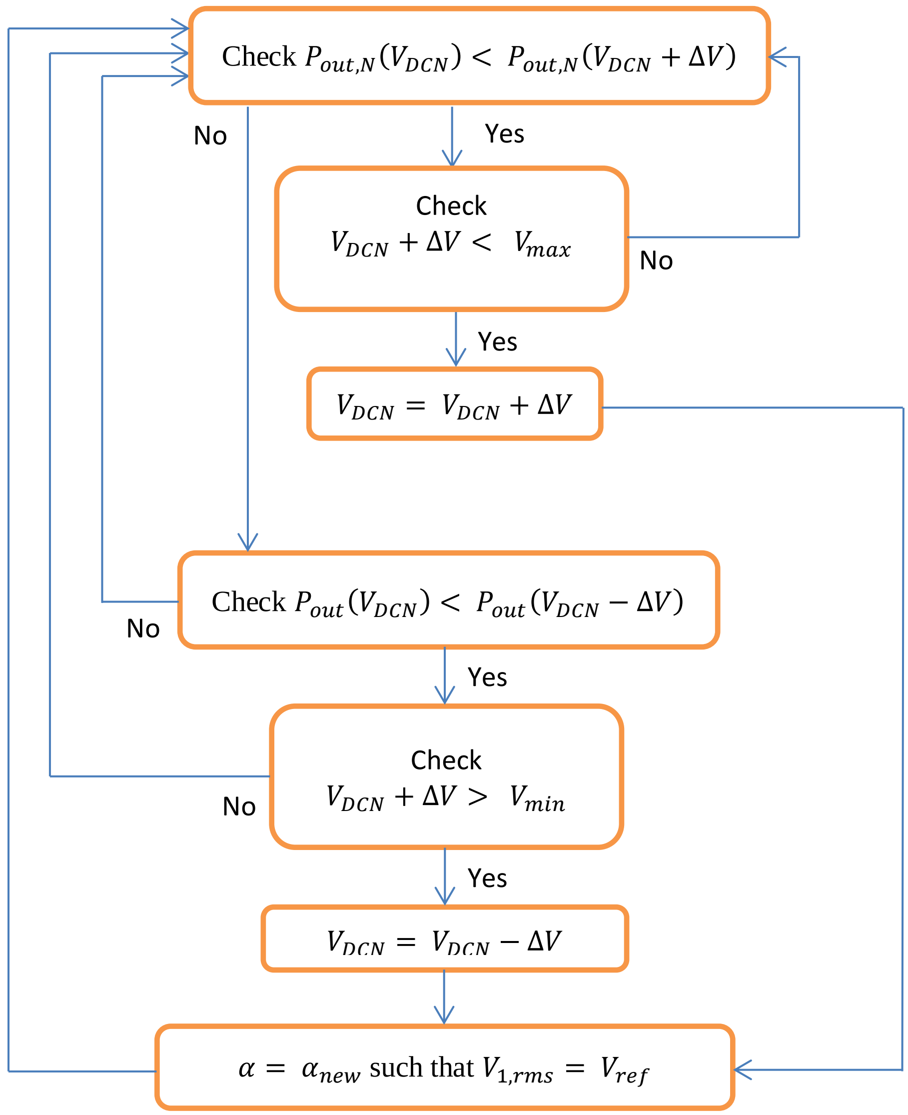

Figure 5.7: Bus Voltage Control Algorithm with Voltage Constraints

The only difference between the two algorithms is that the latter one includes a check to see if changing the bus voltage keeps it within the operating range.

The main drawback of the power maximization approach is that changing DC bus voltages requires adjusting firing angles to keep inverter output voltage constant. These variations can significantly affect harmonic magnitudes. This is a bigger concern if harmonic orders are being eliminated completely since changing firing angles could result in reintroducing the eliminated harmonic orders. As mentioned earlier, a check to ensure that harmonic magnitudes do not exceed limits is thus required when selecting the operating range of the DC bus voltages.

5.4 Objective Function

There is no standard method of selecting firing angles for multilevel inverters. An objective has to be established from which the firing angles as well as voltage levels are derived. Objectives may include minimizing THD, eliminating certain harmonic orders, minimizing lower order harmonics or simply reducing harmonic content to within prescribed limits. It was shown earlier in Table 2.1 that the THD for a square wave inverter is 48.34%. Simply adding an extra voltage level of $0V$ improved the THD of a square wave inverter to 31.08% as shown in Table 2.1. Increasing the number of levels does not necessarily result in improved harmonics profile since stepping up to 5 levels from 3 increased the THD to 31.92% from 31.08% from Table 2.1. This illustrates that selecting the values of firing angles correctly is crucial.

As the number of voltage levels increase, the complexity of optimizing the objective function increases. Computer and iterative methods have to be utilized to find a solution for large number of voltage levels. However, for a PV system, an application for which the practical number of levels is small, optimization objectives are easier to achieve.

An intuitive approach to selecting firing angles is to choose the angles which result in step wave output waveforms resembling the desired filtered waveform as much as possible. For the application of PV inverters, the more the output waveform resembles a sine wave, the lower the harmonic contents would be. Several methods are proposed in Sections 5.5 through Section 5.8 and their performance results are discussed in Chapter VI.

5.5 Equal Angle Approach

One simple approach to choosing firing angles is to equally space them out from $0\ to\ \frac{\pi}{2}$ radians. In general, for an N-level inverter, the firing angle increments can be calculated as follows:

$$\Delta\alpha = \frac{\left(\frac{\pi}{2}\right)}{\left(\frac{N+1}{2}\right)} \qquad (5.2)$$

Using Equation 5.2, for a 5-level inverter, the two firing angles $\alpha_1\ and\ \alpha_2$ would equal $\frac{\pi}{6}\ and\ \frac{2\pi}{6}$ respectively resulting in the plot shown in Figure 5.8.

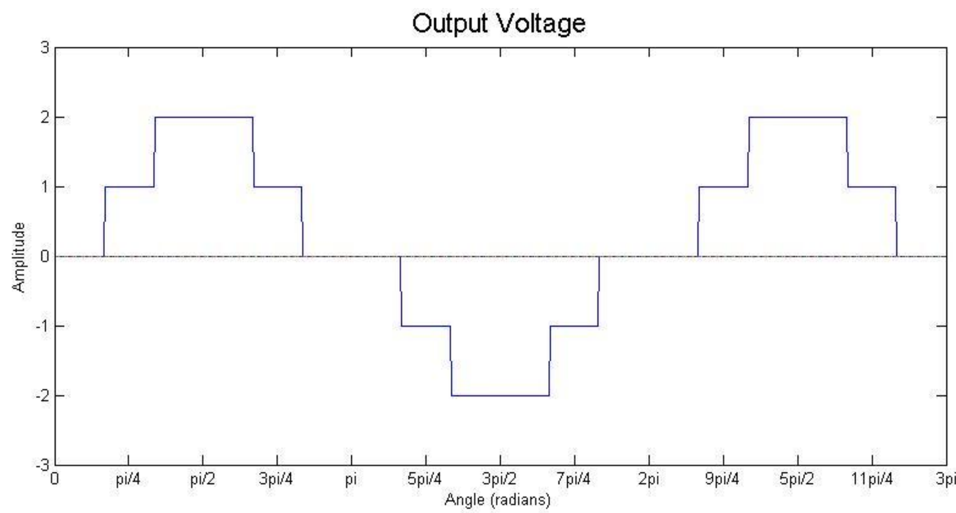

Figure 5.8: 5-level Equal Angle Output Voltage Waveform

As the number of levels increases, the output waveform begins to resemble a triangular waveform. An 11-level, 21-level and 101-level plots are shown in Figure 5.9, Figure 5.10 and Figure 5.11 respectively.

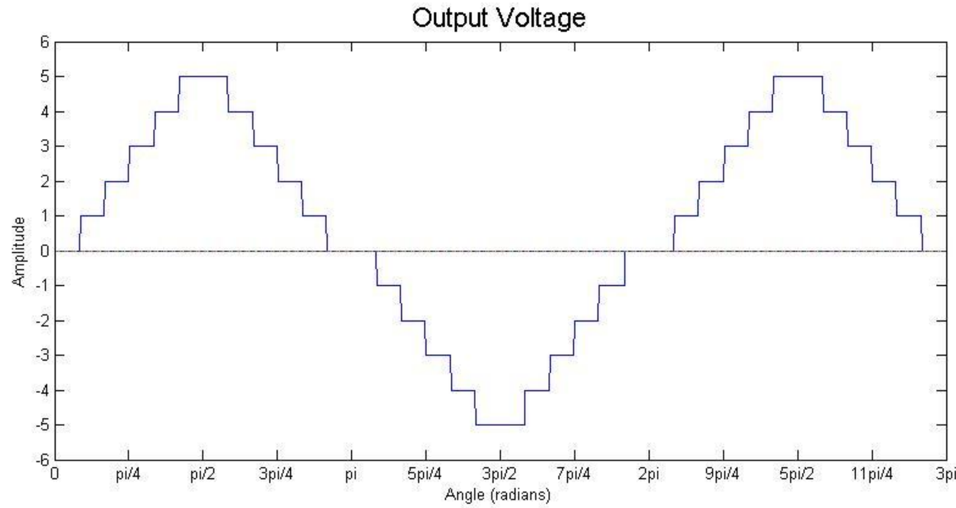

Figure 5.9: 11-level Equal Angle Output Voltage Waveform

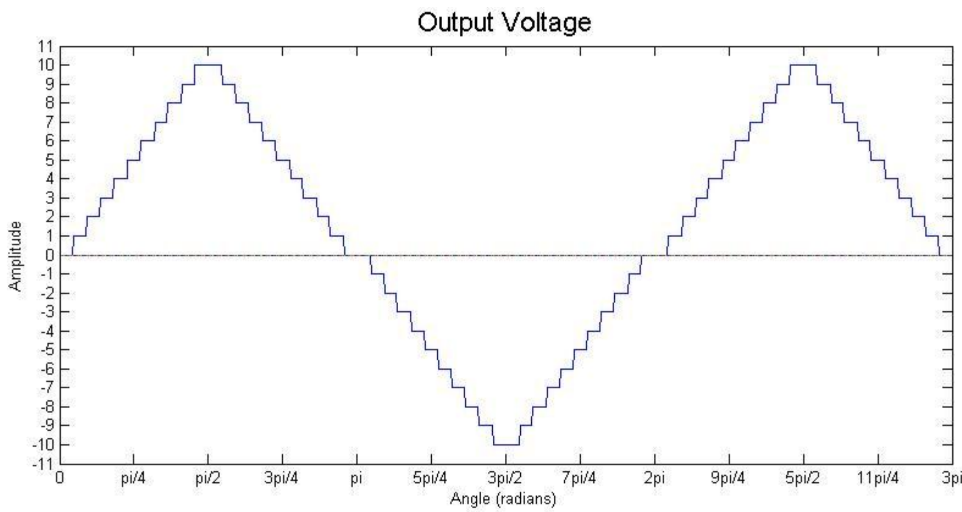

Figure 5.10: 21-level Equal Angle Output Voltage Waveform

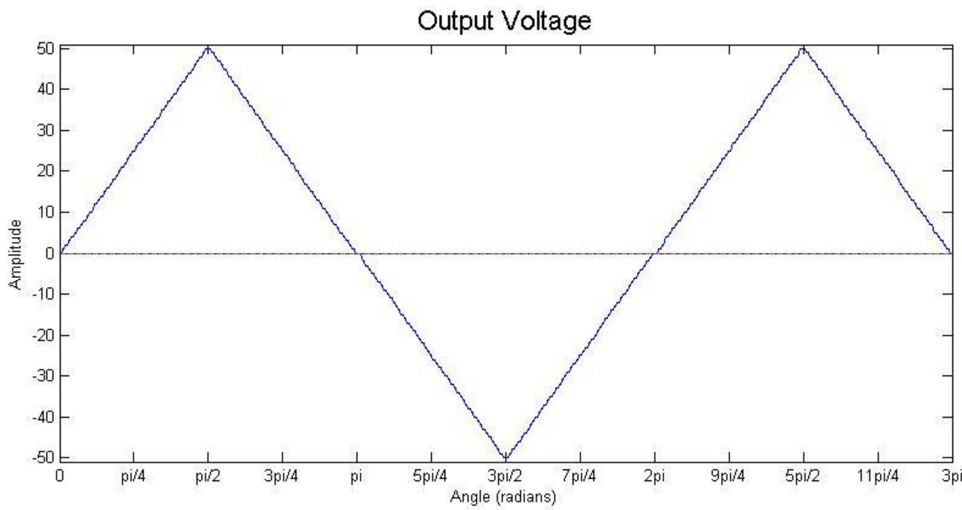

Figure 5.11: 101-level Equal Angle Output Voltage Waveform

The THD of a triangular waveform is 12% which is the value THD for this strategy will approach as the number of levels approach infinity; this is discussed in Chapter VI.

The equal angle approach is limited to minimizing THD to 12%, which is a significant improvement when compared with the performance of a traditional square wave inverter with a THD of 48.34%. The main advantage of utilizing this scheme is that equalizing voltage stress is greatly simplified. Since the firing angles are integer multiples of each other, the appropriate positive and negative voltage firing angles can be paired for each DC voltage bus, making their duty cycles identical. This is discussed in more detail in Chapter VII.

5.6 Elimination of Selected Harmonic Orders

A more practical approach to THD reduction is the elimination of selected harmonic orders. By choosing the firing angles strategically, multiple harmonic orders can be completely eliminated. In high power applications, eliminating a low order harmonic completely would eliminate the need for filtering equipment required for that particular order. It will be shown that instead of eliminating just a single order, integer multiples of that order can also be eliminated. For example, if firing angles are chosen to eliminate the 3rd harmonic order, it can also eliminate the 9th, 15th, 21st, and so on harmonic orders (all triplen harmonics).

Mathematically, an N-level inverter's output voltage Fourier series is expressed as follows:

$$v(t) = \sum_{n=1,3,5\ldots}^{\infty} \left(\frac{4}{n\pi} \left[\sum_{y=1}^{x} V_y \cos\left(\alpha_y n\right) \right] \sin(\mathrm{n\omega t}) \right) \qquad (5.1)$$

The objective function to eliminate the n^{th} harmonic from Equation 5.1 will be:

$$\sum_{y=1}^{x} V_y \cos(\alpha_y\, n) = 0 \qquad (5.2)$$

One method of finding possible solutions to Equation 5.2 is to equate the term $(\alpha_n n)$ to odd integer multiples of $\frac{\pi}{2}$ since cosine of odd integer multiples of $\frac{\pi}{2}$ is equal to zero. This would also result in any integer multiple of the 3rd order frequency, i.e. the 9th, 15th, 21st and so on to be eliminated since those orders are also solutions for Equation 5.2. To illustrate this principle, a 7-level inverter will be used. The output voltage Fourier series of this waveform is expressed as follows:

$$v(t) = \sum_{n=1,3,5\ldots}^{\infty} \left(\frac{4}{n\pi} [V_{DC1} \cos(\alpha_1 n) + V_{DC2} \cos(\alpha_2 n) + V_{DC3} \cos(\alpha_3 n)\,] \sin(\mathrm{n\omega t}) \right) \qquad (5.3)$$

To eliminate the 7th order harmonic, the following objective function can be satisfied:

$$V_{DC1} \cos(7\alpha_1) + V_{DC2} \cos(7\alpha_2) + V_{DC3} \cos(7\alpha_3) = 0 \qquad (5.4)$$

One possible solution is $\alpha_1 = \frac{\pi}{14}$, $\alpha_2 = \frac{3\pi}{14}$ and $\alpha_3 = \frac{5\pi}{14}$. The corresponding waveform and Fourier spectrum of these firing angles are shown in Figure 5.12.

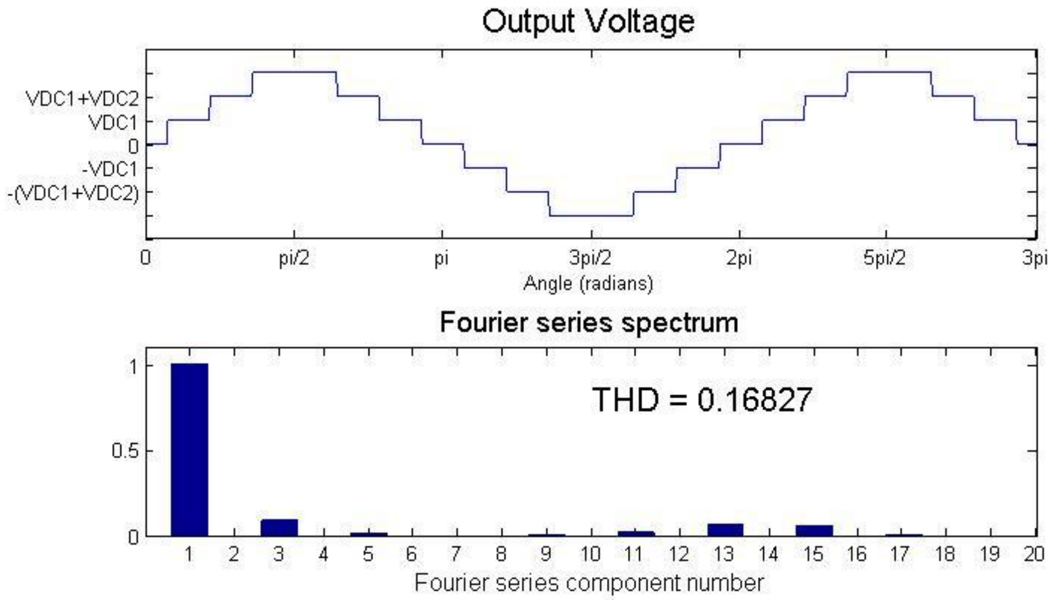

Figure 5.12: Inverter Output Voltage and Fourier Spectrum with 7^{th} Order Elimination

Eliminating the 7^{th} order harmonic also resulted in the elimination of the 14^{th} order harmonic since any harmonic order which is an integer multiple of seven satisfies the objective function and is also eliminated. The performance of inverters with 3^{rd}, 5^{th} and 7^{th} harmonic order elimination is discussed in Chapter VI.

5.7 THD Minimization

The simplest approach to THD minimization is to use optimization techniques. The required equation is obtained by setting the derivative of the THD equation with respect to the firing angles to zero. The minimum value can then be calculated and firing angles selected based on this value. While the solution may not guarantee elimination of particular harmonic orders, it will still yield the lowest THD possible for that particular configuration. For an N-level inverter with x sources, the equation to solve for THD minimization is as follows:

$$\frac{\partial}{\partial \alpha_i}(THD) = 0 \qquad (5.5)$$

$$for\ i = 1, 2, \dots, x$$

For a 3-level inverter with one voltage source, the equation to solve is:

$$\frac{d}{d\alpha_1} THD(\alpha_1) = 0 \qquad (5.6a)$$

$$\frac{d}{d\alpha_1}\left(\sqrt{\left(\frac{V_{rms}}{V_{1,rms}}\right)^2 - 1}\right) = 0 \qquad (5.6b)$$

$$\frac{d}{d\alpha_1}\left(\sqrt{\left(\frac{V_{rms}}{\frac{4V_{DC}}{\sqrt{2}\pi}\cos(\alpha_1)}\right)^2 - 1}\right) = 0; subject\ to\ 0 < \alpha_1 < \frac{\pi}{2} \qquad (5.6b)$$

The solution can be found using a graphical approach. THD can be plotted against the firing angle α_1 from 0˚ to 90˚ with 1˚ increments as shown in Figure 5.13. The results were obtained using the MATLAB code in Appendix A2.

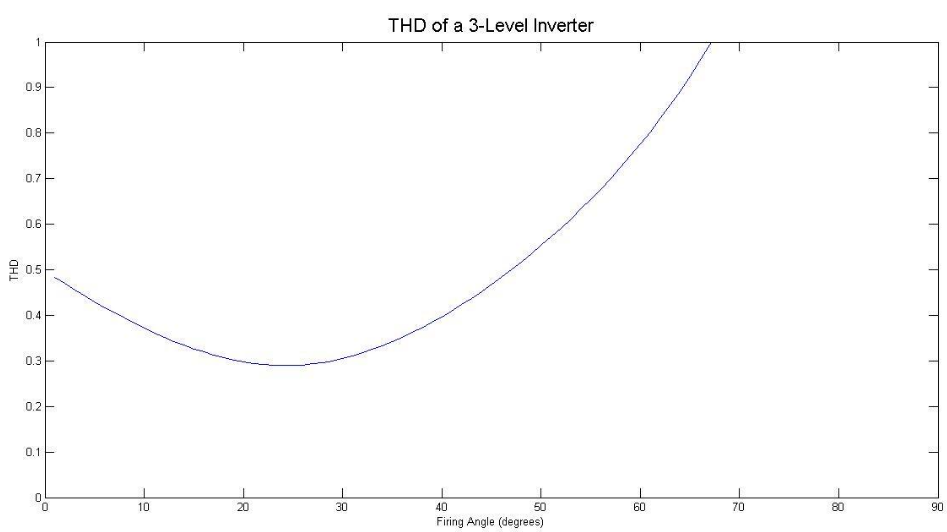

Figure 5.13: Firing Angle vs. THD for a 3-level Inverter

From the plot, it can be seen that the minimum THD value of 28.97% occurs for a firing angle of $\alpha_1 = 23°$.

As the number of levels and thus firing angles increases, the complexity of the derivative increases. For a 5-level inverter, THD is a function of two variables, firing angles α_1 and α_2. Assuming that all DC bus voltages are equal to V_{DC}, the corresponding equation to solve is:

$$\frac{\partial}{\partial \alpha_i}\left(\sqrt{\left(\frac{V_{rms}}{\frac{4V_{DC}}{\sqrt{2}\pi}[\cos(\alpha_1)+\cos(\alpha_2)]}\right)^2 - 1}\right) = 0; \qquad (5.7a)$$

$$for\ i = 1, 2$$

$$subject\ to\ 0 < \alpha_1 < 90^\circ\ AND\ \alpha_1 < \alpha_2 < 90^\circ \qquad (5.7b)$$

Since THD is dependent on two variables, the plot of THD versus the firing angles is three dimensional and is shown in Figure 5.14. The results were obtained using the MATLAB code in Appendix A3.

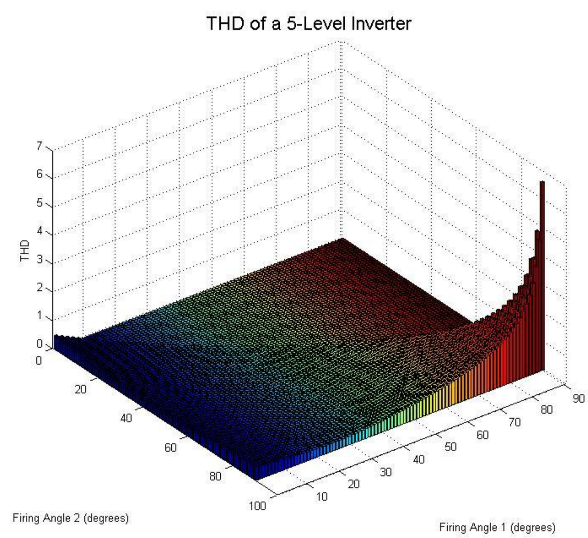

Figure 5.14: Firing Angles vs. THD for a 5-level Inverter

From the plot, it can be seen that a minimum THD value of 16.42% occurs for firing angles $\alpha_1 = 13° \; and \; \alpha_2 = 42°$. THD minimization of inverters with seven levels or more cannot be solved graphically as the number of firing angles increases to three or more. Computer programs calculating THD for all possible combination of firing angles, or iteration methods need to be used to solve for the firing angles.

5.8 Minimization of Selected Harmonics

The concept discussed in Section 5.7 can be extended to target single or multiple harmonic components. If a single harmonic of order n is to be minimized, the corresponding equation to solve is obtained by setting the derivative of the contribution of harmonic order n (Equation 2.29) to zero as follows:

$$\frac{\partial}{\partial \alpha_i}\left(\frac{V_{n,rms}}{V_{1,rms}}\right) = 0 \qquad (5.8)$$

$$for \; i = 1, 2, \dots, x$$

If the distortion contribution of multiple harmonic orders is to be minimized, the objective function is obtained by first finding the expression for the total RMS value of the multiple harmonic orders. The total RMS value of multiple harmonic orders is the square root of the sum of the individual RMS values squared. If harmonic orders of interest are orders a, b and c, their total RMS value is calculated as follows:

$$V_{(a+b+c),rms} = \sqrt{V_{a,rms}^2 + V_{b,rms}^2 + V_{c,rms}^2} \qquad (5.9)$$

In general, the total RMS value of multiple harmonic orders of interest can be calculated as follows:

$$V_{multiple\ orders,rms} = \sqrt{\sum_{n=orders\ of\ interest} V_{n,rms}^2} \qquad (5.10)$$

Using Equation 5.10, the distortion percentage of multiple harmonic orders to distortion can be written as follows:

$$Distortion\ \%\ of\ multiple\ harmonic\ orders = \frac{V_{multiple\ orders,rms}}{V_{1,rms}} \qquad (5.11)$$

Optimization of Equation 5.11 is the objective function for minimizing multiple harmonic orders and can be expressed as follows:

$$\frac{\partial}{\partial \alpha_i}\left(\frac{V_{multiple\ orders,rms}}{V_{1,rms}}\right) = 0 \qquad (5.12)$$

$$for\ i = 1, 2, \ldots, x$$

Using Equation 5.12, if the minimization of the 5th and 7th harmonics is the objective, the following equation is solved to find the minimum magnitude of these harmonic orders:

$$\frac{\partial}{\partial \alpha_i}\left(\frac{\sqrt{V_{5,rms}^2 + V_{7,rms}^2}}{V_{1,rms}}\right) = 0 \qquad (5.13)$$

$$for\ i = 1, 2, \ldots, x$$

To ensure that the magnitudes of other harmonic orders do not increase to undesired levels, constraints can be placed. For the above example if the distortion percentage of the 3rd harmonic is to be limited to 20% and the 9th order to 12.5%, the new equation becomes:

$$\frac{\partial}{\partial \alpha_i}\left(\frac{\sqrt{V_{5,rms}^2 + V_{7,rms}^2}}{V_{1,rms}}\right) = 0 \qquad (5.14a)$$

$$for\ i = 1, 2, \ldots, x$$

$$subject\ to: \left(\frac{V_{3,rms}}{V_{1,rms}}\right) < 0.20\ AND \left(\frac{V_{9,rms}}{V_{1,rms}}\right) < 0.125 \qquad (5.14b)$$

Applying this to a 3-level inverter, if all DC bus voltages are equal to V_{DC}, the equation to solve is:

$$\frac{d}{d\alpha_1}\left(\frac{\sqrt{\left(\frac{4V_{DC}}{5\sqrt{2}\pi}\cos(5\alpha_1)\right)^2 + \left(\frac{4V_{DC}}{7\sqrt{2}\pi}\cos(7\alpha_1)\right)^2}}{\frac{4V_{DC}}{\sqrt{2}\pi}\cos(\alpha_1)}\right) = 0 \qquad (5.15)$$

The solution can be found using a graphical approach. The firing angle will be varied from 0˚ to 90˚ with 1˚ increments, and the total distortion contribution from the 5th and 7th harmonic components will be plotted against the firing angle. The minimum distortion firing angle can be found from the plots illustrated in Figure 5.15. The results were obtained using the MATLAB code in Appendix A4.

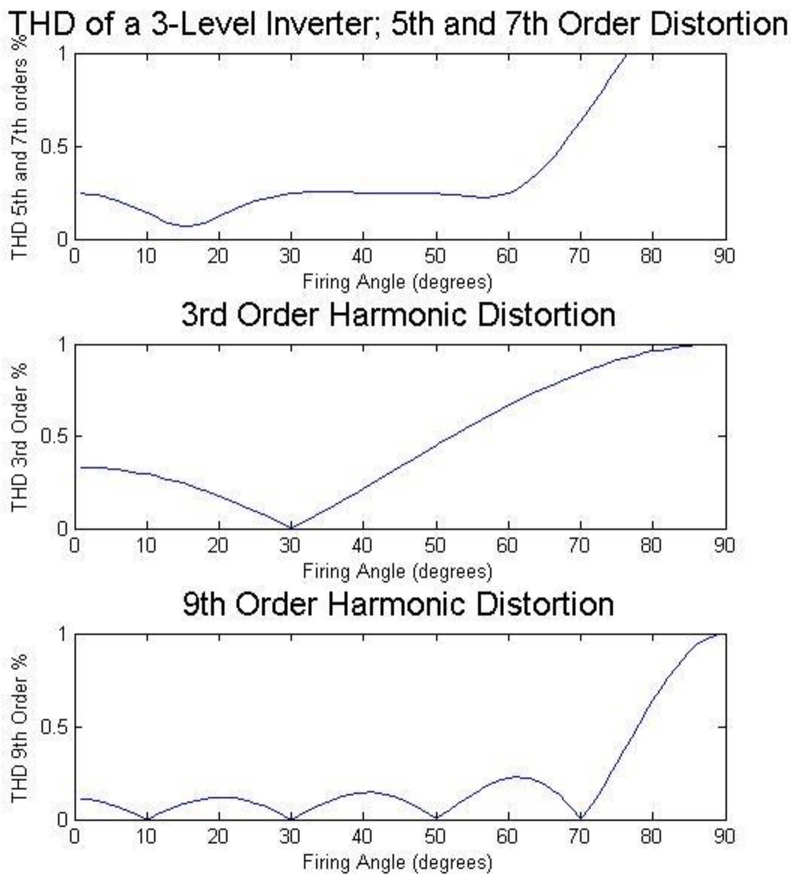

Figure 5.15: 3-level Inverter Output Voltage 5^{th} and 7^{th} Order Minimization

For a firing angle of $\alpha_1 = 15°$, the value of THD is 31.92% and distortion contribution from the 5^{th} and 7^{th} harmonic components is at a minimum value of 6.59%. The corresponding 3^{rd} and 9^{th} order distortion contributions are 24.4% and 8.13% respectively. Since the constraints in Equation 5.14b are not satisfied, a firing angle of $\alpha_1 = 15°$ is not a solution for the objective function.

For a firing of $\alpha_1 = 19°$, THD is 29.77%, distortion contribution from the 3^{rd} and 9^{th} harmonic components are 19.22% and 11.61% respectively. With the constraints in Equation 5.14b satisfied, the minimum value of the distortion contribution from the 5^{th} and 7^{th} harmonic components is 10.47%.

CHAPTER VI

PERFORMANCE RESULTS

6.1 Standards on Harmonics

As mentioned in Section 2.2.3, most industry harmonic limits are derived from IEEE Standard 519 summarized in Table 6.1, and IEC 61000-3 summarized in table 6.2.

Voltage Harmonics		
Voltage at PCC	Individual Limit	THD
<69 kV	3.0	5.0
69-161 kV	1.5	2.5
>161 kV	1.0	1.5

Table 6.1: Voltage Harmonics Limits from the IEEE 519 Standard

Odd harmonics non−multiple of 3			Odd harmonics multiple of 3			Even harmonics		
Harmonic Order h	Harmonic Voltage %		Harmonic Order h	Harmonic Voltage %		Harmonic Order h	Harmonic Voltage %	
	MV	HV-EHV		MV	HV-EHV		MV	HV-EHV
5	5	2	3	4	2	2	1.8	1.4
7	4	2	9	1.2	1	4	1	0.8
11	3	1.5	15	0.3	0.3	6	0.5	0.4
13	2.5	1.5	21	0.2	0.2	8	0.5	0.4
$17 \le h \le 49$	$1.9 \cdot \frac{17}{h} - 0.2$	$1.2 \cdot \frac{17}{h}$	$21 < h \le 45$	0.2	0.2	$10 \le h \le 50$	$0.25 \cdot \frac{10}{h} + 0.22$	$0.19 \cdot \frac{10}{h} + 0.16$

Table 6.2: Voltage Harmonics Limits from the IEC 61000-3 Standard

The goal of the study is to express the reduction in harmonics as a result of utilizing multilevel inverters and comparing the performance results with conventional square wave inverter. Another comparison of interest is multilevel inverters voltage harmonics with IEEE 519 limits.

The magnitude of voltage distortion of multilevel inverters discussed in Chapter V will be presented and then compared with that of square wave inverters. The harmonics will then be compared with IEEE 519 limits.

MATLAB is used to study the harmonic profile of each configuration presented in Chapter V. The code and it's the associated comments are given in Appendix A. The results from MATLAB simulations will be presented as outlined below:

Section 6.2: Performance results of square wave operation, and square wave operation with a firing angle (3-level inverter) for $\alpha = \frac{\pi}{12}, \alpha = \frac{\pi}{6}, \ \alpha = \frac{\pi}{4}, \ \alpha = \frac{\pi}{3}$ and $\alpha = \frac{5\pi}{12}$.

Section 6.3: Performance results for Equal Angle Operation discussed in Section 5.5 for 5, 7, 11, 21 and 101-level inverters.

Section 6.4: Performance results of Elimination of Selected Harmonic Order discussed in Section 5.6. Section 6.4.1 presents the results for a 3-level inverter with the 3rd, 5th and 7th order harmonics eliminated. Section 6.4.2 presents the results for a 5-level inverter with the 5th and 7th order harmonics eliminated, and results for a 7-level inverter with the 7th and 9th level harmonics eliminated.

The performance results of multilevel inverter configurations are presented in terms of the following values obtained from MATLAB simulations:

1- RMS values of odd harmonic components up to the 21st order as per Equation 2.25. These values are normalized with respect to the total output voltage of the inverter (V_{rms}). Even components are not listed since their magnitudes are zero due to waveform symmetry.

2- THD of the inverter output voltage and distortion percentage of odd harmonic components up to the 21st order as per Equation 2.29. Even components are not listed since their magnitudes are zero due to waveform symmetry.

3- Improvement of THD and distortion percentage compared to square wave operation. The improvement will be shown as a percentage of the square wave operation value. These values will be calculated as follows:

$$THD\ Improvement = \frac{(THD_{square\ wave} - THD)}{THD_{square\ wave}} \qquad (6.1)$$

$$Distortion\ \%\ Improvement = \frac{(Distortion\ \%_{square\ wave} - Distortion\ \%)}{Distortion\ \%_{square\ wave}} \qquad (6.2)$$

4- Amount by which IEEE 519 limits are exceeded (hereto designated exceedance) of THD and odd harmonic order distortion percentages up to the 21^{st} order. This will compare how far above or below the limit THD and a particular harmonic order distortion percentage is. These values will be calculated as follows:

$$IEEE519\ Limit\ Exceedance\ of\ THD = (THD) - (Limit) \quad (6.3)$$

$$IEEE519\ Limit\ Exceedance\ of\ Distortion\,\% = (Distortion\,\%) - (Limit) \quad (6.3)$$

5- IEEE 519 limits exceedance in RMS value, normalized to the RMS value of the output voltage up to the 21^{st} order. This value will represent the RMS value of a harmonic order over IEEE 519 limit, per unit of the output voltage. Normalizing the RMS value of the harmonic components make them independent of the magnitude of the input and output voltages. This value will be calculated as follows:

$$RMS\ Value\ of\ the\ n^{th}\ order\ to\ Filter\ (p.u.of\ V_{out}) = \frac{V_{n,rms} - (Limit\ \times\ V_{1,rms})}{V_{out,rms}} \quad (6.4)$$

6.2 Square Wave Operation Results

The following tables document the performance of a square wave inverter as well as that of a square wave inverter with a firing angle α (which is essentially a 3-level inverter.)

RMS VALUES						
Harmonic Order	**Square Wave**	**Square Wave $\alpha_1 = \pi/12$**	**Square Wave $\alpha_1 = \pi/6$**	**Square Wave $\alpha_1 = \pi/4$**	**Square Wave $\alpha_1 = \pi/3$**	**Square Wave $\alpha_1 = 5\pi/12$**
Output Voltage	1.0000	**1.0000**	1.0000	**1.0000**	1.0000	**1.000**
1st	0.9003	**0.9526**	0.9549	**0.9003**	0.7797	**0.5708**
3rd	0.3001	**0.2325**	0.0000	**0.3001**	0.5198	**0.5198**
5th	0.1801	**0.0511**	0.1910	**0.1801**	0.1559	**0.4260**
7th	0.1286	**0.0365**	0.1364	**0.1286**	0.1114	**0.3043**
9th	0.1000	**0.0775**	0.0000	**0.1000**	0.1733	**0.1733**
11th	0.0818	**0.0866**	0.0868	**0.0818**	0.0709	**0.0519**
13th	0.0693	**0.0733**	0.0735	**0.0693**	0.0600	**0.0439**
15th	0.0600	**0.0465**	0.0000	**0.0600**	0.1040	**0.1040**
17th	0.0530	**0.0150**	0.0562	**0.0530**	0.0459	**0.1253**
19th	0.0474	**0.0134**	0.0503	**0.0474**	0.0410	**0.1121**
21st	0.0429	**0.0332**	0.0000	**0.0429**	0.0743	**0.0743**
>21	0.1357	**0.1096**	0.1202	**0.1358**	0.1624	**0.2251**

Table 6.3: RMS Values of Output Voltage and Harmonics for Square Wave Operation

HARMONIC DISTORTION (%)						
Harmonic Order	**Square Wave**	**Square Wave $\alpha_1 = \pi/12$**	**Square Wave $\alpha_1 = \pi/6$**	**Square Wave $\alpha_1 = \pi/4$**	**Square Wave $\alpha_1 = \pi/3$**	**Square Wave $\alpha_1 = 5\pi/12$**
THD	48.34%	**31.92%**	31.08%	**48.34%**	80.31%	**143.86%**
3rd	33.33%	**24.40%**	0.00%	**33.33%**	66.67%	**91.07%**
5th	20.00%	**5.36%**	20.00%	**20.00%**	20.00%	**74.64%**
7th	14.29%	**3.83%**	14.29%	**14.29%**	14.29%	**53.32%**
9th	11.11%	**8.13%**	0.00%	**11.11%**	22.22%	**30.36%**
11th	9.09%	**9.09%**	9.09%	**9.09%**	9.09%	**9.09%**
13th	7.69%	**7.69%**	7.69%	**7.69%**	7.69%	**7.69%**
15th	6.67%	**4.88%**	0.00%	**6.67%**	13.33%	**18.21%**
17th	5.88%	**1.58%**	5.88%	**5.88%**	5.88%	**21.95%**
19th	5.26%	**1.41%**	5.26%	**5.26%**	5.26%	**19.64%**
21st	4.76%	**3.49%**	0.00%	**4.76%**	9.52%	**13.01%**
>21	15.06%	**11.47%**	12.57%	**15.06%**	20.84%	**39.45%**

Table 6.4: THD and Harmonics Distortion % for Square Wave Operation

PERFORMANCE IMPROVEMENT (%)					
Harmonic Order	**Square Wave $\alpha_1 = \pi/12$**	**Square Wave $\alpha_1 = \pi/6$**	**Square Wave $\alpha_1 = \pi/4$**	**Square Wave $\alpha_1 = \pi/3$**	**Square Wave $\alpha_1 = 5\pi/12$**
THD	33.97%	**35.70%**	0.00%	**-66.13%**	-197.60%
3rd	26.79%	**100.00%**	0.00%	**-100.00%**	-173.21%
5th	73.21%	**0.00%**	0.00%	**0.00%**	-273.21%
7th	73.21%	**0.00%**	0.00%	**0.00%**	-273.21%
9th	26.79%	**100.00%**	0.00%	**-100.00%**	-173.21%
11th	0.00%	**0.00%**	0.00%	**0.00%**	0.00%
13th	0.00%	**0.00%**	0.00%	**0.00%**	0.00%
15th	26.79%	**100.00%**	0.00%	**-100.00%**	-173.21%
17th	73.21%	**0.00%**	0.00%	**0.00%**	-273.21%
19th	73.21%	**0.00%**	0.00%	**0.00%**	-273.21%
21st	26.79%	**100.00%**	0.00%	**-100.00%**	-173.21%

Table 6.5: Harmonics Improvement vs. Square Wave for Square Wave Operation

IEEE519 LIMIT EXCEEDANCE (%)						
Harmonic Order	**Square Wave**	**Square Wave $\alpha_1 = \pi/12$**	**Square Wave $\alpha_1 = \pi/6$**	**Square Wave $\alpha_1 = \pi/4$**	**Square Wave $\alpha_1 = \pi/3$**	**Square Wave $\alpha_1 = 5\pi/12$**
THD	43.34%	**26.92%**	26.08%	**43.34%**	75.31%	**138.86%**
3rd	30.33%	**21.40%**	-3.00%	**30.33%**	63.67%	**88.07%**
5th	17.00%	**2.36%**	17.00%	**17.00%**	17.00%	**71.64%**
7th	11.29%	**0.83%**	11.29%	**11.29%**	11.29%	**50.32%**
9th	8.11%	**5.13%**	-3.00%	**8.11%**	19.22%	**27.36%**
11th	6.09%	**6.09%**	6.09%	**6.09%**	6.09%	**6.09%**
13th	4.69%	**4.69%**	4.69%	**4.69%**	4.69%	**4.69%**
15th	3.67%	**1.88%**	-3.00%	**3.67%**	10.33%	**15.21%**
17th	2.88%	**-1.42%**	2.88%	**2.88%**	2.88%	**18.95%**
19th	2.26%	**-1.59%**	2.26%	**2.26%**	2.26%	**16.64%**
21st	1.76%	**0.49%**	-3.00%	**1.76%**	6.52%	**10.01%**

Table 6.6: IEEE519 Limits Exceedance for Square Wave Operation

RMS VALUE OF IEEE519 LIMIT EXCEEDANCE (P.U. OF Vout RMS)						
Harmonic Order	**Square Wave**	**Square Wave $\alpha_1 = \pi/12$**	**Square Wave $\alpha_1 = \pi/6$**	**Square Wave $\alpha_1 = \pi/4$**	**Square Wave $\alpha_1 = \pi/3$**	**Square Wave $\alpha_1 = 5\pi/12$**
THD	0.3902	**0.2566**	0.2491	**0.3902**	0.5873	**0.7925**
3rd	0.2731	**0.2039**	-0.0286	**0.2731**	0.4964	**0.5027**
5th	0.1531	**0.0225**	0.1623	**0.1531**	0.1325	**0.4090**
7th	0.1016	**0.0079**	0.1078	**0.1016**	0.0880	**0.2872**
9th	0.0730	**0.0489**	-0.0286	**0.0730**	0.1499	**0.1562**
11th	0.0548	**0.0580**	0.0582	**0.0548**	0.0475	**0.0348**
13th	0.0422	**0.0447**	0.0448	**0.0422**	0.0366	**0.0268**
15th	0.0330	**0.0179**	-0.0286	**0.0330**	0.0806	**0.0868**
17th	0.0260	**-0.0136**	0.0275	**0.0260**	0.0225	**0.1082**
19th	0.0204	**-0.0151**	0.0216	**0.0204**	0.0176	**0.0950**
21st	0.0159	**0.0046**	-0.0286	**0.0159**	0.0509	**0.0571**

Table 6.7: RMS Value of IEEE519 Limits Exceedance for Square Wave Operation

From the results presented above, it can be seen that employing a firing angle to square wave operation or an additional voltage level of $0V$ can significantly improve harmonic performance of a square wave inverter. The results are consistent with Figure 5.13, which illustrates the THD of a 3-level inverter when THD minimization is the objective.

6.3 Equal Angle Operation Results

The following tables documents the performance of a multilevel inverter using the equal angle approach discussed in Section 5.5.

RMS VALUES						
Harmonic Order	**Square Wave**	**5-levels $\Delta\alpha = \pi/6$**	**7-levels $\Delta\alpha = \pi/8$**	**11-levels $\Delta\alpha = \pi/12$**	**21-Levels $\Delta\alpha = \pi/22$**	**101-levels $\Delta\alpha = \pi/102$**
Output Voltage	1.0000	**1.0000**	1.0000	**1.0000**	1.0000	**1.0000**
1st	0.9003	**0.9526**	0.9691	**0.9807**	0.9878	**0.9924**
3rd	0.3001	**0.2325**	0.2002	**0.1692**	0.1420	**0.1137**
5th	0.1801	**0.0511**	0.0160	**0.0090**	0.0256	**0.0384**
7th	0.1286	**0.0365**	0.0412	**0.0375**	0.0308	**0.0215**
9th	0.1000	**0.0775**	0.0321	**0.0097**	0.0028	**0.0115**
11th	0.0818	**0.0866**	0.0073	**0.0153**	0.0138	**0.0090**
13th	0.0693	**0.0733**	0.0462	**0.0129**	0.0015	**0.0053**
15th	0.0600	**0.0465**	0.0646	**0.0058**	0.0078	**0.0049**
17th	0.0530	**0.0150**	0.0570	**0.0155**	0.0028	**0.0030**
19th	0.0474	**0.0134**	0.0316	**0.0024**	0.0049	**0.0032**
21st	0.0429	**0.0332**	0.0038	**0.0242**	0.0034	**0.0019**
>21	0.1357	**0.1096**	0.0846	**0.0822**	0.0467	**0.0000**

Table 6.8: Output Voltage and Harmonics RMS Values for Equal Angle Operation

HARMONIC DISTORTION (%)						
Harmonic Order	**Square Wave**	**5-levels $\Delta\alpha = \pi/6$**	**7-levels $\Delta\alpha = \pi/8$**	**11-levels $\Delta\alpha = \pi/12$**	**21-Levels $\Delta\alpha = \pi/22$**	**101-levels $\Delta\alpha = \pi/102$**
THD	48.34%	**31.92%**	25.47%	**19.95%**	15.77%	**12.75%**
3rd	33.33%	**24.40%**	20.66%	**17.25%**	14.37%	**11.80%**
5th	20.00%	**5.36%**	1.65%	**0.92%**	2.59%	**3.74%**
7th	14.29%	**3.83%**	4.25%	**3.83%**	3.12%	**2.29%**
9th	11.11%	**8.13%**	3.31%	**0.99%**	0.29%	**1.07%**
11th	9.09%	**9.09%**	0.75%	**1.56%**	1.40%	**0.97%**
13th	7.69%	**7.69%**	4.77%	**1.32%**	0.15%	**0.47%**
15th	6.67%	**4.88%**	6.67%	**0.59%**	0.79%	**0.55%**
17th	5.88%	**1.58%**	5.88%	**1.58%**	0.28%	**0.25%**
19th	5.26%	**1.41%**	3.26%	**0.24%**	0.49%	**0.36%**
21st	4.76%	**3.49%**	0.39%	**2.46%**	0.34%	**0.15%**
>21	15.06%	**11.47%**	8.77%	**8.41%**	4.75%	**1.13%**

Table 6.9: THD and Harmonic Order Distortion % for Equal Angle Operation

PERFORMANCE IMPROVEMENT (%)					
Harmonic Order	**5-levels $\Delta\alpha = \pi/6$**	**7-levels $\Delta\alpha = \pi/8$**	**11-levels $\Delta\alpha = \pi/12$**	**21-Levels $\Delta\alpha = \pi/22$**	**101-levels $\Delta\alpha = \pi/102$**
THD	33.97%	**47.31%**	58.73%	**67.38%**	73.62%
3rd	26.79%	**38.01%**	48.24%	**56.89%**	64.60%
5th	73.21%	**91.76%**	95.40%	**87.05%**	81.29%
7th	73.21%	**70.23%**	73.21%	**78.19%**	83.98%
9th	26.79%	**70.23%**	91.12%	**97.41%**	90.35%
11th	0.00%	**91.76%**	82.84%	**84.59%**	89.29%
13th	0.00%	**38.01%**	82.84%	**98.06%**	93.86%
15th	26.79%	**0.00%**	91.12%	**88.09%**	91.79%
17th	73.21%	**0.00%**	73.21%	**95.17%**	95.73%
19th	73.21%	**38.01%**	95.40%	**90.62%**	93.24%
21st	26.79%	**91.76%**	48.24%	**92.85%**	96.90%

Table 6.10: Harmonics Improvement vs. Square Wave for Equal Angle Operation

IEEE519 LIMIT EXCEEDANCE (%)						
Harmonic Order	**Square Wave**	**5-levels $\Delta\alpha = \pi/6$**	**7-levels $\Delta\alpha = \pi/8$**	**11-levels $\Delta\alpha = \pi/12$**	**21-Levels $\Delta\alpha = \pi/22$**	**101-levels $\Delta\alpha = \pi/102$**
THD	43.34%	**26.92%**	20.47%	**14.95%**	10.77%	**7.75%**
3rd	30.33%	**21.40%**	17.66%	**14.25%**	11.37%	**8.80%**
5th	17.00%	**2.36%**	-1.35%	**-2.08%**	-0.41%	**0.74%**
7th	11.29%	**0.83%**	1.25%	**0.83%**	0.12%	**-0.71%**
9th	8.11%	**5.13%**	0.31%	**-2.01%**	-2.71%	**-1.93%**
11th	6.09%	**6.09%**	-2.25%	**-1.44%**	-1.60%	**-2.03%**
13th	4.69%	**4.69%**	1.77%	**-1.68%**	-2.85%	**-2.53%**
15th	3.67%	**1.88%**	3.67%	**-2.41%**	-2.21%	**-2.45%**
17th	2.88%	**-1.42%**	2.88%	**-1.42%**	-2.72%	**-2.75%**
19th	2.26%	**-1.59%**	0.26%	**-2.76%**	-2.51%	**-2.64%**
21st	1.76%	**0.49%**	-2.61%	**-0.54%**	-2.66%	**-2.85%**

Table 6.11: IEEE519 Limits Exceedance for Equal Angle Operation

RMS VALUE OF IEEE519 LIMIT EXCEEDANCE (P.U. OF Vout RMS)						
Harmonic Order	**Square Wave**	**5-levels $\Delta\alpha = \pi/6$**	**7-levels $\Delta\alpha = \pi/8$**	**11-levels $\Delta\alpha = \pi/12$**	**21-Levels $\Delta\alpha = \pi/22$**	**101-levels $\Delta\alpha = \pi/102$**
THD	0.3902	**0.2565**	0.1983	**0.1456**	0.1064	**0.0768**
3rd	0.2731	**0.2039**	0.1712	**0.1398**	0.1123	**0.0873**
5th	0.1531	**0.0225**	-0.0131	**-0.0204**	-0.0041	**0.0074**
7th	0.1016	**0.0079**	0.0121	**0.0081**	0.0011	**-0.0071**
9th	0.0730	**0.0489**	0.0030	**-0.0197**	-0.0268	**-0.0191**
11th	0.0548	**0.0580**	-0.0218	**-0.0141**	-0.0158	**-0.0201**
13th	0.0422	**0.0447**	0.0171	**-0.0165**	-0.0282	**-0.0251**
15th	0.0330	**0.0179**	0.0355	**-0.0236**	-0.0218	**-0.0243**
17th	0.0260	**-0.0136**	0.0279	**-0.0140**	-0.0268	**-0.0273**
19th	0.0204	**-0.0151**	0.0025	**-0.0271**	-0.0248	**-0.0262**
21st	0.0159	**0.0046**	-0.0253	**-0.0052**	-0.0263	**-0.0283**

Table 6.12: RMS Value of IEEE519 Limits Exceedance for Equal Angle Operation

From the results presented above, it can be seen that as the number of levels with equally spaced firing angles is increased, THD decreases significantly. As mentioned in Section 5.5, as the number of levels approaches infinity, the output waveform approaches a perfect triangular waveform which has a THD of 12%.

6.4 Select Harmonic Order Elimination Results

The following two sections demonstrate the cancellation of the 3rd, 5th and 7th order harmonics for multilevel inverters. Section 6.4.1 demonstrates cancellation of these harmonic orders in 3-level inverters. Section 6.4.2 demonstrates the cancellation of the 5th and 7th order harmonics for 5-level inverters and the 3rd and 7th order harmonics for 7-level inverters.

6.4.1 3-level Inverters Harmonic Order Elimination

RMS VALUES							
Harmonic Order	**Square Wave**	**3-levels 3rd Order Elimination $\alpha_1 = \pi/6$**	**3-levels 5th Order Elimination $\alpha_1 = \pi/10$**	**3-levels 5th Order Elimination $\alpha_1 = 3\pi/10$**	**3-levels 7th Order Elimination $\alpha_1 = \pi/14$**	**3-levels 7th Order Elimination $\alpha_1 = 3\pi/14$**	**3-levels 7th Order Elimination $\alpha_1 = 5\pi/14$**
Output Voltage	1.0000	**1.0000**	1.0000	**1.0000**	1.0000	**1.0000**	1.0000
1st	0.9003	**0.9549**	0.9573	**0.8367**	0.9481	**0.9312**	0.7308
3rd	0.3001	**0.0000**	0.1972	**0.4513**	0.2534	**0.1723**	0.5474
5th	0.1801	**0.1910**	0.0000	**0.0000**	0.0844	**0.2322**	0.2634
7th	0.1286	**0.1364**	0.0845	**0.1934**	0.0000	**0.0000**	0.0000
9th	0.1000	**0.0000**	0.1064	**0.0930**	0.0469	**0.1290**	0.1463
11th	0.0818	**0.0868**	0.0870	**0.0761**	0.0691	**0.0470**	0.1493
13th	0.0693	**0.0735**	0.0455	**0.1041**	0.0729	**0.0716**	0.0562
15th	0.0600	**0.0000**	0.0000	**0.0000**	0.0632	**0.0621**	0.0487
17th	0.0530	**0.0562**	0.0348	**0.0796**	0.0447	**0.0304**	0.0966
19th	0.0474	**0.0503**	0.0504	**0.0440**	0.0222	**0.0611**	0.0693
21st	0.0429	**0.0000**	0.0456	**0.0398**	0.0000	**0.0000**	0.0000
>21	0.1357	**0.1202**	0.1037	**0.1540**	0.1049	**0.1295**	0.1832

Table 6.13: RMS Value of Output Voltage and Harmonics for 3-level Inverter Harmonic Elimination

HARMONIC DISTORTION (%)							
Harmonic Order	**Square Wave**	**3-levels 3^{rd} Order Elimination $\alpha_1 = \pi/6$**	**3-levels 5^{th} Order Elimination $\alpha_1 = \pi/10$**	**3-levels 5^{th} Order Elimination $\alpha_1 = 3\pi/10$**	**3-levels 7^{th} Order Elimination $\alpha_1 = \pi/14$**	**3-levels 7^{th} Order Elimination $\alpha_1 = 3\pi/14$**	**3-levels 7^{th} Order Elimination $\alpha_1 = 5\pi/14$**
THD	48.34%	**31.08%**	30.19%	**65.45%**	33.55%	**39.16%**	93.40%
3rd	33.33%	**0.00%**	20.60%	**53.93%**	26.73%	**18.50%**	74.90%
5th	20.00%	**20.00%**	0.00%	**0.00%**	8.90%	**24.94%**	36.04%
7th	14.29%	**14.29%**	8.83%	**23.11%**	0.00%	**0.00%**	0.00%
9th	11.11%	**0.00%**	11.11%	**11.11%**	4.94%	**13.86%**	20.02%
11th	9.09%	**9.09%**	9.09%	**9.09%**	7.29%	**5.05%**	20.43%
13th	7.69%	**7.69%**	4.75%	**12.45%**	7.69%	**7.69%**	7.69%
15th	6.67%	**0.00%**	0.00%	**0.00%**	6.67%	**6.67%**	6.67%
17th	5.88%	**5.88%**	3.64%	**9.52%**	4.72%	**3.26%**	13.22%
19th	5.26%	**5.26%**	5.26%	**5.26%**	2.34%	**6.56%**	9.48%
21st	4.76%	**0.00%**	4.76%	**4.76%**	0.00%	**0.00%**	0.00%
>21	15.06%	**12.57%**	10.81%	**18.40%**	11.08%	**13.95%**	25.07%

Table 6.14: THD and Harmonics Distortion % for 3-level Inverter Harmonic Elimination

PERFORMANCE IMPROVEMENT (%)						
Harmonic Order	**3-levels 3^{rd} Order Elimination $\alpha_1 = \pi/6$**	**3-levels 5^{th} Order Elimination $\alpha_1 = \pi/10$**	**3-levels 5^{th} Order Elimination $\alpha_1 = 3\pi/10$**	**3-levels 7^{th} Order Elimination $\alpha_1 = \pi/14$**	**3-levels 7^{th} Order Elimination $\alpha_1 = 3\pi/14$**	**3-levels 7^{th} Order Elimination $\alpha_1 = 5\pi/14$**
THD	35.70%	**37.55%**	-35.40%	**30.60%**	18.99%	**-93.21%**
3rd	100.00%	**38.20%**	-61.80%	**19.81%**	44.50%	**-124.70%**
5th	0.00%	**100.00%**	100.00%	**55.50%**	-24.70%	**-80.20%**
7th	0.00%	**38.20%**	-61.80%	**100.00%**	100.00%	**100.00%**
9th	100.00%	**0.00%**	0.00%	**55.50%**	-24.70%	**-80.19%**
11th	0.00%	**0.00%**	0.00%	**19.81%**	44.50%	**-124.70%**
13th	0.00%	**38.18%**	-61.85%	**0.00%**	0.00%	**0.00%**
15th	100.00%	**100.00%**	100.00%	**0.00%**	0.00%	**0.00%**
17th	0.00%	**38.17%**	-293.11%	**19.73%**	44.48%	**-124.79%**
19th	0.00%	**0.00%**	-111.24%	**55.47%**	-24.77%	**-80.30%**
21st	100.00%	**0.00%**	-90.99%	**100.00%**	100.00%	**100.00%**

Table 6.15: Harmonics Improvement vs. Square Wave Operation for 3-level Inverter Harmonic Elimination

IEEE519 LIMIT EXCEEDANCE (%)							
	Square Wave	**3-levels 3^{rd} Order Elimination $\alpha_1 = \pi/6$**	**3-levels 5^{th} Order Elimination $\alpha_1 = \pi/10$**	**3-levels 5^{th} Order Elimination $\alpha_1 = 3\pi/10$**	**3-levels 7^{th} Order Elimination $\alpha_1 = \pi/14$**	**3-levels 7^{th} Order Eliminatio n $\alpha_1 = 3\pi/14$**	**3-levels 7^{th} Order Elimination $\alpha_1 = 5\pi/14$**
THD	43.34%	**26.08%**	25.19%	**60.45%**	28.55%	**34.16%**	88.40%
3rd	30.33%	**-3.00%**	17.60%	**50.93%**	23.73%	**15.50%**	71.90%
5th	17.00%	**17.00%**	-3.00%	**-3.00%**	5.90%	**21.94%**	33.04%
7th	11.29%	**11.29%**	5.83%	**20.11%**	-3.00%	**-3.00%**	-3.00%
9th	8.11%	**-3.00%**	8.11%	**8.11%**	1.94%	**10.86%**	17.02%
11th	6.09%	**6.09%**	6.09%	**6.09%**	4.29%	**2.05%**	17.43%
13th	4.69%	**4.69%**	1.75%	**9.45%**	4.69%	**4.69%**	4.69%
15th	3.67%	**-3.00%**	-3.00%	**-3.00%**	3.67%	**3.67%**	3.67%
17th	2.88%	**2.88%**	0.64%	**6.52%**	1.72%	**0.26%**	10.22%
19th	2.26%	**2.26%**	2.26%	**2.26%**	-0.66%	**3.56%**	6.48%
21st	1.76%	**-3.00%**	1.76%	**1.76%**	-3.00%	**-3.00%**	-3.00%

Table 6.16: IEEE519 Limits Exceedance for 3-level Inverter Harmonic Elimination

RMS VALUE OF IEEE519 LIMIT EXCEEDANCE (P.U. OF Vout RMS)							
Harmonic Order	**Square Wave**	**3-levels 3^{rd} Order Elimination $\alpha_1 = \pi/6$**	**3-levels 5^{th} Order Elimination $\alpha_1 = \pi/10$**	**3-levels 5^{th} Order Elimination $\alpha_1 = 3\pi/10$**	**3-levels 7^{th} Order Elimination $\alpha_1 = \pi/14$**	**3-levels 7^{th} Order Elimination $\alpha_1 = 3\pi/14$**	**3-levels 7^{th} Order Elimination $\alpha_1 = 5\pi/14$**
THD	0.3902	**0.2491**	0.2411	**0.5059**	0.2706	**0.3179**	0.6460
3rd	0.2731	**-0.0286**	0.1685	**0.4262**	0.2250	**0.1443**	0.5255
5th	0.1531	**0.1623**	-0.0287	**-0.0251**	0.0559	**0.2043**	0.2415
7th	0.1016	**0.1078**	0.0558	**0.1683**	-0.0284	**-0.0279**	-0.0219
9th	0.0730	**-0.0286**	0.0777	**0.0679**	0.0184	**0.1011**	0.1244
11th	0.0548	**0.0582**	0.0583	**0.0510**	0.0407	**0.0190**	0.1274
13th	0.0422	**0.0448**	0.0168	**0.0790**	0.0445	**0.0436**	0.0343
15th	0.0330	**-0.0286**	-0.0287	**-0.0251**	0.0348	**0.0341**	0.0268
17th	0.0260	**0.0275**	0.0061	**0.0545**	0.0163	**0.0025**	0.0747
19th	0.0204	**0.0216**	0.0217	**0.0190**	-0.0062	**0.0332**	0.0474
21st	0.0159	**-0.0286**	0.0169	**0.0147**	-0.0284	**-0.0279**	-0.0219

Table 6.17: RMS Value of IEEE519 Limits Exceedance for 3-level Inverter Harmonic Elimination

6.4.2 5-level and 7-level Inverters Harmonic Order Elimination

RMS VALUES							
Harmonic Order	**Square Wave**	**5-levels 5^{th} Order Elimination $\alpha_1 = \pi/10$ $\alpha_2 = 3\pi/10$**	**5-levels 7^{th} Order Elimination $\alpha_1 = \pi/14$ $\alpha_2 = 3\pi/14$**	**5-levels 7^{th} Order Elimination $\alpha_1 = \pi/14$ $\alpha_2 = 5\pi/14$**	**5-levels 7^{th} Order Elimination $\alpha_1 = 3\pi/14$ $\alpha_2 = 5\pi/14$**	**7-levels 7^{th} Order Elimination $\alpha_1 = \pi/14$ $\alpha_2 = 3\pi/14$ $\alpha_3 = 5\pi/14$**	**7-levels 9^{th} Order Elimination $\alpha_1 = \pi/18$ $\alpha_2 = 3\pi/18$ $\alpha_3 = 5\pi/18$**
Output Voltage	1.0000	**1.0000**	1.0000	**1.0000**	1.0000	**1.0000**	1.0000
1st	0.9003	**0.9797**	0.9863	**0.9687**	0.9157	**0.9861**	0.9930
3rd	0.3001	**0.0771**	0.0651	**0.0443**	0.3537	**0.0941**	0.0000
5th	0.1801	**0.0000**	0.0608	**0.1672**	0.0291	**0.0217**	0.0450
7th	0.1286	**0.0330**	0.0000	**0.0000**	0.0000	**0.0000**	0.0262
9th	0.1000	**0.1089**	0.0338	**0.929**	0.0162	**0.0120**	0.0000
11th	0.0818	**0.0891**	0.0178	**0.0121**	0.0965	**0.0257**	0.0167
13th	0.0693	**0.0178**	0.0759	**0.0745**	0.0704	**0.0759**	0.0173
15th	0.0600	**0.0000**	0.0658	**0.0646**	0.0610	**0.0657**	0.0000
17th	0.0530	**0.0136**	0.0115	**0.0078**	0.0624	**0.0166**	0.0584
19th	0.0474	**0.0516**	0.0160	**0.0440**	0.0077	**0.0057**	0.0523
21st	0.0429	**0.0467**	0.0000	**0.0000**	0.0000	**0.0000**	0.0000
>21	0.1357	**0.0895**	0.0857	**0.0000**	0.1155	**0.0842**	0.0672

Table 6.18: RMS Values of Output Voltage and Harmonics for 5/7-level Inverter Harmonic Elimination

HARMONIC DISTORTION (%)							
Harmonic Order	**Square Wave**	**5-levels 5^{th} Order Elimination $\alpha_1 = \pi/10$ $\alpha_2 = 3\pi/10$**	**5-levels 7^{th} Order Elimination $\alpha_1 = \pi/14$ $\alpha_2 = 3\pi/14$**	**5-levels 7^{th} Order Elimination $\alpha_1 = \pi/14$ $\alpha_2 = 5\pi/14$**	**5-levels 7^{th} Order Elimination $\alpha_1 = 3\pi/14$ $\alpha_2 = 5\pi/14$**	**7-levels 7^{th} Order Elimination $\alpha_1 = \pi/14$ $\alpha_2 = 3\pi/14$ $\alpha_3 = 5\pi/14$**	**7-levels 9^{th} Order Elimination $\alpha_1 = \pi/18$ $\alpha_2 = 3\pi/18$ $\alpha_3 = 5\pi/18$**
THD	48.34%	**20.49%**	16.71%	**25.61%**	43.48%	**16.83%**	11.86%
3rd	33.33%	**7.87%**	6.60%	**4.57%**	38.63%	**9.54%**	0.00%
5th	20.00%	**0.00%**	6.16%	**17.26%**	3.18%	**2.20%**	4.53%
7th	14.29%	**3.37%**	0.00%	**0.00%**	0.00%	**0.00%**	2.64%
9th	11.11%	**11.11%**	3.42%	**9.59%**	1.76%	**1.22%**	0.00%
11th	9.09%	**9.09%**	1.80%	**1.25%**	10.53%	**2.60%**	1.68%
13th	7.69%	**1.82%**	7.69%	**7.69%**	7.69%	**7.69%**	1.74%
15th	6.67%	**0.00%**	6.67%	**6.67%**	6.67%	**6.67%**	0.00%
17th	5.88%	**1.39%**	1.17%	**0.81%**	6.82%	**1.68%**	5.88%
19th	5.26%	**5.26%**	1.62%	**4.54%**	0.84%	**0.06%**	5.26%
21st	4.76%	**4.76%**	0.00%	**0.00%**	0.00%	**0.00%**	0.00%
>21	15.06%	**9.21%**	8.67%	**10.90%**	11.10%	**8.53%**	6.71%

Table 6.19: THD and Harmonics Distortion % for 5/7-level Inverter Harmonic Elimination

PERFORMANCE IMPROVEMENT (%)						
Harmonic Order	**5-levels 5^{th} Order Elimination $\alpha_1 = \pi/10$ $\alpha_2 = 3\pi/10$**	**5-levels 7^{th} Order Elimination $\alpha_1 = \pi/14$ $\alpha_2 = 3\pi/14$**	**5-levels 7^{th} Order Elimination $\alpha_1 = \pi/14$ $\alpha_2 = 5\pi/14$**	**5-levels 7^{th} Order Elimination $\alpha_1 = 3\pi/14$ $\alpha_2 = 5\pi/14$**	**7-levels 7^{th} Order Elimination $\alpha_1 = \pi/14$ $\alpha_2 = 3\pi/14$ $\alpha_3 = 5\pi/14$**	**7-levels 9^{th} Order Elimination $\alpha_1 = \pi/18$ $\alpha_2 = 3\pi/18$ $\alpha_3 = 5\pi/18$**
THD	57.61%	**65.43%**	47.02%	**10.05%**	65.18%	**75.47%**
3rd	76.39%	**80.19%**	86.29%	**-15.89%**	71.38%	**100.00%**
5th	100.00%	**69.20%**	13.70%	**84.10%**	89.00%	**77.35%**
7th	76.39%	**100.00%**	100.00%	**100.00%**	100.00%	**81.52%**
9th	0.00%	**69.20%**	13.71%	**84.16%**	89.02%	**100.00%**
11th	0.00%	**80.19%**	86.29%	**-15.83%**	71.40%	**81.52%**
13th	76.39%	**-0.03%**	-0.03%	**0.00%**	0.00%	**77.37%**
15th	100.00%	**0.05%**	0.05%	**0.00%**	0.00%	**100.00%**
17th	76.38%	**80.19%**	86.29%	**-15.99%**	71.43%	**0.00%**
19th	-0.06%	**69.18%**	13.65%	**84.03%**	98.90%	**0.00%**
21st	-0.04%	**100.00%**	100.00%	**100.00%**	100.00%	**100.00%**

Table 6.20: Harmonics Improvement vs. Square Wave for 5/7-level Inverter Harmonic Elimination

IEEE519 LIMIT EXCEEDANCE (%)							
	Square Wave	**5-levels 5^{th} Order Elimination $\alpha_1 = \pi/10$ $\alpha_2 = 3\pi/10$**	**5-levels 7^{th} Order Elimination $\alpha_1 = \pi/14$ $\alpha_2 = 3\pi/14$**	**5-levels 7^{th} Order Elimination $\alpha_1 = \pi/14$ $\alpha_2 = 5\pi/14$**	**5-levels 7^{th} Order Elimination $\alpha_1 = 3\pi/14$ $\alpha_2 = 5\pi/14$**	**7-levels 7^{th} Order Elimination $\alpha_1 = \pi/14$ $\alpha_2 = 3\pi/14$ $\alpha_3 = 5\pi/14$**	**7-levels 9^{th} Order Elimination $\alpha_1 = \pi/18$ $\alpha_2 = 3\pi/18$ $\alpha_3 = 5\pi/18$**
THD	43.34%	**15.49%**	11.71%	**20.61%**	38.48%	**11.83%**	6.86%
3rd	30.33%	**4.87%**	3.60%	**1.57%**	35.63%	**6.54%**	-3.00%
5th	17.00%	**-3.00%**	3.16%	**14.26%**	0.18%	**-0.80%**	1.53%
7th	11.29%	**0.37%**	-3.00%	**-3.00%**	-3.00%	**-3.00%**	-0.36%
9th	8.11%	**8.11%**	0.42%	**6.59%**	-1.24%	**-1.78%**	-3.00%
11th	6.09%	**6.09%**	-1.20%	**-1.75%**	7.53%	**-0.40%**	-1.32%
13th	4.69%	**-1.18%**	4.69%	**4.69%**	4.69%	**4.69%**	-1.26%
15th	3.67%	**-3.00%**	3.67%	**3.67%**	3.67%	**3.67%**	-3.00%
17th	2.88%	**-1.61%**	-1.83%	**-2.19%**	3.82%	**-1.32%**	2.88%
19th	2.26%	**2.26%**	-1.38%	**1.54%**	-2.16%	**-2.94%**	2.26%
21st	1.76%	**1.76%**	-3.00%	**-3.00%**	-3.00%	**-3.00%**	-3.00%

Table 6.21: IEEE519 Limits Exceedance for 5/7-level Inverter Harmonic Elimination

RMS VALUE OF IEEE519 LIMIT EXCEEDANCE (P.U. OF Vout RMS)							
Harmonic Order	**Square Wave**	**5-levels 5^{th} Order Elimination $\alpha_1 = \pi/10$ $\alpha_2 = 3\pi/10$**	**5-levels 7^{th} Order Elimination $\alpha_1 = \pi/14$ $\alpha_2 = 3\pi/14$**	**5-levels 7^{th} Order Elimination $\alpha_1 = \pi/14$ $\alpha_2 = 5\pi/14$**	**5-levels 7^{th} Order Elimination $\alpha_1 = 3\pi/14$ $\alpha_2 = 5\pi/14$**	**7-levels 7^{th} Order Elimination $\alpha_1 = \pi/14$ $\alpha_2 = 3\pi/14$ $\alpha_3 = 5\pi/14$**	**7-levels 9^{th} Order Elimination $\alpha_1 = \pi/18$ $\alpha_2 = 3\pi/18$ $\alpha_3 = 5\pi/18$**
THD	0.3902	**0.1516**	0.1156	**0.1996**	0.3560	**0.1165**	0.0684
3rd	0.2731	**0.0477**	0.0355	**0.0152**	0.3263	**0.0645**	-0.0298
5th	0.1531	**-0.0294**	0.0312	**0.1381**	0.0016	**-0.0079**	0.0152
7th	0.1016	**0.0036**	-0.0296	**-0.0291**	-0.0275	**-0.0296**	-0.0036
9th	0.0730	**0.0795**	0.0042	**0.0638**	-0.0113	**-0.0175**	-0.0298
11th	0.0548	**0.0597**	-0.0118	**-0.0170**	0.0690	**-0.0039**	-0.0131
13th	0.0422	**-0.0116**	0.0463	**0.0455**	0.0430	**0.0463**	-0.0125
15th	0.0330	**-0.0294**	0.0362	**0.0355**	0.0336	**0.0362**	-0.0298
17th	0.0260	**-0.0158**	-0.0181	**-0.0213**	0.0349	**-0.0130**	0.0286
19th	0.0204	**0.0222**	-0.0136	**0.0149**	-0.0199	**-0.0239**	0.0225
21st	0.0159	**0.0173**	-0.0296	**-0.0291**	-0.0275	**-0.0296**	-0.0298

Table 6.22: RMS Value of IEEE519 Limits Exceedance for 5/7-level Inverter Harmonic Elimination

As discussed in Section 5.6, it can be seen that eliminating a select harmonic order eliminates its integer multiples as well. To truly take advantage of this technique, harmonic orders with the highest RMS values cancelled should be targeted since it would eliminate the highest power filter required.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

The biggest drawback in utilizing multilevel inverters is the additional components required and hence cost. When considering the usage of multilevel inverters, the cost of additional components has to be compared with the savings and benefits provided by lower filtering requirements. If the benefit outweighs the additional cost, multilevel inverters are the logical technological choice. For high power three phase applications, however, the cost of using more than 7 voltage levels becomes impractical and expensive; the recommendations in the following sections is thus based on a maximum of seven levels, which would require three separate DC buses.

7.1 The Quick and Economic Fix

For systems using square wave operation already in service, the simplest and most economical fix is to use a 3-level inverter configuration. This is essentially using the same hardware; the only difference is that the voltage level of $0V$ is utilized by applying a firing angle of $23°$ to minimize THD based on the results from Section 5.7. THD is reduced to 28.97% and using Equation 2.18, the resulting duty cycle is 25%. One half of the cycle can be used to supply output power and the other half to charge the battery backup. This approach would lower filtering requirements, without requiring the addition of new components and dedicate a duty cycle of 25% to charging the battery backup.

7.2 Multilevel Inverter Configuration for THD Minimization

As seen from the results in Chapter VI, increasing the number of voltage levels lowers the harmonic content of the output voltage of inverters. For practical reasons, the number of voltage levels will be limited to seven levels as mentioned earlier. The schematic for a 7-level inverter is shown in Figure 7.1.

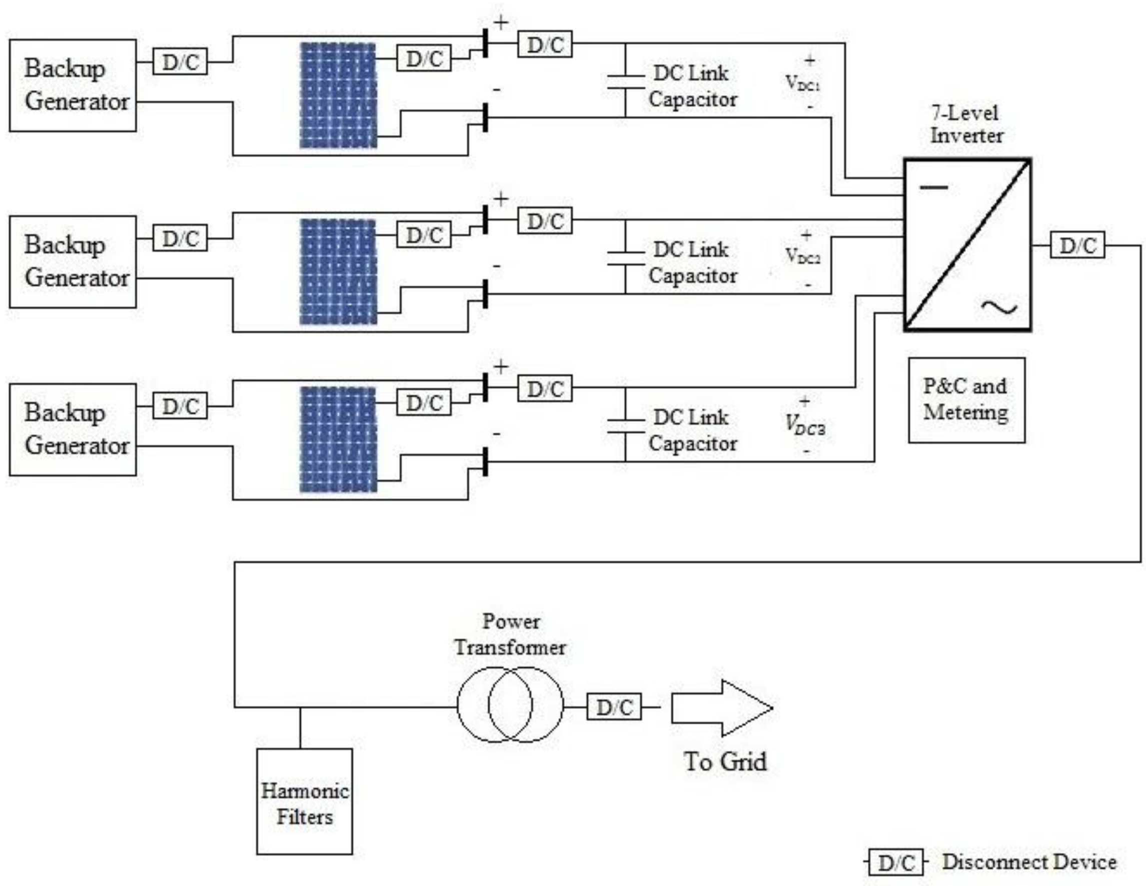

Figure 7.1: 7-level Inverter PV System Schematic

Based on the results from Chapter VI, a 7-level inverter with firing angles $\alpha_1 = \frac{\pi}{18}, \alpha_2 = \frac{3\pi}{18}$ and $\alpha_3 = \frac{5\pi}{18}$ is an ideal choice when utilizing a 7-level inverter for mitigation of low order harmonics. THD is lowered to 11.86% or by 75.47% when compared to square wave operation. Harmonic orders which are integer multiples of three (triplens) are completely eliminated. The 5th, 17th and 19th order harmonics are the only ones surpassing individual order limits up to the 21st order. Distortion from harmonic orders greater than 21 are reduced from 15.06% to 6.72%. The main advantage of this strategy is the complete elimination of harmonics which are integer multiples of three (triplens), which significantly reduces filtering requirements. Figure 7.2 shows the magnitudes of the harmonic components up to the 21st order normalized to the fundamental frequency components for both a square wave and the recommended 7-level inverter.

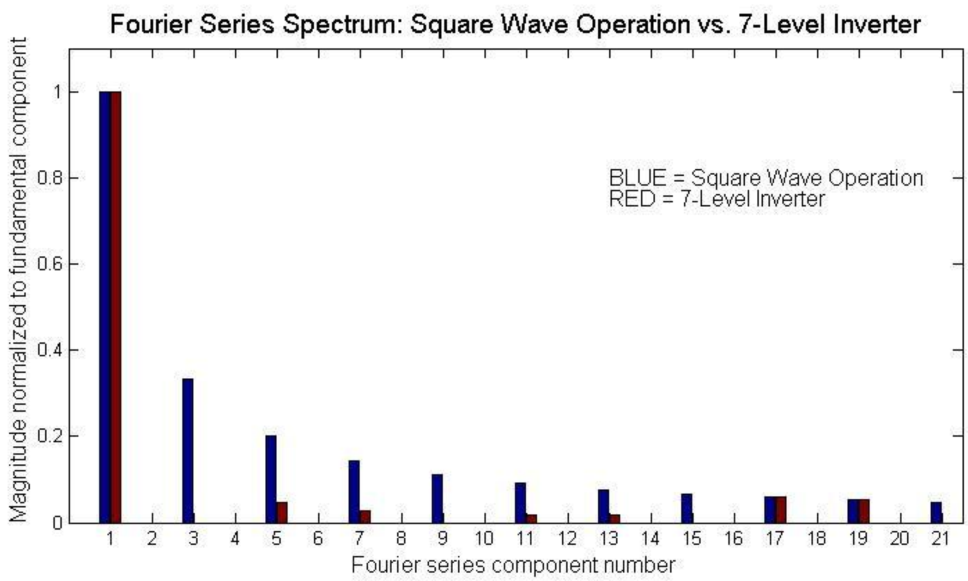

Figure 7.2: Harmonic Spectrums: Square Wave vs.7-level, THD Minimization

7.3 Multilevel Inverter Configuration for Equalizing Voltage Stress

An option which simplifies equalizing voltage stresses in addition to reducing lower order harmonics is the equal angle approach. Using a 7-level inverter will lower the THD by 47.31% to 25.47% when compared to a conventional a square wave inverter. The most significant lower order harmonic is the 3rd order with a distortion percentage of 20.66%. Distortion percentages of the remaining harmonics up to the 21st order are under 3.67%. The 5th, 11th and 21st harmonics are under the individual harmonic limits.

To obtain a 7-level inverter, the PV modules would require to be divided into three arrays with three DC voltage buses as shown in Figure 7.1. The firing angles of the positive and negative cycles of each DC bus are adjusted such that the duty cycles of all DC sources are equal. This can be done by pairing the smallest positive firing angle of a source with the largest negative firing angle, the second smallest firing angle with the second largest firing angle, and so on until one source has the same firing angles for both the positive and negative cycles. For an N-level inverter, the firing angle increments as per Equation 5.2 would be $\Delta\alpha = \frac{\left(\frac{\pi}{2}\right)}{\left(\frac{N+1}{2}\right)}$. The smallest firing angle would be $\alpha_1 = \frac{\left(\frac{\pi}{2}\right)}{\left(\frac{N+1}{2}\right)}$ and the largest firing angle can be calculated as follows:

$$\alpha_N = \left(\frac{N-1}{2}\right) \times \frac{\left(\frac{\pi}{2}\right)}{\left(\frac{N+1}{2}\right)} \qquad (7.1)$$

Each increment from the smallest firing angle will be paired with a decrement from the largest firing angle to form a positive and negative firing angle pair.

Using this principle for a 7-level inverter, which has three firing angles, the firing angle increments as per Equation 5.2, are $\frac{\pi}{8}$. The smallest firing angle is $\frac{\pi}{8}$ and the largest firing $\frac{3\pi}{8}$; the positive and negative firing angles will be paired as per the values in Table 7.1.

PV Array Number	α_+	α_-
1	$\frac{\pi}{8}$	$\frac{3\pi}{8}$
2	$\frac{2\pi}{8}$	$\frac{2\pi}{8}$
3	$\frac{3\pi}{8}$	$\frac{\pi}{8}$

Table 7.1: 7-level Inverter Equal Angle Operation Firing Angles

The voltage waveforms of the output and the individual DC bus contributions as per the firing angles from Table 7.1 are illustrated in Figure 7.3.

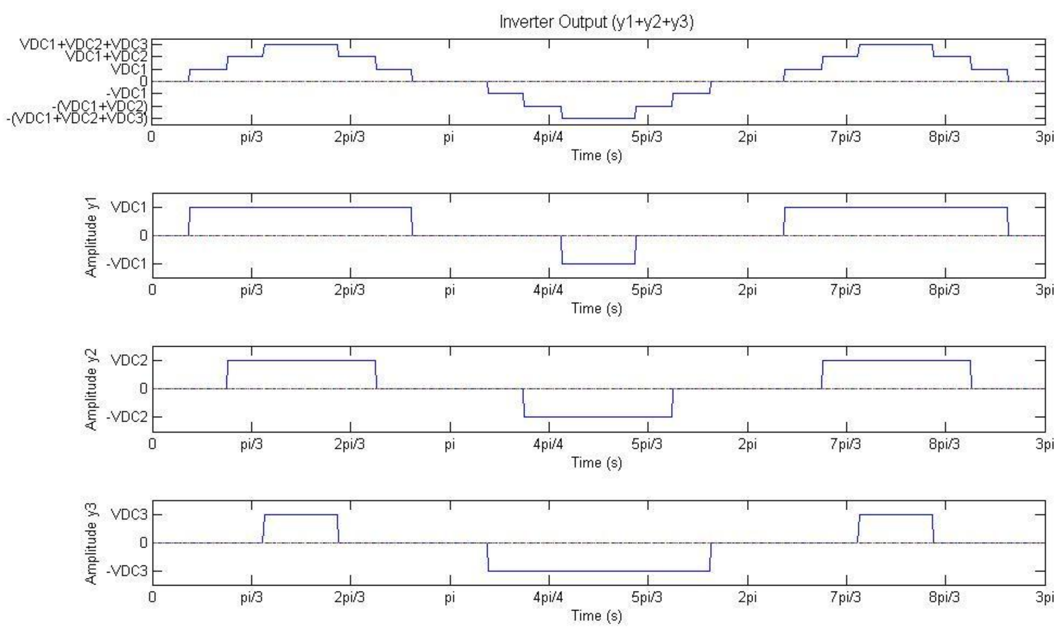

Figure 7.3: 7-level Inverter Output for Equalizing Voltage Stress

All three buses have a combined non-zero duty cycle of 50%. The term non-zero duty cycle will refer to the percentage of the total period for which the value of the waveform has a non-zero value which is calculated as follows:

$$Non - zero\ Duty\ Cycle = \frac{2(\pi - \alpha_1 - \alpha_2)}{2\pi} \qquad (7.2)$$

To ensure that the RMS value of the negative half cycle of the inverter output voltage is equal to the positive half cycle, all three voltages V_{DC1} , V_{DC2} and V_{DC3} have to be equal.

For single phase operation, when the inverter is not drawing current, the generated current can be utilized to charge a backup battery. The magnitudes of the harmonic components up to the 21st order normalized to the fundamental frequency component for both a square wave and the recommended 7-level inverter are shown in Figure 7.4.

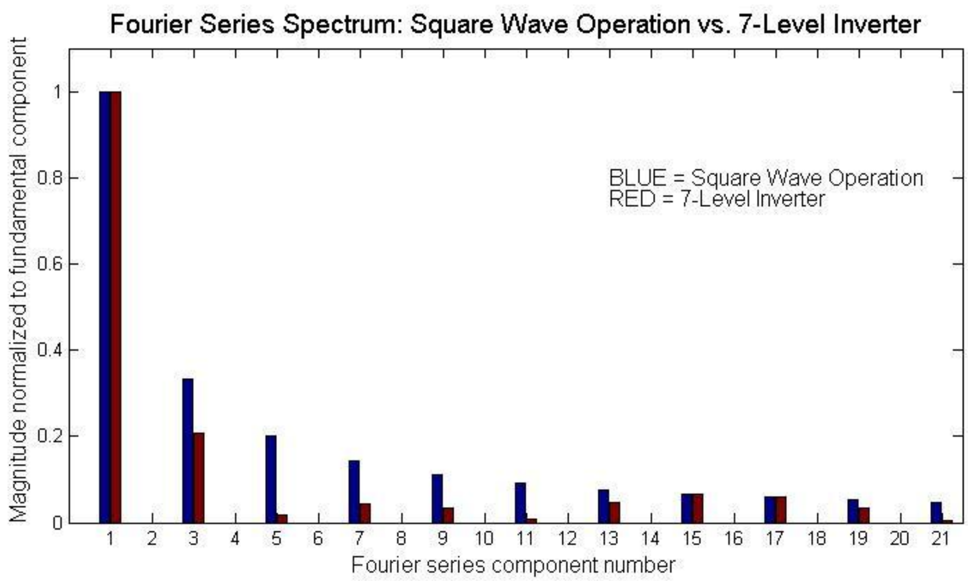

Figure 7.4: Harmonic Magnitudes: Square Wave vs.7-level, Equal Angle

If extended to three phases, assuming that the current drawn from all phases are equal, the total output and individual phase currents of DC bus 1, corresponding to the voltage waveform y1 in Figure 7.3, are shown in Figure 7.5.

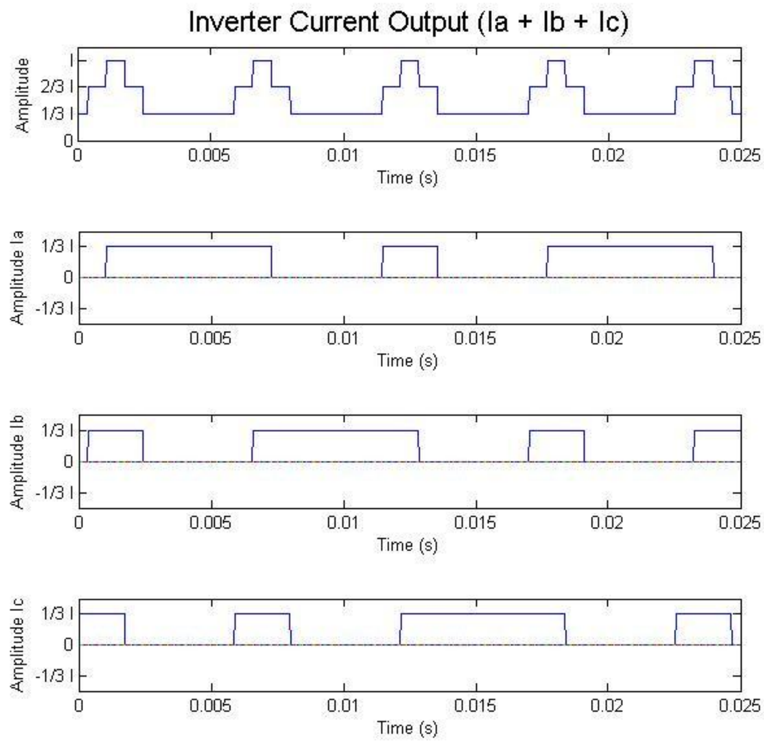

Figure 7.5: 3-Phase DC Bus Output Currents

When PV arrays generate power, the value of the output DC current is I, the magnitude of which depends on the bus voltage V. It is assumed that all three phases draw the same current from the DC bus, which is equal to $\frac{I}{3}$ per phase. When a single phase draws current from the DC bus, the total current fed into the inverter from the bus is $\frac{I}{3}$. Similarly, when two phases draw current from the DC bus, the total current fed into the inverter from the bus is $\frac{2I}{3}$. When current is being drawn by only one or two phases, the remainder of the current I will be referred to as the off-cycle current. The off-cycle current of the DC bus can be dedicated for charging the battery backup system instead of diverting it through shunt resistance. The waveforms of the PV array output, inverter output, and backup battery charging currents are shown in Figure 7.6.

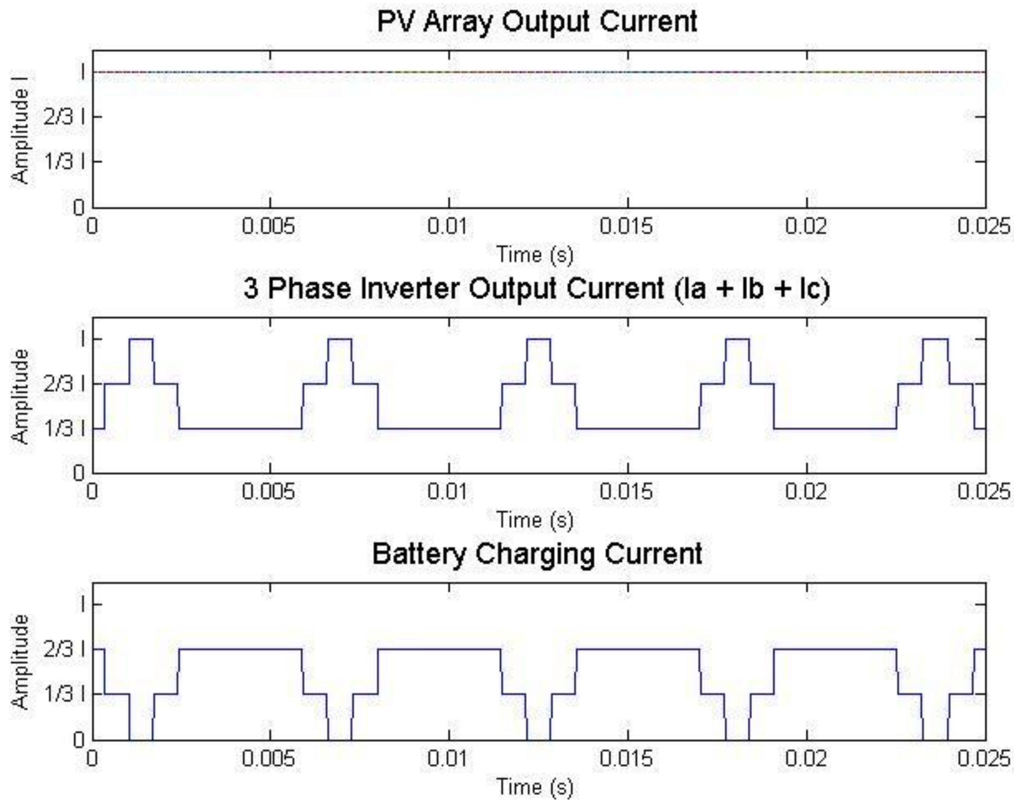

Figure 7.6: 3 Phase Inverter and Battery Charging Currents

The off-cycle current may also be used as a separate voltage source. This could allow the addition of another level without requiring an additional DC bus for it.

This is a contrast from a PV system with a square wave inverter, since the output waveform has a non-zero duty cycle of 100%, i.e. all of the DC voltage bus current I is constantly fed into the inverter. Algorithms to use off cycle currents of DC buses for battery charging or an additional voltage level can be developed further. The intent of this thesis is to introduce the concept for development later.

The main drawback of this system, as is the case for any multi-level inverter scheme, is the additional number of components. The significant reduction in harmonics and a relatively simple control scheme however, make this a very attractive option and the recommended one based on the findings in this thesis.

7.4 Contributions

1- The concept of utilizing equal angle operation for equalizing voltage stress and harmonics mitigation techniques discussed in Chapter V

2- The concept of utilizing current off-cycles for battery charging, extra voltage level or supplying local power supply

3- Metric for measuring RMS value of harmonics exceeding limits normalized to output voltage

7.5 Further Work

The following topics were introduced in this thesis but were not developed further:

1- Protection and Control algorithms for power maximization of a multilevel inverter configuration introduced in Section 5.3

2- Equalizing voltage stress for non-symmetrical firing angles introduced in Section 5.2

3- Algorithm for charging backup batteries with off-cycle current, introduced in Section 7.3

4- Utilizing PWM by using IGBT configurations capable of high frequency switching

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APPENDICES

Appendix A: MATLAB Code

A.1 MATLAB Code: HarmonicAnalysis.m

```
%Parameters
vn = zeros(1,30); %Coefficient of harmonic order n (Vn from Eq 2.21)
vn_rms = zeros(30,1); %RMS value of harmonic order n (Eq 2.23)
vn_distortion = zeros(30,1); %Distortion % of harmonic order n (Eq
2.26)
vn_limit_exceedance = zeros(30,1); %IEEE519 limit exceedance (%) (Sec
6.1)
vn_limit_exceedance_rms = zeros(30,1);%RMS value limit exceedance (Sec
6.1)

t = 0:0.000033:0.05; % Sampling frequency 1 kHz
f = 60; % Voltage waveform frequency in Hz

levels = 3; %Number of DC voltage sources
alpha = zeros (1,levels); %Firing angles


%Variables used for computation
v = 0; %Voltage waveform
cos_term = zeros(1,30); %Cosine term summation of Fourier series
rms_square = 0; %RMS squared of output waveform


%Assignment of firing angles
alpha(1) = pi/18;
alpha(2) = 3*pi/18;
alpha(3) = 5*pi/18;

%{
For the equal angle approach from Section 5.5, the following code can
be
used:

for x = 1:levels
    alpha(s) = (s/(levels+1))*(pi/2);
end
%}


%Output voltage waveform generation
for s = 1:levels
    d1 = (1-(((2*alpha(s))/pi)))*50; %Duty cycle
    vp = (square(2*pi*f*(t-(((50-d1)/200)/f)), d1)+1)/2; %Positive
component
    vn = (-(square(2*pi*f*(t-((((50-d1)/200)+0.5)/f)), d1))-1)/2;
%Negative component
    v = v+vp+vn; %Total waveform
end
```

```
%Computing RMS value of output voltage waveform assuming all DC
voltages
%are the same and equal to 1
for sss = 1:levels

    if sss == levels
        rms_square = rms_square + [([pi/2 - alpha(sss)]*2)/pi*(sss^2)];
    else
        rms_square = rms_square + ([alpha(sss+1)-
alpha(sss)]*2)/pi*(sss^2);
    end
end
rms = sqrt(rms_square);


%Computing harmonic components
for s = 1:30

    for ss = 1:levels
        cos_term (s) = cos_term (s) + cos(s*alpha(ss));%Cosine terms
    end

    vn(s) = abs(((4*cos_term(s))/(s * pi)) * (sin
(s*pi/2)));%Coefficients
    vn_rms(s) = vn(s)/sqrt(2);%RMS value of harmonic order s
    vn_distortion(s) = vn(s)/vn(1); %Distortion % of harmonic order s
    vn_limit_exceedance(s) = vn_distortion(s) - 0.03; %limit exceedance
of harmonic order s
    vn_limit_exceedance_rms(s) = [vn_rms(s) - (0.03*vn_rms(1))];%RMS
value of limit exceedance of harmonic order s

end


%THD Caculation
rms1 = vn_rms(1); %RMS value of fundamental frequency component
thd = sqrt((rms/rms1)^2 - 1);%THD

%thd_square = vn/vn(1);
vn_rms = vn_rms/rms;%Normalizing RMS value of harmonic components to
RMS value of output waveform
thd_exceedance = thd - 0.05; %THD IEEE519 limit exceedance
thd_exceedance_rms = [sqrt(1^2 - vn_rms(1)^2) -
(0.05*vn_rms(1))]/1;%RMS value of THD limit exceedance

%Plotting output voltage waveform
subplot (2,1,1)
plot(t,v,t,0);
axis([0 0.025 -levels-1 levels+1]);
set(gca,'YTick',-3:1:3)
set(gca,'YTickLabel',{'-(VDC1+VDC2+VDC3)','-(VDC1+VDC2)','-
VDC1','0','VDC1','VDC1+VDC2', 'VDC1+VDC2+VDC3'})
title ('Output Voltage')
h = get(gca, 'title');
set(h, 'FontSize', 16)
```

```
xlabel('Time (s)');
ylabel('Amplitude');

%Plotting harmonic magnitude spectrum normalized to fundamental
frequency
%component
subplot(2,1,2)
bar(vn/vn(1))
axis([0 20 0 1.1]);
set(gca,'XTick',[1:20])
title ('Fourier series spectrum')
h = get(gca, 'title');
set(h, 'FontSize', 14)
xlabel('Fourier series component number');
h = get(gca, 'xlabel');
set(h, 'FontSize', 12)
ylabel('Magnitude normalized to fundamental component');
h = get(gca, 'ylabel');
set(h, 'FontSize', 12)
text(15,0.8,['THD = ',
num2str(thd)],'HorizontalAlignment','right','FontSize',16);
```

A.2 MATLAB Code: ThreeLevelTHDminimization.m

```
alpha = 0; %firing angles
rms = 0; %rms value of voltage waveform
thd = zeros(1,90) %total harmonic distortion

for s = 1:90 %varying firing angle from 0 to 90 degrees (0 to pi/2
rad/s)

   %calculating V1,rms
   cos_term =  cos(alpha);
   x = ((4*cos_term)/(pi))/sqrt(2);

   %calculating Vrms
   rms = sqrt(([(pi/2)-alpha]*2)/pi);

   %calculating THD
   thd(s) = sqrt((rms/x)^2 - 1);

   %increasing firing angle by one degree (pi/180 rad/s)
   alpha = alpha + pi/180;
end

%plotting THD vs. firing angle
tt = 1:90;
plot(tt,thd,tt,0);
axis([0 90 0 2.5]);
title ('THD of a 3-Level Inverter')
h = get(gca, 'title');
set(h, 'FontSize', 16)
xlabel('Firing Angle (degrees)');
ylabel('THD');
```

A.3 MATLAB Code: FiveLevelTHDminimization.m

```
levels = 2; %inverter level
alpha = zeros (1,levels); %firing angles
rms_square = 0; %rms value of waveform squared
thd = zeros (90,90); %total harmonic distortion matrix (90x90) matrix

%Firing angles alpha1 and alpha2
alpha(1) = 0;
alpha(2) = 0;

 for xx = 1:90 %varying alpha2 from 0-90 degrees (0 - pi/2 rad/s)

    alpha(1) = 0; %alpha1 reset to 0
    for yy = 1:xx %set alpha1 range from 0 to alpha2

            %calculate the summation of the cosine terms
            cos_term = 0;
            for ss = 1:levels
                cos_term = cos_term + cos(alpha(ss));
            end

            %calculate V1,rms
            x = ((4*cos_term)/(pi))/sqrt(2);

            %calculate Vrms squared
            rms_square = 0;
            for sss = 1:levels
                if sss == levels
                    rms_square = rms_square + [(([pi/2 -
alpha(sss)]*2)/pi*(sss^2)];
                else
                    rms_square = rms_square + ([alpha(sss+1)-
alpha(sss)]*2)/pi*(sss^2);
                end
            end

            rms = sqrt(rms_square); %calculate Vrms
            rms1 = x; %V1,rms
            %calculate THD
            thd (xx,yy) = sqrt((rms/rms1)^2 - 1);

            %increase alpha1 by 1 degree (pi/180 rad/s)
            alpha(1) = alpha(1) + pi/180;
        end

    alpha(2) = alpha(2) + pi/180; %increase alpha2 by 1 degree (pi/180
rad/s)
end

bar3(thd); %plot THD vs firing angles
xlabel('Firing Angle 1 (degrees)');
ylabel('Firing Angle 2 (degrees)');
zlabel('THD');
```

A.4 MATLAB Code: FifthSeventhDistortionPercentageMinimization.m

```
alpha = 0; %firing angle
rms_square = 0; %rms of waveform squared

thd5_7 = zeros(1,90); %Distortion % of the 5th and 7th harmonic orders
thd3 = zeros (1,90); %Distortion % of the 3rd harmonic order
thd9 = zeros (1,90); %Distortion % of the 9th harmonic order

for s = 1:90 %vary firing angle from 0 to 90 degrees (0 to (pi/2 rad/s)

   alpha = alpha + pi/180; %increase firing angle by 1

   %cosine calculationgs
   cos_term =  cos(alpha); %for fundamental frequency
   cos_term3 =  cos(3*alpha)/3; %for 3rd harmonic order
   cos_term5 = cos(5*alpha)/5; %for 5th harmonic order
   cos_term7 = cos(7*alpha)/7; %for 7th harmonic order
   cos_term9 = cos(9*alpha)/9; %for 9th harmonic order

   rms1 = abs(((4*cos_term)/(pi))/sqrt(2)); %V1,rms
   rms3 = abs(((4*cos_term3)/(pi))/sqrt(2)); %V3,rms
   rms5 = abs(((4*cos_term5)/(pi))/sqrt(2)); %V5,rms
   rms7 = abs(((4*cos_term7)/(pi))/sqrt(2)); %V7,rms
   rms9 = abs(((4*cos_term9)/(pi))/sqrt(2)); %V9,rms

   %Distortion percentage of the 5th and 7th orders
   thd5_7(s) = sqrt((rms5)^2 + (rms7)^2)/rms1;

   %Distortion percentage of
   thd3 (s)= rms3/rms1; %3rd harmonic order
   thd9 (s)= rms9/rms1; %9th harmonic order

end

%plot distortion percentage of 5th and 7th harmonic orders
subplot (3,1,1)
tt = 1:90;
plot(tt,thd5_7,tt,0);
axis([0 90 0 1]);
title ('THD of a 3-Level Inverter; 5th and 7th Order Distortion')
h = get(gca, 'title');
set(h, 'FontSize', 16)
xlabel('Firing Angle (degrees)');
ylabel('THD 5th and 7th orders %');

%plot distortion percentage of 3rd harmonic order
subplot (3,1,2)
plot(tt,thd3,tt,0);
axis([0 90 0 1]);
title ('3rd Order Harmonic Distortion')
h = get(gca, 'title');
set(h, 'FontSize', 16)
xlabel('Firing Angle (degrees)');
ylabel('THD 3rd Order %');
```

```
%plot distortion percentage of 9th harmonic order
subplot (3,1,3)
plot(tt,thd9,tt,0);
axis([0 90 0 1]);
title ('9th Order Harmonic Distortion')
h = get(gca, 'title');
set(h, 'FontSize', 16)
xlabel('Firing Angle (degrees)');
ylabel('THD 9th Order %');
```

VITA

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