A CONCENTRIC JUNCTION THERMOPILE

By

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PREFACE

The thermopile is often neglected as a possible source of electrical power because of its excessive size and weight, and its low power output and efficiency. It is the object of this investigation to determine what factors affect the power output, efficiency, size and weight of a thermopile, and to recommend a design that will provide the greatest efficiency and power output with a minimum size and weight.

This thesis represents a part of the research program for the Wright Air Development Center, United States Air Force, in connection with its investigation of unconventional power supplies. My deepest appreciation and gratitude is due Professor Paul A. McCollum, my thesis advisor and competent leader of the project mentioned above, for his excellent advice and conscientious guidance. I am deeply indebted also to Professor C. F. Cameron whose cooperation made this thesis possible. Thanks is extended to K. C. Wehr, Doyle Frazier, and George Ingram for their aid in constructing and testing the experimental thermopile. I also wish to express my gratitude to Mrs. Mildred J. Avery for her conscientious, efficient, and capable service in typing this thesis.

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CHAPTER I

INTRODUCTION

For many years scientists have experimented with the thermoelectric effect as a direct method of converting thermal energy to electrical energy. This method has several advantages that make it worth considering as a source of power. It contains no moving parts, requires no maintenance, and is noiseless. Conversely, it has several disadvantages which seriously limit its use. This method of power generation is very inefficient, has a low power output per pound of weight, and a low power output per cubic foot of volume.

The maximum efficiency that may be obtained from a thermopile may be theoretically calculated from the characteristics of the thermocouple materials. This theoretical maximum efficiency can be improved only by metallurgical research. The resultant efficiency of a practical thermopile, however, seldom approaches the maximum efficiency calculated from the characteristics of its thermocouple materials. A thorough analysis of previous work on thermopiles led this author to believe that much could be done to improve the efficiency and reduce the size and weight of the practical thermopile. The text of this thesis is concerned with the design of the seemingly most efficient, smallest, and lightest thermopile possible using existing thermocouple materials. Many important factors are brought out in developing this optimum design which are not usually considered in thermopile design. The limitation and merit of this optimum thermopile are discussed in the summary and conclusions.

CHAPTER II

DESIGN CONSIDERATIONS

It is a well known fact that the efficiencies of practical thermopiles seldom approach their theoretically calculated maximum efficiencies. Some efficiency is lost through the effects of stray heat losses. The extremely low efficiency observed by T. N. Ewing (1) while testing a copper-copnic thermopile led this author to believe that other factors were involved. The reason for the low efficiency of this copper-copnic thermopile is rather obvious when the characteristics of copper and copnic are compared. Copper has a thermal conductivity of about 10 times that of copnic and a resistivity of about 1/10 the resistivity of copnic. Since the thermocouple elements of this particular thermopile were made of equal lengths and diameters of copper and copnic wire, the copper elements were conducting about 10 times more heat away from the hot junctions than the copnic elements, while the copper elements accounted for only 1/10 of the total thermopile resistance. If copper elements of smaller cross-section were used, the heat input to the thermopile would be greatly reduced without appreciably increasing the internal resistance of the thermopile. Although the power output would be reduced slightly, the input power would be greatly lowered. The result would be a considerable improvement in thermopile efficiency. While increasing the efficiency, this modification would also reduce the weight of the thermopile. Therefore, the key to raising the actual thermopile efficiency to the theoretical maximum appears to rest largely in the proper determination of form factors of thermocouple elements.

Aside from increasing the efficiency, it is believed that proper design will also reduce the excessive size and weight associated with the thermopile. Many previously constructed thermopiles have contained a considerable volume of ceramic materials necessary for thermal insulation and mechanical support of the thermocouple elements. A large reduction of weight and volume could be obtained by making the thermocouple elements occupy more of the thermopile volume. After a detailed study of all possible configurations and arrangements of thermocouple elements, a design composed of metal sheets with concentric junctions seemed to be the best approach to the problem. The concentric thermopile consists of circular sheets of alternate thermocouple materials separated by thin circular sheets of insulating material. These sheets are joined alternately at the outside and inside circumferences as shown in Figure 1. Heat is pro-

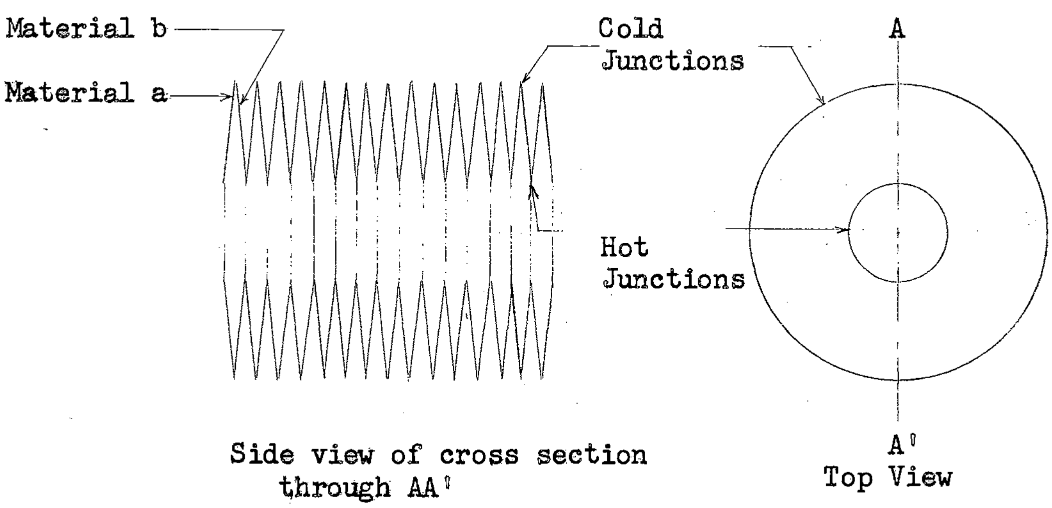


Fig. 1. The Geometry of the Concentric Junction Thermopile.

vided to the hot junctions on the inside of the thermopile while the cold junctions on the outside dissipate heat to the surrounding air. The circular sheet arrangement forms heating and cooling fins to aid in absorbing and dissipating heat at the hot and cold junctions respectively. This design is ideal in that it requires an absolute minimum of insulating material. The concentric junction thermocouple configuration is assumed in all the calculations of the following chapters.

CHAPTER III

DESIGN FOR MAXIMUM EFFICIENCY

The efficiency of any device is defined as the ratio of power output to power input. For extremely inefficient devices, such as the thermopile, it can be shown that maximum efficiency conditions always occur at the point of maximum power output, according to M. Telks. (2). The maximum power output of any electrical device, and therefore the maximum efficiency of a thermopile generator, occurs when the external resistance R_o is equal to the internal resistance of the thermopile R. When operating at maximum efficiency, the current flowing in the thermopile circuit is:

$$I = \frac{E}{R + R_o} = \frac{E}{2R} \tag{1}$$

where I is expressed in amperes
E is the total internally generated emf of the thermopile in volts
R and R_o are expressed in ohms.

The maximum usable power output of a thermopile is therefore:

$$P_o = I^2 R_o = \frac{E^2 R_o}{4R^2} \quad \frac{E^2}{4R} \tag{2}$$

where P_o is expressed in watts.

The total power input to a thermopile is dissipated by heat flow through the thermocouple elements, stray heat losses, and the thermal power required for the total electrical power generated by the pile which is $Pt = 2P_o$. According to the second law of thermodynamics, the thermal power required to generate 2P electrical power is: (2)

$$\frac{2P_oT_h}{T_h - T_c} = \frac{2P_oT_h}{\Delta t} \qquad (3)$$

where T_h is the hot junction temperature in degrees Kelvin
T_c is the cold junction temperature in degrees Kelvin
Δt is the temperature difference in either degrees Centigrade or degrees Kelvin.

If it is assumed that the heat transferred through the thermocouple elements greatly exceeds the stray heat losses, the total power input to the thermopile is:

$$P_i = \frac{2P_oT_h}{\Delta t} + Q \qquad (4)$$

where Q is the heat flow through the thermocouple elements in watts.

By dividing Equation (2) by Equation (4), the efficiency is calculated as:

$$\text{eff.} = \frac{P_o}{P_i} = \frac{1}{\frac{2T_h}{\Delta t} + \frac{4QR}{E2}} \qquad (5)$$

For further analysis of the efficiency equation, the terms Q, R, and E must be calculated in terms of the dimensions and characteristics of the thermocouple elements.

Since the thermocouple elements of the concentric thermopile are connected in series, the total emf may be expressed by:

$$E = ne\Delta t$$

where n is the number of thermocouple elements
e is the average thermopower of a thermocouple element in volts/degree Centigrade.

The calculation of R and Q in terms of the thermocouple dimensions requires special consideration because of the unusual shape of the thermocouple elements. The following calculations refer to Figure 2.

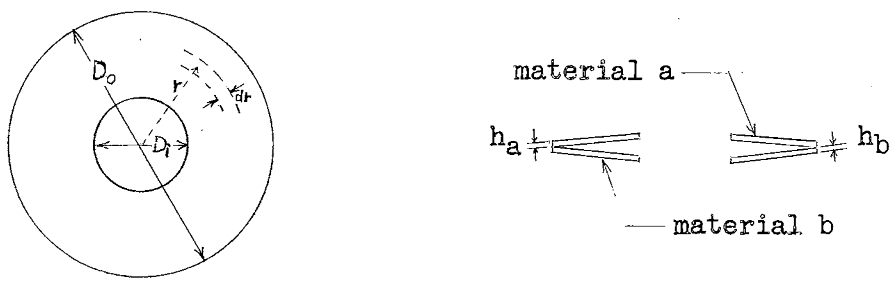


Figure 2. A Concentric Junction Thermocouple Element.

The incremental resistance of material "a" at any distance r is:

$$dR_a = \frac{p_a dr}{A_a} = \frac{p_a dr}{h_a(2\pi r)} \qquad (7)$$

where R is the resistance of material "a" in ohms
p is the resistivity of material "a" in ohm-meters
$A_a = 2\pi r \ h_a$ is the area of material "a" perpendicular to the direction of current flow
h_a and r are thickness and radius of material "a" expressed in meters.

If p_a is assumed to be a constant which is calculated at an arithmetic mean temperature, the total resistance of material "a" may be found easily by integrating Equation (7). The result is:

$$R_a = \int_{D_i/2}^{D_o/2} \frac{p_a dr}{2\pi h_a r} = \frac{p_a}{2\pi h_a} \ln D_o/D_i \qquad (8)$$

If the same procedure is applied to material "b", the total thermopile resistance is:

$$R = n(R_a + R_b) = \frac{n}{2\pi}(p_a/h_a + p_b/h_b) \ln D_o/D_i \qquad (9)$$

The heat flow through the thermopile elements may be calculated easily by Fourier's conduction law. This law for steady state heat transfer may be expressed as: (3)

$$q = \frac{K\Delta t}{\int \frac{dr}{A}} \qquad (10)$$

where
- q is the heat flow in watts
- K is an average value of thermal conductivity based on an arithmetic-mean temperature in watts/meter-deg C
- $A = 2\pi r\, h$ is the area perpendicular to heat flow in square meters
- r is the distance of heat flow in meters.

Applying Equation (10) to material "a", the heat flow through this material is:

$$q_a = \frac{K_a \Delta t}{\int_{D_i/2}^{D_o/2} \frac{dr}{2\pi r h_a}} = \frac{2\pi h_a K_a \Delta t}{\ln D_o/D_i} \qquad (11)$$

If the same procedure is repeated for material "b", the total heat flow through the thermocouple elements of the thermopile is:

$$Q = n(q_a + q_b) = \frac{n2\Delta t\pi}{\ln D_o/D_i}(h_a K_a + h_b K_b) \qquad (12)$$

If the calculated expressions for E, R, and Q are substituted in Equation (5), the general equation for the efficiency of a concentric junction thermopile is:

$$\text{eff} = \frac{1}{\frac{2T_h}{\Delta t} + \frac{4(K_a h_a + K_b h_b)(p_a/h_a + p_b/h_b)}{e^2 \Delta t}} \qquad (13)$$

Notice that the resultant efficiency equation contains only the metal thickness h_a and h_b and is independent of the overall dimensions D_o, D_i and n. The equation may be arranged so that it contains only one variable h_a/h_b. This equation is:

$$\text{eff} = \frac{1}{\frac{2T_h}{\Delta t} + \frac{4(K_a p_a + K_b p_b + K_b p_a (h_b/h_a) + K_a p_b (h_a/h_b)}{e^2 \Delta t}} \qquad (14)$$

Since the thickness ratio h_a/h_b may be varied easily in thermopile construction, it is desirable to determine the value of this ratio for maximum efficiency. The maximum efficiency may be determined by differentiating Equation (14) and equating the results to zero.

$$\frac{d(\text{eff})}{d(h_a/h_b)} = - K_b p_a (h_a/h_b)^2 + K_a p_b = 0$$

$$h_a/h_b = \sqrt{\frac{K_b p_a}{K_a p_b}} \qquad (15)$$

Equation (15) shows that an optimum thickness ratio certainly exists and its value may be calculated from a knowledge of the thermal and electrical characteristics of the materials. If this optimum thickness ratio is substituted in the efficiency expression, the maximum possible efficiency is:

$$eff_{max} = \frac{1}{\frac{2T_h}{\triangle t} + \frac{4(K_a p_a + k_b p_b + 2\sqrt{K_a p_b K_b p_a})}{e^2 \triangle t}} \tag{16}$$

To determine the degree of importance of the optimum thickness ratio, a plot of Equation (14) was made. This graph, Figure 3, illustrates the variation of efficiency with thickness ratio for both iron-copnic and copper-copnic thermopiles. These curves were plotted for the specific case of $T_h = 400$ C and $T_c = 0$ C using data from the International Critical Tables. (4). These curves are of no value if either of these temperatures are changed. This graph clearly illustrates the cause of the inefficient copper-copnic thermopile referred to in the previous chapter. This thermopile was constructed with an area ratio of 1, which, of course, corresponds to a thickness ratio of 1 on Figure 3. If this thermopile had been constructed with an area ratio of 13.7/1, its theoretical efficiency could have been increased from about 0.13 to about 0.5. This increase in efficiency would be accompanied by a considerable decrease in weight and volume of the thermopile.

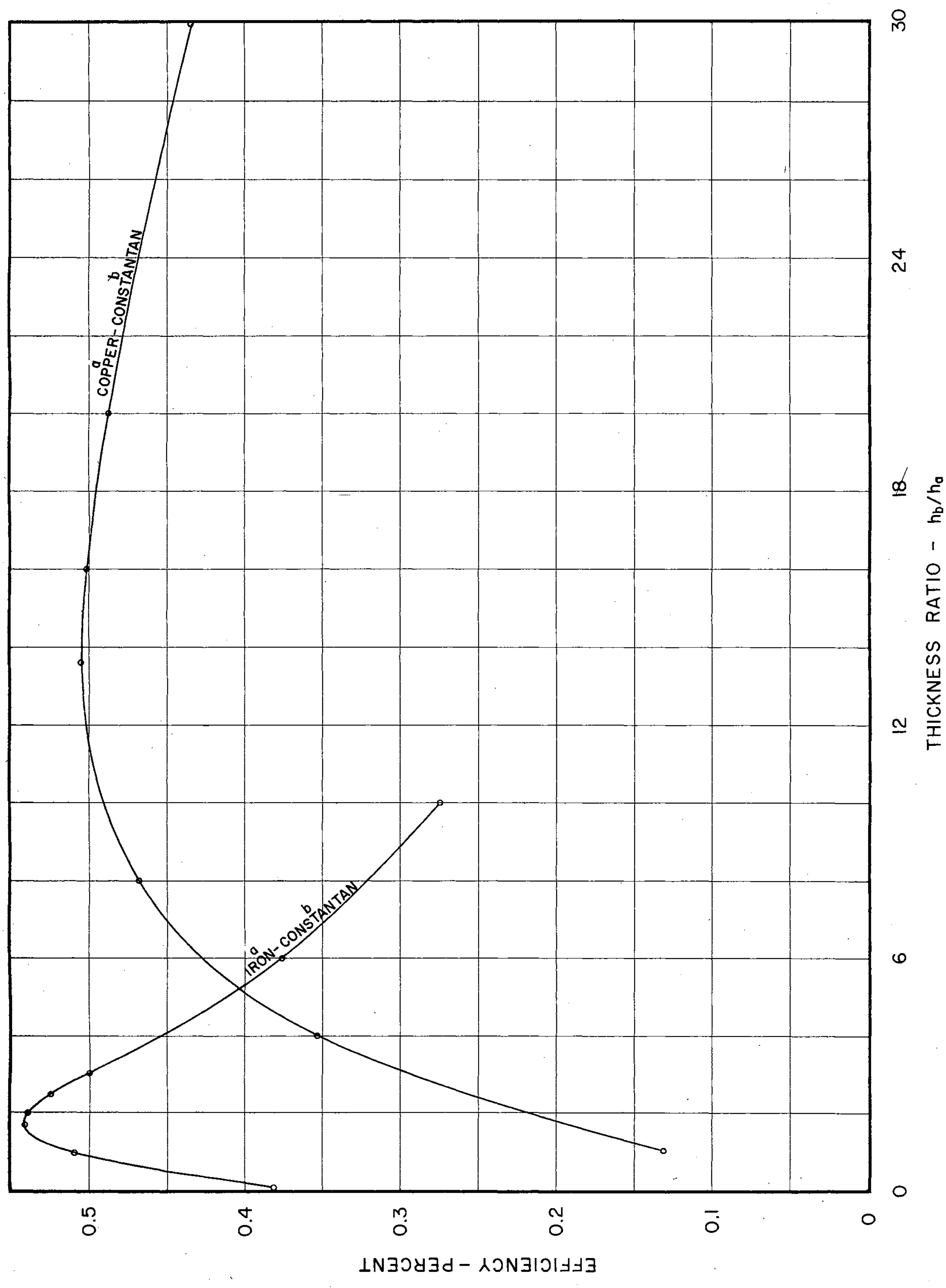


Figure 3. Maximum efficiency vs thickness ratio. Tc = 0 °C, Th = 400 °C.

CHAPTER IV

THE RELATIONS BETWEEN POWER OUTPUT, WEIGHT, AND VOLUME

In the previous chapter it was shown that the efficiency of a concentric junction thermopile was independent of all thermopile dimensions except the thickness ratio of the metals forming its thermocouple elements. If the optimum thickness ratio is used in designing a thermopile, the power output, weight, and volume of the thermopile are determined by all other dimensions. These dimensions are insulation thickness, outside diameter, inside diameter, length, and the thickness of one thermocouple material. For optimum thermopile design, the effects of these dimensions must be known.

The power output and weight of the concentric thermopile of a particular size will be greatly affected by the thickness of the insulation between the thermocouple materials. The effects of insulation thickness on power output may be found by Equation (2). This equation when combined with Equation (6) states that the usable power output of a thermopile is:

$$P_o = \frac{n^2 e^2 \Delta t^2}{4R} \qquad (17)$$

If h_i is defined as the insulation thickness, and H as the total thermopile length, the number of thermocouple elements in the thermopile will be:

$$n = \frac{H}{2h_i + h_a + h_b} \tag{18}$$

Instead of using h_i as an independent variable, an insulation factor defined as:

$$a = h_i/h_b \tag{19}$$

is substituted in Equation (17) along with Equations (18) and (9). The power output in terms of the insulation factor and thermopile dimensions is:

$$P_o = \frac{\pi H e^2 \Delta t^2}{2 \ln D_o/D_i \left[(2a+1)(p_b+p_a h_b/h_a)+p_a+p_b h_a/h_b\right]} \tag{20}$$

Note that the units to be used with H depend only upon the units of resistivity. To develop an expression which is independent of all thermopile dimensions except the insulation factor, a power output factor is defined as:

$$F_p = 1/H \ln D_o/D_i = \frac{\pi e^2 \Delta t^2}{2\left[2a(p_b+p_a h_b/h_a)+(1+h_a/h_b)p_b+(1+h_b/h_a)p_a\right]} \tag{21}$$

A plot of Equation (21) showing the variation in power output factor with the insulation factor for both iron-copnic and copper-copnic thermopiles is shown in Figure 4. These curves were plotted for the same specific set of operating conditions as Figure 3.

Figure 4 may be used to calculate the power output of a particular thermopile as follows. The insulation factor of the thermopile is first

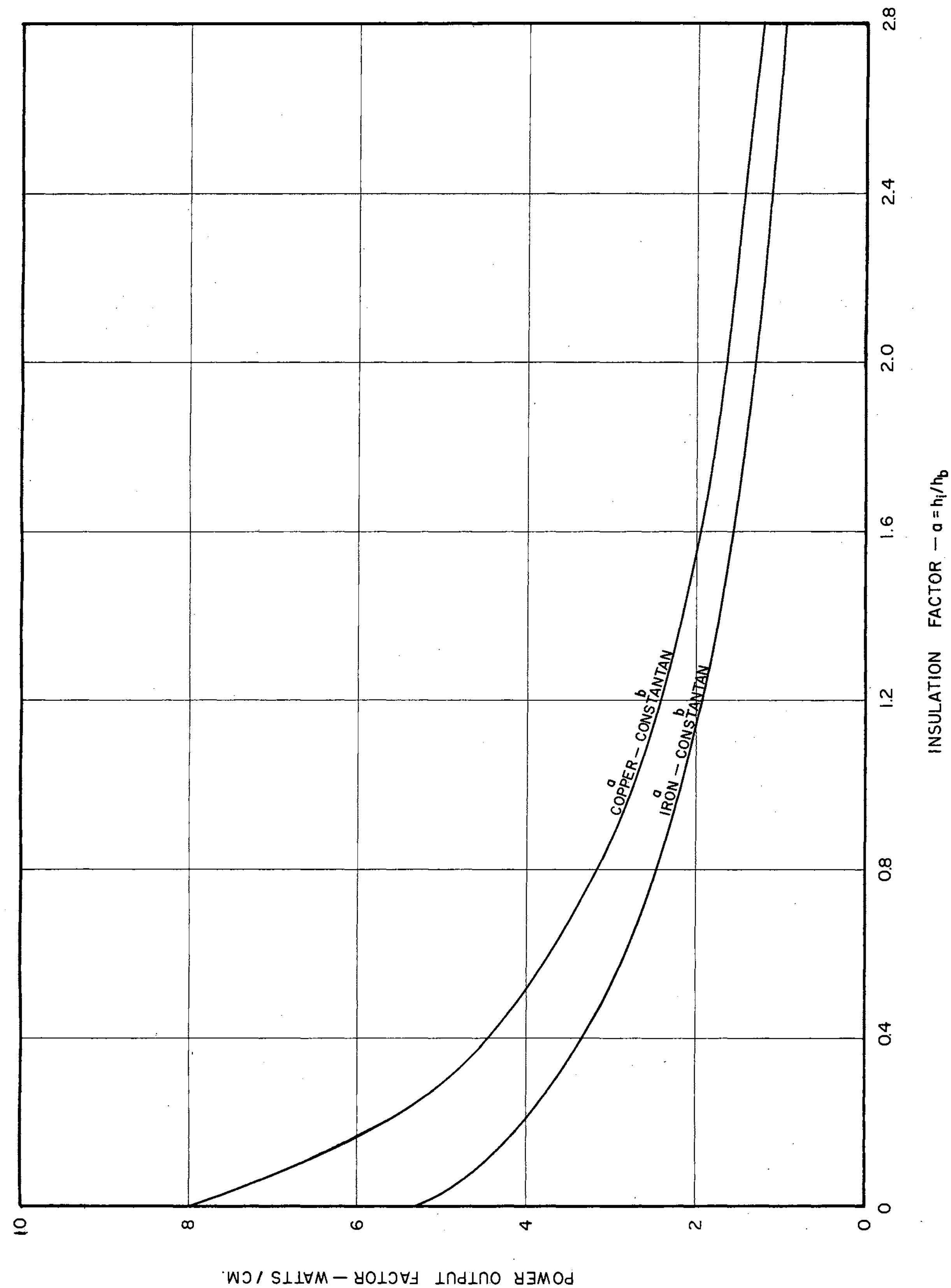


Figure 4. The Effect of Insulation Thickness on Power Output, $T_h = 400^oC$, $T_c = 0^oC$

calculated by Equation (19). The power output factor corresponding to this insulation factor is then read from the appropriate curve on Figure 4. The power output may be calculated from the power output factor by Equation (21). These curves may be used to calculate the power output of a thermopile of any size as long as it is operating at the specified temperatures. New curves must be plotted for other operating conditions.

The total weight of a concentric thermopile may also be expressed as a function of the insulation factor. The total thermopile weight is:

$$W = W_a + W_b + W_i \tag{22}$$

where
- W_a is the weight of material "a" with a density d_a lb/cu meter
- W_b is the weight of material "b" with a density d_b lb/cu meter
- W_i is the weight of the insulating material with a density d_i lb/cu meter.

Since the weights of the individual materials are

$$W_a = d_a \pi/4(D_o^2 - D_i^2)h_a n$$
$$W_b = d_b \pi/4(D_o^2 - D_i^2)h_b n$$
$$W_i = d_i \pi/4(D_o^2 - D_i^2)h_i 2n$$

they may be substituted in Equation (22) along with Equations (18) and (19) to find the total thermopile weight. This is:

$$W = \frac{\pi H(D_o^2 - D_i^2)\left[d + (d_b + 2ad_i)h_b/h_a\right]}{4\left[1 + (1 + 2a)(h_b/h_a)\right]} \tag{23}$$

Note that the units to be used with the thermopile dimensions depends only upon the units of density.

To develop an equation which is independent of the thermopile dimensions, a weight factor is defined as:

$$F_W = \frac{W}{H(D_o^2 - D_i^2)} = \pi/4 \quad \frac{d_a + d_b(h_b/h_a) + 2ad_i)h_b/h_a)}{1 + h_b/h_a + 2a(h_b/h_a)} \qquad (24)$$

The weight factor as a function of insulation factor for iron-copnic and copper-copnic thermopiles is plotted as Figure 5. The procedure for using these curves to determine the weight of a particular thermopile is similar to that used with Figure 4.

Two relatively important factors concerning thermopile design are brought out by Figures 4 and 5. First, for thermopiles occupying the same volume, a considerably larger power output may be obtained by using copper-copnic thermocouple elements instead of iron-copnic elements. Also from Figure 5, it is seen that the higher output copper-copnic thermopile actually weighs less than an iron-copnic thermopile of the same volume if the insulation factor is greater than .14.

For additional information, the effects of the diameter ratio D_o/D_i on power output and weight was determined as follows. If Equations (21) and (24) are solved for P_o and W respectively, these expressions are

$$P_o = \frac{F_p H}{\ln D_o/D_i} \qquad (25)$$

and

$$W = F_w H(D_o^2 - D_i^2) \qquad (26)$$

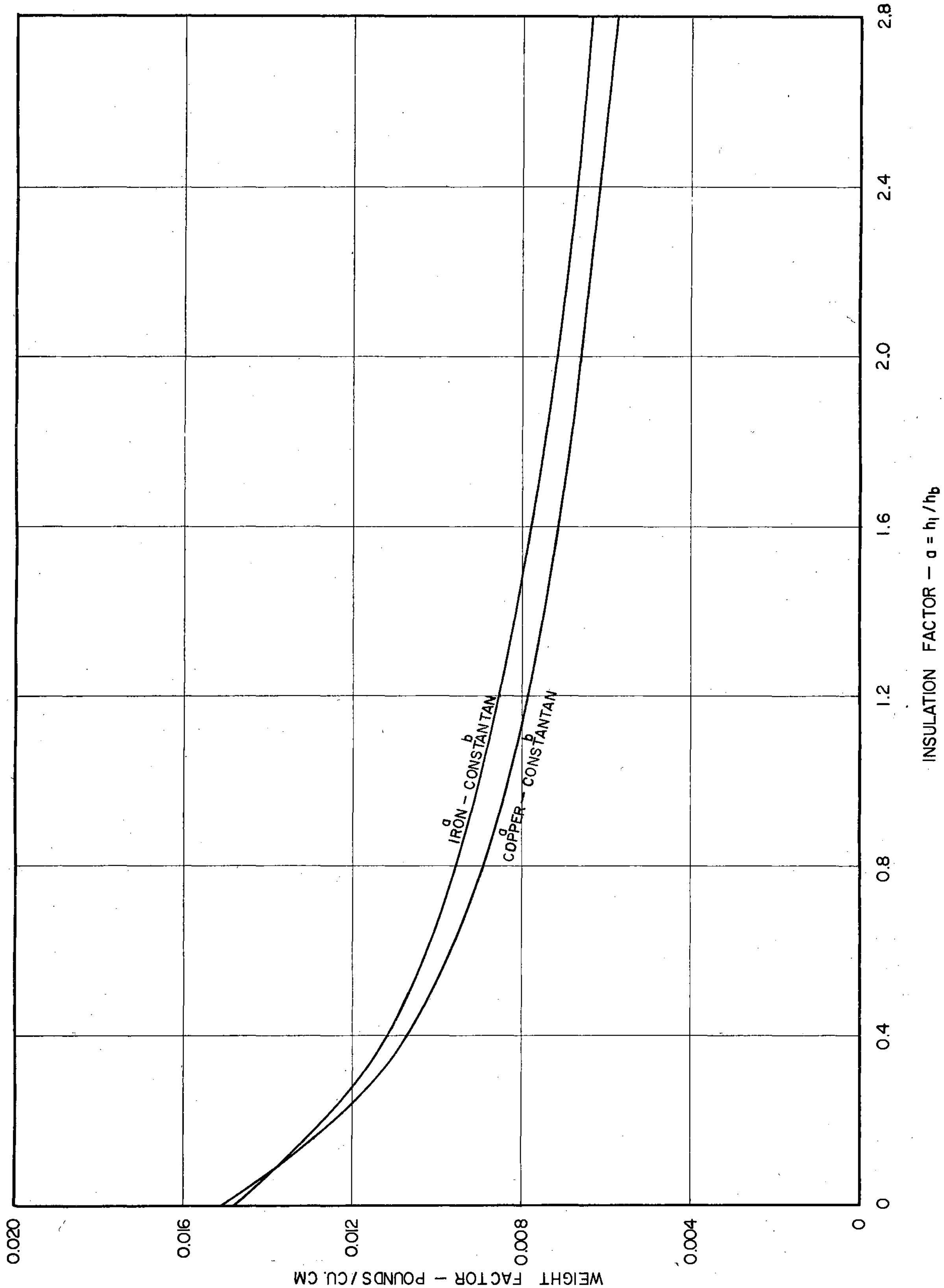


Figure 5. The Effect of Insulation Thickness on Thermopile Weight, T = 400°C., T = 0°C.

The power-weight ratio is found by dividing Equation (25) by Equation (26).

$$\frac{P_o}{W} = \frac{F_p}{F_w\left[\ln D_o/D_i\right]D_i^2\left[(D_o/D_i)^2-1\right]} \qquad (27)$$

If D_o is considered to be the only independent variable, a power-weight factor may be defined as:

$$F_{pw} = \frac{P_oF_w}{WF_p}\,D_i^2 = \frac{1}{\ln D_o/D_i\left[\left(\frac{D_o}{D_i}\right)^2-1\right]} \qquad (28)$$

This power-weight factor is plotted as a function of D_o/D_i in Figure 6. This curve may be used to determine the power-weight ratio of any thermopile as follows. The insulation factor is first calculated by Equation (19). The values of F_p and F_w are then read from Figures 4 and 5 corresponding to this insulation factor. The power-weight factor is read from Figure 6 corresponding to the value of D_o/D_i for the thermopile. The magnitude of P_o/W is then determined by Equation (28).

Figure 6 shows that the power-weight ratio becomes very large and approaches infinity as the outside diameter approaches the inside diameter. From this it would seem that a thermopile could be made very light with a sizable power output. This would be true if not for the difficulties encountered in heat transfer. The problem of heat transfer is taken up in the following pages.

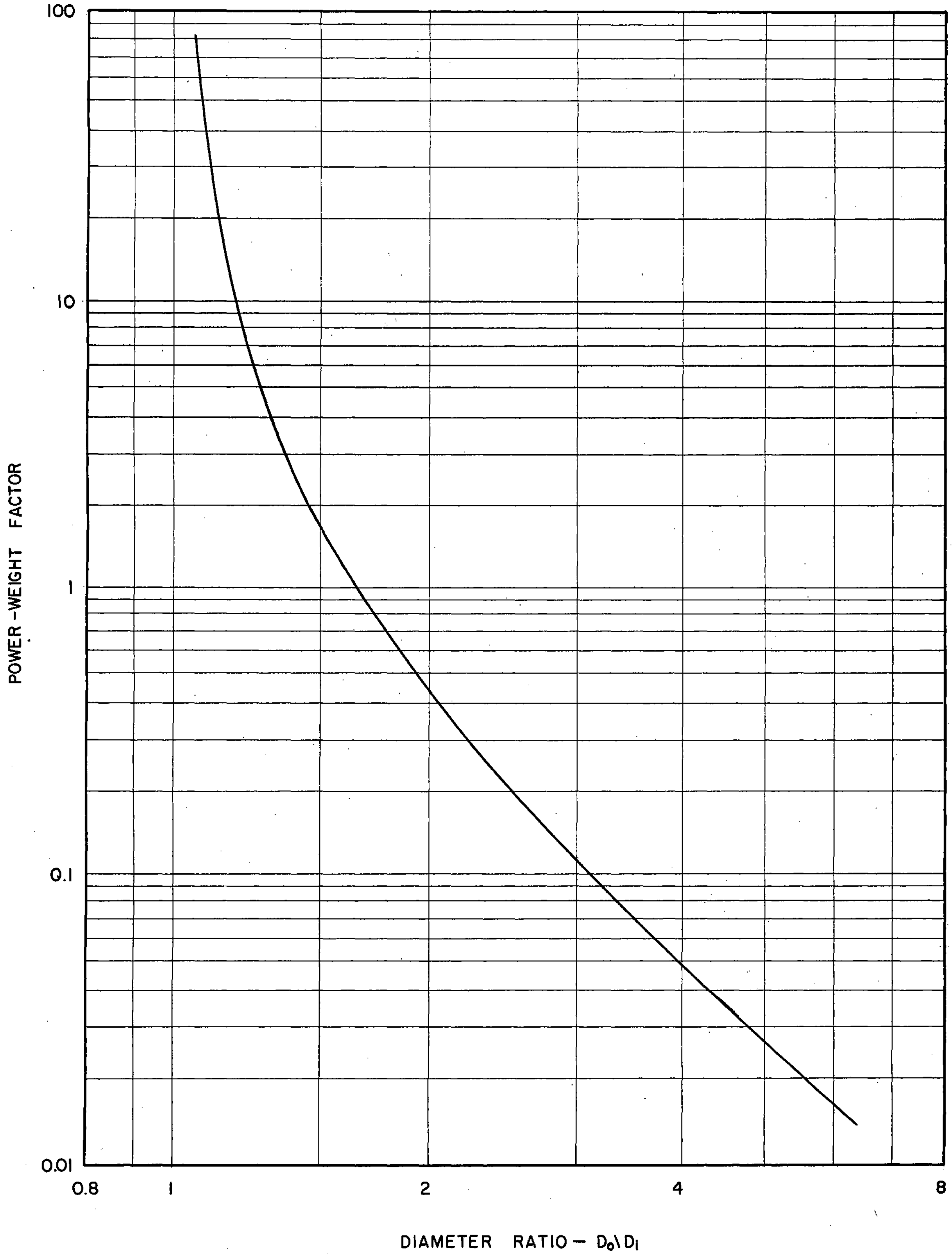


Figure 6. Diameter Ratio Versus Power per Unit Weight

A. Heat Transfer Analysis

The efficiency Equation (13), states that the efficiency of a thermopile is directly proportional to the temperature differential established between the hot and cold junctions. By rearranging Equation (12), this temperature differential may be expressed as:

$$\triangle t = \frac{Q \ln D_o/D_i}{2\pi n(h_a K_a + h_b K_b)} \qquad (29)$$

This equation shows that a reduction in the outside diameter of the thermopile, D_o, which is required for a higher power output per pound, will cause a reduction in temperature differential, and therefore efficiency, if all other factors remain constant. If the same efficiency or temperature differential is to be maintained, a reduction in D_o must be accompanied by an increase in heat flow Q. Therefore, the maximum power output per pound that may be obtained from a thermopile, which is operating at a fixed temperature differential, depends on the magnitude of heat flow that can be maintained between the hot and cold junctions of the thermopile. This heat flow, in turn, depends on the ability of the hot junctions to absorb heat and the cold junctions to dissipate heat. If either the hot or cold junctions of a thermopile are not capable of performing the required heat transfer necessary to maintain the required efficiency, the thermopile will be forced to operate at a smaller temperature differential or efficiency. Since the heat transfer characteristics affect power output, weight, and efficiency, they are one of the most important factors in thermopile design.

Because of the many empirical constants involved, heat transfer calculations must be confined to one specific case where all configurations and dimensions are known. Heat transfer calculations may be used in many different ways for thermopile design depending upon the factors that are initially known. In many cases, trial and error type solutions must be used. In the author's case it was most convenient to design the thermopile so that it could be constructed easily with available materials and then to determine the heating and cooling system requirements by heat transfer analysis.

Because of the availability of 6 inch x .02 inch constantan sheets, the outside diameter of the thermopile was chosen as 6 inches. Anticipating a difficulty in maintaining proper heat transfer, D_o/D_i was made as large as possible by making the inside diameter as small as possible. Since a 3/4 inch diameter globar heating element was to be used, the smallest possible inside diameter was chosen as 1 inch. The experimental thermopile was constructed of iron and constantan sheets with a thickness ratio of 1. Figure 3 shows that this thickness ratio is near the optimum for these materials.

In order to maintain an acceptable efficiency, the first requirement is that the heat flow through the thermocouple elements be sufficient to develop the required temperature differential. The relation between heat flow and temperature differential is given by Equation (12). If the thermopile dimensions are substituted into this equation with proper units, the heat transfer per thermocouple element is:

$$Q / n = q = .178(K_a + K_b)(t_i - t_o) \quad (30)$$

Because of the variation of thermal conductivity with temperature, this equation is very difficult to interpret directly. A graphical interpretation of this equation which illustrates the variation of heat flow per junction with outside temperature for several successive values of inside temperature is shown in Figure 7. The use of Figure 7 to determine temperature differentials will be presented later.

The entire heat flow through a thermocouple element must be dissipated to the surrounding air at the cold junction. The quantity of heat that the cold junction can dissipate depends on the cold junction temperature of the thermopile, the temperature of the surrounding air, and the condition of the surrounding air. The heat flow from a cylindrical body by natural convection into still air is given by the Mark's Mechanical Engineering Handbook as: (5)

$$q = (h_c + h_r)\, A \Delta t \qquad (31)$$

where
- q is the heat transfer in Btu/hr
- A is the area of heat transfer surface in sq. ft.
- $h_c + h_r$ is the combined convection and radiation heat transfer coefficient in Btu per hr sq. ft. per deg. F
- Δt is the temperature differential between the surface and the surrounding air in deg F.

This equation may be modified by simple dimensional analysis to use more convenient units to give:

$$q = 5.69 \times 10^{-4} (h_c + h_r) A (t_o - t_a) \qquad (32)$$

where
- q is the heat transfer in watts
- $h_c + h_r$ are in same units as in Equation (31) for use of M. E. handbook values
- A is expressed in sq. cm.
- t_o is the outside or cold junction temperature of the thermopile in deg. C
- t_a is the temperature of the surrounding air in deg. C

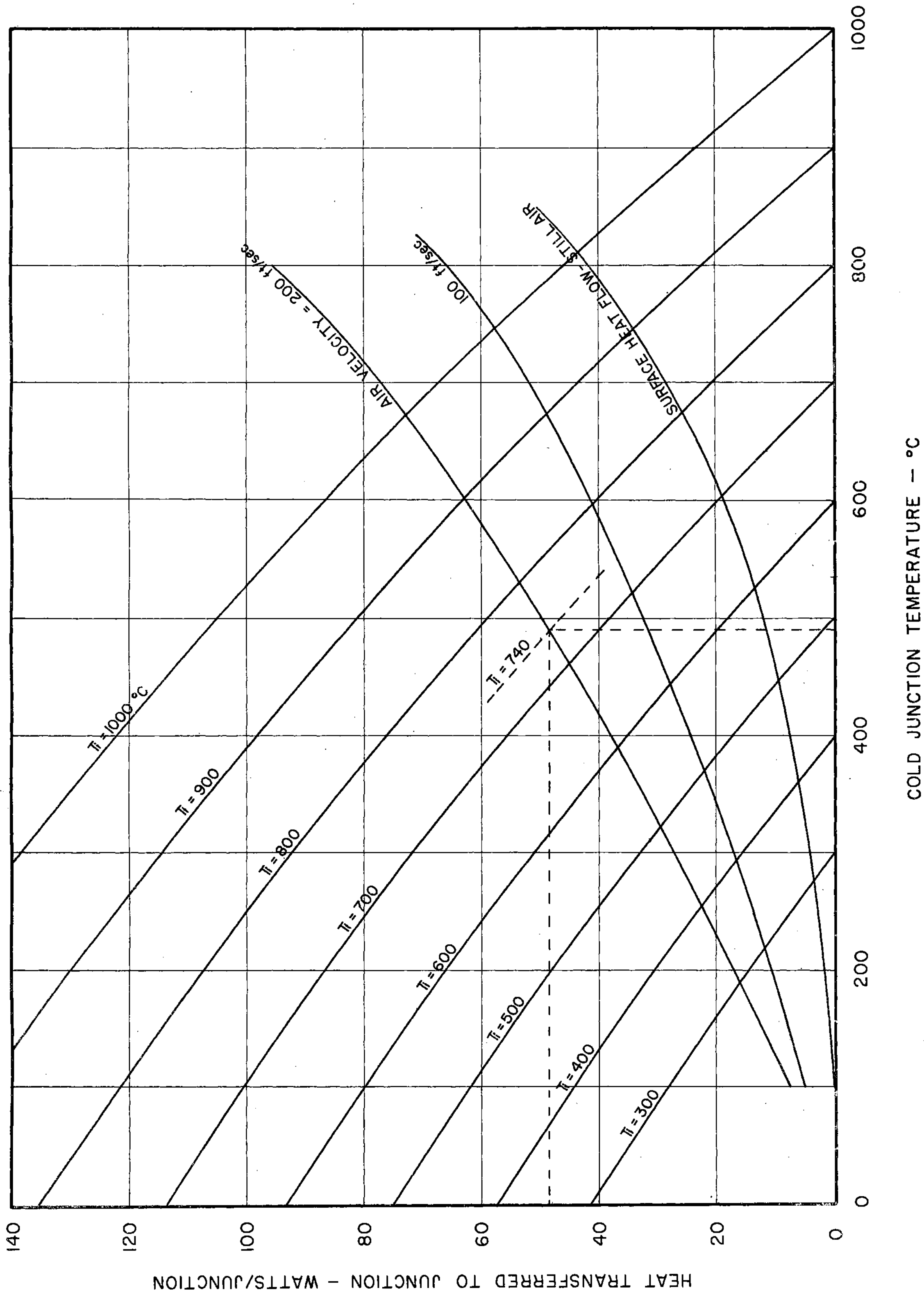


Figure 7. Heat Transfer Characteristics of the Thermopile and Cooling System

For one thermocouple element the area of the heat transfer surface is the exposed outer surface area which, referring to Figure 1, is $A_o = \pi D_o(h_a + h_b)$. Substituting the thermocouple dimensions into Equation (32) the heat transfer per junction may be expressed as:

$$q = 27.7 \times 10^{-4}(h_c + h_r)(t_o - t_a) \tag{33}$$

If the temperature of the surrounding air, t_a, is assumed constant, this equation expresses the heat flow in terms of outside temperature of the thermopile. Since these are the same variables used in plotting Equation (30) this equation may also be plotted on Figure 7. If the thermopile is operating in still air, the operating point of the thermopile must lie on this cooling curve. Since the operating point must also lie on the appropriate t_i curve, the intersection of this curve with the cooling curve must give the operating point of the thermopile. For example, if the inside temperature is 900°C, the intersection of this $t_i = 900°$ curve with the cooling curve shows that 34 watts of heat will be flowing through each thermocouple element and the outside temperature will be 740°C. The thermopile would then be operating at a temperature differential of 900-740 = 160°C. This is a very small temperature differential and would cause very inefficient thermopile operation. A slightly larger temperature differential could be obtained by using an inside temperature of 1000°C but this would not be practical because this temperature is close to the melting point of the copnic. It is obvious that the only way of getting a larger temperature differential is to provide a better cooling system to remove the heat from the cold junctions.

A considerable improvement in heat transfer may be obtained by forcing air past the cold junctions. According to McAdams (3), page 237, the convection heat transfer coefficient for air moving along a flat surface with velocities varying from 16 to 100 fps may be expressed as:

$$h_c = .53V^{.78} \tag{34}$$

where h_c is in Btu per hr. sq. ft. per deg. F
V is the air velocity in ft/sec.

The heat removed from the cold junctions by convection may be calculated easily from the basic equation for convection heat transfer. This equation is identical to Equation (31) except that the approximate radiation coefficient h_r is left out. By the same process used to develop Equation (33), the equation for forced convection heat transfer is:

$$q_c = 27.7 \times 10^{-4}(h_c)(t_o - t_a). = 14.7 \times 10^{-4} V^{.78}(t_o - t_a) \tag{35}$$

In addition to the convection heat transfer calculated by Equation (35) a significant quantity of heat is removed from the cold junctions by radiation. From McAdams (3), page 54, the heat transfer between concentric cylinders is given as:

$$q_r = .172\, A\left[(T_o/100)^4 - (T_s/100)^4\right] \frac{1}{1/p_o + A_o/A_s(1/p_s - 1)} \tag{36}$$

where q_r is the heat transfer by radiation in Btu/hr
T_o is the cold junction temperature in deg. F abs.
T_s is the temperature of the air duct surrounding the thermopile in deg. F abs.

p_o is the emissivity of the cold junction surface
p_s is the emissivity of the air duct absorbing surface
A_o is the area of heat emitting surface of the cold junction in ft^2
A_s is the corresponding heat absorbing surface of the air duct in ft^2.

If heat transfer is assumed to be radially outward from the cold junction surfaces, the area A_o has the same value as that used in Equation (32) and the absorbing surface area will be:

$$A_s = \pi D_s(h_a + h_b)$$

where D_s is the diameter of the air duct in feet, h_a and h_b must also be in feet.

If the thermopile dimensions and the proper emissivity coefficients from McAdams (3), page 46, are substituted into Equation (36), this equation becomes:

$$q_r = .177 \times 10^{-3}\left[(T_o/100)^4 - (T_s/100)^4\right] \qquad (37)$$

where q_r is in watts
T_o and T_s are in deg. F abs.

The total heat dissipated from the cold junctions consists of the sum of the radiation and convection components. If the cooling air temperature, cooling air velocity, and cooling duct surface temperature are constant, the total heat transfer may be calculated from Equations (35) and (37) for successive values of cold junction temperature. This relationship is plotted in Figure 7 for two values of air velocity. The intersection of the 200 ft/sec cooling curve with the 900° inside temperature curve gives a cold junction temperature of 600°C. In this case, the thermopile would be operating with a temperature differential of 400° which is a considerable improvement over the 160° calculated in the previous example for natural convection cooling. Up to now it has been assumed that no heat transfer problem existed at the hot junction. In the above example it is entirely possible that the heating element could not transfer the 63 watts per junction necessary to maintain the 900° hot junction temperature which was assumed. Therefore, a study must be made of the heat transfer between the heating element and the hot junctions.

The magnitude of heat transfer from the heating element to the hot junctions depends on the surface temperature of the heating element. A heating element is capable of transferring an unlimited quantity of heat as long as its surface temperature does not exceed the melting point of the element material. For successful thermopile operation, the heating element must be capable of delivering the heat required for the best possible operating conditions without melting. These optimum conditions may be determined by locating the operating point which will give the greatest temperature differential on Figure 7. In order to determine the heating element requirements, the heat transfer characteristics of the hot junction must be determined.

In the author's case it was proposed that a 3/4 inch diameter globar be used as a heating element. Here the problem was to determine if sufficient heat could be transferred by the globar to the hot junctions without reaching its melting temperature of 3200 deg F. (6) Heat transfer at the hot junction takes place by radiation and by natural convection in the air between the heating element and hot junction. The expression for radiant heat transfer between the heating element and the hot junctions is identical to Equation (36) except that all o subscripts must be changed to e to designate the heating element surface and all s subscripts must be changed to i to designate the inside surface of the thermopile. In this case the emitting surface area per junction is:

$$A_e = \pi D_e(h_a + h_b)$$

where D_e is the diameter of the heating element.

The absorbing surface area per junction is:

$$A_i = \pi D_i(h_a + h_b)$$

Substituting thermopile dimensions and the proper emissivity coefficients from McAdams (3) into Equation (36) with the above stated modifications, the radiant heat transfer per junction is:

$$q_r = 1.89 \times 10^{-5}\left[(T_a/100)^4 - (T_i/100)^4\right] \qquad (38)$$

where q_r is in watts
T_e and T_i are in deg. F abs.

The large temperature differential causes air circulation and, therefore, convection heat transfer between the heating element and the hot junctions. This heat transfer may be calculated from the basic convection equation and Equation (34). The heat transfer per junction may be approximated by:

$$q_c = 3.01 \times 10^{-4} A\, V^{.78}(t_a - t_i) \qquad (39)$$

where q_c is the heat transfer in watts
A is the area of the hot junction heat transfer surface in sq. cm.
t_i is the hot junction temperature in deg. C
V is the estimated velocity of the air moving past the hot junction surface in ft/sec
t_a is the temperature of the moving air in degrees C

If the temperature of the moving air is assumed to be that of the heating element and the velocity of the air is estimated at 20 ft/sec, Equation (39) may be simplified as:

$$q_c = 25.5 \times 10^{-4}(t_e - t_i) \qquad (40)$$

For any particular value of heating element surface temperature and hot junction temperature, the total heat transfer at the hot junction may be calculated by combining the results of Equations (38) and (40). The relationship between heat transfer and heating element surface temperature for hot junction temperatures of 500 and 1000 deg. C are shown in Figure 8.

Figure 8 clearly shows that the globar heating element is not capable of delivering the 63 watts per junction which is necessary to maintain a 400° temperature differential at the operating point used as an example on page 28. In actual operation, the thermopile will have a different operating point which must be determined from Figures 7 and 8.

The operating point for a maximum temperature differential may be determined from Figures 7 and 8 as follows. For maximum heat trans-

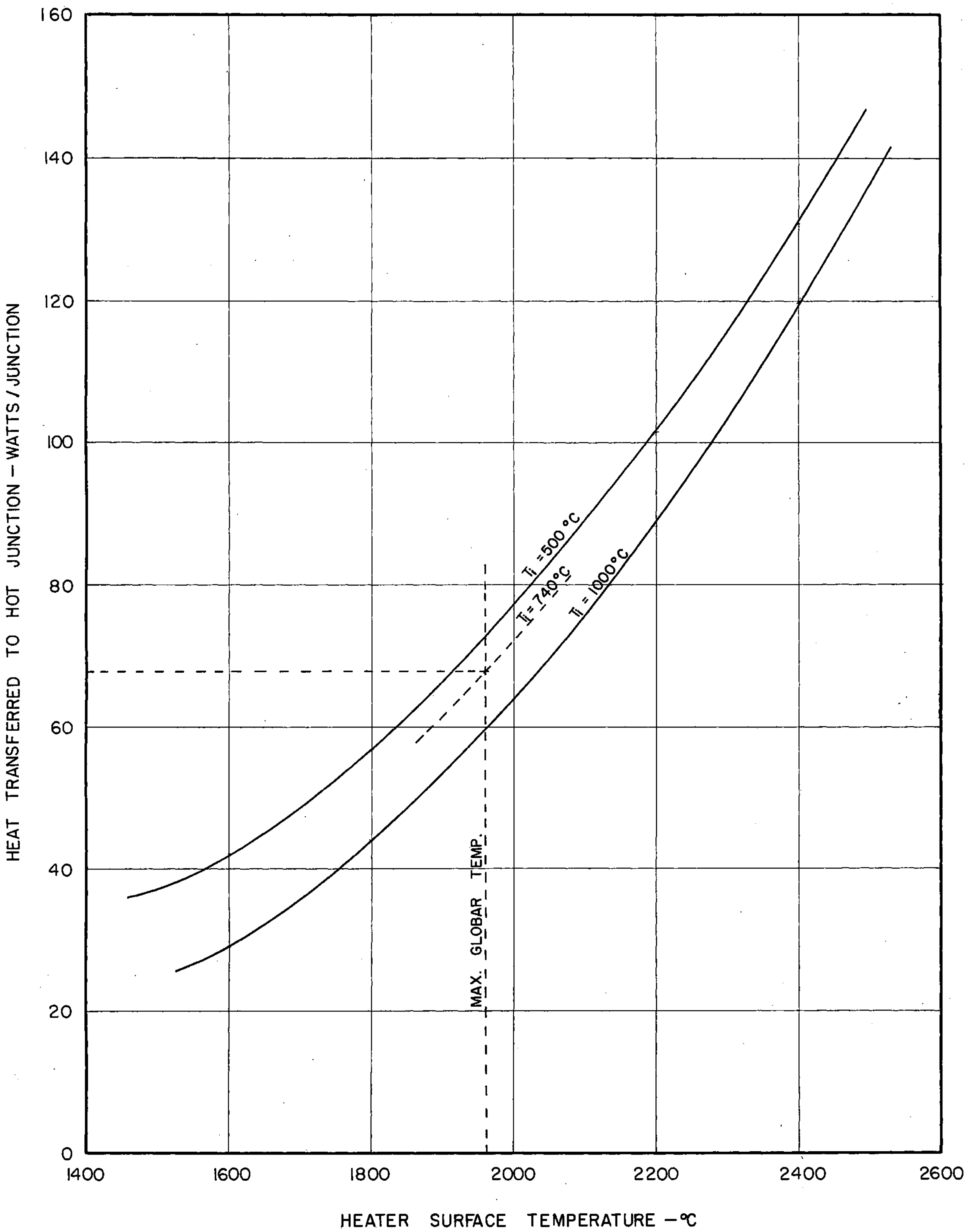


Figure 8. Heat Transfer Characteristics of the Heating System

fer, the heating element surface must be as close as possible to its melting point. This limiting temperature is shown by the dotted line on Figure 8. At this temperature Figure 8 shows that the heating element is capable of delivering 53 watts per junction if the hot junction temperature is 500°C or 41 watts per/junction if the hot junction temperature is 1000°C. Since the hot junction temperature is not known, one must be assumed to determine the heat transfer from Figure 8. If the operating point found on Figure 7 does not agree with this hot junction temperature, successive hot junction temperatures must be tried until an agreement is reached. If a hot junction temperature of 500°C is assumed, Figure 8 shows that the heating element is capable of delivering a maximum heat transfer of 53 watts. This quantity of heat flow must also be the ordinate of the operating point on Figure 7. If the heat is removed from the cold junction by forced air flowing at 200 ft/sec, the operating point must be the point on this cooling curve which has a 53 watt ordinate. This operating point gives an incorrect hot junction temperature of 800°C. If the above procedure is repeated assuming a hot junction temperature of 740°C a nearly correct operating point is located on Figure 7. The graphical construction used in locating this point is shown on Figures 7 and 8. This operating point exhibits an outside or cold junction temperature of 490 C which gives a temperature differential of 740-490 = 250°C. This is the greatest temperature differential that can be obtained from this particular thermopile with globar heating and 200 ft/sec cooling air. A larger cooling air velocity would give somewhat better operation by lowering the hot junction temperature. This would allow a greater heat transfer at the heating element. A considerable improvement in operation could be obtained by using a heating element with a higher melting temperature. A study of all known substances with high melting points which could be used as heating elements revealed that they were all unstable in air at temperatures above 3200°F. Therefore, a thermopile of these particular dimensions appears to be doomed to exhibit a poor efficiency and a low power output.

CHAPTER V

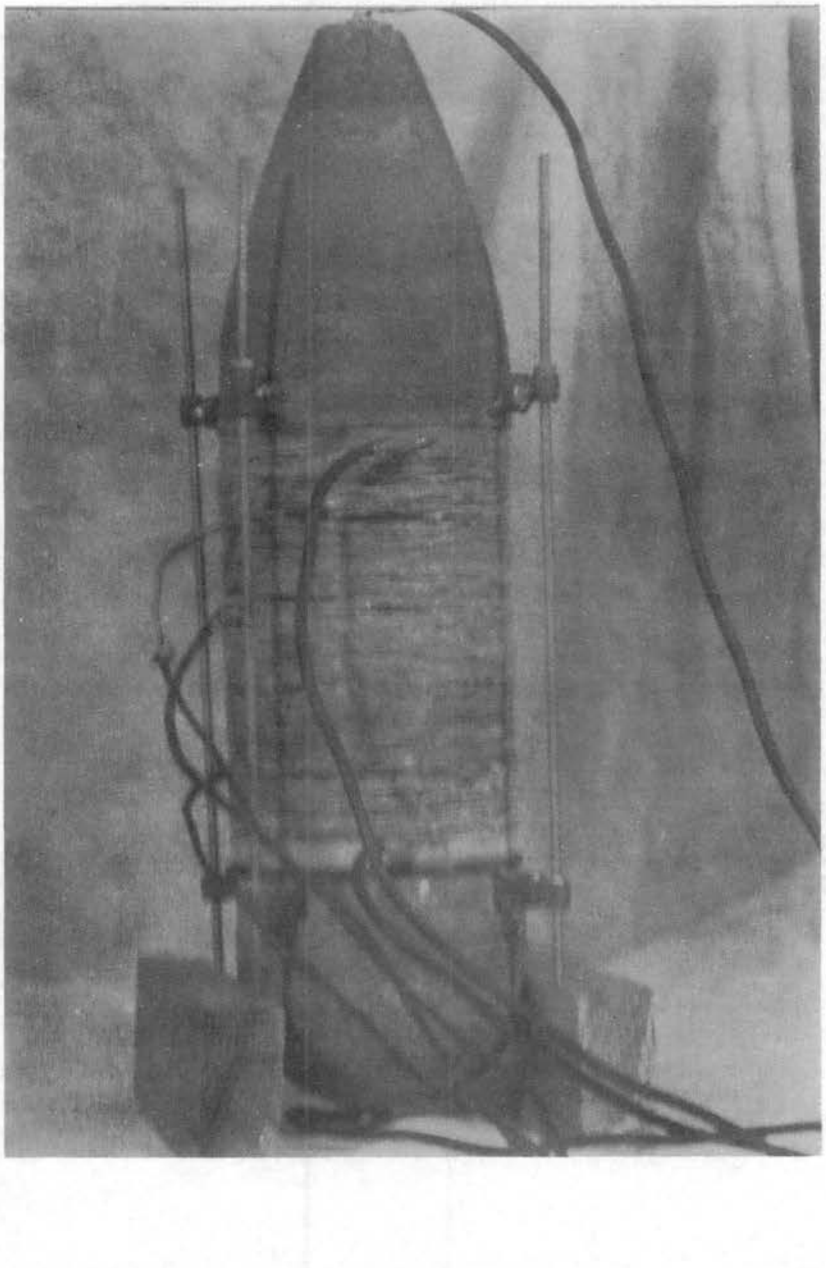
EXPERIMENTAL RESULTS

A complete verification of the theoretical analysis concerning thickness ratio and the power-weight relationships would require the construction of many different thermopiles with different dimensions. Because of time and material limitations a project of this magnitude could not be undertaken. However, one thermopile was constructed using available materials. The experimental results of this thermopile will be compared to those calculated by the theoretical methods of the preceding chapters.

A concentric junction thermopile was constructed of 75 iron sheets and 75 copnic sheets which were alternately arranged and joined as shown in Figure 1. The junctions of the metals were formed by resistance welding. A picture of the completed thermopile is shown in Plate 1. The dimensions of the thermopile were: outside diameter 6 inches, inside diameter 1 inch, copnic thickness .02 inch, iron thickness .02 inch, and insulation thickness .02 inch. If these dimensions are substituted in Equation (18), the total thermopile height is given as 6 inches. Actually, because of distortion of the sheets during welding, the height was about 8 inches. A 3/4 inch x 8 inch globar heating element was used to supply heat to the inside or hot junctions. The heat was removed from the cold junctions by forced air convection which was provided by a propeller fan and air duct. A picture of the completed test arrangement is shown

PLATE 1

THE CONCENTRIC JUNCTION THERMOPILE



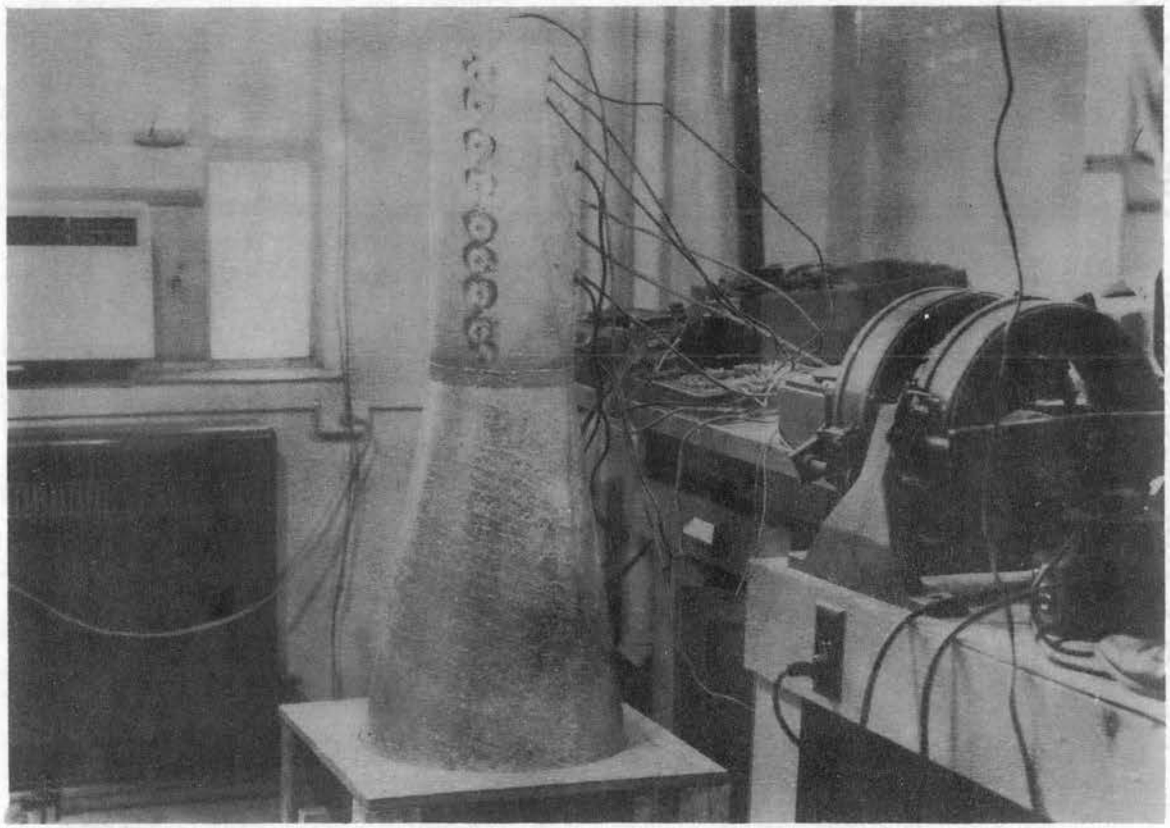
in Plate 2. The fan is located at the extreme bottom of the air duct and forces air upward past the thermopile which is located at the top of the duct.

To determine the characteristics of the thermopile, the power input to the globar was varies from 0 to 4000 watts in steps of 400 watts. At each input setting, after sufficient time was allowed for thermal stabilization, hot junction temperature, cold junction temperature, maximum power output, and maximum efficiency were determined. The results of this test are displayed in Figures 9, 10, and 11. The hot and cold junction temperatures were read directly from calibrated thermocouples which were placed in the thermopile during its construction. The variable nature of the internal resistance of the thermopile and therefore the load resistance for maximum power output, required that the maximum power be determined experimentally for each value of power input. This was done by recording the output voltage of the thermopile for successive values of load resistance while the input was held constant. The power output for each resistance was then found by $P = V^2/R$. The maximum value determined by this method was assumed to be the maximum power output of the thermopile for the particular value of power input. The same procedure was repeated for all other values of power input. The maximum efficiency was determined in each case by dividing this maximum power output by the corresponding power input.

The theoretical characteristics of the thermopile were determined by applying the theoretical analysis of previous chapters to a thermo-

PLATE 2

THE COMPLETE TEST ARRANGEMENT



pile with the dimensions of the experimental thermopile. At an assumed power input, the hot and cold junction temperatures were determined by heat transfer analysis and Figure 7. The maximum efficiency was then determined from Equation (13) using the theoretically calculated temperatures. The theoretical power output was found by multiplying this maximum efficiency by the corresponding value of power input. This procedure was repeated for several power inputs lying in the same range that was used in the experimental tests. These calculated results are displayed on Figures 9, 10, and 11 along with the experimental data.

An examination of Figure 9 reveals that the experimental and calculated values of hot junction temperatures compare rather closely while a large discrepancy is noted between experimental and calculated values of cold junction temperature. At all values of power input, the actual cold junction temperature was found to be considerably less than the theoretically calculated cold junction temperature. Referring to Figure 7, if hot junction temperature and heat flow are fixed, the only way a reduced cold junction temperature could be obtained is for the hot junction temperature curve to have less slope. An examination of Equation (30), which was used to plot Figure 7, shows that a reduction in slope will result only if the thermal conductivity of one or both of the metals is reduced. This means that either the iron or the copnic or both must have a lower thermal conductivity than that assumed in the theoretical calculations. A resistivity test on the iron and constantan revealed that the iron had a considerably higher resistance than that assumed in the theoretical calculations while the resistance of the copnic was nearly the same. According to the Wiedemann-Franz-Lorenz relation, thermal conductivity times resistivity must equal a constant if temperature remains constant. (2). Therefore, an increase in resistivity must be accompanied by a reduction in thermal conductivity. The reduction in cold junction temperature must have been due to the lower value of the iron thermal conductivity. A small portion of the discrepancy in experimental and calculated cold junction temperatures could have been due to the non-uniform temperature distribution along the hot junction and cold junction surfaces. This was caused by the increased temperature of the cooling air as it passed along the cold junction surface and by non-uniform temperature distribution along the heating element surface.

The lower cold junction temperature and therefore larger temperature differential would be expected to cause a higher output and greater efficiency than that which was theoretically calculated. Figures 10, and 11 show that this was not the case. The power output and efficiency of the thermopile were found to be much less than the calculated values. Some of this error could be accounted for by the difference in the thermoelectric characteristics of the iron and copnic which was used in this thermopile from those used in the theoretical calculations. Also, it is believed that some of the thermocouple elements had "shorted" because of distortion of the sheets during the welding process. Stray heat losses, of course, accounted for a considerable part of the error.

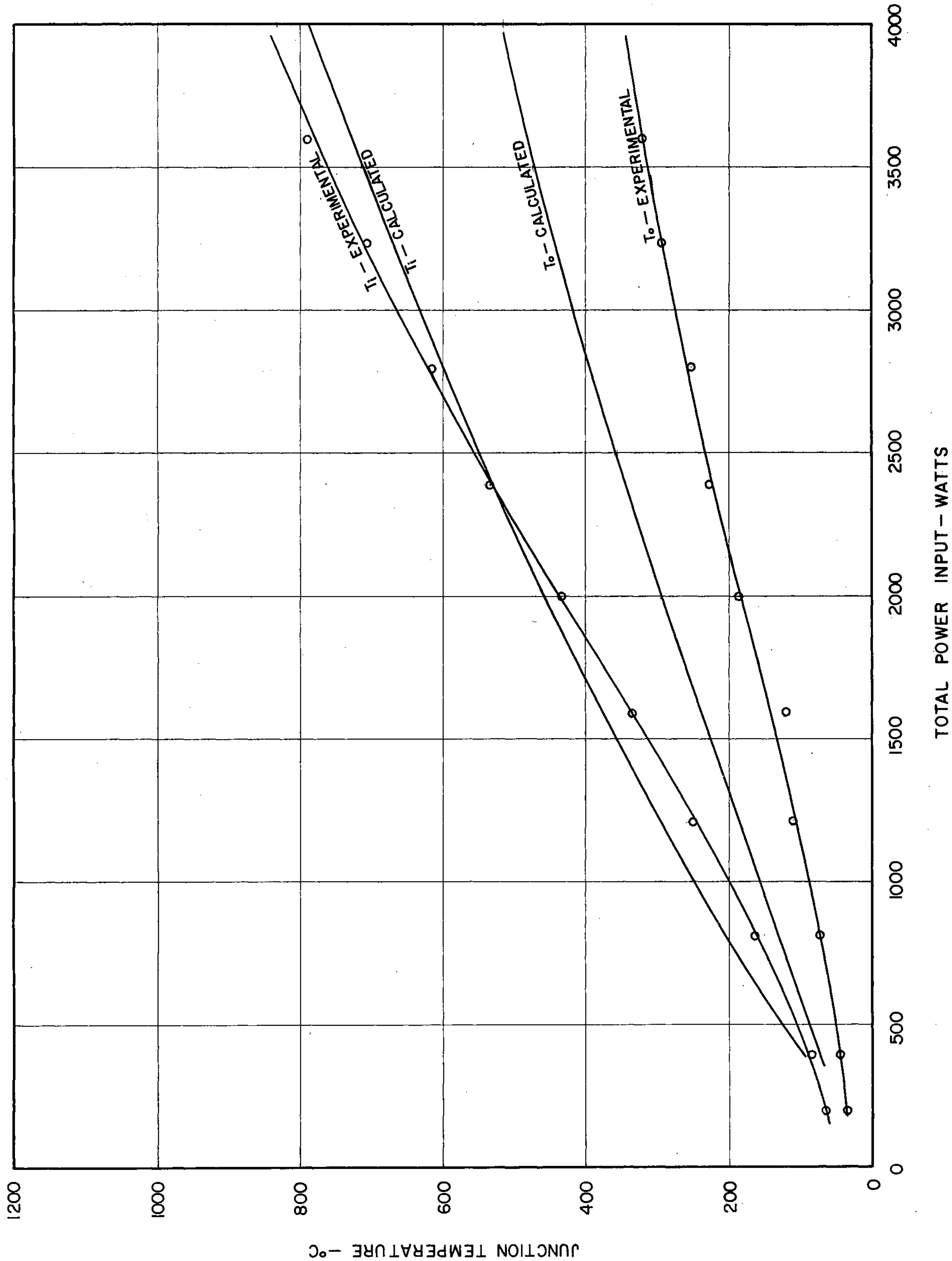


Figure 9. Hot and Cold Junction Temperatures Versus Power Input

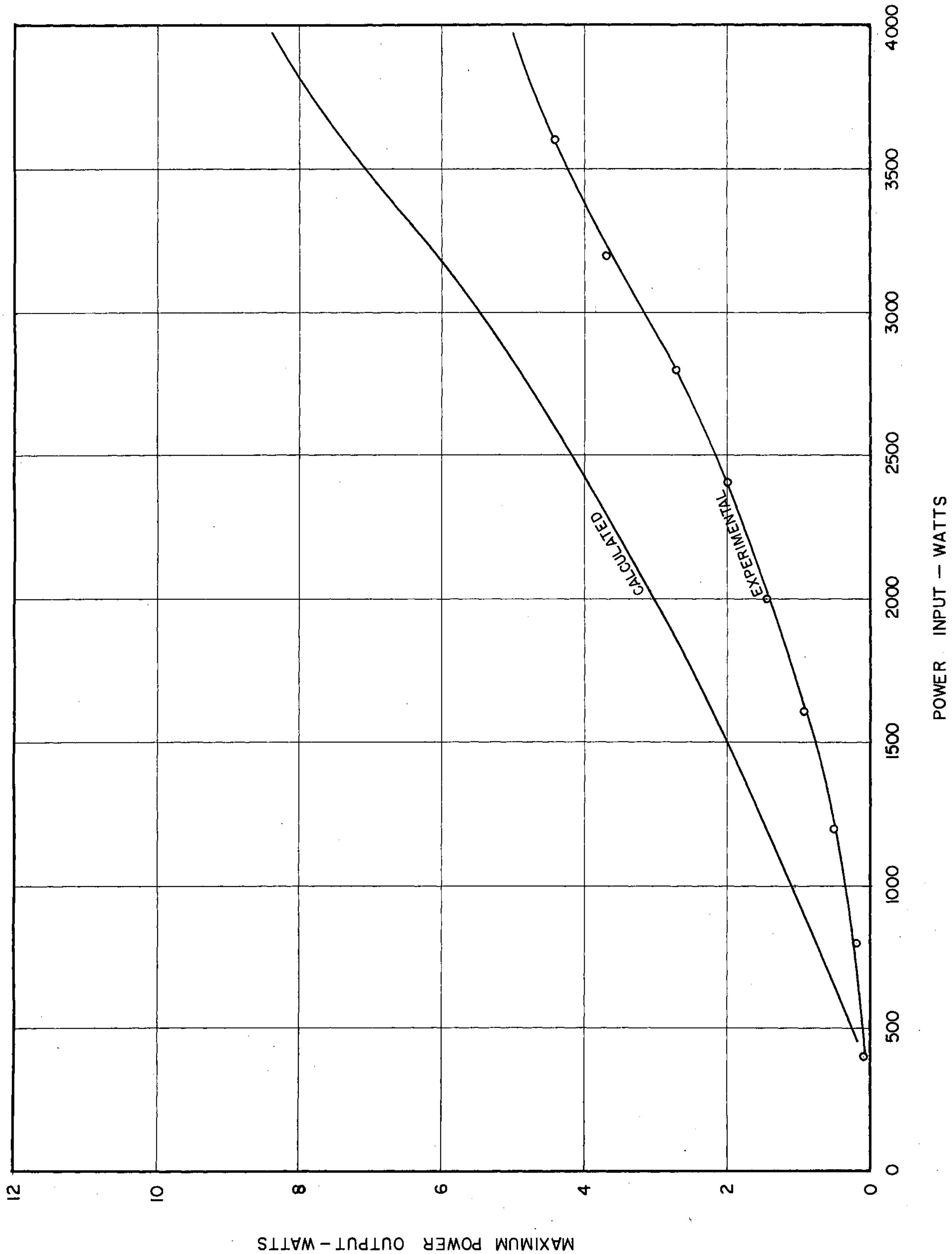


Figure 10. Power Output Versus Power Input

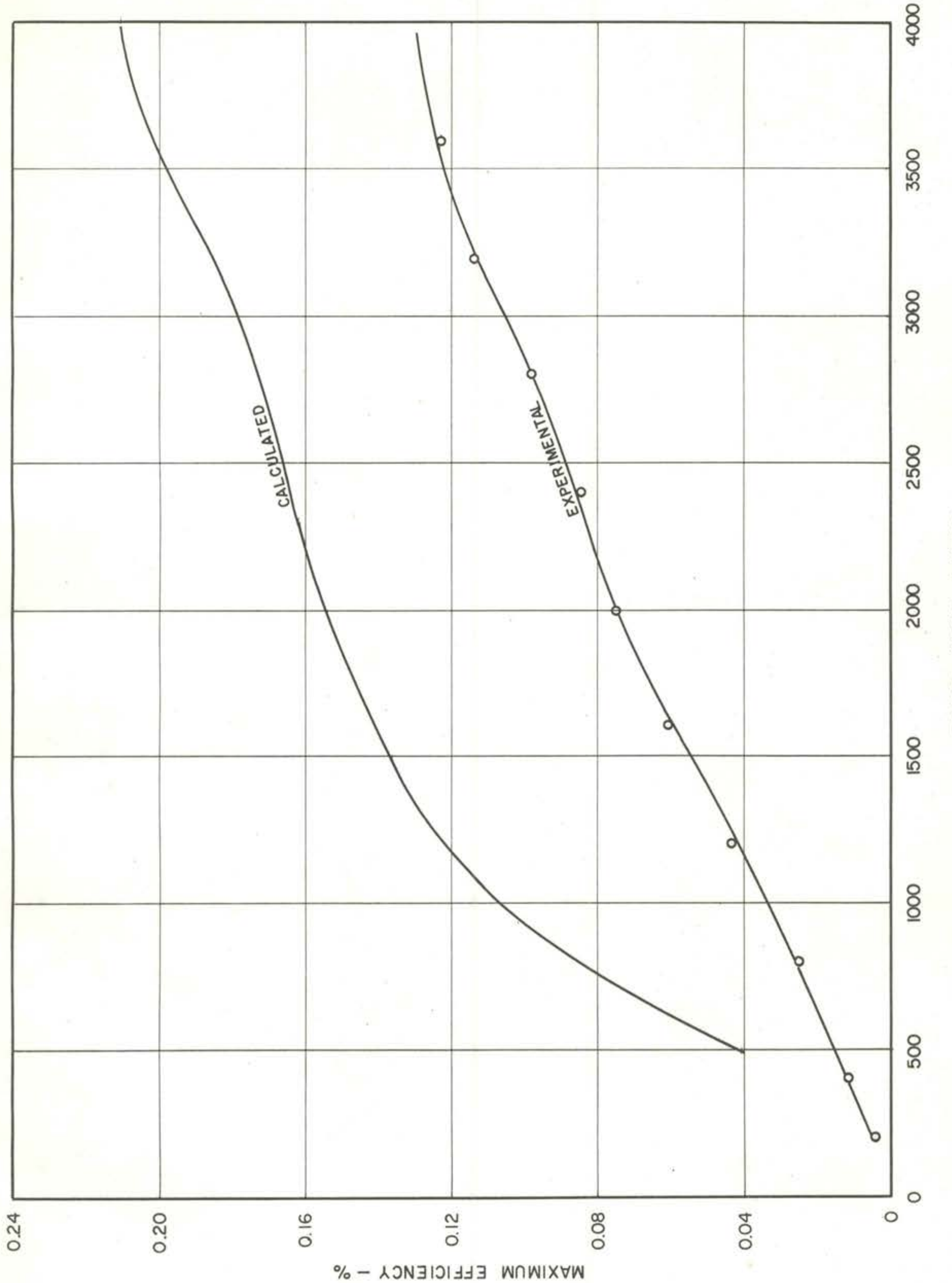


Figure 11. Efficiency Versus Power Input

CHAPTER VI

SUMMARY AND CONCLUSIONS

The calculations and curves presented in the preceding chapters illustrate two very important and often neglected facts concerning thermopile design. The first of these is the importance of maintaining the optimum thickness ratio between thermocouple materials. It can be shown that the thickness ratio is actually the ratio of the areas of the thermocouple materials presented to heat flow. Therefore, an optimum area ratio must exist for all thermopiles regardless of their design or thermocouple configurations. Calculations similar to those presented in Chapter 3 should be an integral part of any thermopile design procedure. It is believed that these optimum area ratio calculations would always reveal that the optimum ratio is entirely dependent upon the constants of the thermocouple materials. Since the constants of all thermocouple materials vary with the operating temperatures, it is necessary to know these temperatures before the optimum ratio can be calculated. This leads to the second important fact concerning thermopile design which is the importance of heat transfer analysis.

Heat transfer difficulties become increasingly significant as attempts are made to increase the power output, reduce the weight, or reduce the volume of the thermopile. If a thermopile is designed to have a very small output and large volume it will have a correspondingly small power input and large heat transfer surfaces. This means that probably it will not be difficult to transfer this small quantity of heat to the hot junctions or from the cold junctions and therefore, nearly any set of operating temperatures can be maintained easily. On the other hand, as the power output of a thermopile is increased and its size is reduced, the quantity of heat which must be transferred to the hot junction and from the cold junctions must become must larger if an acceptable temperature differential or efficiency is to be maintained. The reduction in volume further complicates the problem by reducing the exposed heat transfer surfaces. It was mentioned in Chapter 2 that the concentric junction thermopile design gives the largest power output for its volume. Actually, thermopiles of the concentric junction type can be designed to give enormous power output with a small weight and volume. As an example, calculations will be performed for a copper copnic thermopile about the size of a No. 6 dry cell battery. The outside diameter will be assumed as 2.6 inches, the inside diameter 2.5 inches, and the length 6 inches. The applications of Figures 4 and 5 and Equations (21) and (24) reveal that, with an insulation factor of 0.4, this thermopile would yield an output of 1750 watts while its total weight would be only 0.515 pound. It is theoretically possible for a concentric junction thermopile weighing only $\frac{1}{2}$ pound to deliver nearly 2 K.W. of power! The difficulty or impossibility is in maintaining the temperature differential of 400°C related to Figures 4 and 5. At the least, a temperature differential of this magnitude must be maintained if an acceptable efficiency is to be obtained. The input to this small thermopile, which is necessary to maintain this temperature differential, may be found by dividing the output by the efficiency. This efficiency, assuming optimum thickness ratio, is read from Figure 3 as 0.5. The input would therefore be 1750/.005 or 350,000 watts. Obviously, it is impossible to transfer this large quantity of heat either to the hot junctions or from the cold junctions in a thermopile of this small size. Therefore, the thermopile designer is forced to work from the heat transfer capabilities of the heating element in delivering heat to the hot junctions and the capabilities of the cooling system in removing heat from the cold junctions.

If the heat transfer characteristics of the heating element and cooling system are known, the design of a concentric junction thermopile would consist of juggling the thermopile dimensions until the desired heat flow or temperature differential could be established. This procedure would consist of assuming successive values of thermopile dimensions and calculating the operating temperatures by the methods of Chapter 4 until these temperatures agree with those desired. After the required operating conditions have been reached, the optimum thickness ratio and the efficiency could be found by the methods of Chapter 3. The power output of the thermopile could then be found by multiplying the efficiency by power input.

Essentially, the problem which started as one of designing a thermopile of small size, weight and volume with a large power output, becomes one of decreasing power output, and increasing size, weight, and volume so that heat transfer relationships may be satisfied. The concentric junction design which is theoretically the best possible configuration for a maximum power output with minimum weight and volume is, because of heat transfer complications, forced to become equal to more conventional thermopile designs for use as a low-power source.

It is possible that by using large diameters and suitable heating and cooling fins a concentric junction thermopile could be made to operate successfully and deliver a large power output for its size and weight. The power wasted, however, would be enormous. For example, a 5 K. W. thermopile would require an input of about 1000 K. W. This means 995 K. W. would be wasted. Since efficiencies become increasingly important as the consumed power becomes larger, thermopiles of this design could find use only if its efficiency could be improved. The obvious way of improving the efficiency is to utilize the wasted heat which is being removed from the cold junctions. One method of raising the efficiency of a thermopile would be to use the thermopile as a heat transfer medium in some other thermal system. For example it could be used to conduct heat from a furnace to a hot air heating duct. A diagram of such a system is shown in Figure 12. The concentric junction thermopile, because of its rigidity, and high heat transfer, would be ideal for this process. Another method for improving the efficiency of a concentric junction thermopile would be to return the waste heat removed from the cold junctions to the hot junctions. This could be done as shown in Figure 13.

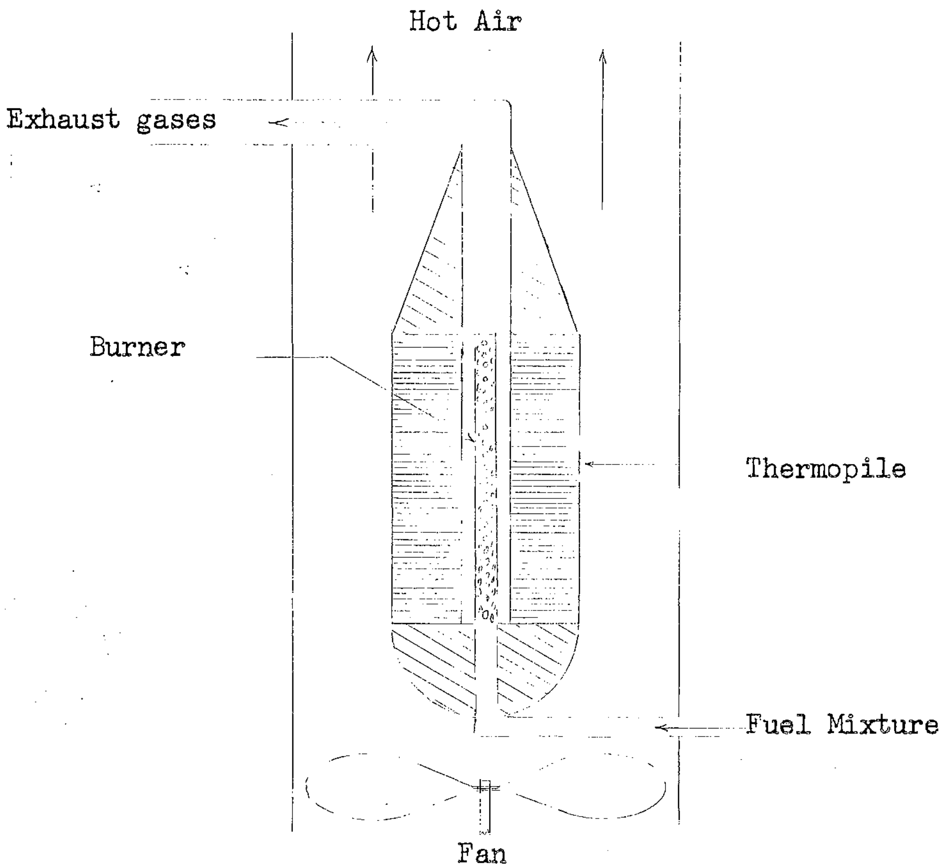


Figure 12. Thermopile Used as a Heat Transfer Medium

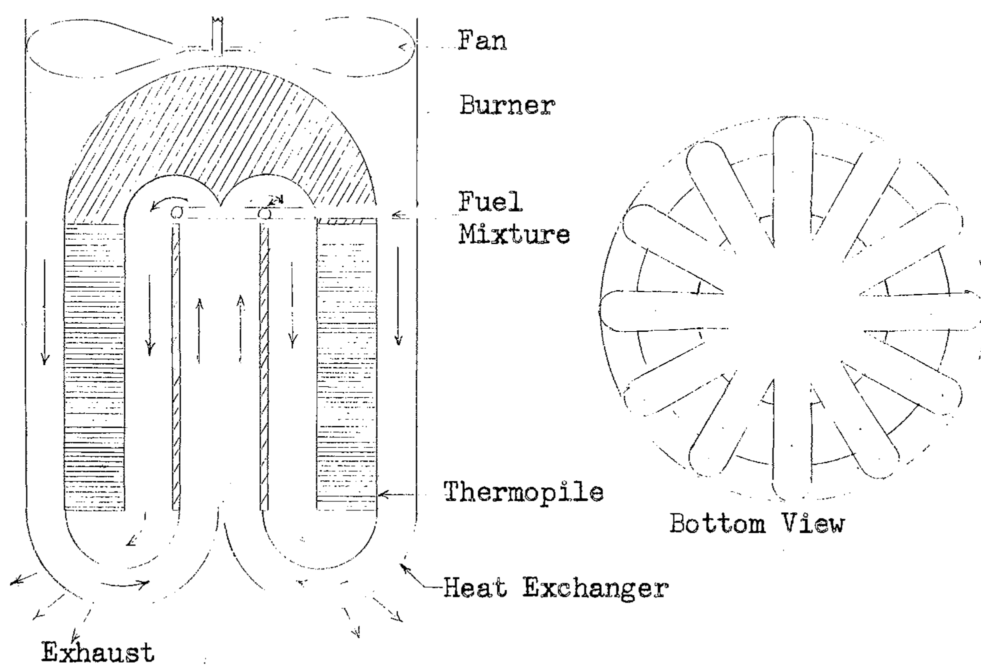


Figure 13. The Regenerative Thermopile

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