ESTIMATES OF A DYNAMIC DEMAND FOR MONEY MODEL WITH DISTRIBUTED LAGS AND AUTOREGRESSIVE ERRORS

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CHAPTER I

INTRODUCTION

Statement of the Problem

A large number of empirical studies concerning the demand function for money have been conducted by both academic and government economists during the period of the 1950's and 1960's. The most important issue in these studies is the identification and measurement of a stable demand function for money. The knowledge of a stable demand function for money is indispensable for a better understanding of behavior in the monetary sector as well as in other sectors of the economy. For example, a stable demand function for money would permit the consequences of shifting the supply function of money to be more easily and accurately predicted. Harry G. Johnson, in his survey of monetary theory, has suggested that the variables on which the demand for money depends and the stability of the demand function for money are among the most urgent issues in the study of monetary economics.[1]

In recent years, econometricians have introduced a number of ingenious models utilizing distributed lags, e.g., Friedman's

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[1] H. G. Johnson, "Monetary Theory and Policy," The American Economic Review, 52 (June 1962), p. 351.

permanent income hypothesis,[2] Cagan's adaptive expectation model[3] and Nerlove's partial adjustment model.[4] But most of the estimations of the demand function for money were made largely on the basis of statistical convenience. Little consideration was given to the problems of inconsistency and inefficiency which may arise from the application of the method of ordinary least squares.

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[2] Friedman constructs the permanent income hypothesis in which the expected value of current measured income (Y^*_p) is revised over time at a rate that is proportional to the difference between expected and actual income

$$\frac{dY^*_p}{dT} = \beta[Y^*(T) - Y^*_p(T)] , \quad 0 < \beta < 1 .$$

He then reduces this differential equation to a simpler form as

$$Y^*_p(T) = \beta \int_{-\infty}^{T} e^{\beta(t-T)} Y^*(t)\, dt .$$

This hypothesis implies that permanent income is a function of all past actual income with an exponentially declining distributed lag.

[3] Cagan suggests a geometrically distributed lag form in his adaptive expectation model. According to his hypothesis, the expected price (P^*) is revised in proportion to the error associated with the previous level of expected prices

$$P^*_t = \beta(P_{t-1} - P^*_{t-1}) - P^*_{t-1} , \quad 0 < \beta < 1 .$$

By repeating the substitution process indefinitely, Cagan derives a general equation of the form

$$P^*_t = \sum_{i=1}^{\infty} \beta(1-\beta)^i P_{t-i} .$$

[4] Nerlove formulates the partial adjustment model in which the desired value of the dependent variable (Y^*_t) depends on the current value of the independent variable:

$$Y^*_t = \alpha + \beta X_t$$

and assumes that the adjustment of the actual value Y_t toward the desired value Y^*_t is not an instantaneous process, but is only a fraction $1 - \lambda$ of the difference between the desired value Y^*_t and the previous actual value Y_{t-1}

$$Y_t - Y_{t-1} = (1-\lambda)(Y^*_t - Y_{t-1}) , \quad 0 \leq \lambda < 1$$

thus

$$Y_t = \alpha(1-\lambda) + \beta(1-\lambda)X_t + \lambda Y_{t-1} .$$

A few examples can be cited from the literature of monetary economics. By applying ordinary least squares regression to United States time series, Meltzer in his "The Demand for Money; the Evidence from Time Series," indicates that he has found some results on several critical issues in monetary theory and policy.[5] However, the Durbin-Watson statistics from the regression show that the time series used by Meltzer are autocorrelated. This implies that Meltzer is likely to obtain a serious underestimate of sampling variances even if the estimates of parameters are unbiased.[6]

Teigen, in his "Demand and Supply Function for Money in the United States; Some Structural Estimates," treates the money supply as an endogenous variable and uses two-stage least squares to derive the conclusion that the variables--the product of the interest rate and national income and national income alone--in the demand function for money give the best results during the post-war period.[7] However, his estimation procedure with simultaneous equations largely ignores problems of lagged adjustment and serial correlation.

A study of the demand for money is also done by Chow.[8] In his

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[5] A.H. Meltzer, "The Demand for Money: The Evidence from the Time Series," The Journal of Political Economy, 71 (June 1963), pp. 219-246.

[6] T.J. Courchene and H.T. Shapiro, "The Demand for Money: A Note from the Time Series," The Journal of Political Economy, 72 (October 1964), pp. 498-503.

[7] R.L. Teigen, "Demand and Supply Functions for Money in the United States: Structural Estimates," Econometrica 32 (October 1964), pp. 476-509.

[8] G.C. Chow, "On the Long-Run and Short-Run Demand for Money," The Journal of Political Economy, 74 (April 1966), pp. 111-131.

"On the Long-Run and Short-Run Demand for Money," he considers the question of lagged dependent variables, by contrasting both permanent income and wealth with current income. He finds that permanent income is more important in the long-run demand for money, and current income is more important in the short-run demand for money. But, again, Chow pays little attention to the statistical problems of autoregressive errors.

In his "The Demand for Money: Speed of Adjustment, Interest Rate and Wealth," de Leeuw employs an alternative estimation technique in an attempt to deal with the problems of serial correlation and lagged adjustment.[9] His analysis is based on three-pass least squares (3-PLS). By using 3-PLS, de Leeuw produces strong evidence supporting the hypothesis that there exists a substantial lag in the adjustment of money holdings to changes in long-run desired holdings. However, the most crucial problem is that the ordinary least squares estimate is inconsistent, and this leads to the second pass estimate being inconsistent which in turn leads to the inconsistent 3-PLS estimate of the speed of adjustment.[10]

Thus, it is possible to conclude that the methods used to estimate the demand function for money still suffer from statistical techniques

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[9] F. de Leeuw, "The Demand for Money: Speed of Adjustment, Interest Rates and Wealth," Monetary Process and Policy: A Symposium, Edited by G. Horwich, Homewood, Illinois, Richard D. Irwin, Inc., 1966, pp. 167-186.

[10] K. F. Wallis, "Lagged Dependent Variables and Serially Correlated Errors: A Reappraisal of Three-pass Least Squares," Review of Economics and Statistics, 49 (November 1967), pp. 555-567.

that are inadequate for solving the problems of consistency and efficiency.

Motive of the Study

The purposes of this study are (1) to introduce a distributed lag model with autoregressive errors which contains two parameters (per capita real income and rate of interest) in the estimation of the demand for money in the United States as well as in Taiwan, (2) to test the validity that the theoretical relationships which have been developed to explain monetary behavior and opinions which have been based on the interpretation of monetary experience in the United States are applicable to the economic structure of Taiwan as well. For the purpose of a comprehensive comparison among the various models, five least squares regressions are also performed. There are (1) Two Lag Model with Autoregressive Errors, (2) Two Lag Model with Independent Errors, (3) Static Model with Autoregressive Errors, (4) First Difference Model with Independent Errors and (5) Static Model with Independent Errors. There are discussed in the third chapter.

Sources of Data and Definitions

In this study, the money, the price index, population and interest rate data are taken from International Financial Statistics and income data from the United Nation's Year Book of National Account Statistics.[11] However, the short-term interest rate and the long-term

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[11] For the purpose of better comparison of the monetary phenomena between the United States and Taiwan, most of the statistical data are taken from these United Nations' publications.

interest rate are taken from the Federal Reserve Bulletin. The time period under study for the United States and Taiwan is from 1956 to 1969 for both annual and quarterly data.[12]

Money, as shown in International Financial Statistics, is defined conventionally as narrow money (currency plus demand deposits). In the case of Taiwan, money comprises the Bank of Taiwan's monetary liabilities to the private sector, demand deposits with commercial banks plus post offices demand deposits. Money, in the case of the United States, covers the total of the public's holdings of currency and demand deposits in the banks.

Quasi-money (time deposits), in the case of Taiwan, comprises the private sector time deposits with the Bank of Taiwan and commercial banks. The time deposits series of the United States covers time and savings deposits at all commercial banks other than those due to domestic commercial banks and the United States government. In this study, both the narrow and the broad concepts of money (money plus quasi-money) are used respectively.

The interest rate in Taiwan is the rate of the Central Bank of China (before July 1, 1961, it is called the discount rate of the Bank of Taiwan) on its called loans and secured advances to banks. The interest rate for the United States is the short-term interest rate which

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[12] By using the quarterly data for the analysis, the numbers of degree of freedom can be increased from 14 to 56 which is expected to improve the results from the distributed lag model.

is the market yield on 90-day U. S. government securities.[13] However, the long-term interest rate which is the United States long-term bond yield is also tested.

The income variable used for the United States and Taiwan is national income which is the sum of the income accruing to factors of production supplied by residents of the given country before deduction of direct taxes.

The price index in Taiwan (defined as cost of living in International Financial Statistics) is a weighted average of 179 commodities selected without regard to the distinction between imports, exports and goods produced for domestic use. This price index mostly refers to foodstuffs, raw materials and simply processed goods in major cities. The price index in the United States includes all goods (goods domestically produced for domestic use and for export and imported goods). The weights of the index attempt to avoid the multiple-counting of commodities at various stages of processing by assigning to raw materials their full value and to commodities at later stages weights equal only to the value added at that stage. Hence the index is relatively heavily weighted with primary goods and relatively lightly weighted with finished goods.

Organization of the Study

The material of this study is arranged under four chapter

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[13] The best performance of the rate of interest in the estimation of the per capita demand for real money in the United States is obtained by using the market yield on 90-day U.S. government securities instead of using the United States long-term bond yield or the discount rate as a proxy variable.

headings. In order to provide a better understanding of estimated demand function for money, a review of the literature concerning hypothetical approaches used in estimating the demand for money is given in Chapter II. In Chapter III, the empirical results from the analytical framework are presented. The final chapter consists of a general summary of the problem and conclusions of the study.

CHAPTER II

THE DEMAND FOR MONEY MODEL

Introduction

Most economic time series tend to be serially correlated. To the extent that this is true, the autocorrelation in the dependent variables reduces the efficiency of regression estimates. In other words, serial correlation in the demand equation for money implies that the number of degrees of freedom available is far less than the number of observations. As a result, the necessary conditions of the Markov theorem of least squares cannot be satisfied and the direct application of ordinary least squares to each equation would not yield consistent estimates.

A considerable amount of work has been devoted to this problem. Various estimating techniques, which all share the property of consistency, have been subsequently devised. Among these methods, two-stage least squares, indirect least squares, and limited information maximum likelihood estimations are commonly used. Others include full information maximum likelihood estimation, unbiased k-class and three-pass least squares.

For the case of a first order autoregressive process which exists in the disturbance term of a single structural equation, Klein has demonstrated that the generalized least squares technique can be employed.[1] However, he did not obtain asymptotically efficient estimators. In addition, the criticism regarding the consistency of the estimates obtained by Klein's method has been raised by Malinvaud.[2] Liviatan has tackled this problem and suggests a procedure which will solve the problem of obtaining consistent estimators.[3] Liviatan's procedure can be interpreted in terms of Theil's "two-stage" least squares technique. It can be also interpreted as an application of the "instrumental variables" technique to the problem of distributed lags with first order autoregressive errors. Hannan has shown that Liviatan's estimator is inefficient.[4] He has proposed an application of the spectral analysis technique which will yield the asymptotically efficient estimator. Nevertheless, Hannan's estimator is essentially an Aitken estimator where the relevant covariance matrix used is consistently estimated. Finally, Wickens has shown that the limited information maximum likelihood method will yield a consistent and asymptotically efficient estimator within the single equation.[5] An obvious line of inquiry regarding the appropriate estimating method,

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[1] L. R. Klein, "The Estimation of Distributed Lags," Econometrica, 26 (October, 1958), pp. 553-565.

[2] E. Malinvaud, "The Estimation of Distributed Lags: A Comment," Econometrica, 29 (July, 1961), pp. 430-433.

[3] N. Liviatan, "Consistent Estimation of Distributed Lags," International Economic Review, 9 (January, 1963), pp. 44-52.

[4] E. J. Hannan, "The Estimation of Relations Involving Distributed Lags," Econometrica, 33 (January, 1965), pp. 206-224.

[5] M. R. Wickens, "The Consistency and Efficiency of Generalized Least Squares in Simultaneous Equation Systems with Autocorrelated Errors," Econometrica, 37 (October, 1969), pp. 651-659.

of course, would be the one which would yield the estimator with the properties that are consistent and efficient.

Derivation of the Demand Function for Money

In this study, the formulation of the model involves an extension of the derivation suggested by Koyck.[6] We can regard the per capita demand for real money as a function of the "normal" or "average" level of per capita real income and the interest rate. The nature of the variables applied in this study is similar to the permanent income hypothesis and the adaptive expectation model, concepts developed by Milton Friedman and Phillip Cagan, respectively. Friedman has argued that per capita demand for real money is determined by per capita permanent real income.[7] Cagan has suggested that an individual's desired real cash balances depend upon his wealth in real terms,

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[6] The growing interest in the application of distributed lag model in the regression analysis of time series has been stimulated by the work of Koyck. As a matter of fact, both Cagan's adaptive expectation model and Nerlove's partial adjustment model are originated from Koyck's study.

The general form of the distributed lag equation for Koyck's model is

$$Y_t = \sum_{i=0}^{\infty} \alpha_i X_{t-i} + u_t ,$$

where Y_t and X_t are observed time series and u_t is a random disturbance. However, Koyck assumes that the distributed lag coefficients α_i decay geometrically, so that the original equation can be expressed as

$$Y_t = \alpha \sum_{i=0}^{\infty} \lambda^i X_{t-i} + u_t .$$

[7] Milton Friedman, "The Demand for Money: Some Theoretical and Empirical Results," The Journal of Political Economy, 67 (August, 1959), pp. 335-336.

his current real income and the expected rate of change in prices.[8] The economic rationale behind this model is that the quantity of real money per capita demanded, like the quantity of consumption services in general, is adapted not only to current values but also to permanent values. The derivation of permanent values in this study follows the Koyck type of distributed lag equation where the coefficients of the distributed lag equation decline in geometric progression in successive periods. This suggests that (1) the current value of the demand for real money per capita is not only related to the values of current real income per capita and the interest rate but also related to the values of these variables at several different times in the past; (2) recent values of real income per capita and the interest rate should be weighted more heavily than those of the remote past. Therefore, according to the hypothesis under investigation, the effects of real income per capita and interest rate changes diminish through time at constant geometric rates.

The basic model can be written as

$$\left(\frac{M}{Np}\right)_t = \gamma_o + \alpha_o \sum_{i=0}^{\infty} \lambda^i \left(\frac{Y}{Np}\right)_{t-i} + \beta_o \sum_{i=0}^{\infty} \mu^i r_{t-i} + e_t \qquad (1)$$

where e_t is an error term,

$$0 < |\lambda| < 1 \quad \text{and} \quad 0 < |\mu| < 1 .$$

In order to reduce equation (1) to a form appropriate for estimation, equation (1) is lagged one period, and multiplied by λ. Thus,

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[8] Phillip Cagan, "The Demand for Currency Relative to Total Money Supply," The Journal of Political Economy, 66 (August, 1958), pp. 305-318.

one obtains:

$$\lambda(\frac{M}{Np})_{t-1} = \gamma_o\lambda + \alpha_o \sum_{i=1}^{\infty} \lambda^i(\frac{Y}{Np})_{t-i} + \lambda\beta_o \sum_{i=1}^{\infty} \mu^{i-1} r_{t-i} + \lambda e_{t-1} \qquad (2)$$

Then equation (2) is subtracted from equation (1), which yields:

$$(\frac{M}{Np})_t = \gamma_o(1-\lambda) + \alpha_o(\frac{Y}{Np})_t + \beta_o r_t + (\mu-\lambda)\beta_o \sum_{i=1}^{\infty} \mu^{i-1} r_{t-i}$$
$$+ \lambda(\frac{M}{Np})_{t-1} + e_t - \lambda e_{t-1} \qquad (3)$$

Next, equation (3) is lagged one period again, and the result is multiplied by μ. Then, the following is obtained:

$$\mu(\frac{M}{Np})_{t-1} = \gamma_o(1-\lambda)\mu + \mu\alpha_o(\frac{Y}{Np})_{t-1} + \mu\beta_o r_{t-1} + (\mu-\lambda)\beta_o \sum_{i=2}^{\infty} \mu^{i-1} r_{t-i}$$
$$+ \lambda\mu(\frac{M}{Np})_{t-2} + \mu e_{t-1} - \lambda\mu e_{t-2} \qquad (4)$$

If equation (4) is subtracted from equation (3), one obtains:

$$(\frac{M}{Np})_t = \gamma_o(1-\lambda)(1-\mu) + \alpha_o(\frac{Y}{Np})_t - \mu\alpha_o(\frac{Y}{Np})_{t-1} + \beta_o r_t - \lambda\beta_o r_{t-1}$$
$$+ (\lambda+\mu)(\frac{M}{Np})_{t-1} - \lambda\mu(\frac{M}{Np})_{t-2} + W_t \qquad (5)$$

where

$$W_t = e_t - (\lambda+\mu)e_{t-1} + \lambda\mu e_{t-2}$$

Equation (5), which is non-linear in the original parameters, and contains lagged values of the dependent variable on the right hand side of the equation, contains serious statistical problems. The direct application of least squares to the equation under the assumption of independent errors will not only be inefficient, but also will be biased when in fact the errors in the equation are correlated.

Therefore, in order to obtain estimators with consistent and efficient properties, it is assumed that the errors follow the first order autoregressive scheme:

$$W_t = \rho W_{t-1} + \epsilon_t \quad \text{and} \quad |\rho| < 1 .$$

where ρ is the autocorrelation coefficient and ϵ_t is assumed to have the following statistical properties

$$\begin{aligned} E[\epsilon_t] &= 0 \\ E[\epsilon_t \epsilon_{t+s}] &= \sigma_\varepsilon^2 \quad \text{for} \quad s = 0 \\ &= 0 \quad \text{for} \quad s \neq 0 \end{aligned}$$

Combining the first order autoregressive errors assumption with equation (5), one obtains:

$$\begin{aligned} (\frac{M}{Np})_t = {} & \gamma_o(1-\lambda)(1-\mu)(1-\rho) + \alpha_o(\frac{Y}{Np})_t - (\mu+\rho)\alpha_o(\frac{Y}{Np})_{t-1} + \mu\rho\alpha_o(\frac{Y}{Np})_{t-2} \\ & + \beta_o r_t - (\lambda+\rho)\beta_o r_{t-1} + \lambda\rho\beta_o r_{t-2} + (\lambda+\mu+\rho)(\frac{M}{Np})_{t-1} \\ & - [(\lambda+\mu)\rho + \lambda\mu](\frac{M}{Np})_{t-2} + \lambda\mu\rho(\frac{M}{Np})_{t-3} + \epsilon_t \end{aligned} \tag{6}$$

where

M_{t-i} = the current and lagged values of the demand for nominal money $(i = 0, 1, 2, 3)$,[9]

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[9] Two concepts of money are used respectively in this study; (1) traditional definition of money (currency plus demand deposits) and (2) Friedman's type of definition of money (currency plus demand deposits and time deposits).

Y_{t-i} = the current and lagged values of measured aggregate income in nominal terms $(i = 0, 1, 2)$,

N_{t-i} = the current and lagged values of population, measured and permanent being taken as identical $(i = 0, 1, 2)$,

r_{t-i} = the current and lagged values of interest rate $(i = 0, 1, 2)$,

p_{t-i} = the current and lagged values of the price index $(i = 0, 1, 2)$,

ϵ_t = the error in the equation

$\gamma_o(1 - \lambda)(1 - \mu)(1 - \rho)$ = the pure constant term

α_o = the parameter for the real income per capita, $(\frac{Y}{Np})_{t-i}$

β_o = the parameter for the interest rate, r_{t-i}

λ = the lag parameter associated with the real income per capita, $(\frac{Y}{Np})_{t-i}$

μ = the lag parameter associated with the interest rate, r_{t-i}

ρ = the first order autocorrelation coefficient

or

$$(\frac{M}{Np})_t = \Pi_0 + \Pi_1(\frac{Y}{Np})_t + \Pi_2(\frac{Y}{Np})_{t-1} + \Pi_3(\frac{Y}{Np})_{t-2} + \Pi_4 r_t + \Pi_5 r_{t-1} + \Pi_6 r_{t-2} + \Pi_7(\frac{M}{Np})_{t-1} + \Pi_8(\frac{M}{Np})_{t-2} + \Pi_9(\frac{M}{Np})_{t-3} + \epsilon_t \qquad (7)$$

where

$$\Pi_0 = \gamma_o(1 - \lambda)(1 - \mu)(1 - \rho)$$

$$\Pi_1 = \alpha_o$$

$$\Pi_2 = -(\mu + \rho)\alpha_o$$

$$\Pi_3 = \mu \rho \alpha_o$$

$$\Pi_4 = \beta_o$$

$$\Pi_5 = -(\lambda + \rho)\beta_o$$

$$\Pi_6 = \lambda \rho \beta_o$$

$$\Pi_7 = (\lambda + \mu + \rho)$$

$$\Pi_8 = [(\lambda + \mu)\rho + \lambda \mu]$$

$$\Pi_9 = \lambda \mu \rho$$

Four other least squares regressions are also derived from equation (6). These are

Two Lag Model with Independent Errors

If it is assumed that the per capita demand for real money depends upon the per capita real income and the rate of interest, and that there are lag coefficients for the explanatory variables and zero autocorrelation coefficient, i.e., $\lambda \neq 0$, $\mu \neq 0$ and $\rho = 0$, then given equation (6), the basic model can be written as

$$(\frac{M}{Np})_t = \gamma_o(1 - \lambda)(1 - \mu) + \alpha_o(\frac{Y}{Np})_t - \mu\alpha_o(\frac{Y}{Np})_{t-1} + \beta_o r_t$$
$$- \lambda\beta_o r_{t-1} + (\lambda + \mu)(\frac{M}{Np})_{t-1} - \lambda\mu(\frac{M}{Np})_{t-2} + \epsilon_t .$$

Static Model with Autoregressive Errors

If it is assumed that the per capita demand for real money depends upon per capita real income and the interest rate, and that the lag coefficients associated with the explanatory variables are zero, i.e., $\lambda = \mu = 0$, then given equation (6), the nonlinear estimation of the static model with first order autoregressive errors is

$$(\frac{M}{Np})_t = \gamma_o(1-\rho) + \alpha_o(\frac{Y}{Np})_t - \rho\alpha_o(\frac{Y}{Np})_{t-1} + \beta_o r_t - \rho\beta_o r_{t-1} + \rho(\frac{M}{Np})_{t-1} + \epsilon_t .$$

First Difference Model with Independent Errors

If it is assumed that the per capita demand for real money depends upon per capita real income and the interest rate, and that the values of lag coefficients are zero and the coefficient of autocorrelation is one, i.e., $\lambda = \mu = 0$ and $\rho = 1$, then given equation (6), the basic model can be written as

$$(\frac{M}{Np})_t - (\frac{M}{Np})_{t-1} = \alpha_o[(\frac{Y}{Np})_t - (\frac{Y}{Np})_{t-1}] + \beta_o(r_t - r_{t-1}) + \epsilon_t .$$

Static Model with Independent Errors

If it is assumed that the per capita demand for real money depends upon current per capita real income and the current interest rate, and that the values of lag and autocorrelation coefficients are zero, i.e., $\lambda = \mu = \rho = 0$, then the basic model can be expressed as

$$(\frac{M}{Np})_t = \gamma_o + \alpha_o(\frac{Y}{Np})_t + \beta_o r_t + \epsilon_t .$$

Statement of Estimation Procedure

Consider the mathematical form of a functional relationship which is assumed to be known and is written as

$$\eta = f(\delta_i ; \xi_j) \tag{8}$$

where δ_i $(i = 1, 2, \ldots, 9)$ are explanatory variables and ξ_j $(j = 1, 2, \ldots, 5)$ are the unknown parameters.[10] The main task is to determine the functional relationship between a response for the demand for money at period t and a number of inputs, i.e., real income per capita, interest rate and the demand for money in the previous periods.

The standard statistical method of estimation for the unknown parameters ξ_j in equation (8) is the method of ordinary least squares (OLS). The virtue of OLS is that the sum of squares of the differences between the observed responses η and their associated inputs δ_i will be minimized (through the regression function (8)). In order to minimize the sum of the squares of the residuals, $\Sigma\varphi^2 = \sum_h [\eta_h - f(\delta_{ih}; \xi_{jh})]^2$, one must consider the first and second derivatives of $f(\delta_i; \xi_j)$. It is assumed that they exist over the region of interest and write them respectively as

$$\frac{\partial f}{\partial \xi_j} = f_j(\delta ; \xi) \; ; \; \frac{\partial^2 f}{\partial \xi_j \, \partial \xi_k} = f_{jk}(\delta ; \xi) \, . \tag{9}$$

For the purpose of minimizing $\Sigma\varphi^2$, the following conditions must be met; given $\Omega = \Sigma\varphi^2$,

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[10] For convenience, the following simplifying notations are introduced:

$\eta = (\frac{M}{Np})_t$, $\delta_1 = (\frac{Y}{Np})_t$, $\delta_2 = (\frac{Y}{Np})_{t-1}$, $\delta_3 = (\frac{Y}{Np})_{t-2}$

$\delta_4 = r_t$, $\delta_5 = r_{t-1}$, $\delta_6 = r_{t-2}$, $\delta_7 = (\frac{M}{Np})_{t-1}$

$\delta_8 = (\frac{M}{Np})_{t-2}$, $\delta_9 = (\frac{M}{Np})_{t-3}$, $\xi_1 = \alpha_o$, $\xi_2 = \beta_o$,

$\xi_3 = \lambda$, $\xi_4 = \mu$ and $\xi_5 = \rho$.

$$\frac{\partial \Omega}{\partial \xi_j} = \Omega_j(\delta ; \xi) = -2 \sum_h [\eta_h - f(\delta_{ih}; \xi_{jh})] f_j(\delta_{ih}; \xi_{jh}) = 0 .$$

and

$$\frac{\partial^2 \Omega}{\partial \xi_j \, \partial \xi_k} = \Omega_{jk}(\delta ; \xi) = -2 \sum_h [\eta_h - f(\delta_{ih}; \xi_{jh})] f_{jk}(\delta_{ih}; \xi_{jh})$$
$$+ 2 \sum_h f_j(\delta_{ih}; \xi_{jh}) f_k(\delta_{ih}; \xi_{jh}) > 0 . \qquad (10)$$

Nevertheless, when the regression function $f(\delta_i ; \xi_j)$ is non-linear in the parameters, direct application of OLS may yield conflicting estimation of the original parameters. This becomes obvious when we observe that the application of OLS to equation (7) will result in nine coefficients being estimated while this original model specifies only five parameters, i.e., α_o, β_o, λ, μ and ρ. Furthermore, if the regression function $f(\delta_i ; \xi_j)$ is non-linear in the parameters, the estimation procedure will complicate matters somewhat from a computational point of view. It is for this reason that the modification of the "Gauss-Newton" method of iterative estimation is applied in this study.[11]

First, choose a vector of initial solutions, ξ_j^o $(j = 1, 2, \ldots, 5)$ for each unknown parameter. These guessed values may depend on past experience with similar problems, scatter diagrams of the observations or from the means of the observed variables. Thus we obtain

$$\xi_j = \xi_j^o + \Delta \xi_j^o \qquad (11)$$

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[11] H. O. Hartley, "The Modified Gauss-Newton Method for the Fitting of Non-linear Regression Functions by Least Squares," Technometrics Vol. 3 (May 1961), pp. 269-280.

where ξ_j^o is an approximation of the true value of the parameter ξ_j; and $\Delta\xi_j^o$ is the difference between the approximation and the true value.

Second, substitution of (11) into (8) gives

$$\eta = f(\delta_i; \xi_j^o + \Delta\xi_j^o) \tag{12}$$

Then, the first order Taylor expansions of the residuals are taken about ξ_j^o which give us the set of linear approximations to the residuals;

$$\eta \doteq f(\delta_i; \xi_j^o) + \sum_j \Delta\xi_j^o \left(\frac{\partial f}{\partial \xi_j}\right)_o + \ldots \tag{13}$$

where $\Delta\xi_j^o = \xi_j - \xi_j^o$, and the symbol $\left(\frac{\partial f}{\partial \xi_j}\right)_o$ is understood to mean the partial derivative of the dependent variable with respect to the j^{th} parameter evaluated at $\xi_j = \xi_j^o$ for all j.

If equation (13) is expressed in the notation of the original model, then the following relations are obtained:

$$\Delta\xi_1^o = \alpha - \alpha^o$$

$$\Delta\xi_2^o = \beta - \beta^o$$

$$\Delta\xi_3^o = \lambda - \lambda^o$$

$$\Delta\xi_4^o = \mu - \mu^o$$

$$\Delta\xi_5^o = \rho - \rho^o$$

$$\left(\frac{\partial f}{\partial \xi_1}\right)_o = \left(\frac{Y}{Np}\right)_t - (\mu^o + \rho^o)\left(\frac{Y}{Np}\right)_{t-1} + \mu^o\rho^o\left(\frac{Y}{Np}\right)_{t-2}$$

$$(\frac{\partial f}{\partial \xi_2})_o = r_t - (\lambda^o + \rho^o)\, r_{t-1} + \lambda^o \rho^o\, r_{t-2}$$

$$(\frac{\partial f}{\partial \xi_3})_o = \beta^o r_{t-1} + \lambda^o \beta^o r_{t-2} + (\frac{M}{Np})_{t-1} + (\rho^o + \mu^o)(\frac{M}{Np})_{t-2} + \mu^o \rho^o (\frac{M}{Np})_{t-3}$$

$$(\frac{\partial f}{\partial \xi_4})_o = \alpha^o(\frac{Y}{Np})_{t-1} + \rho^o \alpha^o(\frac{Y}{Np})_{t-2} + (\frac{M}{Np})_{t-1} + (\rho^o + \lambda^o)(\frac{M}{Np})_{t-2}$$
$$+ \lambda^o \rho^o (\frac{M}{Np})_{t-3}$$

$$(\frac{\partial f}{\partial \xi_5})_o = \alpha^o(\frac{Y}{Np})_{t-1} + \alpha^o \mu^o(\frac{Y}{Np})_{t-2} - \beta^o r_{t-1} + \lambda^o \beta^o r_{t-2} + (\frac{M}{Np})_{t-1}$$
$$+ (\lambda^o + \mu^o)(\frac{M}{Np})_{t-2} + \lambda^o \mu^o (\frac{M}{Np})_{t-3}$$

$$f(\delta_i ; \xi_j^o) = \alpha^o(\frac{Y}{Np})_t - (\mu^o + \rho^o)\alpha^o(\frac{Y}{Np})_{t-1} + \mu^o \rho^o \alpha^o(\frac{Y}{Np})_{t-2} + \beta^o r_t$$
$$- (\lambda^o + \rho^o)\beta^o r_{t-1} + \lambda^o \rho^o \beta^o r_{t-2} + (\lambda^o + \mu^o + \rho^o)(\frac{M}{Np})_{t-1}$$
$$+ [(\lambda^o + \mu^o)\rho^o + \lambda^o \mu^o]\,(\frac{M}{Np})_{t-2} + \lambda^o \mu^o \rho^o (\frac{M}{Np})_{t-3}$$

It is permissible to ignore the terms with derivatives of second or higher order if we assume that the vector of initial values ξ_j^o is a close approximation of the vector of true values ξ_j so that equation (13) can be reduced to

$$\sum_j \Delta \xi_j^o (\frac{\partial f}{\partial \xi_j})_o = \eta - f(\delta_i ; \xi_j^o) \qquad (14)$$

where the left hand side of the equation is a linear combination of known constant $(\frac{\partial f}{\partial \xi_j})_o$ and unknown correction factors $\Delta \xi_j^o$. The right hand side of the equation is the residual of the observed value η from estimated values $f(\delta_i ; \xi_j^o)$.

Third, in order to obtain the values of $\Delta \xi_j^o$, we compute the regression of $\eta - f(\delta_i; \xi_j^o)$ on the $(\frac{\partial f}{\partial \xi_j})_o$ by the ordinary linear regression method. If the estimated values of $\Delta \xi_j^o$ are not small enough, then the process will be repeated by using the successive point; i. e.,

$$\eta = f(\delta_i; \xi_j^o + \Delta \hat{\xi}_j^o) \tag{15}$$

where the values of $\Delta \hat{\xi}_j^o$ are the least-squares estimates of $\Delta \xi_j^o$. Finally, the iteration will be continued until the correction factors $\Delta \hat{\xi}_j^N$ $(N = 0, 1, 2, \ldots)$ become very close to zero.

Given the assumptions that the δ_{ih} are bounded and the ϵ_h are normally distributed, it is likely that the final set of estimates, $\eta = f(\delta_i; \xi_j^o + \Delta \hat{\xi}_j^N)$, are maximum likelihood estimates which possess the properties of consistency and asymptotic normality. In other words, if the estimates of ξ_j computed from the initial least squares estimation of equation (7) do not yield the minimum residual sum of squares, the iterative process will be repeated until the procedure converges to the set of estimated parameters which give the smallest residual sum of squares, or the maximum likelihood solution.

Theoretical Properties of the Estimated Function

The non-linear least squares estimating technique employed in this study is based on a maximum likelihood technique which is a modification of the Gauss-Newton method. Given a non-linear regression model,

$$\eta_h = f(\delta_{ih}; \xi_{jh}) + \epsilon_h \qquad \begin{aligned} h &= 1, 2, \ldots, t \\ i &= 1, 2, \ldots, \ell \\ j &= 1, 2, \ldots, m \end{aligned} \tag{16}$$

where δ_i denotes the fixed input vector of i elements while ξ is an m-element unknown parameter vector with element ξ_j, and ϵ_h is a set of t independent error residuals, the estimation of ξ_j can be accomplished by applying the non-linear least squares method. In order to guarantee the minimization of sum of squares of residuals, three normal conditions are required on the function, $\eta_h = f(\delta_{ih}; \xi_{jh})$.[12]

First, there exist first and second derivatives of the function $\Omega(\delta_i; \xi_j)$ with respect to the ξ, i.e.,

$$\frac{\partial \Omega}{\partial \xi_j} = \Omega_j(\delta_i; \xi_j) = -2 \sum_h \{\eta_h - f(\delta_{ih}; \xi_{jh})\} f_j(\delta_{ih}; \xi_{jh})$$

and

$$\frac{\partial^2 \Omega}{\partial \xi_j \partial \xi_h} = \Omega_{jk}(\delta_i; \xi_j) = -2 \sum_h \{\eta_h - f(\delta_{ih}; \xi_{jh})\} f_{jk}(\delta_{ih}; \xi_{jh})$$

$$+ 2 \sum_h f_j(\delta_{ih}; \xi_{jh}) f_k(\delta_{ih}; \xi_{jh}). \tag{17}$$

This assumption will assure the continuity of the function $f(\delta_{ih}; \xi_{jh})$ for all of ξ_j.

Secondly, there exists a non-trivial set of θ_j, $(j = 1, 2, \ldots, m)$ with $\Sigma\, \theta_j^2 > 0$, such that

$$\sum_h \{\sum_j \theta_j f_j(\delta_{ih}; \xi_{jh})\}^2 > 0 \tag{18}$$

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[12] H. O. Hartley, *op. cit.*, pp. 270-271.

for the observed vectors δ_i and for all ξ_j in a bounded convex set S of the parameter space $\xi_1, \xi_2, \xi_3, \ldots, \xi_m$. This assumption will provide the condition of non-degeneracy of rank in the least squares regression.

Thirdly, in order to assure the convergence of the iterative process of solution from the application of the Modified Gauss-Newton method, it is assumed that there exists a vector ξ_j^o in the interior of S such that

$$\Omega(\delta_{ih}; \xi_{jh}) < \liminf_{\overline{S}} \Omega(\delta_i; \xi) \tag{19}$$

where inf implies greatest lower bound and $\overline{S}$ is the complement to S, or $J = m+1, m+2, \ldots, \infty$.

Under these assumptions, the proof of convergence is derived by Hartley.[13] He starts with the elements ξ_j^o of the vector ξ^o which is expected to be proportional to the solutions ϕ_j of the Gauss-Newton equation.

By substituting a multiple 1st order Taylor expansion of $f(\delta_{ih}; \xi_{jh})$ at $\xi_j = \xi_j^o$, he obtains the linear least squares equations, namely;

$$2 \sum_k \{\sum_h f_j(\delta_{ih}; \xi_{jh}^o) f_k(\delta_{ih}; \xi_{jh}^o)\} \phi_k = 2 \sum_h \{\eta_h - f(\delta_{ih}; \xi_{jh})\} f_j(\delta_{ih}; \xi_{jh}). \tag{20}$$

Because the determinant of the linear equations (16) has rank m, this system of linear equations can be solved and yields elements ϕ_j of the vector ϕ as solutions.

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[13] Ibid., p. 271.

Now, consider the following function,

$$\Omega(\delta_i ; \xi_j^o + v\,\phi_j) \ . \qquad \text{For} \quad 0 \leq v \leq 1 \tag{21}$$

and define a vector

$$\xi^1 = \xi^o + v'\,\phi \tag{22}$$

where v' implies the value of v for which $\Omega(\delta_i ; \xi_j^o + v\,\phi_j)$ is a minimum on the interval $0 \leq v \leq 1$, then it is clear that

$$\Omega(\delta_i ; \xi_j^1) \leq \Omega(\delta_i ; \xi_j^o) < \lim\inf \Omega(\delta_i ; \xi) \tag{23}$$

so that ξ_j^1 also lies in the interior of S. If the preceding computation is repeated at ξ^1 and so on, a sequence of vector ξ^n, $(n = 1, 2, \ldots)$ will be derived, and all of these vector will also be within the bounded convex set S.

Let $\lim\limits_{n \to \infty} \xi^n \to \xi^*$, it is clear that

$$\lim_{n \to \infty} \Omega(\delta_i ; \xi_j^n) = \Omega(\delta_i ; \xi_j^*) \leq \Omega(\delta_i ; \xi_j^o) < \underset{S}{\underline{\lim}}\,\inf \Omega(\delta_i ; \xi) \tag{24}$$

and ξ_j^* will be also in the interior of S.

Next, for the purpose of minimizing the sum of squares of residuals, the first partials Ω_j at the point ξ^* are set to zero. If ϕ^* is the solution of the equation, then it is possible to obtain

$$2 \sum_k \{ \sum_h f_j(\delta_{ih} ; \xi_{jh}^*)\, f_k(\delta_{ih} ; \xi_{jh}^*) \} \phi^*$$
$$= 2 \sum_h \{ \eta_h - f(\delta_{ih} ; \xi_{jh}^*) \} f_j(\delta_{ih} ; \xi_{jh}^*) \ . \tag{25}$$

From the equations (17), (18), (19) and (25), it can be shown that

$$\lim_{n' \to \infty} \phi_j^{n'} = \phi_j^* \tag{26}$$

and

$$\sum_j \Omega_j(\delta_{ih}; \xi_{jh}^*)\phi_j^* = -2 \sum_h \{\sum_j f_j(\delta_{ih}; \xi_{jh}^*)\phi_j^*\}^2 < 0 \tag{27}$$

provided

$$\sum_j \phi_j^{*2} > 0 \tag{28}$$

Hartley has shown that, in order to avoid the contradiction against the statement that the $\Omega(\delta_{ih}; \xi^n)$ of the original sequence n converges to ϕ^*, which would also be the limit of the subsequence $\Omega(\delta_{ih}; \xi_j^{n'})$, the condition

$$\sum_j \Omega_j^2(\delta_{ih}; \xi_j^*) = 0 \tag{29}$$

must be met.[14] Therefore, it is concluded that a subsequence $\xi_j^{n'}$ of the sequence of vectors ξ_j^n converges to a solution ξ_j^* of the least squares equations, i.e.,

$$\Omega_j(\delta_{ih}; \xi_{jh}^*) = 0 \quad j = 1, 2, \ldots, m . \tag{30}$$

Theoretical Development of Estimation Techniques in Monetary Literature

The choice of estimation procedure is not only important but also critical in answering the questions of unbiasedness, efficiency and consistency in the estimation of the demand functions for money.

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[14] Ibid., p. 272.

An examination of Meltzer's demand for money study illustrates the pitfalls of his deep involvement in the statistical problems, such as multicollinearity and serial correlation.[15] His basic model of the demand for money is

$$M = \alpha R^{\beta} W_n^{\gamma} \tag{31}$$

where M represents the quantity of money demanded. R is the rate of interest and W_n is non-human wealth. α, β and γ are parameters to be estimated. Equation (31) can be written in logarithm form:

$$\ell n M = \ell n \alpha + \beta \ell n R + \gamma \ell n W_n \tag{32}$$

Two concepts of money stock variables (M_1 = currency plus demand deposits, M_2 = currency plus demand deposits and time deposits) are used. Several alternative measures of non-human wealth and Durand's basic yield on twenty-year corporate bonds for the interest rate variable are employed in Meltzer's study. He applies ordinary least squares regression to the United States time series and obtains the following conclusions:[16] (1) The demand function for money is homogeneous of first degree in prices and financial assets; (2) The demand function for money balances M_1 is at least as stable as the demand function involving broader definition of money M_2; (3) The demand function for money is more stable when the function is formulated in terms of a wealth constraint rather than an income constraint; (4) The

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[15] A. H. Meltzer, "The Demand for Money: The Evidence from the Time Series." The Journal of Political Economy, 71 (June, 1963) pp. 219-246.

[16] Ibid., pp. 244-246.

wealth elasticity increases when the definition of money is broadened to include time deposits. Nevertheless, Meltzer's empirical results reflect serious statistical problems, the problems of serial correlation and multicollinearity. Meltzer claims that, when both real income and real wealth are included in his demand function for money, the income variable fails to show its statistical significance. But his statistical results may be attributed to the fact that the lack of explanatory power is due to the multicollinearity which is implicitly involved in a high correlation between income and wealth. All of these results cast some doubt on Meltzer's empirical conclusion that the wealth variable is the best constraint in the demand function for money. By means of the criteria of "t" values or standard errors of estimate, Meltzer also concludes that the elasticity of broadly defined money balances with respect to wealth is substantially above the elasticity of narrowly defined money balances with respect to wealth. But these results again may be attributable to the autocorrelation in the disturbance terms.

In their comments on Meltzer's study, Courchene and Shapiro assert that Meltzer's money demand function is not in fact stable because his parameter estimates for different time periods are not generated from the same underlying population.[17] In addition, to deal with the question of the presence of autocorrelation in the regression analysis, Courchene and Shapiro analyze results of Meltzer's demand function for money with the application of Durbin-Watson test and find

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[17] T. J. Courchene and H. T. Shapiro, "The Demand for Money: A Note from the Time Series," The Journal of Political Economy, 72 (October, 1964), pp. 498-503.

that Meltzer's evidence indeed presents the serial correlation in the residuals.[18] The problem of the presence of autocorrelation with the application of ordinary least squares to estimate the parameter in the regression analysis will cause an understatement of sampling variances.[19] Meltzer later concedes the existence of serial correlation in his extensive analyses and then presents a more refined estimates of his wealth hypothesis. He finds less serial correlation and more evidence on the problem of stability at the cost of greater multicollinearity.[20]

Teigen illustrates his simultaneous equation system by the use of a supply-demand model, similar in type to the following:

Demand for Money (Md)	$Md = M[(r\,Y), Y]$
Supply of Money (Ms)	$Ms = \frac{M}{M^*} = x(r, r_c)$
Equilibrium	$Md = Ms$

where M is the sum of currency in the hands of the public and private demand deposits at member and non-member banks, M^* is the maximum stock of money based upon the reserves supplied by the Federal Reserve System, r represents the return to commercial banks from making loan and r_c is a measure of the cost of lending

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[18] Ibid., pp. 499.

[19] J. Johnston, Econometric Methods, New York: McGraw-Hill Book Company, Inc., 1963, pp. 177-179.

[20] A.H. Meltzer, "A Little More Evidence from the Time Series," The Journal of Political Economy, 72 (October, 1964), pp. 506-507.

for commercial banks.[21] He demonstrates that if there exists a supply function for money, then it is possible to estimate the coefficients of the demand and the supply functions of money jointly, so that it would be possible to take into consideration the interdependence among the variables. Teigen proceeds to treat the money supply as an endogenous rather than exogenous variable, and deals with a structural model as a distinct improvement over the single-equation analysis of the demand for money.[22] He argues that the single equation regression for the coefficients of the demand for money is often biased, and that the two-stage least squares method, which is applied to a structural model with money supply endogenously determined, will yield consistency in the parameters estimated.[23] In other words, he believes that a logical way of eliminating the bias in the estimation of demand function for money is to estimate the coefficients of the demand and the supply functions of money jointly by two-stage least squares, so that it would take account of the interdependence of the functions.

It is well known that ordinary least squares should not be directly applied to the model where the random disturbance term is correlated with one or more independent variables or where two or more independent variables are jointly dependent.[24] Teigen employs two-stage least squares to estimate the coefficients of the money demand and

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[21] R. L. Teigen, "Demand and Supply Functions for Money in the United States: Structural Estimates," Econometrica, 32 (October, 1964), pp. 476-509.

[22] Ibid., pp. 500-502.

[23] Ibid., pp. 503-505.

[24] J. Johnston, op. cit., pp. 177-179.

money supply functions simultaneously. In the first part of the two-stage procedure, the endogenous variables are regressed on all of the exogenous variables. In the second part of the procedure, the estimates variables in the first stage are substituted for the endogenous variables, and ordinary least squares is applied.

Teigen estimates a structural model of money supply and demand by two-stage least squares and sums up with the following conclusions:[25] (1) The variables--the product of the interest rate and national income and national income alone--in the demand function for money give the best results during the postwar period, (2) A log-linear function is judged to be the best functional form during the prewar years, (3) His structural estimates support the hypothesis that the supply of money should be treated endogenously, (4) The short term rate of interest links the supply function of money to the rest of the model. Finally, (5) for the postwar years, his estimates indicate no difference in the extent of response to movements in the interest and discount rates. But for the prewar period, however, the coefficient of the interest rate in the supply function is more than twice as large in absolute value as the coefficient of the discount rate. It should be pointed out that the two-stage least squares method only gives us an estimator with the property of consistency. The estimator is generally inefficient.[26]

Chow argues that one of the major weakness in the available theoretical formulations of demand functions for money seems to be the

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[25] R. L. Teigen, op. cit., pp. 505-506.

[26] P. J. Dhrymes, Econometrics: Statistical Foundations and Applications, New York: Harper & Row, 1970, pp. 176-182.

failure to distinguish between a long-run equilibrium demand and a short-run demand function for money.[27] Therefore Chow's model consists of a "long-run" component and a "short-run" component of the demand function for money; i.e.,

$$M_t - M_{t-1} = \alpha(M_t^* - M_{t-1}) + \beta(Y_t - Y_{t-1}) \tag{34}$$

where $M_t - M_{t-1}$ is a change in the money demanded, $\alpha(M_t^* - M_{t-1})$ is a fraction of the difference between the so-called equilibrium demand for money and the money demanded in the previous period, and $\beta(Y_t - Y_{t-1})$ is a fraction of change in total assets. The equilibrium demand for money is derived from the following functional relationship:

$$M_t^* = b_0 + b_1 Y_t + b_2 r_t \tag{35}$$

where M^* is the estimated equilibrium demand for money, Y is the relevant income or wealth variable, r is the rate of interest, and the subscript t denotes time.

Chow applies ordinary least squares regression to the United States time series data and estimates the short-run as well as long-run demand function for money. The estimates obtained by Chow not only support the hypothesis of a long lag in adjustment, but they also support the view that permanent income explains the total stock of money demanded better than current income does when short-run adjustments are not allowed for. In other words, Chow concludes that

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[27] G. C. Chow, "On the Long-Run and Short-Run Demand for Money," The Journal of Political Economy, 74 (April, 1966), pp. 111-131.

permanent income is more important than current income in the long-run framework, and current income is more important in short-run change in demand for money.[28] All of these economic interpretations are actually based upon statistical evidence that (1) permanent income turns out to yield a slightly higher regression coefficient than does the non-human wealth measure, (2) the coefficient of the permanent income variable has smaller value of standard error, (3) the coefficient for current income is positive and significant.[29] In order to illustrate the usefulness of Chow's demand function for money, some econometric problems, such as estimating the parameters of a distributed lag model and testing serial correlation in the presence of a lagged dependent variable, are carefully examined. In the case of positive autocorrelation among the errors, the direct application of the ordinary least squares method on Chow's demand function would underestimate the standard errors of the coefficient even if the coefficients of the regression are still unbiased.[30] On the other hand, in the case of the lagged dependent variable, the application of the ordinary least squares method to estimate distributed lag models would lead to inconsistent estimates, particularly when the model involves the property of autocorrelation in the disturbance terms.[31] For these reasons, Chow's estimates of the short-run demand for

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[28] Ibid., p. 127.

[29] Ibid., pp. 118-125.

[30] J. Johnston, op. cit., p. 179.

[31] P. Rao and R. L. Miller, Applied Econometrics, California: Wadsworth Publishing Company, Inc., 1971, pp. 165-173.

money cannot be reliably used since his empirical results are in fact inconsistent estimates.

In his paper, de Leeuw demonstrates that it is important to integrate the problem of speeds of adjustment into a theory of demand for money.[32] The key assumption in de Leeuw's theoretical scheme is that the public adjusts its demand for money during the period by a constant fraction (1 - k) of the discrepancy between income or interest rate x_t and its existing stock of money demanded in the previous period M_{t-1}. Mathematically

$$M_t - M_{t-1} = (1-k)(b'x_t - M_{t-1}) + u_t \tag{36}$$

where M is real per capita money demanded, b' is a vector of coefficients, x is a vector of some factors which influence demand for money, (1 - k) is a measure of the speed of adjustment and u_t is the unexplained residual. He adds M_{t-1} into both sides of equation (36) and obtains

$$M_t = kM_{t-1} + (1-k)\,b'x_t + u_t \,. \tag{37}$$

Due to the statistical problems which exist in the estimation procedures of ordinary least squares and two-stage least squares, an upward bias in the coefficient on the lagged stock and thus downward bias in the

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[32] F. de Leeuw, "The Demand for Money: Speed of Adjustment, Interest Rates, and Wealth," Monetary Process and Policy: A Symposium, Edited by G. Horwich, Homewood, Illinois, Richard D. Irwin, Inc., 1966.

speed of adjustment will arise.[33] de Leeuw employs another estimating technique, three-pass least squares, to remove the bias in the estimation of k. He completes the specification of his model by invoking an assumption with respect to the probability distribution of random error u_t. He assumes that the errors follow a Markov process of the form

$$u_t = \rho u_{t-1} + \epsilon_t \tag{38}$$

where ϵ_t is a random normal variable with zero mean, constant variance σ_ϵ^2 and serially independent. He then substitutes equation (38) into equation (37), so that

$$M_t = kM_{t-1} + (1-k)b'x_t + \rho u_{t-1} + \epsilon_t \tag{39}$$

If u_{t-1} is known, the application of the ordinary least squares to equation (39) would result in consistent, asymptotically efficient estimates although small-sample bias would still remain. Nonetheless, the values of u_{t-1} are not available, thus ordinary least squares could not be applied to equation (39) without introducing bias into the estimation of speed of adjustment. In fact, the idea of the three-pass least squares is to obtain the estimated values of u_{t-1} by the process of successive approximation, and then to apply ordinary least squares to equation (39). In other words, de Leeuw in the first-pass applies ordinary least squares to equation (37) so that the coefficient estimates k and b', and regression residuals

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[33] P. H. Hendershott, "The Demand for Money: Speed of Adjustment, Interest Rates, and Wealth--A Sequel," Staff Economic Studies, Washington, D. C.: Board of Governors of the Federal Reserve System, 1966.

$$u_t^* = M_t - kM_{t-1} - (1-k)b'x_t \tag{40}$$

are calculated. A new dependent variable $M_t - (1-k)b'x_t$ then is calculated. In the second-pass, he applies ordinary least squares to the equation

$$M_t - (1-k)b'x_t = k^{**}M_{t-1} + \rho u_{t-1}^* + \theta M_{t-2} + \epsilon_t \tag{41}$$

so that he would be able to obtain the estimated values of k^{**}, ρ and θ. Among these three estimators, only k^{**} is used, where the double asterisks indicate that it is obtained in the second-pass. Finally, a new estimate of the u-series is obtained,

$$u_{t-1}^{**} = M_{t-1} - k^{**}M_{t-2} - (1-k)b'x_{t-1} \tag{42}$$

and in the third-pass, the estimates of the coefficients of the equation

$$M_t = kM_{t-1} + (1-k)b'x_t + \rho u_{t-1}^{**} + \epsilon_t \tag{43}$$

are obtained through the application of the ordinary least squares. These estimated values of the parameters from the three-pass least squares technique are consistent only if x_t is serially uncorrelated, and only if the estimates of $(1-k)b'$ are consistent in the first-pass of estimation. If $(1-k)b'$ are not consistently estimated in the first pass, u_{t-1}^* will not be a consistent estimate of u_{t-1} and the final estimates of the coefficient of equation (43) will not be consistent. Wallis has indicated that three-pass least squares estimates are in general not consistent.[34] The results of the Monte Carlo method suggest that when the estimated values of parameters from three-pass least squares are consistent, they are still less efficient than other estimation procedures such as the generalized least squares method.[35] This suggests that the de Leeuw hypothesis of a substantial lag in the adjustment of money holding and the estimating technique employed by him need to be reconsidered.

Supplementary Remarks on Various Estimation Techniques

These supplementary remarks discuss the advantages and disadvantages of the various estimation techniques mentioned in the previous section of this chapter.

In a single equation model, the ordinary least squares method of estimation is appropriate only when the usual conditions for identification are satisfied and the error terms have met the provisions required by the Markov theorem. However, if there is a correlation between the disturbance term and the explanatory variable of a regression equation, or if the error terms are autocorrelated, the ordinary least squares method does not yield the best unbiased estimates. Without any modification, A.H. Meltzer has applied ordinary least squares to his basic model of demand for money, which is the Static Model with Independent Errors, the model where the values of lag and

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[34] K. F. Wallis, "Lagged Dependent Variables and Serially Correlated Errors: A Reappraisal of Three-pass Least Squares," Review of Economics and Statistics, 49 (November, 1967), pp. 555-567.

[35] Ibid., pp. 566-567.

autocorrelation coefficients are assumed to be zero.[36]

For the fundamental problems in this situation, two alternative estimation methods are generally suggested, provided the appropriate conditions are satisfied. First, in order to cope with the problem of correlation which exists between the error term and some of the independent variables caused by the simultaneous satisfaction of all the equations of the model, two-stage least squares, which was used by R. L. Teigen,[37] is often employed.[38] The method of estimation mainly attempts to "purge" the explanatory variables subject to error on the right hand side of the equation of their error components by replacing these variables with suitable estimates which may be derived from a separate least squares regression. If these independent variables can be somehow "purged" of error component by two-stage least squares, the least squares estimation of a single equation should be able to provide "reasonable" estimates of the parameters. As long as the reduced-form equations do not involve any endogenous variable in the independent variables of a regression equation, it should be possible to estimate "purified" endogenous variables from the estimated reduced-form equations. However, it should be noted that the method of two-stage least squares will fail if the usual conditions for identification are not satisfied or the error term in the second-stage application of least squares does not meet the usual conditions required by the Markov theorem. The method of estimation fails for the reason that

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[36] A. H. Meltzer, op. cit., pp. 219-246.

[37] R. L. Teigen, op. cit., pp. 476-509.

[38] P. Rao and R. L. Miller, op. cit., pp. 212-215.

two-stage least squares is studied under the assumption that error terms in the two equations are independently distributed. As a matter of fact, in some econometric models, this is rather a restrictive assumption. In the estimation of demand functions from time series data, there are many situations where the error disturbances of a system of regression equations exhibit serial correlation. Although the technique of two-stage least squares adapted by R. L. Teigen in the estimation of the demand function for money is an improved estimation method over the method of ordinary least squares employed by A. H. Meltzer, without a proper transformation of autoregressive errors, the direct application of two-stage least squares inevitably yields in estimates which are inconsistent and inefficient.

Second, in some econometric studies, the specification that the error terms in each observation are drawn independently of other terms may be inappropriate. If the error terms are serially dependent, the estimates by ordinary least squares are not the minimum-variance unbiased estimates of the parameters. In order to cope with this problem, an alternative estimation procedure, i.e., generalized least squares, is suggested.[39] The generalized least squares estimates of the coefficients are obtained by applying ordinary least squares estimation to the regression equation in which all of the variables, dependent and independent, are transformed. The variation of the value of the autocorrelation coefficient of the Two Lag Model with Autoregressive Errors provides two types of specification. The first specification is

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[39] Henri Theil, Principles of Econometrics, New York: John Wiley & Sons, Inc., 1971, pp. 236-243.

the Static Model with Autoregressive Errors where the lag coefficients associated with the explanatory variables are zero and the autocorrelation coefficient is bounded between zero and one. The estimated parameter of the autoregressive process is consistently estimated on the basis of a regression on these residuals. The second specification is the First Difference Model with Independent Errors where the values of the lag coefficients are zero and the coefficient of autocorrelation is one. G. C. Chow has estimated the demand function for money by using the ordinary least squares on the first difference model.[40] The disadvantage of applying the first difference technique has been shown by Rao and Miller to be that if the assumed value of autocorrelation coefficient is different from the true value of the parameter, the loss in precision of the estimation from the transformed variable can be extremely large.[41] The modified Gauss-Newton method of estimation provides the best estimated values of coefficient by the iterative process which will be repeated until the procedure converges to the set of estimated parameters which give the smallest residual sum of squares.[42]

The problem of specification bias arises from incorrect specification of a given model. This may be because any given linear model could be generated by a somewhat more general non-linear model. In face of the difficulty of building up a complete theory and the little knowledge of the properties of actual demand function for money, an

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[40] G. C. Chow, op. cit., pp. 111-131.

[41] P. Rao and R. L. Miller, op. cit., pp. 74-75.

[42] H. O. Hartley, op. cit., pp. 269-280.

operational procedure to guard against this specification bias is to fit several versions of a given equation by exploring the versatile nature of the basic model, i. e., Two Lag Model with Autoregressive Errors, and examining the statistical results of estimation from a variety of assumptions regarding the values of lag and autocorrelation coefficients. Some of these versions are non-linear and any non-linear specification generally tends to make the estimation either unobtained or only very approximately obtainable by the least squares estimation. Hence, the method of recursive improvement of an approximate initial estimate must be incorporated. This is exactly what the modified Gauss-Newton method of estimation involves.

Summary

This chapter has been devoted to a presentation of the major theoretical and empirical contributions in the recent development of lags in the demand for money. It is known that the direct application of ordinary least squares on the distributed lag model as well as the serial correlation model would not yield consistent and efficient estimates. For this reason, considerable attention should be given to an alternative estimating method, e. g., the Modified Gauss-Newton method, which would provide the statistical results with the minimum sum of squares of the residuals. It means that the chief merit of the procedure employed in this study is that it works in the case of the distributed lag model or the serial correlation model, and it yields maximum likelihood estimators that possess the properties of consistency and asymptotic efficiency and sufficiency.

CHAPTER III

ESTIMATION OF THE MODEL

Introduction

The statistical procedures used to estimate the demand function for money in the current literature of monetary economics are often unable to provide the estimated coefficients with the properties of consistency and efficiency. In this study, the modified Gauss-Newton method is used in an effort to obtain consistent and efficient estimators.

The model is based on the modified Keynesian approach to the problem of the demand for money. This implies that on the one hand the demand for money is attributed to the transaction motive and the speculative motive. On the other hand the demand for money is expressed in terms of the Koyck distributed lag model.[1] The economic rationale behind this hypothesis is that changes in per capita real income and the rate of interest do not produce their effects only in the current period. The explanatory variables, per capita real income and the rate of interest, may be incorporated with the concept of distributed lag model for the reason that people do not fully adjust their actual holdings of real cash balances to their desired level in the current period. Instead, it takes several periods for them to complete

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[1] L. M. Koyck, Distributed Lags and Investment Analysis. Amsterdam: North Holland Publishing Company, 1954.

the adjustment when there are changes in per capita real income and the rate of interest.

The derivation of the coefficients of demand for money follows the Koyck distributed lag equation where the coefficients of the distributed lag equation decline in geometric progression in successive periods. This suggests that (1) the current value of the demand for money is not only related to the values of current per capita real income and the rate of interest, but also related to the values of these variables at several different times in the past; and (2) recent values of per capita real income and the rate of interest should be weighted more heavily than those of the remote past. Therefore, the values of the coefficients of explanatory variables are derived from an exponentially weighted average of current and past levels of per capita real income and the rate of interest.

By the transformation given in the previous chapter, the model becomes

$$\begin{aligned}(\tfrac{M}{Np})_t = {} & \gamma_o(1-\lambda)(1-\mu)(1-\rho) + \alpha_o(\tfrac{Y}{Np})_t - (\mu+\rho)\alpha_o(\tfrac{Y}{Np})_{t-1} + \mu\rho\alpha_o(\tfrac{Y}{Np})_{t-2} \\ & + \beta_o r_t - (\lambda+\rho)\beta_o r_{t-1} + \lambda\rho\beta_o r_{t-2} + (\lambda+\mu+\rho)(\tfrac{M}{Np})_{t-1} \\ & - [(\lambda+\mu)\rho + \lambda\mu](\tfrac{M}{Np})_{t-2} + \lambda\mu\rho(\tfrac{M}{Np})_{t-3} + \epsilon_t . \qquad (1)\end{aligned}$$

The error term, ϵ, has the following stochastic assumptions: (1) ϵ is a random real variable and ϵ_t is random for every t; (2) ϵ_t, for every t, has zero expected value, $E[\epsilon_t] = 0$; (3) the variance of ϵ_t is constant, and $E[\epsilon_t \epsilon_t] = \sigma_\epsilon^2 < \infty$ for all t; and (4) the random terms of different time periods are independent. This implies that in each period the random terms act independently of their behavior in all previous and subsequent periods, $E[\epsilon_t \epsilon_{t-s}] = 0$ for all t and for $s \neq 0$. In other words, the second and the third assumptions state that the variance matrix of the disturbances has σ_ϵ^2 in each element of its leading diagonal and zero elsewhere. Therefore, ϵ is distributed independently over time so that it shows no serial correlation, but with constant variance.

The problem of finding an empirical definition of money is also an essential issue in the theory of demand for money. Since the choices between money defined to exclude time deposits and money defined to include them have not yet been satisfactorily settled, both definitions of money have been employed in this study.[2]

All of the functional forms and statistical tests undertaken in this study are based on the application of the single equation approach. The models are deliberately specified in such a way that the supply function of money shifts independently of the demand function along a stable demand surface. This means that the supply function of money contains at least one variable that does not appear in the demand function for money so that this variable shifts the supply function of money around over time. For this reason, it is possible to obtain observations at different points on the demand function for money. For example, the level of reserves made available by the Central Bank to the commercial banking system often plays a prominent role in the theory of the supply of money and does not appear in the theory of the demand for money.

The analysis proceeds in the following manner. First, equation (1) is estimated. This is the basic model of the dissertation, the

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[2] The reader who is interested in pursuing this matter will find a good reference over the definition of money in Laidler (1969).

model that allows for separate adjustment lags for per capita real income and the rate of interest as well as for autocorrelation. This model is designated as the Two Lag Model with Autoregressive Errors. The following model is equation (1) when $\rho = 0$, the Two Lag Model with Independent Errors. The third model that is investigated consists of setting $\lambda = \mu = 0$ in equation (1)--the Static Model with Autoregressive Errors. The fourth model constrains ρ in equation (1) to be equal to unity; this is the First Difference Model with Independent Errors. The last model that is estimated is the Static Model with Independent Errors, the model in which $\lambda = \mu = \rho = 0$.

Two Lag Model With Autoregressive Errors

The basic model in this study is one in which the per capita demand for real money depends upon per capita real income and the rate of interest. If it is assumed that there are lag coefficients associated with the explanatory variables and that the autocorrelation coefficient is bounded between zero and one, i.e., $\lambda \neq 0$, $\mu \neq 0$, and $0 < \rho \leq 1$, then the basic model is written as

$$\begin{aligned}(\frac{M}{Np})_t = {} & \gamma_o(1-\lambda)(1-\mu)(1-\rho) + \alpha_o(\frac{Y}{Np})_t - (\mu+\rho)\alpha_o(\frac{Y}{Np})_{t-1} + \mu\rho\alpha_o(\frac{Y}{Np})_{t-2} \\ & + \beta_o r_t - (\lambda+\rho)\beta_o r_{t-1} + \lambda\rho\beta_o r_{t-2} + (\lambda+\mu+\rho)(\frac{M}{Np})_{t-1} \\ & - [(\lambda+\mu)\rho + \lambda\mu](\frac{M}{Np})_{t-2} + \lambda\mu\rho(\frac{M}{Np})_{t-3} + \epsilon_t \qquad (2)\end{aligned}$$

where $\frac{M}{Np}$ is per capita demand for real money, $\frac{Y}{Np}$ is per capita real income, r is the rate of interest, the parameters, α_o and β_o, indicate the magnitude of the respective influences of the impact of per capita real income and the rate of interest on the per capita demand for real money, and γ_o is a constant term. All other notation and the statistical conditions are defined as before.

The method of estimation consists of choosing the values of α_o, β_o, λ, μ, and ρ which minimize the sum of squares of the residuals. Theoretically, the estimators obtained by means of the modified Gauss-Newton method of iterative estimation are consistent and sufficient.[3] In addition, if ϵ's are normally distributed, these estimators are also maximum likelihood estimators.[4]

The theory of demand for money usually demonstrates that per capita demand for real money is positively related to per capita real income. With regard to the rate of interest, it is expected to have a negative influence on the per capita demand for real money.

In order to choose an appropriate interest rate variable, two variables were tested; the short-term interest rate which is the market yield on 90-day United States government securities and the long-term interest rate which is the United States long-term bond yield. To test whether the short-term or the long-term interest rate has stronger influence on the per capita demand for real money, regressions of annual and quarterly data for the period between 1956 and 1969 were run on these two variables with alternative definitions of money.

The summary of the regression results for the Two Lag Model with Autoregressive Errors for the United States are reported in

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[3] H. O. Hartley, "The Modified Gauss-Newton Method for Fitting of Non-Linear Regression Functions by Least Squares," Technometrics, 3 (May, 1961), pp. 269-280.

[4] H. O. Hartley and A. Booker, "Nonlinear Least Squares Estimation," Annual of Mathematical Statistics, 36 (April, 1965), pp. 638-650.

TABLE I

PARAMETERS OF DEMAND FOR MONEY IN THE UNITED STATES UNDER TWO LAG MODEL WITH AUTOREGRESSIVE ERRORS USING SHORT-TERM INTEREST RATE

Dependent Variable	Constant	Estimates of the Parameters		Estimates of Lag and Autocorrelation Coefficients			$\bar{R}^2$	Durbin-Watson Statistic
	γ_o	α_o	β_o	λ	μ	ρ		DW
Annual Data (1956-1969)								
$\frac{M_1}{Np}$	5,854.28	8.897 (6.298)	-0.114 (1.607)	-0.252 (0.221)	0.137 (5.029)	0.141 (5.232)	0.375	0.57
$\frac{M_2}{Np}$	-222.06	7.012* (0.603)	-0.596* (0.163)	0.127 (0.065)	0.445* (0.070)	-0.264 (0.300)	0.967	2.13
Quarterly Data (1956-1969)								
$\frac{M_1}{Np}$		39.449* (0.868)	-0.114 (0.417)	-0.004 (0.019)	-0.257 (0.140)	1##	0.673	2.08
$\frac{M_2}{Np}$	9.57	5.427* (0.152)	-0.066** (0.028)	0.010 (0.026)	0.747* (0.138)	0.023 (0.192)	0.989	1.99

TABLE II

PARAMETERS OF DEMAND FOR MONEY IN THE UNITED STATES UNDER TWO LAG MODEL WITH AUTOREGRESSIVE ERRORS USING LONG-TERM INTEREST RATE

Dependent Variable	Constant	Estimates of the Parameters		Estimates of Lag and Autocorrelation Coefficients			$\bar{R}^2$	Durbin-Watson Statistic DW
	γ_o	α_o	β_o	λ	μ	ρ		
Annual Data (1956-1969)								
$\frac{M_1}{Np}$	5,656.25	8.825 (9.780)	0.120 (3.897)	-0.369^{**} (0.173)	0.194 (20.183)	0.200 (20.467)	0.249	0.78
$\frac{M_2}{Np}$	41.11	6.389^{*} (0.697)	-0.920^{**} (0.362)	0.026 (0.139)	-0.045 (0.645)	-0.008 (0.854)	0.964	1.97
Quarterly Data (1956-1969)								
$\frac{M_1}{Np}$		37.344^{*} (1.687)	1.480 (1.137)	0.003 (0.021)	-0.206 (0.146)	$1^{\#\#}$	0.684	2.05
$\frac{M_2}{Np}$	27.55	5.390^{*} (0.156)	-0.092 (0.049)	0.002 (0.026)	0.737 (0.167)	0.087 (0.215)	0.989	2.00

Tables I and II. The standard errors are presented in parentheses below the estimated coefficients. An estimated coefficient is considered statistically significant if its accompanying "t" statistic is 1.96 or larger. The notations * and ** indicate the estimated coefficients which are significantly different from zero at the one and the five percent levels respectively. The notation, ##, implies that the value of the autocorrelation coefficient is fixed at unity whenever the coefficient has been discovered to exceed one after the regression. Since the constant term in equation (1) is $\gamma_o(1-\lambda)(1-\mu)(1-\rho)$, the value of γ_o is not shown in the table whenever the value of λ, μ or ρ is fixed a priori at unity. The adjusted multiple correlation coefficient $\bar{R}^2$ instead of the multiple correlation coefficient R^2 is used in this study. The value of the adjusted multiple correlation coefficient $\bar{R}^2$ is defined as

$$\bar{R}^2 = 1 - (1 - R^2)\ \frac{n-1}{n-k} \tag{3}$$

where n is the number of observations and k is the number of parameters. In general, the value of R^2 is larger than the value of $\bar{R}^2$ except when $k = 1$ in which case $R^2 = \bar{R}^2$. The use of R^2 has some disadvantages, because when the number of parameters (k) is larger compared with the number of observations (n), the mean squares of the residual tends to be on the low side and hence it gives an overly optimistic picture of the performance of the explanatory variable.[5]

The statistical evidence for the United States shows that, for the

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[5] H. Theil, Principle of Econometrics. New York: John Wiley and Sons, Inc., 1971, pp. 178-179.

entire period, the per capita real income has a positive coefficient which is statistically significant and the rate of interest has a coefficient which is statistically insignificant or slightly less than zero. Most of the equations are sufficiently well specified to pass the Durbin-Watson test for autocorrelation. The adjusted coefficient of determination $\bar{R}^2$ is larger with the short-term interest rate than with the long-term interest rate. The $\bar{R}^2$ statistic, a measure of the percent of the variation in the dependent variable which is explained by variation in the independent variables, ranges from 0.38 to 0.99 for the short-term interest rate and from 0.25 to 0.99 for the long-term interest rate. Thus if the quarterly data of the short-term interest rate are used in the independent variable, the specification of the Two Lag Model with Autoregressive Errors explains 99 percent of the variance in the level of the per capita demand for real money in the United States when the regression is based on the broad definition of money.

Since a higher t-value implies a higher level of confidence in the estimated regression coefficient, the upward change in the t-value of the explanatory variable can be interpretated as an indication of greater reliability of the estimated value of demand for money. Examination of the statistical results indicates a greater t-value for the regression coefficients of the short-term interest rate than for the long-term interest rate. A strong empirical relationship between per capita demand for real money and per capita real income and the short-term interest rate points to the conclusion that per capita real income and the short-term interest rate can and should play a prominent role in the estimation of per capita demand for real money provided that the broad definition of money is used in the regression.

Thus, if the statistical significance of the coefficients of the short-term interest rate or the long-term interest rate is measured by the t-value of the coefficients, the short-term interest rate variable in general is more statistically significant and more reliable in that its relationship to per capita demand for real money is more predictable. From the statistical evidence, it is found that the t-value of the short-term interest rate is substantially larger than the t-value of the long-term interest rate, particularly when per capita demand for real money (broadly defined) is used as the dependent variable.

From these statistical results, two distinct conclusions are made. First, the short-term interest rate is the appropriate rate in the estimation of the demand function for money in the United States. For this reason, the short-term interest rate is used as the rate of interest variable in the remainder of the study. Second, the evidence indicates that money should be defined to include time deposits. That is, the best results are obtained when money is defined as M_2.

The statistical evidence for Taiwan is presented in Table III. The results from the regression covering data for the period between 1956 and 1969 show the anticipated signs for the coefficients of per capita real income and the interest rate variables. If annual data are used, the per capita demand for real money is positively related to the per capita real income. The coefficients of per capita real income are also significantly different from zero at the one percent level for both definitions of money. Although the coefficients of the rate of interest have the correct sign, the estimated coefficients of the rate of interest are not significantly different from zero. If quarterly data for the same period are used for the analysis, the coefficients of per

TABLE III

PARAMETERS OF DEMAND FOR MONEY IN TAIWAN UNDER TWO LAG MODEL WITH AUTOREGRESSIVE ERRORS

Dependent Variable	Constant	Estimates of the Parameters		Estimates of Lag and Autocorrelation Coefficients			$\bar{R}^2$	Durbin-Watson Statistic DW
	γ_o	α_o	β_o	λ	μ	ρ		
Annual Data (1956-1969)								
$\frac{M_1}{Np}$	-1,202.69	23.909* (2.883)	-14.859 (9.749)	0.048 (0.118)	-0.021 (0.272)	0.413 (0.336)	0.988	2.29
$\frac{M_2}{Np}$	-6,022.70	54.325* (16.335)	-35.225 (22.755)	0.222 (0.240)	0.218 (0.386)	-0.086 (0.501)	0.994	2.53
Quarterly Data (1956-1969)								
$\frac{M_1}{Np}$	-1,773.37	11.877* (1.484)	4.643* (1.562)	0.562* (0.060)	-0.650* (0.109)	0.570* (0.072)	0.989	1.81
$\frac{M_2}{Np}$		24.317* (4.135)	-16.302* (5.700)	-0.194* (0.064)	-0.202** (0.105)	1##	0.996	1.86

capita real income variable still show the appropriate sign for both definitions of money, and they are significantly different from zero at the one percent level. If annual data are used, the interest rate variable fails to show its importance in the determination of the demand for money. However, if quarterly data for the broad definition of money are used in the dependent variable, the estimated coefficients of interest rate variable not only provide the expected sign, but also show their statistical significance. The Durbin-Watson statistics for both annual and quarterly data do not permit accepting the hypothesis that the time series under the Two Lag Model with Autoregressive Errors is serially correlated. This implies that the incorporation of autocorrelation coefficient into the model indeed provides an appropriate functional form where the residual terms are serially independent. The $\bar{R}^2$ statistic ranges from 0.988 to 0.996.

The elasticities of per capita demand for real money with respect to per capita real income and the rate of interest are shown in Table IV. These elasticities are calculated at the mean. The results indicate that the elasticity of per capita demand for real money, when money is broadly defined, with respect to per capita real income is close to be unity using the short-term interest rate as a proxy variable. In the United States, most of the elasticities of per capita demand for real money with respect to the rate of interest are less than 10 percent except when annual data for broadly defined money are used in the dependent variable. On the other hand, the situation is quite different in Taiwan. The elasticities of per capita demand for real money with respect to per capita real income and the rate of interest are much larger than those in the United States. This implies

TABLE IV

ELASTICITIES OF PER CAPITA DEMAND FOR REAL MONEY WITH RESPECT TO PER CAPITA REAL INCOME AND THE RATE OF INTEREST

Variable	Per Capita Real Income	The Rate of Interest	Variable	Per Capita Real Income	The Rate of Interest
For the United States			For Taiwan		
Annual Data (1956-1969)			Annual Data (1956-1969)		
$\frac{M_1}{Np}$	0.288	-0.005	$\frac{M_1}{Np}$	1.525	-0.319
$\frac{M_2}{Np}$	1.329	-0.156	$\frac{M_2}{Np}$	17.194	-3.755
Quarterly Data (1956-1969)			Quarterly Data (1956-1969)		
$\frac{M_1}{Np}$	1.262	-0.005	$\frac{M_1}{Np}$	0.796	0.108
$\frac{M_2}{Np}$	1.025	-0.018	$\frac{M_2}{Np}$	7.609	-1.766

that per capita demand for real money in Taiwan is highly responsive to the changes in per capita real income and the rate of interest, particularly when the broad definition of money is used in the dependent variable. The elasticities range from 7.6 to 17.2 for the per capita real income variable and from 1.8 to 3.8 for the interest rate variable. The greater than unity income elasticity of demand for real cash balances in Taiwan illustrates the existence of a plausible monetary phenomenon in less developed countries. Demonstration effects and inefficient credit facilities lead people to increase their demand for real cash balances. Thus, according to the estimates, a one percent increase in per capita real income results in a 7.6 percent increase in per capita demand for real money.

In order to measure the relative promptness of influence of per capita real income and the rate of interest on the per capita demand for real money, the lag coefficients for the Two Lag Model with Autoregressive Errors are presented in Tables V and VI. In all instances, except in the case of annual data with narrowly defined money for the United States, the interest rate variable has a consistently faster influence on the per capita demand for real money than the per capita real income. Five out of eight lag coefficients of interest rate variable show the completion of more than fifty percent of adjustment in the first two periods. On the other hand, only three out of eight lag coefficients of per capita real income variable complete more than fifty percent of adjustment in the first two periods.

It is found that the theory of demand for money provides a solid theoretical base for a reasonably good empirical explanation of per capita demand for real money in the past fourteen years. The major

TABLE V

THE LAGS STRUCTURE OF PER CAPITA REAL INCOME UNDER TWO LAG MODEL WITH AUTOREGRESSIVE ERRORS

Variable	t	t-1	t-2	$\sum_{i=0}^{2} t-i$	Variable	t	t-1	t-2	$\sum_{i=0}^{2} t-i$
For the United States					For Taiwan				
Annual Data (1956-1969)					Annual Data (1956-1969)				
$\frac{M_1}{Np}$	0.252	0.188	0.141	0.581	$\frac{M_1}{Np}$	0.048	0.002	0.000	0.050
$\frac{M_2}{Np}$	0.127	0.111	0.097	0.335	$\frac{M_2}{Np}$	0.222	0.173	0.134	0.529
Quarterly Data (1956-1969)					Quarterly Data (1956-1969)				
$\frac{M_1}{Np}$	0.004	0.004	0.004	0.012	$\frac{M_1}{Np}$	0.562	0.246	0.108	0.916
$\frac{M_2}{Np}$	0.010	0.010	0.010	0.030	$\frac{M_2}{Np}$	0.194	0.156	0.126	0.476

TABLE VI

THE LAGS STRUCTURE OF THE INTEREST RATE UNDER TWO LAG MODEL WITH AUTOREGRESSIVE ERRORS

Variable	t	t-1	t-2	$\sum_{i=0}^{2} t-i$	Variable	t	t-1	t-2	$\sum_{i=0}^{2} t-i$
For the United States					For Taiwan				
Annual Data (1956-1969)					Annual Data (1956-1969)				
$\frac{M_1}{Np}$	0.137	0.118	0.102	0.357	$\frac{M_1}{Np}$	0.021	0.021	0.020	0.062
$\frac{M_2}{Np}$	0.445	0.247	0.137	0.829	$\frac{M_2}{Np}$	0.218	0.170	0.133	0.521
Quarterly Data (1956-1969)					Quarterly Data (1956-1969)				
$\frac{M_1}{Np}$	0.257	0.191	0.142	0.590	$\frac{M_1}{Np}$	0.650	0.228	0.080	0.958
$\frac{M_2}{Np}$	0.747	0.189	0.048	0.984	$\frac{M_2}{Np}$	0.202	0.161	0.129	0.492

factors determining the per capita demand for real money are shown to be per capita real income and the rate of interest. An increase in per capita real income tends to increase the per capita demand for real money while a higher interest rate tends to depress the per capita demand for real money.

Given the broad definition of money, the best results are obtained by regressing per capita demand for real money on per capita real income and the short-term interest rate. The coefficients for the per capita real income and the interest rate variables are statistically significant and have expected signs. The per capita real income and the interest rate explain 97 to 99 percent of the variance of the change in per capita demand for real money.

Finally, the empirical estimation of this model indicates that the per capita real income and the short-term interest rate are dominant factors in the determination of per capita demand for real money, and the statistical estimation of the demand for money relationship leads to the conclusion that per capita demand for real money is influenced mainly by per capita real income and the rate of interest, i.e., the short-term interest rate in the United States and the discount rate in Taiwan.

The following paragraphs briefly explore the versatile nature of the basic model and examine the results of estimation from a variety of assumptions regarding the values of lag and autocorrelation coefficients.

Two Lag Model with Independent Errors

If it is assumed that the per capita demand for real money depends upon per capita real income and the rate of interest, and that there are lag coefficients for the explanatory variables and zero autocorrelation coefficient, i.e., $\lambda \neq 0$, $\mu \neq 0$, and $\rho = 0$, then equation (1) reduces to the simple specification of a distributed lag model with two parameters:

$$\begin{aligned}(\frac{M}{Np})_t = {} & \gamma_o(1-\lambda)(1-\mu) + \alpha_o(\frac{Y}{Np})_t - \mu\alpha_o(\frac{Y}{Np})_{t-1} + \beta_o r_t \\ & - \lambda\beta_o r_{t-1} + (\lambda+\mu)(\frac{M}{Np})_{t-1} - \lambda\mu(\frac{M}{Np})_{t-2} + \epsilon_t . \qquad (4)\end{aligned}$$

All of the notation and the statistical conditions are defined as before. This specification implies that the per capita demand for real money depends not only on current values of per capita real income and the rate of interest, but also on the lagged values of per capita real income, the rate of interest and the previous per capita demand for real money.[6] Since the influence of changes in per capita real income and the rate of interest are spread over several periods of time, the distributed lag effects of variables $\frac{Y}{Np}$ and r on the dependent variable $\frac{M}{Np}$ are described by the various products of these parameters. As indicated above, equation (4) is non-linear in the parameters. The modified Gauss-Newton method of iterative estimation is therefore used.

The statistical results for the Two Lag Model with Independent Errors are presented in Table VII for the United States and in Table VIII for Taiwan. All of the notation and the statistical tests are defined as those in the previous section.

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[6] The model used in this section has been employed by Humberger (1966), Nerlove (1958) and Friedman (1957).

TABLE VII

PARAMETERS OF DEMAND FOR MONEY IN THE UNITED STATES UNDER TWO LAG MODEL WITH INDEPENDENT ERRORS

Dependent Variable	Constant	Estimates of the Parameters		Estimates of Lag and Autocorrelation Coefficients			$\bar{R}^2$	Durbin-Watson Statistic DW
	γ_o	α_o	β_o	λ	μ	ρ		
Annual Data (1956-1969)								
$\frac{M_1}{Np}$	1,368.20	12.790**	-1.628**	0.290	0.578**		0.799	1.78
		(6.916)	(0.836)	(0.317)	(0.321)			
$\frac{M_2}{Np}$	-191.83	7.149*	-0.564*	0.029	0.292		0.976	2.12
		(1.421)	(0.206)	(0.248)	(0.334)			
Quarterly Data (1956-1969)								
$\frac{M_1}{Np}$	924.96	6.760*	-0.128	0.158	0.823*		0.948	2.20
		(2.036)	(0.118)	(0.157)	(0.064)			
$\frac{M_2}{Np}$	-0.918	1.800*	-0.096*	0.693*	0.697*		0.996	2.31
		(0.523)	(0.041)	(0.089)	(0.111)			

TABLE VIII

PARAMETERS OF DEMAND FOR MONEY IN TAIWAN UNDER TWO LAG MODEL WITH INDEPENDENT ERRORS

Dependent Variable	Constant	Estimates of the Parameters		Estimates of Lag and Autocorrelation Coefficients			$\bar{R}^2$	Durbin-Watson Statistic
	γ_o	α_o	β_o	λ	μ	ρ		DW
Annual Data (1956-1969)								
$\frac{M_1}{Np}$	7,663.84	13.277** (6.180)	-31.375* (7.682)	-0.437** (0.240)	0.952* (0.064)		0.987	2.39
$\frac{M_2}{Np}$		15.178 (13.832)	-15.081* (18.501)	-0.508 (0.307)	1##		0.991	2.52
Quarterly Data (1956-1969)								
$\frac{M_1}{Np}$	1,445.71	6.349 (5.000)	-5.840** (3.044)	-0.357* (0.132)	0.999* (0.024)		0.988	1.86
$\frac{M_2}{Np}$		18.612* (7.695)	-10.208** (5.070)	-0.384* (0.125)	1##		0.997	1.89

The Durbin-Watson statistics for both the United States and Taiwan indicate no autocorrelation. The coefficient of autocorrelation is assumed to be zero under the model considered. Somewhat surprisingly perhaps, the Durbin-Watson statistic indicates the pluasibility of the assumption in that the statistic does not allow acceptance of the hypothesis that autocorrelation exists. This means that the results of independent errors are obtained without using the technique of Aitken's generalized least squares method.

The empirical results for the United States during the period from 1956 to 1969, either by annual or quarterly data, show that the signs of the coefficients for per capita real income and the interest rate variables are as expected on a priori grounds. The per capita real income and the interest rate variables play a prominent role in explaining the behavior of per capita demand for real money when either annual or quarterly data for broadly defined money are employed in the regression analysis. On the other hand, when quarterly data for narrowly defined money are used, the rate of interest fails to be an important determinant in the demand for money even though the sign of the coefficient is negative. This implies that the rate of interest in this case is not an important factor in the estimation of the per capita demand for real money in the United States. The coefficients of per capita real income variable have correct sign and are significantly different from zero at the one percent level. The specification of the Two Lag Model with Independent Errors for the United States explains 80 to 99 percent of the variance in the level of the per capita demand for real money.

The empirical findings for Taiwan are presented in Table VIII. The statistical results give more satisfactory evidence for the interest rate variable than for the per capita real income variable. The per capita demand for real money is negatively related to the rate of interest under the various types of definition of the dependent variable. All of the estimated parameters are significantly different from zero at either the one or the five percent level. The sign of the coefficients for the per capita real income variable is also the correct one. Nonetheless, the per capita real income variable shows its significant impact on the per capita demand for real money at the one percent level only when the quarterly data of broadly defined money are used in the regression. If annual data are employed, the narrowly defined money would give better results in explaining the behavior of per capita demand for real money. The specification of the Two Lag Model with Independent Errors for Taiwan explains 99 percent of the variance of the level of the per capita demand for real money.

It is concluded that the Two Lag Model with Independent Errors solves the problem of autocorrelated errors in the disturbance terms for both countries. The model also gives results similar to the more general model of the previous section. However, the per capita real income variable is an important factor in the United States while the interest rate variable gives more consistent results in Taiwan.

Static Model with Autoregressive Errors

The problem of autocorrelation in disturbance terms arises most frequently in the estimation of demand functions from time series data. In order to improve the consistency and the efficiency of the estimators, as compared with the estimators obtained from the application of the ordinary least squares regression, the information of the autoregressive structure of the disturbances has been incorporated into the demand function for money.[7]

If it is assumed that the per capita demand for real money depends upon per capita real income and the rate of interest, and that the lag coefficients associated with the explanatory variables are zero, i.e., $\lambda = \mu = 0$, then given equation (1), the basic model can be written as

$$\left(\frac{M}{Np}\right)_t = \gamma_o(1-\rho) + \alpha_o\left(\frac{Y}{Np}\right)_t - \rho\alpha_o\left(\frac{Y}{Np}\right)_{t-1} + \beta_o r_t - \rho\beta_o r_{t-1} + \rho\left(\frac{M}{Np}\right)_{t-1} + \epsilon_t \tag{5}$$

which is equivalent to

$$\left(\frac{M}{Np}\right)_t = \gamma_o + \alpha_o\left(\frac{Y}{Np}\right)_t + \beta_o r_t + \rho\left\{\left(\frac{M}{Np}\right)_{t-1} - \left[\gamma_o + \alpha_o\left(\frac{Y}{Np}\right)_{t-1} + \beta_o r_{t-1}\right]\right\} + \epsilon_t . \tag{6}$$

All of the definition of the terms and the statistical conditions are defined as before.

The incorporation of an autocorrelation coefficient into the demand function for money has two important properties.[8] First, the parameters are estimated by utilizing the information of the autoregressive structure. Second, a term,

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[7] The model used in this section has be suggested by Johnston (1963) and it has been used previously by Klein (1958) and Cochrane and Orcutt (1949).

[8] J. Johnston, Econometric Methods. New York: McGraw-Hill Book Company, Inc., 1963, p. 196.

$$\rho\{(\frac{M}{Np})_{t-1} - [\gamma_o + \alpha_o(\frac{Y}{Np})_{t-1} + \beta_o r_{t-1}]\}$$

which is the product of ρ and the estimated disturbance in the previous period, is added to the Static Model with Independent Errors. As Johnston indicates, the obtained estimators are likely to be unbiased because the approach takes account of recent disturbances, and the estimators will be asymptotically efficient.[9] In addition, since the value of ρ is usually unknown, the initial value of ρ is applied and an iterative procedure of the modified Gauss-Newton method is employed in order to assure that the use of the estimated ρ minimize the sum of the squares of residuals.

The results of the estimation are presented in Table IX for the United States and in Table X for Taiwan. All of the notation and the statistical tests are defined as those in the second section of this chapter.

In the case of the United States, the sign of the coefficients for the rate of interest is not consistently negative. All of the regressions of quarterly data show that the estimated parameters for the rate of interest are not significantly different from zero. This implies that the rate of interest as an explanatory variable is not an important determinant in the theory of demand for money if quarterly data of the time series are used. The per capita demand for real money is positively related to per capita real income and the coefficients of the income variable are also significantly different from zero at the one percent level. The Durbin-Watson statistic shows that, except in the

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[9] Ibid., pp. 196-197.

TABLE IX

PARAMETERS OF DEMAND FOR MONEY IN THE UNITED STATES UNDER STATIC MODEL WITH AUTOREGRESSIVE ERRORS

Dependent Variable	Constant	Estimates of the Parameters		Estimates of Lag and Autocorrelation Coefficients			$\bar{R}^2$	Durbin-Watson Statistic
	γ_o	α_o	β_o	λ	μ	ρ		DW
Annual Data (1956-1969)								
$\frac{M_1}{Np}$	1,937.50	11.539^{*} (3.938)	-1.038^{**} (0.496)			0.635^{**} (0.246)	0.818	1.97
$\frac{M_2}{Np}$	-101.73	6.188^{*} (0.461)	-0.377^{**} (0.160)			0.020 (0.377)	0.972	1.68
Quarterly Data (1956-1969)								
$\frac{M_1}{Np}$	1,073.71	6.270^{*} (1.240)	0.084 (0.182)			0.831^{*} (0.047)	0.947	1.78
$\frac{M_2}{Np}$		1.626^{*} (0.590)	-0.053 (0.047)			$1^{\#\#}$	0.993	1.18

TABLE X

PARAMETERS OF DEMAND FOR MONEY IN TAIWAN UNDER STATIC MODEL WITH AUTOREGRESSIVE ERRORS

Dependent Variable	Constant γ_o	Estimates of the Parameters α_o	Estimates of the Parameters β_o	Estimates of Lag and Autocorrelation Coefficients λ	Estimates of Lag and Autocorrelation Coefficients μ	Estimates of Lag and Autocorrelation Coefficients ρ	$\overline{R}^2$	Durbin-Watson Statistic DW
Annual Data (1956-1969)								
$\frac{M_1}{Np}$	-904.92	24.598* (1.618)	-16.453 (9.811)			0.433 (0.323)	0.982	2.12
$\frac{M_2}{Np}$	-13,331.85	68.872* (2.800)	-30.498 (19.931)			-0.161 (0.332)	0.989	2.10
Quarterly Data (1956-1969)								
$\frac{M_1}{Np}$	-1,232.91	24.262* (1.042)	-8.092 (5.323)			0.580* (0.121)	0.987	2.16
$\frac{M_2}{Np}$		20.037* (8.070)	-14.214** (6.988)			$1^{\#\#}$	0.996	2.83

case of quarterly data for broadly defined money, the Static Model with Autoregressive Errors solves the problem of autocorrelation. The specification of the model for the United States explains 82 to 99 percent of the variance in the level of per capita demand for real money.

The statistical evidence for Taiwan during the period between 1956 and 1969, either for annual or quarterly data, seem to confirm the common view positing a positive relationship between the per capita demand for real cash balances and per capita real income and a negative relationship between the per capita demand for real cash balances and the interest rate variable. The per capita demand for real money is positively related to per capita real income for both definitions of money. Although per capita real income plays an important role in explaining the behavior in the monetary sector at the one percent significance level, narrowly defined money always results in estimators with a smaller standard error of estimates. The sign of the coefficients for the rate of interest is the correct one. Nonetheless, only quarterly data of broadly defined money in per capita real terms show that the coefficients of the interest rate variable are significantly different from zero at the five percent level. The Durbin-Watson statistic suggests that the direct application of the iterative procedure of the modified Gauss-Newton method to the transformed variable $(\frac{M}{Np})_t - \rho(\frac{M}{Np})_{t-1}$, $(\frac{Y}{Np})_t - \rho(\frac{Y}{Np})_{t-1}$ and $r_t - \rho r_{t-1}$ solves the problem of autocorrelated disturbances in time series data. The specification of the model for Taiwan explains 99 percent of the variance in the level of the per capita demand for real money.

In comparing the statistical results between the United States and Taiwan, all of the demand functions for money in Taiwan reveal that the role of per capita real income is as expected and the signs of all of the coefficients of per capita real income and the interest rate variables are the correct ones. The coefficients of the per capita real income variable are significantly different from zero at the one percent level. The Durbin-Watson statistics also reveal that there is no positive autocorrelation in the data of the transformed variables. The empirical results in the United States suggest that the explanatory variables affect the behavior of per capita demand for real money in the expected direction. However, on the whole, the Static Model with Autoregressive Errors performs better in the estimation of the demand function for money in Taiwan than in the United States.

First Difference Model with Independent Errors

In many cases, due to the absence of any information about the value of the autocorrelation coefficient ρ, the following alternative approaches are suggested.[10] These are: (1) estimate the parameters from the transformed variables using estimated ρ;[11] and (2) use the

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[10] P. Rao and R. L. Miller, Applied Econometrics. California: Wadsworth Publishing Company, Inc., 1971, pp. 74-75.

[11] To estimate the parameter of the autoregressive process, consider $w_h = \eta_h - f(\delta_{ih}; \xi_{jh})$, $h = 1, 2, \ldots, t$; $i = 1, 2, \ldots, \ell$; and $j = 1, 2, \ldots, n$, where w, η, δ and ξ are defined in Chapter II. The estimated parameter, $\hat{\rho}$, of the autoregressive process can be consistently estimated on the basis of a regression on these residuals, and the regression equation $\hat{w}_h = \rho \hat{w}_{h-1} + \epsilon_h$ gives

$$\hat{\rho} = \frac{\sum_{h=1}^{t} w_h w_{h-1}}{\sum_{h=1}^{t} w_h^2} .$$

The single equation estimation procedure used to obtain $\hat{\rho}$ could be criticized for being inefficient, but the estimated ρ is consistent.

first difference technique. If the explanatory variables have high serial correlation which is generally the case with economic time series data, the gain in the precision of estimation might become substantial by using the first difference technique to estimate the coefficients of the demand function for money. To deal with the problem of autocorrelated disturbances, the application of the least squares regression of the first difference of the series provides two advantages.[12] First, by reducing the correlation among the explanatory variables, it helps to identify their independent effect. Second, the bias resulting from the omission of the slowly moving variable from a regression equation is reduced.[13]

If it is assumed that the per capita demand for real money depends upon per capita real income and the rate of interest, and that the values of lag coefficients are zero and the coefficient of autocorrelation is one, i. e., $\lambda = \mu = 0$ and $\rho = 1$, then given equation (1) the basic model can be written as

$$\left(\frac{M}{Np}\right)_t - \left(\frac{M}{Np}\right)_{t-1} = \alpha_o\left[\left(\frac{Y}{Np}\right)_t - \left(\frac{Y}{Np}\right)_{t-1}\right] + \beta_o(r_t - r_{t-1}) + \epsilon_t \tag{7}$$

or

$$\left(\frac{M}{Np}\right)_t = \alpha_o\left(\frac{Y}{Np}\right)_t - \alpha_o\left(\frac{Y}{Np}\right)_{t-1} + \beta_o r_t - \beta_o r_{t-1} + \left(\frac{M}{Np}\right)_{t-1} + \epsilon_t . \tag{8}$$

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[12] M. J. Hamburger, "The Demand for Money by Households, Money Substitutes, and Monetary Policy," The Journal of Political Economy, 74 (December, 1966), p. 604.

[13] Ibid., pp. 604-605.

The economic interpretation of the equation (7) or (8) is that in each period people revise their notions as to the per capita demand for real money in proportion to the difference between the measured or observed value of the explanatory variable and its previous value.[14]

The statistical results of the First Difference Model with Independent Errors are presented in Table XI for the United States and in Table XII for Taiwan. The notation, #, implies that the value of autocorrelation coefficient is fixed at unity before the regression. Since the constant term in equation (1) is $\gamma_o(1 - \lambda)(1 - \mu)(1 - \rho)$, the value of γ_o is not shown in the table whenever the value of ρ is fixed a priori at unity. All of the other notation and the statistical conditions are defined as before.

In the case of the demand function for money in the United States, the Durbin-Watson statistic indicates that the First Difference Model with Independent Errors works satisfactorily except for the case of quarterly data with broadly defined money. The coefficients of per capita real income have the appropriate sign and are significantly different from zero at the one percent level. The regression provides satisfactory results when annual data are used. Most of the available evidence suggests that the sign of the coefficients for the rate of interest is as expected. The quarterly data for broadly defined money give the wrong sign. If quarterly data are used, the coefficients of the interest rate variable show no significant difference from zero.

In the case of the demand function for money in Taiwan, the signs of all of the coefficients are theoretically correct; however, only

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[14] The model used in this section has been used by Chow (1966), Laidler (1966) and Humberger (1966).

TABLE XI

PARAMETERS OF DEMAND FOR MONEY IN THE UNITED STATES UNDER FIRST DIFFERENCE MODEL WITH INDEPENDENT ERRORS

Dependent Variable	Constant	Estimates of the Parameters		Estimates of Lag and Autocorrelation Coefficients			$\bar{R}^2$	Durbin-Watson Statistic
	γ_o	α_o	β_o	λ	μ	ρ		DW
Annual Data (1956-1969)								
$\frac{M_1}{Np}$		22.444^* (5.239)	-0.949^{**} (0.417)			$1^{\#}$	0.803	1.94
$\frac{M_2}{Np}$		6.143^* (1.870)	-0.357^{**} (0.149)			$1^{\#}$	0.957	1.95
Quarterly Data (1956-1969)								
$\frac{M_1}{Np}$		6.286^* (2.354)	0.046 (0.187)			$1^{\#}$	0.933	1.62
$\frac{M_2}{Np}$		1.639^* (0.572)	-0.053 (0.046)			$1^{\#}$	0.994	1.18

TABLE XII

PARAMETERS OF DEMAND FOR MONEY IN TAIWAN UNDER FIRST DIFFERENCE MODEL WITH INDEPENDENT ERRORS

Dependent Variable	Constant	Estimates of the Parameters		Estimates of Lag and Autocorrelation Coefficients			$\bar{R}^2$	Durbin-Watson Statistic
	γ_o	α_o	β_o	λ	μ	ρ		DW
Annual Data (1956-1969)								
$\frac{M_1}{Np}$		22.882* (5.635)	-18.786** (7.970)			1#	0.977	2.50
$\frac{M_2}{Np}$		42.384* (13.111)	-31.567 (18.544)			1#	0.984	2.07
Quarterly Data (1956-1969)								
$\frac{M_1}{Np}$		10.373** (4.929)	-8.138 (4.574)			1#	0.986	2.67
$\frac{M_2}{Np}$		28.500* (8.016)	-12.597 (7.439)			1#	0.996	2.44

the coefficients of the per capita real income variable are significantly different from zero at least at the five percent level. In other words, when annual data between 1956 and 1969 are used, the coefficients for the per capita real income variable are significantly different from zero at the one percent level. But if quarterly data during the same period are employed, the broadly defined money gives results with smaller standard errors of estimates. The Durbin-Watson statistic indicates no autocorrelation. This implies that the first difference technique solves the problem of autocorrelation when the transaction and the speculative motives for holding cash balances are regarded as dominant factors in the estimation of per capita demand for real money.

The evidence shows that per capita real income, as an explanatory variable, plays a significant role in the estimation of the demand for money for both countries. If quarterly data are used, the coefficients for the rate of interest are not significantly different from zero. So far as the Durbin-Watson statistics are concerned, the regression in general gives satisfactory results for both the United States and Taiwan. This implies that the use of the first difference technique solves the problems of autocorrelated errors and yields the minimum variance and consistent estimates of the coefficients. The signs of the coefficients for per capita real income and the rate of interest are more consistent in their directions in Taiwan than in the United States. For either annual or quarterly data during the period between 1956 and 1969, a distinct negative relationship between the per capita demand for real money and the rate of interest can be observed in Taiwan. However, the statistical evidence for the United States supports Friedman's conclusion that the rate of interest fails to be an important determinant of the demand for money when quarterly data are used for the analysis.[15] In the case of quarterly data with the narrow definition of money, the sign of the coefficients for the rate of interest in the United States is not as predicted, i.e., it shows a positive relationship between the rate of interest and the per capita demand for real money.

Static Model with Independent Errors

The model is a simple textbook version of the demand function for money in which the per capita demand for real money depends upon per capita real income and the rate of interest.[16] Assume that the values of lag and autocorrelation coefficients are zero, i.e., $\lambda = \mu = \rho = 0$. Then, given equation (1), the basic model can be expressed by the following equation:

$$\left(\frac{M}{Np}\right)_t = \gamma_o + \alpha_o\left(\frac{Y}{Np}\right)_t + \beta_o r_t + \epsilon_t \tag{9}$$

The simple version of the per capita demand function for real money is applied to the quarterly data of the times series for the United States for the period between 1956 and 1969. The empirical evidence in Table XIII shows that the sign of the coefficients for the per capita real income variable is consistent with what monetary theory predicts. However, the direction of the sign of the coefficients for the rate of interest is uncertain. The estimated coefficients for the rate of

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[15] M. Friedman, "The Demand for Money: Some Theoretical and Empirical Results," The Journal of Political Economy, 67 (August, 1959), pp. 327-351.

[16] The simple version of the demand for money used in this section is also employed by Meltzer (1963).

TABLE XIII

PARAMETERS OF DEMAND FOR MONEY IN THE UNITED STATES UNDER STATIC MODEL WITH INDEPENDENT ERRORS

Dependent Variable	Constant	Estimates of the Parameters		Estimates of Lag and Autocorrelation Coefficients			$\bar{R}^2$	Durbin-Watson Statistic
	γ_o	α_o	β_o	λ	μ	ρ		DW
Annual Data (1956-1969)								
$\frac{M_1}{Np}$	6,749.82	4.833^{**} (2.398)	0.232 (0.827)				0.625	1.21
$\frac{M_2}{Np}$	-102.98	6.182^{*} (0.418)	-0.374^{**} (0.144)				0.982	1.81
Quarterly Data (1956-1969)								
$\frac{M_1}{Np}$	7,123.52	2.491^{**} (1.197)	0.911^{*} (0.366)				0.580	0.22
$\frac{M_2}{Np}$	-78.59	5.896^{*} (0.201)	-0.212^{*} (0.061)				0.980	0.67

interest are statistically significant with the expected sign only when the narrowly defined money is used in the demand function for money. To a large extent, the Durbin-Watson statistic fails to meet the crucial requirement that the disturbance terms be randomly distributed except in the case where annual data for broadly defined money are used as the dependent variable. This means that there exists a needlessly large sample variance in the estimated coefficient. In other words, when annual data instead of quarterly data from the time series during the period between 1956 and 1969 are used, most of the Durbin-Watson statistics, except narrowly defined money in per capita real terms, show lack of any autocorrelation in the disturbance terms. The coefficients of the rate of interest are statistically significant from zero at the five percent level.

The empirical results of the per capita demand for real money in Taiwan during the same period are presented in Table XIV. By regressing per capita demand for real money on per capita real income and the rate of interest, either based on annual or quarterly data, all of the coefficients of per capita real income and the interest rate variables have the expected signs. This means that an increase in per capita real income will lead to a higher level of per capita demand for real money while an increase in the rate of interest will lower the level of per capita demand for real money. However, only the coefficients of per capita real income variable are significantly different from zero at the one percent level, while that for the interest rate variable are statistically significant only in the quarterly data with the broad definition of money. In other words, if the annual or the quarterly data for the narrow definition of money are used, the

TABLE XIV

PARAMETERS OF DEMAND FOR MONEY IN TAIWAN UNDER STATIC MODEL WITH INDEPENDENT ERRORS

Dependent Variable	Constant	Estimates of the Parameters		Estimates of Lag and Autocorrelation Coefficients			$\bar{R}^2$	Durbin-Watson Statistic
	γ_o	α_o	β_o	λ	μ	ρ		DW
Annual Data (1956-1969)								
$\frac{M_1}{Np}$	-3,206.78	25.019* (1.386)	-6.873 (0.272)				0.986	1.42
$\frac{M_2}{Np}$	-11,564.90	68.771* (2.725)	-29.533 (18.236)				0.993	2.28
Quarterly Data (1956-1969)								
$\frac{M_1}{Np}$	-3,961.81	24.793* (0.718)	-3.070 (4.259)				0.983	0.89
$\frac{M_2}{Np}$	-10,706.89	66.568* (1.453)	-31.223* (8.616)				0.991	0.76

rate of interest fails to show statistical significance at the 95 percent of confidence level, i.e., no significant relationship between the per capita demand for real money and the rate of interest is found. By using quarterly data in the regression analysis, the Durbin-Watson statistic indicates that the residuals of the regression equations are highly autocorrelated. However, the Durbin-Watson statistic has been improved when the quarterly data for the broad definition of money are employed. All of the estimated regression coefficients for per capita real income have the positive sign and have a high statistical significance in most cases, and the estimated coefficients for the rate of interest have the negative sign in all cases but generally are of low statistical significance. Thus, the statistical results show no significant impact of the rate of interest on the per capita demand for real money during the period between 1956 and 1969.

The conclusions from the preceding argument can be summarized as follows: when a simple version of the model is fitted to the quarterly or the annual data for either the United States or Taiwan, the regression with a larger number of observations provides evidence of autocorrelation. The rate of interest in general fails to be statistically significant. Empirical results also show that the signs of the structural coefficients are nearly what might be expected on a priori grounds. The better estimation of the structure is found by using a broader definition of money. Furthermore, the sign and significant level of each estimated coefficient are similar in both countries. This suggests that the theoretical and empirical work developed in the United States can be applied to the monetary phenomenon in Taiwan.

Summary

The empirical results, which are generated by means of the modified Gauss-Newton method, for various models of the demand for money are presented in this chapter. The basic model is one in which the per capita demand for real money depends not only on the current values of per capita real income and the rate of interest, but also on the values of these explanatory variables in the previous periods. The actual functional relationships are represented by the concept of an exponentially distributed lag.

The main conclusions that may be drawn from these statistical results are as follows: the evidence from the basic model of this study, i.e., the Two Lag Model with Autoregressive Errors, strongly support the hypothesis that the per capita demand for real money in Taiwan depends not only upon the current values of per capita real income and the rate of interest, but also on the values of per capita real income and the rate of interest of the previous periods. The sign of the coefficients for the per capita real income variable is the appropriate one, as expected on a priori grounds for all types of regression models. The estimated coefficients of the per capita real income variable are also significantly different from zero at least at the five percent level.

As the measures of the influences of the rate of interest on the per capita demand for real money in the United States, two variables were tested: the short-term interest rate which is the market yield on 90-day United States government securities and the long-term interest rate which is the United States long-term bond yield. The statistical evidence indicated that the influence of the short-term interest rate is more certain than that of the long-term interest rate. Generally speaking, the sign of the coefficients of the interest rate variable is not always as predicted and most of the estimated coefficients are too small to have significant influence on the behavior of the demand for money. However, the role of the interest rate in the demand function for money is statistically significant in both countries when the broad definition of money is used as the dependent variable.

Finally, the incorporation of an autocorrelation coefficient into the regression, from the Durbin-Watson statistical criteria viewpoint, indeed reduces serial correlation in the disturbance terms.

CHAPTER IV

SUMMARY AND CONCLUSIONS

Many of the statistical methods used to estimate the coefficients of the demand function for money in the current literature of monetary economics are econometrically inappropriate. For this reason, the modified Gauss-Newton method of iterative estimation is used. It has the following desirable properties: (1) the computational procedure is convergent for a finite sample size, and (2) the resulting estimators are consistent and asymptotically efficient as the sample size approaches infinity.[1]

Five functional forms are estimated for the demand for money in the United States and Taiwan. When data from the time series are used in the Static Model with Independent Errors, the most commonly used empirical demand for money model, the statistical results fail to meet the crucial requirement of the Durbin-Watson test. This implies that the hypothesis of random disturbance in the error terms must be rejected. The incorporation of an autocorrelation coefficient into the regression model improves the estimates because higher values of the Durbin-Watson statistic are obtained. However, the estimated coefficients from the application of the generalized least squares method are

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[1] H. O. Hartley and A. Booker, "Nonlinear Least Squares Estimation," Annual of Mathematical Statistics, 36 (April, 1965), pp. 638-650.

more satisfactory than the ones from the use of the first difference technique.

The formulation of the model in this study is derived from an extension of the distributed lag model suggested by Koyck.[2] The per capita demand for real money is considered as a function of per capita real income and the rate of interest. In other words, the current value of the per capita demand for real money is related to the values of current per capita real income and the interest rate as well as the values of both explanatory variables at several different times in the past. This distributed lag model also implies that recent values of these variables are weighted more heavily than those of the remote past. The effects of per capita real income and the interest rate changes diminish through time at constant geometric rates.

Some selective features of statistical evidence from the static and dynamic models are as follows:

1. By implementing the modified Gauss-Newton method of nonlinear regression, the empirical results, on the whole, can be considered as very satisfactory. The standard errors of estimates for most of the coefficients are reasonably small as compared with the values of their estimated coefficients. This implies that the estimation method employed in this study provides estimated coefficients with the asymptotically efficient property.

2. The empirical evidence suggests that the significance of explanatory variables in the demand function for money depends not

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[2] L. M. Koyck, Distributed Lags and Investment Analysis. Amsterdam: North Holland Publishing Company, 1954.

only on the functional form of the model, but also on the nature of the variable used.

3. The sign of the coefficients for per capita real income variable, in general, is consistent with what the monetary theory predicts. Most of the estimated coefficients of per capita real income variable are significantly different from zero at the one or the five percent level. This implies that the per capita real income variable occupies a prominent role in the estimation of demand function for money.

4. The interst rate variable does not always provide a correct sign. The estimated "t" values for the coefficients of the variable are often less than the critical value. The best performance of the rate of interest in the estimation of the per capita demand for real money in the United States is obtained by using market yield of 90-day United States government securities as a proxy variable to measure the response of the demand for money. In addition, the broad definition of money often produces the estimated parameters which are statistically significant at least at five percent level.

5. Given the broad definition of money, the elasticities of per capita demand for real money with respect to per capita real income in the United States are found to be close to unity. On the other hand, most of the interest rate elasticities are less than ten percent. The per capita demand for real money in Taiwan is highly responsive to the changes of per capita real income and the rate of interest. It is found that the elasticities of per capita demand for real money range from 7.6 to 17.2 for the per capita real income variable and from 1.8 to 3.8 for the interest rate variable.

6. The specification of the Two Lag Model with Autoregressive Errors, based on the quarterly data of broadly defined money, explains 99 percent of the variance in the level of the per capita demand for real money in the United States as well as in Taiwan.

Finally, the statistical evidence for the United States and Taiwan is quite similar in the significance of the estimated coefficients as well as in the direction of the signs. It is therefore concluded that the theoretical relationship which has been developed to explain the monetary experience in the United States are also applicable to the economic structure in Taiwan.

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