### **Damped Harmonic Oscillator**

$$T = 2$$

Period of the oscillator if the friction is zero

$$\gamma := 0.35$$

Measure of the friction

$$\omega := \frac{2 \cdot \pi}{T} = 3.142$$

$$\omega_1 := \sqrt{\omega^2 - \gamma^2} = 3.122$$

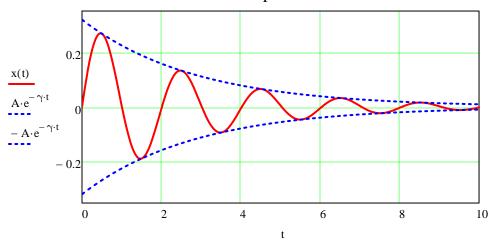
$$A := \frac{1}{\omega_1}$$

$$\varphi := \frac{-\pi}{2}$$

Constants of integration (follow from the initial conditions)

$$x(t) := A \cdot e^{-\gamma \cdot t} \cdot \cos(\omega_1 \cdot t + \phi)$$

# **Underdamped Oscillation**



#### The critically damped case:

$$\gamma_2 \coloneqq \omega$$

$$B := 0$$
  $C := 1$ 

These follow from the initial conditions

$$\mathbf{x}_{\mathbf{c}}(t) \coloneqq (\mathbf{B} + \mathbf{C} \cdot \mathbf{t}) \cdot \mathbf{e}^{-\gamma_2 \cdot t}$$

### The overdamped case:

$$\gamma_2 := 2 \cdot \omega$$

$$\gamma_3 := 2 \cdot \omega$$
  $\qquad \omega_2 := \sqrt{{\gamma_3}^2 - \omega^2} = 5.441$ 

$$A_2 := \frac{1}{2 \cdot \omega_2} = 0.092$$

$$A_1 := -A_2$$

$$\mathbf{x}_{0}(\mathsf{t}) \coloneqq \left(\mathbf{A}_{1} \cdot \mathbf{e}^{-\omega_{2} \cdot \mathsf{t}} + \mathbf{A}_{2} \cdot \mathbf{e}^{\omega_{2} \cdot \mathsf{t}}\right) \cdot \mathbf{e}^{-\gamma_{3} \cdot \mathsf{t}}$$

# Examples starting from equilibrium

