

Reflection of a wave on a string (Problem 15-40)

$$\underbrace{A}_{\text{WW}} := 1 \quad k_1 := \frac{2 \cdot \pi}{0.2} = 31.416 \quad k_2 := \frac{2 \cdot \pi}{0.1} = 62.832 \quad \underbrace{T}_{\text{WW}} := 0.1 \quad \omega := \frac{2 \cdot \pi}{T} = 62.832$$

$$v_1 := \frac{\omega}{k_1} = 2 \quad v_2 := \frac{\omega}{k_2} = 1 \quad \text{Velocity for } x < 0 \text{ and for } x > 0$$

$$A_R := A \cdot \left(\frac{v_2 - v_1}{v_2 + v_1} \right) = -0.333 \quad \text{Reflected amplitude}$$

$$A_T := A \cdot \left(\frac{2 \cdot v_2}{v_1 + v_2} \right) = 0.667 \quad \text{Transmitted amplitude}$$

$$\frac{1}{v_1} \cdot A_R^2 + \frac{1}{v_2} \cdot A_T^2 = 0.5$$

$$\frac{1}{v_1} \cdot A^2 = 0.5$$

This is a check that the power of the reflected plus transmitted waves is equal to the power of the incident wave. Energy is conserved!

$$D(x, t) := \text{if}(x < 0, A \cdot \sin(k_1 \cdot x - \omega \cdot t), 0) \quad \text{Incident wave}$$

$$D_R(x, t) := \text{if}(x < 0, A_R \cdot \sin(-k_1 \cdot x - \omega \cdot t), 0) \quad \text{Reflected wave. (I use a different sign convention here compared with 15-40, so that } A_R \text{ is negative for the reflected wave when inverted by reflection.)}$$

$$D_T(x, t) := A_T \cdot \sin(k_2 \cdot x - \omega \cdot t) \quad \text{Transmitted wave}$$

$$D_{\text{total}}(x, t) := \text{if}(x < 0, D(x, t) + D_R(x, t), D_T(x, t)) \quad \text{Total wave, for plotting}$$

$$t := \text{FRAME} \cdot 0.1 \cdot T \quad \text{Time steps set up for animation of the plot}$$



