Addition of Harmonic Waves (musical instruments)

Specify the harmonic amplitudes in dB for each instrument (See Figure 16-14)

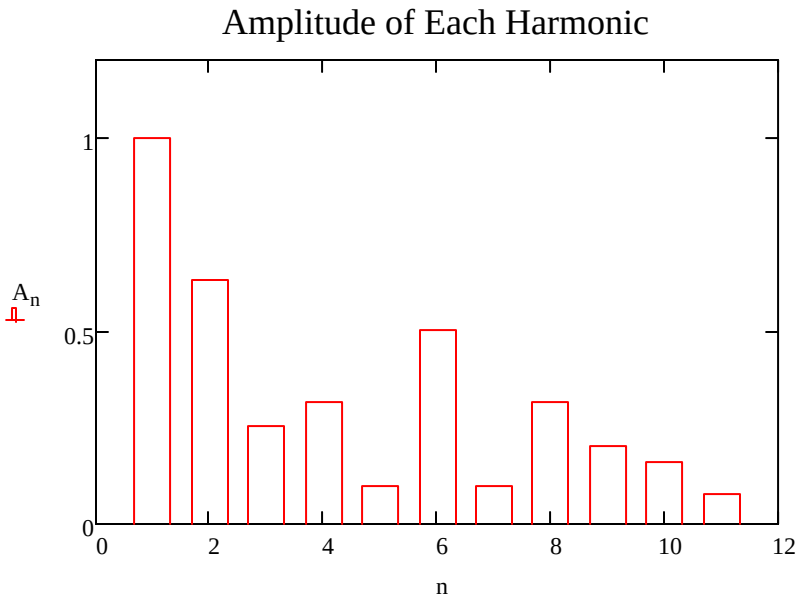
$$\beta_{violin} := \begin{pmatrix} 0 \\ -4 \\ -7 \\ -8 \\ -10 \\ -12 \\ -80 \\ -80 \\ -80 \\ -80 \\ -80 \end{pmatrix} \quad \beta_{piano} := \begin{pmatrix} 0 \\ -2 \\ -6 \\ -5 \\ -10 \\ -3 \\ -10 \\ -5 \\ -7 \\ -8 \\ -11 \end{pmatrix} \quad \beta_{clarinet} := \begin{pmatrix} 0 \\ -80 \\ -3 \\ -80 \\ -10 \\ -80 \\ -6 \\ -4 \\ -3 \\ -3 \\ -6 \end{pmatrix}$$

$$A := 10^{\frac{\beta_{piano}}{10}} =$$

	1
1	1
2	0.631
3	0.251
4	0.316
5	0.1
6	0.501
7	0.1
8	0.316
9	0.2
10	0.158
11	0.079

Amplitude of each harmonic for the chosen instrument

$n := 1, 2 .. 11$ Eleven harmonics total



Plot the displacement versus time

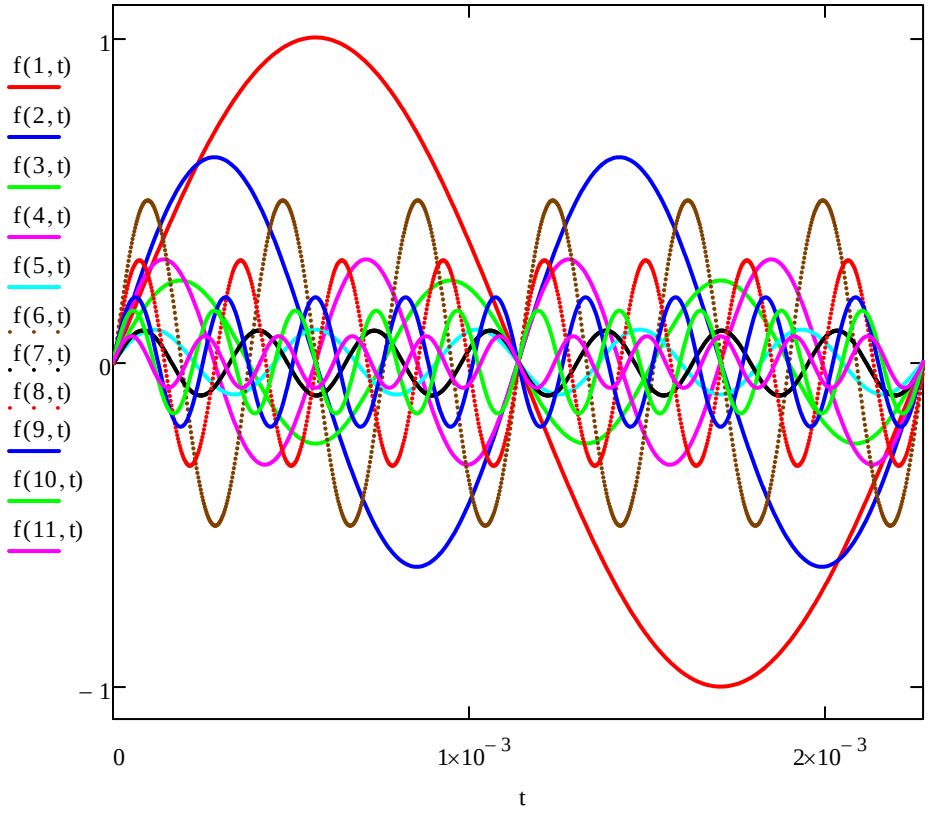
$f_1 := 440$ $\quad T := \frac{1}{f_1} = 2.273 \times 10^{-3}$ Fundamental frequency and period

$\omega_n := 2 \cdot \pi \cdot n \cdot f_1$ Angular frequency of each harmonic

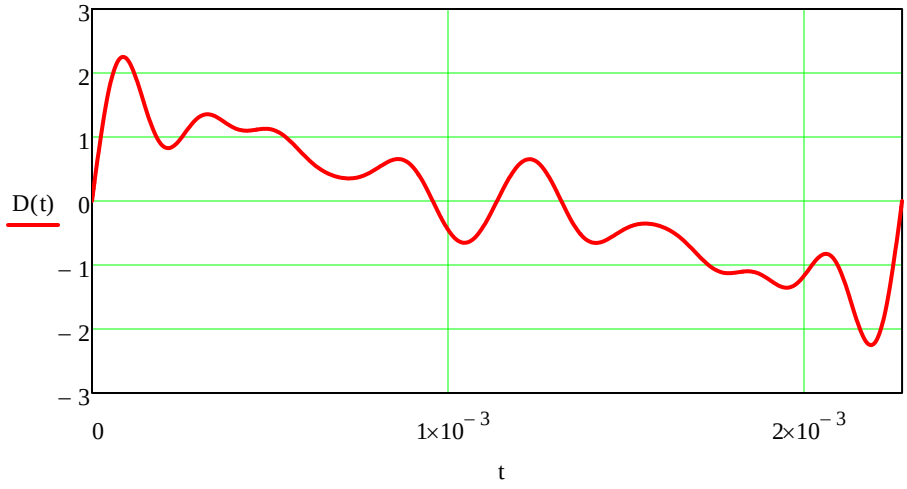
$f(n,t) := A_n \cdot \sin(\omega_n \cdot t)$ The individual sine waves

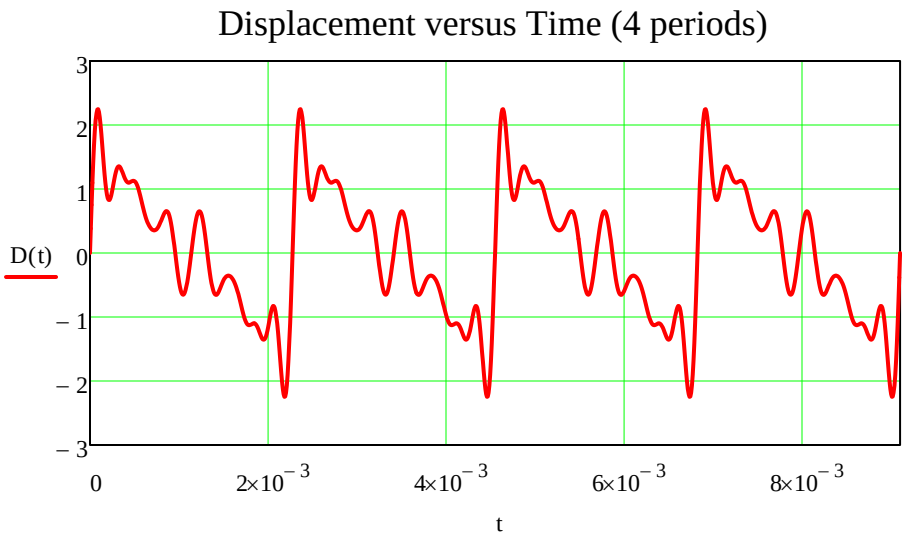
$D(t) := \sum_n f(n,t)$ Sum together the 11 sine waves

Individual Harmonics



Displacement versus Time (one period)




Displacement versus Time (4 periods)
D(t)
3
2
1
0
– 1
– 2
– 3
0
2×10^– 3
4×10^– 3
6×10^– 3
8×10^– 3
t