Physics 5B

Lecture 7, January 25, 2012

Chapter 14,

- Pendulum
- Damped Harmonic Oscillator
- Resonance

First Midterm Exam

- Next Monday, January 30
- Covers the first three homework assignments, through Section 14-5.
  - Note that the 3rd assignment is due Friday evening. Solutions will be posted afterwards.
- A practice exam is posted on eCommons
- An ordinary calculator is allowed, but no notes, computers, tablets, telephones
  - Do not store notes in the calculator memory!
- DRC in Thimann Labs 397

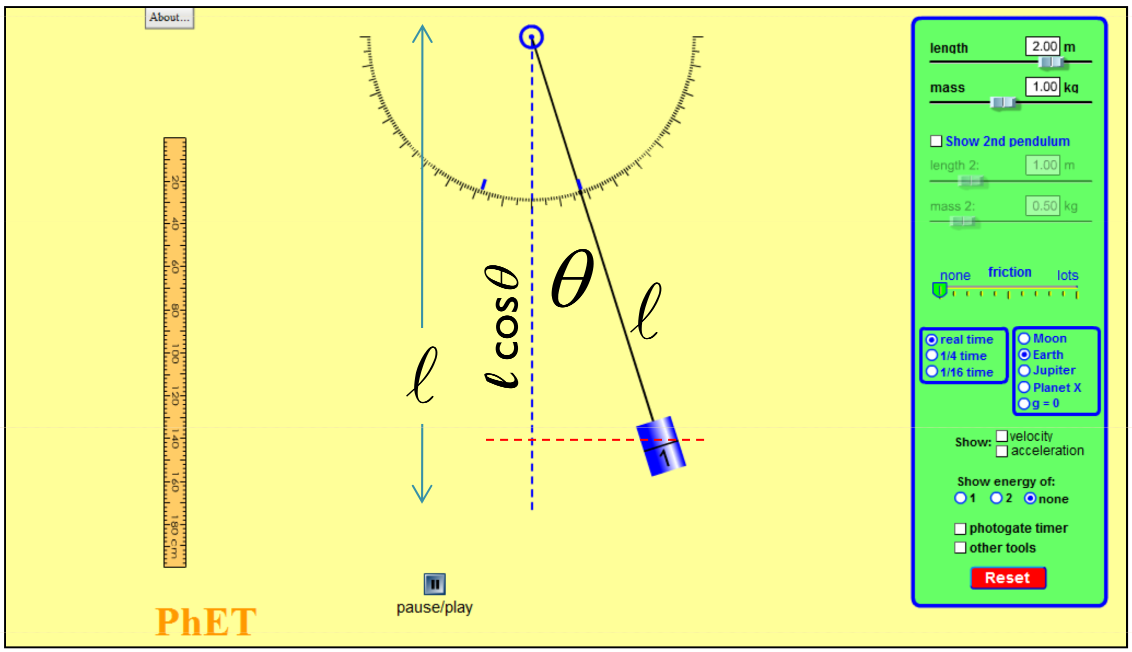
Simple Plane Pendulum (Energy)

$$E = \tfrac{1}{2} m v^2 + m g \ell (1 - \cos\theta)$$

$\cos\theta \approx 1 - \tfrac{1}{2}\theta^2$ for small angles

$$E \approx \tfrac{1}{2} m \ell^2 \dot{\theta}^2 + \tfrac{1}{2} m g \ell \theta^2$$

for small angles looks like a S.H.O.



$$\omega \approx \sqrt{\frac{g}{\ell}}$$

http://phet.colorado.edu/en/simulation/pendulum-lab

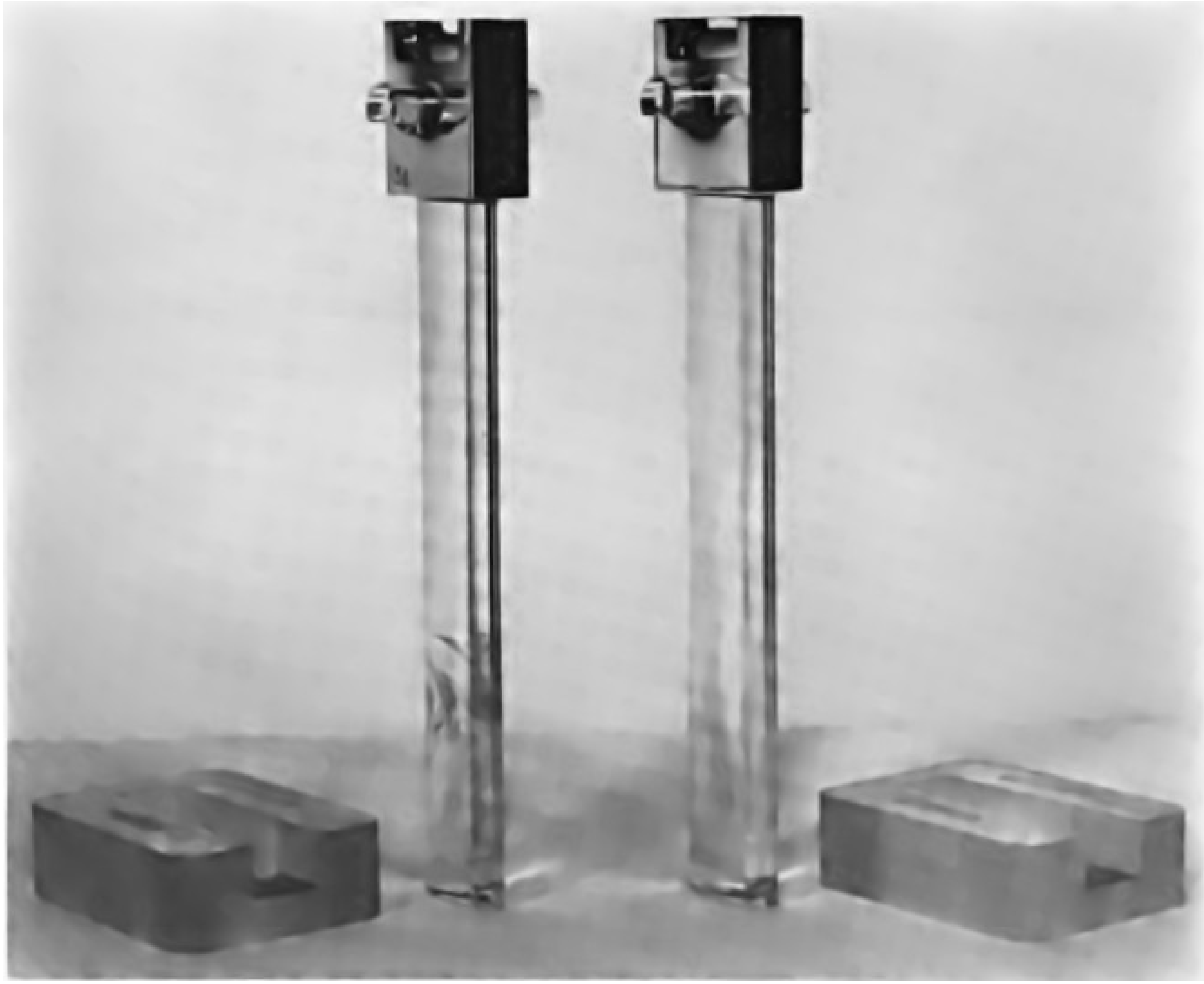
Simple Plane Pendulum

$$E \approx \frac{1}{2} m\left(\ell \dot{\theta}\right)^2 + \frac{1}{2}\frac{mg}{\ell}(\ell\theta)^2 \qquad \text{Pendulum}$$

$$E = \frac{1}{2} mv^2 + \frac{1}{2} kx^2 \qquad \text{Mass on spring}$$

$$\text{For pendulum } k \to \frac{mg}{\ell}, \text{ so } \omega \to \sqrt{\frac{(mg/\ell)}{m}} = \sqrt{\frac{g}{\ell}}$$

But the pendulum is only approximately a simple harmonic oscillator, and the approximation is good only when the amplitude is small (e.g. a few degrees).



Quartz pendulums used in the Gulf gravimeter, 1929. Accurate to better than one part in 10 million.

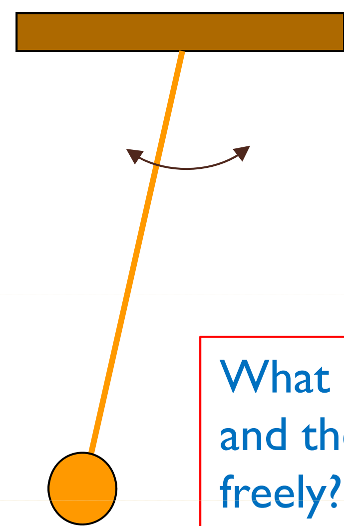
A pendulum is hanging vertically from the ceiling of an elevator. Initially the elevator is at rest and the period of the pendulum is T. Then the elevator accelerates upward. During the acceleration the period of the pendulum becomes

A. greater than T

B. equal to T

C. less than T

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$



What if the cable is cut and the elevator falls freely? Then what will the period be?

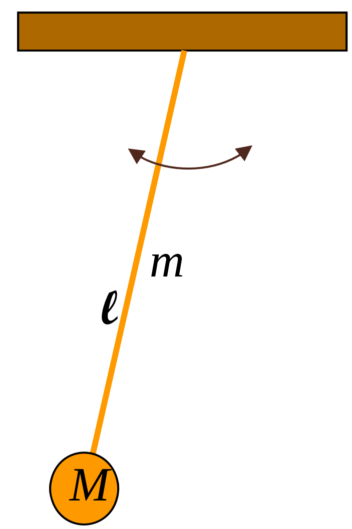
You feel heavier in an elevator accelerating upward. That is due to the floor pushing upward with greater force, but it is equivalent to standing on a planet with a greater gravitational acceleration.

Problem 14-48

A physical pendulum consists of a tiny bob of mass M and a uniform cord of mass m and length ℓ.

a) Find a formula for the frequency ω using the small angle approximation.

b) How does this compare with the formula for a simple pendulum (ignoring the mass of the cord)?



Damped Harmonic Motion

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

Simple ***model*** for the friction

The solution depends on how the damping constant compares with the spring constant and mass:

Let $\omega \equiv \sqrt{\frac{k}{m}}$ $\qquad \gamma \equiv \frac{b}{2m}$

$\gamma < \omega \qquad x(t) = Ae^{-\gamma t}\cos(\omega' t + \phi) \qquad$ "underdamped"

$\gamma = \omega \qquad x(t) = (B + C \cdot t)e^{-\gamma t} \qquad$ "critically damped"

$\gamma > \omega \qquad x(t) = \left(A_1 e^{\omega'' t} + A_2 e^{-\omega'' t}\right) \cdot e^{-\gamma t} \qquad$ "overdamped"

where $\quad \omega' \equiv \sqrt{\omega^2 - \gamma^2} \qquad \omega'' \equiv \sqrt{\gamma^2 - \omega^2}$

Underdamped Oscillator

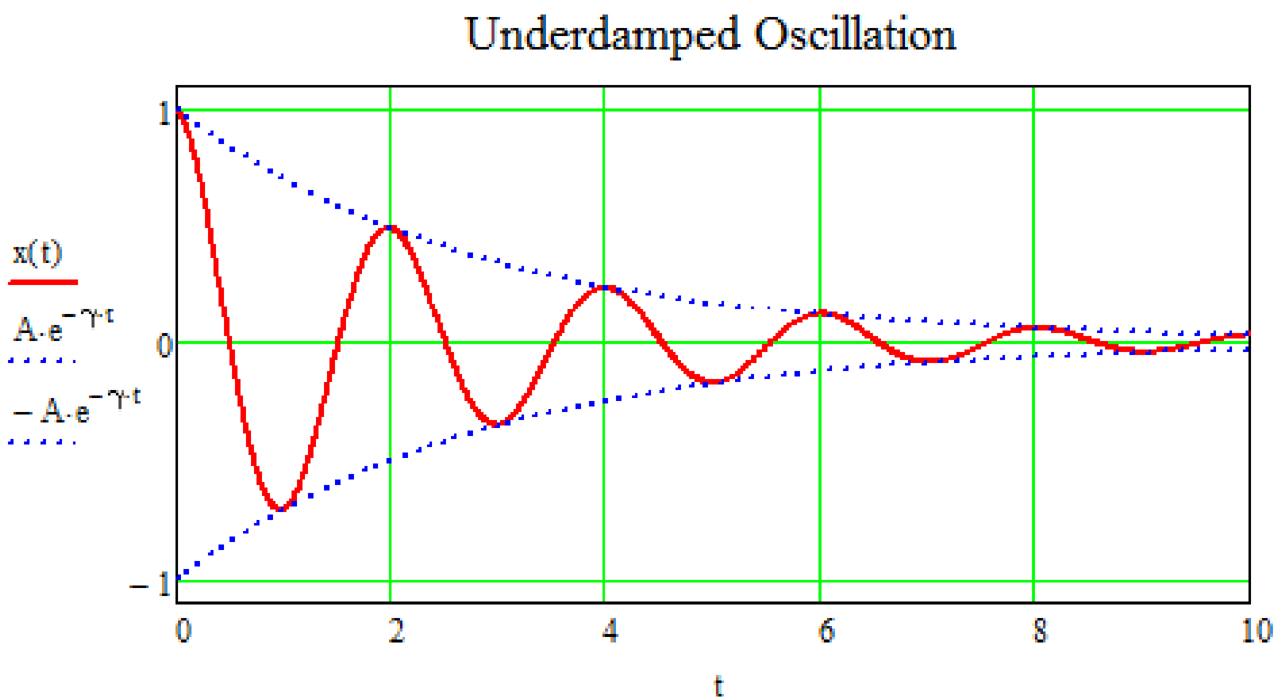
$\gamma := 0.35 \qquad T := 2 \qquad A := 1 \qquad \varphi := 0$

$$\omega := \frac{2\cdot\pi}{T} = 3.142$$

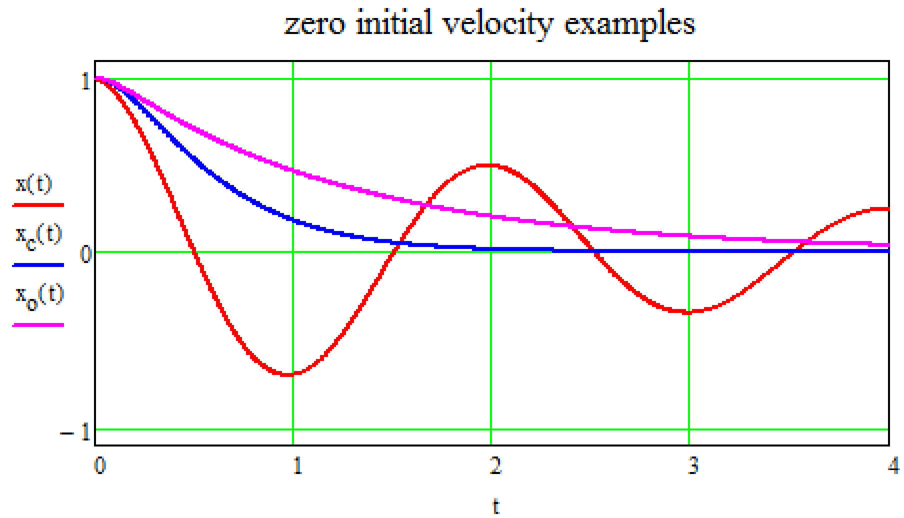
$$\omega_1 := \sqrt{\omega^2 - \gamma^2} = 3.122$$

$$x(t) := A\cdot e^{-\gamma\cdot t}\cdot\cos(\omega_1\cdot t + \varphi)$$

http://phy.hk/wiki/englishhtm/Damped.htm



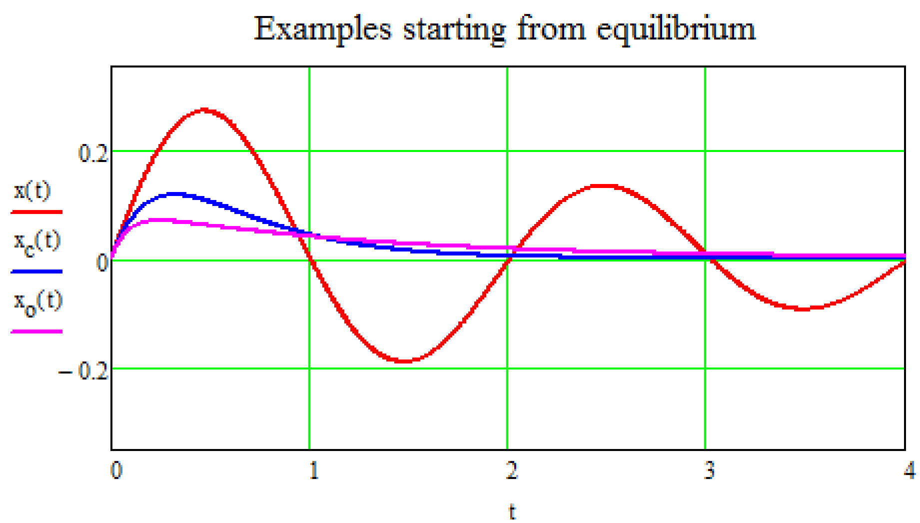
Damped Oscillator



Underdamped

Critically damped

Overdamped



Example plots for two different initial conditions.

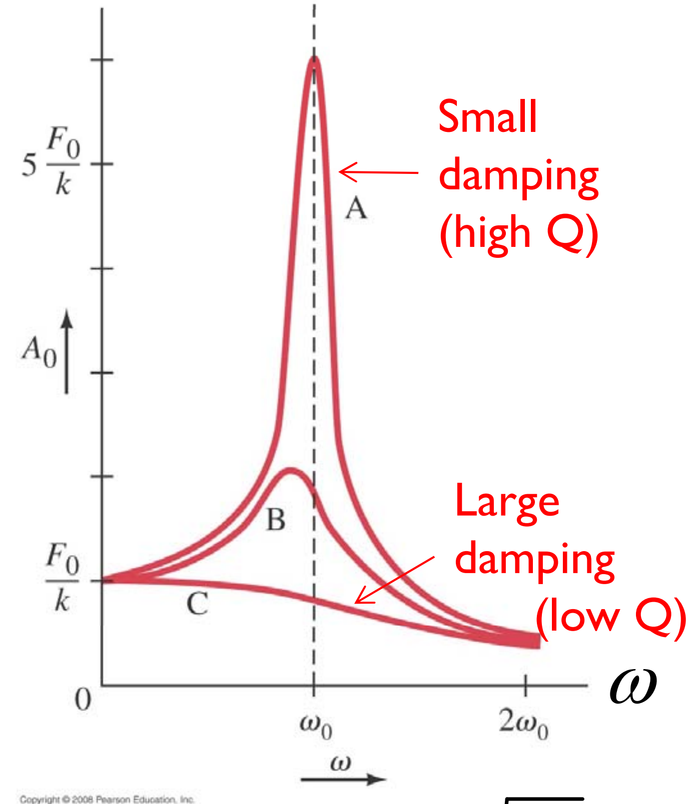
Resonance Demonstration

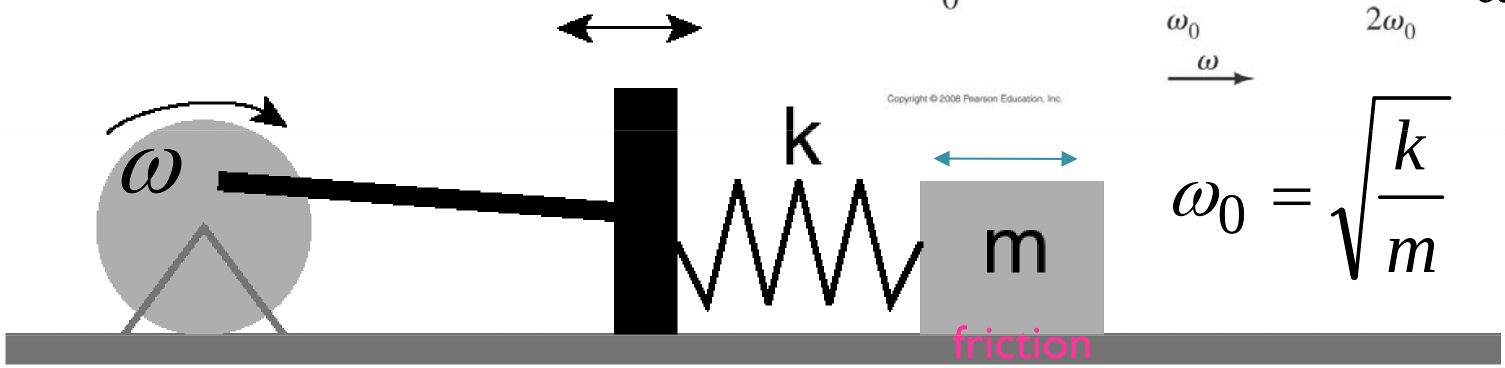
Important! Resonance is one of the most important concepts in all of physics. Examples are ubiquitous.

Forced Damped Oscillator

Mechanical example: move the wall to which the spring is attached back and forth sinusoidally.

Eventually the mass will oscillate with the driving frequency ω, ***not*** the natural oscillator frequency ω_0.





$$\omega_0 = \sqrt{\frac{k}{m}}$$

http://www.walter-fendt.de/ph14e/resonance.htm

Forced Damped Oscillator

- Important points!
  - After start-up transients have died out, the response is at the same frequency as the drive.
  - The response amplitude is small far from resonance and large at resonance.
    - For high Q (small damping) the response can get huge at resonance, when the drive operates at the natural frequency of the oscillator.
  - The phase difference between drive and response is
    - zero far below resonance (drive and oscillator move together).
    - 90 degrees at resonance (e.g. the oscillator is at its maximum amplitude when the drive passes through its equilibrium position).
    - 180 degrees far above resonance (drive and oscillator move opposite each other).
  - The width of the resonant curve (amplitude vs frequency) is
    - wide for large damping (low Q).
    - narrow for small damping (high Q).