PLASTIC ANALYSIS OF TWO HINGED CIRCULAR ARCHES

By

ROBERT C. CORNFORTH

Bachelor of Architectural Engineering

Oklahoma State University

Stillwater, Oklahoma

1961

Submitted to the faculty of the Graduate School of
the Oklahoma State University
in partial fulfillment of the requirements
for the degree of
MASTER OF ARCHITECTURAL ENGINEERING
August, 1964

PLASTIC ANALYSIS OF TWO HINGED
CIRCULAR ARCHES

Thesis Approved:

Thesis Adviser

W. G. Chamberlain

Dean of the Graduate School

PREFACE

Plastic Analysis of Two Hinged Arches was selected as a thesis project through discussion with Professor Louis O. Bass of the School of Architecture at Oklahoma State University. Because of the possibility for a savings in time to the engineer and a savings in material it was felt that a method of designing arches by plastic analysis would be worth investigating.

I wish to express my appreciation to the following persons for their help and guidance while I was working on this thesis project:

To Professor Bass, whose technical advice and suggestions were very beneficial.

To Professor F. Cuthbert Salmon, Head of the School of Architecture, for his help in attaining a graduate assistantship. which made my graduate study possible.

To Professor S. Bart Childs of the School of Civil Engineering, whose generous contribution of his time and ability was an invaluable help with the computer programing.

To Miss Ann Caskey, who did an excellent typing job.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

EQUATIONS	COMPUTER PROGRAM	
a	A(I)	Horizontal distance from left end to point load
	ACC	Accuracy
	ADEM	Denominator
	ANUM	Numerator
b		Horizontal distance from right end to point load
	BMD	Bending moment due to dead load
	BMDR	Bending moment due to drift load
	BMH	Plastic bending moment factor
	BML	Bending moment due to live load
	BMP	Bending moment due to point load
	BMTOT	Summation of bending moments
	BMW	Bending moments due to wind load
c		Horizontal distance from arch center line to point load
d		Vertical distance from arch center of curvature to point load
e	E1	Vertical distance from arch center of curvature to base line of arch
f		Vertical distance from the base line to the point load
F(L)		Function of the load L.
H	H(I)	Horizontal reaction

EQUATIONS	COMPUTER PROGRAM	
H_s		Shear due to the horizontal reaction
H_t		Thrust due to the horizontal reaction
	ISW	Geometry Calculation Indicator
L	SPAN	Arch span
	M	Infulence Line Designator
M		Bending moment
	NP	Number of Point Loads to be considered
P	WP(I)	Point load
R	R1	Arch radius of curvature
S	S1	One half the span
T	RISE	Height of the arch
	THRST	Thrust
V_l	VL(I)	Left end vertical reaction
V_r		Right end vertical reaction
V_s		Shear due to vertical reaction
V_t		Thrust due to vertical reaction
w_d	WD	Dead load
w_{dr}	WDR	Drift or snow load
w_l	WL	Live load
w_i	W(I)	Wind load
w_i'		ASCE Wind Load
x	X(I)	Horizontal distance to segment
x_h	XH	Horizontal distance to plastic hinge
y	Y(I)	Vertical distance to segment
y_h	YH	Vertical distance to plastic hinge

EQUATIONS	COMPUTER PROGRAM	
α	ALFA	Angle measured from horizontal to left end of arch
β		Angle measured from horizontal to the right end of the arch
γ	GAMMA	Angle measure from right end of arch to point load
θ	THETA (I)	Angle from horizontal to the point load
θ_1	THTA 1	Angle from horizontal to the 1/4 point of the arch
θ_2	THTA 2	Angle from horizontal to the 3/4 point of the arch
		Angle from horizontal to segment being considered
$d\rho$	DRHO	Increment of the angle
ϕ	PHI	Central angle of the arch
	AHPH	One half phi

CHAPTER I

INTRODUCTION

Plastic analysis of steel structures has been primarily limited to continuous beams, frames, and bents composed of straight members. This thesis by a theoretical approach applies the principles of plastic analysis to statically indeterminate arches. Analysis was limited to two hinged circular arches of constant cross section, for dead load, full live load, drift or snow load, point load, and wind load conditions. Computations using symmetrical loads were made for several arches of different rise to span ratios to check the equations.

The general structure is shown in Figure 1.

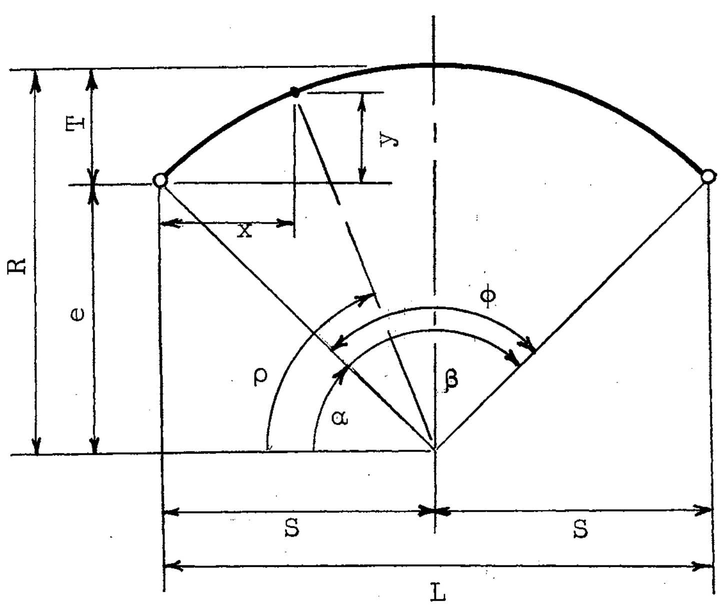


Figure 1

Because of its speed and accuracy, a digital computer was used to make the calculations for the arch analysis. Initially an IBM 1620 Computer was used, but the final program was written for the IBM 1410 Computer.

The necessary equations for vertical reactions and horizontal reactions were derived as shown in Appendix A. Then all the equations required to analyze an arch were converted into Fortran computer language. The computer program is discussed in Chapter II.

The computer program is written so that the span and rise of the arch and the values of any loads to be considered are the only data that must be determined in advance. The computer will then compute all required geometry, reactions and moments.

1. Load conditions. Dead load, live load, drift load, point load, and wind load are the five load conditions considered. It is recognized by most building codes that it would be practically impossible to have the maximum value of all five loads acting simultaneously. Therefore, various combinations of loads are allowed by different codes. The individual designer will have to determine the proper load values in advance according to local codes or practice.

Another factor which must be considered in plastic design of steel structures is cyclic or repeated loading. Plastic design is not allowed when an excessive number of cycles or repetitions of a critical load are expected.[1] The individual designer must therefore predict the number of repetitions of a critical load cycle that may be expected during the life of the structure and accordingly decide if plastic design is allowable.

---

[1] Lambert Tall et al., Structural Steel Design (New York, 1964), p. 167.

The number of repetitions considered excessive varies among codes and also according to the type structure.

2. Determination of Plastic Moments. In plastic design it is assumed that a sufficient number of plastic hinges are allowed to develop to form a collapse condition. The necessary plastic hinges to form a collapse condition or mechanism with a two hinged circular arch are shown in Figures 2 and 3.

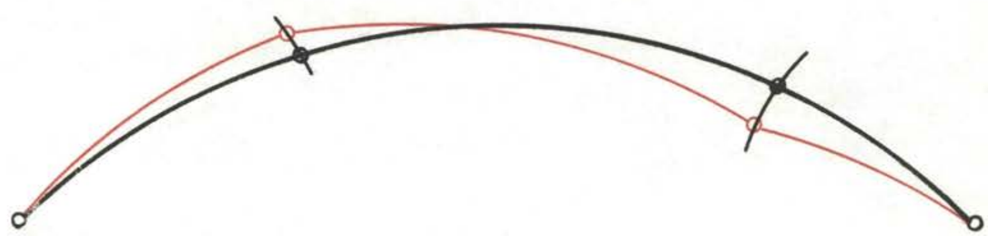


Figure 2

A minimum of four hinges must be present to have a collapse mechanism. In the case of the two hinged arch under consideration, a minimum of two plastic hinges must be developed. The two plastic hinges must form at points of maximum moment and opposite sign. Thus in the case of a symmetrical load on the structure it is theoretically possible that five hinges may be necessary before a collapse mechanism is obtained. Two plastic hinges could form simultaneously at the points of maximum negative moment before allowing a plastic hinge to develop at the point of maximum positive moment. (In this case the first peak will be in the negative range.)

Table I shows assumptions made relative to the locations of the

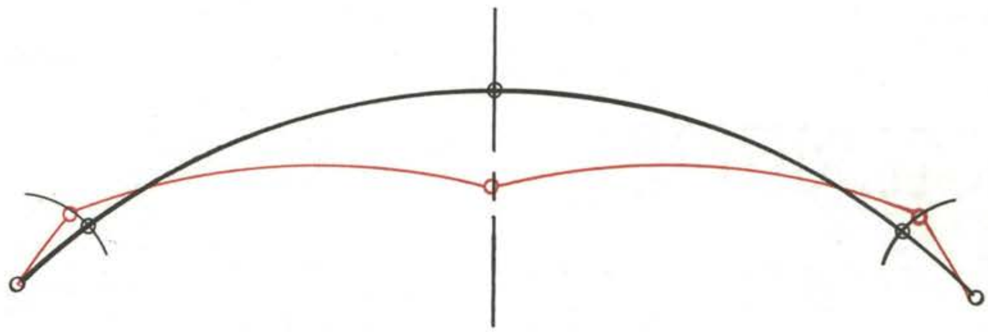


Figure 3

plastic hinges formed under the different loading conditions. These locations are predicted on the basis of data obtained from elastic analysis of arches by Bradley (2) and refer to loads applied individually.

TABLE I

Load	Location of 1st Plastic Hinge	Location of 2nd Plastic Hinge
Point Load	At Point of Load	Near 1/4 point of side opposite load
Live Load	Near ends of arch 2 to 3/20 up from spring line	At mid-point of arch
Dead Load	Same as full uniform live load	
Drift Load	Near 1/4 point of side opposite load	Near 1/4 point of side under load

Note: 1/4 points and mid-point reference entire arch length and are measured along the arch axis.

The Maximum moment value (either positive or negative) and its location is determined by elastic analysis. This location is then assumed to be the point where the first plastic hinge will form. It is also assumed that the plastic moment value will lie between the maximum elastic moment and the largest moment of opposite sign.

The following assumptions are made as a basis for allowing the distribution of the moments in such a way that a collapse mechanism would be obtained. Assume a section is selected for the arch that is slightly smaller than that required to resist the maximum elastic moment. Now when the maximum plastic moment value for this section is reached at some point, no further moment resistance is possible at this point. Also note that this will occur prior to application of the full load. As the remainder of the load is applied, theoretically a plastic hinge will develop at this point causing an increase in stress in another portion of the arch. When the load is sufficient a second plastic hinge will eventually form creating a collapse mechanism. This is similar to the way a collapse mechanism is formed in a frame.

The usual approach to plastic design is to increase the load by the load factor of the section and use the increased load to determine the plastic moment values for the collapse mechanism of the structure being considered.

The approach used here is to determine a plastic moment value based on the actual load values and then multiply these moment values by the load factor of the section, using this final moment as the basis for design.

After the first plastic hinge is allowed to develop an arch condition similar to the one shown in Figure 11-a would be developed and new reactions must be determined. Note that in no way are any of the loads altered in any form. As in elastic analysis the vertical reactions may be determined by summing moments about the end points. Since no loads were altered it is obvious that the vertical reactions remain unchanged.

The arch is now broken into two free body diagrams as shown in Figure 11-b. The moment that exists at the point of the hinge after the hinge has developed will be the plastic moment value. Now moments are summed about the point of the plastic hinge using the plastic moment and new values are found for the horizontal reactions. Under these conditions all reactions are computed from equations of statics.

Using the new values for horizontal reactions, it is now possible to compute a new set of moment values across the arch.

The first plastic hinge will remain at the location of the maximum elastic moment. The second plastic hinge will form near the point of the largest elastic moment of opposite sign. When the arch is divided into a small number of segments the second hinge will probably be located at this same point.

3. Shear and Thrust. Shear and Thrust should be considered in the design of all arches, but it is more important in some cases. As the rise to span ratio becomes smaller, shear and thrust become more critical. Shear and thrust values at each point on an arch are determined by resolving the reactions into the proper components.

Tall (5) has a discussion of the requirements in plastic analysis for selecting a section based on combined stresses.

The reactions have to be adjusted appropriately according to the load under consideration. All loads, except wind, act in a vertical direction only and thus only the vertical reaction is affected. Wind load however acts perpendicular to the arch axis and affects both the horizontal and the vertical reactions, when resolving them into shear and thrust components.

Figure 4 shows generally how shear and thrust values are computed. The specific equations for shear and thrust for each load condition are shown later.

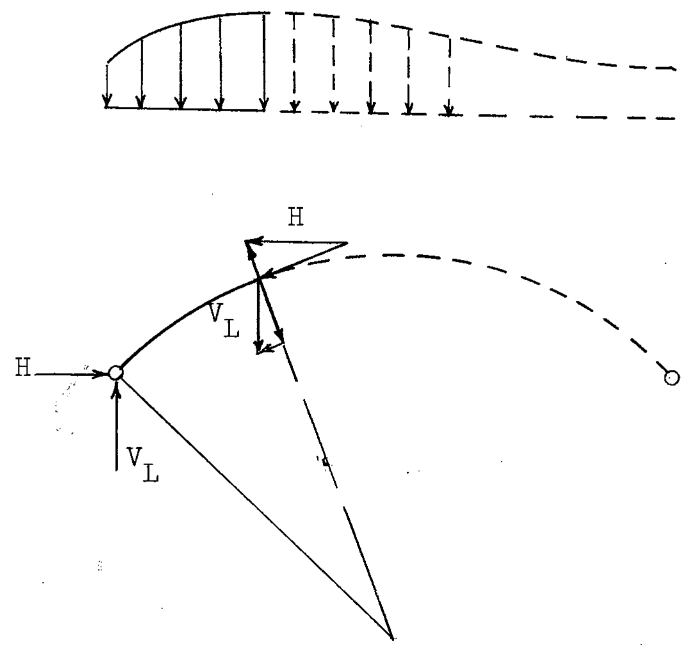


Figure 4

$V_s = (V - F(L)) \sin \rho$

$V_t = (V - F(L)) \cos \rho$

$H_s = H \cos \rho$

$H_t = H \sin \rho$

$\text{Shear} = V_s - H_s$

$\text{Thrust} = V_t + H_t$

The following figures show the loading conditions that are considered in this thesis and give the equations necessary to analyze an arch load.

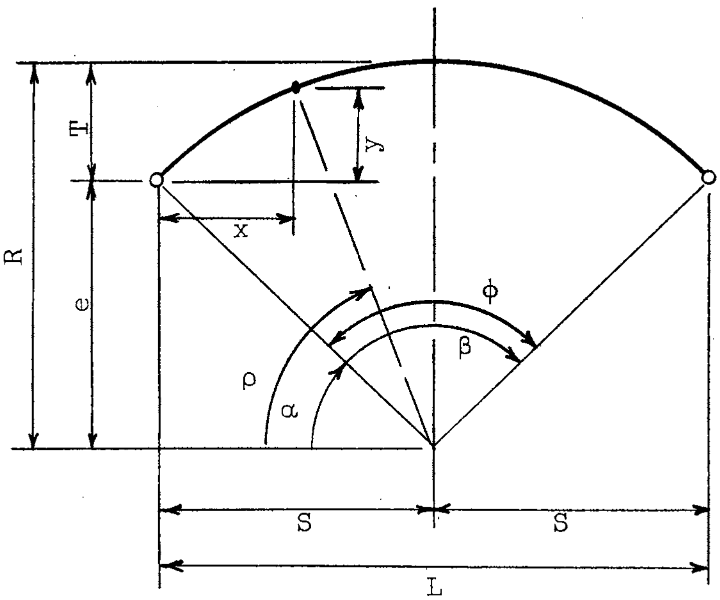


Figure 5

Geometry

Given: Values for L and T

$$S = \frac{L}{2}$$

$$R = \frac{T^2 + S^2}{2T}$$

$$e = R - T$$

$$\phi = \sin^{-1} \frac{S}{R}$$

$$\alpha = \sin^{-1} \frac{e}{R}$$

$$\beta = \alpha + \phi$$

ρ varies from α to β

$$x = S - R \cos \rho$$

$$y = R \sin \rho - e$$

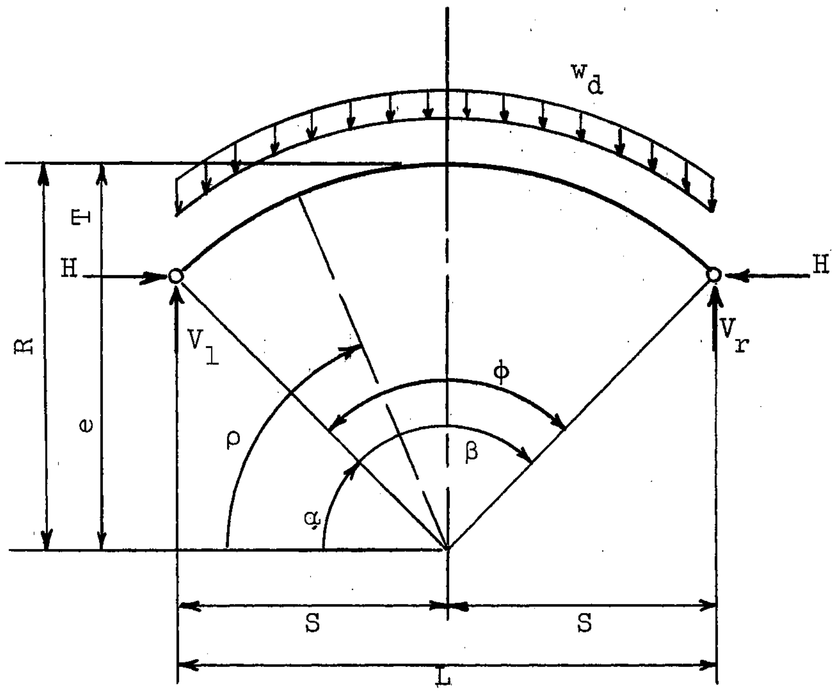


Figure 6

Dead Load

a. Special Geometry

$$\bar{x} = \frac{y}{\rho - \alpha} - R\cos\rho$$

b. Reactions

$$V_l = V_r = \frac{R\, w_d \phi}{2}$$

$$H = \frac{w_d\, R\left[\phi(-9e^2 + S^2 - 2S\, e\, \phi) + 18\, e\, S\right]}{4\left[\phi\left(\frac{R^2}{2} + e^2\right) - 3\, e\, S\right]}$$

c. Moments

$$M = V_l x - Hy - w_d R(\rho - \alpha)\,\bar{x}$$

d. Shear and Thrust

$$S = \left[V_l - R\, w_d(\rho - \alpha)\right]\sin\rho - H\cos\rho$$

$$T = \left[V_l - R\, w_d(\rho - \alpha)\right]\cos\rho + H\sin\rho$$

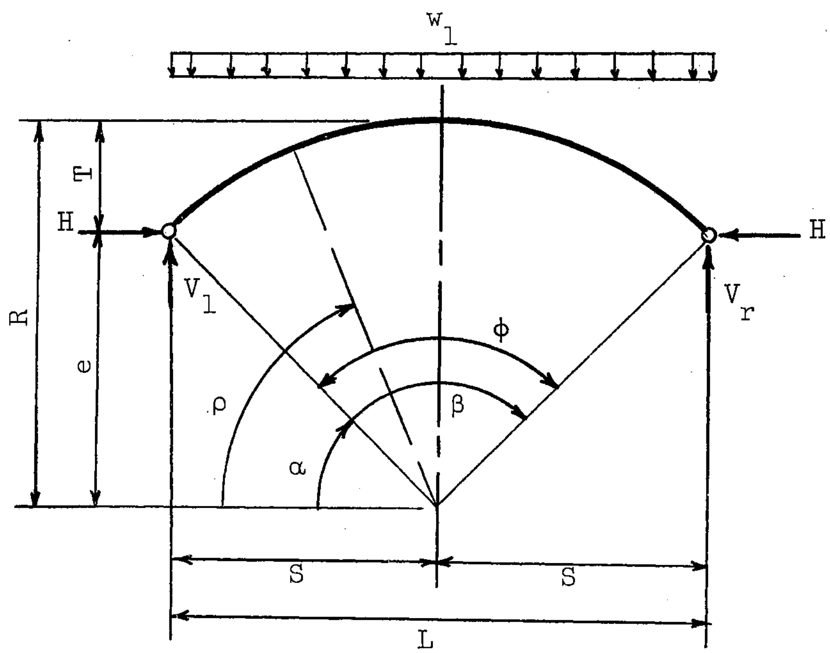


Figure 7

Live Load

a. Special Geometry

None

b. Reactions

$$V_l = V_r = w_l\, S$$

$$H = \frac{w_l}{2}\left[\frac{\frac{4}{3} S^3 + e\phi(\frac{R^2}{2} - S^2) - e^2 S}{\phi(\frac{R^2}{2} + e^2) - 3e\, S}\right]$$

c. Moments

$$M = V_l X - Hy - \frac{w_l x^2}{2}$$

d. Shear and Thrust

$$S = (V_l - w_l x) \sin\rho - H \cos\rho$$

$$T = (V_l - w_l x) \cos\rho - H \sin\rho$$

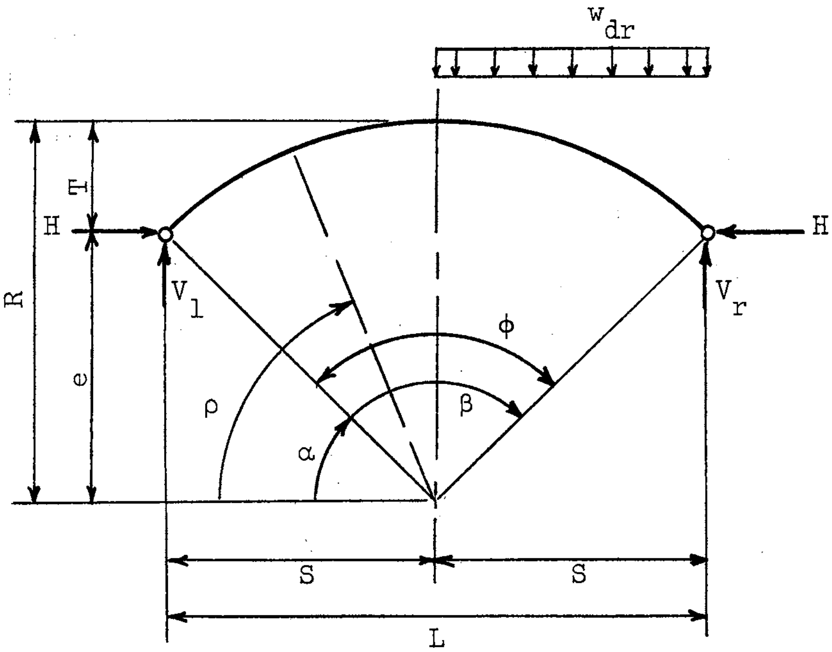


Figure 8

Drift Load

a. Special Geometry

None

b. Reactions

$$V_l = \frac{w_{dr} l}{8}$$

$$V_r = \frac{3w_{dr} l}{8}$$

$$H = w_{dr} \left[\frac{\frac{S^3}{3} + \frac{e\phi}{4}\left(\frac{R^2}{2} - S^2\right) - \frac{e^2 S}{4}}{\phi\left(\frac{R^2}{2} + e^2\right) - 3e\, S} \right]$$

c. Moments

$$M = V_l x - Hy \qquad (0 \rightarrow \frac{\phi}{2})$$

$$M = V_l x - Hy \qquad (\frac{\phi}{2} \rightarrow \phi)$$

d. Shear and Thrust

$$S = V_1 \sin \rho - H \cos \rho \qquad (0 \rightarrow \frac{\phi}{2})$$

$$S = [V_1 - w_{dr}(x - S)] \sin \rho - H \cos \rho \qquad (\frac{\phi}{2} \rightarrow \phi)$$

$$T = V_1 \cos \rho + H \sin \rho \qquad (0 \rightarrow \frac{\phi}{2})$$

$$T = [V_1 - w_{dr}(x - S)] \cos \rho + H \sin \rho \qquad (\frac{\phi}{2} \rightarrow \phi)$$

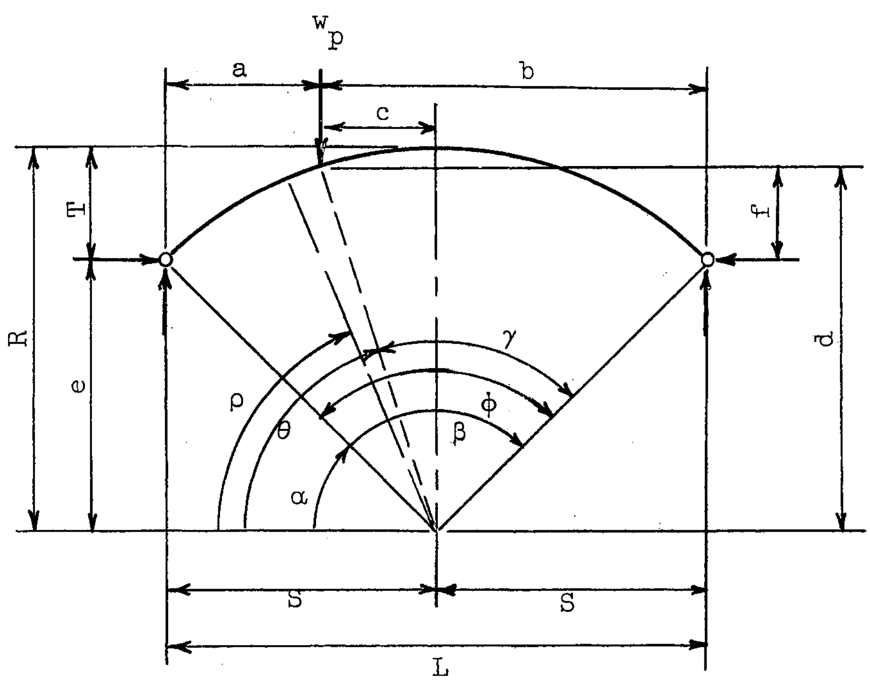


Figure 9

Point Load

a. Special Geometry

a = Given

$c = S - a$

$b = S + c$

$d = (R^2 - c^2)^{1/2}$

$f = d - e$

$\theta = \tan^{-1} \frac{c}{d} \qquad \phi = \beta - \theta$

b. Reactions

$$V_l = \frac{w_p b}{l}$$

$$V_r = \frac{w_p a}{l}$$

$$H = w_p \left[\frac{\frac{b}{2}(2S - e\phi) - bc - \frac{f^2}{2} + e\,c}{\phi(\frac{R^2}{2} + e^2) - 3e\,S} \right]$$

c. Moments

$$M = V_l x - Hy \qquad (x \leq a)$$

$$M = V_l x - Hy - w_p(x - a) \qquad (x > a)$$

d. Shear and Thrust

$$S = V_l \sin\rho - H\cos\rho \qquad (x \leq a)$$

$$S = (V_l - P)\sin\rho - H\cos\rho \qquad (x > a)$$

$$T = V_l \cos\rho + H\sin\rho \qquad (x \leq a)$$

$$T = (V_l - P)\cos\rho - H\cos\rho \qquad (x > a)$$

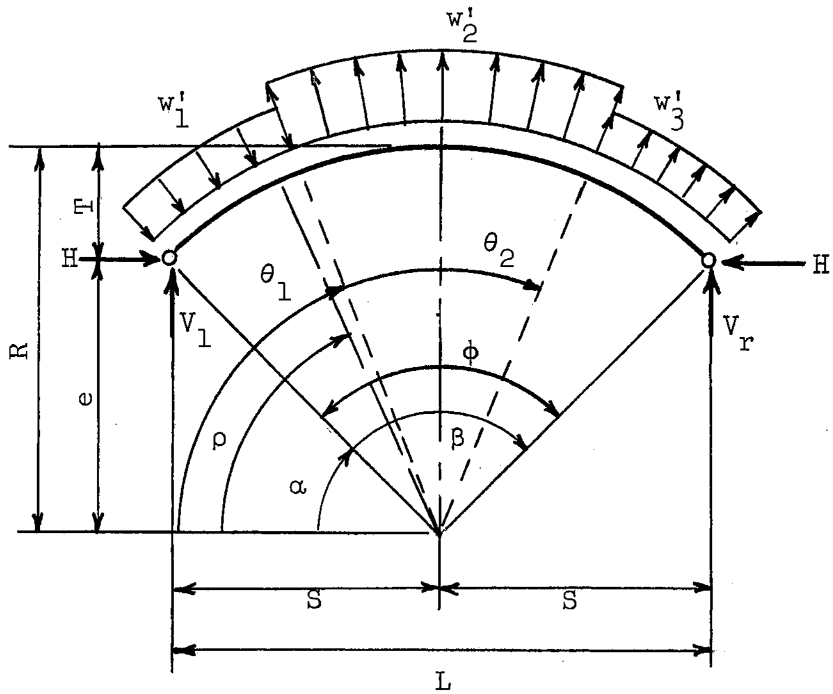


Figure 10

Wind Load

a. Special Geometry

$$\theta_1 = \alpha + \frac{\phi}{4} \qquad w_1 = w_1'$$

$$\theta_2 = \alpha + \frac{3\phi}{4} \qquad w_2 = w_1' + w_2'$$

$$w_3 = w_2' - w_3'$$

b. Reactions

$$V_1 = w_1 S - \frac{2w_2 R^2 \sin^2 (\frac{3\phi}{8})}{L} + \frac{2w_3 R^2 \sin^2 (\frac{\phi}{8})}{L}$$

$$V_r = V - V_1$$

$$H_1 = w_1 \left[\frac{2S^3 - 2R^2 S - eS^2 \phi + \frac{3eR^2\phi}{2} - e^2 S}{\phi(\frac{R^2}{2} + e^2) - 3eS} \right]$$

$$H_2 = w_2 R^2 \left[\frac{\frac{2\sin^2(\frac{3\phi}{8})}{L}(2S^2 - eS\phi) - S + \frac{3e\phi}{4} - R\cos\theta_1 - \frac{e^2}{2R}\cos\theta_1 - \frac{eS}{2R}\sin\theta_1 + \frac{3R\phi}{8}\sin\theta_1}{\phi(\frac{R^2}{2} + e^2) - 3eS} \right]$$

$$H_3 = w_3 R^2 \left[\frac{\frac{2\sin^2(\frac{\phi}{8})}{L}(2S^2 - eS\phi) - S + \frac{e\phi}{4} - R\cos\theta_2 - \frac{e^2}{2R}\cos\theta_2 - \frac{eS}{2R}\sin\theta_2 + \frac{R}{8}\sin\theta_2}{\phi(\frac{R^2}{2} + e^2) - 3eS} \right]$$

$$H = H_1 - H_2 + H_3$$

c. Moments

$$M_1 = V_{L1}x - H_1 y - 2w_1 R^2 \sin^2\left(\frac{\rho - \alpha}{2}\right)$$

$$M_2 = V_{L2}x - H_2 y \qquad (\rho \leq \theta_1)$$

$$M_2 = V_{L2}x - H_2 y - 2w_2 R^2 \sin^2\left(\frac{\rho - \theta_1}{2}\right) \qquad (\rho > \theta_1)$$

$$M_3 = V_{L3}x - H_3 y \qquad (\rho \leq \theta_2)$$

$$M_3 = V_{L3}x - H_3 y - 2w_3 R^2 \sin^2\left(\frac{\rho - \theta_2}{2}\right) \qquad (\rho > \theta_2)$$

$$M = M_1 - M_2 + M_3$$

d. Shear and Thrust

$$VK_1 = V_L - w_1 R(\cos\alpha - \cos\rho)$$

$$VK_2 = VK_1 + w_2 R(\cos\theta_1 - \cos\rho)$$

$$VK_3 = VK_2 - w_3 R(\cos\theta_2 - \cos\rho)$$

$HK_1 = H + w_1R\ (\sin\rho - \sin\alpha)$

$HK_2 = HK_1 - w_2R\ (\sin\rho - \sin\theta_1)$

$HK_3 = HK_2 + w_3R\ (\sin\rho - \sin\theta_2)$

$S = VK_1 \sin\rho - HK_1 \cos\rho \qquad (\alpha < \rho \le \theta_1)$

$S = VK_2 \sin\rho - HK_2 \cos\rho \qquad (\theta_1 < \rho \le \theta_2)$

$S = VK_3 \sin\rho - HK_3 \cos\rho \qquad (\theta_2 < \rho \le \beta)$

$T = VK_1 \cos\rho + HK_1 \sin\rho \qquad (\alpha < \rho \le \theta_1)$

$T = VK_2 \cos\rho + HK_2 \sin\rho \qquad (\theta_1 < \rho \le \theta_2)$

$T = VK_3 \cos\rho + HK_3 \sin\rho \qquad (\theta_2 < \rho \le \beta)$

Free Body Diagrams

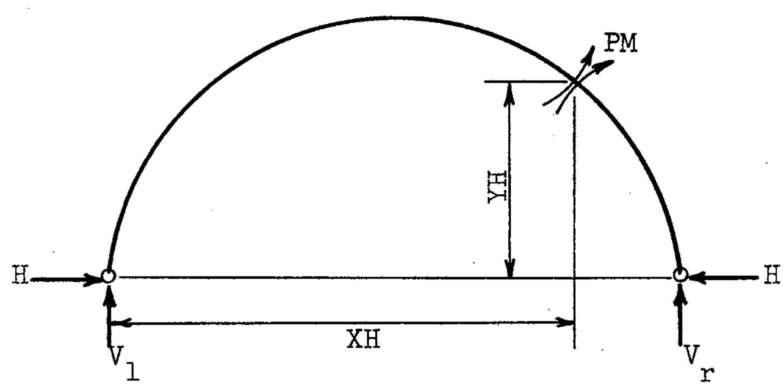


Figure 11-a

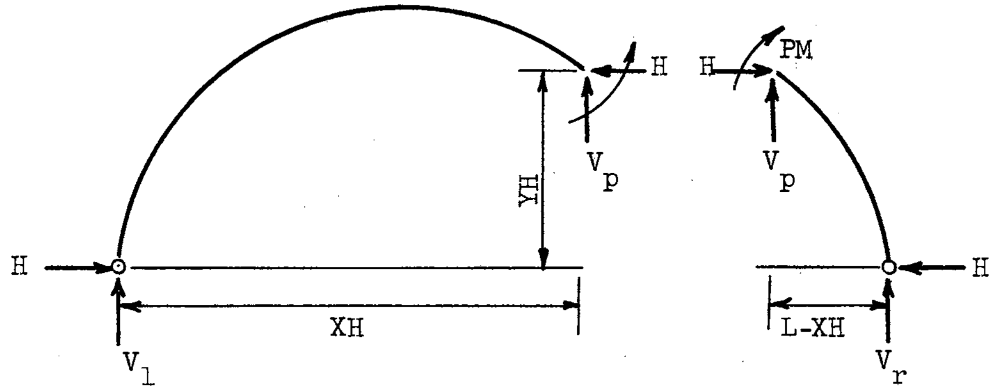


Figure 11-b

Known: Find: H for WD, WP, WL, WDR, WW

VL
VR
XH
YH
PM

For geometry not shown here refer to original geometry, Figure 5.

The PM used in the following equations for H would be the PM due to the individual load. Because the PM found by the computer is for combined loading, the program considers the effects of PM on the H values in one equation, (BMH = PM Y(I)/YH). The individual equations for H as written in the program will differ from those shown in this section because PM is removed from the equations in the program.

Dead Load

$$H_1 = \frac{(V_1)(x_h) - PM - (w_d)(R)(\rho - \alpha)\bar{x}}{Y_h}$$

Live Load

$$H_2 = \frac{(V_1)(x_h) - PM - \frac{(w_1)(x_h)^2}{2}}{Y_h}$$

Drift Load

on Right Side

$$H_3 = \frac{(V_1)(x_h) - PM}{Y_h} \qquad (x_h < S)$$

$$H_3 = \frac{(V_1)(x_h) - PM - \frac{(w_{dr})(x_h - S)^2}{2}}{Y_h} \qquad (x_h > S)$$

on Left Side

$$H_3 = \frac{(V_1)(x_h) - PM - (w_{dr})(S)(x_h - \frac{S}{2})}{Y_h} \qquad (x_h > S)$$

$$H_3 = \frac{(V_1)(x_h) - PM - \frac{(w_{dr})(x_h)^2}{2}}{Y_h} \qquad (x_h < S)$$

The manner in which drift load is applied on the left side by the computer makes the equations shown here for drift load on the left side unnecessary in the computer program.

Point Load

$$H_4 = \frac{(V_1)(x_h) - PM}{Y_h} \qquad (4 \geq x_h)$$

$$H_4 = \frac{(V_1)(x_h) - PM - w_p\ (x_h - 4)}{Y_h}$$

Wind Load

$$H_{w1} = \frac{V_1 x_h - PM + 2w_1\ R^2 \sin^2 (\frac{\rho - \alpha}{2})}{Y_h} \qquad (\rho \leq \theta_1)$$

$$H_{w2} = H_{w1} - \frac{2w_2\ R^2 \sin^2 \frac{\rho - \theta_1}{2}}{Y_h} \qquad (\theta_1 < \rho \leq \theta_2)$$

$$H_{w3} = H_{w2} + \frac{2w_3\ R^2 \sin^2 \frac{\rho - \theta_2}{2}}{Y_h} \qquad (\theta_2 < \rho \leq \beta)$$

CHAPTER II

COMPUTER PROGRAM

To analyze a given arch with this program, data cards must be prepared with values for the span, rise, loads, number and location of point loads, and the degree of accuracy desired in the plastic moments. The computer will determine all other necessary information. Note that the program is written for a maximum of twenty point loads and that three wind load values are required. The wind load equations are based on the A.S.C.E. recommendations for wind loads on curved roofs (6).

In determining the plastic moments and the shear and thrust values, the computer follows the same general line of reasoning as is discussed in Chapter I.

First reactions are determined for the elastic case and a set of elastic bending moments are computed. The maximum moment is found and this point is recorded as the location of the first plastic hinge.

A trial plastic moment value is then selected by the machine and a new set of reactions are calculated using the plastic moment value at the point of the plastic hinge. Then a new set of moment values are computed across the arch. The trial plastic moment values are selected by averaging the latest plastic moment with the current largest moment of opposite sign. New plastic moment values will be selected and checked in this manner until two maximum moments of equal value and opposite sign have been found. The program is written to compute these two values to within the accuracy specified on the data card.

The location of the second plastic hinge and the two plastic moment values are now recorded.

Next the shear and thrust values are computed.

The results are recorded in a tabulated form. The x and y coordinates of the segments are recorded first, followed by the elastic moment values, and finally the shear, thrust, and plastic moment values.

Tables showing the wind load factors that should be used, if the A.S.C.E. recommendations are followed, are given in Appendix B.

A flow diagram and a listing of the actual computer program as written in Fortran IV are shown in Appendix C.

CHAPTER III

EXAMPLE PROBLEM

An example arch was designed for the following conditions:

Spacing 16' - 0 on center

Span 100' - 0

Rise 25' - 0

Rigid metal deck roofing fastened to arches so as to provide lateral support.

Arches are two hinged with a constant circular radius.

Loading:

Dead Load - 250 lbs./ft.

Live Load - 400 lbs./ft.

Drift Load - 400 lbs./ft.

Wind Load - 320 lbs./ft.

Point Loads - 1000 lbs. at 35' - 0

1000 lbs. at 75' - 0

Combinations of Loads[1]:

$(w_d + w_l) + P$

$(w_d + w_{dr}) + P$

$0.75(w_d + 1/2w_l + w_i) + P$

$0.75(w_d + w_{dr} + w_i) + P$

$0.75(w_d + w_l + 1/3w_i) + P$

---

[1]Lothers, John E., Advanced Design in Structural Steel (Englewood Cliffs, 1960) p. 200.

The load values that were punched in the cards are shown on pages 24 - 26 with the results that were determined for each load condition. The critical load condition was found to be $(w_d + w_{dr}) + P$ and the complete set of results is shown for this condition.

Design of sections based on plastic and elastic design are shown for comparison of the two methods. A load factor of 1.80 was assumed for the design of an arch by plastic analysis.

The computer results were used as follows in selecting arch sections:

Plastic Design:

Load Factor (LF) = 1.80

Plastic Bending Moment (M'_p) = 71.38 Kip - ft.

$$M_p = M'_p \times LF = 71.38 \times 1.80$$

$$= 128.48 \text{ Kip - ft.}$$

$$Z_{req'd} = \frac{M_p \times 12 \text{ in./ft.}}{36 \text{ K si}} = \frac{128.48 \times 12}{36}$$

$$= 42.82 \text{ in.}^3$$

Select a 16 B 26 Section

Elastic Design:

Maximum Elastic Moment (M) = 77.06 Kip - ft.

$$S_{req'd} = \frac{M \times 12 \text{ in./ft.}}{24 \text{ K si}} = \frac{77.06 \times 12}{24}$$

$$= 38.53 \text{ in.}^3$$

Select a 14 WF 30 Section

The two sections selected show that a lighter section can be obtained from plastic design. The analysis shown is obviously not complete, since an actual design must consider the distance between lateral bracing, width-thickness ratios of compression elements, and other requirements of the code.

POSITION	X	Y	RHO IN DEGREES
0	.00	.00	36.86
1	3.68	4.46	42.18
2	7.77	8.57	47.49
3	12.22	12.28	52.80
4	16.99	15.57	58.12
5	22.04	18.40	63.43
6	27.34	20.74	68.74
7	32.83	22.59	74.06
8	38.47	23.92	79.37
9	44.21	24.73	84.68
10	49.99	25.00	89.99
11	55.78	24.73	95.31
12	61.52	23.92	100.62
13	67.16	22.59	105.93
14	72.65	20.74	111.25
15	77.95	18.40	116.56
16	83.00	15.57	121.87
17	87.77	12.28	127.19
18	92.22	8.57	132.50
19	96.31	4.46	137.81
20	99.99	.00	143.13

ISW	SPAN	RISE	WD	WL	WDR	NP	W1	W2	W3	M	ACC
2	100.00	25.00	.25	.00	.40	2	.00	.00	.00	1	50.00

I = 1 WP = 1.00 A = 35.00

I = 2 WP = 1.00 A = 75.00

POSITION	BMD	BML	BMDR	BMP	BMW	BMH	BMTOT
0	.00	.00	.00	.00	.00	.00	.00
1	-7.44	.00	-24.62	-2.07	.00	.00	-34.14
2	-10.63	.00	-43.78	-3.35	.00	.00	-57.77
3	-10.64	.00	-57.30	-3.83	.00	.00	-71.78
4	-8.46	.00	-65.09	-3.50	.00	.00	-77.06
5	-5.00	.00	-67.06	-2.36	.00	.00	-74.44
6	-1.05	.00	-63.20	-.43	.00	.00	-64.69
7	2.73	.00	-53.55	2.27	.00	.00	-48.54
8	5.82	.00	-38.18	2.26	.00	.00	-30.09
9	7.83	.00	-17.23	.72	.00	.00	-8.68
10	8.52	.00	9.11	-.17	.00	.00	17.46
11	7.83	.00	33.93	-.43	.00	.00	41.33
12	5.82	.00	50.49	-.03	.00	.00	56.28
13	2.73	.00	59.16	1.00	.00	.00	62.90
14	-1.05	.00	60.69	2.68	.00	.00	62.32
15	-5.00	.00	56.19	2.04	.00	.00	53.22
16	-8.46	.00	47.08	-.10	.00	.00	38.51
17	-10.64	.00	35.02	-1.39	.00	.00	22.99
18	-10.63	.00	21.86	-1.80	.00	.00	9.42
19	-7.44	.00	9.53	-1.33	.00	.00	.74
20	.00	.00	.00	-.00	.00	.00	.00

THE HINGE IS AT 4 XH = 16.99 YH = 15.57

THE HINGE IS AT 13 XH = 67.16 YH = 22.59

THE FIRST PLASTIC MOMENT IS -71.22

THE SECOND PLASTIC MOMENT IS 71.38

POSITION	SHEAR	THRUST	BENDING MOMENT
0	-6.55	30.39	.00
1	-4.68	29.80	-32.46
2	-2.96	29.12	-54.55
3	-1.41	28.40	-67.17
4	-.00	27.64	-71.22
5	1.25	26.87	-67.53
6	2.38	26.12	-56.90
7	3.40	25.39	-40.06
8	3.33	24.51	-21.11
9	4.14	23.96	.59
10	4.89	23.47	26.84
11	3.30	23.27	50.61
12	1.76	23.55	65.26
13	.37	24.30	71.38
14	-.76	25.49	70.11
15	-2.49	27.49	60.13
16	-2.88	29.44	44.35
17	-2.81	31.61	27.60
18	-2.25	33.91	12.64
19	-1.17	36.26	2.42
20	.43	38.55	.00

END REACTIONS

LEFT END		RIGHT END	
VERTICAL	20.38	VERTICAL	30.58
HORIZONTAL	23.47	HORIZONTAL	23.47

ISW	SPAN	RISE	WD	WL	WDR	NP	W1	W2	W3	M	ACC
	1100.00	25.00	.25	.40	.00	2	.00	.00	.00		1 50.00

I = 1 WP = 1.00 A = 35.00

I = 2 WP = 1.00 A = 75.00

THE HINGE IS AT 3 XH = 12.22 YH = 12.28

THE HINGE IS AT 10 XH = 49.99 YH = 25.00

THE FIRST PLASTIC MOMENT IS -33.46

THE SECOND PLASTIC MOMENT IS 33.27

ISW	SPAN	RISE	WD	WL	WDR	NP	W1	W2	W3	M	ACC
	2100.00	25.00	.19	.15	.00	2	.08	.29	.10		1 50.00

I = 1 WP = 1.00 A = 35.00

I = 2 WP = 1.00 A = 75.00

THE HINGE IS AT 4 XH = 16.99 YH = 15.57

THE HINGE IS AT 17 XH = 87.77 YH = 12.28

THE FIRST PLASTIC MOMENT IS 22.68

THE SECOND PLASTIC MOMENT IS -22.67

ISW	SPAN	RISE	WD	WL	WDR	NP	W1	W2	W3	M	ACC
	2100.00	25.00	.19	.00	.30	2	.08	.29	.10		1 50.00

I = 1 WP = 1.00 A = 35.00

I = 2 WP = 1.00 A = 75.00

THE HINGE IS AT 14 XH = 72.65 YH = 20.74

THE HINGE IS AT 6 XH = 27.34 YH = 20.74

THE FIRST PLASTIC MOMENT IS 30.27

THE SECOND PLASTIC MOMENT IS -30.27

ISW	SPAN	RISE	WD	WL	WDR	NP	W1	W2	W3	M	ACC
	2100.00	25.00	.19	.30	.00	2	.03	.10	.04		1 50.00

I = 1 WP = 1.00 A = 35.00

I = 2 WP = 1.00 A = 75.00

THE HINGE IS AT 17 XH = 87.77 YH = 12.28

THE HINGE IS AT 9 XH = 44.21 YH = 24.73

THE FIRST PLASTIC MOMENT IS -22.73

THE SECOND PLASTIC MOMENT IS 22.55

CHAPTER IV

SUMMARY AND CONCLUSIONS

One of the biggest advantages of plastic design of steel frames is that it renders a statically indeterminate structure statically determinate in most cases. This is a result of being able to accurately predict the locations of the plastic hinges that will form a collapse mechanism. A savings in the weight of steel is also an advantage normally found in plastic design.

The method of plastic design for two hinged circular arches as presented here does not have the advantage of changing the arch to a statically determinate structure. However, a savings in steel can be realized with this method, because a smaller section is obtained for a given arch condition, when the section is picked based on plastic design principles.

Use of a computer to perform the calculations aids tremendously in the plastic design of a two hinged circular arch as presented here. To perform the necessary calculations by hand would consume an unreasonable amount of time, as is the case with almost any arch structure.

The computer program presented can be adapted, with minor changes when necessary, to nearly any condition for a two hinged arch with the end points supported at equal elevations.

1. Suggestions for Future Study. Because this thesis work is entirely theoretical, actual testing is needed to verify if this type

of structure will perform under load as predicted here.

If through testing and further theoretical work a correlation can be found between the rise to span ratio, the load condition, and the points where the plastic hinges form, then a big advantage could be obtained with plastic design by making a two hinged arch statically determinate. To render one of these arches statically determinate would simplify the design of an arch as well as reduce the amount of time consumed in design. If the locations of the plastic hinges could be predicted accurately in advance, the plastic design of an arch would be similar to plastic design of a steel frame composed of straight members.

SELECTED BIBLIOGRAPHY

1. Beedle, Lynn S. Plastic Design of Steel Frames. New York: John Wiley and Sons, Inc., 1958.

2. Bradley, Jerrold F. "Analysis of Two Hinged Circular Arches by Electronic Computer". (Unpublished Thesis, Oklahoma State University, 1963.)

3. Lothers, John E. Advanced Design in Structural Steel. Englewood Cliffs: Prentice-Hall, Inc., 1960.

4. Parcel, John I. and Moorman, Robert B. B. Analysis of Statically Indeterminate Structures. New York: John Wiley and Sons, Inc., 1955.

5. Tall, Lambert et al., Structural Steel Design. New York: The Ronald Press Co., 1964.

6. "Wind Forces", Final Report of the Task Committee on Wind Forces, Transactions Paper No. 3269, A.S.C.E., Vol. 126, Part II, 1961.

APPENDIX A

DERIVATIONS

A two hinged circular arch is statically indeterminate to the first degree. The vertical reactions (V) are easily found by statics, by summing moments about the hinged ends of the arch. But the horizontal reactions (H) cannot be found by statics and were found based on the principal of virtual work. When the vertical and horizontal reactions have been evaluated the bending moments, shears and thrusts may be determined by statics.

For two hinged circular arches with rise to span ratios of 1/8 or greater, no significant error is introduced when axial shortening or normal force is ignored in the derivation of the equations for H. An error of approximately 2 per cent is created when the rise to span ratio is 1/8. For any rise to span ratio of less than 1/8 normal force should be considered.[1]

Only the derivations of the wind load equations are shown. Equations for the other loads were derived and shown by Bradley (2). Bradley's derivations use a slightly different nomenclature and the equations appear different in final form, but the basic principles are the same.

---

[1]John I. Parcel and Robert B. B. Moorman, Analysis of Statically Indeterminate Structures. (New York 1955) p. 464.

Derivation of the horizontal reactions is based on the following conditions and/or assumptions:

1. The Arch is of constant cross section and homogeneous material (E I is a constant) and a constant circular radius.

2. The end conditions are such that the arch will act as though it is hinged at both ends.

3. The material of the arch conforms to Hooke's Law, stating that stress is proportional to strain, and that all deformation and stress is within the elastic limit.

4. Effects of temperature change, displacement of supports, and change in length of the center line of the arch due to longitudinal compression are neglected.

5. The radius of curvature of the arch is large in comparison to the depth of the cross section of the member.

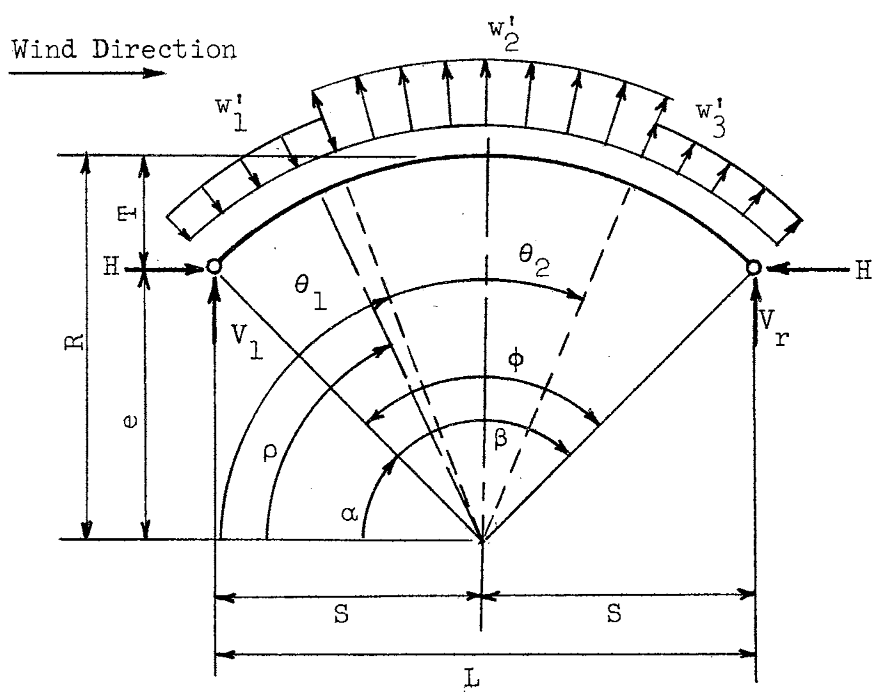


Figure 12 a

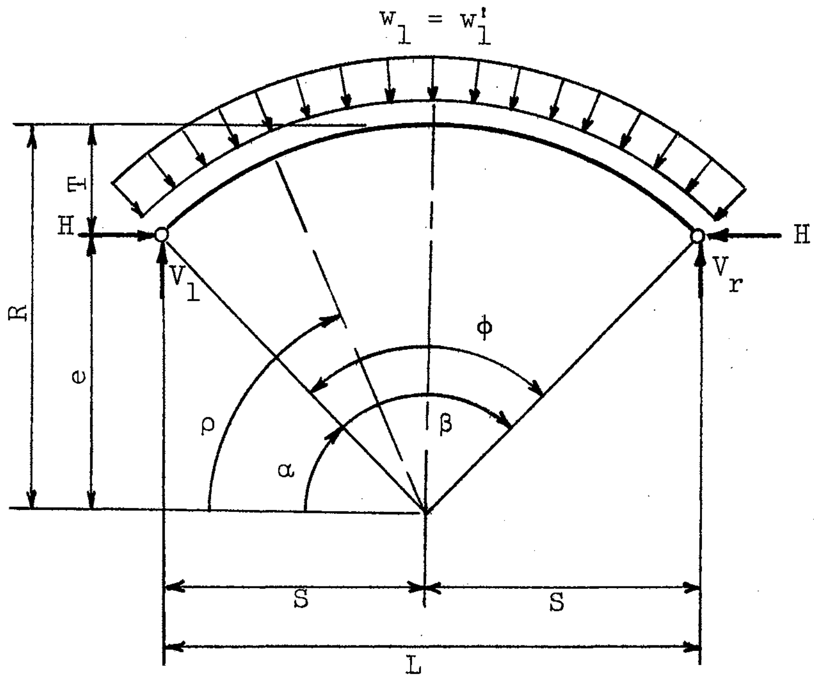


Figure 12 b

1.) $\Sigma M_R = 0$

$$V_{L1}L - 2w_1 R^2 \sin^2 (\frac{\phi}{2}) = 0$$

$$V_{L1} = \frac{2w_1 R^2 \sin^2 (\frac{\phi}{2})}{L}$$

$$V_{L1} = \frac{2w_1 S^2}{L}$$

$$V_{L1} = w_1 S$$

2.) $\Sigma V = 0$

$$V_{L1} + V_{R1} - 2w_1 R \sin (\frac{\phi}{2}) = 0$$

$$V_{R1} = w_1 2R \sin (\frac{\phi}{2}) - w_1 S$$

$$V_{R1} = 2w_1 S - w_1 S$$

$$V_{R1} = w_1 S$$

3.) $\Sigma H = 0$

$$H_{L1} = H_{R1} = H_1$$

4.) $\Sigma M \frac{\delta m}{\delta H_1} \frac{\Delta S}{EI} = 0 = \Delta_{LX}$

$$\Delta S = R\ d\rho$$

$$M_{(\alpha\ -\ \beta)} = V_{L1}x - H_1 y - 2w_1 R^2 \sin^2\left(\frac{\rho - \alpha}{2}\right)] \frac{\delta m}{\delta H_1}$$

$$\frac{\delta m}{\delta H_1} = -y$$

$$0 = \int_\alpha^\beta V_{L1} xy\ d\rho - \int_\alpha^\beta H_1 y^2\ d\rho - \int_\alpha^\beta 2w_1 R^2 y \sin^2\left(\frac{\rho - \alpha}{2}\right) d\rho$$

$$x = S - R\cos\rho$$

$$y = R\sin\rho - e$$

$$sy = SR\sin\rho - eS - R^2 \sin\rho\cos\rho + eR\cos\rho$$

$$y^2 = R^2 \sin^2\rho - 2eR\sin\rho + e^2$$

Let $\left(\frac{\rho - \alpha}{2}\right) = K_1$

$$y \sin^2 K_1 = R \sin\rho \sin^2 K_1 - e \sin^2 K_1$$

$$\sin^2 K_1 = \sin^2\left(\frac{\rho - \alpha}{2}\right) = 1/2 - 1/2 \cos(\rho - \alpha)$$

$$\cos(\rho - \alpha) = \cos\rho\cos\alpha + \sin\rho\sin\alpha$$

$$\sin^2 K_1 = 1/2 - \cos\rho\cos\alpha - \sin\rho\sin\alpha$$

$$V_{L1}\int_\alpha^\beta xy\ d\rho = V_{L1}\int_\alpha^\beta (SR\sin\rho - eS - R^2\sin\rho\cos\rho + eR\cos\rho)\ d\rho$$

$$= V_{L1}\left[- SR\cos\rho - eS\rho - \frac{R^2\sin^2\rho}{2} + eR\sin\rho\right]_\alpha^\beta$$

$$= V_{L2}\ [S^2 + S^2 - eS\phi]$$

$$w_1 S\ [2S^2 - eS\phi]$$

$$H_1 \int_\alpha^\beta y^2\, d\rho = H_1 \int_\alpha^\beta (R^2 \sin^2 \rho - 2eR \sin \rho + e^2)\, d\rho$$

$$= H_1 \left[\frac{R^2}{2} (\rho - \sin \rho \cos \rho) + 2eR \cos \rho + e^2 \rho \right]_\alpha^\beta$$

$$= H_1 \left[\frac{R^2}{2} \phi - 3eS + e^2 \right]$$

$$= H_1 \left[\phi(\frac{R^2}{2} + e^2) - 3eS \right]$$

$$2w_1 R^2 \int_\alpha^\beta y \sin^2 (\frac{\rho - \alpha}{2})\, d\rho$$

$$= 2w_1 R^2 \int_\alpha^\beta 1/2(R \sin \rho - R \sin \rho \cos \rho \cos \alpha - R \sin^2 \rho \sin \alpha - e$$

$$+ e \cos \rho \cos \alpha + e \sin \rho \sin \alpha)\, d\rho$$

$$= w_1 R^2 \Big[- R \cos \rho - \frac{R \sin^2 \rho}{2} \cos \alpha - \frac{R}{2} (\rho - \sin \rho \cos \rho) \sin \alpha$$

$$- e\rho + e \sin \rho \cos \alpha - e \cos \rho \sin \alpha \Big]_\alpha^\beta$$

$$R \cos \beta = - S \qquad R \cos \alpha = S$$

$$R \sin \beta = e \qquad R \sin \alpha = e$$

$$= w_1 R^2 [S + S - \frac{e \sin \beta \cos \alpha}{2} + \frac{e \sin \alpha \cos \alpha}{2} - (\frac{R\beta}{2} - \frac{R\alpha}{2}) \sin \alpha$$

$$+ (\frac{e \cos \beta}{2} - \frac{e \cos \alpha}{2}) \sin \alpha - e\beta + e\alpha + e \sin \beta \cos a$$

$$- e \sin \alpha \cos \alpha - e \cos \beta \sin \alpha + e \cos \alpha \sin \alpha]$$

$$\sin \alpha = \frac{e}{R} \qquad \sin \beta = \frac{e}{R}$$

$$\cos \alpha = \frac{S}{R} \qquad \cos \beta = - \frac{S}{R}$$

$$= w_1R^2\left[2S - \frac{e}{2}\cdot\frac{e}{R}\cdot\frac{S}{R} + \frac{e}{2}\cdot\frac{e}{R}\cdot\frac{S}{R} - \frac{R}{2}\cdot\frac{e}{R} + \frac{e}{2}\cdot\left(-\frac{S}{R}\right)\cdot\frac{e}{R}\right.$$

$$\left. - \frac{e}{2}\cdot\frac{S}{R}\cdot\frac{e}{R} - e\phi + e\cdot\frac{e}{R}\cdot\frac{S}{R} - e\left(-\frac{S}{R}\right)\cdot\frac{e}{R} + e\cdot\frac{S}{R}\cdot\frac{e}{R}\right]$$

$$= w_1R^2\left[2S - \frac{3e}{2} + \frac{e^2S}{R^2}\right]$$

$$H_1 = \frac{w_1S(2S^2 - eS\rho) - w_1R^2\left(2S - \frac{3e}{2} + \frac{e^2S}{R^2}\right)}{\left(\frac{R^2}{2} + e^2\right) - 3eS}$$

$$H_1 = \frac{w_1\left[2S^3 - 2R^2S - eS^2 + \frac{3eR^2}{2} - e^2S\right]}{\phi\left(\frac{R^2}{2} + e^2\right) - 3eS}$$

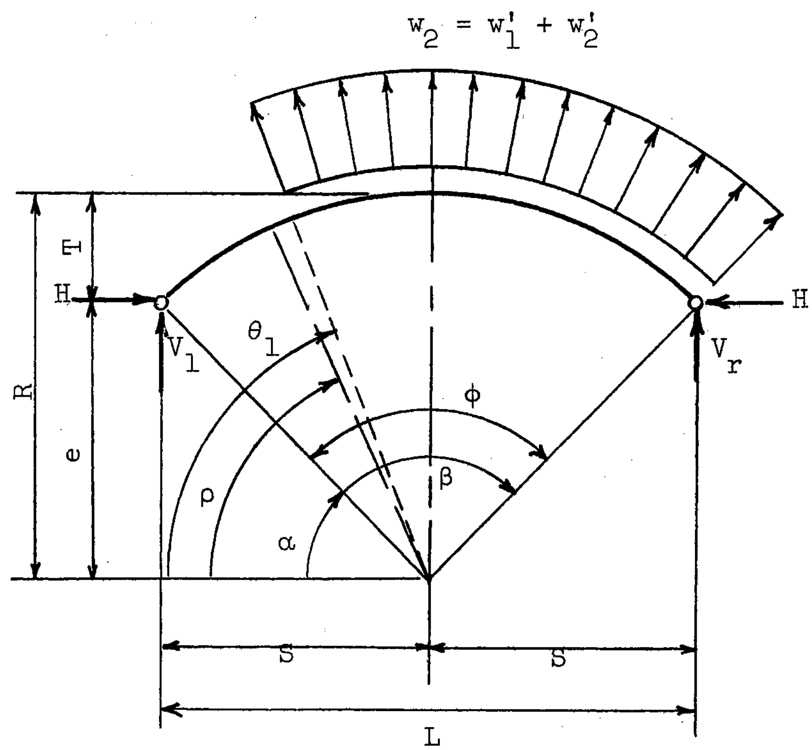


Figure 12 c

1.) $\Sigma M_R = 0$

$$V_{L2}L - 2w_2R^2 \sin^2 \left(\frac{\beta - \theta_1}{2}\right) = 0$$

$$\frac{\beta - \theta_1}{2} = \frac{3\phi}{8}$$

$$V_{L2} = \frac{2w_2R^2 \sin^2 \left(\frac{3\phi}{8}\right)}{L}$$

2.) $\Sigma V = 0$

$$V_{L2} + V_{R2} - 2w_2R \sin \left(\frac{3\phi}{8}\right) \sin \left(\pi - \frac{\beta + \theta_1}{2}\right)$$

$$V_{R2} = 2w_2R \sin \left(\frac{3\phi}{8}\right) \sin \left(\pi - \frac{\beta + \theta_1}{2}\right) - \frac{2w_2R^2 \sin^2\left(\frac{3\phi}{8}\right)}{L}$$

3.) $\Sigma H = 0$

$$H_{L2} = H_{R2} = H_2$$

4.) $\Sigma M \frac{\delta m}{\delta H_2} \frac{\Delta S}{EI} = 0 = \Delta_{LX}$

$$\Delta S = R \, d\rho$$

$$\left. M_{(\alpha - \theta_1)} = V_{L2}x - H_2y \right\} \frac{\delta m}{\delta H_2}$$

$$\left. M_{(\theta_1 - \beta)} = V_{L2}x - H_2y - 2w_2R^2 \sin^2 \left(\frac{\rho - \theta_1}{2}\right) \right\} \frac{\delta m}{\delta H_2}$$

$$\frac{\delta m}{\delta H_2} = - y$$

$$0 = \int_{\alpha}^{\beta} V_{L2} xy \, d\rho - \int_{\alpha}^{\beta} H_2 y^2 \, d\rho - \int_{\alpha}^{\beta} 2w_2 R^2 y \sin^2 \left(\frac{\rho - \theta_1}{2}\right) d\rho$$

$$x = S - R \cos \rho$$

$$y = R \sin \rho - e$$

$$xy = SR \sin \rho - eS - R^2 \sin \rho \cos \rho + eR \cos \rho$$

$$y^2 = R^2 \sin^2 \rho - 2eR \sin \rho + e^2$$

$$\text{Let } \left(\frac{\rho - \theta_1}{2}\right) = K_2$$

$$y \sin^2 K_2 = R \sin \rho \sin^2 K_2 - e \sin^2 K_2$$

$$\sin^2 K_2 = \sin^2 \left(\frac{\rho - \theta_1}{2}\right) = 1/2 - 1/2 \cos (\rho - \theta_1)$$

$$\cos (\rho - \theta_1) = \cos \rho \cos \theta_1 + \sin \rho \sin \theta_1$$

$$\sin^2 K_2 = 1/2 - \cos \rho \cos \theta_1 - \sin \rho \sin \theta_1$$

For derivation of V_{L2} and H_2 terms of the equation see page 33.

$$V_{L2} \int_{\alpha}^{\beta} xy \, d\rho = \frac{2w_2 R^2 \sin^2 \left(\frac{3\phi}{8}\right)}{L} \left[2S^2 - eS\phi\right]$$

$$H_2 \int_{\alpha}^{\beta} y^2 \, d\rho = H_2 \left[\phi\left(\frac{R^2}{2} + e^2\right) - 3eS\right]$$

$$2w_2 R^2 \int_{\alpha}^{\beta} y \sin^2 \left(\frac{\rho - \theta_1}{2}\right) d\rho =$$

$$2w_2 R^2 \int_{\alpha}^{\beta} 1/2(R \sin \rho - R \sin \rho \cos \rho \cos \theta_1 - R \sin^2 \rho \sin \theta_1 - e$$

$$+ e \cos \rho \cos \theta_1 + e \sin \rho \sin \theta_1) \, d\rho$$

$$= w_2 R^2 \Big[- R \cos \rho - \frac{R \sin^2 \rho \cos \theta_1}{2} - \frac{R}{2}(\rho - \sin \rho \cos \rho) \sin \theta_1$$

$$- e\rho + e \sin \rho \cos \theta_1 - e \cos \rho \sin \theta_1 \Big]_{\theta_1}^{\beta}$$

$$= w_2 R^2 \Big[S + R \cos \theta_1 - \frac{e \sin \beta \cos \theta_1}{2} + \frac{R \sin^2 \theta_1 \cos \theta_1}{2}$$

$$- \frac{3R\phi}{8} \sin \theta_1 + \frac{e}{2} \cos \beta \sin \theta_1 - \frac{R \sin^2 \theta_1 \cos \theta_1}{2}$$

$$- \frac{3e\phi}{4} + e \sin \beta \cos \theta_1 - e \sin \theta_1 \cos \theta_1$$

$$- e \cos \beta \sin \theta_1 + e \cos \theta_1 \sin \theta_1 \Big]$$

$$\cos \beta = - \frac{S}{R}$$

$$\sin \beta = \frac{e}{R}$$

$$= w_2 R_2 \Big[S - \frac{3e\phi}{4} + R \cos \theta_1 + \frac{e^2}{2R} \cos \theta_1 + \frac{eS}{2R} \sin \theta_1$$

$$- \frac{3R\phi}{8} \sin \theta_1 \Big]$$

$$H_2 = \frac{w_2 R^2 \left[\frac{2 \sin^2 \left(\frac{3\phi}{8}\right)}{L} (2S^2 - eS\rho) - S + \frac{3e\phi}{4} - R \cos \theta_1 - \frac{e^2}{2R} \cos \theta_1 - \frac{eS}{2R} \sin \theta_1 + \frac{3R\phi}{8} \sin \theta_1 \right]}{\phi \left(\frac{R^2}{2} + e^2 \right) - 3eS}$$

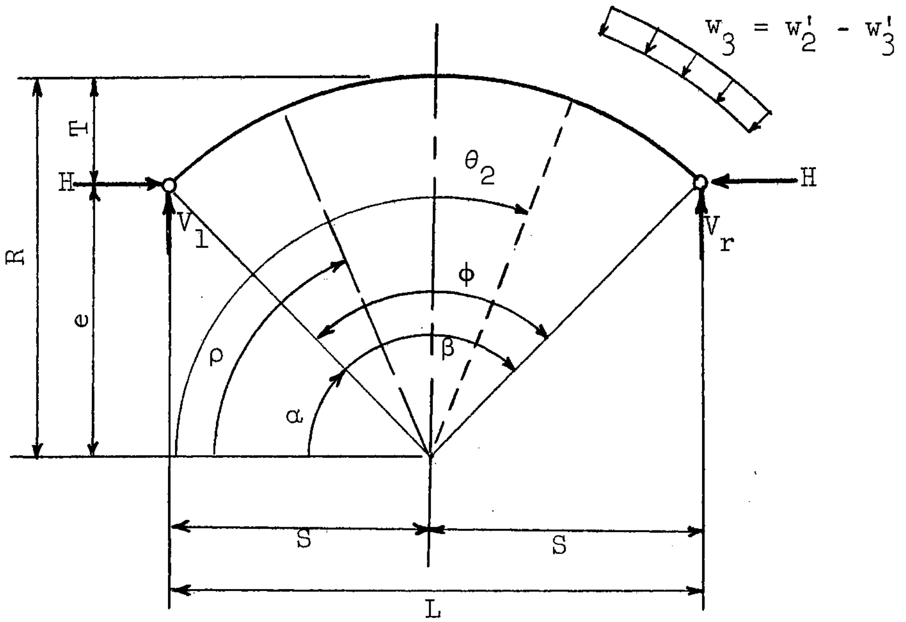


Figure 12 d

1.) $\Sigma M_2 = 0$

$$V_{L3}L - 2w_3R^2 \sin^2 \left(\frac{\beta - \theta_2}{2}\right) = 0$$

$$\frac{\beta - \theta_1}{2} = \frac{\phi}{8}$$

$$V_{L3} = \frac{2w_3R^2 \sin^2 (\frac{\phi}{8})}{L}$$

2.) $\Sigma V = 0$

$$V_{L3} + V_{R3} - 2w_3R \sin \frac{\phi}{8} \sin \left(\pi - \frac{\beta + \theta_2}{2}\right)$$

$$V_{R3} = 2w_3R \sin \frac{}{8} \sin \left(\pi - \frac{\beta + \theta_2}{2}\right) - \frac{2w_3R^2 \sin^2(\frac{\phi}{8})}{L}$$

3.) $\Sigma H = 0$

$$H_{L3} = H_{R3} = H_3$$

4.) $\Sigma M \frac{\delta m}{\delta H_3} \frac{\Delta S}{EI} = 0 = \Delta'_{LX}$

$$\Delta S = R \, d\rho$$

$$M_{(\alpha - \theta_2)} = V_{L3}x - H_3 y \Big\} \frac{\delta m}{\delta H_3}$$

$$M_{(\theta_2 - \beta)} = V_{L3}x - H_3 y - 2w_3R^2 \sin^2\left(\frac{\rho - \theta_2}{2}\right) \Big\} \frac{\delta m}{\delta H_3}$$

$$0 = \int_\alpha^\beta V_{L3}xy \, d\rho - \int_\alpha^\beta H_3 y^2 \, d\rho - \int_{\theta_2}^\beta 2w_3R^2 y \sin^2\left(\frac{\rho - \theta_2}{2}\right) d\rho$$

Derivation of the equation for H_3 is the same as for H_2 except the load is applied only from θ_2 to β.

$$V_{L3} \int_\alpha^\beta xy \, d\rho = \frac{2w_3R^2 \sin^2\left(\frac{\phi}{8}\right)}{L} \left[2S^2 - eS\phi\right]$$

$$H_3 \int_\alpha^\beta y^2 \, d\rho = H_3 \left[\phi\left(\frac{R^2}{2} + e^2\right) - 3eS\right]$$

$$2w_3R^2 \int_{\theta_2}^\beta y \sin^2\left(\frac{\rho - \theta_2}{2}\right) d\rho =$$

$$w_3R^2 \left[S - \frac{e\phi}{4} + R\cos\theta_2 + \frac{e^2}{2R}\cos\theta_2 + \frac{eS}{2R}\sin\theta_2 - \frac{R\phi}{8}\sin\theta_2\right]$$

$$H_3 = \frac{w_3R^2 \left[\frac{2\sin^2\left(\frac{\phi}{8}\right)}{L}(2S^2 - 2S\ ) - S + \frac{e}{4} - R\cos\theta_2 - \frac{e^2}{2R}\cos\theta_2 - \frac{eS}{2R}\sin\theta_2 + \frac{R\phi}{8}\sin\theta_2\right]}{\phi\left(\frac{R^2}{2} + e^2\right) - 3eS}$$

$$H_w = H_1 - H_2 + H_3$$

Moments

$$M_1 = V_{L1}x - H_1y - 2w_1R^2 \sin^2 \left(\frac{\rho - \alpha}{2}\right) \qquad (\alpha - \beta)$$

$$M_2 = V_{L2}x - H_2y \qquad (\alpha - \theta_1)$$

$$M_2 = V_{L2}x - H_2y - 2w_2R^2 \sin^2 \left(\frac{\rho - \theta_1}{2}\right) \qquad (\theta_1 - \beta)$$

$$M_3 = V_{L3}x - H_3y \qquad (\alpha - \theta_2)$$

$$M_3 = V_{13}x - H_3y - 2w_3R^2 \sin^2 \left(\frac{\rho - \theta_2}{2}\right) \qquad (\theta_2 - \beta)$$

$$M = M_1 - M_2 + M_3$$

APPENDIX B

WIND LOAD FACTORS

Tables 2 and 3 give the wind load factors for some rise to span ratios of an arch. Some factors given by the A.S.C.E. must be modified to satisfy the wind load equations as written for the computer. A check of the derivation of the wind load equations will explain why this is necessary. The A.S.C.E. factors for the windward 1/4 (W1) of a roof need not be adjusted. The factors for the central 1/2 (W2) are determined by summing the absolute values for the windward 1/4 and the central 1/2. The factor for the leeward 1/4 (W3) is determined by subtracting the leeward 1/4 factor from the A.S.C.E. factor for the central 1/2. The factors are applied to the wind load value and the values found are punched in the data card as positive values. The program accounts for part of the wind load being a suction or negative force.

There is an exception for certain rise to span ratios or arches supported above ground level, where a suction force is created across the entire arch. In this case a negative value for W1 must be punched in the data card. The factor for W1 is now subtracted from the factor for W2. W2 is still used as a positive value. The factor for W3 is subtracted from the factor for W1 and this factor is also punched in the data card as a positive value.

TABLE II

WIND LOAD FACTORS

For Arches Supported at Ground Level

Rise/span	Windward 1/4	Central 1/2	Leeward 1/4
1/2 (0.500)	0.700	1.900(-1.200)	0.700(-0.500)
1/3 (0.333)	0.450	1.480(-1.030)	0.530(-0.500)
1/4 (0.250)	0.340	1.290(-0.950)	0.450(-0.500)
1/5 (0.200)	0.280	1.180(-0.900)	0.400(-0.500)
1/6 (0.166)	0.230	1.096(-0.866)	0.366(-0.500)
1/7 (0.143)	0.190	1.033(-0.843)	0.343(-0.500)
1/8 (0.125)	0.170	0.995(-0.825)	0.325(-0.500)

TABLE III

WIND LOAD FACTORS

For Arches Supported above Ground Level

Rise/span	Windward 1/4	Central 1/2	Leeward 1/4
1/2 (0.500)	0.700	1.900(-1.200)	0.700(-0.500)
1/3 (0.333)	0.250	1.280(-1.030)	0.530(-0.500)
1/4 (0.250)	0.000	0.950(-0.950)	0.450(-0.500)
1/5 (0.200)	-0.900	0.000(-0.900)	0.400(-0.500)
1/6 (0.166)	-0.866	0.000(-0.866)	0.366(-0.500)
1/7 (0.143)	-0.843	0.000(-0.843)	0.343(-0.500)
1/8 (0.125)	-0.825	0.000(-0.825)	0.325(-0.500)

If the A.S.C.E. recommendations for wind load are followed these are the factors that should be used with this computer program. The A.S.C.E. factors are shown in parentheses.

APPENDIX C

COMPUTER PROGRAM

To utilize the computer program it is essential that the correct number of data cards be prepared according to a rigid set of rules. Sample data cards are shown in Figure 14.

The first card must contain a number punched in column six which specifies the number of sets of data to be considered.

The next data card will contain the information denoted in statement number one of the program.

ISW, 4*, must always contain the digit 1 with the first set of data so that the geometry of the structure will be computed. If it is not necessary to compute new geometry for the next set of data a 2 is punched in this position. ISW must always be either a 1 or a 2.

Span, 5-10, and rise, 11-16, values must be punched in the proper units, usually in feet.

WD, 17-22, WL, 23-28, and WDR, 29-34, are where the values for dead load, live load, and drift load respectively are to be punched. The program equations are written for the drift load applied on the right half of the arch. A drift load may be applied on the left side by adding the drift load value to the live load value and then using a negative value for drift load.

---

*The numbers after the symbols indicate the card columns within which the values must be punched.

NP, 40, indicates the number of points loads that will be considered with this set of data. The number of point loads may vary from zero to twenty.

W1, 41-46, W2, 47-52, and W3, 53-58, are the three values that are necessary for a wind load. (See Appendix B)

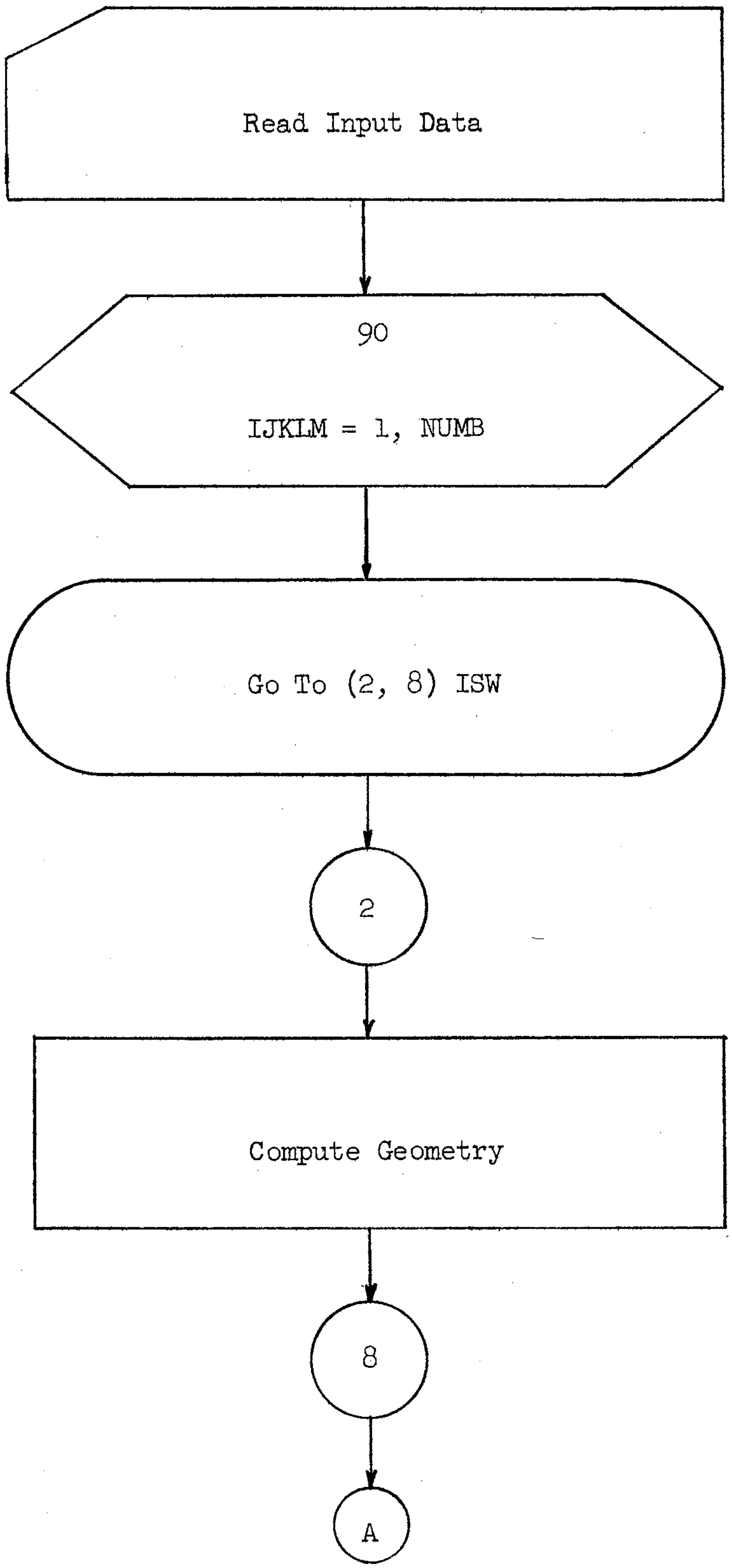
M, 64, must always contain either a 1 or a 2. Normally M will contain a 1 and will have no effect on the program. If a 2 is used values will be determined from which influence lines may be plotted.

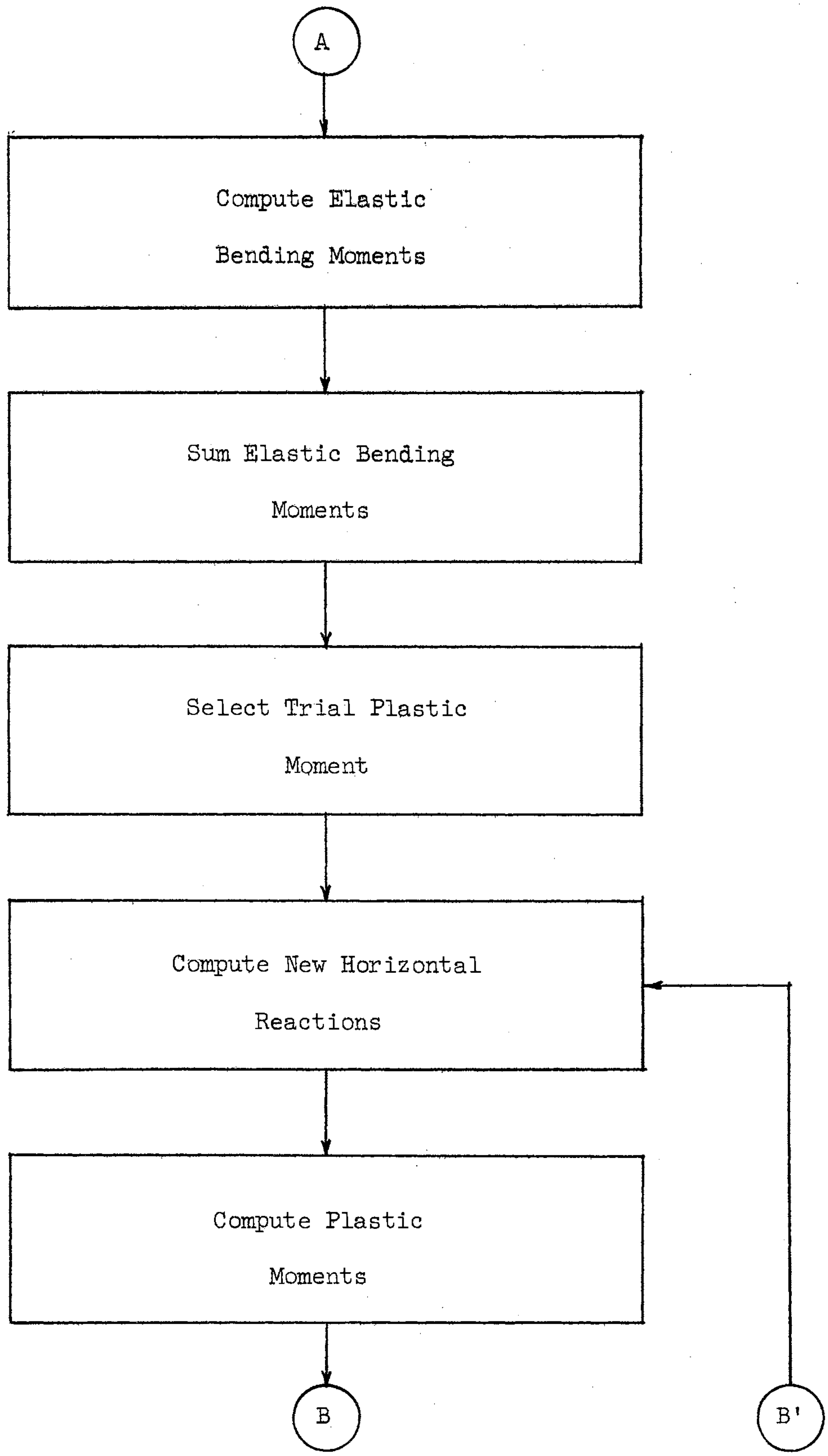
ACC, 65-70, designates the accuracy to which the plastic moments will be computed. The accuracy is one part in the number specified. If ACC is set equal to 50.00, the accuracy requested is 1 part in 50.00 or 2 per cent.

The data card just discussed will be followed by a data card for each point load to be considered. Each card will contain the point load value, WP, 1-10, and the distance from the left end, A, 10-20. The cards must be placed in order beginning with the point load on the left.

Figure 13 shows a flow diagram of the computer program, which will help in following the program procedure.

The last part of the appendix is a listing of the actual computer program as written in Fortran IV.


Read Input Data
90
IJKLM = 1, NUMB
Go To (2, 8) ISW
2
Compute Geometry
8
A


A
Compute Elastic Bending Moments
Sum Elastic Bending Moments
Select Trial Plastic Moment
Compute New Horizontal Reactions
Compute Plastic Moments
B
B'

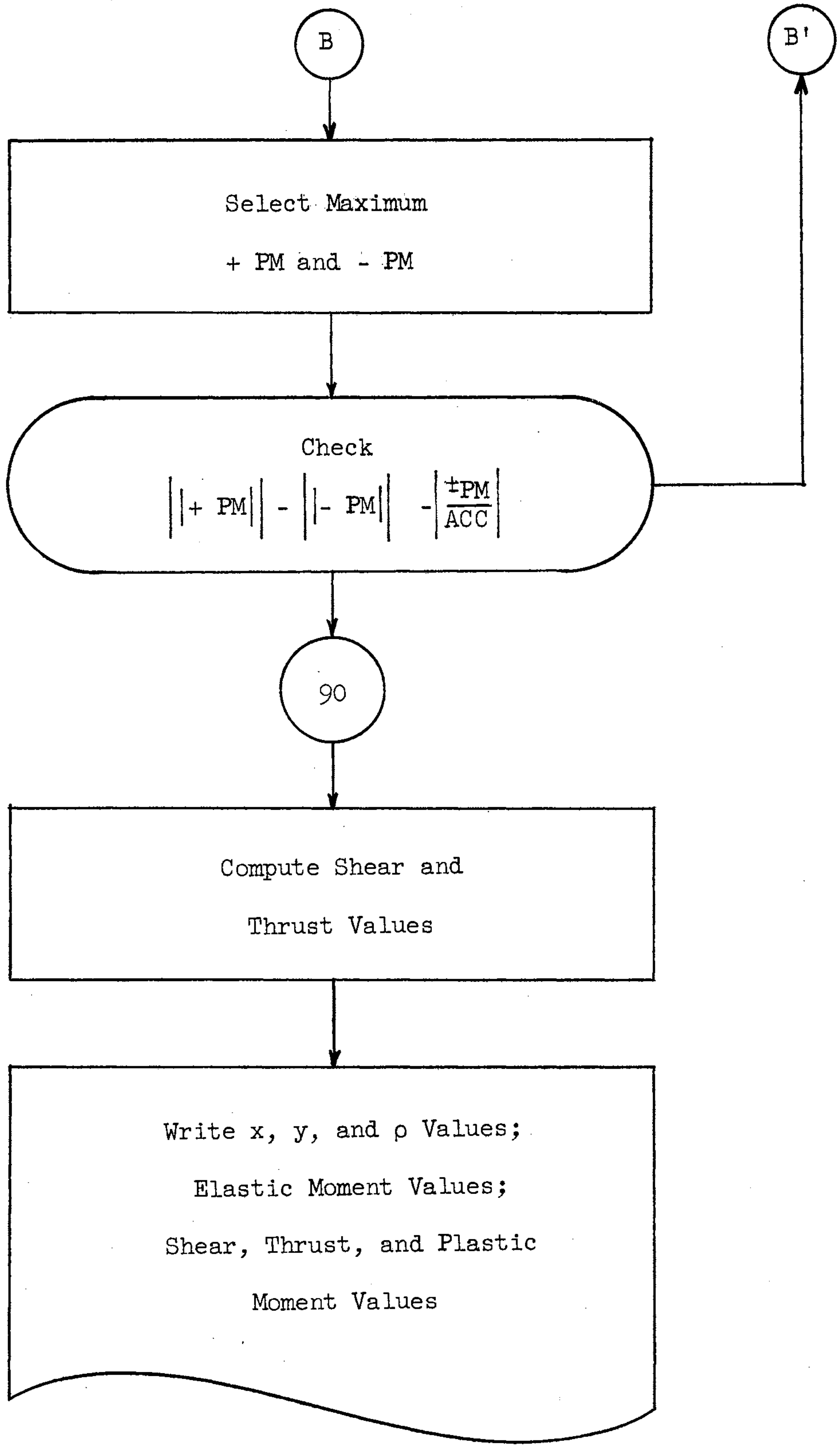

B
B'
Select Maximum
+ PM and - PM
Check
||+ PM| - |- PM|| - |±PM/ACC|
90
Compute Shear and
Thrust Values
Write x, y, and ρ Values;
Elastic Moment Values;
Shear, Thrust, and Plastic
Moment Values

Figure 13

```
1.00      75.00
2100.00 25.00  0.19  0.30   0.0      1  0.03  0.10  0.04      1 50.00
1.00      75.00
2100.00 25.00  0.19   0.0  0.30      1  0.08  0.29  0.10      1 50.00
1.00      75.00
2100.00 25.00  0.19  0.15   0.0      1  0.08  0.29  0.10      1 50.00
1.00      75.00
2100.00 25.00  0.25   0.0  0.40      1   0.0   0.0   0.0      1 50.00
1.00      75.00
1100.00 25.00  0.25  0.40   0.0      1   0.0   0.0   0.0      1 50.00
5
```

FORTRAN STATEMENT

IDENTIFICATION

```
MON$$      JOB  250540001THESIS
MON$$      ASGN MGO,A2
MON$$      ASGN MJB,A3
MON$$      ASGN MW1,A4
MON$$      ASGN MW2,A5
MON$$      ASGN MW3,A6
MON$$      MODE GO,TEST
MON$$      EXEQ FORTRAN,SOF,SIU,,,,,THESIS
      DIMENSION X(21), Y(21),  RHQ(21), BMTOT(21) ,WP1(20),A1(20)
      DIMENSION THETA(20)
   98 FORMAT (I6)
   99 FORMAT (1H1)
  100 FORMAT (I4,5F6.2,I6,3F6.2,I6,F6.2)
  101 FORMAT (/72H   ISW  SPAN  RISE    WD    WL    WDR     NP    W1    W2
     1    W3     M     ACC)
  102 FORMAT (I6,5F6.2,I6,3F6.2,I6,F6.2)
  103 FORMAT (2F10.0)
  104 FORMAT (/65H POSITION     BMD     BML     BMDR     BMP     BMW
     1BMH   BMTOT)
  105 FORMAT (15X,5H  I =,I4,8H    WP =,F8.2,8H      A =,F8.2)
  106 FORMAT(/27X,1HX,14X,1HY,4X,14HRHO IN DEGREES)
  109 FORMAT(/10X,15HTHE HINGE IS AT,I3,5H  XH=,F7.2,5H  YH=,F7.2)
  110 FORMAT(/10X,27HTHE FIRST PLASTIC MOMENT IS,F8.2,//10X,28HTHE SECON
     1D PLASTIC MOMENT IS,F8.2)
  111 FORMAT (1X,I8,7F8.2)
  113 FORMAT(I15,3F15.2)
      READ (1,98) NUMB
      REWIND 6
      WRITE (6) NUMB
      DO 90 IJKLM = 1,NUMB
    1 READ (1,100) ISW, SPAN, RISE,WD,  WL, WDR,NP,W1, W2, W3, M, ACC
      YH = 0.0
      PM = YH
      ICGT = 1
      H2 =0.0
      H3 = 0.0
      H4 = 0.0
      HW = 0.0
      VL2 = 0.0
      VL3 = 0.0
      VL4 = 0.0
      VLW = 0.0
      GO TO (2,8), ISW
    2 S1 = SPAN*0.5
      S2 = S1*S1
      S3 = S2*S1
      R1 = (RISE*RISE+S2)/(2.0*RISE)
      R2 = R1*R1
      E1 = R1 - RISE
      E2 = E1*E1
      AHPH = ATAN(S1/E1)
      PHI=2.0*AHPH
      X(1) = 0.0
      Y(1) =  0.0
```

```
      ALFA = 1.5707963- AHPH
      THTA1=ALFA+0.5*AHPH
      THTA2=THTA1+AHPH
      DRHO = 0.1*AHPH
      RHQ(1) = ALFA
      DO 5  I =2,21
      I1 = I - 1
      RHQ(I) = RHQ(I1) + DRHO
      RHO = RHQ(I)
      X(I) = S1 - R1*COS(RHO)
    5 Y(I) = R1*SIN(RHO) - E1
      WRITE(3,99)
      WRITE(3,106)
      DO 6 I = 1,21
      I1 = I-1
      RHDEG = 57.29578*RHQ(I)
    6 WRITE(3,113)I1,X(I),Y(I),RHDEG
    8 VL1=R1*WD*PHI*0.5
      WRITE (3,99)
      WRITE (3,101)
      WRITE(3,102) ISW, SPAN, RISE,WD,  WL, WDR,NP,W1, W2, W3, M, ACC
      ADEM = 2.0*AHPH *(R2*0.5+E2)-3.0*E1*S1
      ANUM=PHI*(-9.0*E2+S2-2.0*S1*E1*PHI)+18.0*E1*S1
      H1=WD*R1*0.25*ANUM/ADEM
      IF (WL.EQ.0.0) GO TO 9
      VL2 = WL*S1
      ANUM = 1.3333333*S3+E1*2.0*AHPH*(R2*0.5-S2)-E2*S1
      H2 = WL*ANUM*0.5/ADEM
    9 IF (WDR.EQ.0.0) GO TO 10
      VL3 = WDR*S1*0.25
      ANUM = 8.0*S3-3.0*E1*2.0*AHPH*(S2-E2)-6.0*E2*S1
      H3 = WDR*ANUM/(24.0*ADEM)
   10 IF (NP .EQ. 0)  GO TO 11
      READ (1,103)    (WP1(I),A1(I),I=1,NP)
      WRITE (3,105)(I,WP1(I),A1(I),I=1,NP)
      VL4 = 0.0
      H4 = 0.0
      DO 20 I=1,NP
      WP = WP1(I)
      A = A1(I)
      C = S1-A
      B = C+S1
      D = SQRT (R2 - C*C)
      THETA (I) = ATAN(C/D)
      F = D - E1
      GAMMA = THETA(I) + AHPH
      THETA(I) = 1.5707963 - THETA(I)
      ANUM = 0.5*B*(2.0*S1-E1*2.0*AHPH)-B*C-F*F/2.0+E1*C*GAMMA
      H4 = WP*ANUM/ADEM + H4
   20 VL4 = WP*B/SPAN + VL4
   11 IF (W3.EQ.0.0) GO TO 12
      VW1=W1*S1
      VW2=(2.0*W2*R2*SIN(0.375*PHI)*SIN(0.375*PHI))/SPAN
      VW3=(2.0*W3*R2*SIN(0.125*PHI)*SIN(0.125*PHI))/SPAN
```

```
      VWL=VW1-VW2+VW3
      ANUM=2.0*S3-2.0*R2*S1-E1*S2*PHI+1.5*E1*R2*PHI-E2*S1
      HW1=W1*ANUM/ADEM
      ARJ = 0.375*PHI
      ARG = THTA1
      COEF1 = 0.75
      COEF = 0.375
      ANUM1=(SIN(ARJ)**2)*(SPAN-E1*PHI)-S1+COEF1*E1*PHI-R1*COS(ARG)
      ANUM2=-(0.5*E1*S1*SIN(ARG))/R1+COEF*R1*PHI*SIN(ARG)
      ANUM3=-(0.5*E2*COS(ARG))/R1
      ANUM=ANUM1+ANUM2+ANUM3
      HW2=W2*R2*ANUM/ADEM
      ARJ = 0.125*PHI
      ARG = THTA2
      COEF1 = 0.25
      COEF = 0.125
      ANUM1=(SIN(ARJ)**2)*(SPAN-E1*PHI)-S1+COEF1*E1*PHI-R1*COS(ARG)
      ANUM2=-(0.5*E1*S1*SIN(ARG))/R1+COEF*R1*PHI*SIN(ARG)
      ANUM3=-(0.5*E2*COS(ARG))/R1
      ANUM=ANUM1+ANUM2+ANUM3
      HW3=W3*R2*ANUM/ADEM
      HW=HW1-HW2+HW3
   12 BMD = 0.0
      BML = BMD
      BMP = BMD
      BMW = BMD
      BMDR = BMD
      BMH =  BMD
   29 I1 = 0
      I = 1
      BMTOT(I) = 0.0
      WRITE (3,104)
      WRITE(3,111)I1,BMD, BML, BMDR, BMP, BMW, BMH , BMTOT(I)
   30 DO 50  I=2,21
      I1 = I-1
      GLE = RHQ(I)
      IF(WD.EQ.0.0) GO TO 41
      XBAR=-R1*COS(GLE)+Y(I)/(GLE-ALFA)
      BMD=VL1*X(I)-H1*Y(I)-WD*R1*(GLE-ALFA)*XBAR
   41 IF (WL.EQ.0.0) GO TO 42
      BML     = VL2*X(I)-H2*Y(I)-WL*X(I)*X(I)*0.5
   42 IF (NP.EQ. 0) GO TO 43
       BMP     = VL4*X(I) - H4*Y(I)
      DO 25 JJ=1,NP
      IF (GLE.LE. THETA(JJ))GO TO 43
      A = A1(JJ)
      WP = WP1(JJ)
   25 BMP = BMP-WP*(X(I)-A)
   43 IF (WDR .EQ. 0.0) GO TO 39
   44 BMDR = VL3*X(I) - H3*Y(I)
      IF (S1.GE.X(I)) GO TO 39
   40 BMDR = BMDR-WDR*(X(I)-S1)*(X(I)-S1)*0.5
   39 IF (W3.EQ.0.0) GO TO 46
      BMW1 = VW1*X(I)-HW *Y(I)-2.0*W1*R2*SIN(0.5*(GLE-ALFA))**2
```

```
      BMW2=VW2*X(I)
      BMW3=VW3*X(I)
      IF (GLE.LT.THTA1) GO TO 45
      BMW2=BMW2-2.0*W2*R2*SIN(0.5*(GLE-THTA1))*SIN(0.5*(GLE-THTA1))
      IF (GLE.LT.THTA2) GO TO 45
      BMW3 = BMW3-2.0*W3*R2*SIN(0.5*(GLE-THTA2))**2
   45 BMW=BMW1-BMW2+BMW3
   46 IF (PM.EQ.0.0) GO TO 47
      BMH = PM*Y(I)/YH
   47 BMTOT(I) = BMD+BML+BMP+BMW+BMDR+BMH
      GO TO (49,50,50),ICGT
   49 WRITE(3,111)I1,BMD, BML, BMDR, BMP, BMW, BMH , BMTOT(I)
   50 CONTINUE
      GO TO (51,1),M
   51 CHECK = BMTOT(1)
      DO 60  I=2,20
      IF (PM*BMTOT(I) .GT. 0.0) GO TO 60
      IF(ABS(CHECK).GT.ABS(BMTOT(I)))GO TO 60
   55 CHECK = BMTOT(I)
      IJ = I
   60  CONTINUE
      GO TO (63,65,64),ICGT
   63 PM = CHECK
   64 ICGT = ICGT + 1
      RH = RHQ(IJ)
      XH = X(IJ)
      YH = Y(IJ)
      GO TO (66,66,67,67),ICGT
   66 YH1 = YH
      XH1 = XH
      RH1 = RH
   67 IJ = IJ - 1
      WRITE (3,109) IJ, XH, YH
      GO TO (1,71,1,85),ICGT
   65 IABC = ACC*CHECK/PM + ACC
      IF (IABC .EQ. 0) ICGT = 3
   70 PM = (PM-CHECK)*0.5
      GO TO 30
   71 IF (WD .EQ. 0.0) GO TO 73
      XBAR = -R1*COS(RH) + YH/(RH-ALFA)
      H1=(VL1*XH-WD*R1*(RH-ALFA)*XBAR)/YH
   73 IF (WL .EQ. 0.0) GO TO 75
   74 H2 = (VL2*XH-   WL*XH*XH/2.0)/YH
   75 IF (WDR .EQ. 0.0 ) GO TO 77
      IF(XH.GT.S1) GO TO 76
      H3=VL3*XH/YH
      GO TO 77
   76 H3 = (VL3*XH-   WDR*(XH-S1)*(XH-S1)/2.0)/YH
   77 IF (NP.EQ.0) GO TO 79
      H4 = VL4*XH/YH
      DO 84 KK = 1,NP
      IF (RH.LE.THETA(KK)) GO TO 79
   84 H4 = H4 - WP1(KK)*(XH - A1(KK))/YH
   79 IF (W3.EQ.0.0) GO TO 30
```

```
      HW=(VWL*XH-2.0*W1*R2*SIN(0.5*(RH-ALFA))**2)/YH
      IF (RH.LE.THTA1) GO TO 30
      HW=HW+(2.0*W2*R2*SIN(0.5*(RH-THTA1))*SIN(0.5*(RH-THTA1)))/YH
      IF (RH.LE.THTA2) GO TO 30
      HW=HW-(2.0*W3*R2*SIN(0.5*(RH-THTA2))*SIN(0.5*(RH-THTA2)))/YH
      GO TO 30
   85 WRITE (3,110) PM, CHECK
      WRITE (6) X,Y,RHQ,THETA,THTA1,THTA2,VL1,VL2,VL3,VWL,PM,XH1,YH1,VL4
      WRITE (6) ALFA,H1,H2,H3,H4,HW,RH,BMTOT,S1,R1,WD,WL,W1,W2,W3,A,PHI
   90 WRITE (6)  WDR,AHPH,A1,WP1,NP
      CALL EXIT
      END
     MON$$      EXEQ LINKLOAD
                PHASEPROGRAM
                CALL THESIS
     MON$$      EXEQ PROGRAM,MJB

     5
  1100.00 25.00  0.25  0.40   0.0      1    0.0    0.0    0.0      1 50.00
     1.00     75.00
  2100.00 25.00  0.25   0.0  0.40      1    0.0    0.0    0.0      1 50.00
     1.00     75.00
  2100.00 25.00  0.19  0.15   0.0      1   0.08   0.29   0.10      1 50.00
     1.00     75.00
  2100.00 25.00  0.19   0.0  0.30      1   0.08   0.29   0.10      1 50.00
     1.00     75.00
  2100.00 25.00  0.19  0.30   0.0      1   0.03   0.10   0.04      1 50.00
     1.00     75.00
     MON$$      JOB   250540001THESIS
     MON$$      ASGN MGO,A2
     MON$$      ASGN MJB,A3
     MON$$      ASGN MW1,A4
     MON$$      ASGN MW2,A5
     MON$$      ASGN MW3,A6
     MON$$      MODE GO,TEST
     MON$$      EXEQ FORTRAN,SOF,SIU,,,,,THESIS
      DIMENSION THETA(20)
      DIMENSION X(21), Y(21),  RHQ(21), BMTOT(21) ,WP1(20),A1(20)
   99 FORMAT (1H1)
  112 FORMAT(7X,8HPOSITION,10X,5HSHEAR,9X,6HTHRUST,4X,14HBENDING MOMENT)
  113 FORMAT(I15,3F15.2)
  115 FORMAT(/10X,13HEND REACTIONS,/10X,8HLEFT END,12X,9HRIGHT END)
  116 FORMAT(10X,10HVERTICAL  ,F8.2,3X,10HVERTICAL  ,F8.2)
  117 FORMAT(10X,10HHORIZONTAL,F8.2,3X,10HHORIZONTAL,F8.2)
      REWIND 6
      READ (6) NUMB
      DO 202  KKK = 1, NUMB
      READ  (6) X,Y,RHQ,THETA,THTA1,THTA2,VL1,VL2,VL3,VWL,PM,XH1,YH1,VL4
      READ  (6) ALFA,H1,H2,H3,H4,HW,RH,BMTOT,S1,R1,WD,WL,W1,W2,W3,A,PHI
      READ  (6)  WDR,AHPH,A1,WP1,NP
      WRITE (3,99)
      WRITE (3,112)
      HP = -PM/YH1
      VS1 = 0.0
```

```
      VS2 = 0.0
      VS3 = 0.0
      VS4 = 0.0
      VSW = 0.0
      HS1 = 0.0
      HS2 = 0.0
      HS3 = 0.0
      HS4 = 0.0
      HSW = 0.0
      HPS = 0.0
      VT1 = 0.0
      VT2 = 0.0
      VT3 = 0.0
      VT4 = 0.0
      VTW = 0.0
      HT1 = 0.0
      HT2 = 0.0
      HT3 = 0.0
      HT4 = 0.0
      HTW = 0.0
      HPT = 0.0
      HCON = 0.0
      VCON = 0.0
      SHEAR = 0.0
      THRST = 0.0
      DO 200 I = 1,21
      GLE = RHQ(I)
      IF (WD.EQ.0.0) GO TO 118
      HS1 = H1*COS(GLE)
      VS1 = (VL1-R1*WD*(GLE-ALFA))*SIN(GLE)
      HT1 = H1*SIN(GLE)
      VT1 = (VL1-R1*WD*(GLE-ALFA))*COS(GLE)
  118 IF (WL.EQ.0.0) GO TO 119
      HS2 = H2*COS(GLE)
      VS2 = (VL2-WL*X(I))*SIN(GLE)
      HT2 = H2*SIN(GLE)
      VT2 = (VL2-WL*X(I))*COS(GLE)
  119 IF (WDR.EQ.0.0) GO TO 129
      HS3 = H3*COS(GLE)
      VS3 = VL3*SIN(GLE)
      ACON = ALFA + AHPH
      IF(GLE.LE.ACON)GO TO 120
      VS3 = VS3-(WDR*(X(I)-S1))*SIN(GLE)
  120 HT3 = H3*SIN(GLE)
      VT3 = VL3*COS(GLE)
      IF(GLE.LE.ACON)GO TO 129
      VT3 = VT3-(WDR*(X(I)-S1))*COS(GLE)
  129 IF (NP.EQ. 0) GO TO 149
      HS4 = H4*COS(GLE)
      VS4 = VL4*SIN(GLE)
      HT4 = H4*SIN(GLE)
      VT4 = VL4*COS(GLE)
      DO 148 JKL = 1,NP
      WP = WP1(JKL)
```

```
    IF(GLE.LE.THETA(JKL))GO TO 149
    VS4 = VS4-WP*SIN(GLE)
148 VT4 = VT4-WP*COS(GLE)
149 IF (W3.EQ.0.0) GO TO 161
150 HCON = HW+W1*R1*(SIN(GLE)-SIN(ALFA))
    VCON = VWL-W1*R1*(COS(ALFA)-COS(GLE))
    IF (GLE.LE.THTA1) GO TO 160
    HCON = HCON-W2*R1*(SIN(GLE)-SIN(THTA1))
    VCON = VCON + W2*R1*(COS(THTA1)-COS(GLE))
    IF (GLE.LE.THTA2) GO TO 160
    HCON = HCON+W3*R1*(SIN(GLE)-SIN(THTA2))
    VCON = VCON-W3*R1*(COS(THTA2)-COS(GLE))
160 HSW = HCON*COS(GLE)
    VSW = VCON*SIN(GLE)
    HTW = HCON*SIN(GLE)
    VTW = VCON*COS(GLE)
161 HPS = HP*COS(GLE)
    HPT = HP*SIN(GLE)
    SHEAR = VS1+VS2+VS3+VS4+VSW-HS1-HS2-HS3-HS4-HSW-HPS
    THRST = VT1+VT2+VT3+VT4+VTW+HT1+HT2+HT3+HT4+HTW+HPT
    I1 = I-1
200 WRITE (3,113) I1,SHEAR,THRST,BMTOT(I)
    VL = VL1+VL2+VL3+VL4+VWL
    HL = H1+H2+H3+H4+HW+HP
    VR = -SHEAR*SIN(ALFA)+THRST*COS(ALFA)
    HR =  SHEAR*COS(ALFA)+THRST*SIN(ALFA)
    WRITE(3,115)
    WRITE(3,116)VL,VR
202 WRITE(3,117)HL,HR
    CALL EXIT
    END
   MON$$      EXEQ LINKLOAD
              PHASEPROGRAM
              CALL THESIS
   MON$$      EXEQ PROGRAM,MJB
```

VITA

Robert C. Cornforth

Candidate for the Degree of

Master of Architectural Engineering

Thesis: PLASTIC ANALYSIS OF TWO HINGED CIRCULAR ARCHES

Major Field: Architectural Engineering (Structures)

Biographical:

Personal Data: Born in Guthrie, Oklahoma, February 18, 1937, the son of Louis C. and Lillian L. Cornforth.

Education: Graduated from Classen High School, Oklahoma City, Oklahoma, in May, 1955. Received the degree of Bachelor of Architectural Engineering from Oklahoma State University in May, 1961. Completed requirements for the Master of Architectural Engineering degree in August, 1964.

Professional Experience: R. T. Mitchel Construction Company, Oklahoma City, Oklahoma, from June, 1959 to March, 1961. United States Army from March, 1961 to January, 1963. Graduate assistant, School of Architecture, Oklahoma State University, from January, 1963 to January, 1964. Structural Engineer, Sorey, Hill, and Sorey, Architects and Engineers, Oklahoma City, Oklahoma from June, 1963 to present.

Organizations: Chi Epsilon, Sigma Tau, Blue Key.