TABLES AND NOMOGRAPHIC CHARTS FOR THE PRELIMINARY ANALYSIS OF RIGID FRAMES WITH CURVED MEMBERS

By

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TABLES AND NOMOGRAPHIC CHARTS FOR

THE PRELIMINARY ANALYSIS OF RIGID

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PREFACE

This study is the first part of a series, for the development of charts and tables, for preliminary analysis of rigid frames with curved members, directed by Professor Jan J. Tuma.

The writer wishes to acknowledge his indebtedness to Professor Tuma for his guidance and assistance in preparation of this thesis and for acting as the writer's major adviser, to Dr. John W. Hamblen for the use of the Oklahoma State University Computing Center facilities and for his advice in their use, to the staff of the School of Civil Engineering for their guidance and instruction, and to the Continental Oil Company for the scholarship which made this year of graduate study possible.

D. C. C.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF CHARTS

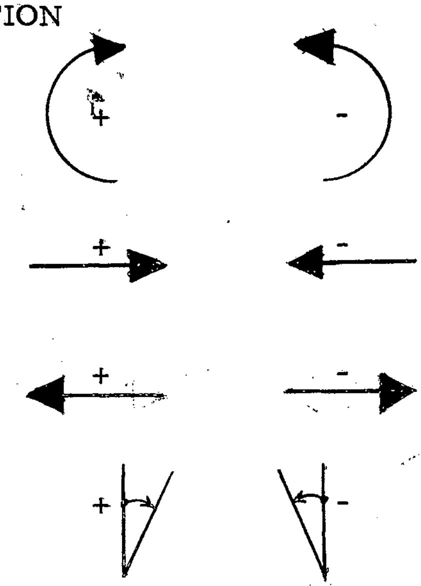
LIST OF FIGURES

NOMENCLATURE

M	End moment
Q	End moment coefficient
L	Length of span
H	Horizontal thrust
V	Shear
w	Intensity of load
I	Moment of inertia
E	Modulus of elasticity
r	Girder height
h	Column length
α	Column length parameter
β	Girder height parameter
θ	Angular rotation of a joint
Δx	Linear displacement in horizontal direction
e	Distance to elastic center from lowest part of girder
BM	Bending moment of basic structure
$\overline{A}$	Elastic area $= \int \frac{ds}{EI}$
$\overline{I}_{xx}$	Moment of inertia of elastic area about horizontal axis $= \int y^2 \frac{ds}{EI}$
$\overline{I}_{yy}$	Moment of inertia of elastic area about vertical axis $= \int x^2 \frac{ds}{EI}$
$\overline{P}$	Elastic weight $= \int \frac{BM\ ds}{EI}$
$\overline{M}_{xx}$	Static moment of elastic weight about horizontal axis $= \int \frac{BM\ y\ ds}{EI}$
$\overline{M}_{yy}$	Static moment of elastic weight about vertical axis $= \int \frac{BM\ x\ ds}{EI}$

SIGN CONVENTION

1. Moments
2. Horizontal thrusts and shears
3. Horizontal displacements
4. Angular rotations



PART I

INTRODUCTION

The analysis of continuous rigid frames with curved members, by energy or deformation methods, is a laborious and time consuming procedure. The difficulty arises in the preparation and solution of a system of simultaneous equations. In this study, tables and nomographic charts, for the preliminary analysis of two, three, and four span rigid frames (Fig. 1 a, b, c) are presented. The tables and charts were evaluated by the use of slope-deflection equations, as developed by Ungson[4], Parcel and Moorman[2], and Havner[1].

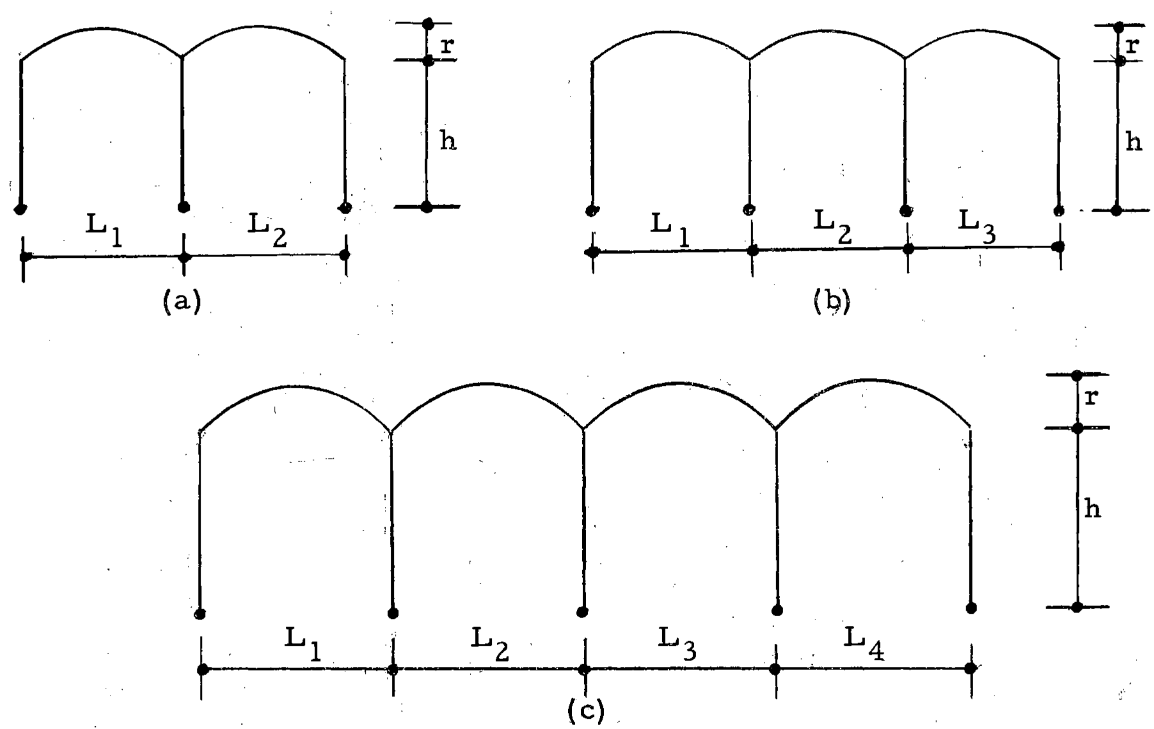


Fig. 1
Typical Rigid Frames with Curved Members

Frames, with girders of circular and parabolic shape and straight vertical columns, were considered. The column length and girder rise, were expressed as a function of the span length, and denoted by αL and βL respectively. The values of the parameters α and β were chosen to cover a range of practical limits.

The solution of the numerical equations was done on the IBM 650 digital computer.

Examples for the use of the charts and tables are given in Part IV.

The nomenclature and sign convention were chosen to conform with that generally used in the application of the slope-deflection method, and are stated in the introductory part of this thesis.

PART II

EQUATIONS OF DEFORMATION

1. Slope-Deflection Equations

The general slope deflection equations for a curved member of constant cross section, with axis curved in a circular arc, as shown in Fig. 2, are:

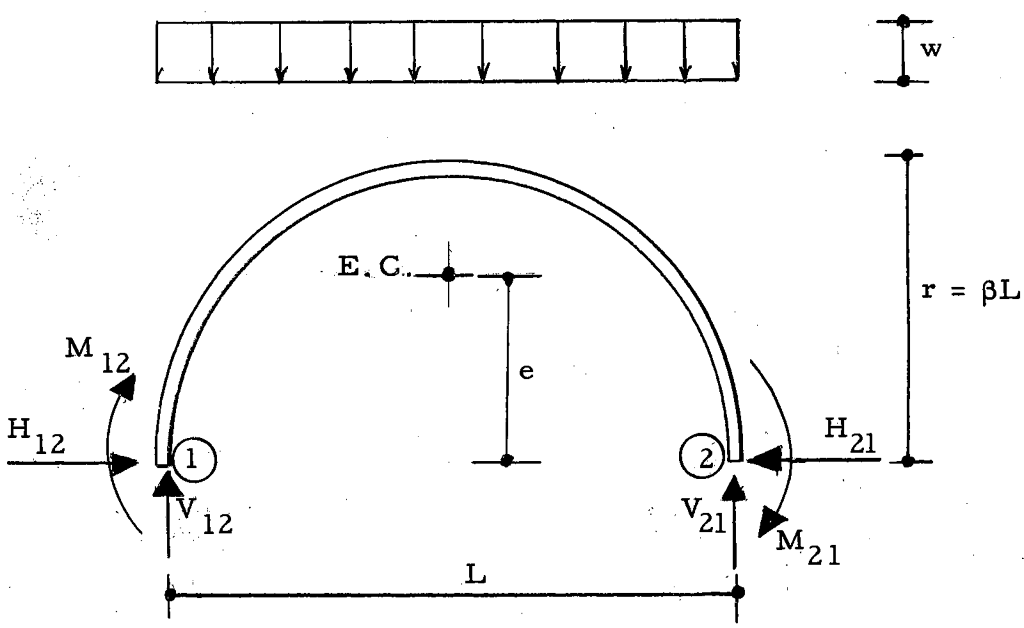


Fig. 2
Uniformly Loaded Curved Member of Circular Shape

$$M_{12} = \frac{EI}{L}\,\theta_1\left(\frac{L}{\bar{A}} + \frac{L^3}{4\bar{I}_{yy}} + \frac{e^2L}{\bar{I}_{xx}}\right) - \frac{EI}{L}\,\theta_2\left(\frac{L^3}{4\bar{I}_{yy}} - \frac{L}{\bar{A}} - \frac{e^2L}{\bar{I}_{xx}}\right)$$

$$+\left(\frac{erL}{\bar{I}_{xx}}\right)\frac{EI}{Lr}\left(\Delta_{2x} - \Delta_{1x}\right) - BM_x - \left(\frac{\bar{P}}{\bar{A}} + \frac{\bar{M}_{yy}}{\bar{I}_{yy}}\,\frac{L}{2} + \frac{e\bar{M}_{xx}}{\bar{I}_{xx}}\right) \qquad (1)$$

$$M_{21} = \frac{EI}{L}\theta_2\left(\frac{L}{\overline{A}} + \frac{L^3}{4\overline{I}_{yy}} + \frac{e^2L}{\overline{I}_{xx}}\right) - \frac{EI}{L}\theta_1\left(\frac{L^3}{4\overline{I}_{yy}} - \frac{L}{\overline{A}} - \frac{e^2L}{\overline{I}_{xx}}\right)$$

$$- \left(\frac{erL}{\overline{I}_{xx}}\right)\frac{EI}{Lr}(\Delta_{2x} - \Delta_{1x}) - BM_x - \left(\frac{\overline{P}}{\overline{A}} + \frac{\overline{M}_{yy}}{\overline{I}_{yy}}\frac{L}{2} + \frac{e\overline{M}_{xx}}{\overline{I}_{xx}}\right) \qquad (2)$$

$$H_{12} = H_{21} = \frac{EI}{Lr}\left(\frac{erL}{\overline{I}_{xx}}\right) - \frac{EI\theta_2}{Lr}\left(\frac{erL}{\overline{I}_{xx}}\right) + \frac{EI}{Lre}(\Delta_{2x} - \Delta_{1x})\left(\frac{erL}{\overline{I}_{xx}}\right)$$

$$+ \frac{\overline{M}_{xx}}{\overline{I}_{xx}} \qquad (3)$$

The stiffness and carry-over factors, that are stated in column analogy terms, are denoted by a constant (C), and the fixed end moments and fixed end thrusts are expressed as a function of wL .

$$\frac{L}{\overline{A}} + \frac{L^3}{4\overline{I}_{yy}} + \frac{e^2L}{\overline{I}_{xx}} = C_1$$

$$\frac{L^3}{4\overline{I}_{yy}} - \frac{L}{\overline{A}} - \frac{e^2L}{\overline{I}_{xx}} = C_2$$

$$\frac{erL}{\overline{I}_{xx}} = C_4$$

$$BM_x - \left(\frac{\overline{P}}{\overline{A}} + \frac{\overline{M}_{yy}}{\overline{I}_{yy}}\frac{L}{2} + \frac{e\overline{M}_{xx}}{\overline{I}_{xx}}\right) = \text{Fixed End Moment} = iwL^2$$

$$\frac{\overline{M}_{xx}}{\overline{I}_{xx}} = \text{Fixed End Thrust} = jwL$$

The slope deflection equations in terms of the parameter β are:

$$M_{12} = C_1\frac{EI}{L}\theta_1 - C_2\frac{EI}{L}\theta_2 + \frac{C_4}{\beta}\frac{EI}{L^2}(\Delta_{2x} - \Delta_{1x}) + iwL^2 \qquad (1a)$$

$$M_{21} = C_1 \frac{EI}{L}\theta_2 - C_2 \frac{EI}{L}\theta_1 - \frac{C_4}{\beta}\frac{EI}{L^2}(\Delta_{2x} - \Delta_{1x}) - iwL^2 \qquad (2a)$$

$$H_{12} = H_{21} = \frac{C_4}{\beta}\frac{EI}{L^2}\theta_1 - \frac{C_4}{\beta}\frac{EI}{L^2}\theta_2 + \frac{C_4}{e\beta^2}\frac{EI}{L^3}(\Delta_{2x} - \Delta_{1x}) + jwL \qquad (3a)$$

The values of C_1, C_2, C_4, i, j and e are taken from the book, Analysis of Statically Indeterminate Structures, by Parcel and Moorman[2].

The general slope deflection equations for a curved member, with axis curved as a second degree parabola, and so proportioned that the moment of inertia varies as the secant of the angle of inclination of the arch axis with the horizontal as shown in Fig. 3 are:

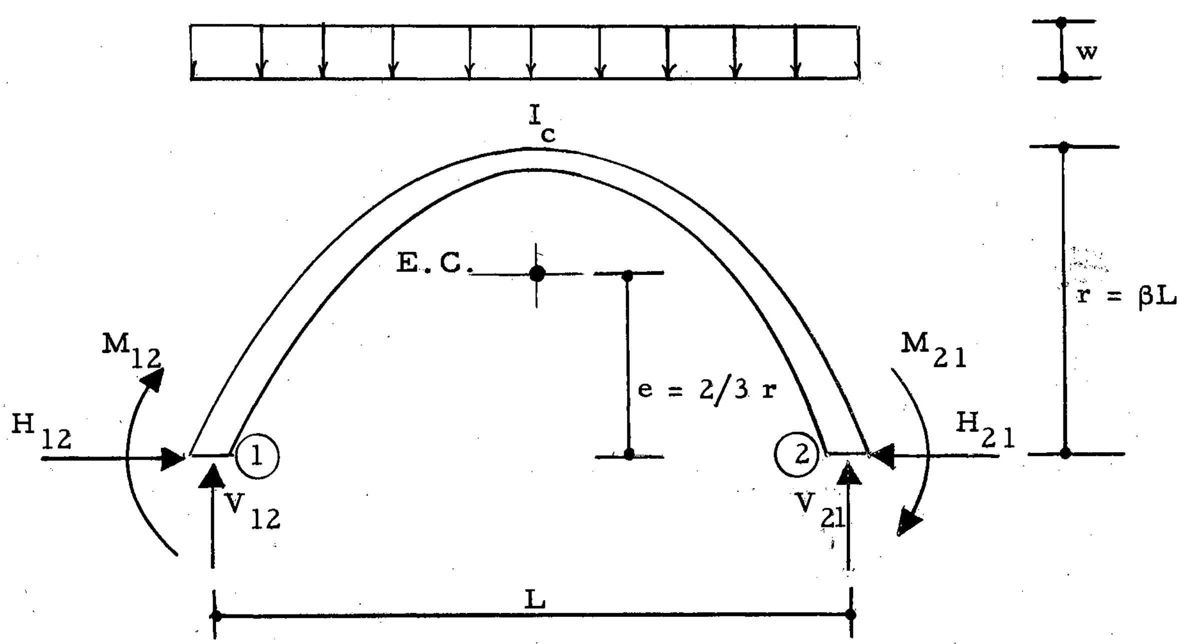


Fig. 3
Uniformly Loaded Curved Member of Parabolic Shape

$$M_{12} = 9 \frac{EI_c}{L} \theta_1 - 3 \frac{EI_c}{L} \theta_2 + \frac{7.5}{r} \frac{EI_c}{L} (\Delta_{2x} - \Delta_{1x}) \quad (4)$$

$$M_{21} = 9 \frac{EI_c}{L} \theta_2 - 3 \frac{EI_c}{L} \theta_1 - \frac{7.5}{r} \frac{EI_c}{L} (\Delta_{2x} - \Delta_{1x}) \quad (5)$$

$$H_{12} = H_{21} = \frac{7.5}{r} \frac{EI_c}{L} \theta_1 - \frac{7.5}{r} \frac{EI_c}{L} \theta_2 + \frac{7.5}{2/3\, r^2} \frac{EI_c}{L} (\Delta_{2x} - \Delta_{1x})$$

$$+ \frac{wL^2}{8r} \quad (6)$$

The slope deflection equations in terms of the parameter β are:

$$M_{12} = 9 \frac{EI_c}{L} \theta_1 - \frac{3EI_c}{L} \theta_2 + \frac{7.5}{\beta} \frac{EI_c}{L^2} (\Delta_{2x} - \Delta_{1x}) \quad (4a)$$

$$M_{21} = 9 \frac{EI_c}{L} \theta_2 - \frac{3EI_c}{L} \theta_1 - \frac{7.5}{\beta} \frac{EI_c}{L^2} (\Delta_{2x} - \Delta_{1x}) \quad (5a)$$

$$H_{12} = H_{21} = \frac{7.5}{\beta} \frac{EI_c}{L^2} \theta_1 - \frac{7.5}{\beta} \frac{EI_c}{L^2} \theta_2 + \frac{7.5}{2/3\, \beta^2} \frac{EI_c}{L^3} (\Delta_{2x} - \Delta_{1x})$$

$$+ \frac{wL}{8\beta} \quad (6a)$$

The slope deflection equations, for a straight column of constant cross section as shown in Fig. 4, are:

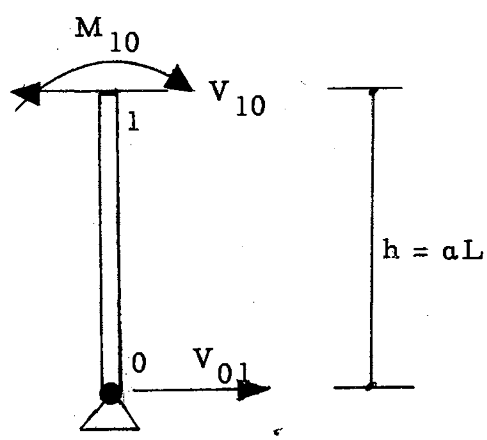


Fig. 4
Column of Constant Moment of Inertia

$$M_{10} = 3 \frac{EI}{h} \theta_1 + 3 \frac{EI}{h} \frac{\Delta_{1x}}{h} \tag{7}$$

$$M_{01} = 0 \tag{8}$$

$$V_{10} = V_{01} = \frac{3EI}{h^2} \theta_1 + \frac{3EI}{h^3} \Delta_{1x} \tag{9}$$

The slope deflection equations in terms of the parameter α are:

$$M_{10} = \frac{3}{\alpha} \frac{EI}{L} \theta_1 + \frac{3}{\alpha^2} \frac{EI}{L} \frac{\Delta_{1x}}{L} \tag{7a}$$

$$M_{01} = 0 \tag{8a}$$

$$V_{10} = V_{01} = \frac{3}{\alpha^2} \frac{EI}{L^2} \theta_1 + \frac{3}{\alpha^3} \frac{EI}{L^2} \frac{\Delta_{1x}}{L} \tag{9a}$$

2. Conditions of Static Equilibrium

Three conditions of static equilibrium can be written for any joint of a continuous rigid frame. These conditions are:

Equilibrium of Moments

$$\Sigma M = 0 \tag{10}$$

Equilibrium of Horizontal Forces

$$\Sigma F_x = 0 \tag{11}$$

Equilibrium of Vertical Forces

$$\Sigma F_y = 0 \tag{12}$$

Assuming that vertical displacement will not take place, condition (12) is not required for the solution.

Applications of equations (10) and (11) to a typical joint of a rigid frame, Fig. 5, results in the following equations of equilibrium:

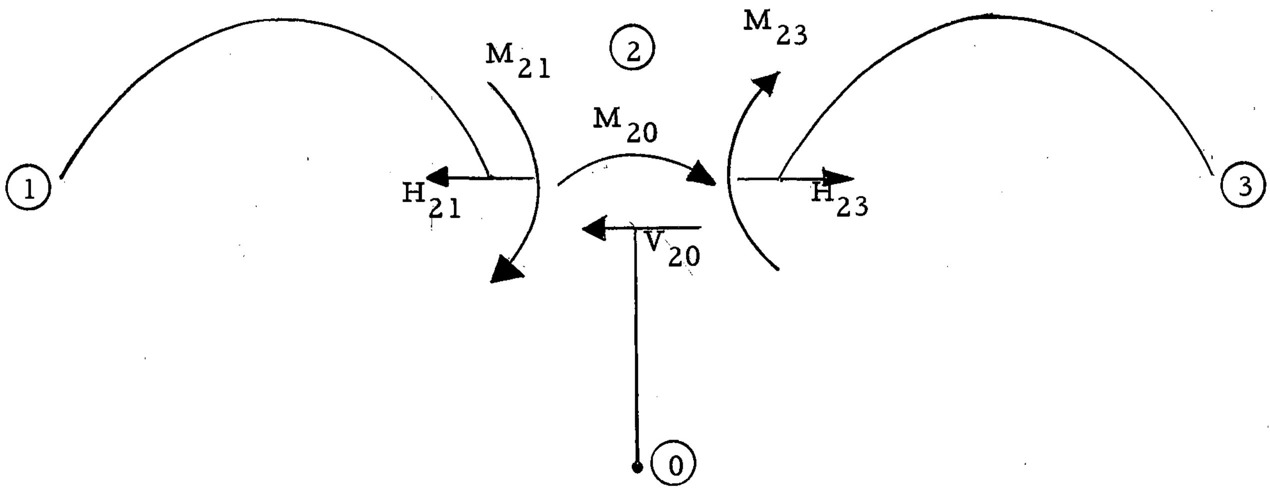


Fig. 5
Typical Joint of a Rigid Frame

$$\Sigma M = 0 : M_{21} + M_{20} + M_{23} = 0 \qquad (10a)$$

$$\Sigma F_{2x} = 0 : H_{21} + V_{20} - H_{23} = 0 \qquad (11a)$$

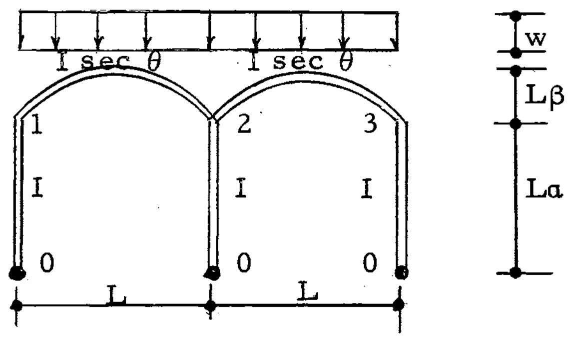
3. Matrix Tables

Using equations (10a) and (11a) in conjunction with the slope-deflection equations, a matrix of equations can be formed for the solution of the unknown rotation and displacement equivalents. There will be as many matrix equations as there are unknown deformations.

The matrices, for two, three, and four span symmetrical rigid frames, uniformly loaded, and with circular and parabolic girders, are given in Tables I, II, and III. The moment of inertia of the girders and the columns are equal.

TABLE I - MATRIX FOR TWO SPAN FRAME

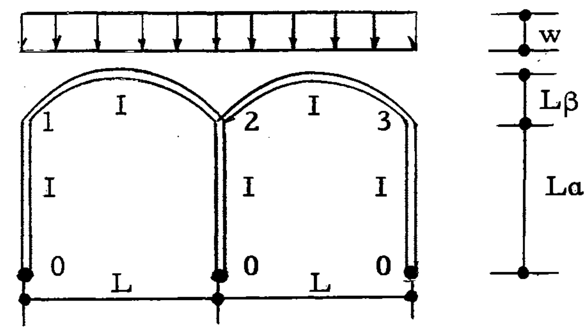
Parabolic Girder



$\theta_1 = -\theta_3 \qquad \theta_2 = 0 \qquad I_c = I$

$\Delta_{1x} = -\Delta_{3x} \qquad \Delta_{2x} = 0$

Circular Girder



$\theta_1 = -\theta_3 \qquad \theta_2 = 0$

$\Delta_{1x} = -\Delta_{3x} \qquad \Delta_{2x} = 0$

Matrix Constants (Parabolic Girder)

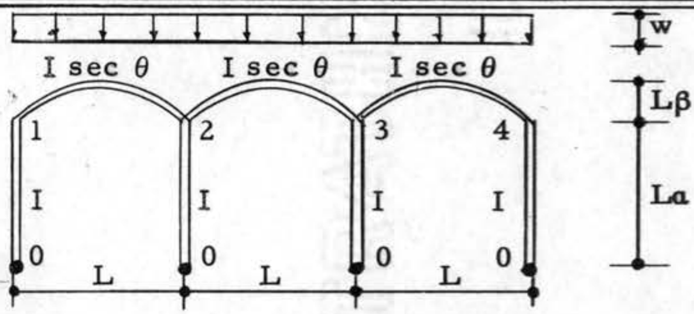
	$\frac{EI_c}{L}\theta_1$	$\frac{EI_c}{L}\Delta_1$	wL^2
$\Sigma M_1 = 0$	$9 + \frac{3}{\alpha}$	$\frac{3}{\alpha^2} - \frac{7.5}{\beta}$	0
$\Sigma F_{1x} = 0$	$\frac{7.5}{\beta} - \frac{3}{\alpha^2}$	$-\frac{11.25}{\beta^2} + \frac{3}{\alpha^3}$	$\frac{1}{8\beta}$

Matrix Constants (Circular Girder)

	$\frac{EI}{L}\theta_1$	$\frac{EI}{L}\Delta_1$	wL^2
$\Sigma M_1 = 0$	$C_1 + \frac{3}{\alpha}$	$\frac{3}{\alpha^2} - \frac{C_4}{\beta}$	i
$\Sigma F_{1x} = 0$	$\frac{C_4}{\beta} - \frac{3}{\alpha^2}$	$\frac{C_4}{e\beta^2} - \frac{3}{\alpha^3}$	j

TABLE II MATRIX FOR THREE SPAN FRAMES

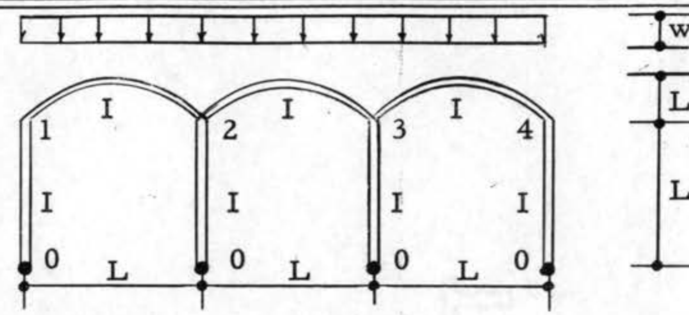
Parabolic Girder



$\theta_1 = -\theta_4$ $\qquad \theta_2 = -\theta_3$ $\qquad I_c = I$

$\Delta_1 = -\Delta_4$ $\qquad \Delta_2 = -\Delta_3$

Circular Girder



$\theta_1 = -\theta_4$ $\qquad \theta_2 = -\theta_3$

$\Delta_1 = -\Delta_4$ $\qquad \Delta_2 = -\Delta_3$

Matrix Constants (Parabolic Girder)

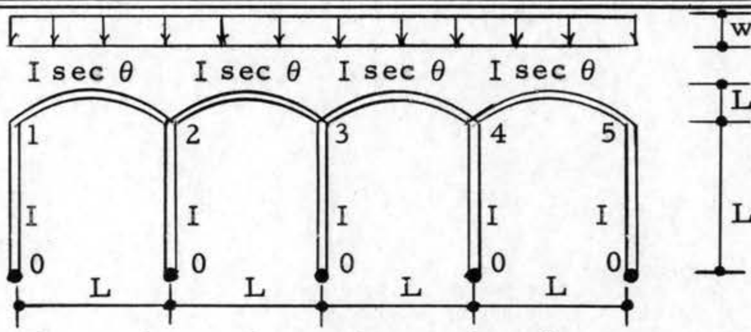
	$\frac{EI_c}{L}\theta_1$	$\frac{EI_c}{L}\theta_2$	$\frac{EI_c}{L}\Delta_1$	$\frac{EI_c}{L}\Delta_2$	wL^2
$\Sigma M_1=0$	$9 + \frac{3}{\alpha}$	-3	$\frac{3}{\alpha^2} - \frac{7.5}{\beta}$	$\frac{7.5}{\beta}$	0
$\Sigma M_2=0$	-3	$\frac{3}{\alpha} + 21$	$\frac{7.5}{\beta}$	$\frac{3}{\alpha^2} - \frac{22.5}{\beta}$	0
$\Sigma F_{1x}=0$	$\frac{7.5}{\beta} - \frac{3}{\alpha^2}$	$-\frac{7.5}{\beta}$	$-\frac{11.25}{\beta^2} - \frac{3}{\alpha^3}$	$\frac{11.25}{\beta^2}$	$\frac{1}{8\beta}$
$\Sigma F_{2x}=0$	$\frac{7.5}{\beta}$	$\frac{3}{\alpha^2} - \frac{22.5}{\beta}$	$-\frac{11.25}{\beta^2}$	$\frac{3}{\alpha^3} + \frac{33.75}{\beta^2}$	0

Matrix Constants (Circular Girder)

	$\frac{EI}{L}\theta_1$	$\frac{EI}{L}\theta_2$	$\frac{EI}{L}\Delta_1$	$\frac{EI}{L}\Delta_2$	wL^2
$\Sigma M_1=0$	$C_1 + \frac{3}{\alpha}$	$-C_2$	$\frac{3}{\alpha^2} - \frac{C_4}{\beta}$	$\frac{C_4}{\beta}$	i
$\Sigma M_2=0$	$-C_2$	$2C_1 + C_2 + \frac{3}{\alpha}$	$\frac{C_4}{\beta}$	$\frac{3}{\alpha^2} - \frac{3C_4}{\beta}$	0
$\Sigma F_{1x}=0$	$\frac{C_4}{\beta} - \frac{3}{\alpha^2}$	$-\frac{C_4}{\beta}$	$-\frac{C_4}{e\beta^2} - \frac{3}{\alpha^3}$	$\frac{C_4}{e\beta^2}$	j
$\Sigma F_{2x}=0$	$\frac{C_4}{\beta}$	$\frac{3}{\alpha^2} - \frac{3C_4}{\beta}$	$-\frac{C_4}{e\beta^2}$	$\frac{3}{\alpha^3} + \frac{3C_4}{e\beta^2}$	0

TABLE III - MATRIX FOR FOUR SPAN FRAME

Parabolic Girder



$\theta_1 = -\theta_5 \quad \theta_2 = -\theta_4 \quad \theta_3 = 0$

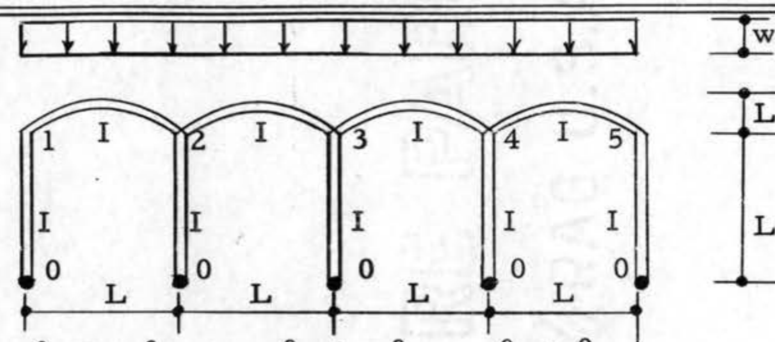
$\Delta_1 = -\Delta_5 \quad \Delta_2 = -\Delta_4 \quad \Delta_3 = 0$

$I_c = I$

Matrix Constants

	$\frac{EI_c}{L}\theta_1$	$\frac{EI_c}{L}\theta_2$	$\frac{EI_c}{L}\Delta_1$	$\frac{EI_c}{L}\Delta_2$	wL^2
$\Sigma M_1=0$	$9 + \frac{3}{\alpha}$	-3	$\frac{3}{\alpha^2} - \frac{7.5}{\beta}$	$\frac{7.5}{\beta}$	0
$\Sigma M_2=0$	-3	$18 + \frac{3}{\alpha}$	$\frac{7.5}{\beta}$	$\frac{3}{\alpha^2} - \frac{15}{\beta}$	0
$\Sigma F_{1x}=0$	$\frac{7.5}{\beta} - \frac{3}{\alpha^2}$	$-\frac{7.5}{\beta}$	$-\frac{3}{\alpha^3} - \frac{11.25}{\beta^2}$	$\frac{11.25}{\beta^2}$	$\frac{1}{8\beta}$
$\Sigma F_{2x}=0$	$\frac{7.5}{\beta}$	$\frac{3}{\alpha^2} - \frac{15}{\beta}$	$-\frac{11.25}{\beta^2}$	$\frac{3}{\alpha^3} + \frac{22.5}{\beta^2}$	0

Circular Girder



$\theta_1 = -\theta_5 \quad \theta_2 = -\theta_4 \quad \theta_3 = 0$

$\Delta_1 = -\Delta_5 \quad \Delta_2 = -\Delta_4 \quad \Delta_3 = 0$

Matrix Constants

	$\frac{EI}{L}\theta_1$	$\frac{EI}{L}\theta_2$	$\frac{EI}{L}\Delta_1$	$\frac{EI}{L}\Delta_2$	wL^2
$\Sigma M_1=0$	$C_1 + \frac{3}{\alpha}$	$-C_2$	$\frac{3}{\alpha^2} - \frac{C_4}{\beta}$	$\frac{C_4}{\beta}$	i
$\Sigma M_2=0$	$-C_2$	$\frac{3}{\alpha} + 2C_1$	$\frac{C_4}{\beta}$	$\frac{3}{\alpha^2} - \frac{2C_4}{\beta}$	0
$\Sigma F_{1x}=0$	$\frac{C_4}{\beta} - \frac{3}{\alpha^2}$	$-\frac{C_4}{\beta}$	$-\frac{3}{\alpha^3} + \frac{C_4}{e\beta^2}$	$\frac{C_4}{e\beta^2}$	j
$\Sigma F_{2x}=0$	$\frac{C_4}{\beta}$	$\frac{3}{\alpha^2} - \frac{2C_4}{\beta}$	$-\frac{C_4}{e\beta^2}$	$\frac{3}{\alpha^3} + \frac{2C_4}{e\beta^2}$	0

4. Solution of Matrices

The matrix constants were evaluated on a desk calculator for all combinations of α and β. The values of α and β used were:

$$\alpha = .2, .4, .5, .6, .8, 1.0$$

$$\beta \text{ for circular girder} = .1, .15, .20, .25, .35$$

$$\beta \text{ for parabolic girder} = .1, .2, .3, .4, .5$$

The matrix equations were solved for the rotations and displacement equivalents on the IBM 650 digital computer.

Each matrix equation was punched into standard 8-10 word machine data cards, with its identification, in floating point form. The data cards were then read into the computer with a program for solution of simultaneous equations. The program used was for the solution of linear equations using Cholesky's scheme.

This program is on file, in the program library at Oklahoma State University Computing Center and with International Business Machines Corporation, under the name of "Equ. Solv." This program will solve a series of five equations with five unknowns. The computer output data gives the values of the rotations and displacement equivalents with its identification.

The values of rotation and displacement equivalents from the computer are used in the slope-deflection equation for solution of the end moment coefficients. These end moment coefficients are presented in tables and nomographic charts in Part III of this thesis.

PART III

TABLES AND CHARTS

1. General Notes

The tables and charts for end moment coefficients presented in this section are divided into the following groups:

Group 2 - Two Span Frames

Group 3 - Three Span Frames

Group 4 - Four Span Frames

Each group is further divided into two subgroups:

B. Frames with Parabolic Girders

C. Frames with Circular Girders

Each table's symbol is composed of two terms: First a capital letter "B" or "C" indicating the subgroup; next, an Arabic number indicating the number of spans. The symbols on the charts contain an additional number indicating sequence of end moments from left to right.

2. Contents of Tables and Charts

Each table and chart is composed of the following major parts:

1. Description of frame and table number.

2. Illustration of frame: A figure containing symbols for all structural elements such as symbols for joints, lengths, moment of inertia, intensity and type of loading.

3. Column parameter α .

4. Girder height parameter β .

5. Moment coefficient (Q) : function of α and β .

6. End moments: Formulas for determining the final end moments: $M = Q\,wL^2$.

The number of horizontal rows in the tables is equivalent to the number of α values used in the solution. The number of vertical rows is equal to the number of β values used. The charts contain five curves, one for each β value used in solution.

3. Steps of Procedure

1. Select the table or chart corresponding to the case under consideration and adjust the symbols to those shown on chart or table.

2. Determine the values of α and β .

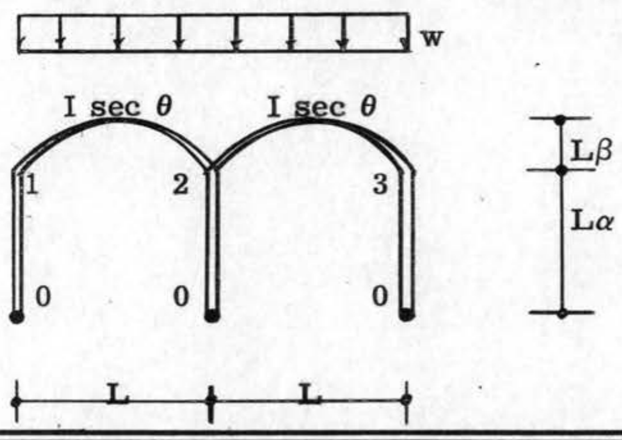
3. Determine the moment coefficient (Q) by the use of α and β from Step 2.

4. Compute final end moments by the use of formulas given in charts and tables.

4. Interpolation

The end moments can be determined very efficiently from the tables if the values of α and β are the same as those tabulated. Interpolation for values of α and β , other than those in the table, can be done more accurately from the charts.

TWO SPAN FRAME PARABOLIC GIRDER TABLE B-2

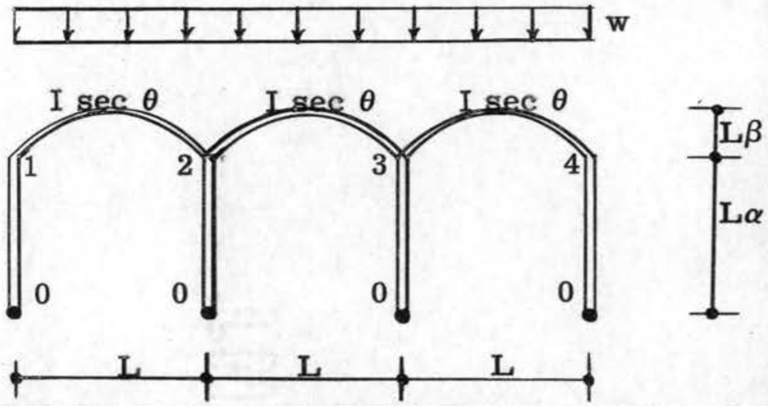


$$M_{12} = -wL^2\, Q_{12}$$

$$M_{21} = +wL^2\, Q_{21}$$

	Q_{12}						Q_{21}				
α \ β	0.1	0.2	0.3	0.4	0.5	α \ β	0.1	0.2	0.3	0.4	0.5
0.2	.06250	.05392	.04605	.03972	.03472	0.2	.06250	.04412	.03289	.02570	.02083
0.4	.05652	.05556	.05307	.05000	.04682	0.4	.08261	.06944	.05866	.05000	.04307
0.5	.05258	.05298	.05203	.05033	.04824	0.5	.08820	.07732	.06776	.05957	.05263
0.6	.04891	.05000	.05000	.04927	.04808	0.6	.09239	.08333	.07500	.06752	.06090
0.8	.04267	.04422	.04510	.04545	.04540	0.8	.09664	.09184	.08560	.07955	.07393
1.0	.03772	.03926	.04040	.04118	.04167	1.0	.10237	.09752	.09268	.08793	.08333

THREE SPAN FRAME PARABOLIC GIRDERS TABLE B-3



$$M_{12} = -wL^2 Q_{12}$$

$$M_{21} = +wL^2 Q_{21}$$

$$M_{23} = -wL^2 Q_{23}$$

$$M_{20} = +wL^2 Q_{20}$$

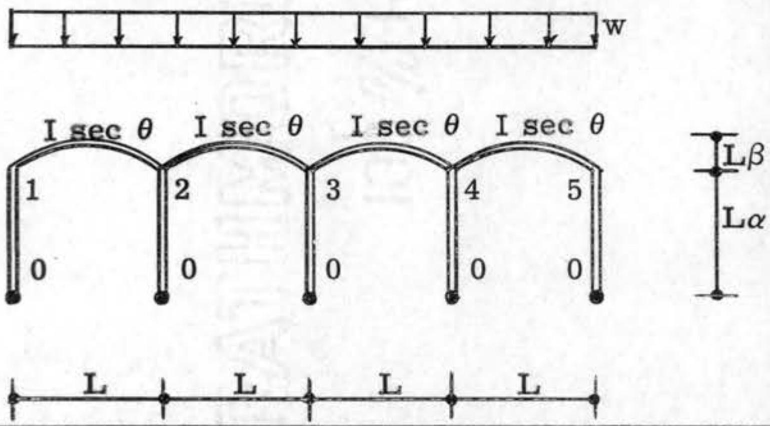
Q_{12}					
α \ β	0.1	0.2	0.3	0.4	0.5
0.2	.06757	.05823	.04914	.04191	.03631
0.4	.06044	.06034	.05781	.05428	.05054
0.5	.05617	.05750	.05680	.05495	.05252
0.6	.05230	.05425	.05463	.05397	.05262
0.8	.04586	.04809	.04941	.04991	.05002
1.0	.04081	.04288	.04442	.04548	.04613

Q_{21}					
α \ β	0.1	0.2	0.3	0.4	0.5
0.2	.05405	.03389	.02324	.01717	.01341
0.4	.07629	.06103	.04898	.03981	.03288
0.5	.08173	.06937	.05860	.04966	.04236
0.6	.08545	.07540	.06612	.05789	.05077
0.8	.09010	.08324	.07648	.07000	.06395
1.0	.09280	.08790	.08290	.07792	.07306

TABLE B-3 (CONTINUED)

	Q_{23}						Q_{20}				
α \ β	0.1	0.2	0.3	0.4	0.5	α \ β	0.1	0.2	0.3	0.4	0.5
0.2	.06081	.06177	.02988	.02245	.01760	0.2	+.00676	+.00788	+.00663	+.00528	+.00419
0.4	.07562	.06448	.05429	.04572	.03876	0.4	-.00067	+.00344	+.00531	+.00591	+.00588
0.5	.07886	.07038	.06190	.05421	.04748	0.5	0.00287	+.00100	+.00329	+.00454	+.00512
0.6	.08112	.07442	.06749	.06081	.05465	0.6	-.00433	-.00099	+.00137	+.00292	+.00388
0.8	.08416	.07970	.07490	.07000	.06517	0.8	-.00594	-.00355	-.00155	+.00000	+.00122
1.0	.08620	.08300	.07951	.07586	.07214	1.0	-.00660	-.00490	-.00338	-.00206	-.00923

FOUR SPAN FRAME PARABOLIC GIRDERS TABLE B-4



$$M_{12} = -wL^2 Q_{12}$$
$$M_{21} = +wL^2 Q_{21}$$
$$M_{20} = +wL^2 Q_{20}$$
$$M_{23} = -wL^2 Q_{23}$$
$$M_{32} = +wL^2 Q_{32}$$

Q_{12}

α \ β	0.1	0.2	0.3	0.4	0.5
0.2	.07177	.06114	.05088	.04297	.03700
0.4	.06335	.06412	.06133	.05723	.05290
0.5	.05841	.06088	.06041	.05831	.05547
0.6	.05403	.05716	.05804	.05741	.05586
0.8	.04687	.05013	.05212	.05307	.05323
1.0	.04137	.04428	.04646	.04797	.04892

Q_{21}

α \ β	0.1	0.2	0.3	0.4	0.5
0.2	.04732	.02707	.01787	.01306	.01021
0.4	.07301	.05530	.04230	.03313	.02665
0.5	.07958	.06488	.05273	.04317	.03580
0.6	.08415	.07205	.06124	.05260	.04447
0.8	.09000	.08162	.07348	.06589	.05898
1.0	.09354	.08748	.08136	.07537	.06963

TABLE B-4 (CONTINUED)

Q_{23}

α \ β	0.1	0.2	0.3	0.4	0.5
0.2	.06230	.04135	.02875	.02119	.01640
0.4	.07793	.06588	.05457	.04518	.03772
0.5	.08135	.07224	.06287	.04532	.04694
0.6	.08381	.07669	.06901	.06154	.05468
0.8	.08726	.08254	.07725	.07174	.06629
1.0	.08969	.08629	.08245	.07835	.07413

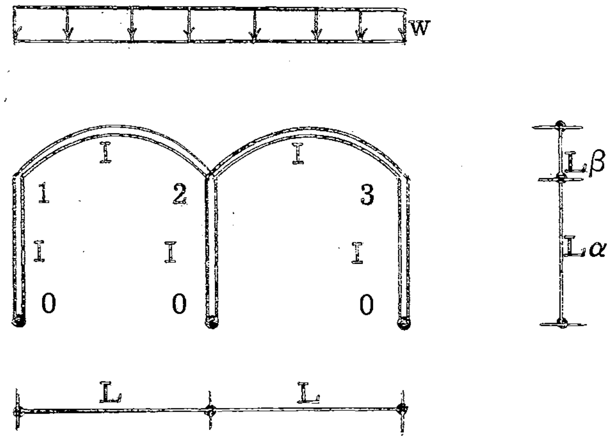
Q_{32}

α \ β	0.1	0.2	0.3	0.4	0.5
0.2	.05047	.02891	.01798	.01218	.00882
0.4	.06897	.05471	.04251	.03313	.02618
0.5	.07229	.06159	.05123	.04227	.03493
0.6	.07415	.06606	.05759	.04963	.04259
0.8	.07585	.07097	.06542	.05966	.05402
1.0	.07640	.07324	.06951	.06544	.06122

Q_{20}

α \ β	0.1	0.2	0.3	0.4	0.5
0.2	+.01498	+.01428	+.01088	+.00814	+.00619
0.4	+.00492	+.01059	+.01227	+.01205	+.01107
0.5	+.00177	+.00735	+.01015	+.01115	+.01114
0.6	-.00035	+.00464	+.00777	+.00948	+.01022
0.8	-.00274	+.00092	+.00377	+.00586	+.00730
1.0	-.00385	-.00119	+.00109	+.00298	+.00450

TWO SPAN FRAME CIRCULAR GIRDERS TABLE C-2



$$M_{12} = -wL^2\,Q_{12}$$

$$M_{21} = +wL^2\,Q_{21}$$

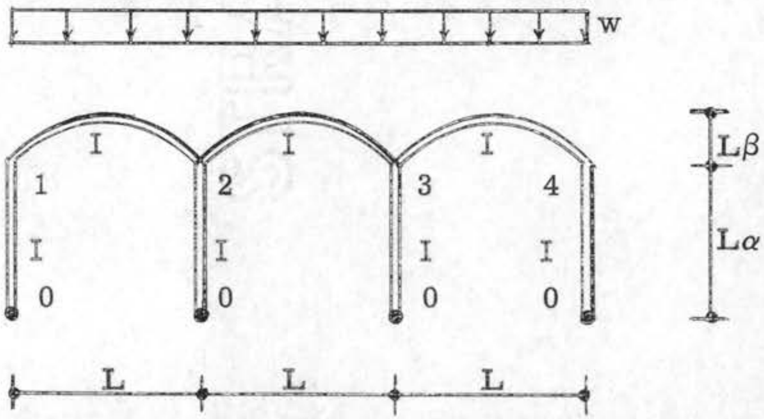
Q_{12}

α \ β	0.1	0.15	0.20	0.25	0.35
0.2	.06226	.05762	.05293	.04839	.03938
0.4	.05661	.05601	.05537	.05339	.05895
0.5	.05278	.05289	.05324	.05271	.04960
0.6	.04886	.04969	.05064	.05074	.04906
0.8	.04307	.04386	.04537	.04619	.04640
1.0	.03816	.03901	.04071	.04184	.04309

Q_{21}

α \ β	0.1	0.15	0.20	0.25	0.35
0.2	.06149	.05013	.04111	.03370	.02095
0.4	.08111	.07216	.06445	.05677	.04156
0.5	.08656	.07851	.07165	.06429	.04890
0.6	.09025	.08329	.07715	.07016	.05484
0.8	.09647	.09001	.08499	.07861	.06373
1.0	.10044	.09451	.09029	.08437	.06997

THREE SPAN FRAME CIRCULAR GIRDER TABLE C-3



$$M_{12} = -wL^2 Q_{12}$$
$$M_{21} = +wL^2 Q_{21}$$
$$M_{23} = -wL^2 Q_{23}$$
$$M_{20} = +wL^2 Q_{20}$$

Q_{12}

α \ β	0.1	0.15	0.20	0.25	0.35
0.2	.06733	.06254	.05732	.05222	.04228
0.4	.06048	.06042	.06011	.05871	.05356
0.5	.05630	.05689	.05765	.05732	.05432
0.6	.05251	.05336	.05472	.05506	.05367
0.8	.04615	.04709	.04897	.04999	.05055
1.0	.04036	.04199	.04400	.04526	.04678

Q_{21}

α \ β	0.1	0.15	0.20	0.25	0.35
0.2	.05307	.04034	.03076	.02325	.01070
0.4	.07497	.06509	.05639	.04796	.03157
0.5	.08035	.07186	.06429	.05636	.03984
0.6	.08404	.07663	.07008	.06276	.04662
0.8	.08868	.08271	.07770	.07149	.05661
1.0	.09022	.08628	.08230	.07690	.06327

TABLE C-3 (CONTINUED)

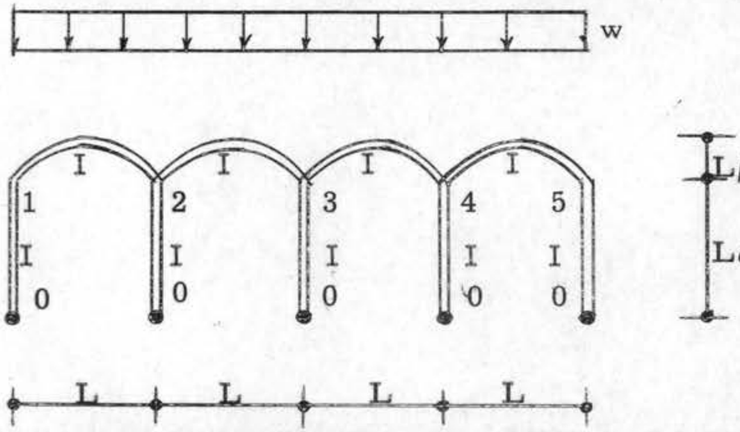
Q_{23}

α \ β	0.1	0.15	0.20	0.25	0.35
0.2	.06003	.04869	.03918	.03134	.01776
0.4	.07464	.06769	.06091	.05400	.03948
0.5	.07782	.07211	.06647	.06036	.04662
0.6	.08004	.07514	.07034	.06489	.05201
0.8	.08303	.07908	.07535	.07081	.05937
1.0	.08355	.08158	.07847	.07447	.06401

Q_{20}

α \ β	0.1	0.15	0.20	0.25	0.35
0.2	+.00696	+.00836	+.00843	+.00809	+.00706
0.4	-.00034	+.00261	+.00452	+.00604	+.00791
0.5	-.00252	+.00025	+.00218	+.00399	+.00678
0.6	-.00400	-.00149	+.00027	+.00213	+.00450
0.8	-.00565	-.00363	-.00235	-.00068	+.00275
1.0	-.00665	-.00471	-.00383	-.00243	+.00073

FOUR SPAN FRAME | CIRCULAR GIRDER | TABLE C-4



$$M_{12} = -wL^2 \; Q_{12}$$

$$M_{21} = +wL^2 \; Q_{21}$$

$$M_{23} = -wL^2 \; Q_{23}$$

$$M_{2.0} = +wL^2 \; Q_{20}$$

$$M_{32} = +wL^2 \; Q_{32}$$

Q_{12}

α \ β	0.1	0.15	0.20	0.25	0.35
0.2	.07154	.06626	.06032	.05463	.04387
0.4	.06342	.06405	.06399	.06258	.05701
0.5	.05858	.05994	.06117	.06150	.05809
0.6	.05427	.05589	.05779	.05852	.05746
0.8	.04721	.04880	.05121	.05269	.05397
1.0	.04175	.04316	.04562	.04732	.04967

Q_{21}

α \ β	0.1	0.15	0.20	0.25	0.35
0.2	.04630	.03304	.02373	.01672	.00508
0.4	.07163	.06026	.05043	.05128	.02423
0.5	.07812	.06828	.05953	.05135	.03290
0.6	.08264	.07403	.06638	.05810	.04039
0.8	.08844	.08154	.07562	.06850	.05191
1.0	.09196	.08603	.08133	.07516	.05986

TABLE C-4 (CONTINUED)

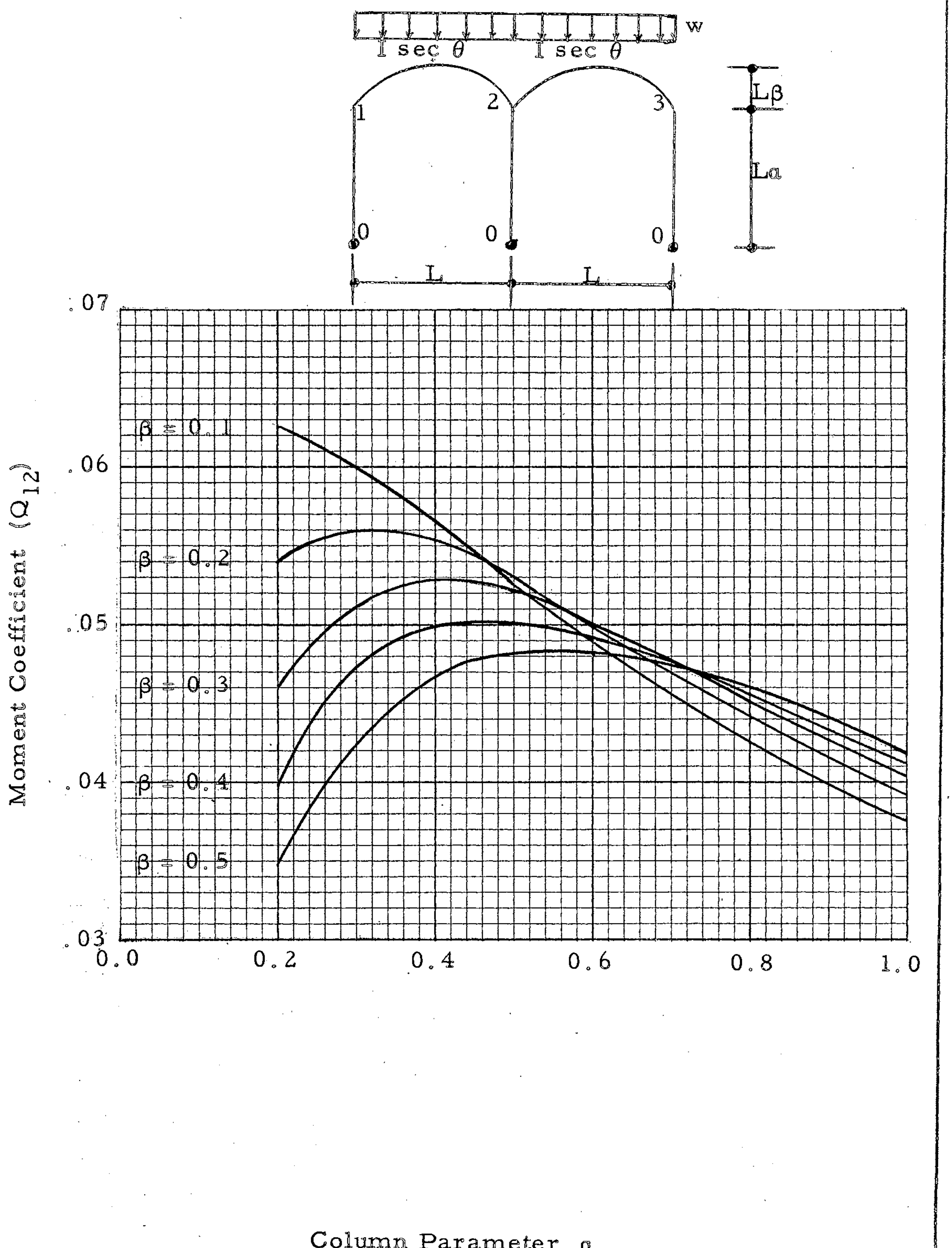
Q_{23} α \ β	0.1	0.15	0.20	0.25	0.35
0.2	.06153	.04909	.03875	.03039	.01617
0.4	.07689	.06954	.06224	.05478	.03919
0.5	.08023	.07418	.06821	.06185	.04695
0.6	.08260	.07738	.07235	.06652	.05280
0.8	.08595	.08165	.07772	.07285	.06070
1.0	.08830	.08436	.08114	.07680	.06564

Q_{32} α \ β	0.1	0.15	0.20	0.25	0.35
0.2	.04969	.03630	.02603	.01811	.00486
0.4	.06814	.05969	.05132	.04314	.02664
0.5	.07149	.06498	.05818	.05135	.03519
0.6	.07339	.06825	.06273	.05658	.04194
0.8	.07517	.07113	.06789	.06332	.05120
1.0	.07578	.07326	.07040	.06685	.05675

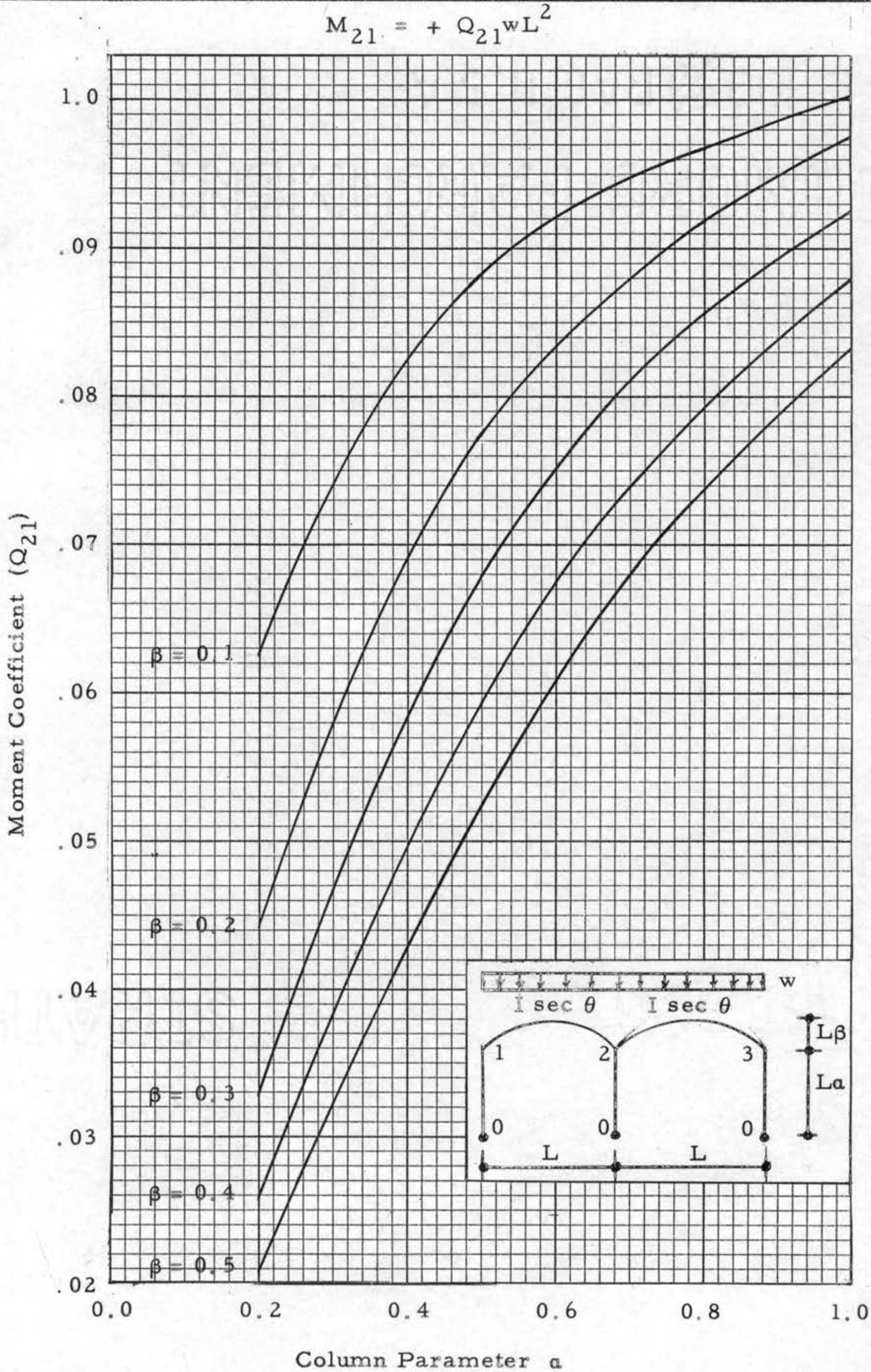
Q_{20} α \ β	0.1	0.15	0.20	0.25	0.35
0.2	+.01523	+.01606	+.01503	+.01367	+.01109
0.4	+.00526	+.00928	+.01181	+.01349	+.01495
0.5	+.00211	+.00591	+.00867	+.01050	+.01405
0.6	-.0004	+.00335	+.00596	+.00842	+.01241
0.8	-.00249	+.00012	+.00211	+.00435	+.00088
1.0	-.00365	-.00166	-.00019	+.00166	+.00577

Two Span Frame | Parabolic Girder | Chart B-20

$M_{12} = - Q_{12} wL^2$

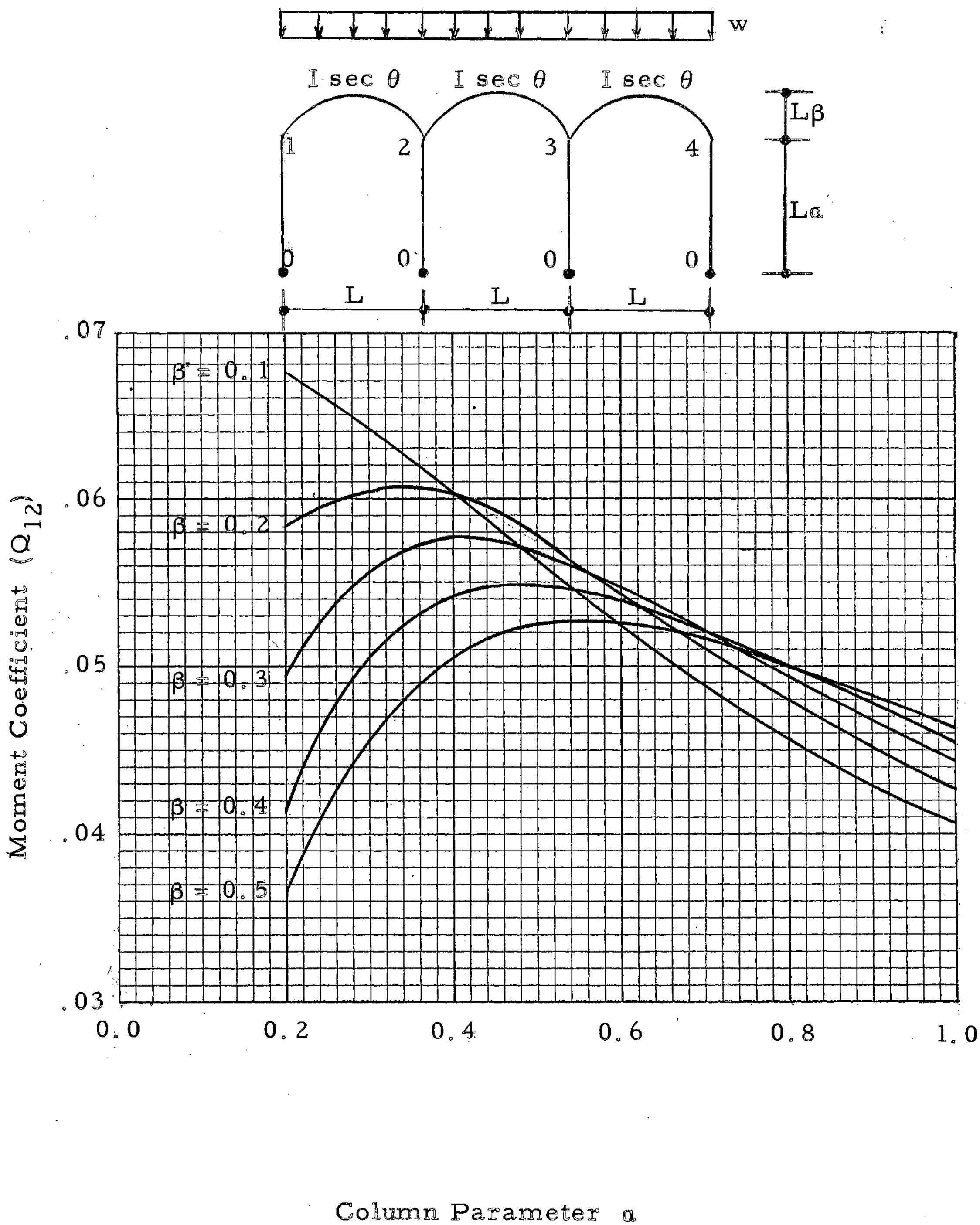


Two Span Frame | Parabolic Girder | Chart B-21



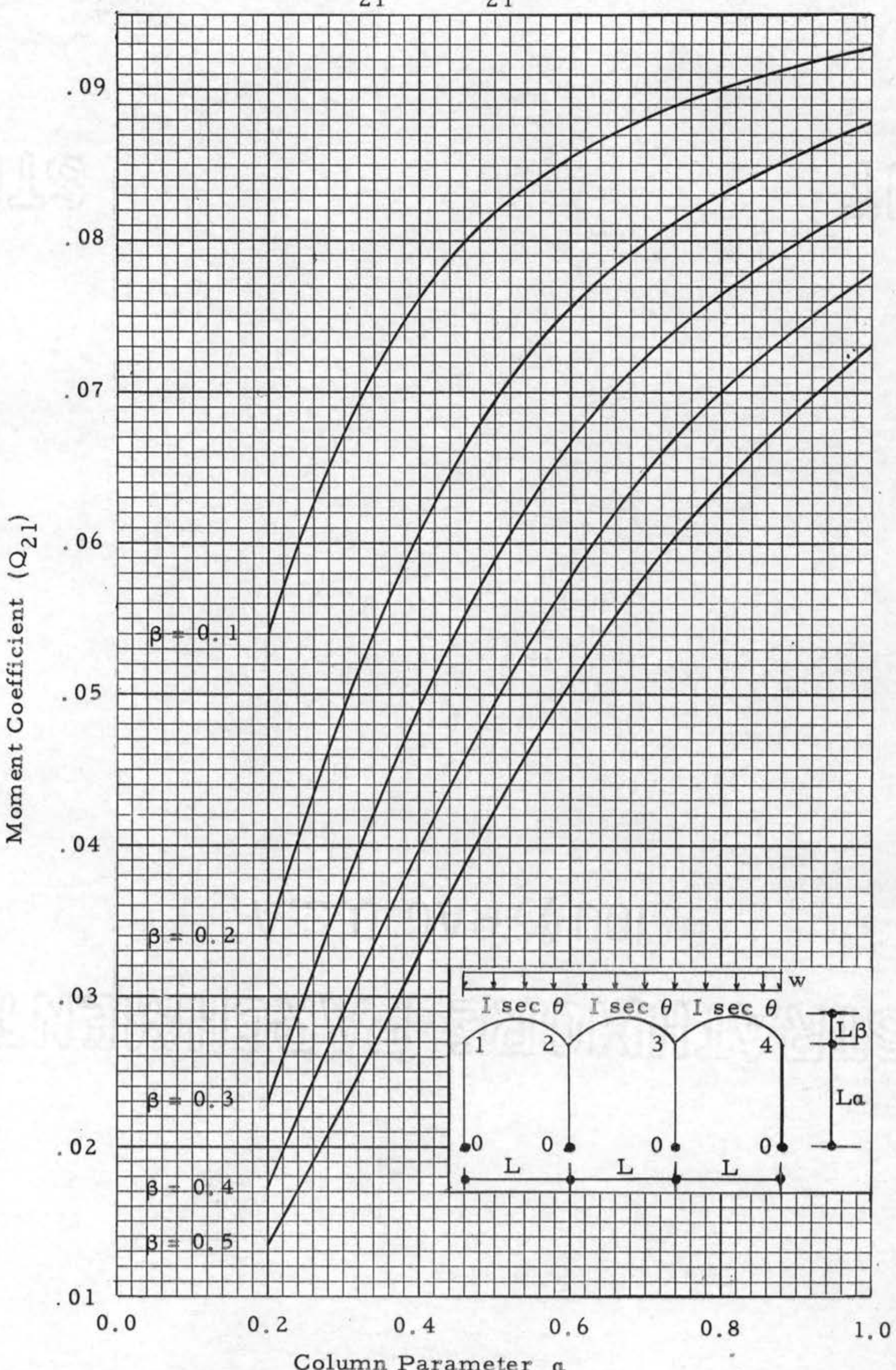
Three Span Frame   Parabolic Girder   Chart B-30

$$M_{12} = -Q_{12} w L^2$$



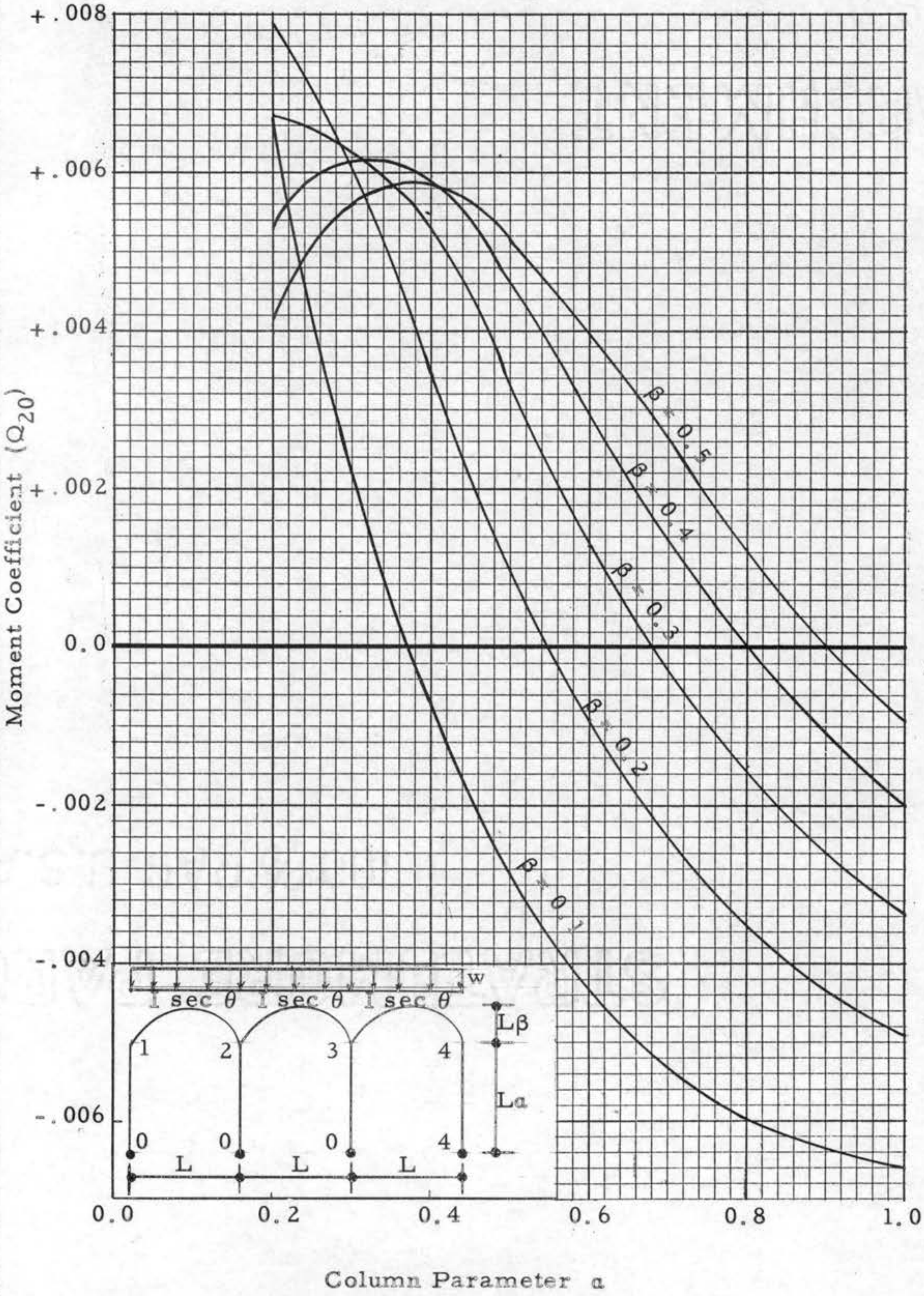
Three Span Frame | Parabolic Girder | Chart B-31

$$M_{21} = + Q_{21} w L^2$$



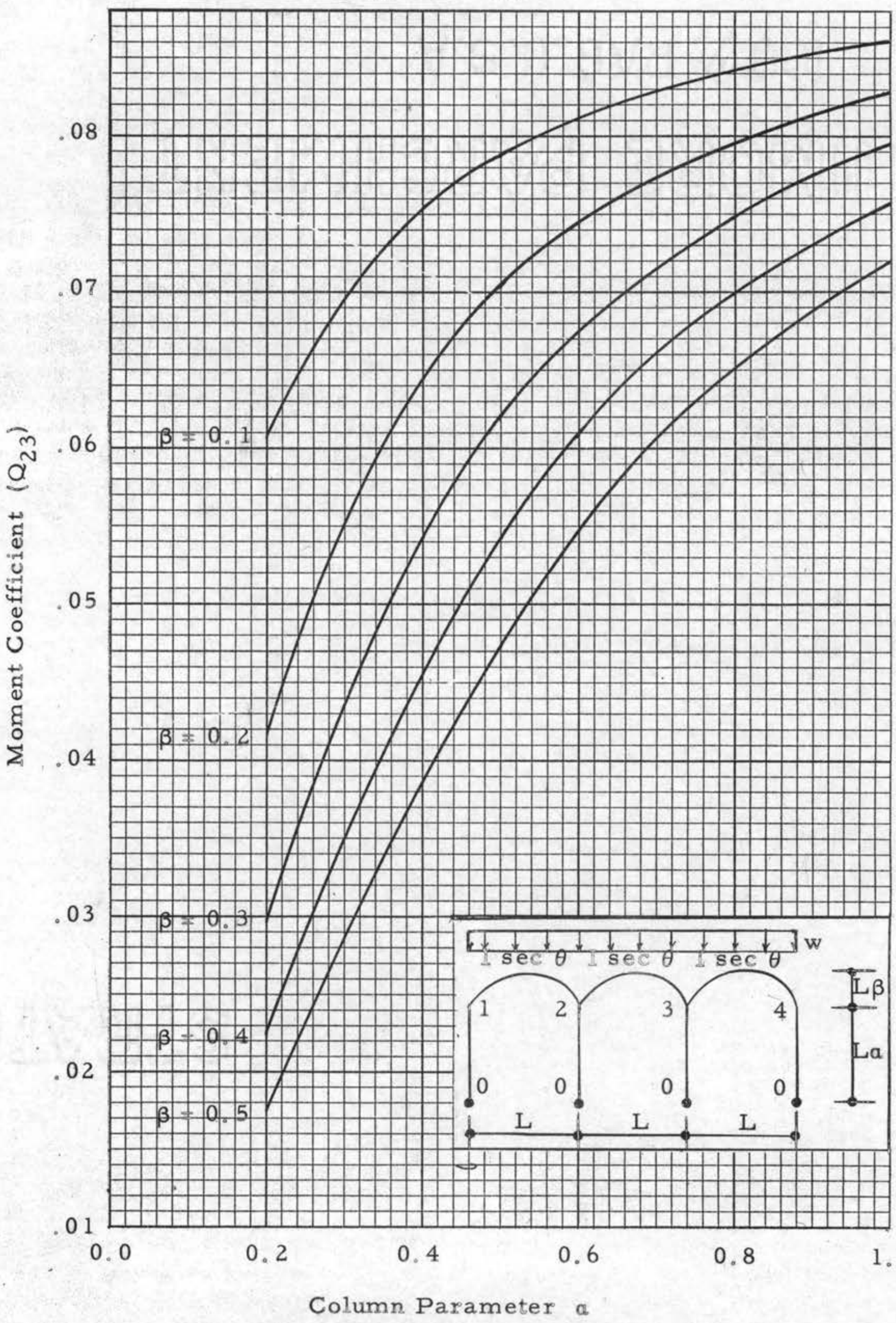
Three Span Frame | Parabolic Girder | Chart B-32

$$M_{20} = + Q_{20} w L^2$$



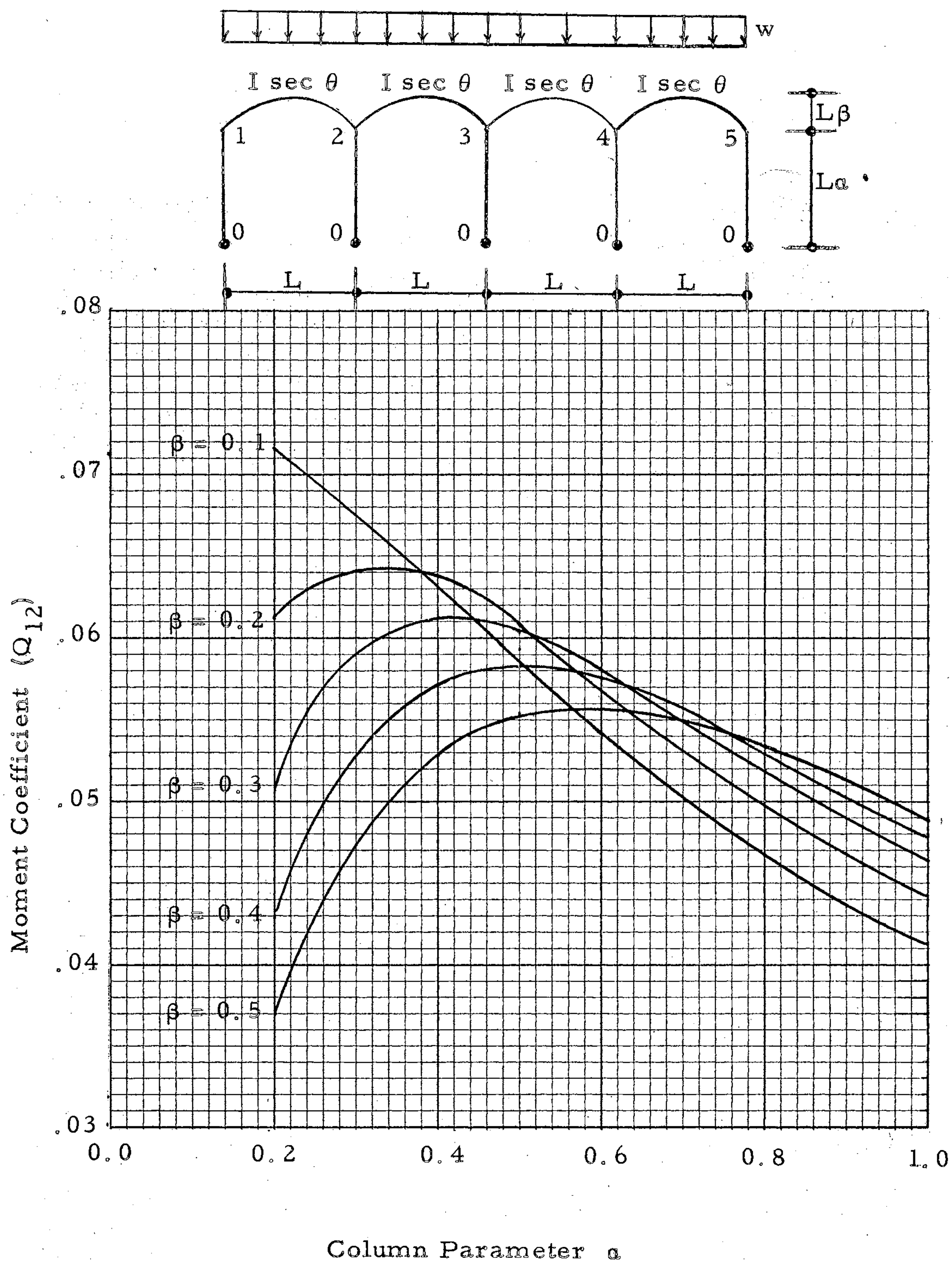
Three Span Frame | Parabolic Girder | Chart B-33

$$M_{23} = -Q_{23} w L^2$$

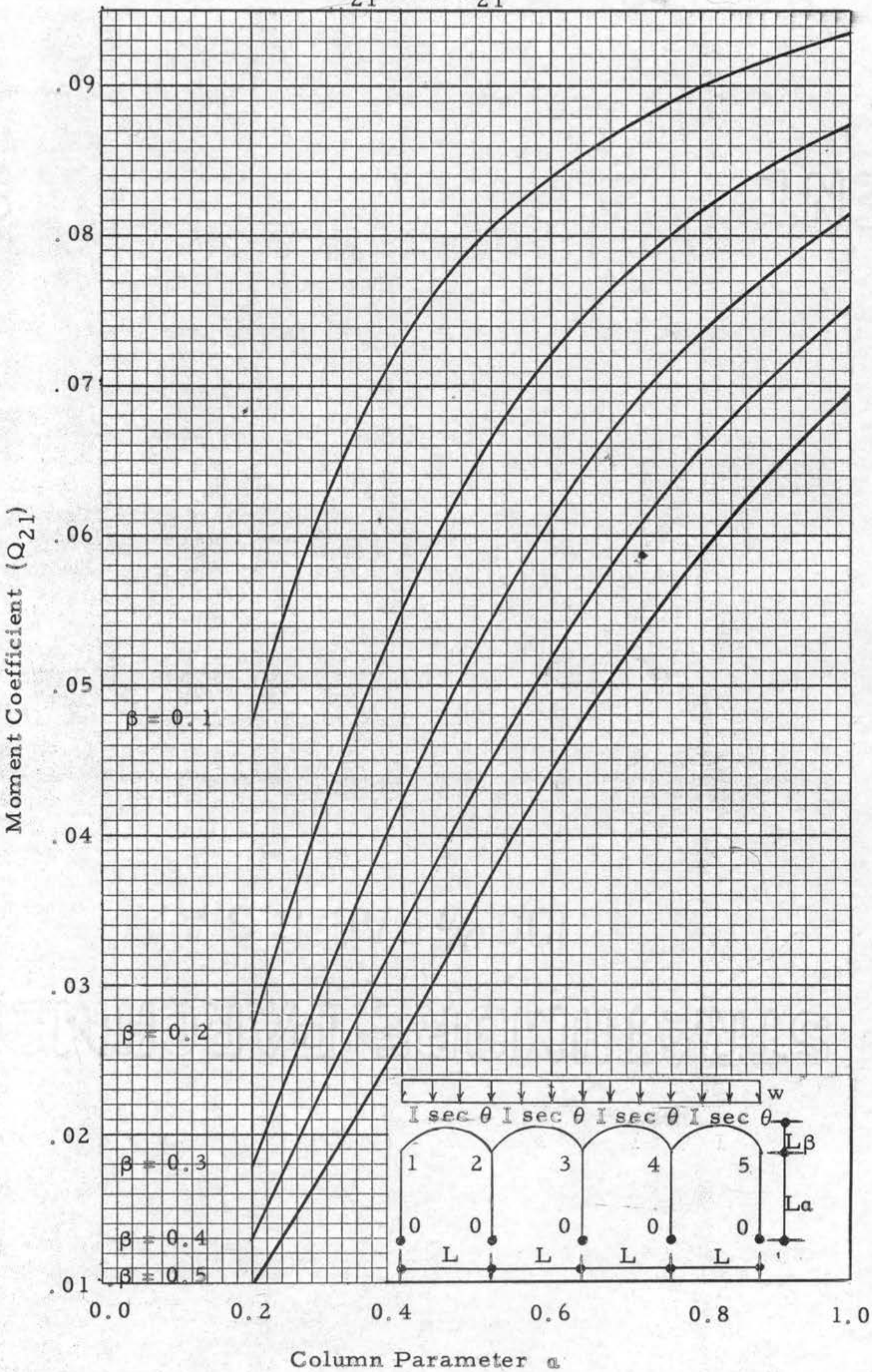


Four Span Frame · Parabolic Girder Chart B-40

$$M_{12} = -Q_{12} w L^2$$

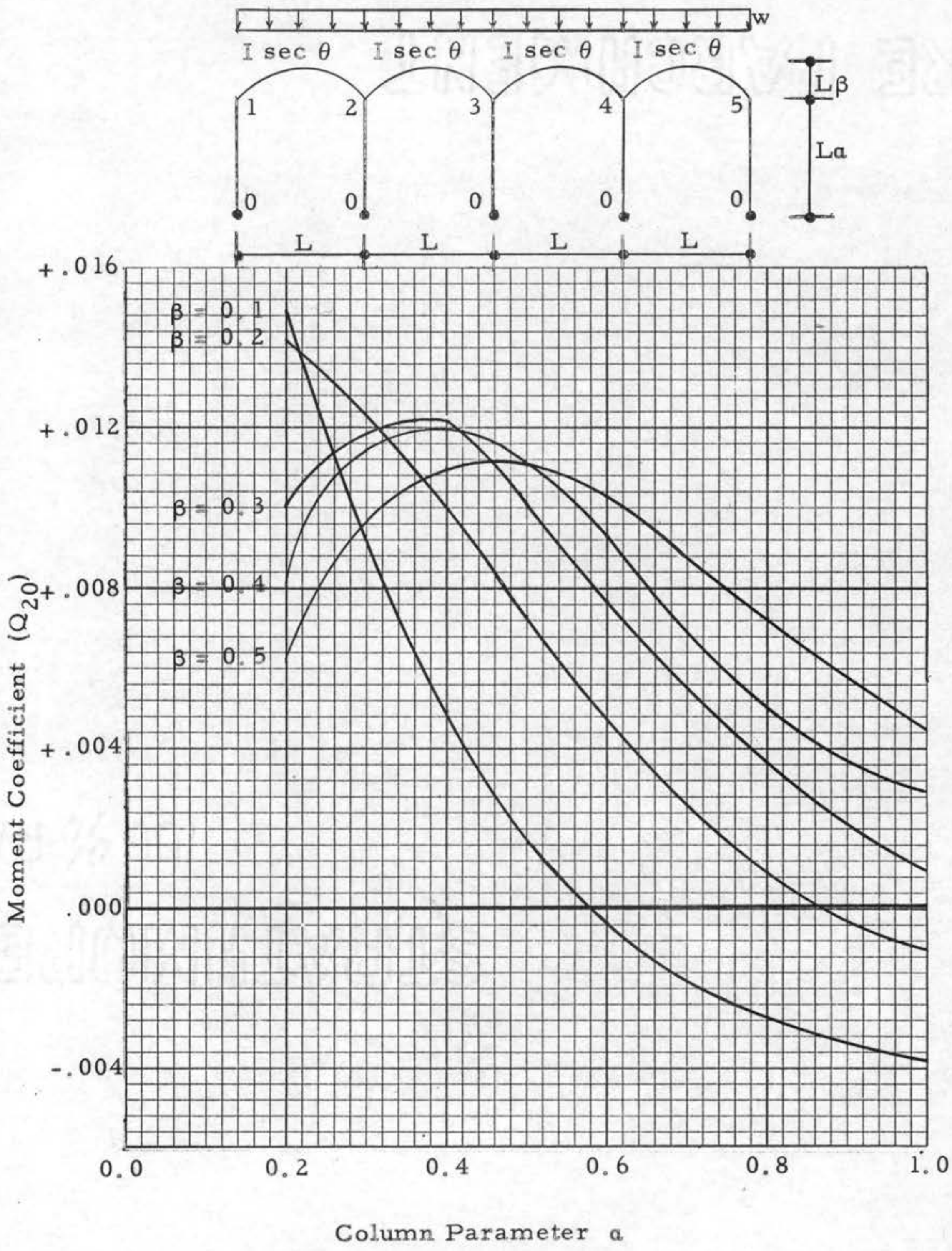


$M_{21} = + Q_{21} wL^2$


.09
.08
.07
.06
.05
.04
.03
.02
.01
Moment Coefficient (Q_{21})
β = 0.1
β = 0.2
β = 0.3
β = 0.4
β = 0.5
0.0
0.2
0.4
0.6
0.8
1.0
Column Parameter α
w
I sec θ
I sec θ
I sec θ
I sec θ
1
2
3
4
5
0
0
0
0
0
Lβ
Lα
L
L
L
L

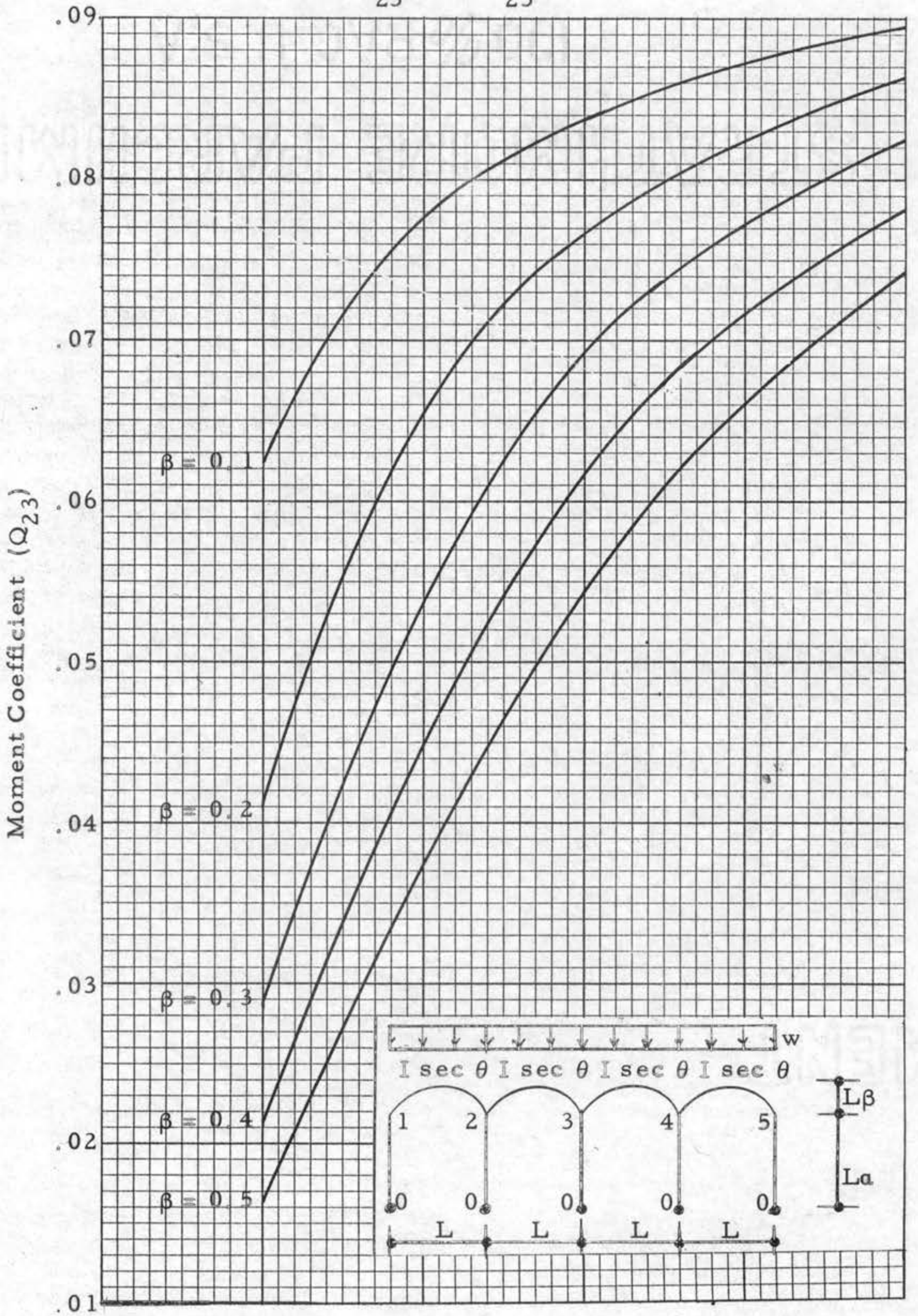
Four Span Frame | Parabolic Girder | Chart B-42

$$M_{20} = + Q_{20} w L^2$$



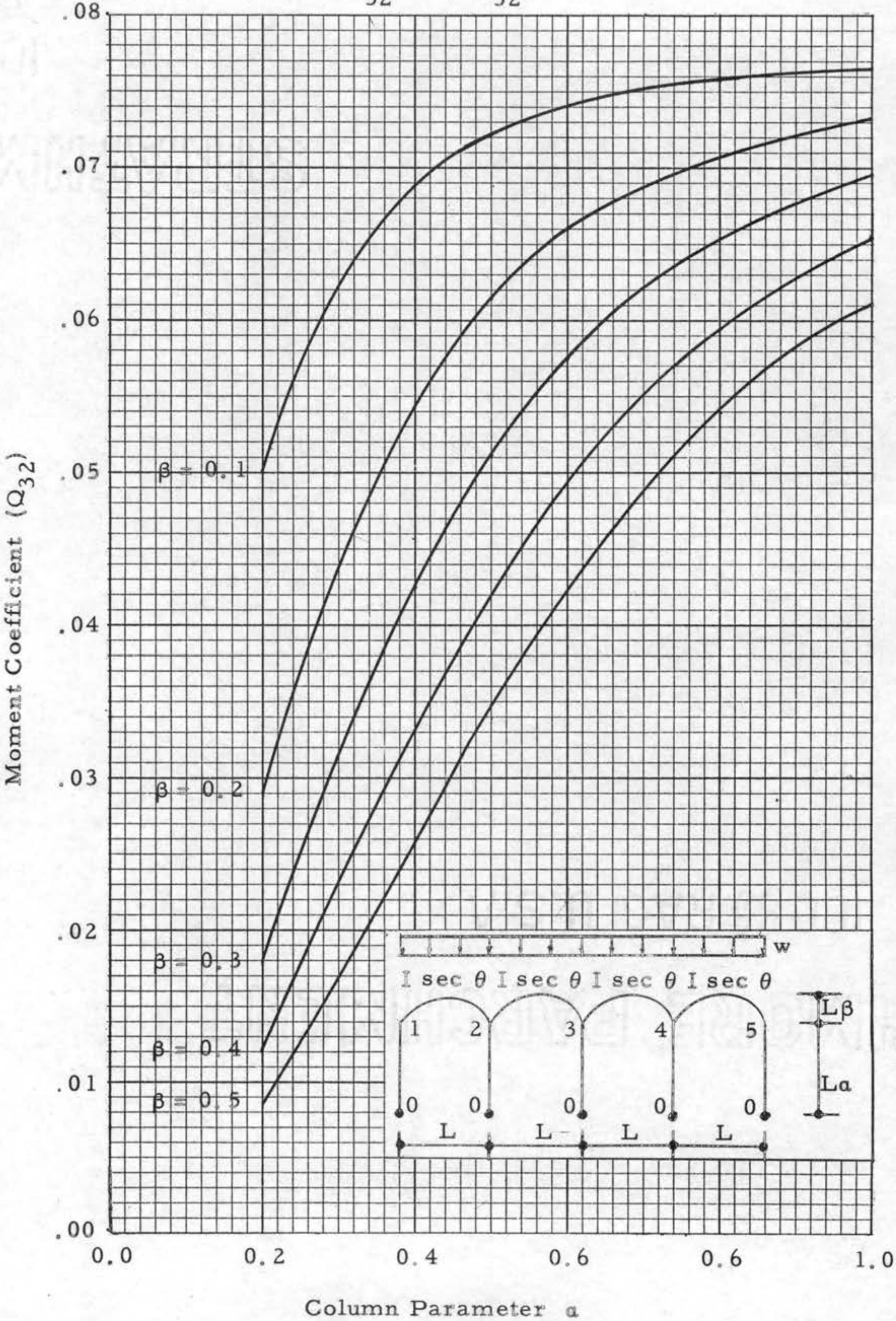
Four Span Frame | Parabolic Girder | Chart B-43

$$M_{23} = -Q_{23} w L^2$$



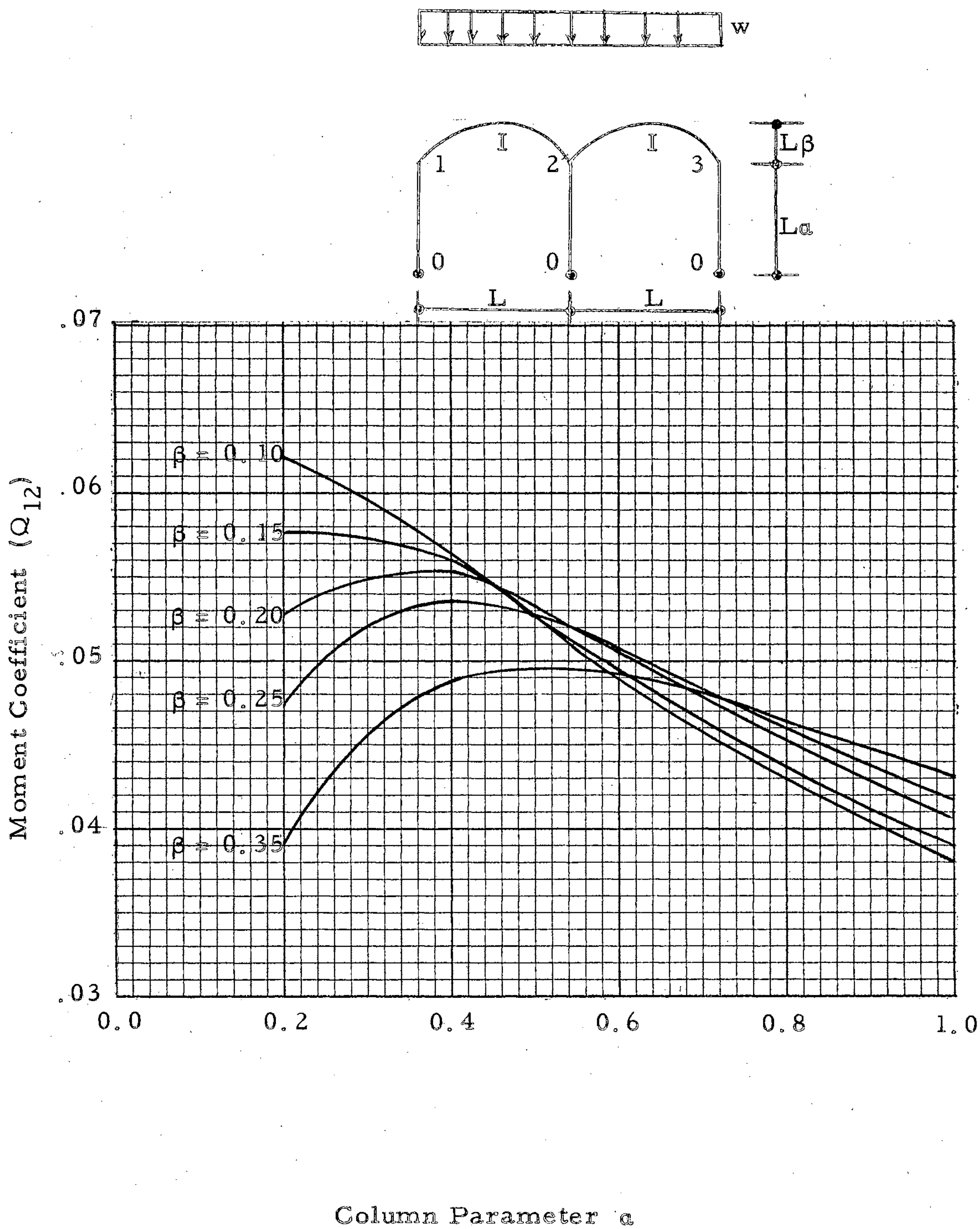
Four Span Frame | Parabolic Girder | Chart B-44

$M_{32} = + Q_{32}wL^2$

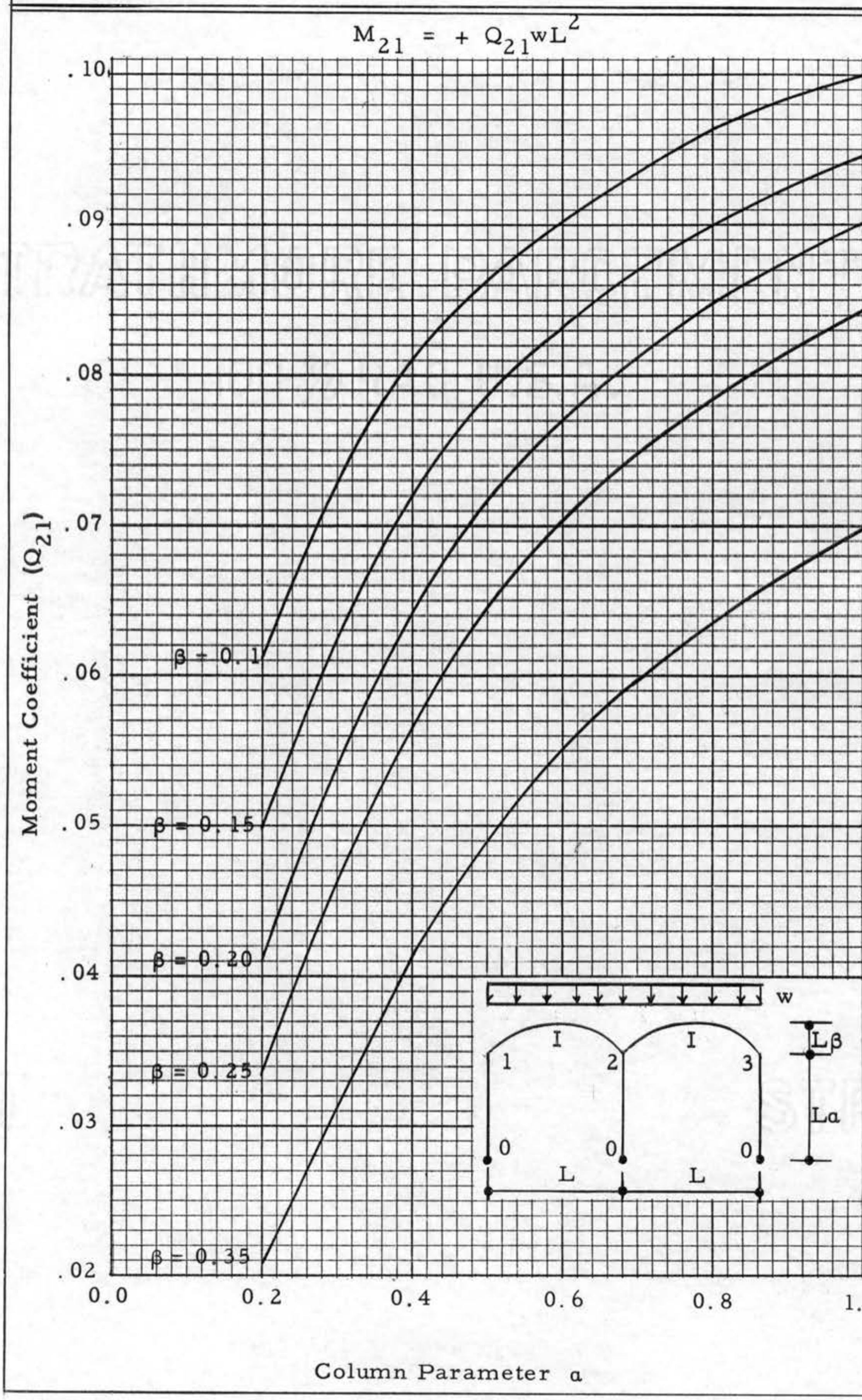


Two Span Frame | Circular Girder | Chart C-20

$$M_{12} = - Q_{12} w L^2$$

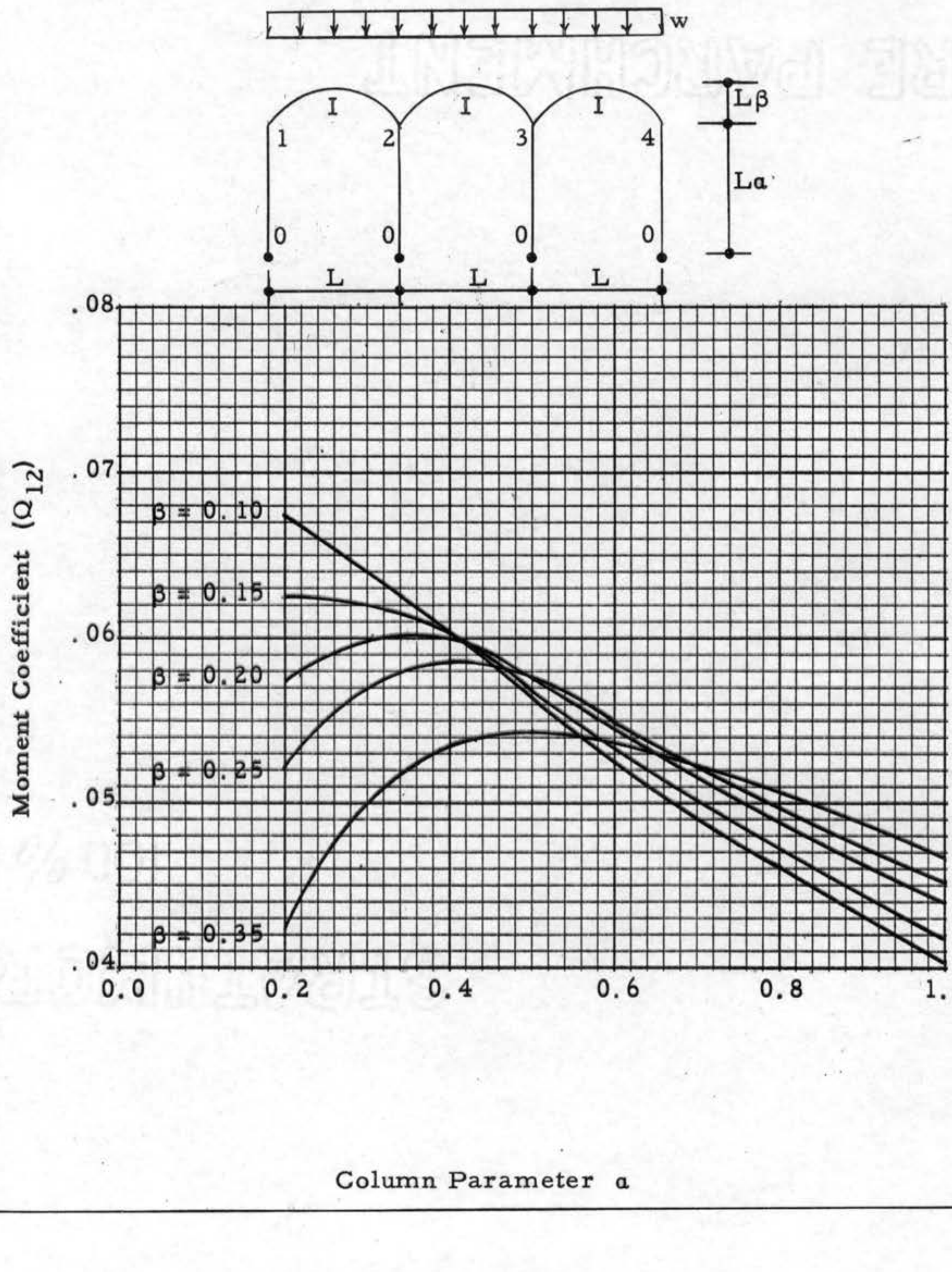


Two Span Frame | Circular Girder | Chart C-21

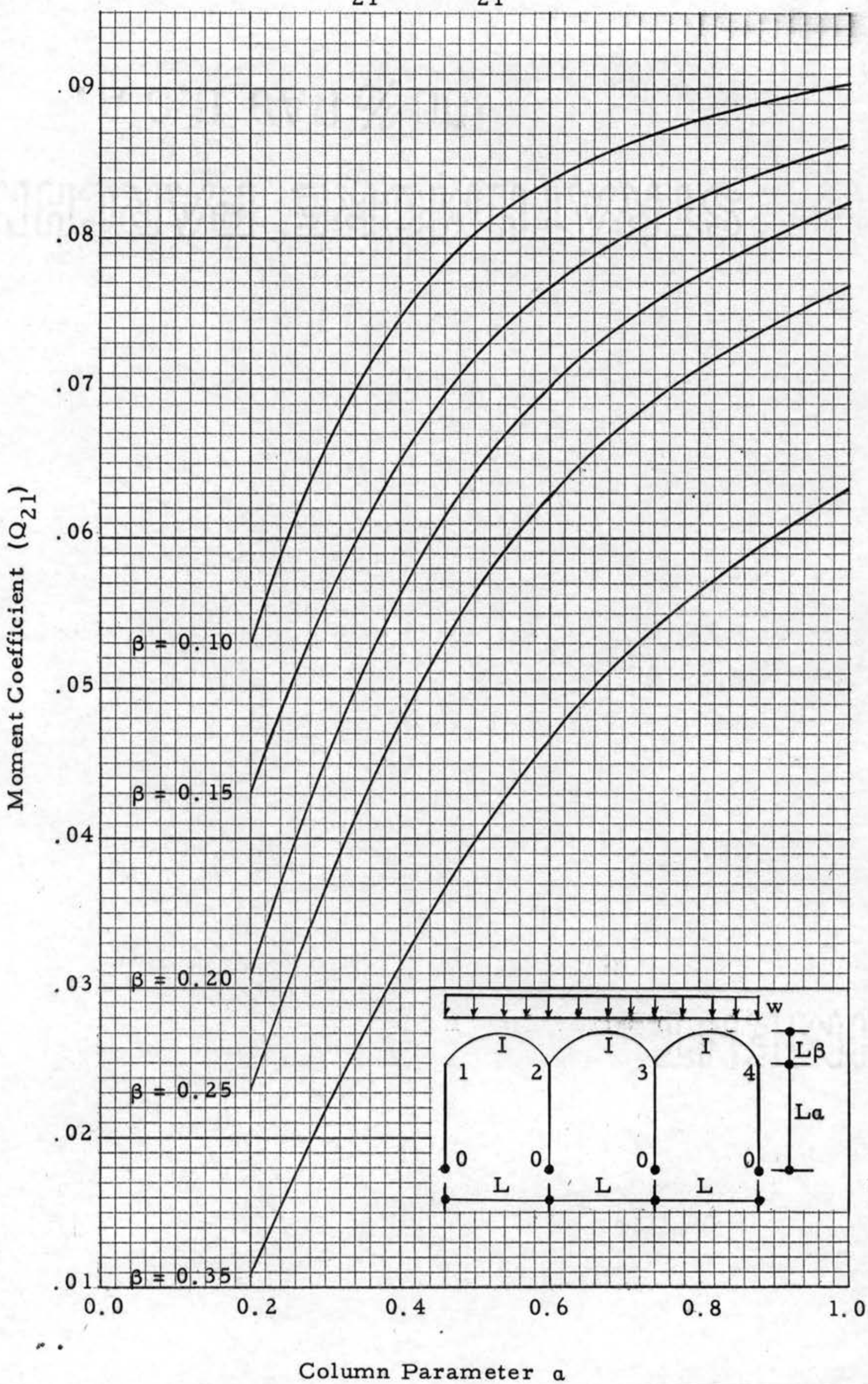


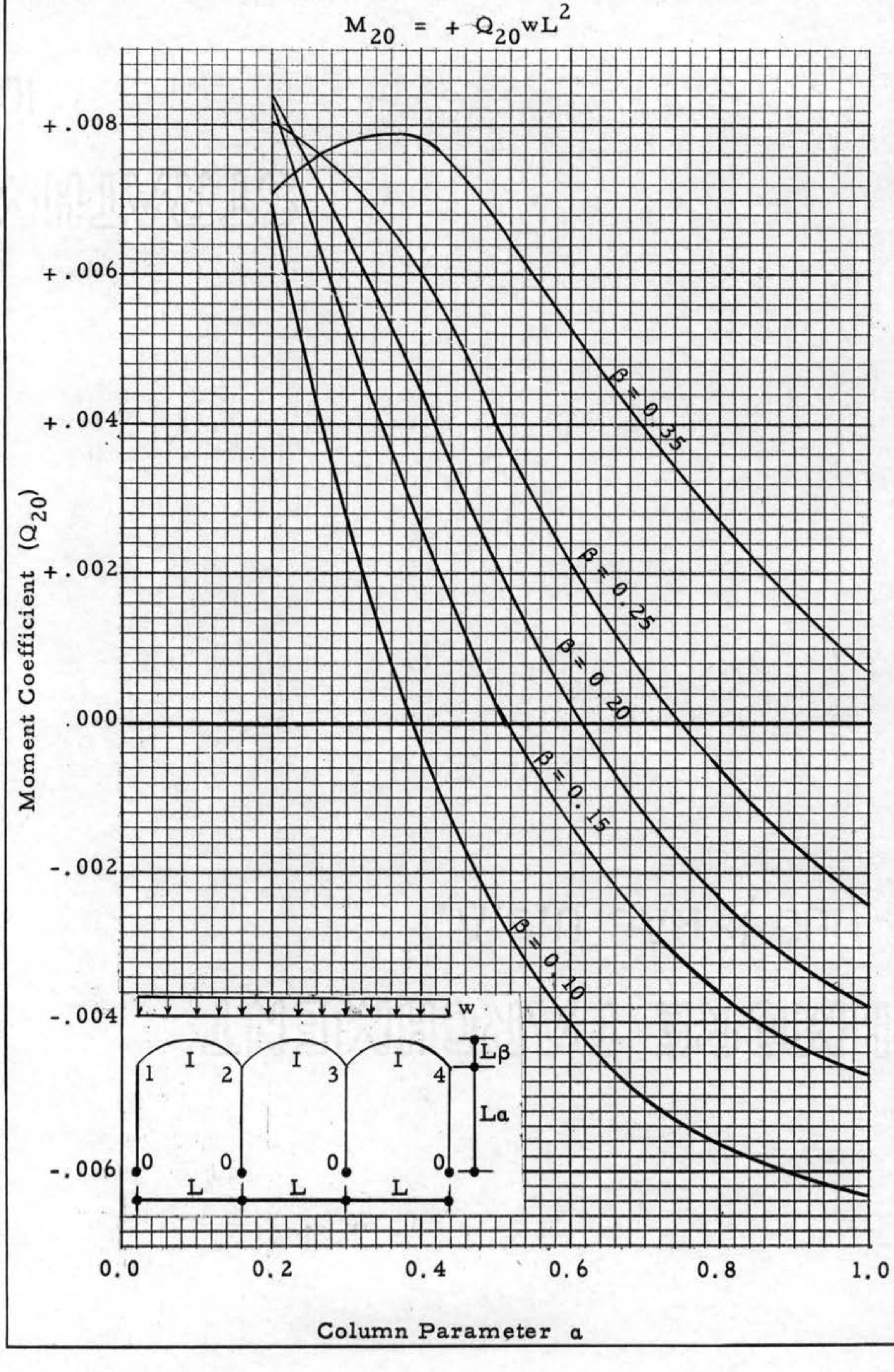
Three Span Frame | Circular Girder | Chart C-30

$$M_{12} = -Q_{12} w L^2$$



$M_{21} = + Q_{21} w L^2$

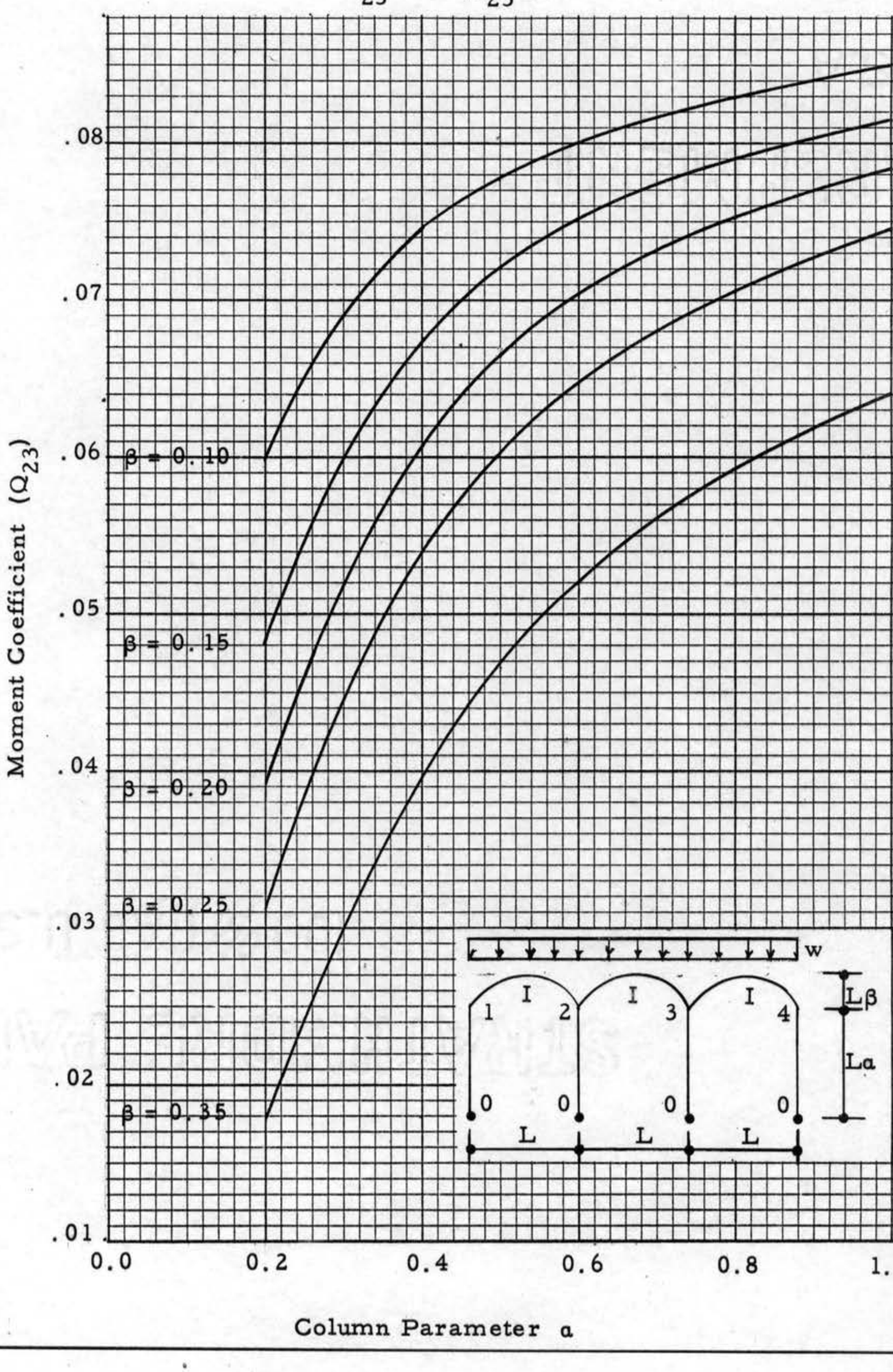

Moment Coefficient (Q_{21})
.09
.08
.07
.06
.05
.04
.03
.02
.01
β = 0.10
β = 0.15
β = 0.20
β = 0.25
β = 0.35
0.0
0.2
0.4
0.6
0.8
1.0
w
I
I
I
1
2
3
4
Lβ
Lα
0
0
0
0
L
L
L
Column Parameter α



M20 = + Q20wL2
+.008
+.006
+.004
+.002
.000
-.002
-.004
-.006
Moment Coefficient (Q20)
β = 0.35
β = 0.25
β = 0.20
β = 0.15
β = 0.10
w
Lβ
La
I
1
2
3
4
0
L
0.0
0.2
0.4
0.6
0.8
1.0
Column Parameter a

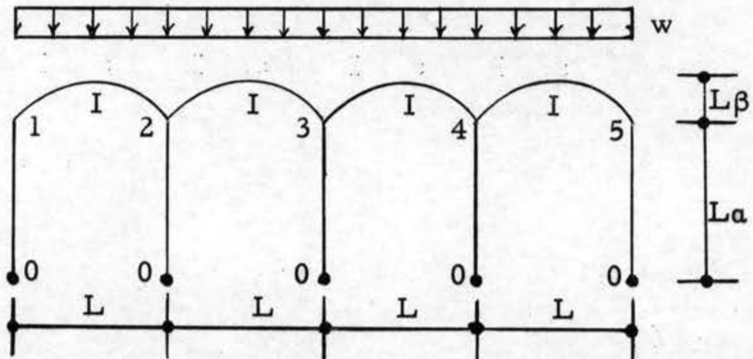
Three Span Frame | Circular Girder | Chart C-33

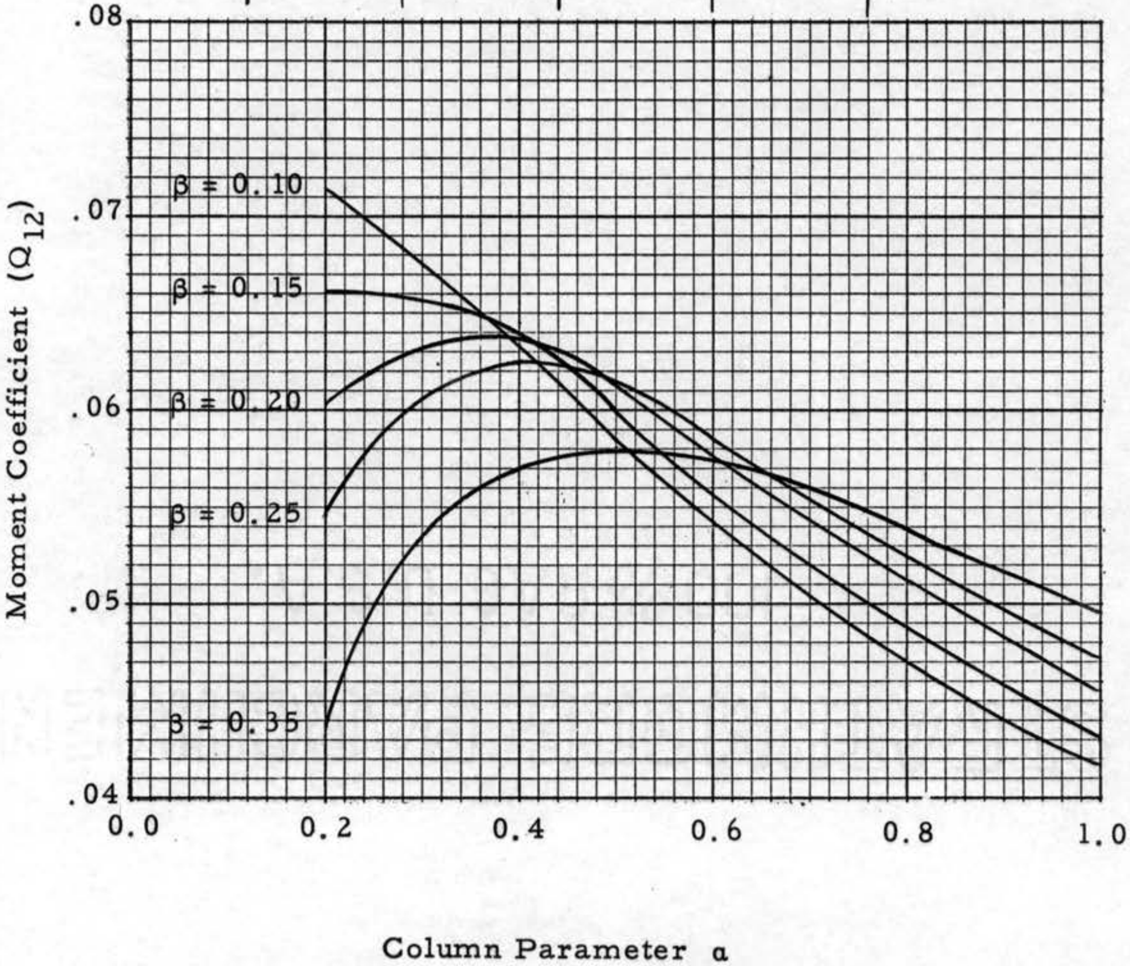
$$M_{23} = -Q_{23}wL^2$$



Four Span Frame | Circular Girder | Chart C-40

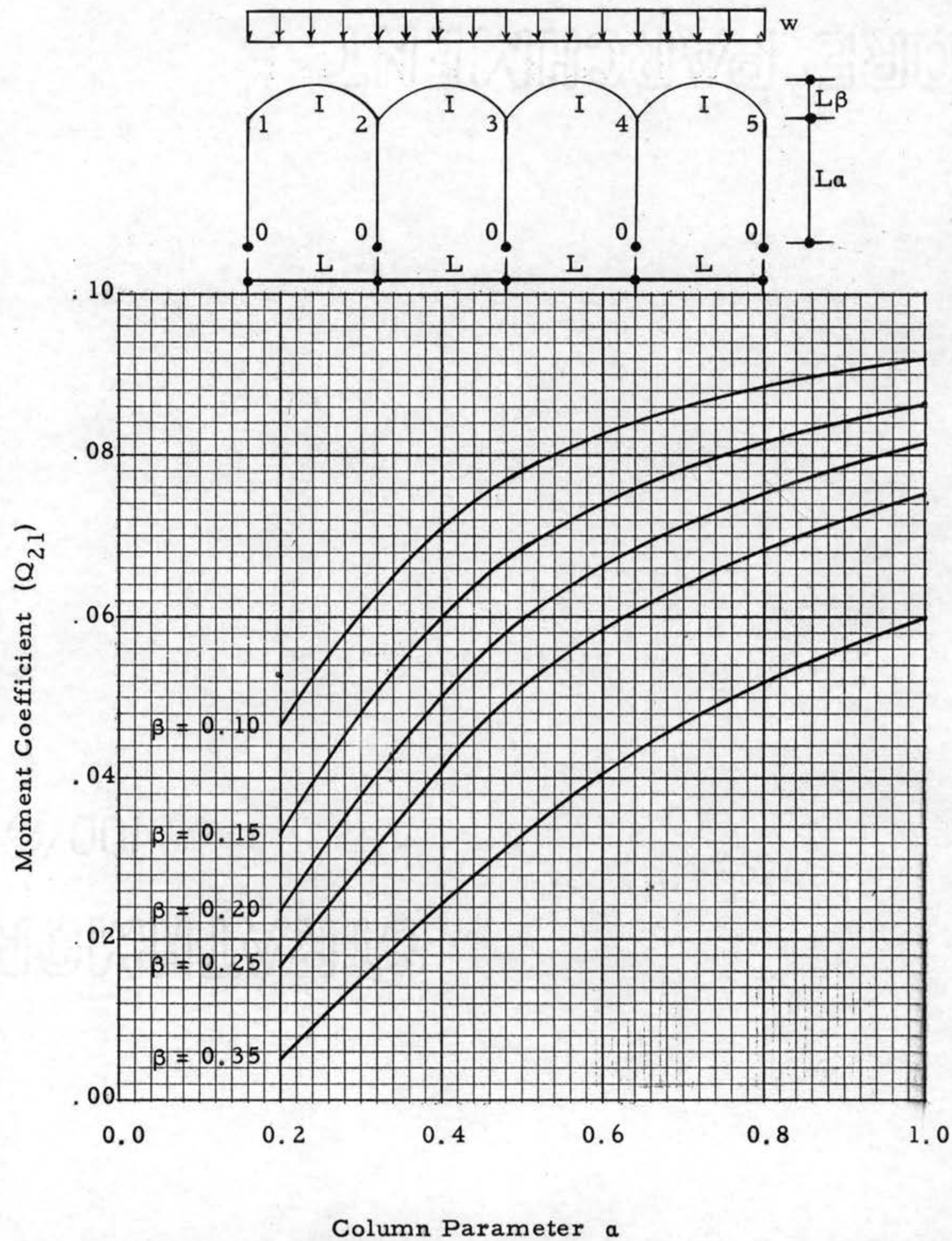
$$M_{12} = -Q_{12}wL^2$$





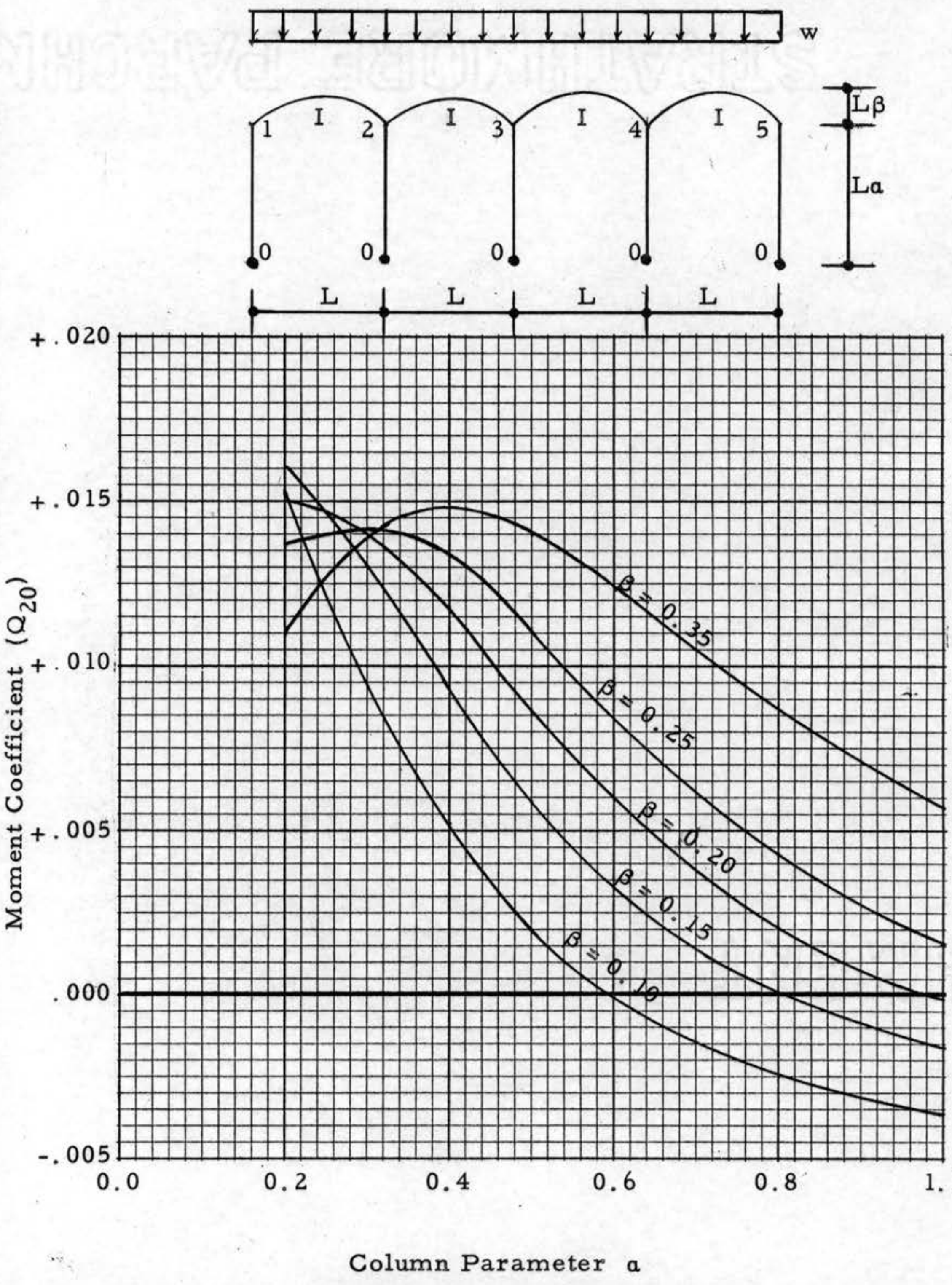
Four Span Frame | Circular Girder | Chart C-41

$$M_{21} = + Q_{21} w L^2$$

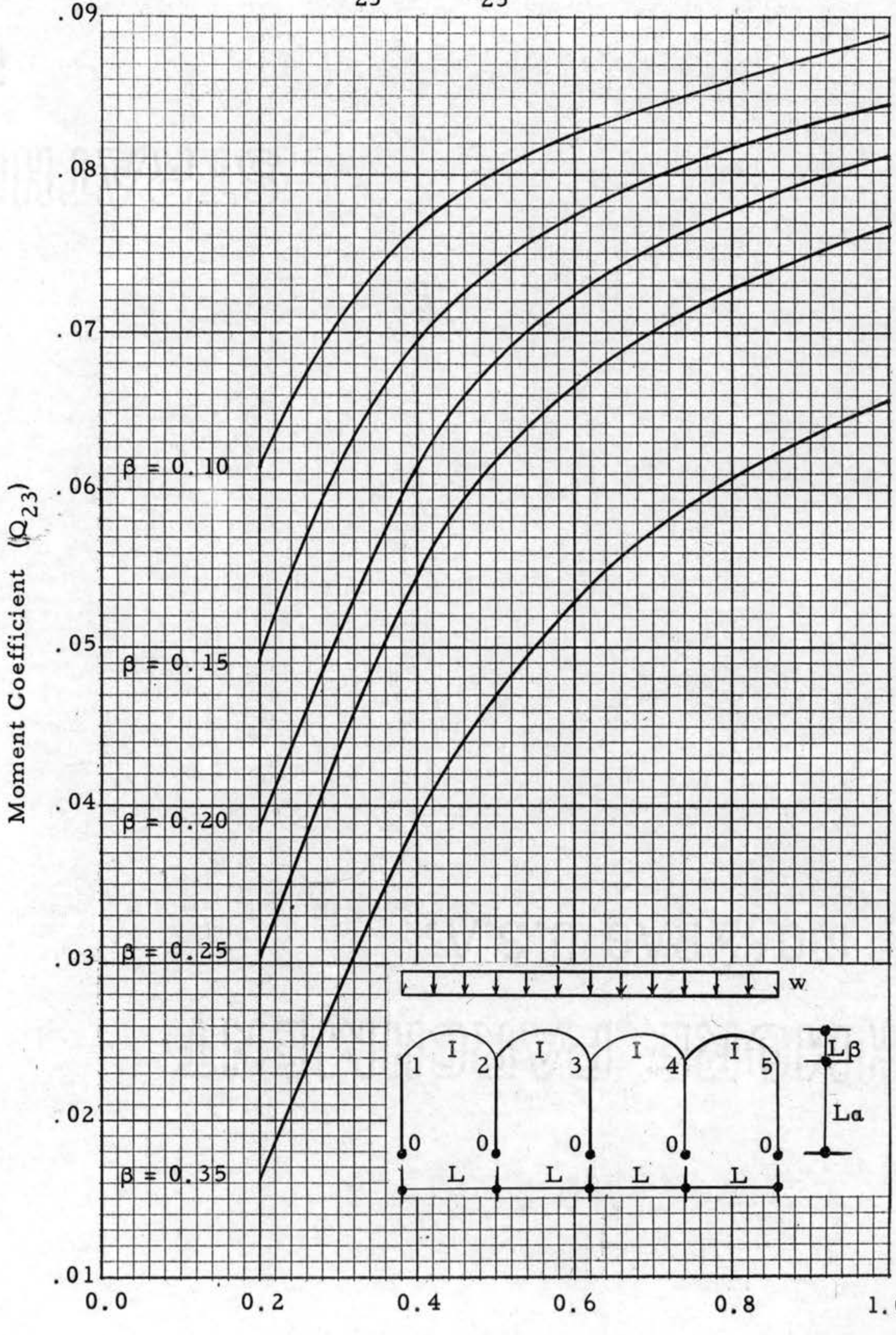


Four Span Frame | Circular Girder | Chart C-42

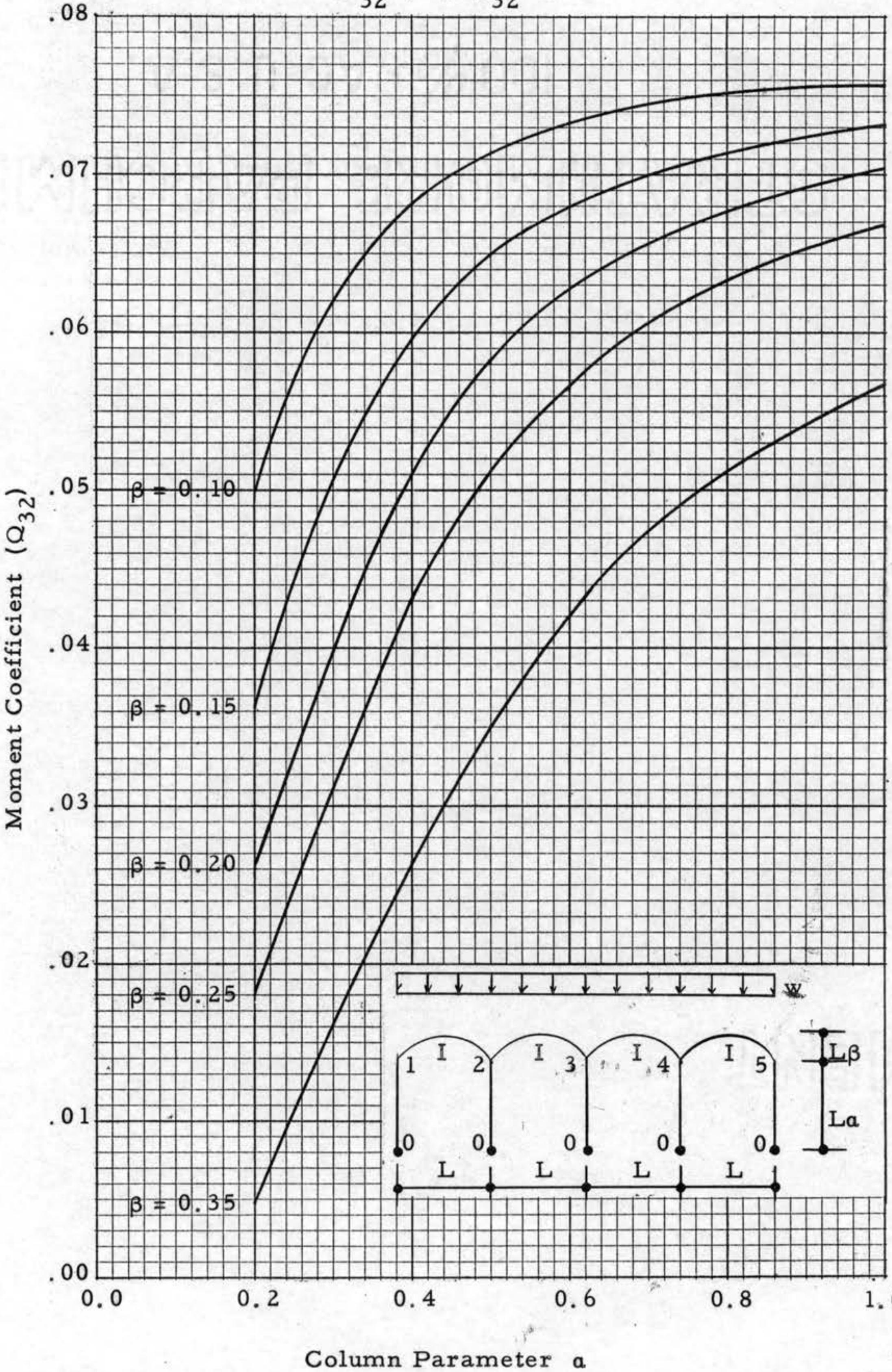
$$M_{20} = + Q_{20} w L^2$$



$M_{23} = -Q_{23} w L^2$


.09
.08
.07
.06
.05
.04
.03
.02
.01
Moment Coefficient (Q23)
β = 0.10
β = 0.15
β = 0.20
β = 0.25
β = 0.35
w
I
I
I
I
1
2
3
4
5
Lβ
La
0
0
0
0
0
L
L
L
L
0.0
0.2
0.4
0.6
0.8
1.0
Column Parameter α

$$M_{32} \mp + Q_{32} w L^2$$



PART IV

ILLUSTRATIVE EXAMPLES

First Example

A three span rigid frame, with dimensions and loads, as shown (Fig. 6), is considered. The modulus of elasticity is assumed constant for all members. The girder is of parabolic shape and the moment of inertia varies as the secant of the angle of inclination of the arch axis with the horizontal. The columns are straight and of constant cross section. The dimensions are chosen such that α and β will correspond to values listed in the tables. The values as determined by moment distribution are given for comparison.

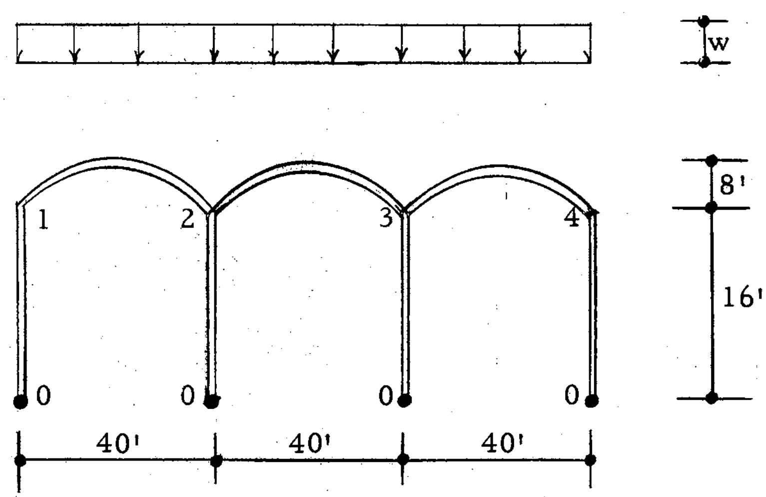


Fig. 6
Uniformly Loaded, Three Span Frame with Parabolic Girder

$\alpha = 16/40 = 0.4$ $\beta = 8/40 = 0.2$ $w = 1000^{\#}/ft.$

From Table B-2:

$Q_{12} = + .06034$ $\qquad$ $Q_{20} = + .00344$

$Q_{21} = + .06103$ $\qquad$ $Q_{23} = + .06448$

Final Moments:

	Tables	By Moment Distribution
M_{12}	= - 96, 544 #ft.	- 96, 300 #ft.
M_{21}	= + 97, 684 #ft.	+ 97, 100 #ft.
M_{23}	= - 103, 152 #ft.	- 102, 900 #ft.
M_{20}	= + 5, 504 #/ft.	+ 5, 800 #/ft.

Second Example

A three span frame, with dimensions and loads, as shown (Fig. 7) is considered. The modulus of elasticity is assumed to be constant for all members. The girders are of circular shape and constant moment of inertia. The columns are straight and of constant moment of inertia. The dimensions are chosen such that α and β do not have values found in the tables. The nomographic charts will be used for interpolation for the end moment coefficient.

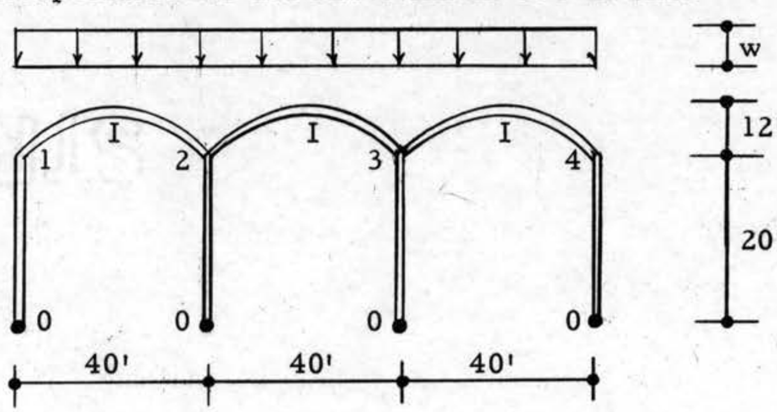


Fig. 7
Uniformly Loaded Three Span Frame with Circular Girder

$\alpha = 20/40 = 0.5$ $\qquad$ $\beta = 12/40 = .03$ $\qquad$ $w = 1000^{\#}/ft.$

From Charts:

$$C\text{-}30 \; ; \; Q_{12} = + .0561$$

$$C\text{-}31 \; ; \; Q_{21} = + .04825$$

$$C\text{-}32 \; ; \; Q_{20} = + .00468$$

$$C\text{-}33 \; ; \; Q_{23} = + .05290$$

Final Moments:

	Charts	Moment Distribution
M_{12}	= - 89, 760#ft.	- 89, 500 #ft.
M_{21}	= + 77, 200#ft.	+ 76, 800 #ft.
M_{20}	= + 7, 488#ft.	+ 7, 100 #ft.
M_{23}	= - 84, 640#ft.	- 83, 400 #ft.

PART V

SUMMARY AND CONCLUSIONS

The tables and charts, for the preliminary analysis of rigid frames with curved members, were evaluated by the slope-deflection method. The slope-deflection equations (Eq. 1a, 2a, 3a, 4a, 5a, 6a, 7a, 8a, 9a) were stated in terms of the parameters α and β. A matrix was formed by the use of the slope-deflection equations and the conditions of static equilibrium. The matrices (Table I, II, III) were evaluated numerically and solved on the IBM 650 digital computer. The moment coefficients were then evaluated with the slope-deflection equations. By two example problems, the results obtained from the tables and charts were compared with those obtained by moment distribution.

The charts and tables provide a fast and accurate method for determining the redundant end moments of two, three, and four span rigid frames having curved girders, columns with hinged base, and a uniform load.

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