ANALYSIS OF MULTISTORY TOWERS WITH INTERNAL TIES BY CARRY-OVER MOMENTS

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1948

Submitted to the Faculty of the Graduate School
of the Oklahoma State University
in partial fulfillment of
the requirements for
the degree of
MASTER OF SCIENCE
May, 1960

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Thesis Approved:

Thesis Adviser

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PREFACE

The material presented in this thesis is the out-growth of the seminar lectures presented by Professor Jan J. Tuma in Spring 1959. The Literature Survey and the general theory recorded in the introduction and Parts 1, 2, 3, and 4 were prepared by Professor Tuma. Application of this general theory to the rectangular towers was reported by Heller (29).

The writer's contribution is the derivation of carry-over constants for eight special cases and preparation of numerical examples.

The writer wishes to express his indebtedness and gratitude to Professor Jan J. Tuma for his invaluable aid and guidance in preparing this thesis and for acting as the writer's major adviser.

The writer also expresses his appreciation to Dr. K.S. Havner for his advice and thorough reading of the manuscript, to Mr. Glenn D. Houser for his kind help in preparing the sketches and tables, to Mr. Jorge H. Tolaba for his help in checking the numerical examples and to Miss Willie Cannaday for her careful typing of the manuscript.

J. T. T.

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NOMENCLATURE

h_j . . . . Height of story i-j

L_j . . . . Length of column i-j

d_j . . . . One-half of length of girder j-j'

e_j . . . . One-half of the difference in length between girders i-i' and j-j'

T_j . . . . Length of tie j

α_j . . . . Inclination of tie j

ω_j . . . . Inclination of column i-j

θ'_j . . . . Slope of members at j due to symmetrical system load

θ''_j . . . . Slope of members at j due to anti-symmetrical system load

Δ_j . . . . Relative displacement between points i and j

Δ_{jj} . . . . Relative displacement between points j and j'

ψ_j . . . . $\frac{\Delta_j}{L_j}$ for trapezoidal panels or $\frac{\Delta_j}{h_j}$ for rectangular panels

ψ_{jj} . . . . $\frac{\Delta_{jj}}{2d_j}$

P_j . . . . One-half of external, horizontal panel load at j

v_{ij} . . . . End shear of member i-j

V_j . . . . Story shear due to one-half of the loads at j

R_j . . . . Axial force of tie j

$R_{jx}(R_{jy})$ . . . . . . Horizontal (Vertical) component of R_j

$M_{ij}^{(I)}$ . . . . End moment of member i-j due to symmetrical system load

$M_{ij}^{(II)}$ . . . . End moment of member i-j due to antisymmetrical system load

K_{ij} . . . . Stiffness factor of member i-j

K_{ij}^* . . . . Modified stiffness factor of member i-j for antisymmetrical system

$CK_{ij} = C_{ij}K_{ij}$ . . . Carry-over stiffness factor of member i-j

CK_{ij}^* . . . . Modified carry-over stiffness factor of member i-j for anitsymmetrical system

C_{ij} . . . . Carry-over factor of member i-j

S_{ij} . . . . $K_{ij}\ (1 + C_{ij})$

$D_{ij}^{(I)}$ . . . . Distribution factor, $\frac{K_{ij}}{\Sigma K_i}$, for symmetrical system

$D_{ij}^{(II)}$ . . . . Modified distribution factor, $\frac{K_{ij}^*}{\Sigma K_i^*}$, for antisymmetrical system

$C_{ij}^{(I)}D_{ij}^{(I)}$ . . . . . . Carry-over distribution factor for symmetrical system

$C_{ij}^{(II)}D_{ij}^{(II)}$ . . . . . Modified carry-over distribution factor for antisymmetrical system

$r_{ij}^{(I)}$ . . . . Joint moment carry-over factor from i to j for symmetrical system

$r_{ij}^{(II)}$ . . . . Joint moment carry-over factor from i to j for antisymmetrical system

$FM_{jj}^{(I)}$ . . . . Fixed-end moment of girder j-j′ due to symmetrical system load

$FM_{jj}^{(II)}$ . . . . Fixed-end moment of girder j-j′ due to antisymmetrical system load

$FM_{ij}^*(FM_{jj}^*)$ . . . . Modified fixed-end moment of member i-j (jj′), antisymmetrical system

$m_j^{(I)}$ . . . . Starting moment for symmetrical system at joint j

$m_j^{(II)}$ . . . . Starting moment for antisymmetrical system at joint j

SM_{0j} . . . . Static load moment about 0_j

JM_j . . . . Joint moment at joint j

$Q_{jx}(Q_{jy})$ . . . . . . Horizontal (Vertical) spring constant of tie j

INTRODUCTION

The analysis of multi-story, one bay, symmetrical frames with vertical or inclined legs by means of energy or slope deflection methods is a lengthy and laborious procedure. The main difficulty lies in the geometry of deformation, preparation of a system of simultaneous equations and solution of this system for a large number of unknowns.

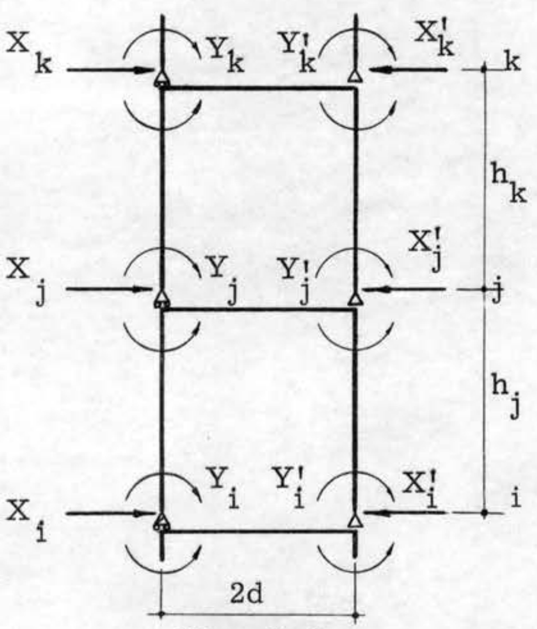


Fig. 0-1
Multistory Two Column Symmetrical Frame with Rectangular Panels. Basic Structure - Simple Two-Legged Frame.

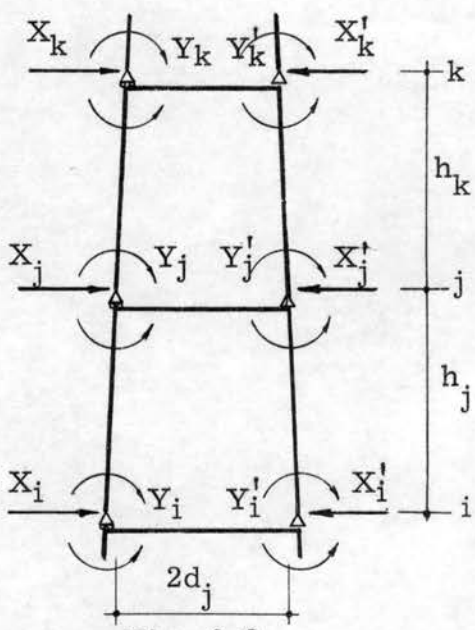


Fig. 0-2
Multistory Two Column Symmetrical Frame with Trapezoidal Panels. Basic Structure - Simple Two-Legged Frame.

The energy solution of rectangular frames was introduced by Müller-Breslau (1) and extended to trapezoidal frames by Beyer (2). Both investigators took advantage of the symmetry and antisymmetry of the loaded frame and used two-legged, simple frames as basic structures (Figs. 0-1, 2).

Parcel and Moorman (3) taking also advantage of symmetry and anti-symmetry selected normal forces and shearing forces at the center of each girder (Figs. 0-3, 4) as unknowns and developed deformation equations in terms of these forces.

The application of slope deflection method was discussed in details by Mann (4), Kruck (5), Amerikian (6), Bažant (7) and others.

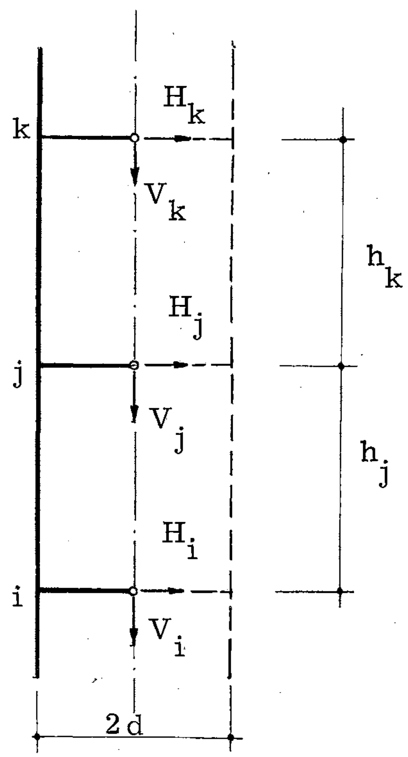


Fig. 0-3

Multistory Two Column Symmetrical Frame with Rectangular Panels. Basic Structure - Cantilever Beam

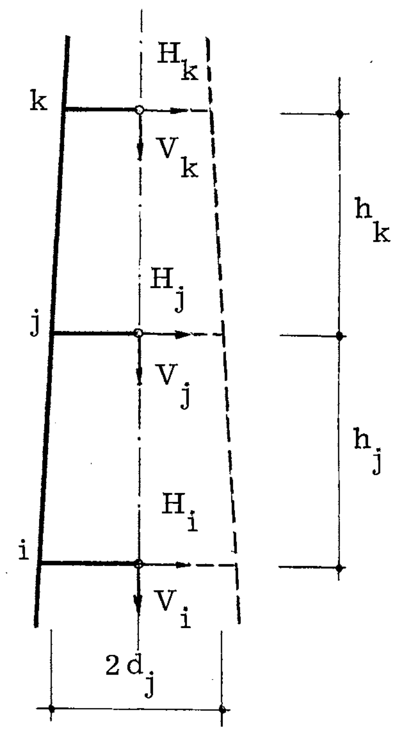


Fig. 0-4

Multistory Two Column Symmetrical Frame with Trapezoidal Panels. Basic- Structure - Cantilever Beam.

The modern philosophy of structural analysis was introduced to the solution of this problem by Čališev (8), Cross (9), and Grinter (10). Although these methods are powerful tools in solution of frames with sway prevented, they offer new difficulties when applied to frames with translating joints. The influence of sidesway must be either assumed in terms of independent Δ's and calculated by means of shear equations as shown by Maugh (11) or corrected by combined procedure of balancing moments and shears in alternate cycles as recommended by Morris (12).

Independent contributions to the numerical solutions of highly indeterminate frame structures with joints free to rotate and translate have been presented by Klouček (13), Dašek (14), Kani (15), Moliotis (16) and others. An excellent comparative study of methods for analyzing frame structures was prepared by Bažant (17) and Worch (18).

In studying the elastic instability of multi-story frames Perri (19) and Kavanagh (20) developed a new form of moment distribution procedure which includes the balancing of moments due to rotation and translation and represents an important innovation in this field. Independently Kupferschmit (21), Grinter and Tsao (22) developed the same procedure which became gradually known as "the cantilever moment distribution method." Parcel and Moorman (23), Kazda (24), Cook (25, 26), Heller (27, 28) and others applied this method to a large number of problems. The advantages and limitations of the cantilever moment distribution method have been summarized by Pei (29).

Goldberg (30) suggested a solution of one-bay, multi-story, rectangular, symmetrical frames by means of three-slope equations and proposed two iterative procedures for the solution of the slope matrix. Modifications and some other possibilities were demonstrated in discussion to Goldberg's paper by Nubar (31), Sobotka (32) and Chang (33).

Cross (34) in reviewing various techniques of moment distribution observed that:

"To one who is familiar with the process of moment distribution it soon becomes evident that it is not necessary to write the distributed moments each time. We may write only the moment carried over, and at the end of the procedure we may find the totals of the original fixed-end moments and the moments carried over, and distribute the unbalanced total. The physical significance of the procedure is somewhat obscured, but time is saved in the computation."

Dašek (35, 36) simplified and improved this special form of moment distribution and demonstrated its application on many examples.

The extension of the carry-over moment procedure (37, 38, 39, 40, 41, 42) to the analysis of multi-story, one bay, symmetrical frames with vertical or inclined columns, with or without ties and acted upon by a general system of loads is introduced here. The initial structure is resolved into symmetrical and anti-symmetrical systems and general three-slope equations for each system are derived. The slope equations are then transformed into joint moment equations from which the carry-over functions are developed.

The frame members may be of constant or variable cross-section and the deformation of the frame may be caused by transverse loads, change in temperature, displacement of supports or applied couples.

The numerical procedure has following characteristics:

a) One assumed starting moment is computed at each joint.

b) One final joint moment is obtained at each joint.

c) No distribution of unbalances is required during the carry-over procedure.

d) The successive approaching to the final values is performed by means of carry-over factors only.

e) The procedure is self checking.

This study is restricted to planar frames and the customary assumptions of the rigid frame analysis are introduced. The deformations of frame members due to shear and axial forces are assumed to be small and are neglected. The ties are elastic ties, resisting tension only and unable to resist bending or compression. The deformations of these ties are very small. Thus it may be assumed that the initial slope of the tie remains the same after the displacement of the joint takes place. The sign convention of the slope-deflection method is adopted. All forces and displacements acting to right and upward are positive. All clockwise moments and angular rotations are positive. The subsequent discussion is divided into six parts. The statement of the problem and the derivation of the fundamental functions are presented in the first four parts. Special cases are defined and tabulated in Part V. Two numerical problems are introduced in Part VI. The physical interpretation and algebraic procedure are discussed in the last parts.

1. STATEMENT OF PROBLEM

A multi-story two column symmetrical frame, with inclined legs and diagonal flexible ties, acted upon by a general system of loads is considered (Fig. 1-1). The cross-sections of the members are symmetrically variable with respect to the axis of symmetry of the frame. Since the structure is symmetrical but unsymmetrically loaded, the resolution of loads into symmetrical and antisymmetrical system offers many advantages, as has been shown by Andreé (43), Müller-Breslau (1), Beyer (2), Newell (44), Naylor (45), Pei (46) and others.

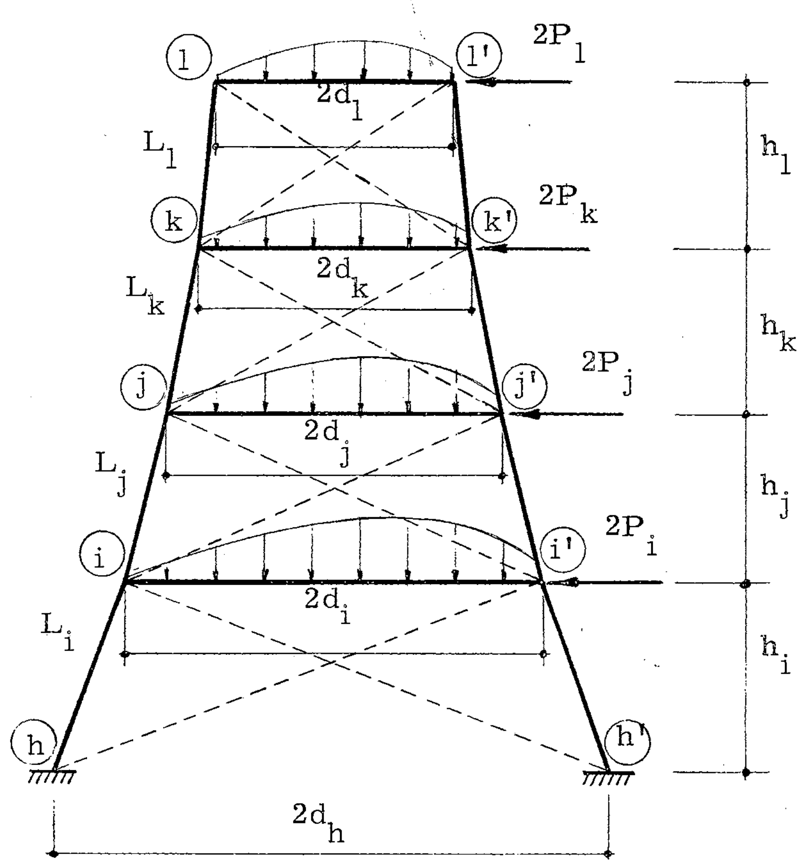


Fig. 1-1

Multistory Symmetrical Frame with Flexible Ties and Inclined Legs, Unsymmetrically Loaded

2. SYMMETRICAL SYSTEM -- TRAPEZOIDAL OR RECTANGULAR PANELS

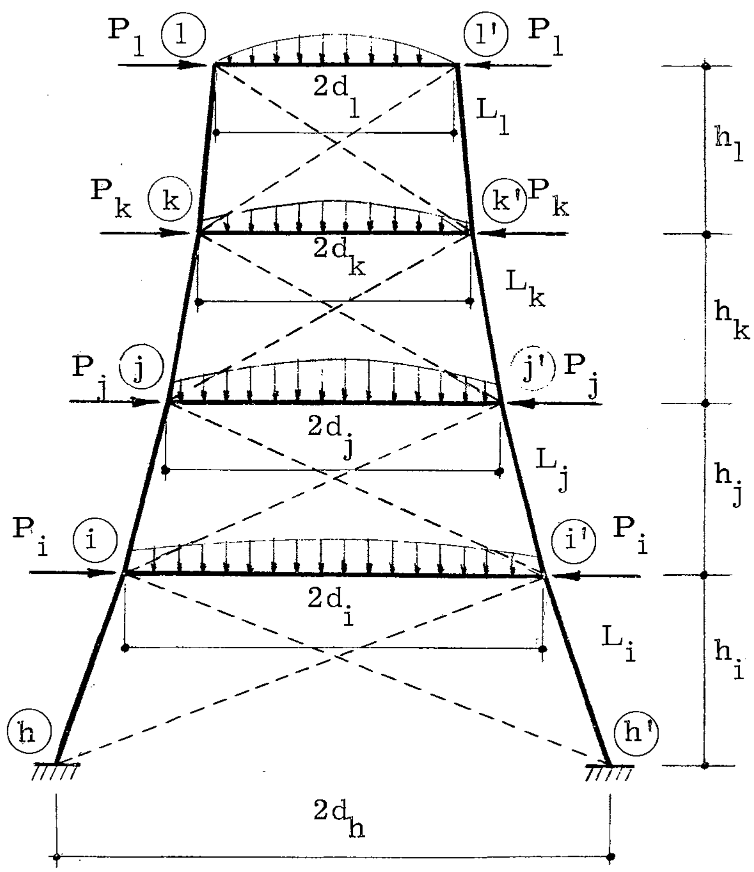


Fig. 2-1

Multistory Symmetrical Frame with Flexible Ties and Inclined Legs, Symmetrically Loaded

Due to the symmetry of the system the joint rotations of the left side are symmetrical with their right side counterparts and no translation takes place. The slope deflection equations in terms of the moment distribution nomenclature for the portion $\overline{ijk}$ (any portion) are:

$$\left.\begin{aligned}
M_{kj}^{(I)} &= K_{kj}\,\theta'_k + CK_{jk}\,\theta'_j \\
M_{jk}^{(I)} &= K_{jk}\,\theta'_j + CK_{kj}\,\theta'_k \\
M_{jj}^{(I)} &= K_{jj}\,(1-C_{jj})\,\theta'_j + FM_{jj}^{(I)} \\
M_{ji}^{(I)} &= K_{ji}\,\theta'_j + CK_{ij}\,\theta'_i \\
M_{ij}^{(I)} &= K_{ij}\,\theta'_i + CK_{ji}\,\theta'_j \ .
\end{aligned}\right\} \qquad (2\text{-}1)$$

Where

$$\left.\begin{aligned}
CK_{jk} &= CK_{kj} = C_{jk}K_{jk} = C_{kj}K_{kj} \\
CK_{ij} &= CK_{ji} = C_{ij}K_{ij} = C_{ji}K_{ji} \ .
\end{aligned}\right\} \qquad (2\text{-}2)$$

From the equilibrium of joint j:

$$M_{ji}^{(I)} + M_{jj}^{(I)} + M_{jk}^{(I)} = 0 \qquad (2\text{-}3)$$

or in terms of equations (2-1)

$$\theta'_k\,CK_{kj} + \theta'_j\,\Sigma K_j + \theta'_i\,CK_{ij} = -\,FM_{jj}^{(I)} \qquad (2\text{-}4)$$

where

$$\Sigma K_j = K_{jk} + K_{jj}\,(1-C_{jj}) + K_{ji} \ .$$

Equation (2-4) is the three-slope equation for a symmetrical system.

With new notation:

$$JM_k^{(I)} = \theta'_k \underbrace{\left[K_{kl} + K_{kk}\,(1-C_{kk}) + K_{kj}\right]}_{\Sigma K_k} \qquad (2\text{-}5a)$$

$$JM_j^{(I)} = \theta'_j \underbrace{\left[K_{jk} + K_{jj}\,(1-C_{jj}) + K_{ji}\right]}_{\Sigma K_j} \qquad (2\text{-}5b)$$

$$JM_i^{(I)} = \theta'_i \left[\underbrace{K_{ij} + K_{ii}(1-C_{ii}) + K_{ih}}_{\Sigma K_i} \right] \tag{2-5}$$

and

$$m_j^{(I)} = - FM_{jj}^{(I)} \tag{2-6}$$

$$\left.\begin{aligned} r_{ij}^{(I)} &= - \frac{CK_{ij}}{\Sigma K_j} \\ r_{kj}^{(I)} &= - \frac{CK_{kj}}{\Sigma K_k} \end{aligned}\right\} \tag{2-7}$$

the three-slope equation (2-4) becomes the three-joint-moment equation:

$$JM_j^{(I)} = r_{kj}^{(I)} JM_k^{(I)} + m_j^{(I)} + r_{ij}^{(I)} JM_i^{(I)} . \tag{2-8}$$

This new form of equation (2-4) consists of the starting moment $m_j^{(I)}$, the carry-over factor $r_{ij}^{(I)}$, $r_{kj}^{(I)}$ and the redundant moments $JM_i^{(I)}$, $JM_j^{(I)}$ and $JM_k^{(I)}$. The similarity of equation (2-8) with equation (11) of Tuma's recent papers (37, 41) is well apparent. The physical interpretation of parameters $m_j^{(I)}$, $r_{ij}^{(I)}$ and $r_{kj}^{(I)}$ follows:

a. <u>Starting Moment $m_j^{(I)}$</u> is the moment at j due to loads, required to hold the joint j in equilibrium, if the joints i and k are fixed.

b. <u>Carry-over Factor $r_{ij}^{(I)}$ ($r_{kj}^{(I)}$)</u> is the moment at j due to $JM_i^{(I)}$ = + 1 ($JM_k^{(I)}$ = + 1), if the joint k (i) is fixed.

The slope deflection equations (2-1) in terms of equations (2-5, 6, 7) and with similar notation become:

$$
\left.
\begin{aligned}
M_{kj}^{(I)} &= D_{kj}^{(I)}\, JM_k^{(I)} + C_{jk}^{(I)}\, D_{jk}^{(I)}\, JM_j^{(I)} \\
M_{jk}^{(I)} &= D_{jk}^{(I)}\, JM_j^{(I)} + C_{kj}^{(I)}\, D_{kj}^{(I)}\, JM_k^{(I)} \\
M_{jj}^{(I)} &= D_{jj}^{(I)}\, JM_j^{(I)} + FM_{jj}^{(I)} \\
M_{ji}^{(I)} &= D_{ji}^{(I)}\, JM_j^{(I)} + C_{ij}^{(I)}\, D_{ij}^{(I)}\, JM_i^{(I)} \\
M_{ij}^{(I)} &= D_{ij}^{(I)}\, JM_i^{(I)} + C_{ji}^{(I)}\, D_{ji}^{(I)}\, JM_j^{(I)} \; .
\end{aligned}
\right\} \quad (2\text{-}9)
$$

3. ANTISYMMETRICAL SYSTEM -- TRAPEZOIDAL PANELS

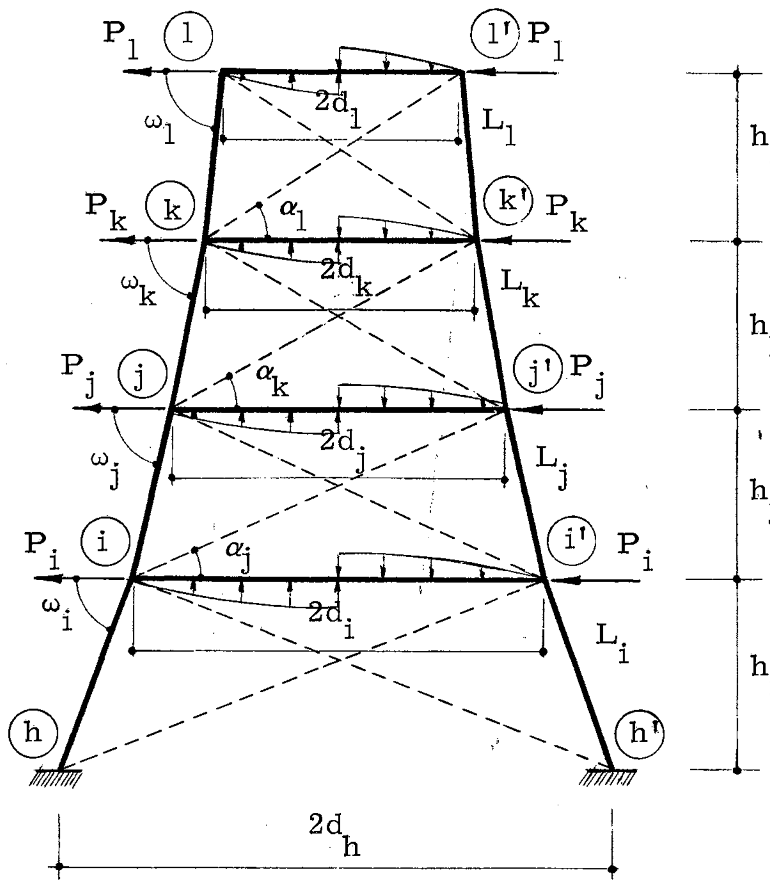


Fig. 3-1

Multi-Story Symmetrical Frame with Flexible Ties and Inclined Legs, Antisymmetrically Loaded

Due to the antisymmetry of the system the joint rotations and translations of the left side are antisymmetrical with their right side counterparts. The slope deflection equations in terms of the moment distribution nomenclature for the portion $\overline{ijk}$ (any portion) are:

$$\left.\begin{aligned}
M_{kj}^{(II)} &= K_{kj}\theta''_{k} + CK_{jk}\theta''_{j} + S_{kj}\psi_{k} \\
M_{jk}^{(II)} &= K_{jk}\theta''_{j} + CK_{kj}\theta''_{k} + S_{jk}\psi_{k} \\
M_{jj}^{(II)} &= K_{jj}\,(1+C_{jj})\,\theta''_{j} - S_{jj}\psi_{jj} + FM_{jj}^{(II)} \\
M_{ji}^{(II)} &= K_{ji}\theta''_{j} + CK_{ij}\theta''_{i} + S_{ji}\psi_{j} \\
M_{ij}^{(II)} &= K_{ij}\theta''_{i} + CK_{ji}\theta''_{j} + S_{ij}\psi_{j} \ .
\end{aligned}\right\} \qquad (3\text{-}1)$$

Where

$$\left.\begin{aligned}
S_{kj} &= K_{kj} + CK_{jk} \\
S_{jk} &= K_{jk} + CK_{kj} \\
S_{jj} &= K_{jj} + CK_{jj} \\
S_{ji} &= K_{ji} + CK_{ij} \\
S_{ij} &= K_{ij} + CK_{ji} \ .
\end{aligned}\right\} \qquad (3\text{-}2)$$

$FM_{jj}^{(II)}$ = fixed end moment due to loads.

From the equilibrium of joint j:

$$M_{jk}^{(II)} + M_{jj}^{(II)} + M_{ji}^{(II)} = 0, \qquad (3\text{-}3)$$

or in terms of equations (3-1)

$$\theta''_{k}\ CK_{kj} + \theta''_{j}\,\Sigma K_{j} + \theta''_{i}\ CK_{ij} + S_{jk}\psi_{k} - S_{jj}\psi_{jj} + S_{ji}\psi_{j} + FM_{jj}^{(II)} = 0. \qquad (3\text{-}4)$$

Where

$$\Sigma K_j = K_{jk} + K_{jj} (1+C_{jj}) + K_{ji} .$$

The six-slope equation (3-4) contains redundant slopes (θ'')'s and ψ's. First the relationship of ψ's to the deformation of flexible ties must be determined. Then the elimination of ψ's and transformation of the six-slope equation into a three-slope equation is shown. Finally the three-joint moment equation similar to equation (2-8) is derived.

The deformation curve of a typical panel (iji'j') is shown in a very exaggerated shape (Fig. 3-2). The translation of joints j and j' is defined by displacements Δ_j and $\Delta_{jj'}$. The elongation of the elastic tie j,

$$\Delta T_j = \frac{R_j T_j}{A_j E} = R_j \lambda_j . \tag{3-5}$$

Where

R_j = Axial force

T_j = Length

A_j = Cross-sectional area

E = Modulus of elasticity

λ_j = Axial flexibility.

The relationship between the translatory displacement Δ_j and the elongation of the tie ΔT_j is given by the geometry of Fig. (3-3).

$$\underbrace{R_j \lambda_j}_{\Delta T_j} = \underbrace{\frac{2h_j d_i}{T_j}}_{\Delta_j \sin(\alpha_j + \omega_j)} \psi_j . \tag{3-6a}$$

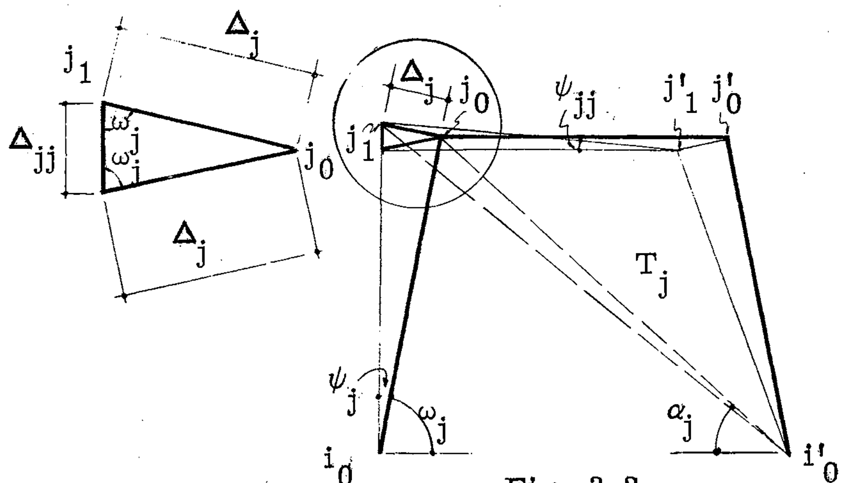


Fig. 3-2

Translocation Diagram

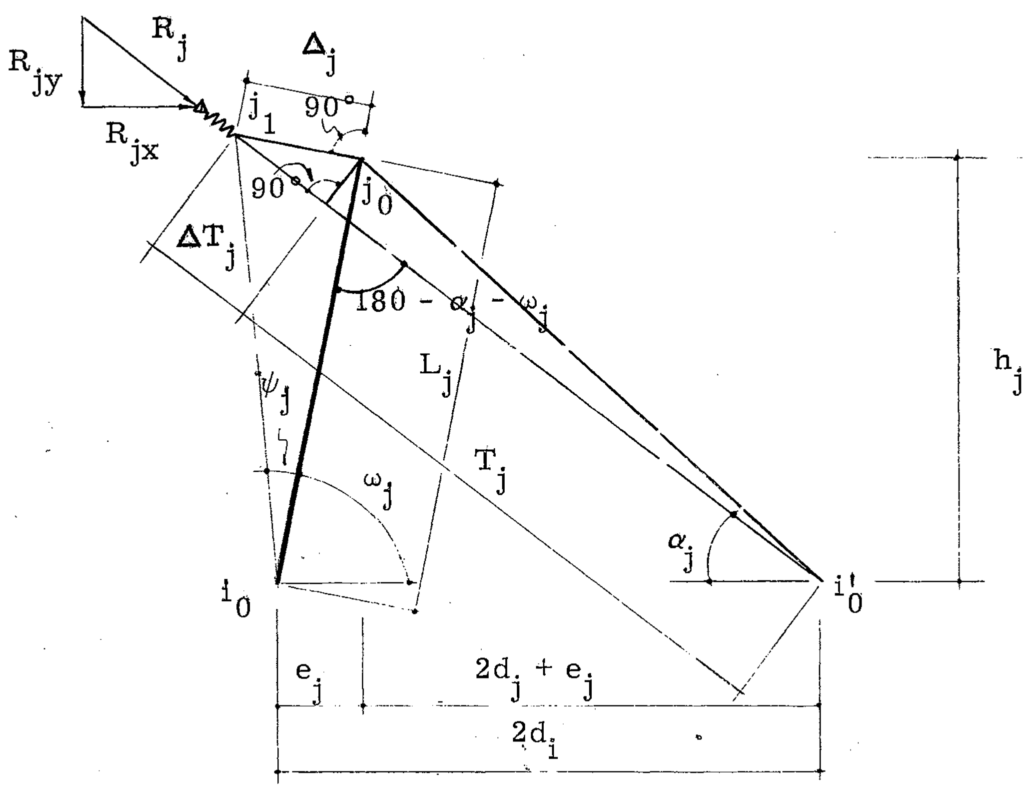


Fig. 3-3

Equivalent Spring

From which the axial force in tie j,

$$R_j = \underbrace{\frac{2h_j d_i}{T_j \lambda_j}}_{Q_j} \psi_j \quad . \qquad (3\text{-}6b)$$

In the derivation of equations (3-6a) following identities are used:

$$\sin(\alpha_j + \omega_j) = (\underbrace{\frac{h_j}{T_j}\frac{e_j}{L_j}}_{\sin\alpha_j \cos\omega_j} + \underbrace{\frac{2d_j + e_j}{T_j}\frac{h_j}{L_j}}_{\cos\alpha_j \sin\omega_j})$$

and

$$\Delta_j = L_j \psi_j \quad .$$

The active tie j is replaced by an elastic spring, with spring force R_j and spring constant Q_j. The component forces of the spring j are then from equation (3-6b)

$$\left.\begin{aligned} R_{jx} &= R_j \cos\alpha_j = (\underbrace{\frac{2d_i}{\lambda_j} \sin\alpha_j \cos\alpha_j}_{Q_{jx}}) \psi_j \ , \\ R_{jy} &= R_j \sin\alpha_j = (\underbrace{\frac{2d_i}{\lambda_j} \sin^2\alpha_j}_{Q_{jy}}) \psi_j \ . \end{aligned}\right\} \qquad (3\text{-}7)$$

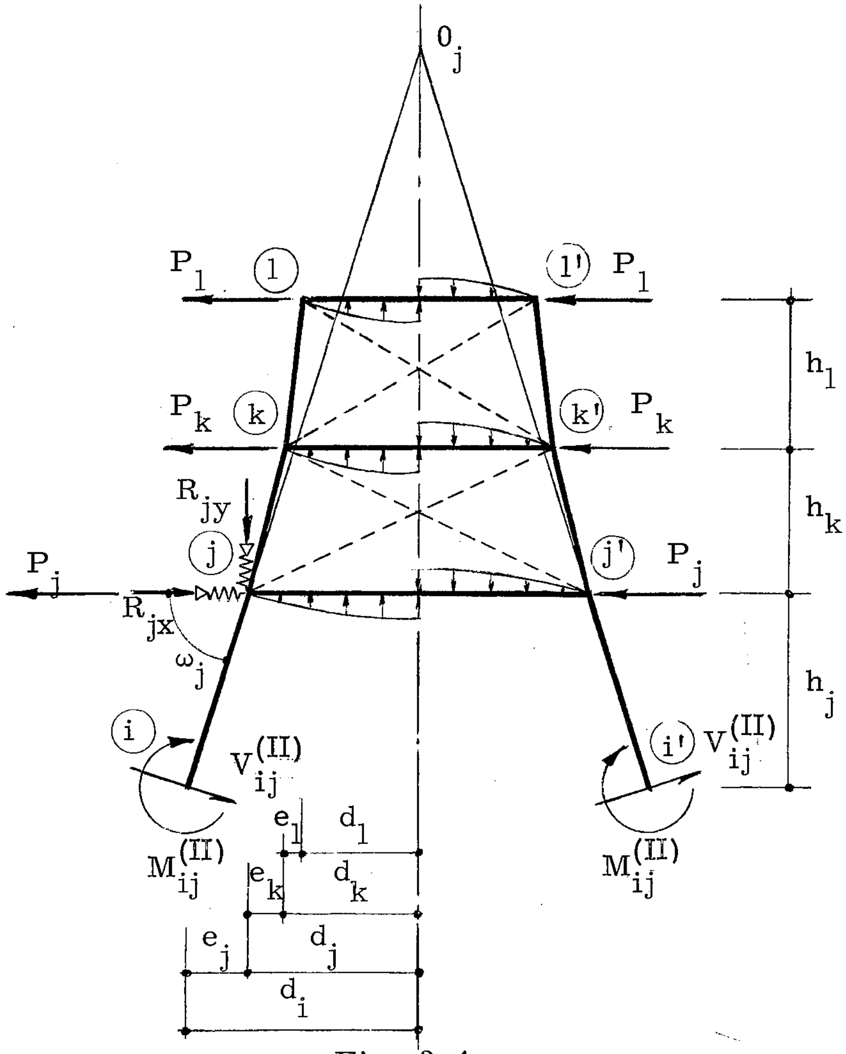


Fig. 3-4

Free Body Above i-i'

From Fig. 3-4

$$\Sigma M_{0j} = SM_{0j}^{(II)} - 2V_{ij}^{(II)} \frac{d_i}{\cos\omega_j} - R_{jx} \frac{d_j}{\cos\omega_j} \sin\omega_j$$

$$- R_{jy} d_j + 2M_{ij}^{(II)} = 0 \qquad (3\text{-}8)$$

where

$SM_{0j}^{(II)}$ = Static load moment about 0_j

$$V_{ij}^{(II)} = \frac{M_{ij}^{(II)} + M_{ji}^{(II)}}{L_j}$$

and $M_{ij}^{(II)}$, $M_{ji}^{(II)}$, R_{jx} and R_{jy} are as given by equations (3-1) and (3-7) respectively. Equation (3-8) is expressed in terms of θ''_i, θ''_j and ψ_j, from which,

$$\psi_j = -\frac{d_j}{N_j}(S_{ij}\theta''_i + S_{ji}\theta''_j) + \frac{e_j}{N_j}(K_{ij}\theta''_i + CK_{ji}\theta''_j) + \frac{e_j}{2N_j} SM_{0j}^{(II)} \qquad (3\text{-}9)$$

in which

$$N_j = d_j S_{ij} + d_i S_{ji} + \frac{L_j d_i}{2}(Q_{jx}\sin\omega_j + Q_{jy}\cos\omega_j) .$$

Similarly

$$\psi_k = -\frac{d_j}{N_k}(S_{jk}\theta''_j + S_{kj}\theta''_k) + \frac{e_k}{N_k}(K_{jk}\theta''_j + CK_{kj}\theta''_k) + \frac{e_k}{2N_k} SM_{0k}^{(II)} \qquad (3\text{-}10)$$

in which

$$N_k = d_k S_{jk} + d_j S_{kj} + \frac{L_k d_k}{2}(Q_{kx}\sin\omega_k + Q_{ky}\cos\omega_k) .$$

From Fig. 3-2

$$\psi_{jj} = \frac{e_j}{d_j}\psi_j \qquad (3\text{-}11)$$

With notations assembled in Table 5-1 the six slope equation (3-4) becomes

$$\theta''_k CK^*_{kj} + \theta''_j \Sigma K^*_j + \theta''_i (CK^*_{ij} + CK^*_{ijj}) = - \Sigma FM^*_j \qquad (3\text{-}12)$$

This equation is the three-slope equation for an antisymmetrical system.

Denoting

$$\left.\begin{aligned} JM_k^{(II)} &= \theta''_k \underbrace{(K^*_{kl} + K^*_{kkj} + K^*_{kj})}_{\Sigma K^*_k} \\ JM_j^{(II)} &= \theta''_j \underbrace{(K^*_{jk} + K^*_{jji} + K^*_{ji})}_{\Sigma K^*_j} \\ JM_i^{(II)} &= \theta''_i \underbrace{(K^*_{ij} + K^*_{iih} + K^*_{ih})}_{\Sigma K^*_i} \end{aligned}\right\} \qquad (3\text{-}13)$$

and

$$m_j^{(II)} = - \Sigma FM^*_j \qquad (3\text{-}14)$$

$$\left.\begin{aligned} r_{kj}^{(II)} &= - \frac{CK^*_{kj}}{\Sigma K^*_k} \\ r_{ij}^{(II)} &= - \frac{CK^*_{ij} + CK^*_{ijj}}{\Sigma K^*_i} \end{aligned}\right\} \qquad (3\text{-}15)$$

the three slope equation (3-12) becomes a three-joint moment equation.

$$JM_j^{(II)} = r_{kj}^{(II)} JM_k^{(II)} + m_j^{(II)} + r_{ij}^{(II)} JM_i^{(II)} . \qquad (3\text{-}16)$$

This new form of equation (3-4) consists again of the starting moment $m_j^{(II)}$, the carry-over factor $r_{ij}^{(II)}$, $r_{ji}^{(II)}$ and the redundant moments $JM_i^{(II)}$, $JM_j^{(II)}$ and $JM_k^{(II)}$. The similarity of equation (3-16) with equation (11) of Tuma's recent papers (37, 41) is also well apparent. The physical interpretation of these parameters follows:

a. Starting Moment $m_j^{(II)}$ is the moment at j due to loads, required to hold the joint j in equilibrium, if the joints i and k are fixed against rotation but free to translate.

b. Carry-over Factor $r_{ij}^{(II)}$ $(r_{kj}^{(II)})$ is the moment at j due to $JM_i^{(II)}$ $(JM_k^{(II)}) = +1$, if the joint k (i) is fixed against rotation but free to translate.

The slope deflection equations (3-1) in terms of equations (3-13, 14, 15) and with similar notation become:

$$\left.\begin{aligned}
M_{kj}^{(II)} &= D_{kj}^{(II)} JM_k^{(II)} + C_{jk}^{(II)} D_{jk}^{(II)} JM_j^{(II)} + FM_{kj}^* \\
M_{jk}^{(II)} &= D_{jk}^{(II)} JM_j^{(II)} + C_{kj}^{(II)} D_{kj}^{(II)} JM_k^{(II)} + FM_{jk}^* \\
M_{jj}^{(II)} &= D_{jji}^{(II)} JM_j^{(II)} + C_{ijj}^{(II)} D_{ijj}^{(II)} JM_i^{(II)} + FM_{jj}^* \\
M_{ji}^{(II)} &= D_{ji}^{(II)} JM_j^{(II)} + C_{ij}^{(II)} D_{ij}^{(II)} JM_i^{(II)} + FM_{ji}^* \\
M_{ij}^{(II)} &= D_{ij}^{(II)} JM_i^{(II)} + C_{ji}^{(II)} D_{ji}^{(II)} JM_j^{(II)} + FM_{ij}^* \ .
\end{aligned}\right\} \quad (3\text{-}17)$$

Two new symbols in equations (3-17) are:

$$D_{jji}^{(II)} = \frac{K_{jji}^*}{\Sigma K_j^*} \qquad\Bigg|\qquad C_{ijj}^{(II)} D_{ijj}^{(II)} = \frac{CK_{ijj}^*}{\Sigma K_i^*} \quad (3\text{-}18)$$

4. ANTISYMMETRICAL SYSTEM -- RECTANGULAR PANELS

In the case of a multi-story symmetrical one-bay frame with vertical columns and antisymmetrical load

$L_j = h_j$	$\omega_j = 90°$
$\Delta_j = \psi_j h_j$	$d_j = d$.

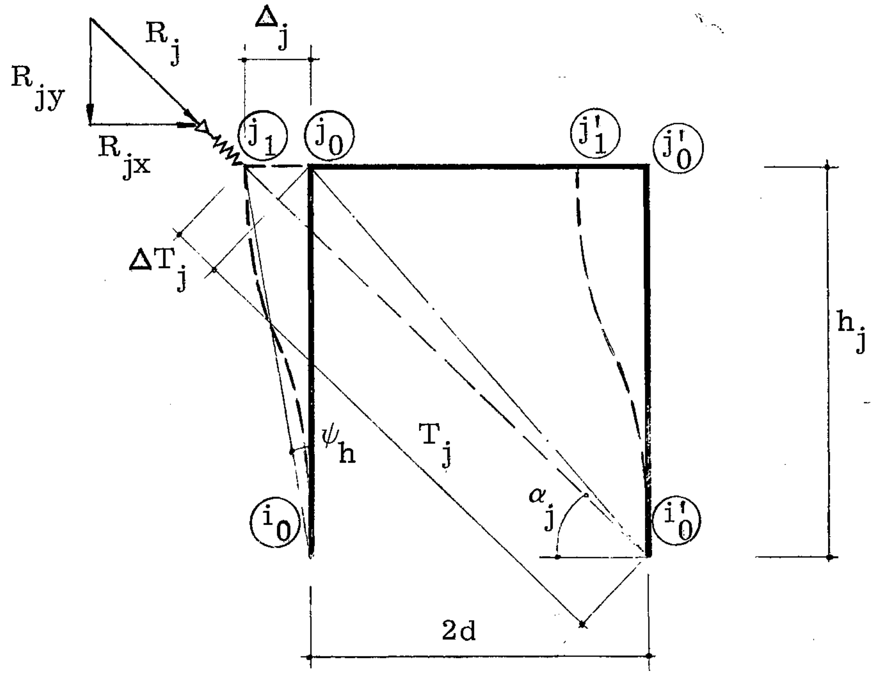


Fig. 4-1
Translocation Diagram

The relationship between Δ_j and ΔT_j is given by the geometry of Fig. 4-1 (compare with Fig. 3-3 and equation 3-6a).

$$\underbrace{\Delta T_j}_{\frac{R_j T_j}{A_j E}} = \underbrace{\Delta_j \cos\alpha_j}_{\psi_j h_j \frac{2d}{T_j}} \quad . \tag{4-1a}$$

From which

$$R_j = \frac{2h_j d}{T_j \lambda_j} \psi_j \quad . \tag{4-1b}$$

Because there is no vertical displacement of joint j, the horizontal component of the spring force R_j is the only action to be considered

$$R_{jx} = \underbrace{\frac{4h_j d^2}{T_j^2 \lambda_j}}_{Q_{jx}} \psi_j \quad . \tag{4-2}$$

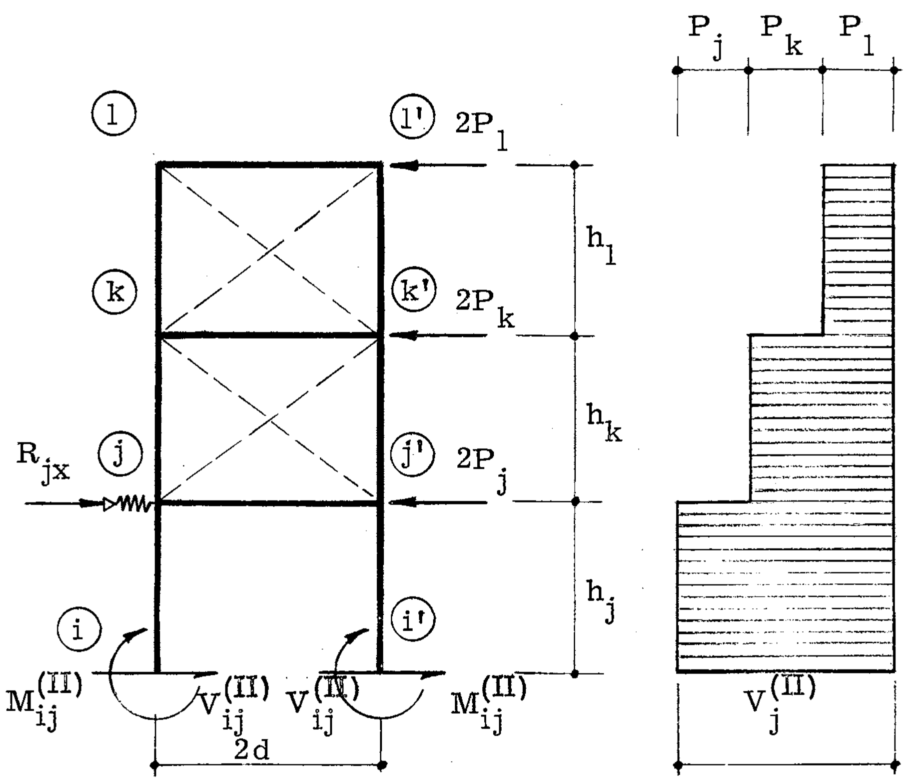


Fig. 4-2
Free Body ii'

For the elimination of ψ_j, the typical story shear equation may be used (Fig. 4-2).

$$\Sigma F_{jx} = 0 \quad \Big| \qquad - 2\overleftarrow{V_j^{(II)}} + 2\overrightarrow{V_{ij}^{(II)}} + \overrightarrow{R_{jx}} = 0. \tag{4-3}$$

Where V_j is the story shear due to one half of the horizontal load. From equation (4-3)

$$\psi_j = \frac{1}{N_j} (V_j^{(II)} h_j - S_{ij}\theta''_i - S_{ji}\theta''_j) \tag{4-4}$$

and

$$N_j = S_{ij} + S_{ji} + \frac{1}{2} Q_{jx} h_j . \tag{4-5}$$

Similar is the derivation of ψ_k and N_k. The constants for this type of frames are recorded in Part 5, Tables 4-5, 6, 7. 8.

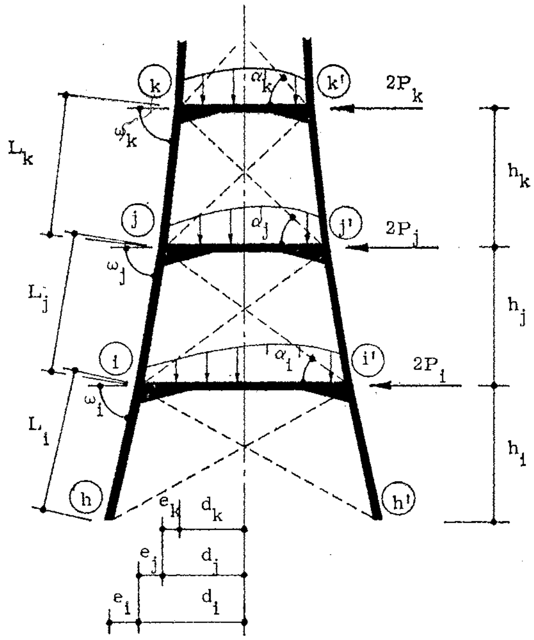
5. TYPICAL CASES

Constants for eight special cases are recorded in the following tables:

Table 5-1:	Constants for Trapezoidal Panels, Variable Cross-Section, with Ties;
Table 5-2:	Constants for Trapezoidal Panels, Variable Cross-Section, without Ties;
Table 5-3:	Constants for Trapezoidal Panels, Constant Cross-Section, with Ties;
Table 5-4:	Constants for Trapezoidal Panels, Constant Cross-Section, without Ties;
Table 5-5:	Constants for Rectangular Panels, Variable Cross-Section, with Ties;
Table 5-6:	Constants for Rectangular Panels, Variable Cross-Section, without Ties;
Table 5-7:	Constants for Rectangular Panels, Constant Cross-Section, with Ties;
Table 5-8:	Constants for Rectangular Panels, Constant Cross-Section, without Ties.

Table 5-1 Constants for Trapezoidal Panels, Variable Cross-Section, with Ties

TYPICAL PORTION:



FIXED END MOMENTS:

$$FM_{jj}^{(I)'} = FM_{jj}^{(I)}$$

$$FM_{jk}^{*} = \frac{S_{jk}}{2N_k} e_k SM_{0k}^{(II)}$$

$$FM_{jj}^{*} = -\frac{S_{jj}}{2N_j} \frac{e_j^2}{d_j} SM_{0j}^{(II)} + FM_{jj}^{(II)}$$

$$FM_{ji}^{*} = \frac{S_{ji}}{2N_j} e_j SM_{0j}^{(II)}$$

CONSTANTS:

$$C_{kj}^{(I)} = C_{kj}K_{kj}$$

$$K_{jk}^{(I)} = K_{jk}$$

$$K_{jj}^{(I)} = K_{jj}(1 - C_{jj})$$

$$K_{ji}^{(I)} = K_{ji}$$

$$CK_{ij}^{(I)} = C_{ij}K_{ij}$$

$$CK_{kj}^{*} = K_{kj}\left[C_{kj} - \frac{S_{jk}}{N_k}(d_k C_{kj} + d_j)\right]$$

$$K_{jk}^{*} = K_{jk}\left[1 - \frac{S_{jk}}{N_k}(d_j C_{jk} + d_k)\right]$$

$$K_{jji}^{*} = S_{jj}\left[1 + \frac{K_{ji}}{N_j}\frac{e_j}{d_j}(d_j C_{ji} + d_i)\right]$$

$$K_{ji}^{*} = K_{ji}\left[1 - \frac{S_{ji}}{N_j}(d_j C_{ji} + d_i)\right]$$

$$CK_{ij}^{*} = K_{ij}\left[C_{ij} - \frac{S_{ji}}{N_j}(d_i C_{ij} + d_j)\right]$$

$$CK_{ijj}^{*} = S_{jj}\left[\frac{K_{ij}}{N_j}\frac{e_j}{d_j}(d_i C_{ij} + d_j)\right]$$

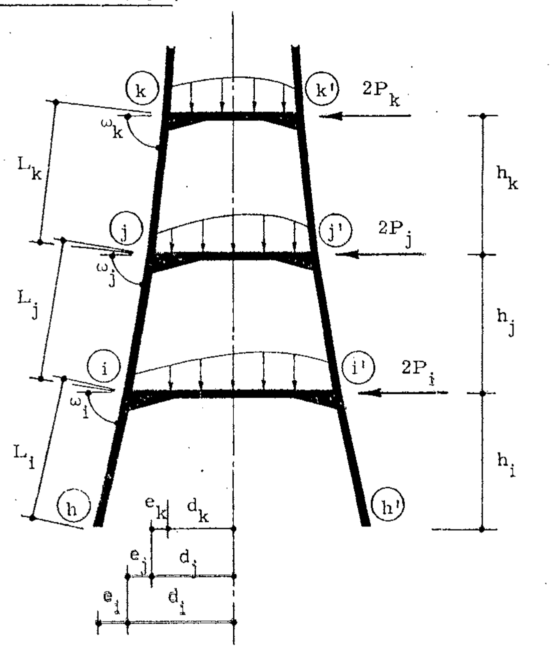
EQUIVALENTS:

$$N_k = d_k S_{jk} + d_j S_{kj} + \frac{d_k}{2}(Q_{kx}h_k + Q_{ky}e_k) \quad\Big|\quad N_j = d_j S_{ij} + d_i S_{ji} + \frac{d_j}{2}(Q_{jx}h_j + Q_{jy}e_j)$$

$S_{jk} = K_{jk}(1 + C_{jk})$	$S_{jj} = K_{jj}(1 + C_{jj})$	$S_{ji} = K_{ji}(1 + C_{ji})$
$S_{kj} = K_{kj}(1 + C_{kj})$		$S_{ij} = K_{ij}(1 + C_{ij})$
$Q_{kx} = \frac{2d_j}{\lambda_k}\sin\alpha_k\cos\alpha_k$	$Q_{ky} = \frac{2d_j}{\lambda_k}\sin^2\alpha_k$	$\lambda_k = \frac{T_k}{A_k E}$
$Q_{jx} = \frac{2d_i}{\lambda_j}\sin\alpha_j\cos\alpha_j$	$Q_{jy} = \frac{2d_i}{\lambda_j}\sin^2\alpha_j$	$\lambda_j = \frac{T_j}{A_j E}$

Table 5-2 Constants for Trapezoidal Panels, Variable Cross-Section, without Ties

TYPICAL PORTION:



FIXED END MOMENTS:

$$FM_{jj}^{(I)} = FM_{jj}^{(I)}$$

$$FM_{jk}^{*} = \frac{S_{jk}}{2N_k} e_k SM_{0k}^{(II)}$$

$$FM_{jj}^{*} = -\frac{S_{jj}}{2N_j} \frac{e_j^2}{d_j} SM_{0j}^{(II)} + FM_{jj}^{(II)}$$

$$FM_{ji}^{*} = \frac{S_{ji}}{2N_j} e_j SM_{0j}^{(II)}$$

CONSTANTS:

$$CK_{kj}^{(I)} = C_{kj}K_{kj}$$

$$K_{jk}^{(I)} = K_{jk}$$

$$K_{jj}^{(I)} = K_{jj}(1 - C_{jj})$$

$$K_{ji}^{(I)} = K_{ji}$$

$$CK_{ij}^{(I)} = C_{ij}K_{ij}$$

$$CK_{kj}^{*} = K_{kj}\left[C_{kj} - \frac{S_{jk}}{N_k}(d_k C_{kj} + d_j)\right]$$

$$K_{jk}^{*} = K_{jk}\left[1 - \frac{S_{jk}}{N_k}(d_j C_{jk} + d_k)\right]$$

$$K_{jji}^{*} = S_{jj}\left[1 - \frac{K_{ji}}{N_j}\frac{e_j}{d_j}(d_j C_{ji} + d_i)\right]$$

$$K_{ji}^{*} = K_{ji}\left[1 - \frac{S_{ji}}{N_j}(d_j C_{ji} + d_i)\right]$$

$$CK_{ij}^{*} = K_{ij}\left[C_{ij} - \frac{S_{ji}}{N_j}(d_i C_{ij} + d_j)\right]$$

$$CK_{ijj}^{*} = S_{jj}\left[\frac{K_{ij}}{N_j}\frac{e_j}{d_j}(d_i C_{ij} + d_j)\right]$$

EQUIVALENTS:

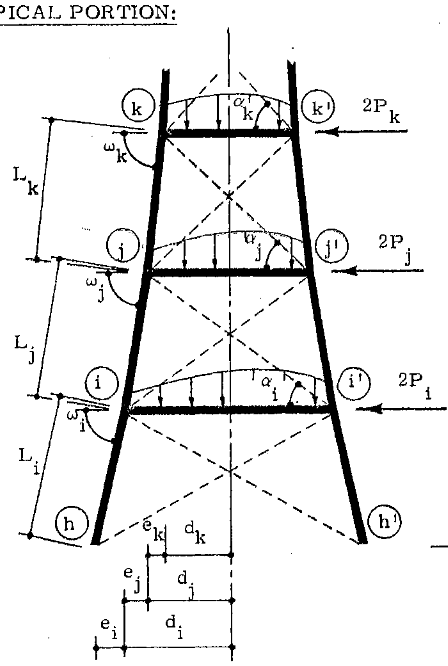
$$N_k = d_k S_{jk} + d_j S_{kj} \qquad N_j = d_j S_{ij} + d_i S_{ji}$$

$$S_{jk} = K_{jk}(1 + C_{jk}) \qquad S_{jj} = K_{jj}(1 + C_{jj}) \qquad S_{ji} = K_{ji}(1 + C_{ji})$$

$$S_{kj} = K_{kj}(1 + C_{kj}) \qquad S_{ij} = K_{ij}(1 + C_{ij})$$

Table 5-3 Constants for Trapezoidal Panels, Constant Cross-Section, with Ties

TYPICAL PORTION:



FIXED END MOMENTS:

$$FM^{(I)}_{jj} = FM^{(I)}_{jj}$$

$$FM^*_{jk} = \frac{3EI_k}{N_k L_k} e_k \, SM^{(II)}_{0k}$$

$$FM^*_{jj} = -\frac{3EI_{jj}}{N_j L_j} \frac{e_j^2}{d_j} SM^{(II)}_{0j} + FM^{(II)}_{jj}$$

$$FM^*_{ji} = \frac{3EI_j}{N_j L_j} e_j \, SM^{(II)}_{0j}$$

CONSTANTS:

$$CK^{(I)}_{kj} = \frac{2EI_k}{L_k}$$

$$K^{(I)}_{jk} = \frac{4EI_k}{L_k}$$

$$K^{(I)}_{jj} = \frac{EI_{jj}}{d_j}$$

$$K^{(I)}_{ji} = \frac{4EI_j}{L_j}$$

$$CK^{(I)}_{ij} = \frac{2EI_j}{L_j}$$

$$CK^*_{kj} = \frac{4EI_k}{L_k}\left[\frac{1}{2} - \frac{6EI_k}{N_k L_k}\left(\frac{d_k}{2} + d_j\right)\right]$$

$$K^*_{jk} = \frac{4EI_k}{L_k}\left[1 - \frac{6EI_k}{N_k L_k}\left(\frac{d_j}{2} + d_k\right)\right]$$

$$K^*_{jji} = \frac{3EI_{jj}}{d_j}\left[1 + \frac{4EI_j}{N_j L_j}\frac{e_j}{d_j}\left(\frac{d_j}{2} + d_i\right)\right]$$

$$K^*_{ji} = \frac{4EI_j}{L_j}\left[1 - \frac{6EI_j}{N_j L_j}\left(\frac{d_j}{2} + d_i\right)\right]$$

$$CK^*_{ij} = \frac{4EI_j}{L_j}\left[\frac{1}{2} - \frac{6EI_j}{N_j L_j}\left(\frac{d_i}{2} + d_j\right)\right]$$

$$CK^*_{ijj} = \frac{3EI_{jj}}{d_j}\left[\frac{4EI_j}{N_j L_j}\frac{e_j}{d_j}\left(\frac{d_i}{2} + d_j\right)\right]$$

EQUIVALENTS:

$$N_k = \frac{6EI_k}{L_k}(d_k + d_j) + \frac{d_k}{2}(Q_{kx}h_k + Q_{ky}e_k)$$

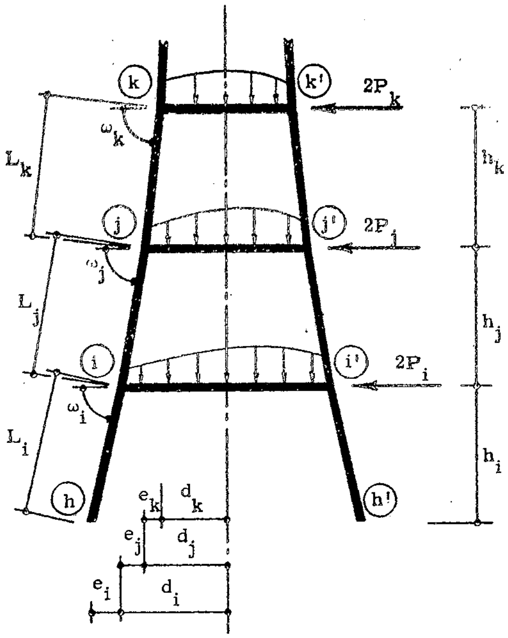
$$N_j = \frac{6EI_j}{L_j}(d_j + d_i) + \frac{d_i}{2}(Q_{jx}h_j + Q_{jy}e_j)$$

$$Q_{kx} = \frac{2d_j}{\lambda_k}\sin\alpha_k \cos\alpha_k \qquad Q_{ky} = \frac{2d_j}{\lambda_k}\sin^2\alpha_k \qquad \lambda_k = \frac{T_k}{A_k E}$$

$$Q_{jx} = \frac{2d_i}{\lambda_j}\sin\alpha_j \cos\alpha_j \qquad Q_{jy} = \frac{2d_i}{\lambda_j}\sin^2\alpha_j \qquad \lambda_j = \frac{T_j}{A_j E}$$

Table 5-4	Constants for Trapezoidal Panels, Constant Cross-Section, without Ties

TYPICAL PORTION:



FIXED END MOMENTS:

$$FM_{jj}^{(I)} = FM_{jj}^{(I)}$$

$$FM_{jk}^{*} = \frac{e_k}{2(d_j + d_k)} SM_{0k}^{(II)}$$

$$FM_{jj}^{*} = -\frac{1}{4}\,\frac{I_{jj}}{I_j}\,\frac{e^2_j}{d^2_j}\,\frac{L_j}{(d_j + d_i)}\,SM_{0j}^{(II)} + FM_{jj}^{(II)}$$

$$FM_{ji}^{*} = \frac{e_j}{2(d_j + d_i)} SM_{0j}^{(II)}$$

CONSTANTS:

$$CK_{kj}^{(I)} = \frac{2EI_k}{L_k}$$

$$K_{jk}^{(I)} = \frac{4EI_k}{L_k}$$

$$K_{jj}^{(I)} = \frac{EI_{jj}}{d_j}$$

$$K_{ji}^{(I)} = \frac{4EI_j}{L_j}$$

$$CK_{ij}^{(I)} = \frac{2EI_j}{L_j}$$

$$CK_{kj}^{*} = -\frac{2EI_k}{L_k}\,\frac{d_j}{(d_k + d_j)}$$

$$K_{jk}^{*} = \frac{2EI_k}{L_k}\,\frac{d_j}{(d_j + d_k)}$$

$$K_{jji}^{*} = \frac{2EI_{jj}}{d^2_j}\,\frac{(d_j^2 + d_i d_j + d_i^2)}{(d_j + d_i)}$$

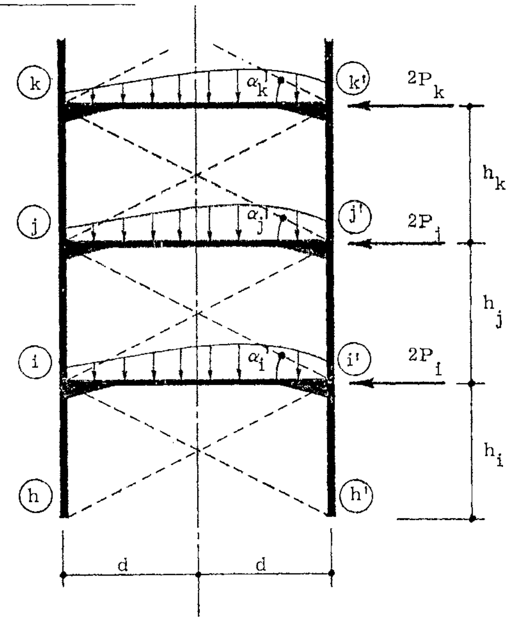
$$K_{ji}^{*} = \frac{2EI_j}{L_j}\,\frac{d_j}{(d_j + d_i)}$$

$$CK_{ij}^{*} = -\frac{2EI_j}{L_j}\,\frac{d_j}{(d_i + d_j)}$$

$$CK_{ijj}^{*} = \frac{EI_{jj}}{d^2_j}\,\frac{(d_i^2 + d_i d_j - 2d_j^2)}{(d_i + d_j)}$$

Table 5-5 — Constants for Rectangular Panels, Variable Cross-Section, with Ties

TYPICAL PORTION:



FIXED END MOMENTS:

$$FM^{(I)}_{jj} = FM^{(I)}_{jj}$$

$$FM^{*}_{jk} = \frac{S_{jk}}{N_k} V_k h_k$$

$$FM^{*}_{jj} = FM^{(II)}_{jj}$$

$$FM_{ji} = \frac{S_{ji}}{N_j} V_j h_j$$

CONSTANTS:

$$CK^{(I)}_{kj} = C_{kj} K_{kj}$$

$$K^{(I)}_{jk} = K_{jk}$$

$$K^{(I)}_{jj} = K_{jj} (1 - C_{jj})$$

$$K^{(I)}_{ji} = K_{ji}$$

$$CK^{(I)}_{ij} = C_{ij} K_{ij}$$

$$CK^{*}_{kj} = K_{kj} \left[C_{kj} - \frac{S_{jk}}{N_k} (C_{kj} + 1) \right]$$

$$K^{*}_{jk} = K_{jk} \left[1 - \frac{S_{jk}}{N_k} (C_{jk} + 1) \right]$$

$$K^{*}_{jj} = S_{jj}$$

$$K^{*}_{ji} = K_{ji} \left[1 - \frac{S_{ji}}{N_j} (C_{ji} + 1) \right]$$

$$CK^{*}_{ij} = K_{ij} \left[C_{ij} - \frac{S_{ji}}{N_j} (C_{ij} + 1) \right]$$

EQUIVALENTS:

$$N_k = S_{kj} + S_{jk} + \tfrac{1}{2} Q_{kx} h_k \qquad N_j = S_{ji} + S_{ij} + \tfrac{1}{2} Q_{jx} h_j$$

$$S_{jk} = K_{jk} (1 + C_{jk}) \qquad S_{jj} = K_{jj} (1 + C_{jj}) \qquad S_{ji} = K_{ji} (1 + C_{ji})$$

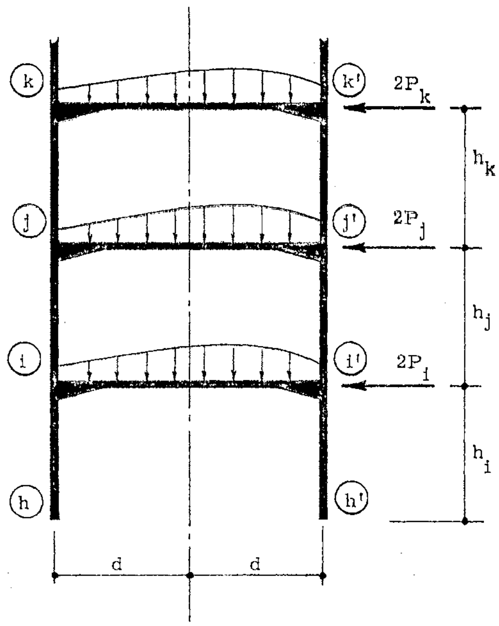
$$S_{kj} = K_{kj} (1 + C_{kj}) \qquad S_{ij} = K_{ij} (1 + C_{ij})$$

$$Q_{kx} = \frac{h_k}{\lambda_k} \cos^2 \alpha_k \qquad \lambda_k = \frac{T_k}{A_k E} \qquad V_k = \sum_k^n P$$

$$Q_{jx} = \frac{h_j}{\lambda_j} \cos^2 \alpha_j \qquad \lambda_j = \frac{T_j}{A_j E} \qquad V_j = \sum_j^n P$$

Table 5-6 — **Constants for Rectangular Panels, Variable Cross-Section, without Ties**

TYPICAL PORTION:



FIXED END MOMENTS:

$$FM_{jj}^{(I)} = FM_{jj}^{(I)}$$

$$FM_{jk}^{*} = \frac{S_{jk}}{S_{jk} + S_{kj}} V_k h_k$$

$$FM_{jj}^{*} = FM_{jj}^{(II)}$$

$$FM_{ji}^{*} = \frac{S_{ji}}{S_{ji} + S_{ij}} V_j h_j$$

CONSTANTS:

$$CK_{kj}^{(I)} = C_{kj} K_{kj} \qquad CK_{kj}^{*} = K_{kj} \left(\frac{S_{kj} C_{kj} - S_{jk}}{S_{jk} + S_{kj}} \right)$$

$$K_{jk}^{(I)} = K_{jk} \qquad K_{jk}^{*} = K_{jk} \left(\frac{S_{kj} - S_{jk} C_{jk}}{S_{jk} + S_{kj}} \right)$$

$$K_{jj}^{(I)} = K_{jj} (1 - C_{jj}) \qquad K_{jj}^{*} = S_{jj}$$

$$K_{ji}^{(I)} = K_{ji} \qquad K_{ji}^{*} = K_{ji} \left(\frac{S_{ij} - S_{ji} C_{ji}}{S_{ji} + S_{ij}} \right)$$

$$CK_{ij}^{(I)} = C_{ij} K_{ij} \qquad CK_{ij}^{*} = K_{ij} \left(\frac{S_{ij} C_{ij} - S_{ji}}{S_{ji} + S_{ij}} \right)$$

EQUIVALENTS:

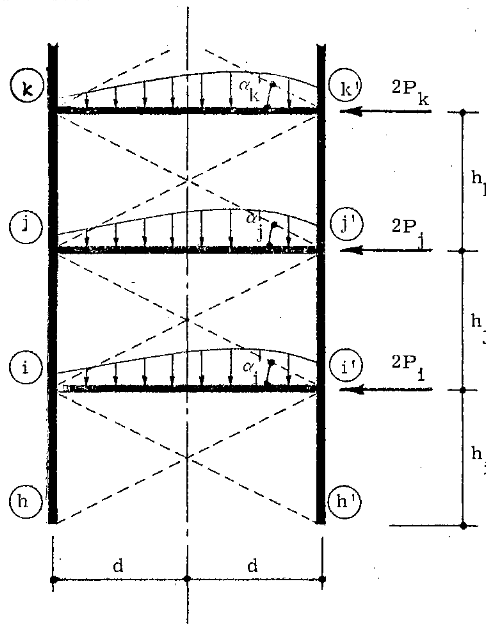
$$S_{ji} = K_{jk} (1 + C_{jk}) \qquad S_{jj} = K_{jj} (1 + C_{jj}) \qquad S_{ji} = K_{ji} (1 + C_{ji})$$

$$S_{kj} = K_{kj} (1 + C_{kj}) \qquad S_{ij} = K_{ij} (1 + C_{ij})$$

$$V_k = \sum_{k}^{n} P \qquad V_j = \sum_{j}^{n} P$$

Table 5-7 Constants for Rectangular Panels, Constant Cross-Section, with Ties

TYPICAL PORTION:



FIXED END MOMENTS:

$$FM_{jj}^{(I)} = FM_{jj}^{(I)}$$

$$FM_{jk}^{*} = \frac{6EI_k}{N_k} V_k$$

$$FM_{jj}^{*} = FM_{jj}^{(II)}$$

$$FM_{ji}^{*} = \frac{6EI_j}{N_j} V_j$$

CONSTANTS:

$$CK_{kj}^{(I)} = \frac{2EI_k}{h_k} \qquad CK_{kj}^{*} = \frac{4EI_k}{h_k^2}\left(\frac{h_k}{2} - \frac{9EI_k}{N_k}\right)$$

$$K_{jk}^{(I)} = \frac{4EI_k}{h_k} \qquad K_{jk}^{*} = \frac{4EI_k}{h_k^2}\left(h_k - \frac{9EI_k}{N_k}\right)$$

$$K_{jj}^{(I)} = \frac{EI_{jj}}{d} \qquad K_{jj}^{*} = \frac{3EI_{jj}}{d}$$

$$K_{ji}^{(I)} = \frac{4EI_j}{h_j} \qquad K_{ji}^{*} = \frac{4EI_j}{h_j^2}\left(h_j - \frac{9EI_j}{N_j}\right)$$

$$CK_{ij}^{(I)} = \frac{2EI_j}{h_j} \qquad CK_{ij}^{*} = \frac{4EI_j}{h_j^2}\left(\frac{h_j}{2} - \frac{9EI_j}{N_j}\right)$$

EQUIVALENTS:

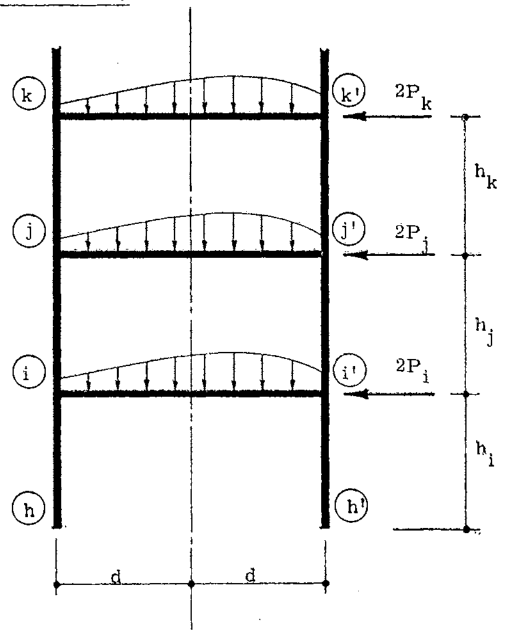
$$N_k = \frac{12EI_k}{h_k} + \frac{1}{2} Q_{kx} h_k \qquad N_j = \frac{12EI_j}{h_j} + \frac{1}{2} Q_{jx} h_j$$

$$Q_{kx} = \frac{h_k}{\lambda_k} \cos^2 \alpha_k \qquad Q_{jx} = \frac{h_j}{\lambda_j} \cos^2 \alpha_j$$

$$V_k = \sum_{k}^{n} P \qquad V_j = \sum_{j}^{n} P$$

Table 5-8	Constants for Rectangular Panels, Constant Cross-Section, without Ties

TYPICAL PORTION:



FIXED END MOMENTS:

$$FM_{jj}^{(I)} = FM_{jj}^{(I)}$$

$$FM_{jk}^{*} = \frac{1}{2} V_k h_k$$

$$FM_{jj}^{*} = FM_{jj}^{(II)}$$

$$FM_{ji}^{*} = \frac{1}{2} V_j h_j$$

CONSTANTS:

$$CK_{kj}^{(I)} = \frac{2EI_k}{h_k} \qquad CK_{kj}^{*} = -\frac{EI_k}{h_k}$$

$$K_{jk}^{(I)} = \frac{4EI_k}{h_k} \qquad K_{jk}^{*} = \frac{EI_k}{h_k}$$

$$K_{jj}^{(I)} = \frac{EI_{jj}}{d} \qquad K_{jj}^{*} = \frac{3EI_{jj}}{d}$$

$$K_{ji}^{(I)} = \frac{4EI_j}{h_j} \qquad K_{ji}^{*} = \frac{EI_j}{h_j}$$

$$CK_{ij}^{(I)} = \frac{2EI_j}{h_j} \qquad CK_{ij}^{*} = -\frac{EI_j}{h_j}$$

EQUIVALENTS:

$$V_k = \sum_{k}^{n} P \qquad V_j = \sum_{j}^{n} P$$

6. NUMERICAL PROCEDURE

The application of the carry-over joint moment procedure to the analysis of one-bay multi-story frames is illustrated by two numerical examples. All values are given in feet, kips and kip-feet.

Example 1. A four story, one-bay, symmetrical frame with inclined columns and loaded as shown is analyzed (Fig. 6-1).

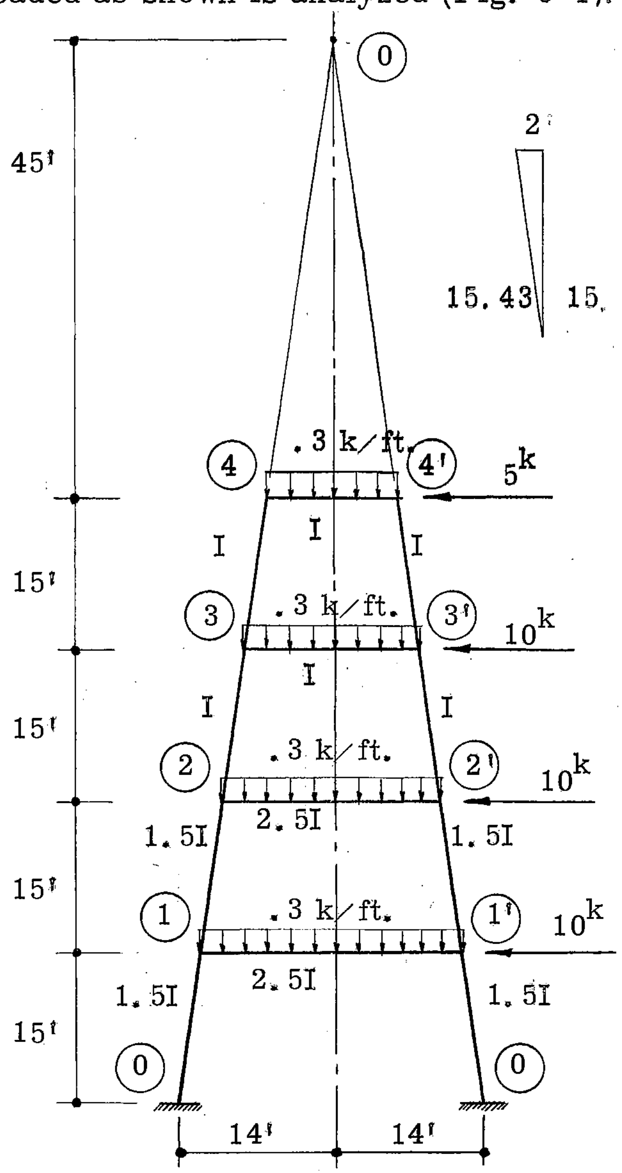


Fig. 6-1

Four Story Symmetrical Frame
with Bottom Fixed

The given system of loads is resolved into a symmetrical and antisymmetrical system (Fig. 6-2a, b)

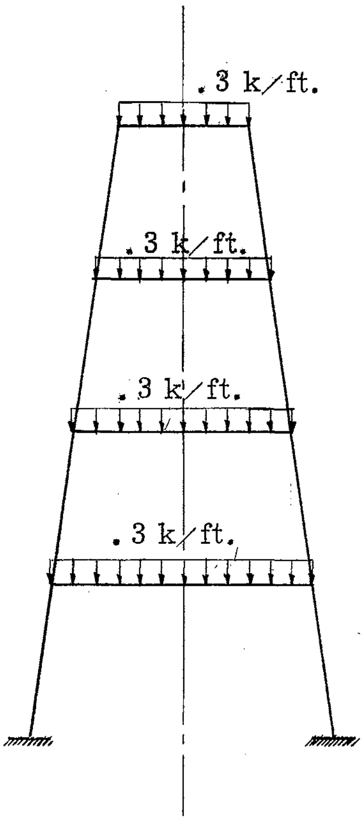


Fig. 6-2a
Symmetrical System

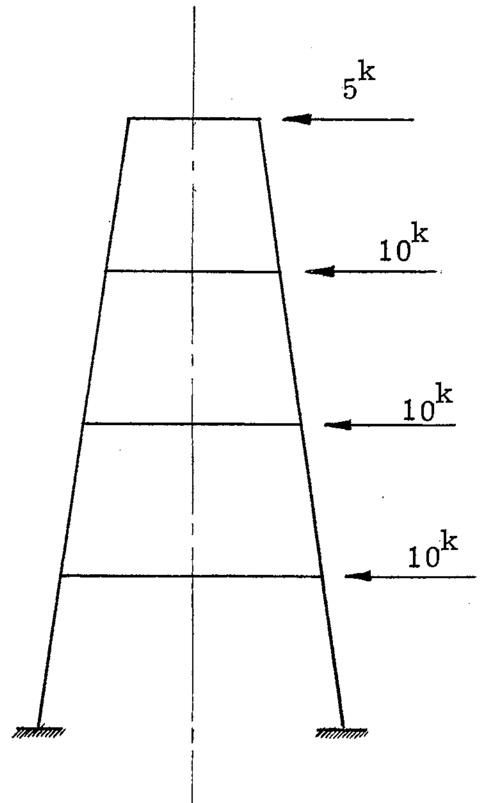


Fig. 6-2b
Antisymmetrical System

(I) Symmetrical System

(a) Carry-over constants are calculated by means of formulas shown in Table 5-4. Their numerical values are recorded in Table 6-1.

(b) Carry-over procedure is performed in Table 6-2 by means of carry-over factors. Each final moment is equal to the sum of all values in the respective column.

Table 6-1			Carry-Over Constants Symmetrical System				Example 1	
MEMBER	$K^{(I)}$	$\Sigma K^{(I)}$	$D^{(I)}$	$CK^{(I)}$	$CD^{(I)}$	$r^{(I)}$	$FM^{(I)}$	$m^{(I)}$
44	.167EI	.432EI	.386	.083EI			- 3.6	+ 3.6
43	.265EI		.614	.132EI	.307	- .307		
34	.265EI	.655EI	.404	.132EI	.202	- .202		+ 6.4
33	.125EI		.192	.062EI			- 6.4	
32	.265EI		.404	.132EI	.202	- .202		
23	.265EI	.911EI	.290	.132EI	.145	- .145		+10.0
22	.250EI		.274	.125EI			-10.0	
21	.396EI		.436	.198EI	.218	- .218		
12	.396EI	1.000EI	.396	.198EI	.198	- .198		+14.4
11	.208EI		.208	.104EI			-14.4	
10	.396EI		.396	.198EI	.198	- .198		

Table 6-2	Carry-Over Procedure Symmetrical System		Example 1	
JOINT	1	2	3	4
$r^{(I)}$	-.198	-.218 -.145	-.202 -.202	-.307
$m^{(I)}$	+14.4	+10.0	+6.4	+3.6
		- 1.29 - 2.85		-1.29
	- 1.28		- .85 - .71	
		+ .32 + .25		+ .32
	- .12		- .08 - .10	
		+ .04 + .02		+ .04
Σ	+13.00	+ 6.49	+4.66	+2.67

Table 6-3				Distribution Table Symmetrical System							Example 1	
JOINT	0	1			2			3			4	
JM	.000	+13.00			+6.49			+4.66			+2.67	
ENDS	01	10	11	12	21	22	23	32	33	34	43	44
D	.000	.396	.208	.396	.436	.274	.290	.404	.192	.406	.614	.386
CD	.000	+.198	.000	+.198	+.218	.000	+.145	+.202	.000	+.202	+.307	.000
(D) (JM)		+5.15	+2.70	+5.15	+2.83	+1.78	+1.88	+1.88	+.89	+1.89	+1.64	+1.03
(CD) (JM)	+2.57			+1.41	+2.57		+ .94	+ .94		+ .82	+ .94	
FM			-14.4			-10.0			-6.4			-3.6
M	+2.57	+5.15	-11.70	+6.56	+5.40	-8.22	+2.82	+2.82	-5.51	+2.71	+2.58	-2.57

(c) Numerical control is performed by means of equation (2-8).

$$JM_4 = - .202\,(4.66) + 3.6 = + 2.66$$

$$JM_3 = - .307\,(2.67) + 6.4 - .145\,(6.49) = + 4.64$$

$$JM_2 = - .202\,(4.66) + 10 - .198\,(13.00) = + 6.49$$

$$JM_1 = - .218\,(6.49) + 14.4 = + 13.00$$

(d) Final moments are obtained from equations (2-9).

$$M_{44}^{(I)} = .386\,(2.67) - 3.60 = - 2.57$$

$$M_{43}^{(I)} = .614\,(2.67) + .202\,(4.66) = + 2.58$$

$$M_{34}^{(I)} = .406\,(4.66) + .307\,(2.67) = + 2.71$$

$$M_{33}^{(I)} = .192\,(4.66) - 6.40 = - 5.51$$

$$M_{32}^{(I)} = .404\,(4.66) + .145\,(6.49) = + 2.82$$

$$M_{23}^{(I)} = .290\,(6.49) + .202\,(4.66) = + 2.82$$

$$M_{22}^{(I)} = .274\,(6.49) - 10.00 = - 8.22$$

$$M_{21}^{(I)} = .436\,(6.49) + .198\,(13.00) = + 5.40$$

$$M_{12}^{(I)} = .396\,(13.00) + .218\,(6.49) = + 6.56$$

$$M_{11}^{(I)} = .208\,(13.00) - 14.40 = - 11.70$$

$$M_{10}^{(I)} = .396\,(13.00) = + 5.15$$

$$M_{01}^{(I)} = .198\,(13.00) = + 2.57$$

(e) Alternate calculation of final moments may be performed in tabular form as shown in Table 6-3 .

Table 6-4 — Carry-Over Constants, Antisymmetrical System — Example 1

MEMBER	K*	ΣK*	D*	CK*	CD*	$r^{(II)}$	FM*	$m^{(II)}$
44-3	.587EI	.644EI	.911				- 6.75	- 9.32
43	.057EI		.089	-.076EI	-.118	+.118	+16.07	
3-44		.559EI		+.079EI	+.141	-.039		- 51.06
34	.076EI		.136	-.057EI	-.102		+16.07	
33-2	.424EI		.758				-10.84	
32	.059EI		.106	-.073EI	-.131	+.131	+45.83	
2-33		.990EI		+.045EI	+.045	+.015		- 99.37
23	.073EI		.074	-.059EI	-.060		+45.83	
22-1	.827EI		.835				-18.05	
21	.090EI		.091	-.108EI	-.109	+.109	+71.59	
1-22		.878EI		+.073EI	+.083	+.019		-150.11
12	.108EI		.123	-.090EI	-.102		+71.59	
11-0	.678EI		.772				-16.67	
10	.092EI		.105	-.107EI	-.101		+95.19	

Table 6-5 — Carry-Over Procedure, Antisymmetrical System — Example 1

JOINT	1	2	3	4
$r^{(II)}$	+.019	+.109 +.015	+.131 -.039	+.118
$m^{(II)}$	-150.11	-99.37	-51.06	-9.32
		- 2.85 - 6.69		+1.99
	- 11.87		- .86 - 1.63	
		- .23 - .33		+ .10
	- .06		+ .01 - .01	
	-162.04	-109.47	-53.55	-7.23

Table 6-6 — Distribution Table, Antisymmetrical System — Example 1

JOINTS	0	1			2			3			4	
JM	.000	-162.04			-109.47			-53.55			-7.23	
ENDS	01	10	11	12	21	22	23	32	33	34	43	44
D	.000	.105	.772	.123	.091	.835	.074	.106	.758	.136	.089	.911
CD	.000	-.101	+.083	-.102	-.109	+.045	-.060	-.131	+.141	-.102	-.118	.000
(D) (JM)		-17.01	-125.09	-19.93	-9.96	-91.41	-8.10	-5.68	-40.59	-7.28	-.63	-6.59
(CD) (JM)	+16.37			+11.93	+16.53	-13.45	+7.02	+6.57	-4.93	+ .85	+5.46	-7.55
FM	+95.19	+95.19	- 16.67	+71.59	+71.59	-18.05	+45.83	+45.83	-10.84	+16.07	+16.07	-6.75
M	+111.56	+78.18	-141.76	+63.59	+78.15	-122.91	+44.75	+46.72	-56.36	+ 9.64	+20.90	-20.89

(II) Antisymmetrical System

(a) Carry-over constants are calculated by means of formulas assembled in Table 5-4. Their numerical equivalents are recorded in Table 6-4.

(b) Carry-over procedure is performed in Table 6-5 by means of carry-over factors. Each final moment is equal to the sum of all values in the respective column.

(c) Numerical control is performed by means of equation (3-16).

$$JM_4 = -9.32 - .039\,(-53.55) = -7.23$$

$$JM_3 = +.118\,(-7.23) - 51.06 + .015\,(-109.47) = -53.55$$

$$JM_2 = +.131\,(-53.55) - 99.37 + .019\,(-162.04) = -109.46$$

$$JM_1 = +.109\,(-109.47) - 150.11 = -162.04$$

(d) Final moments are obtained from equations (3-17).

$$M_{44}^{(II)} = .911\,(-7.23) + .141\,(-53.55) - 6.75 = -20.89$$

$$M_{43}^{(II)} = .087\,(-7.23) - .102\,(-53.55) + 16.07 = +20.90$$

$$M_{34}^{(II)} = .136\,(-53.55) - .118\,(-7.23) + 16.07 = +9.64$$

$$M_{33}^{(II)} = .758\,(-53.55) + .045\,(-109.47) - 10.84 = -56.36$$

$$M_{32}^{(II)} = .106\,(-53.55) - .060\,(-109.47) + 45.83 = +46.72$$

$$M_{23}^{(II)} = .074\,(-109.47) - .131\,(-53.55) + 45.83 = +44.75$$

$$M_{22}^{(II)} = .835\,(-109.47) + .083\,(-162.04) - 18.05 = -122.91$$

$$M_{21}^{(II)} = .091\,(-109.47) - .102\,(-162.04) + 71.59 = +78.15$$

$$M_{12}^{(II)} = .123\,(-162.04) - .109\,(-109.47) + 71.59 = +\,63.59$$

$$M_{11}^{(II)} = .772\,(-162.04) - 16.67 = -\,141.76$$

$$M_{10}^{(II)} = .105\,(-162.04) + 95.19 = +\,78.18$$

$$M_{01}^{(II)} = -\,.101\,(-162.04) + 95.19 = +\,111.56$$

(e) Alternate calculations of final moments may be performed in tabular form as shown in Table 6-6 .

(III) Initial System

The final moments due to the initial system are obtained by superposition of final moments (I) and (II).

$M_{44'} = -\,2.57 - 20.89 = -\,23.46$ $\quad$ $M_{4'4} = +\,2.57 - 20.89 = -\,18.32$

$M_{43} = +\,2.58 + 20.90 = +\,23.48$ $\quad$ $M_{4'3'} = -\,2.58 + 20.90 = +\,18.32$

$M_{34} = +\,2.71 + 9.64 = +\,12.35$ $\quad$ $M_{3'4'} = -\,2.71 + 9.64 = +\,6.93$

$M_{33'} = -\,5.51 - 56.32 = -\,61.83$ $\quad$ $M_{3'3} = +\,5.51 - 56.32 = -\,50.81$

$M_{32} = +\,2.82 + 46.71 = +\,49.53$ $\quad$ $M_{3'2'} = -\,2.82 + 46.71 = +\,43.89$

$M_{23} = +\,2.82 + 44.75 = +\,47.57$ $\quad$ $M_{2'3'} = -\,2.82 + 44.75 = +\,41.93$

$M_{22'} = -\,8.22 - 122.91 = -\,131.13$ $\quad$ $M_{2'2} = +\,8.22 - 122.91 = -\,114.69$

$M_{21} = +\,5.40 + 78.15 = +\,83.55$ $\quad$ $M_{2'1'} = -\,5.40 + 78.15 = +\,72.75$

$M_{12} = +\,6.56 + 63.59 = +\,70.15$ $\quad$ $M_{1'2'} = -\,6.56 + 63.59 = +\,57.03$

$M_{11'} = -\,11.70 - 141.76 = -\,153.46$ $\quad$ $M_{1'1} = +\,11.70 - 141.76 = -\,130.06$

$M_{10} = + 5.15 + 78.18 + 83.33$ $\qquad$ $M_{1'0'} = - 5.15 + 78.18 + 73.03$

$M_{01} = + 2.57 + 111.56 + 114.13$ $\qquad$ $M_{0'1'} = - 2.57 + 111.56 + 108.99$

Example 2. A four story, one-bay, symmetrical frame with vertical columns and flexible ties with horizontal loads applied at joints is analyzed (Fig. 6-3). The loaded frame forms an antisymmetrical system and no resolution is necessary.

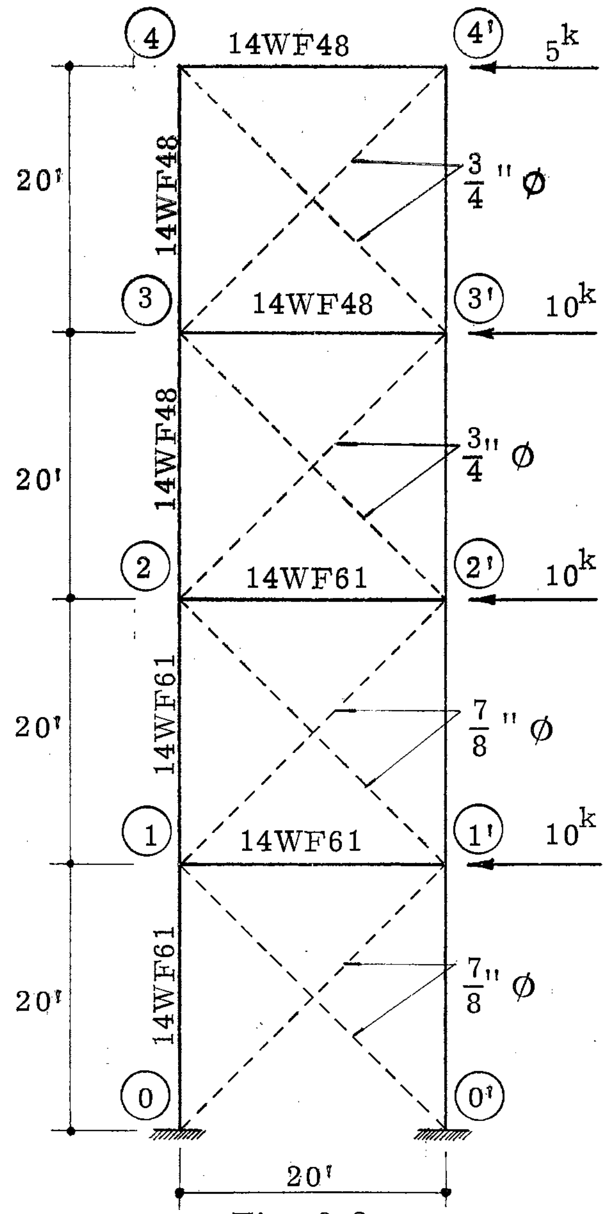


Fig. 6-3

Four Story Symmetrical Frame with Flexible Ties.
Columns Fixed at Bottom

Table 6-7 Carry-Over Constants Example 2

MEMBER	K*	ΣK*	D*	CK*	CD*	$r^{(II)}$	FM*	$m^{(II)}$
44	12.12EI	16.8EI	.722					-28.28
43	4.68EI		.278	.616EI	.037	-.037	28.28	
34	4.68EI	21.48E	.218	.616EI	.029	-.029	28.28	-113.12
33	12.12EI		.564					
32	4.68EI		.218	.616EI	.029	-.029	84.84	
23	4.68EI	26.97E	.174	.616EI	.023	-.023	84.84	-224.20
22	16.04EI		.594					
21	6.25EI		.232	.875EI	.032	-.032	139.36	
12	6.25EI	28.54E	.219	.875EI	.031	-.031	139.36	-334.46
11	16.04EI		.562					
10	6.25EI		.219	.875EI	.031	-.031	195.10	

Table 6-8 Carry-Over Procedure Example 2

JOINT	1	2	3	4
$r^{(II)}$	-.031	-.032 -.023	-.029 -.029	-.037
$m^{(II)}$	-334.46	-224.20	-113.12	-28.28
	+ 7.17		+ 5.16 + 1.05	
		+ 10.14 + 3.10		+ 3.10
	- .42		- .11 - .30	
		+ .01 + .01		+ .01
Σ	-327.71	-210.94	-107.32	-25.17

Table 6-9 Distribution Table Example 3

JOINTS	0	1			2			3			4	
JM	.000	-327.1			-210.93			-107.34			-25.17	
ENDS	01	10	11	12	21	22	23	32	33	34	43	44
D	.000	.219	.562	.219	.232	.594	.174	.218	.564	.218	.278	.722
CD	.000	+.031	+.000	+.031	+.032	.000	+.023	+.029	.000	+.029	+.037	.000
(D) (JM)		-71.77	-184.17	-71.77	-48.94	-125.30	-36.70	-23.40	-60.53	-23.40	-7.00	-18.17
(CD) (JM)	-10.16			- 6.75	-10.16		- 3.11	- 4.85		- .93	-3.11	
FM	+195.10	+195.10		+139.36	+139.36		+84.84	+84.84		+28.28	+28.28	
M	+184.94	+123.33	-184.17	+60.84	+80.26	-125.30	+45.03	+56.59	-60.53	+3.95	+18.17	-18.17

The numerical procedure is the same as in the preceding example.

(a) Carry-over constants (Table 5-7): Table 6-7.

(b) Carry-over procedure: Table 6-8.

(c) Numerical control (Equations 3-16):

$$JM_4 = -28.28 - .029\,(-107.32) = -25.17$$

$$JM_3 = -.037\,(-25.17) - 113.12 - .023\,(-210.94) = -107.34$$

$$JM_2 = -.029\,(-107.32) - 224.20 - .031\,(-327.71) = -210.93$$

$$JM_1 = -.032\,(-210.94) - 334.46 = -327.71$$

(d) Final moments (Equations 3-17)

$$M_{44'} = M_{4'4} = .722\,(-25.17) = -18.17$$

$$M_{43} = M_{4'3'} = .278\,(-25.17) + 28.28 + .029\,(-107.32) = +18.17$$

$$M_{34} = M_{3'4'} = .218\,(-107.32) + .037\,(-25.17) + 28.28 = +3.95$$

$$M_{33'} = M_{3'3} = .564\,(-107.32) = -60.53$$

$$M_{32} = M_{3'2'} = .218\,(-107.32) + .023\,(-210.94) + 84.84 = +56.59$$

$$M_{23} = M_{2'3'} = .174\,(-210.94) + .029\,(-107.32) + 84.84 = +45.03$$

$$M_{22'} = M_{2'2} = .594\,(-210.94) = -125.30$$

$$M_{21} = M_{2'1'} = .232\,(-210.94) + .031\,(-327.71) + 139.36 = +80.26$$

$$M_{12} = M_{1'2'} = .219\,(-327.71) + .032\,(-210.94) + 139.36 = +60.84$$

$$M_{11'} = M_{1'1} = .562\,(-327.71) = -184.17$$

$$M_{10} = M_{1'0'} = .219\,(-327.71) + 195.10 = +123.33$$

$$M_{01} = M_{01}'' = .031\,(-327.71) + 195.10 = +\ 184.94$$

(e) Alternate calculation of final moments may be performed in tabular form as shown in Table 6-9 .

7. PHYSICAL INTERPRETATION

The numerical procedure applied in the two preceding examples may be interpreted in terms of the physical action of the frame as follows:

(1) All joints are fixed against rotation but free to translate.

(2) Fixed end moments due to loads create at each joint an unbalance ΣFM_j which is counteracted by a joint starting moment of equal value and opposite sense m_j.

(3) The starting joint moment at j cause simultaneously new unbalances at the adjacent joints ($-r_{ji}m_j$ and $-r_{jk}m_j$) which must be counterbalanced by new joint moments at those joints ($r_{ji}m_j$ and $r_{jk}m_j$).

(4) Because of the nature of the carry-over factors, the carry-over joint moments ($r_{ji}m_j$ and $r_{jk}m_j$) are at the same time new starting moments.

(5) The step (3) is repeated in a number of cycles till the continuity of the elastic curve is established.

8. ALGEBRAIC CARRY-OVER PROCEDURE

When the convergency of the numerical carry-over procedure is too slow or when various load conditions must be considered, the numerical labor can be reduced by using the method of moment coefficients in algebraic form. The theory of algebraic carry-over procedure is discussed elsewhere (37).

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