ANALYSIS OF VIERENDEEL TRUSSES
WITH INCLINED CHORDS BY
CARRY-OVER MOMENTS

By

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PREFACE

The material presented in this thesis is a continuation of the Carry-Over Joint Moment Method to cover Vierendeel trusses with inclined members. The method was introduced originally by Professor Jan J. Tuma. Others have applied the method to many types of structures.

I wish to express my indebetedness and gratitude to Professor Tuma, not only for his invaluable aid and guidance in preparing this thesis, but also for his kind guidance as my major advisor.

I also wish to thank the staff of the School of Civil Engineering for the valuable instruction given me.

I furthermore wish to express gratitude to Mrs. Virginia Schenandoah for her careful typing of the manuscript, and to Eldon J. Hardy for his kind help in preparing the sketches.

L. J. D.

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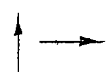
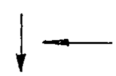
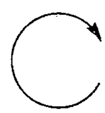
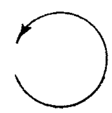
NOMENCLATURE

h_{jn}	Height of vertical member jn
L_j	Length of chord i-j
d_j	Length of panel $\overline{jimn}$
g_p	Amount of rise in chord n-p
ω_j	Inclination of chord i-j from vertical
α_j	Inclination of chord j-k from horizontal
θ_j	Slope of members at j
Δ_j	Relative displacement between points j and k
Δ_{jn}	Relative displacement between points j and n
ψ_j	$\frac{\Delta_i}{L_j}$
ψ_{jn}	$\frac{\Delta_{jn}}{L_{jn}}$
V_{ij}	End shear of member i-j at i
V_j	Shear in Panel just left of member j-n
FV_{ij}	Fixed end shear of member i-j at i
K_{ij}	Stiffness factor of member i-j at i
K^*_{ij}	Modified stiffness factor of member i-j at i
$CK_{ij} = C_{ij}K_{ij}$	Carry-over stiffness factor of member i-j at i
CK^*_{ij}	Modified carry-over stiffness factor of member i-j at i
C_{ij}	Carry-over factor of member i-j at i
S_{ij}	$K_{ij}(1 + C_{ij})$
D_{ij}	Modified distribution factor, $\frac{K^*_{ij}}{\Sigma K^*_i}$

$C^{*}_{ij}D^{*}_{ij}$	Modified carry-over distribution factor
r_{ij}	Joint moment carry-over factor from i to j
M_{ij}	End moment of member i-j at i
FM_{ij}	Fixed end moment of member i-j at i
FM^{*}_{ij}	Modified fixed end moment of member i-j at i
m_j	Starting moment at joint j
JM_j	Joint moment at joint j
SM_{Oj}	Static load moment about Oj
N_{ij}	Normal force on member i-j at i

SIGN CONVENTION

Forces.................positive  negative 

Moments...................positve  negative 

CHAPTER I

INTRODUCTION

The Vierendeel truss was introduced to the engineering world by Arthur Vierendeel, a University of Louvain professor in Louvain, Belgium. The engineers at that time thought such a truss was of no value; however, Professor Vierendeel proved the usefulness of the truss to the engineering world. The first Vierendeel trusses were bridge trusses built in Belgium and her territories. As soon as the usefulness of the truss became evident, the popularity of the truss began to spread among the engineers throughout the world. The Vierendeel truss was first introduced in the United States about 1930 in building foundations.

The first Vierendeel truss was an experimental bridge truss built at Tervueren, Belgium in 1896 and 1897. The bridge was tested to complete failure to confirm Professor Vierendeel's stress analysis theory. The method of analysis is known as Vierendeel's rigid-joint principle (7)[1].

After thirty years of usage Professor Vierendeel changed his method of calculation because it was too laborious and involved (7). There has been numerous methods of analyses derived since Professor Vierendeel's first method, and some are mentioned in the following paragraphs.

---

[1]Note: Numbers in parantheses, refer to numbered references in Selected Bibliography.

The method of successive eliminations of unknowns in slope deflection procedure was extended to Vierendeel trusses by Wilbur (8), Maugh (1), and others.

A mathematical theory of design was discussed by Bateman (10), Sakai (11), Muls (12), and others.

The method of using the points of contraflexure as a means of stress analysis was discussed by Frocht and Leven (13), Bales (14), deMeding (15), Decarpentrie (16), and others. Photoelasticity was one method used to find the point of contraflexure.

The method of moment distribution for solving rigid frames was introduced by Cross (17). The modified form of moment distribution was applied to Vierendeel trusses by Wix and Dornau (19), Lightfoot (2), Krausche (34), Naylor (4), Mijling (20), Matheson (5), Grinter and Tsao (21), and others.

The extension of the carry-over moment procedure (22, 23, 24, 25, 26, 27, 28, 29) to the analysis of Vierendeel trusses with inclined chord members is introduced in this paper. The study is restricted to one span, planar truss and to the customary assumptions of rigid frame analysis. The assumptions are: deformations of frame members due to axial and shear forces are small and neglected, all displacements and forces acting to the right or upward are positive, all clockwise angular rotations and moments are positive and all joints are fixed against rotation but free to translate.

The paper is divided into three parts. The first part defines the problem. The second part with the derivation of fundamental functions. A numerical problem demonstrated the procedure in the third part. Finally, the results are discussed, and a conclusion is drawn.

CHAPTER II

DEFINITION OF PROBLEM

The problem is defined as a Vierendeel truss with inclined chords acted upon by a general system of loads (Figure 2-1). The truss is analyzed as a rigid frame by panels in which each member is considered as a primary structural unit as has been shown by Wix and Dornau (20), Lightfoot (2), Matheson (5), Naylor (4), Tsao (6), Grinter and Tsao (21), and others.

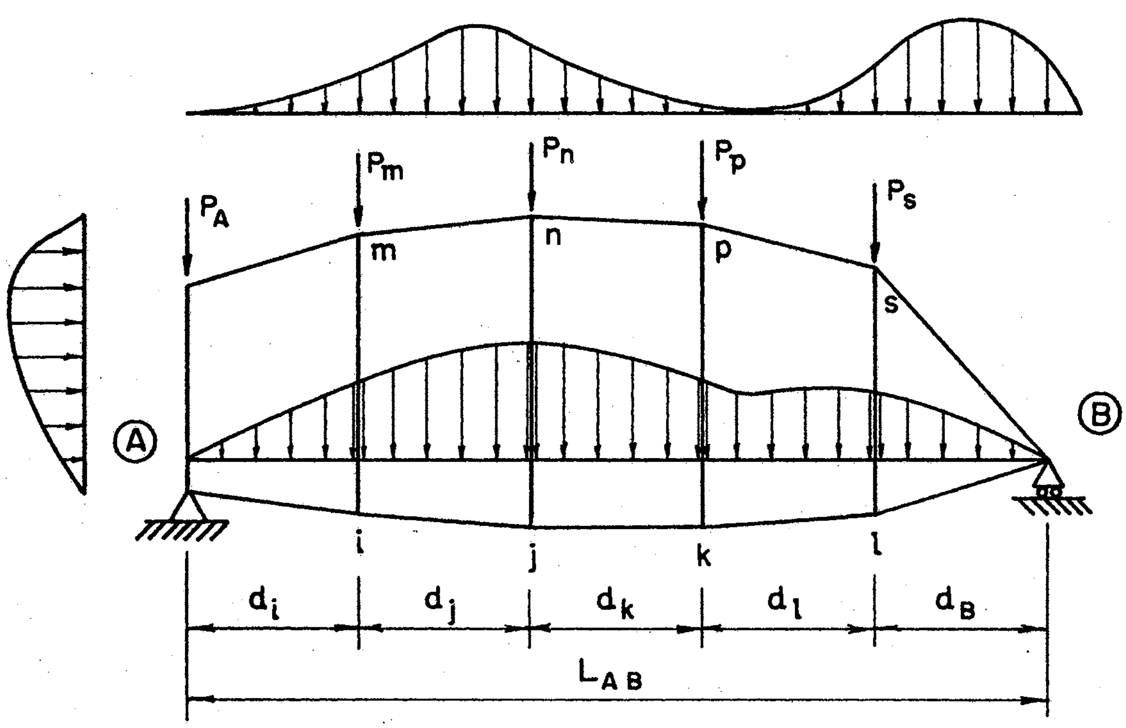


Figure 2-1. Vierendeel Truss with Inclined Members--General System of Loads.

CHAPTER III

INCLINED MEMBERS - TRAPEZOIDAL PANELS

The carry-over joint moment equations are derived for the general case. A Five panel Vierendeel truss with unsymmetrical loading is used in the derivation of the carry-over joint moment equation as shown in Figure 3-1.

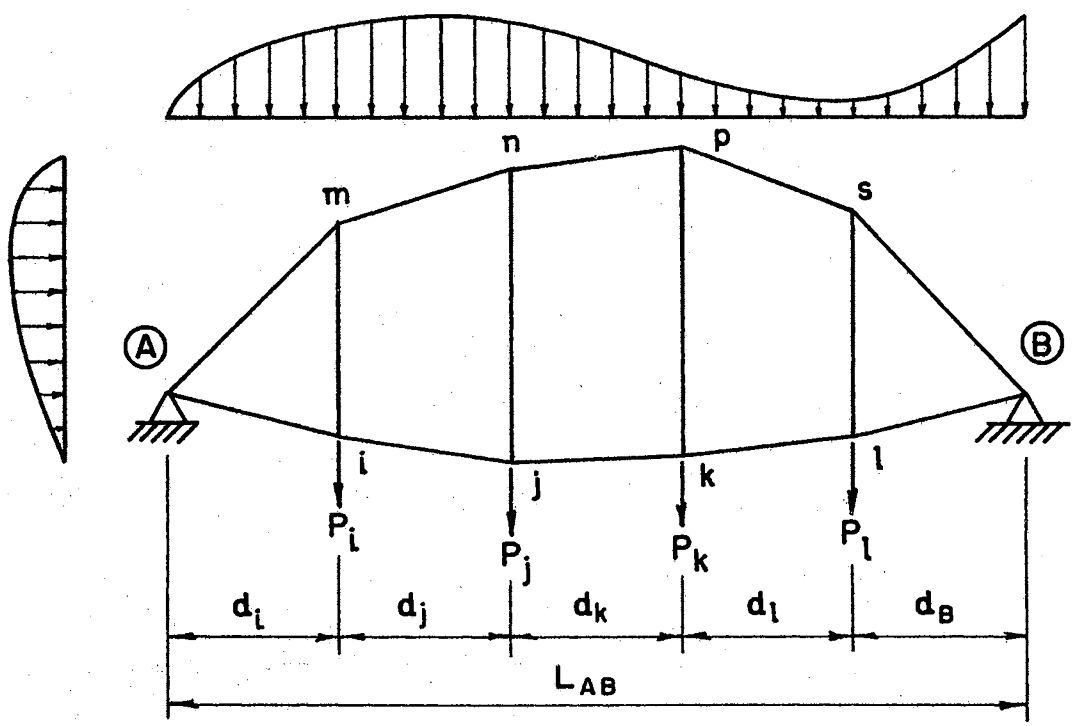


Figure 3-1. Vierendeel Truss with Inclined Members--Unsymmetrical Loading.

In the case of moment distribution there is a distribution and carry-over procedure for every independent displacement plus a distribution and carry-over procedure for the loads on the truss. There is also the problem of analyzing properly the sloped members for moment distribution, and various methods are in existance to make structures aptable to moment distribution.

In the carry-over joint moment analysis there is only one carry-over procedure and the effect of the sloped members on other members in the truss is included in this analysis; thereby, eliminating any approximations to compensate for the deflections caused by sloped members in the truss. The elimination of the displacement terms from the slope deflection equations simplifies the analysis and reduces the analysis time.

The elimination of the displacement terms from the slope deflection equations is accomplished by means of the geometry of the structure and of the equilibrium equations. The displacement equations are derived in terms of joint rotations and are subsequently substituted into joint moment equation which is the summation of the moments at any joint on the truss. This new equation is termed the joint moment equation which defines the starting moments and the carry-over constants. Substitution of the displacement equations into the slope deflections equations determines the new modified slope deflection equations which defines the modified distribution factors and the modified carry-over distribution factors.

After the derivation is complete there are physical interpretations of each new term in the joint moment equations.

In order to facilitate the derivation of the joint moment equation, the slope deflection equations are written using moment distribution nomenclature for panels $\overline{ijkmnp}$ or any similar panels. The equations are:

$$\left.\begin{aligned}
M_{kj} &= K_{kj}\theta_k + CK_{jk}\theta_j + S_{kj}\psi_k + FM_{kj} \\
M_{jk} &= K_{jk}\theta_j + CK_{kj}\theta_k + S_{jk}\psi_k + FM_{jk} \\
M_{jn} &= K_{jn}\theta_j + CK_{nj}\theta_n - S_{jn}\frac{\Delta_{jn} + \Delta_{kp} + \cdots}{h_{jn}} + FM_{jn} \\
M_{ji} &= K_{ji}\theta_j + CK_{ij}\theta_i + S_{ji}\psi_j + FM_{ji} \\
M_{ij} &= K_{ij}\theta_i + CK_{ji}\theta_j + S_{ij}\psi_j + FM_{ij}
\end{aligned}\right\} \quad (3\text{-}1)$$

$$\left.\begin{aligned}
M_{pn} &= K_{pn}\theta_p + CK_{np}\theta_n + S_{pn}\psi_p + FM_{pn} \\
M_{np} &= K_{np}\theta_n + CK_{pn}\theta_p + S_{np}\psi_p + FM_{np} \\
M_{nj} &= K_{nj}\theta_n + CK_{jn}\theta_j - S_{nj}\frac{\Delta_{jn} + \Delta_{kp} + \cdots}{h_{jn}} + FM_{nj} \\
M_{nm} &= K_{nm}\theta_n + CK_{mn}\theta_m + S_{nm}\psi_n + FM_{nm} \\
M_{mn} &= K_{mn}\theta_m + CK_{nm}\theta_n + S_{mn}\psi_n + FM_{mn}.
\end{aligned}\right\} \quad (3\text{-}2)$$

The S's terms are defined as:

$$\left.\begin{aligned}
S_{kj} &= K_{kj} + CK_{jk} = K_{kj} + CK_{kj} = K_{kj}(1 + C_{kj}) \\
S_{jk} &= K_{jk} + CK_{kj} = K_{jk} + CK_{jk} = K_{jk}(1 + C_{jk}) \\
S_{jn} &= K_{jn} + CK_{nj} = K_{jn} + CK_{jn} = K_{jn}(1 + C_{jn}) \\
S_{ji} &= K_{ji} + CK_{ij} = K_{ji} + CK_{ji} = K_{ji}(1 + C_{ji}) \\
S_{ij} &= K_{ij} + CK_{ji} = K_{ij} + CK_{ij} = K_{ij}(1 + C_{ij}) \\
S_{pn} &= K_{pn} + CK_{np} = K_{pn} + CK_{pn} = K_{pn}(1 + C_{pn}) \\
S_{np} &= K_{np} + CK_{pn} = K_{np} + CK_{np} = K_{np}(1 + C_{np}) \\
S_{nj} &= K_{nj} + CK_{jn} = K_{nj} + CK_{nj} = K_{nj}(1 + C_{nj}) \\
S_{nm} &= K_{nm} + CK_{mn} = K_{nm} + CK_{nm} = K_{nm}(1 + C_{nm}) \\
S_{mn} &= K_{mn} + CK_{nm} = K_{mn} + CK_{mn} = K_{mn}(1 + C_{mn})
\end{aligned}\right\} \quad (3\text{-}3)$$

The FM's terms are defined as:

FM_{kj} = fixed end moment due to loads at k in member $\overline{kj}$

FM_{jk} = fixed end moment due to loads at j in member $\overline{kj}$

FM_{ji} = fixed end moment due to loads at j in member $\overline{ji}$

FM_{ij} = fixed end moment due to loads at i in member $\overline{ji}$

FM_{pn} = fixed end moment due to loads at p in member $\overline{pn}$

FM_{np} = fixed end moment due to loads at n in member $\overline{pn}$

FM_{nm} = fixed end moment due to loads at n in member $\overline{nm}$

FM_{mn} = fixed end moment due to loads at m in member $\overline{nm}$.

The third term in the cross-member end moment equation contains deflections from other cross-members. This makes the solubility of the cross-member end moment equation very difficult owing to the lack of knowledge which cross-member deflection influences the cross-member $\overline{jn}$. The influence of the deflection in cross-member $\overline{jn}$ on the deflections of cross-members $\overline{im}$ and $\overline{lh}$ is shown in Figure 3-2. From Figure 3-2 it is apparent that the amount of deflection of any cross-member caused by the deflection of any one of the other cross-members is very difficult, if not impossible, to determine.

In Figure 3-2 the joints are assumed fixed against rotation, and the deflections are greatly magnified to show the effects of the displacements. The Vierendeel Truss is shown as a tower rather than a simple span truss because the translation of a tower shows more clearly the deflections caused by the cross-members.

There are many methods in existance to combat the redundant deflections. However, this thesis used the method that gives the same

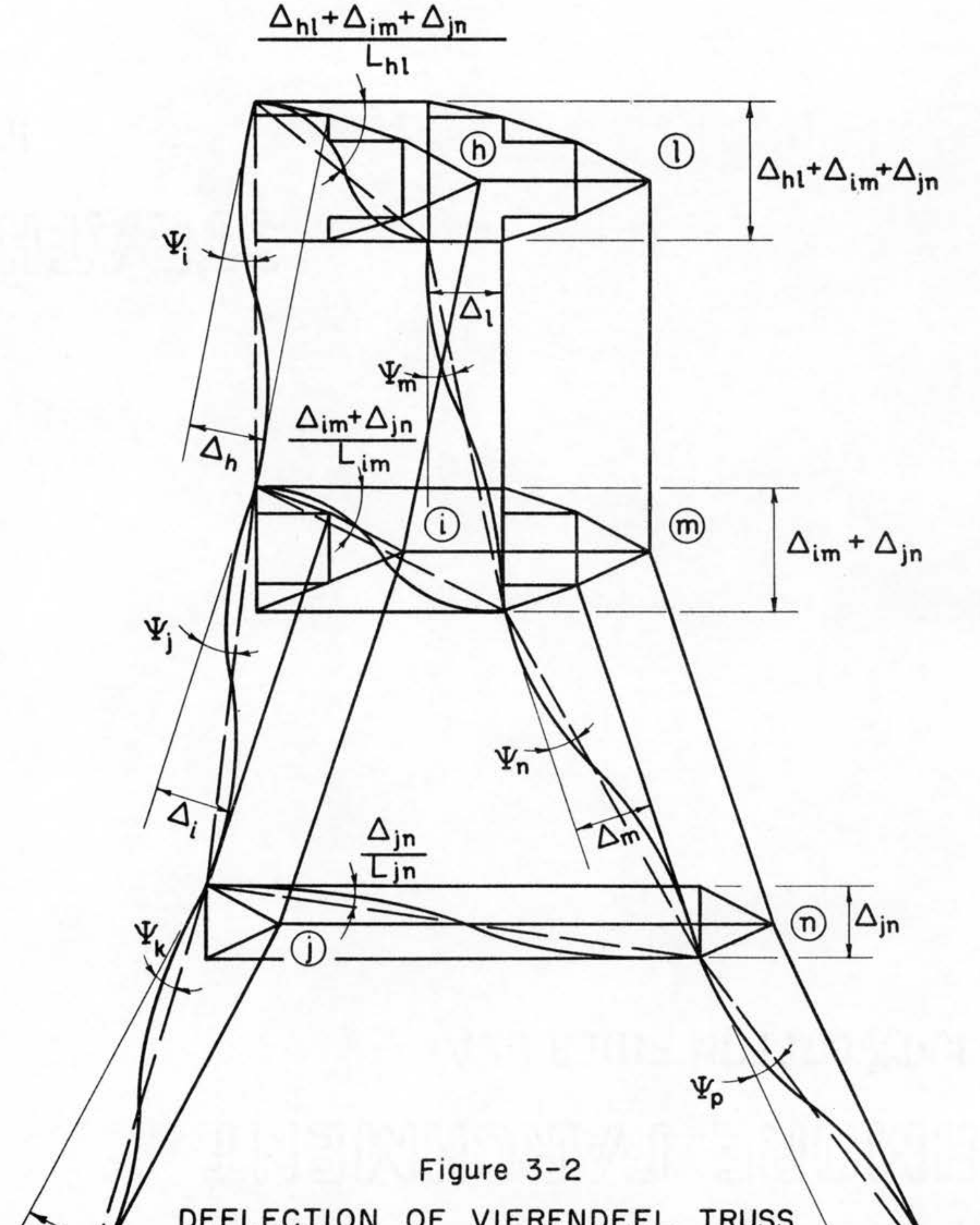


Figure 3-2

DEFLECTION OF VIERENDEEL TRUSS WITH JOINTS FIXED AGAINST ROTATION accuracy as the moment distribution method. In using this method the cross-members are held rigid and not allowed to deflect, but the sloped-members are allowed to deflect due to load and to absorb the rotation caused by the displacement of the cross-member.

To help visualize what is occurring in the truss, three diagrams are used--Figures 3-3, 3-4, and 3-5.

While holding the cross-members rigid and applying a load to panel $\overline{jknp}$ only, the truss is deflected as shown in Figure 3-3. Because cross-member $\overline{jn}$ is held rigid, there are no deflections in any members in the other panels of the truss, only an angular rotation of ψ_{jn} of the panels above cross-member $\overline{jn}$. Because the two side members $\overline{jk}$ and $\overline{np}$ are sloped, the cross-member $\overline{jn}$ is rotated through an angle of ψ_{jn} due the nature of the trapezoidal panel. The two sloped members $\overline{jk}$ and $\overline{np}$ absorb the angular rotation ψ_{jn} of cross-member $\overline{jn}$ in addition to angular rotation ψ_j and ψ_p respectively.

Again holding the cross-members rigid and applying a load to panels $\overline{ijmn}$ and $\overline{jknp}$ such that the load in panel $\overline{jknp}$ is the same as loaded previously, the truss is deflected as shown in Figure 3-4. Again, there are no deflections in any member of the top two panels owing to the deflection of panel $\overline{jknp}$. Also, there are no deflections in any members of panel $\overline{hilm}$ owing to the deflection of panel $\overline{ijmn}$ for reasons previously stated for panel $\overline{jknp}$. There are no deflections in any members in panel $\overline{jknp}$ owing to deflection of panel $\overline{ijmn}$ because cross-member $\overline{jn}$ is kept rigid; thereby, preventing any rotation of joints j and n to cause any carry-over of any deflections from any member in panel $\overline{ijmn}$ to any member or members in panel $\overline{jknp}$.

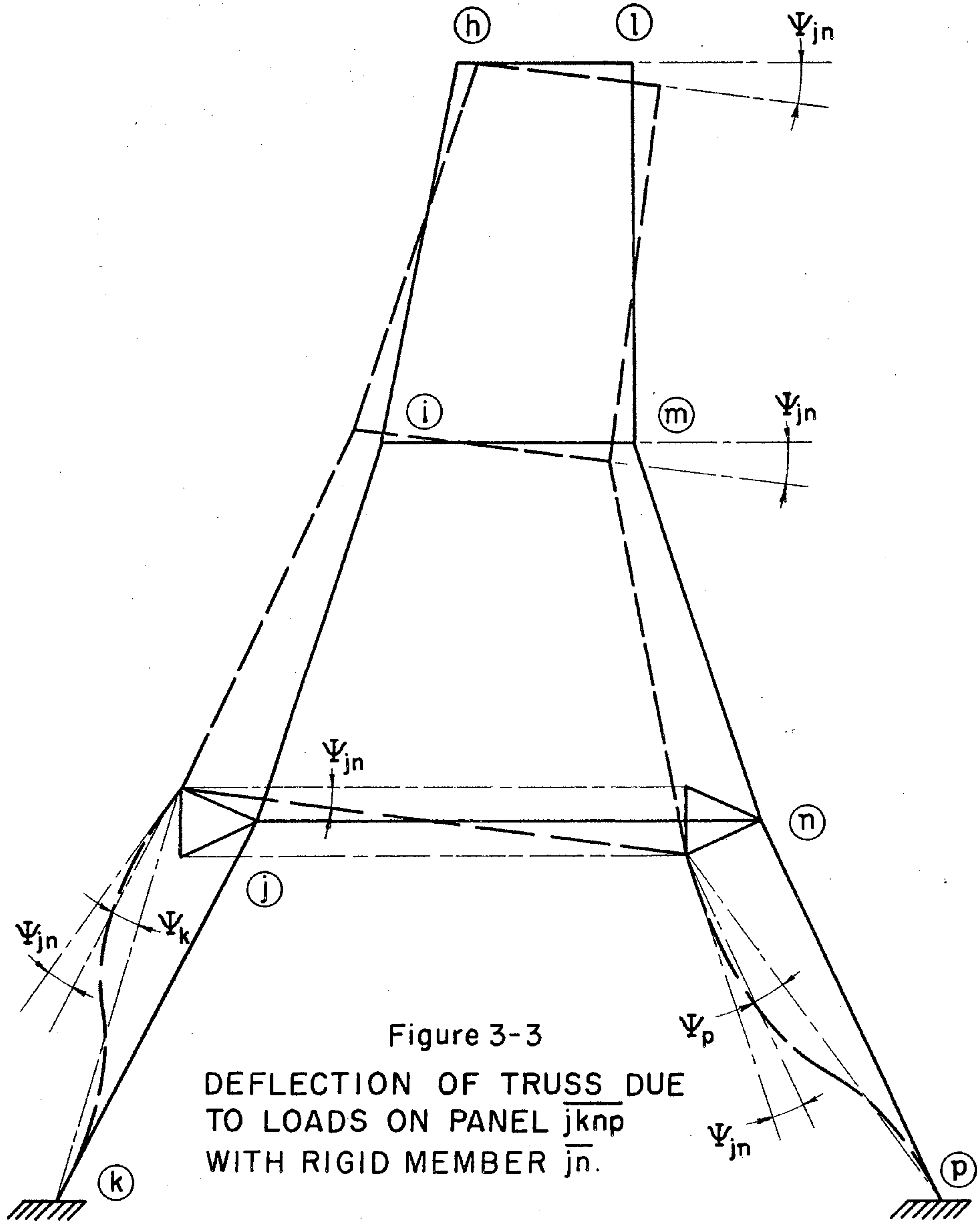


Figure 3-3

DEFLECTION OF TRUSS DUE TO LOADS ON PANEL $\overline{jknp}$ WITH RIGID MEMBER $\overline{jn}$.

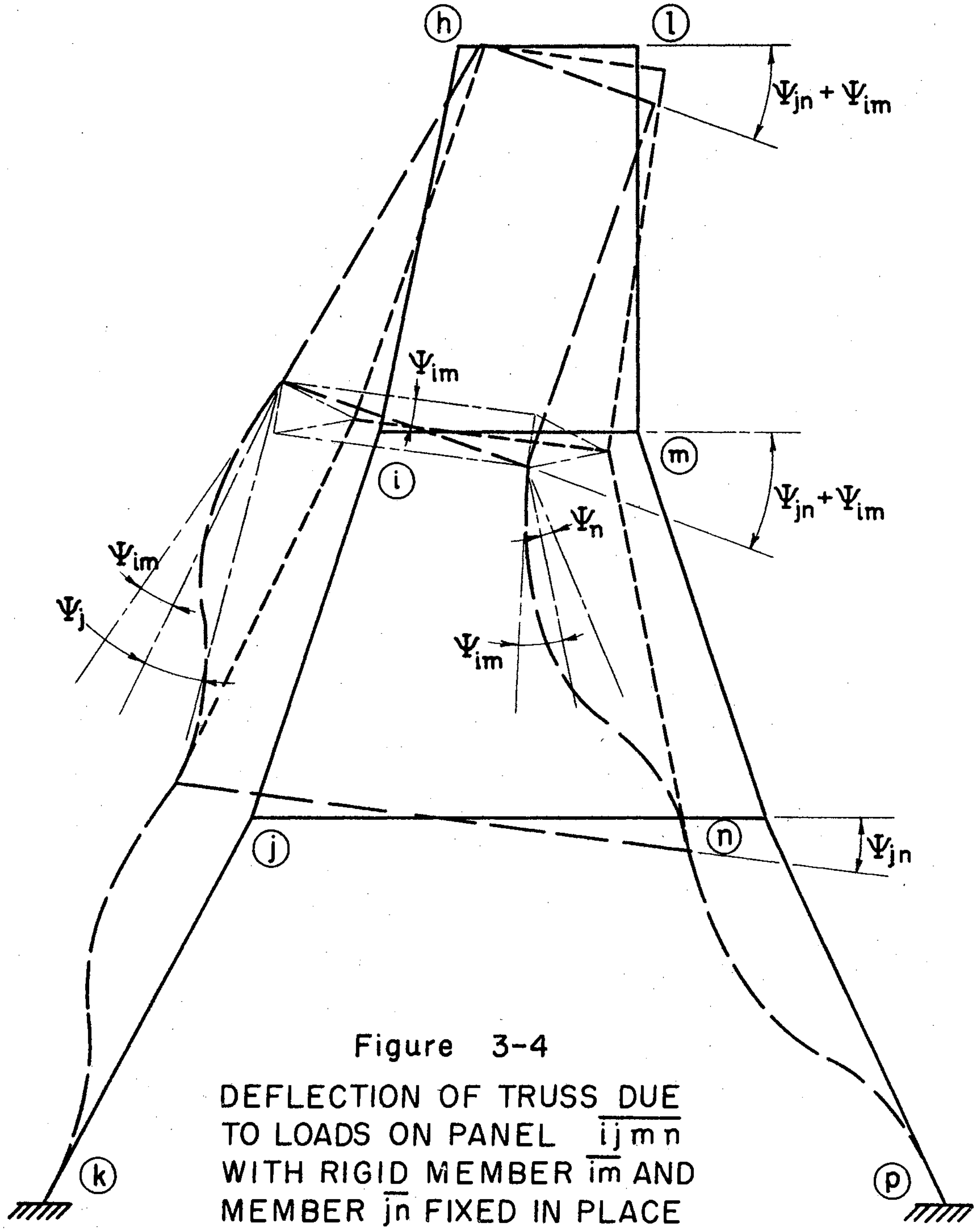


Figure 3-4

DEFLECTION OF TRUSS DUE TO LOADS ON PANEL $\overline{ijmn}$ WITH RIGID MEMBER $\overline{im}$ AND MEMBER $\overline{jn}$ FIXED IN PLACE

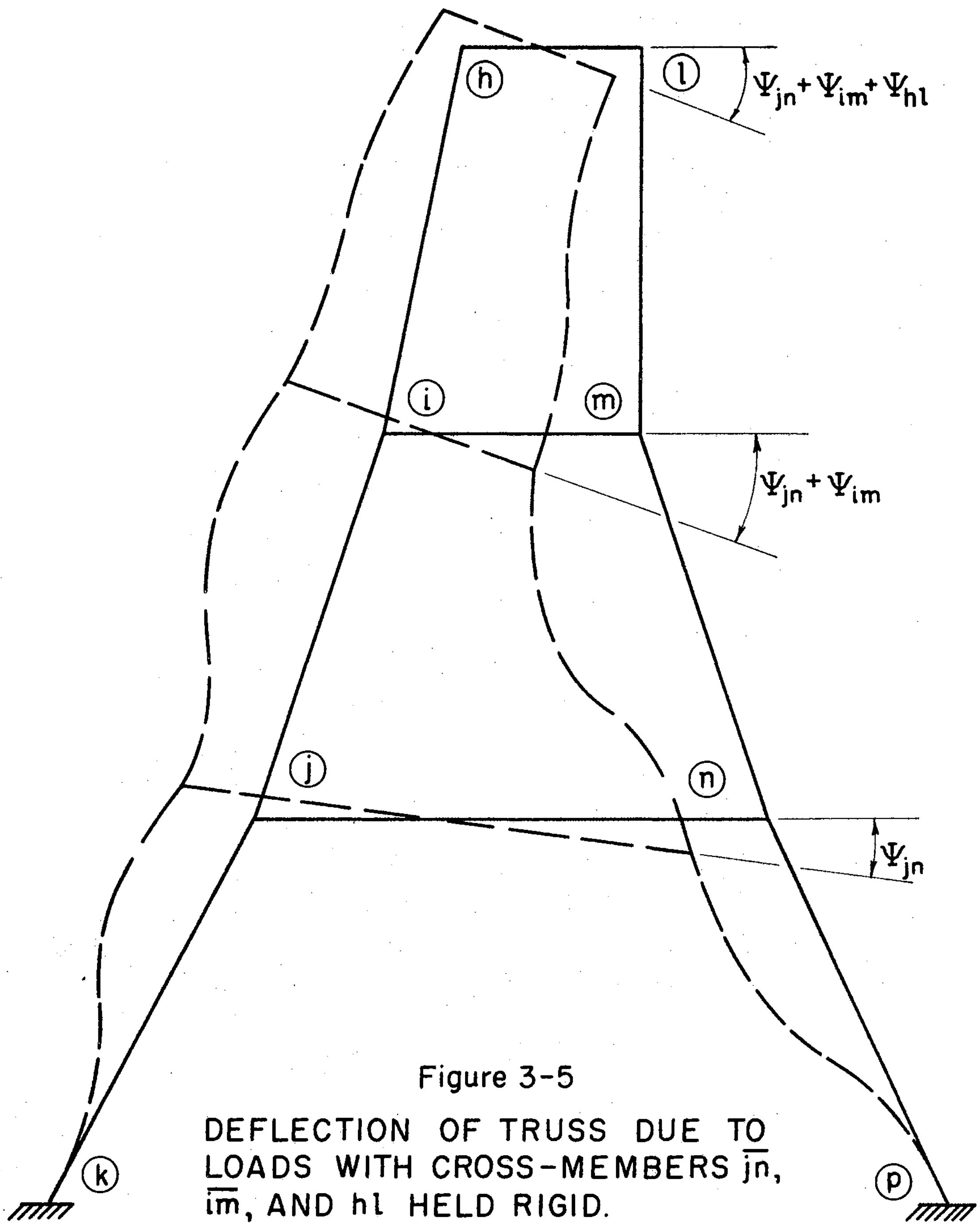


Figure 3-5

DEFLECTION OF TRUSS DUE TO LOADS WITH CROSS-MEMBERS $\overline{jn}$, $\overline{im}$, AND hl HELD RIGID.

The drawing in Figure 3-5 is the general shape of the truss if all cross-members are rigid and if all panels are loaded. With cross-member $\overline{kp}$ anchored in position, the cross-members have the previous angle of rotation for the preceding cross-member as well as its own.

To include this method in the analysis it is necessary to change one of the original assumptions stated in the introduction which is all joints are fixed against rotation but free to translate. The new assumption is that all cross-members are completely rigid but free to translate. Releasing the cross-members from the rigid state causes the joints of the truss to rotate and to carry-over the deflections and rotations from one member to the other members which is accomplished in the carry-over procedure.

This action eliminates the third term in the end moment equations of the cross-members and adds one term to the end moment equations for the sloped members. Equations (3-1 and (3-2) now become:

$$\left.\begin{aligned}
M_{kj} &= K_{kj}\theta_k + CK_{jk}\theta_j + CK_{jk}\psi_{jn} + S_{kj}\psi_k + FM_{kj} \\
M_{jk} &= K_{jk}\theta_j + CK_{kj}\theta_k + K_{jk}\psi_{jn} + S_{jk}\psi_j + FM_{jk} \\
M_{jn} &= K_{nj}\theta_j + CK_{nj}\theta_n + FM_{jn} \\
M_{ji} &= K_{ji}\theta_j + CK_{ij}\theta_i + CK_{ij}\psi_{im} + S_{ji}\psi_j + FM_{ji} \\
M_{ij} &= K_{ij}\theta_i + CK_{ji}\theta_j + K_{ij}\psi_{im} + S_{ij}\psi_j + FM_{ij}
\end{aligned}\right\} \qquad (3\text{-}4)$$

$$\left.\begin{aligned}
M_{pn} &= K_{pn}\theta_p + CK_{np}\theta_n + CK_{np}\psi_{jn} + S_{pn}\psi_p + FM_{pn} \\
M_{np} &= K_{np}\theta_n + CK_{pn}\theta_p + K_{np}\psi_{jn} + S_{np}\psi_p + FM_{np} \\
M_{nj} &= K_{nj}\theta_n + CK_{jn}\theta_j + FM_{nj} \\
M_{nm} &= K_{nm}\theta_n + CK_{mn}\theta_m + CK_{mn}\psi_{im} + S_{nm}\psi_n + FM_{nm} \\
M_{mn} &= K_{mn}\theta_n + CK_{nm}\theta_n + K_{mn}\psi_{im} + S_{mn}\psi_n + FM_{mn}
\end{aligned}\right\} \qquad (3\text{-}5)$$

From the geometry of Figure 3-6 and Figure 3-7, it is evident that

$$\left.\begin{aligned} \psi_k &= \frac{\Delta_j}{L_k} \\ \psi_{jn} &= \frac{\Delta_{jn}}{h_{jn}} \\ \psi_p &= \frac{\Delta_n}{L_p} \end{aligned}\right\} \qquad (3\text{-}6)$$

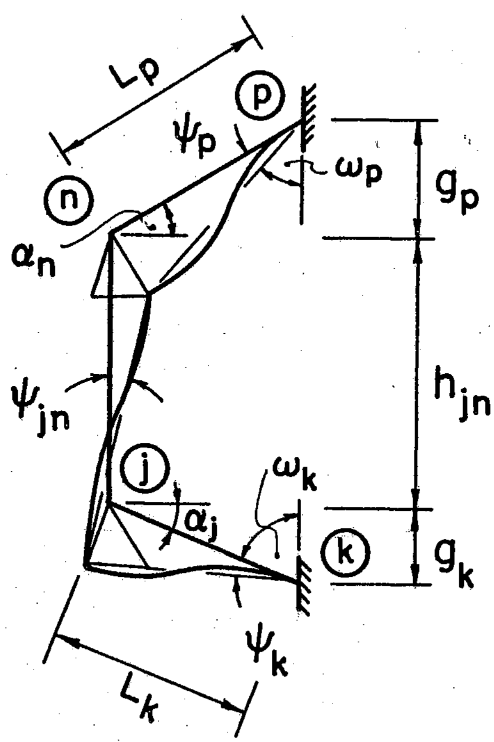


Figure 3-6. Deformation of Panel $\overline{jknp}$ due to Displacements.

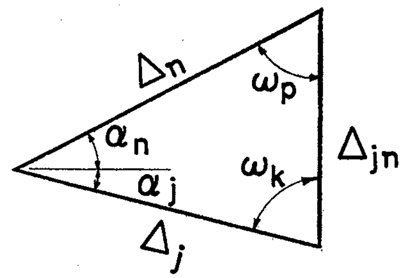


Figure 3-7. Enlargement of the Deflections of Joints j and n in Panel $\overline{jknp}$.

Enlarging the deflections as shown in Figure 3-7 and using the sine law, the following relationship is derived:

$$\psi_k = \psi_p \tag{3-7}$$

Using trigonometry and Figure 3-7 the following relationship is derived:

$$\psi_{jn} = \psi_k \frac{(g_p + g_k)}{h_{jn}} \tag{3-8}$$

Now substituting Equations (3-7) and (3-8) into Equations (3-4) and (3-5), the following equations are derived:

$$\left.\begin{aligned}
M_{kj} &= K_{kj}\theta_k + CK_{jk}\theta_j + \left[S_{kj} + CK_{jk}\frac{(g_p + g_k)}{h_{jn}}\right]\psi_k + FM_{kj} \\
M_{jk} &= K_{jk}\theta_j + CK_{kj}\theta_k + \left[S_{jk} + K_{jk}\frac{(g_p + g_k)}{h_{jn}}\right]\psi_k + FM_{jk} \\
M_{jn} &= K_{jn}\theta_j + CK_{nj}\theta_n + FM_{jn} \\
M_{ji} &= K_{ji}\theta_j + CK_{ij}\theta_i + \left[S_{ji} + CK_{ij}\frac{(g_n + g_j)}{h_{im}}\right]\psi_j + FM_{ji} \\
M_{ij} &= K_{ij}\theta_i + CK_{ji}\theta_j + \left[S_{ij} + K_{ij}\frac{(g_n + g_j)}{h_{im}}\right]\psi_j + FM_{ij}
\end{aligned}\right\} \tag{3-9}$$

$$
\left.
\begin{aligned}
M_{pn} &= K_{pn}\theta_p + CK_{np}\theta_n + \left[S_{pn} + CK_{np}\frac{(g_p + g_k)}{h_{jn}}\right]\psi_k + FM_{pn} \\
M_{np} &= K_{np}\theta_n + CK_{pn}\theta_p + \left[S_{np} + K_{np}\frac{(g_p + g_k)}{h_{jn}}\right]\psi_k + FM_{np} \\
M_{nj} &= K_{nj}\theta_n + CK_{jn}\theta_j + FM_{nj} \\
M_{nm} &= K_{nm}\theta_n + CK_{mn}\theta_m + \left[S_{nm} + CK_{mn}\frac{(g_n + g_j)}{h_{im}}\right]\psi_j + FM_{nm} \\
M_{mn} &= K_{mn}\theta_m + CK_{nm}\theta_n + \left[S_{mn} + K_{mn}\frac{(g_n + g_j)}{h_{im}}\right]\psi_j + FM_{mn}
\end{aligned}
\right\} \quad (3\text{-}10)
$$

In order to eliminate the displacement terms ψ's from the slope-deflection equations, the end moments and shears are summed about Oj in Figure 3-8. The following equation is derived from the summation of moments about Oj.

$$
SM_{oj} - V_{kj}f_{ok} - V_{pn}f_{op} + M_{kj} + M_{pn} = 0 \quad (3\text{-}11)
$$

where

$$
V_{kj} = \frac{M_{jk} + M_{kj}}{L_k} + FV_{kj}
$$

$$
V_{pn} = \frac{M_{np} + M_{pn}}{L_p} + FV_{pn}
$$

Before substituting the end moments into Equation (3-11), the f_{ok} and f_{op} terms are eliminated by using geometry and trigonometry in Figure 3-8 to derive the following equation.

$$
\left.
\begin{aligned}
f_{ok} &= \frac{h_{kp}\, L_k}{g_k + g_p} \\
f_{op} &= \frac{h_{kp}\, L_p}{g_k + g_p}
\end{aligned}
\right\} \quad (3\text{-}12)
$$

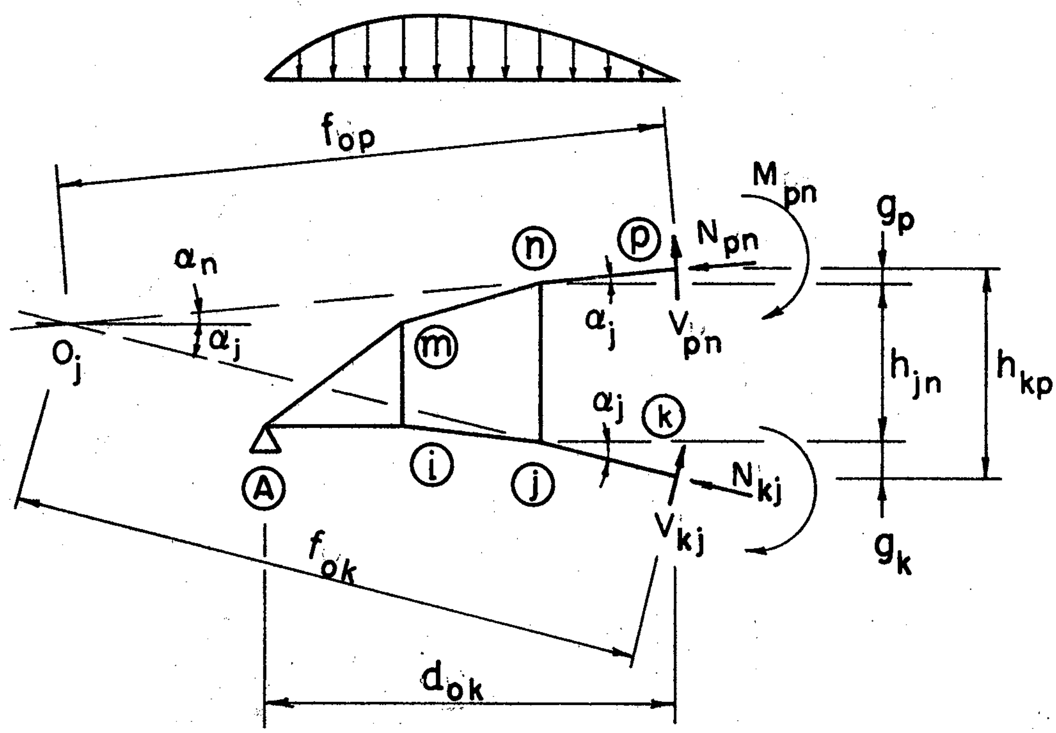


Figure 3-8. Free Body Left of $\overline{kp}$.

The end moment Equations (3-9) and (3-10) and Equation (3-12) are substitued into Equation (3-11). After collecting terms and solving for the displacement term ψ_k, the following equation is derived.

$$\psi_k = -\frac{K_{kj}}{Q_k}(h_{jn} + C_{kj}\, h_{kp})\theta_k - \frac{K_{jk}}{Q_k}(h_{kp} + C_{jk}\, h_{jn})\theta_j$$

$$- \frac{K_{pn}}{Q_k}(h_{jn} + C_{pn}\, h_{kp})\theta_p - \frac{K_{np}}{Q_k}(h_{kp} + C_{np}\, h_{jn})\theta_n$$

$$- \frac{1}{Q_k}\Big[(FM_{kj} + FM_{pn})h_{jn} + (FM_{jk} + FM_{np})h_{kp}$$

$$+ (FV_{kj}\, L_k + FV_{pn}\, L_p)h_{kp} - SM_{oj}(g_p + g_k)\Big] \qquad (3\text{-}13)$$

where

$$Q_k = (S_{kj} + S_{pn})h_{jn} + (S_{jk} + S_{np})h_{kp}$$

$$+ \frac{g_p + g_k}{h_{jn}} \left[(CK_{jk} + CK_{np})h_{jn} + (K_{jk} + K_{np})h_{kp}\right] \qquad (3\text{-}14)$$

Now taking a free body through panel $\overline{ijmn}$ and using the previous reasoning used in solving for ψ_k to solve for ψ_j, the following equation is derived.

$$\psi_j = -\frac{K_{ji}}{Q_j}(h_{im} + C_{ji}\, h_{jn})\theta_j - \frac{K_{ij}}{Q_j}(h_{jn} + C_{ij}\, h_{im})\theta_i$$

$$- \frac{K_{nm}}{Q_j}(h_{im} + C_{nm}\, h_{jn})\theta_n - \frac{K_{mn}}{Q_j}(h_{jn} + C_{mn}\, h_{im})\theta_m$$

$$- \frac{1}{Q_j}\Big[(FM_{ji} + FM_{nm})h_{im} + (FM_{ij} + FM_{mn})h_{jn}$$

$$+ (FV_{ji}\, L_j + FV_{nm}\, L_n)h_{jn} - SM_{oi}(g_n + g_j)\Big] \qquad (3\text{-}15)$$

where

$$Q_j = (S_{ji} + S_{nm})h_{im} + (S_{ij} + S_{mn})h_{jn}$$

$$+ \frac{g_n + g_j}{h_{im}} \left[(CK_{ij} + CK_{mn})h_{im} + (K_{ij} + K_{mn})h_{jn}\right] \qquad (3\text{-}16)$$

Now the displacement terms are derived in terms of joint rotations and are substituted into Equations (3-9) and (3-10). Subsequently, the following equations are derived.

$$M_{kj} = K^*_{kj}\, \theta_j + CK^*_{jk}\, \theta_j + CK^*_{pk}\, \theta_p + CK^*_{nk}\, \theta_n + FM^*_{kj} \qquad (3\text{-}17)$$

where

$$K^*_{kj} = K_{kj}\left[1 - (h_{jn} + C_{kj}\, h_{kp})\, T_{kj}\right] \tag{3-18}$$

$$CK^*_{jk} = K_{jk}\left[C_{jk} - (h_{kp} + C_{jk}\, h_{jn})\, T_{kj}\right] \tag{3-19}$$

$$CK^*_{pk'} = K_{pn}\,(h_{jn} + C_{pn}\, h_{kp})\, T_{kj} \tag{3-20}$$

$$CK^*_{nk} = K_{np}\,(h_{kp} + C_{np}\, h_{jn})\, T_{kj} \tag{3-21}$$

$$FM^*_{kj} = -\,T_{kj}\Big[(FM_{jk} + FM_{np})h_{kp} + (FM_{kj} + FM_{pn})h_{jn} + (FV_{kj}\, L_k + FV_{pn}\, L_p)h_{kp} - SM_{oj}\,(g_p + g_k)\Big] + FM_{kj} \tag{3-22}$$

$$T_{kj} = \frac{1}{Q_k}\left[\, S_{kj} + CK_{jk}\,\frac{g_p + g_k}{h_{jn}}\right] \tag{3-23}$$

$$M_{jk} = K^*_{jk}\,\theta_j + CK^*_{kj}\,\theta_k + CK^*_{nj''}\,\theta_n + CK^*_{pj}\,\theta_p + FM^*_{jk} \tag{3-24}$$

where

$$K^*_{jk} = K_{jk}\left[1 - (h_{kp} + C_{jk}\, h_{jn})\, T_{jk}\right] \tag{3-25}$$

$$CK^*_{kj} = K_{kj}\left[C_{kj} - (h_{jn} + C_{kj}\, h_{kp})\, T_{jk}\right] \tag{3-26}$$

$$CK^*_{nj''} = -\,K_{np}\,(h_{kp} + C_{np}\, h_{jn})\, T_{jk} \tag{3-27}$$

$$CK^*_{pj} = -\,K_{pn}\,(h_{jn} + C_{pn}\, h_{kp})\, T_{jk} \tag{3-28}$$

$$FM^*_{jk} = -\,T_{jk}\Big[(FM_{jk} + FM_{np})h_{kp} + (FM_{kj} + FM_{pn})h_{jn} + (FV_{kj}\, L_k + FV_{pn}\, L_p)h_{kp} - SM_{oj}\,(g_p + g_k)\Big] + FM_{jk} \tag{3-29}$$

$$T_{jk} = \frac{1}{Q_k}\left[S_{jk} + K_{jk}\,\frac{g_p + g_k}{h_{jn}}\right] \qquad (3\text{-}30)$$

$$M_{jn} = K^*_{jn}\,\theta_j + CK^*_{nj}\,\theta_n + FM^*_{jn} \qquad (3\text{-}31)$$

where

$$K^*_{jn} = K_{jn} \qquad (3\text{-}32)$$

$$CK^*_{jn} = CK^*_{jn} \qquad (3\text{-}33)$$

$$FM^*_{jn} = FM_{jn} \qquad (3\text{-}34)$$

$$M_{ji} = K^*_{ji}\,\theta_j + CK^*_{ij}\,\theta_i + CK^*_{nj'}\,\theta_n + CK^*_{mj}\,\theta_m + FM^*_{ji} \qquad (3\text{-}35)$$

where

$$K^*_{ji} = K_{ji}\left[1 - (h_{im} + C_{ji}\,h_{jn})\,T_{ji}\right] \qquad (3\text{-}36)$$

$$CK^*_{ij} = K_{ij}\left[C_{ij} - (h_{jn} + C_{ij}\,h_{im})\,T_{ji}\right] \qquad (3\text{-}37)$$

$$CK^*_{nj'} = K_{nm}\,(h_{im} + C_{nm}\,h_{jn})\,T_{ji} \qquad (3\text{-}38)$$

$$CK^*_{mj} = -\,K_{mn}(h_{jn} + C_{mn}\,h_{im})\,T_{ji} \qquad (3\text{-}39)$$

$$FM^*_{ji} = -\,T_{ji}\left[(FM_{ij} + FM_{mn})h_{jn} + (FM_{ji} + FM_{nm})h_{im} + (FV_{ji}\,L_j + FV_{nm}\,L_n)h_{jn} - SM_{oi}(g_n + g_j)\right] + FM_{ji} \qquad (3\text{-}40)$$

$$T_{ji} = \frac{1}{Q_j}\left[S_{ji} + CK_{ij}\,\frac{g_n + g_j}{h_{im}}\right] \qquad (3\text{-}41)$$

$$M_{ij} = K^*_{ij}\,\theta_i + CK^*_{ji}\,\theta_j + CK^*_{mi''}\,\theta_m + CK^*_{ni}\,\theta_n + FM^*_{ij} \qquad (3\text{-}42)$$

where

$$K^*_{ij} = K_{ij}\left[1 - (h_{jn} + C_{ij}\,h_{im})\,T_{ij}\right] \qquad (3\text{-}43)$$

$$CK^*_{ji} = K_{ji}\left[C_{ji} - (h_{im} + C_{ji}\,h_{jn})\,T_{ij}\right] \qquad (3\text{-}44)$$

$$CK^{*}_{mi'} = - K_{mn} (h_{jn} + C_{mn} h_{im}) T_{ij} \qquad (3\text{-}45)$$

$$CK^{*}_{ni} = - K_{nm} (h_{im} + C_{nm} h_{jn}) T_{ij} \qquad (3\text{-}46)$$

$$FM^{*}_{ij} = - T_{ij} \Big[(FM_{ij} + FM_{mn}) h_{jn} + (FM_{ji} + FM_{nm}) h_{im} + (FV_{ji} L_j + FM_{nm} L_n) h_{jn} - SM_{oi} (g_n + g_j) \Big] + FM_{ij} \qquad (3\text{-}47)$$

$$T_{ij} = \frac{1}{Q_j} \Big[S_{ij} + K_{ij} \frac{g_n + g_j}{h_{im}} \Big] \qquad (3\text{-}48)$$

$$M_{pn} = K^{*}_{pn} \theta_p + CK^{*}_{np} \theta_n + CK^{*}_{kp'} \theta_k + CK^{*}_{jp} \theta_j + FM^{*}_{pn} \qquad (3\text{-}49)$$

where

$$K^{*}_{pn} = K_{pn} \Big[1 - (h_{jn} + C_{pn} h_{kp}) T_{pn} \Big] \qquad (3\text{-}50)$$

$$CK^{*}_{np} = K_{np} \Big[C_{np} - (h_{kp} + C_{np} h_{jn}) T_{pn} \Big] \qquad (3\text{-}51)$$

$$CK^{*}_{kp'} = - K_{kj} (h_{jn} + C_{kj} h_{kp}) T_{pn} \qquad (3\text{-}52)$$

$$CK^{*}_{jp} = - K_{jk} (h_{kp} + C_{jk} h_{jn}) T_{pn} \qquad (3\text{-}53)$$

$$FM^{*}_{pn} = - T_{pn} \Big[(FM_{jk} + FM_{np}) h_{kp} + (FM_{kj} + FM_{pn}) h_{jn} + (FV_{kj} L_k + FV_{pn} L_p) h_{kp} - SM_{oj} (g_p + g_k) \Big] + FM_{pn} \qquad (3\text{-}54)$$

$$T_{pn} = \frac{1}{Q_p} \Big[S_{pn} \quad CK_{np} \frac{g_p + g_k}{h_{jn}} \Big] \qquad (3\text{-}55)$$

$$M_{np} = K^{*}_{np} \theta_n + CK^{*}_{pn} \theta_p + CK^{*}_{jn'} \theta_j + CK^{*}_{kn} \theta_k + FM^{*}_{np} \qquad (3\text{-}56)$$

where

$$K^{*}_{np} = K_{np} \Big[1 - (h_{kp} + C_{np} h_{jn}) T_{np} \Big] \qquad (3\text{-}57)$$

$$CK^*_{pn} = K_{pn} \left[C_{pn} - (h_{jn} + C_{pn} h_{kp}) T_{np} \right] \quad (3\text{-}58)$$

$$CK^*_{jn''} = - K_{jk} (h_{kp} - C_{jk} h_{jn}) T_{np} \quad (3\text{-}59)$$

$$CK^*_{kn} = - K_{kj} (h_{jn} - C_{kj} h_{kp}) T_{np} \quad (3\text{-}60)$$

$$FM^*_{np} = - T_{np} \left[(FM_{jk} + FM_{np}) h_{kp} + (FM_{kj} + FM_{pn}) h_{jn} + (FV_{kj} L_k + FV_{pn} L_p) h_{kp} - SM_{oj} (g_p + g_k) \right] + FM_{np} \quad (3\text{-}61)$$

$$T_{np} = \frac{1}{Q_p} \left[S_{np} + K_{np} \frac{g_p + g_k}{h_{jn}} \right] \quad (3\text{-}62)$$

$$M_{nj} = K^*_{nj} \theta_n + CK^*_{jn} \theta_j + FM^*_{nj} \quad (3\text{-}63)$$

where

$$K^*_{nj} = K_{nj} \quad (3\text{-}64)$$

$$CK^*_{jn} = CK_{jn} \quad (3\text{-}65)$$

$$FM^*_{nj} = FM_{nj} \quad (3\text{-}66)$$

$$M_{nm} = K^*_{nm} \theta_n + CK^*_{mn} \theta_m + CK^*_{jn'} \theta_j + CK^*_{in} \theta_i + FM^*_{nm} \quad (3\text{-}67)$$

where

$$K^*_{nm} = K_{nm} \left[1 - (h_{im} + C_{nm} h_{jn}) T_{nm} \right] \quad (3\text{-}68$$

$$CK^*_{mn} = K_{mn} \left[C_{mn} - (h_{jn} + C_{mn} h_{im}) T_{nm} \right] \quad (3\text{-}69)$$

$$CK^*_{jn'} = - K_{ji} (h_{im} + C_{ji} h_{jn}) T_{nm} \quad (3\text{-}70)$$

$$CK^*_{in} = - K_{ij} (h_{jn} + C_{ij} h_{im}) T_{nm} \quad (3\text{-}71)$$

$$FM^*_{nm} = - T_{nm} \Big[(FM_{ij} + FM_{mn}) h_{jn} + (FM_{ji} + FM_{nm}) h_{im} + (FV_{ji} L_j + FV_{nm} L_n) h_{jn} - SM_{oi} (g_n + g_j) \Big] + FM_{nm} \qquad (3\text{-}72)$$

$$T_{nm} = \frac{1}{Q_n} \left[S_{nm} + CK_{mn} \frac{g_n + g_j}{h_{im}} \right] \qquad (3\text{-}73)$$

$$M_{mn} = K^*_{mn} \theta_m + CK^*_{nm} \theta_n + CK^*_{im''} \theta_i + CK^*_{jm} \theta_j + FM^*_{mn} \qquad (3\text{-}74)$$

where

$$K^*_{mn} = K_{mn} \left[1 - (h_{jn} + C_{mn} h_{im}) T_{mn} \right] \qquad (3\text{-}75)$$

$$CK^*_{nm} = K_{nm} \left[C_{nm} - (h_{im} + C_{nm} h_{jn}) T_{mn} \right] \qquad (3\text{-}76)$$

$$CK^*_{im''} = - K_{ij} (h_{jn} + C_{ij} h_{im}) T_{mn} \qquad (3\text{-}77)$$

$$CK^*_{jm} = - K_{ji} (h_{im} + C_{ji} h_{jn}) T_{mn} \qquad (3\text{-}78)$$

$$FM^*_{mn} = - T_{mn} \Big[(FM_{ij} + FM_{mn}) h_{jn} + (FM_{ji} + FM_{nm}) h_{im} + (FV_{ji} L_j + FV_{nm} L_n) h_{jn} - SM_{oi} (g_n + g_j) \Big] + FM_{mn} \qquad (3\text{-}79)$$

$$T_{mn} = \frac{1}{Q_n} \left[S_{mn} + K_{mn} \frac{g_n + g_j}{h_{im}} \right] \qquad (3\text{-}80)$$

The task of eliminating the displacement terms is accomplished, and the end moment equations are now in terms of joint rotations and fixed end moments. The next step is to devise a method to solve the end moment equations for rotations - θ's. This is accomplished in the same manner as has been done in previous papers on carry-over joint moment analysis for other types of structures.

The first step is the derivation of the joint-moment equation. The purpose of the derivation is to define the carry-over values and the starting moments.

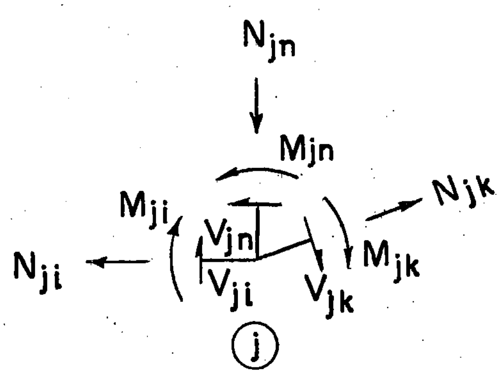


Figure 3-9. Free Body of Joint j.

The summation of moments about joint j under static loads is equal to zero.

$$M_{ji} + M_{jn} + M_{jk} = 0 \qquad (3\text{-}81)$$

Substituting Equations (3-24), (3-31), and (3-35) into Equation (3-81) gives the following equation.

$$CK^*_{kj}\,\theta_k + CK^*_{pj}\,\theta_p + CK^*_{ij}\,\theta_i + CK^*_{mj}\,\theta_m$$

$$+ \Sigma CK^*_{nj}\,\theta_n + \Sigma K^*_j\,\theta_j + \Sigma FM^*_j = 0 \qquad (3\text{-}82)$$

where

$$\Sigma CK^*_{nj} = CK^*_{nj} + CK^*_{nj} + CK^*_{nj}{}'$$

$$\Sigma K^*_j = K^*_{ji} + K^*_{jn} + K^*_{jk}$$

$$\Sigma FM^*_j = FM^*_{ji} + FM^*_{jn} + FM^*_{jk}$$

Equation (3-82) is sufficient to determine the carry-over values and starting moments, but the value of any joint rotation is minute and is very awkward to use in the carry-over procedure. Multiplying the joint rotation value by a large number such as the summation of a new modified stiffness factors for that specific joint does accomplish this purpose. The new term is named joint moment whence this type of analysis derived its name. The joint moment equations are:

$$\left.\begin{aligned} JM_k &= \theta_k\ \Sigma K_k^* \\ JM_j &= \theta_j\ \Sigma K_j^* \\ JM_i &= \theta_i\ \Sigma K_i^* \\ JM_p &= \theta_p\ \Sigma K_p^* \\ JM_n &= \theta_n\ \Sigma K_n^* \\ JM_m &= \theta_m\ \Sigma K_m^* \end{aligned}\right\} \qquad (3\text{-}83)$$

Substituting Equations (3-83) into Equation (3-82) gives the following equation

$$\begin{aligned} JM_j = r_{kj}\ JM_k + m_j + r_{ij}\ JM_i + r_{pj}\ JM_p \\ + r_{nj}\ JM_n + r_{mj}\ JM_m \end{aligned} \qquad (3\text{-}84)$$

where

$$r_{kj} = -\frac{CK_{kj}^*}{\Sigma K_k^*} \qquad (3\text{-}85)$$

$$r_{ij} = -\frac{CK_{ij}^*}{\Sigma K_i^*} \qquad (3\text{-}86)$$

$$r_{nj} = -\frac{\Sigma CK_{nj}^*}{\Sigma K_n^*} \qquad (3\text{-}87)$$

$$r_{mj} = -\frac{CK^*_{mj}}{\Sigma K^*_m} \tag{3-88}$$

$$r_{pj} = -\frac{CK^*_{pj}}{\Sigma K^*_p} \tag{3-89}$$

$$m_j = -\Sigma FM^*_j \tag{3-90}$$

The joint moment Equation (3-84) is similar to the joint moment equations in other apers on carry-over joint moments for other types of structures.

The end moment equations are still in terms of joint rotations thereby creating the necessity for expressing the end moment equation in terms of joint moments. This is accomplished by substituting Equations (3-83) into Equations (3-17), (3-24), (3-31), (3-35), (3-42), (3-49), (3-56), (3-63), (3-67), and (3-72) creating the following equations.

$$M_{kj} = D^*_{kj}\ JM_k + C^*_{jk}\ D^*_{jk}\ JM_j + C^*_{pk'}\ D^*_{pk'}\ JM_p$$

$$+ C^*_{nk}\ D^*_{nk}\ JM_n + FM^*_{kj} \tag{3-91}$$

where

$$D^*_{kj} = \frac{K^*_{kj}}{\Sigma K^*_k} \tag{3-92}$$

$$C^*_{jk}\ D^*_{jk} = \frac{CK^*_{jk}}{\Sigma K^*_j} \tag{3-93}$$

$$C^*_{pk'}\ D^*_{pk'} = \frac{CK^*_{pk'}}{\Sigma K^*_p} \tag{3-94}$$

$$C^*_{nk}\ D^*_{nk} = \frac{CK^*_{nk}}{\Sigma K^*_n} \tag{3-95}$$

$$M_{jk} = D^*_{jk}\ JM_j + C^*_{kj}\ D^*_{kj}\ JM_k + C^*_{nj''}\ D^*_{nj''}\ JM_n$$

$$+ C^*_{pj}\ D^*_{pj}\ JM_p + FM^*_{jk} \qquad (3\text{-}96)$$

where

$$D^*_{jk} = \frac{K^*_{jk}}{\Sigma K^*_j} \qquad (3\text{-}97)$$

$$C^*_{kj}\ D^*_{kj} = \frac{CK^*_{kj}}{\Sigma K^*_k} \qquad (3\text{-}98)$$

$$C^*_{nj''} D^*_{nj''} = \frac{CK^*_{nj''}}{\Sigma K^*_n} \qquad (3\text{-}99)$$

$$C^*_{pj}\ D^*_{pj} = \frac{CK^*_{pj}}{\Sigma K^*_p} \qquad (3\text{-}100)$$

$$M_{jn} = D^*_{jn}\ JM_j + C^*_{nj}\ D^*_{nj}\ JM_n + FM^*_{jn} \qquad (3\text{-}101)$$

where

$$D^*_{jn} = \frac{K^*_{jn}}{\Sigma K^*_j} \qquad (3\text{-}102)$$

$$C^*_{nj}\ D^*_{nj} = \frac{CK^*_{nj}}{\Sigma K^*_n} \qquad (3\text{-}103)$$

$$M_{ji} = D^*_{ji}\ JM_j + C^*_{ij}\ D^*_{ij}\ JM_i + C^*_{nj'} D^*_{nj'} JM_n$$

$$+ C^*_{mj}\ D^*_{mj}\ JM_m + FM^*_{ji} \qquad (3\text{-}104)$$

where

$$D^*_{ji} = \frac{K^*_{ji}}{\Sigma K^*_j} \qquad (3\text{-}105)$$

$$C^*_{ij}\ D^*_{ij} = \frac{CK^*_{ij}}{\Sigma K^*_i} \qquad (3\text{-}106)$$

$$C^*_{nj'} D^*_{nj'} = \frac{CK^*_{nj'}}{\Sigma K^*_n} \qquad (3\text{-}107)$$

$$C^*_{mj} \ D^*_{mj} = \frac{CK_{mj}}{\Sigma K^*_n} \qquad (3\text{-}108)$$

$$M_{ij} = D^*_{ij} \ JM_i + C^*_{ji} \ D^*_{ji} \ JM_j + C^*_{mi''} D^*_{mi''} JM_m$$

$$+ C^*_{nj} \ D^*_{nj} \ JM_n + FM^*_{ij} \qquad (3\text{-}109)$$

where

$$D^*_{ij} = \frac{K^*_{ij}}{\Sigma K^*_i} \qquad (3\text{-}110)$$

$$C^*_{ji} \ D^*_{ji} = \frac{CK^*_{ji}}{\Sigma K^*_j} \qquad (3\text{-}111)$$

$$C^*_{mi''} D^*_{mi''} = \frac{CK^*_{mi''}}{\Sigma K^*_m} \qquad (3\text{-}112)$$

$$C^*_{nj} \ D^*_{nj} = \frac{CK^*_{nj}}{\Sigma K^*_n} \qquad (3\text{-}113)$$

$$M_{pn} = D^*_{pn} \ JM_p + C^*_{np} \ D^*_{np} \ JM_n + C^*_{kp'} D^*_{kp'} JM_k$$

$$+ C^*_{jp} \ D^*_{jp} \ JM_j + FM^*_{pn} \qquad (3\text{-}114)$$

where

$$D^*_{pn} = \frac{K^*_{pn}}{\Sigma K^*_p} \qquad (3\text{-}115)$$

$$C^*_{np} \ D^*_{np} = \frac{CK^*_{np}}{\Sigma K^*_n} \qquad (3\text{-}116)$$

$$C^*_{kp'} D^*_{kp'} = \frac{CK^*_{kp'}}{\Sigma K^*_k} \qquad (3\text{-}117)$$

$$C^*_{jp}\ D^*_{jp} = \frac{CK^*_{jp}}{\Sigma K^*_j} \tag{3-118}$$

$$M_{np} = D^*_{np}\ JM_n + C^*_{pn}\ D^*_{pn}\ JM_p + C^*_{jn''} D^*_{jn''} JM_j$$
$$+ C^*_{kn}\ D^*_{kn}\ JM_k + FM^*_{np} \tag{3-119}$$

where

$$D^*_{np} = \frac{K^*_{np}}{\Sigma K^*_n} \tag{3-120}$$

$$C^*_{pn}\ D^*_{pn} = \frac{CK^*_{pn}}{\Sigma K^*_p} \tag{3-121}$$

$$C^*_{jn''} D^*_{jn''} = \frac{CK^*_{jn''}}{\Sigma K^*_j} \tag{3-122}$$

$$C^*_{kn}\ D^*_{kn} = \frac{CK^*_{kn}}{\Sigma K^*_k} \tag{3-123}$$

$$M_{nj} = D^*_{nj}\ JM_n + C^*_{jn}\ D^*_{jn}\ JM_j + FM^*_{nj} \tag{3-124}$$

where

$$D^*_{nj} = \frac{K^*_{nj}}{\Sigma K^*_n} \tag{3-125}$$

$$C^*_{jn}\ D^*_{jn} = \frac{CK^*_{jn}}{\Sigma K^*_j} \tag{3-126}$$

$$M_{nm} = D^*_{nm}\ JM_n + C^*_{mn}\ D^*_{mn}\ JM_m + C^*_{jn'}\ D^*_{jn'}\ JM_j$$
$$+ C^*_{in}\ D^*_{in}\ JM_i + FM^*_{nm} \tag{3-127}$$

where

$$D^*_{nm} = \frac{K^*_{nm}}{\Sigma K^*_n} \tag{3-128}$$

$$C^*_{mn}\ D^*_{mn} = \frac{CK^*_{mn}}{\Sigma K^*_m} \tag{3-129}$$

$$C^*_{jn'}\ D^*_{jn'} = \frac{CK^*_{jn'}}{\Sigma K^*_j} \tag{3-130}$$

$$C^*_{in}\ D^*_{in} = \frac{CK^*_{in}}{\Sigma K^*_i} \tag{3-131}$$

$$M_{mn} = D^*_{mn}\ JM_m + C^*_{nm}\ D^*_{nm}\ JM_n + C^*_{im''}D^*_{im''}JM_i + C^*_{jm}\ D^*_{jm}\ JM_j + FM^*_{mn} \tag{3-132}$$

where

$$D^*_{mn} = \frac{K^*_{mn}}{\Sigma K^*_m} \tag{3-133}$$

$$C^*_{nm}\ D^*_{nm} = \frac{CK^*_{nm}}{\Sigma K^*_n} \tag{3-134}$$

$$C^*_{im''}D^*_{im''} = \frac{CK^*_{im''}}{\Sigma K^*_i} \tag{3-135}$$

$$C^*_{jm}\ D^*_{jm} = \frac{CK^*_{jm}}{\Sigma K^*_j} \tag{3-136}$$

Ths completes the derivation for Vierendeel truss with sloped members by carry-over joint moments.

The physical interpretation of each new parameter in the joint moment and end moment equations is given in the following two paragraphs.

The starting moment m_j is the joing moment at j due to loads on the truss, if the joints i, k, m, n, and p are fixed against rotations but free to translate. The carry-over factor r_{kj} is the joint moment at j due to JM_k = 1 and no applied loads on panels $\overline{ijkmnp}$; if joints i, m, n, and p are fixed against rotation but free to translate. The carry-over factor rij is the joint moment at i due to JM_i = 1 and no applied loads on panels $\overline{ijkmnp}$; if joints k, m, n, and p are fixed against rotation but free to translate. The carry-over factor r_{mj} is the joint moment at j due to JM_m = 1 and no applied loads on panels $\overline{ijkmnp}$; if joints i, k, n, and p are fixed against rotation but free to translate. The carry-Over factor r_{nj} is the joint moment at j due to JM_n = 1 and no applied loads on panels $\overline{ijkmnp}$; if joints, i, k, m, and p are fixed against rotation but free to translate The carry-over factor r_{pj} is the joint moment at j due to JM_p = 1 and no applied loads on panels $\overline{ijkmnp}$; if joints i, k, m, and n are fixed against rotation but free to translate.

The terms in the new modified end moment equations are defined in this paragraph. The modified distribution D^*_{jk} is the end moment of member jk at j due to JM_j = 1 with all other joint moments equal to zero and with no loads on the truss. The modified carry-over distribution C^*_{kj} D^*_{kj} is the end moment of member jk at j due to JM_k = 1 with all other joint moments equal to zero and with no loads on the truss. The modified carry-over distribution $C^*_{nj}{}''$ $D^*_{nj}{}''$ is the end moment of member jk at j due to JM_n = 1 with all other joint moments equal to zero and with no loads on the truss. The carry-over distribution C^*_{pj} D^*_{pj} is the end moment of jk at j due to JM_p = 1 with all other joint moments equal to zero and with no loads on the truss. The modified fixed end moment FM^{*}_{jk} is the end moment of member jk at j due to loads on the truss and all joint moments equal to zero.

With the completion of the derivation and physical interpretation of the carry-over joint moment equations, the next step is to give the analysis procedure.

The first step is to determine the elastic constants-stiffness factors, carry-over factors, and sidesway stiffness factors. The second step is to determine the equivalent values - the derived terms Q and T. The third step is to solve for modified stiffness factors and modified carry-over stiffness factors. The fourth step is to solve for modified distribution factors, modified carry-over distribution factors, and joint moment carry-over factors. The fifth step is to solve for fixed end moments, static load moments, and fixed end shears. The sixth step is to solve for modified fixed end moments and starting moments. The seventh step is the carry-over procedure. The eighth step is the numerical check. The ninth and last step is to solve for the end moments of every member in the structure. The above procedure is illustrated in two examples in the next chapter.

As noted in the two examples in the next chapter, the convergency of the carry-over procedure is very rapid for the carry-over joint moment method as compared to the moment distribution method. Considerable additional time is saved because only one carry-over procedure is necessary in this analysis as compared to a carry-over procedure for each independent translation in the moment distribution method.

There is no loss of accuracy in the carry-over joint moment analysis as compared to the moment distribution analysis because the derivation of the carry-over joint moment equation used the moment distribution's slope deflection equations in deriving the carry-over joint moment equation. The derivation was accomplished solely through algebraic and trigonometric means with the same assumptions used in the moment distribution method. There has been no simplifying assumption used to make this analysis possible.

CHAPTER IV

NUMERICAL PROCEDURE

The numerical procedure of the carry-over joint moment analysis for the Vierendeel truss with inclined members is demonstrated in the two examples in this chapter. All values are given in feet, kips, and kip-feet.

Example No. 1

A five panel Vierendeel truss with inclined top chords (Figure 4-1) is to be analyzed by the carry-over joint moment method. The equations used in determining the constants used in this example are referenced back to the derivation. The equations for the parallel portion $\overline{CDJL}$ of the truss are referenced to Samuel's (29) Thesis. The results of this example problem can be compared to Example No. 1 in S. L. Lee and F. P. Weisinger, "Veriendeel Bents with Nonprismatic Members," Proceedings, ASCE, Vol. 85, No. ST10, December, 1959, pp. 55-74.

1) Elastic Constants

a) Stiffness Factors

All stiffness factors are assumed to be $4^{Ft\text{-}K}$.

b) Carry-over Values

All carry-over values are assumed to be 0.5.

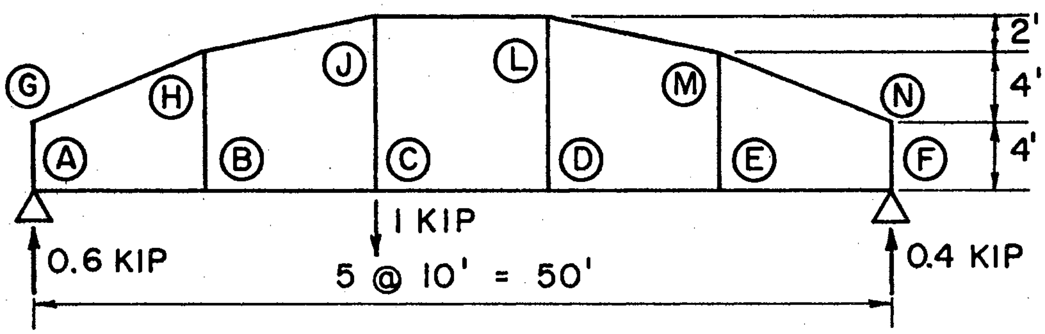


Figure 4-1. Single Span Vierendeel Truss with Inclined Chords.

c) Sideway Stiffness Factors

All sideway stiffness factors are assumed to be $6.0^{Ft\text{-}K}$.

2) Equivalents

a) Q Values

$$Q_j = (S_{ji} + S_{nm})\, h_{im} + (S_{ij} + S_{mn})\, h_{jn}$$

$$+ \frac{g_n + g_j}{h_{im}} \left[(CK_{ij} + CK_{mn})\, h_{im} + (K_{ij} + K_{mn}\; h_{jn}) \right] \qquad (3\text{-}16)$$

Q_B = 224.00	Q_H = 224.00
Q_C = 244.00	Q_J = 244.00
Q_D = 244.00	Q_L = 244.00
Q_E = 224.00	Q_M = 224.00

b) T Values

$$T_{jk} = \frac{1}{Q_K} \left[S_{jk} + K_{jk} \frac{g_p + g_k}{h_{jn}} \right] \qquad (3\text{-}30)$$

$$T_{ji} = \frac{1}{Q_j} \left[S_{ji} + CK_{ij} \frac{g_n + g_j}{h_{im}} \right] \qquad (3\text{-}41)$$

$T_{AB} = 5/112$	$T_{GH} = 5/112$
$T_{BA} = 1/28$	$T_{HG} = 1/28$
$T_{BC} = 7/224$	$T_{HJ} = 7/224$
$T_{CB} = 65/2440$	$T_{JH} = 65/2440$
$T_{DE} = 65/2440$	$T_{LM} = 65/2440$
$T_{ED} = 7/224$	$T_{ML} = 7/224$
$T_{EF} = 1/28$	$T_{MN} = 1/28$
$T_{FE} = 5/112$	$T_{NM} = 5/112$

3) Modified Stiffness Constants

a) Modified Stiffness Factors

1) Panels with inclined members

$$K^*_{jk} = K_{jk} \left[1 - T_{jk} (h_{kp} + C_{jk} h_{jn} \right] \qquad (3\text{-}25)$$

$$K^*_{jn} = K_{jn} \qquad (3\text{-}32)$$

$$K^*_{ji} = K_{ji} \left[1 - T_{ji} (h_{im} + C_{ji} h_{jn} \right] \qquad (3\text{-}36)$$

$K_{AB} = 2.2143$	$K_{FE} = 2.2143$
$K_{AG} = 4.0000$	$K_{FN} = 4.0000$
$K_{GA} = 4.0000$	$K_{NF} = 4.0000$
$K_{GH} = 2.2143$	$K_{NM} = 2.2143$
$K_{BA} = 2.8572$	$K_{EF} = 2.8572$
$K_{BH} = 4.0000$	$K_{EM} = 4.0000$
$K_{BC} = 2.3934$	$K_{ED} = 2.3934$
$K_{HG} = 2.8572$	$K_{MN} = 2.8572$
$K_{HB} = 4.0000$	$K_{ME} = 4.0000$

$K_{HJ} = 2.3934$ $\qquad$ $K_{ML} = 2.3934$

$K_{CB} = 2.6148$ $\qquad$ $K_{DE} = 2.6148$

$K_{JH} = 2.6148$ $\qquad$ $K_{LM} = 2.6148$

2) Panels with Parallel Members

{Equations from Samuel's Thesis (29)}

$$K^*_{jn} = K_{jn} \quad \Big| \quad K^*_{jk} = K_{jk} - \frac{S^2_{jk}}{S_{jk} + S_{kj} + S_{np} + S_{pn}}$$

$K^*_{CJ} = 4.00$ $\qquad$ $K^*_{DL} = 4.00$

$K^*_{CD} = 2.50$ $\qquad$ $K^*_{DC} = 2.50$

$K^*_{JC} = 4.00$ $\qquad$ $K^*_{LD} = 4.00$

$K^*_{JL} = 2.50$ $\qquad$ $K^*_{LJ} = 2.50$

3) Summation of K*'s

$\Sigma K^*_A = \Sigma K^*_G = 6.2143$

$\Sigma K^*_B = \Sigma K^*_H = 9.2506$

$\Sigma K^*_C = \Sigma K^*_J = 9.1148$

$\Sigma K^*_D = \Sigma K^*_L = 9.1148$

$\Sigma K^*_E = \Sigma K^*_M = 9.2506$

$\Sigma K^*_F = \Sigma K^*_N = 6.2143$

b) Modified Carry-over Stiffness Factors

1) Panels with Inclined Members

$$CK^*_{kj} = K_{kj} \left[C_{kj} - T_{jk} (h_{jn} + C_{kj}\, h_{kp})\right] \qquad (3\text{-}26)$$

$$CK^*_{nj} = CK_{nj} \qquad (3\text{-}33)$$

$$CK^*_{ij} = I_{ij} \left[C_{ij} - T_{ji} (h_{jn} + C_{ij} h_{im}) \right] \quad (3\text{-}37)$$

$$CK^*_{nj}{}'' = - K_{np} (h_{kp} + C_{np} h_{jn}) T_{jk} \quad (3\text{-}27)$$

$$CK^*_{pj} = - K_{pn} (h_{jn} + C_{pn} h_{kp}) T_{jk} \quad (3\text{-}28)$$

$$CK^*_{nj}{}' = - K_{nm} (h_{im} + C_{nm} h_{jn}) T_{ji} \quad (3\text{-}38)$$

$$CK^*_{mj} = - K_{mn} (h_{jn} + C_{mn} h_{im}) T_{ji} \quad (3\text{-}39)$$

$CK^*_{AB} = .57143$

$CK^*_{AG} = 2.000$

$CK^*_{AH} = -1.4286$

$CK^*_{AG}{}'' = -1.7857$

$CK^*_{GA} = 2.0000$

$CK^*_{GH} = .57143$

$CK^*_{GB} = -1.4286$

$CK_{GA}{}'' = -1.7857$

$CK^*_{BA} = .57143$

$CK^*_{BH} = 2.0000$

$CK^*_{BC} = .50820$

$CK^*_{BJ} = -1.4918$

$CK^*_{BH}{}'' = -1.6066$

$CK^*_{BH}{}' = -1.1429$

$CK^*_{BG} = -1.4286$

$CK^*_{HG} = .57143$

$CK^*_{HB} = 2.0000$

$CK^*_{HJ} = .50820$

$CK^*_{FE} = .57143$

$CK^*_{FN} = 2.0000$

$CK^*_{FM} = -1.4286$

$CK^*_{FN}{}'' = -1.7857$

$CK^*_{NF} = 2.0000$

$CK^*_{NM} = .57143$

$CK^*_{NE} = -1.4286$

$CK_{NF}{}'' = -1.7857$

$CK^*_{EF} = .57143$

$CK^*_{EM} = 2.0000$

$CK^*_{ED} = .50820$

$CK^*_{EL} = -1.4918$

$CK^*_{EM}{}'' = -1.6066$

$CK^*_{EM}{}' = -1.1429$

$CK^*_{EN} = -1.4286$

$CK^*_{MN} = .57143$

$CK^*_{ME} = 2.0000$

$CK^*_{ML} = .50820$

$CK^*_{HC} = -1.4918$	$CK^*_{MD} = -1.4918$
$CK^*_{HB''} = -1.6066$	$CK^*_{ME''} = -1.6066$
$CK_{HB'} = -1.1429$	$CK_{ME'} = -1.1429$
$CK^*_{HA} = -1.4286$	$CK^*_{MF} = -1.4286$
$CK^*_{CB} = .50820$	$CK^*_{DE} = .50820$
$CK^*_{CJ'} = -1.3852$	$CK^*_{DL'} = -1.3852$
$CK^*_{CH} = -1.4918$	$CK^*_{DM} = -1.4918$
$CK^*_{JH} = .50820$	$CK^*_{LM} = .50820$
$CK^*_{JC'} = -1.3852$	$CK^*_{LD'} = -1.3852$
$CK^*_{JB} = -1.4918$	$CK^*_{LE} = -1.4918$

2) Panels with Parallel Members

Equations from Samuel's Thesis (29)

$$CK^{*(jn)}_{jn} = CK^*_{jn} = CK_{jn}$$

$$CK^*_{jk} = CK_{jk} - \frac{S_{kj}\,S_{jk}}{S_{kj} + S_{jk} + S_{pn} + S_{np}}$$

$$CK^*_{jp} = -\frac{S_{jk}\,S_{pn}}{S_{jk} + S_{kj} + S_{np} + S_{pn}}$$

$$CK^{*(np)}_{jn} = CK^*_{jn} = -\frac{S_{jk}\,S_{np}}{S_{jk} + S_{kj} + S_{np} + S_{pn}}$$

$CK^*_{CJ} = 2.0000$	$CK^*_{DL} = 2.0000$
$CK^*_{CD} = .50000$	$CK^*_{DC} = .50000$
$CK^*_{CL} = -1.5000$	$CK^*_{DJ} = -1.5000$
$CK_{CJ''} = -1.5000$	$CK_{DL''} = -1.5000$

$CK^*_{JC} = 2.0000$

$CK^*_{JL} = .50000$

$CK^*_{JD} = -1.5000$

$CK^*_{JC''} = -1.5000$

$CK^*_{LD} = 2.0000$

$CK^*_{LJ} = .50000$

$CK^*_{LC} = -1.5000$

$CK^*_{LD''} = -1.5000$

4) Modified Distribution Constants

a) Modified Distribution Factors

$$D^*_{jk} = \frac{K^*_{jk}}{\Sigma K^*_j} \qquad (3\text{-}97)$$

$D^*_{AB} = .35632$

$D^*_{AG} = .64368$

$D^*_{GA} = .64368$

$D^*_{GH} = .35632$

$D^*_{BA} = .30887$

$D^*_{BH} = .43240$

$D^*_{BC} = .25873$

$D^*_{HG} = .30887$

$D^*_{HB} = .43240$

$D^*_{HJ} = .25873$

$D^*_{CB} = .28687$

$D^*_{CJ} = .43885$

$D^*_{CD} = .27428$

$D^*_{JH} = .28687$

$D^*_{JC} = .43885$

$D^*_{JL} = .27428$

$D^*_{FE} = .35632$

$D^*_{FN} = .64368$

$D^*_{NF} = .64368$

$D^*_{NM} = .35632$

$D^*_{EF} = .30887$

$D^*_{EM} = .43240$

$D^*_{ED} = .25873$

$D^*_{MN} = .30887$

$D^*_{ME} = .43240$

$D^*_{ML} = .25873$

$D^*_{DE} = .28687$

$D^*_{DL} = .43885$

$D^*_{DC} = .27428$

$D^*_{LM} = .28687$

$D^*_{LD} = .43885$

$D^*_{LJ} = .27428$

b) Modified Carry-over Distribution Factors

$$C^*_{jk}\ D^*_{jk} = \frac{CK^*_{jk}}{\Sigma K^*_j} \qquad (3\text{-}93)$$

$$C^*_{ji}\ D^*_{ji} = \frac{CK^*_{ji}}{\Sigma K^*_j} \qquad (3\text{-}111)$$

$$C^*_{jp}\ D^*_{jp} = \frac{CK^*_{jp}}{\Sigma K^*_j} \qquad (3\text{-}118)$$

$$C^*_{jn''} D^*_{jn''} = \frac{CK^*_{jn''}}{\Sigma K^*_j} \qquad (3\text{-}122)$$

$$C^*_{jn}\ D^*_{jn} = \frac{CK^*_{jn}}{\Sigma K^*_j} \qquad (3\text{-}126)$$

$$C^*_{jn'}\ D^*_{jn'} = \frac{CK^*_{jn'}}{\Sigma K^*_j} \qquad (3\text{-}130)$$

$$C^*_{jm}\ D^*_{jm} = \frac{CK^*_{jm}}{\Sigma K^*_j} \qquad (3\text{-}136)$$

$C^*_{AB}\ D^*_{AB} = +.091954$

$C^*_{AH}\ D^*_{AH} = -.22989$

$C^*_{AG''} D^*_{AG''} = -.28735$

$C^*_{AG}\ D^*_{AG} = +.32184$

$C^*_{GH}\ D^*_{GH} = +.091954$

$C^*_{GB}\ D^*_{GB} = -.22989$

$C^*_{GA''} D^*_{GA''} = -.28735$

$C^*_{GA}\ D^*_{GA} = +.32184$

$C^*_{BC}\ D^*_{BC} = +.054937$

$C^*_{BJ}\ D^*_{BJ} = -.16127$

$C^*_{BH''} D^*_{BH''} = -.17368$

$C^*_{BH'} D^*_{BH'} = -.12355$

$C^*_{BG}\ D^*_{BG} = -.15443$

$C^*_{BA}\ D^*_{BA} = +.061772$

$C^*_{BH}\ D^*_{BH} = +.21620$

$C^*_{HJ}\ D^*_{HJ} = +.054937$

$C^*_{FE}\ D^*_{FE} = +.091954$

$C^*_{FM}\ D^*_{FM} = -.22989$

$C^*_{FN''} D^*_{FN''} = -.28735$

$C^*_{FN}\ D^*_{FN} = +.32184$

$C^*_{NM}\ D^*_{NM} = +.091954$

$C^*_{NE}\ D^*_{NE} = -.22989$

$C^*_{NF''} D^*_{NF''} = -.28735$

$C^*_{NF}\ D^*_{NF} = +.32184$

$C^*_{ED}\ D^*_{ED} = +.054937$

$C^*_{EL}\ D^*_{EL} = -.16127$

$C^*_{EM''} D^*_{EM''} = -.17368$

$C^*_{EM'} D^*_{EM'} = -.12355$

$C^*_{EN}\ D^*_{EN} = -.15443$

$C^*_{EF}\ D^*_{EF} = +.061772$

$C^*_{EM}\ D^*_{EM} = +.21620$

$C^*_{ML}\ D^*_{ML} = +.54937$

$C^*_{HC}\ D^*_{HC} = -.16127$

$C^*_{HB}{}''D^*_{HB}{}'' = -.17368$

$C^*_{HB}\ D^*_{HB} = +.21620$

$C^*_{HB}{}'D^*_{HB}{}' = -.12355$

$C^*_{HA}\ D^*_{HA} = -.15443$

$C^*_{HG}\ D^*_{HG} = +.061772$

$C^*_{CB}\ D^*_{CB} = +.055755$

$C^*_{CH}\ D^*_{CH} = -.16367$

$C^*_{CJ}{}'D^*_{CJ}{}' = -.151973$

$C^*_{CJ}\ D^*_{CJ} = +.21942$

$C^*_{CJ}{}''D^*_{CJ}{}'' = -.16457$

$C^*_{CL}\ D^*_{CL} = -.16457$

$C^*_{CD}\ D^*_{CD} = +.054856$

$C^*_{JH}\ D^*_{JH} = +.055755$

$C^*_{JB}\ D^*_{JB} = -.16367$

$C^*_{JC}{}'D^*_{JC}{}' = -.15197$

$C^*_{JC}\ D^*_{JC} = +.21942$

$C^*_{JC}{}''D^*_{JC}{}'' = -.16457$

$C^*_{JD}\ D^*_{JD} = -.16457$

$C^*_{JL}\ D^*_{JL} = +.054856$

$C^*_{MD}\ D^*_{MD} = -.16127$

$C^*_{ME}{}''D^*_{ME}{}'' = -.17368$

$C^*_{ME}\ D^*_{ME} = +.21620$

$C^*_{ME}{}'D^*_{ME}{}' = -.12355$

$C^*_{MF}\ D^*_{MF} = -.15443$

$C^*_{MN}\ D^*_{MN} = +.061772$

$C^*_{DE}\ D^*_{DE} = +.055755$

$C^*_{DM}\ D^*_{DM} = -.16367$

$C^*_{DL}{}'D^*_{DL}{}' = -.151973$

$C^*_{DL}\ D^*_{DL} = +.21942$

$C^*_{DL}{}''D^*_{DL}{}'' = -.16457$

$C^*_{DJ}\ D^*_{DJ} = -.16457$

$C^*_{DC}\ D^*_{DC} = +.054856$

$C^*_{LM}\ D^*_{LM} = +.055755$

$C^*_{LE}\ D^*_{LE} = -.16367$

$C^*_{LD}{}'D^*_{LD}{}' = -.15197$

$C^*_{LD}\ D^*_{LD} = +.21942$

$C^*_{LD}{}''D^*_{LD}{}'' = -.16457$

$C^*_{LC}\ D^*_{LC} = -.16457$

$C^*_{LJ}\ D^*_{LJ} = +.054856$

c) Joint Moment Carry-over Factors

$$r_{ji} = -\,C^*_{ji}\,D^*_{ji} \qquad (3\text{-}85,111)$$

$$r_{jn} = -(C^*_{jn}{}'D^*_{jn}{}' + C^*_{jn}\,D^*_{jn} + C^*_{jn}{}''D^*_{jn}{}'') \qquad (3\text{-}87,\ 99,\ 103,\ 107)$$

$r_{AB} = -.091954$	$r_{FE} = -.091954$
$r_{AH} = +.22989$	$r_{FM} = +.22989$
$r_{AG} = -.03449$	$r_{FN} = -.03449$
$r_{GH} = -.091954$	$r_{NM} = -.091954$
$r_{GB} = +.22989$	$r_{NE} = +.22989$
$r_{GA} = -.03449$	$r_{NF} = -.03449$
$r_{BC} = -.054937$	$r_{ED} = -.054937$
$r_{BJ} = +.16127$	$r_{EL} = +.16127$
$r_{BH} = +.08103$	$r_{EM} = +.08103$
$r_{BG} = +.15443$	$r_{EN} = +.15443$
$r_{BA} = -.061772$	$r_{EF} = -.061772$
$r_{HJ} = -.054937$	$r_{ML} = -.054937$
$r_{HC} = +.16127$	$r_{MD} = +.16127$
$r_{HB} = +.08103$	$r_{ME} = +.08103$
$r_{HA} = +.15443$	$r_{MF} = +.15443$
$r_{HG} = -.061772$	$r_{MN} = -.061772$
$r_{CD} = -.054856$	$r_{DC} = -.054856$
$r_{CL} = +.16457$	$r_{DJ} = +.16457$
$r_{CJ} = +.09712$	$r_{DL} = +.09712$
$r_{CH} = +.16367$	$r_{DM} = +.16367$
$r_{CB} = -.055755$	$r_{DE} = -.055755$
$r_{JL} = -.054856$	$r_{LJ} = -.054856$
$r_{JD} = +.16457$	$r_{LC} = +.16457$
$r_{JC} = +.09712$	$r_{LD} = +.09712$
$r_{JB} = +.16367$	$r_{LE} = +.16367$
$r_{JH} = -.055755$	$r_{LM} = -.055755$

5) Load Constants

a) Fixed End Moments

All Fixed End Moments are zero.

b) Static Load Moments

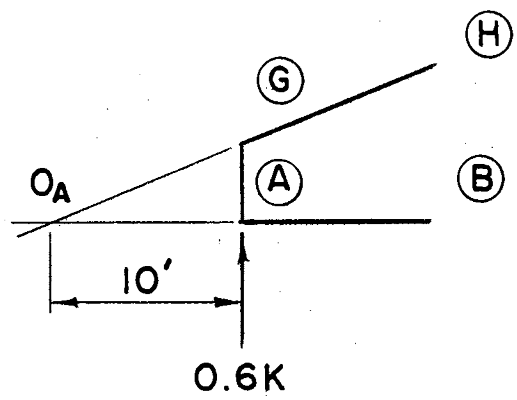


Figure 4-2. Free Body Left of $\overline{BH}$.

$SM_{OA} = SM_{OG} = -6.000$

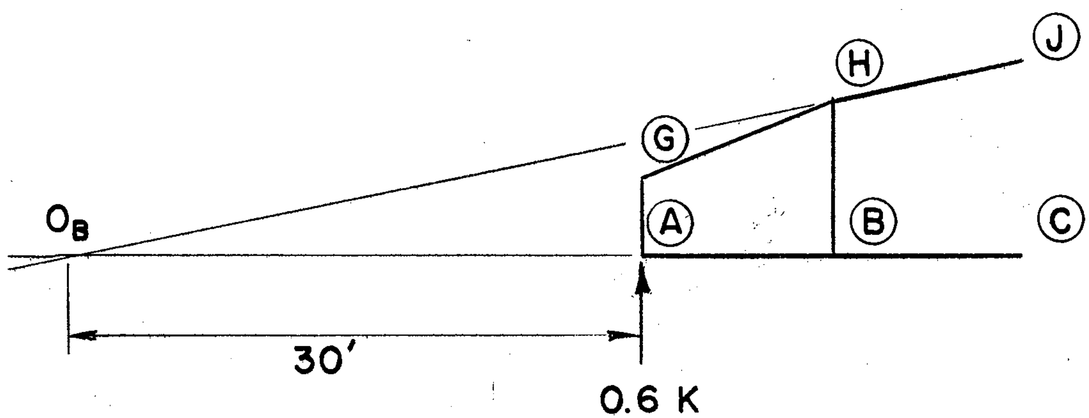


Figure 4-3. Free Body Left of $\overline{CJ}$.

$SM_{OB} = SM_{OH} = -18.000$

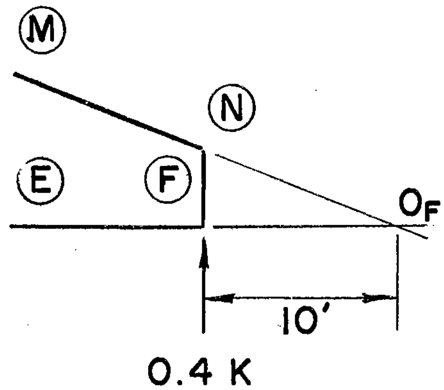


Figure 4-4. Free Body Right of $\overline{ME}$.

$SM_{OF} = SM_{ON} = +4.000$

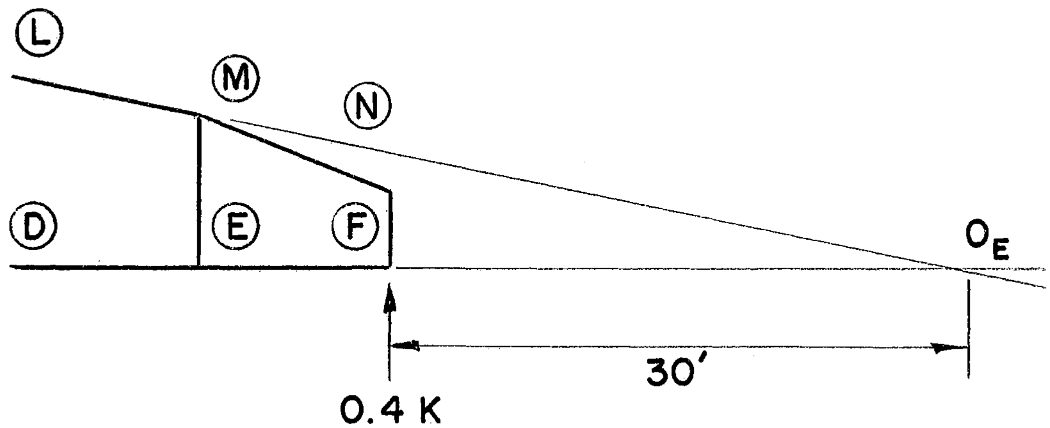


Figure 4-5. Free Body Right of $\overline{LD}$.

$SM_{OE} = SM_{OM} = +12.000$

c) Panel $\overline{CDJL}$ Shear

$$V_C = V_J = -0.4$$

d) Fixed End Shears

All fixed end shears are zero.

6) Modified Fixed End Moment Constants

a) Modified Fixed End Moments

1) Panel with sloped members

$$FM^*_{jk} = FM_{jk} - T_{jk}\left[(FM_{jk} + FM_{np})\, h_{kp} + (FM_{kj} + FM_{pn})\, h_{jn} + (FV_{kj}\, L_k + FV_{pn}\, I_p)\, h_{kp} - SM_{oj}\,(g_p + g_k)\right] \qquad (3\text{-}29)$$

$$FM^*_{jn} = FM_{jn} \qquad (3\text{-}24)$$

$$FM^*_{ji} = FM_{ji} - T_{ji}\left[(FM_{ij} + FM_{mn})\, h_{jn} + (FM_{ji} + FM_{nm})\, h_{im} + (FV_{ji}\, I_j + FV_{nm}\, I_n)\, h_{jn} - SM_{oi}\,(g_n + g_j)\right] \qquad (3\text{-}40)$$

$FM^*_{AB} = -1.0714$

$FM^*_{AG} = 0$

$FM^*_{BA} = -.85714$

$FM^*_{BH} = 0$

$FM^*_{BC} = -1.0328$

$FM^*_{CB} = -.95902$

$FM^*_{DE} = +.63934$

$FM^*_{GH} = -1.0714$

$FM^*_{GA} = 0$

$FM^*_{HG} = -.85714$

$FM^*_{HB} = 0$

$FM^*_{HJ} = -1.0328$

$FM^*_{JH} = -.95902$

$FM^*_{LM} = +.63934$

$FM^*_{ED} = + .68852$ $FM^*_{ML} = + .68852$

$FM^*_{EM} = 0$ $FM^*_{ME} = 0$

$FM^*_{EF} = + .57143$ $FM^*_{MN} = + .57143$

$FM^*_{FE} = + .71429$ $FM^*_{NM} = + .71429$

$FM^*_{FN} = 0$ $FM^*_{NF} = 0$

2) Panels with Parallel Members

{Equations from Samuel's Thesis (29)}

$$FM^*_{jk} = FM_{jk} - S_{jk}\left[\frac{FM_{jk}+FM_{kj}+FM_{pn}+FM_{np}+(BV_{pn}+BV_{kj}+V_k)L_k}{S_{jk}+S_{kj}+S_{np}+S_{pn}}\right]$$

$$FM^*_{jn} = 0$$

$$FM^*_{ji} = FM_{ji} - S_{ji}\left[\frac{FM_{ij}+FM_{ji}+FM_{mn}+FM_{nm}+(BV_{nm}+BV_{ji}+V_j)L_j}{S_{ji}+S_{ij}+S_{nm}+S_{mn}}\right]$$

$FM^*_{CJ} = 0$ $FM^*_{JC} = 0$

$FM^*_{CD} = +1.000$ $FM^*_{JL} = +1.000$

$FM^*_{DC} = +1.000$ $FM^*_{LJ} = +1.000$

$FM^*_{DL} = 0$ $FM^*_{LD} = 0$

b) Starting Moments

$$m_j = - \Sigma FM^*_j \qquad (3\text{-}90)$$

$m_A = +1.0714$ $m_G = +1.0714$

$m_B = +1.8899$ $m_H = +1.8899$

$m_C = - .04098$ $m_J = - .04098$

$m_D = -1.6393$ $m_L = -1.6393$

$m_E = -1.2600$ $m_M = -1.2600$

$m_F = - .71429$ $m_N = - .71429$

TABLE 4-1

7) CARRY-OVER PROCEDURE

M_A	M_B	M_C	M_D	M_E	M_F	M_G	M_H	M_J	M_L	M_M	M_N
$.\bar{0}91954$ → B .22989 → H $.\bar{0}3449$ → G	$.\bar{0}54937$ → C .16127 → J .08103 → H .15443 → G A ← $.\bar{0}61772$	$.\bar{0}54856$ → D .16457 → L .09712 → J .16367 → H B ← $.\bar{0}55755$	$.\bar{0}55755$ → E .16367 → M .09712 → L .16457 → J C ← $.\bar{0}54856$	$.\bar{0}61772$ → F .15443 → N .08103 → M .16127 → L D ← $.\bar{0}54937$	$.\bar{0}3449$ → N .22989 → M E ← $.\bar{0}91954$	$.\bar{0}91954$ → H B ← .22989 A ← $.\bar{0}3449$	$.\bar{0}54937$ → J C ← .16127 B ← .08103 A ← .15443 G ← $.\bar{0}61772$	$.\bar{0}54856$ → L D ← .16457 C ← .09712 B ← .16367 H ← $.\bar{0}55755$	$.\bar{0}55755$ → M E ← .16367 D ← .09712 C ← .16457 J ← $.\bar{0}54856$	$.\bar{0}61772$ → N F ← .15443 E ← .08103 D ← .16127 L ← $.\bar{0}54937$	F ← $.\bar{0}3449$ E ← .22989 M ← $.\bar{0}91954$
1.0714	1.8899	$.\bar{0}4098$	$\bar{1}.6393$	$\bar{1}.2600$	$.\bar{7}1429$	1.0714	1.8899	$.\bar{0}4098$	$\bar{1}.6393$	$\bar{1}.2600$	$.\bar{7}1429$
$.\bar{0}3695$ $.\bar{1}1674$ .29186	$.\bar{0}9852$ .24630 .15314 .00228 $.\bar{0}0671$	$.\bar{1}0383$ .30478 $.\bar{0}0398$ .08993 $.\bar{2}6978$	$.\bar{0}0225$ .00674 $.\bar{1}5921$ .06922 $.\bar{2}0320$	.09140 $.\bar{2}6830$ $.\bar{1}0210$ .06568 $.\bar{1}6421$	.07783 $.\bar{1}9458$ .02464	$.\bar{0}3695$ .29186 $.\bar{1}1674$	.24630 .09852 .15314 $.\bar{0}0671$ .00228	.30478 $.\bar{1}0383$ $.\bar{0}0398$ $.\bar{2}6978$ .08993	$.\bar{0}0674$ .00225 $.\bar{1}5921$ $.\bar{2}0320$ .06922	$.\bar{2}6830$ .09140 $.\bar{1}0210$ $.\bar{1}6421$ .06568	$.\bar{1}9458$ .07783 .02464
.1382	.2965	.0171	$.\bar{2}977$	$.\bar{3}775$	$.\bar{0}921$	.1382	.2965	.0171	$.\bar{2}977$	$.\bar{3}775$	$.\bar{0}921$
$.\bar{0}1832$ $.\bar{0}0477$ .04579	$.\bar{0}1271$ $.\bar{0}0095$ .03177 .02403 .00280	$.\bar{0}1629$ .01633 .04782 .00166 $.\bar{0}4899$	$.\bar{0}0094$ .02074 .00281 $.\bar{0}2891$ $.\bar{0}6088$	.01660 .00847 $.\bar{0}4872$ $.\bar{0}3059$ $.\bar{0}2117$	.02332 $.\bar{0}5830$ .00318	$.\bar{0}0477$ .04579 $.\bar{0}1832$	.03177 .02403 .00280 $.\bar{0}1271$ $.\bar{0}0095$	.04782 .00166 $.\bar{0}4899$ $.\bar{0}1629$ .01633	.00281 $.\bar{0}2891$ $.\bar{0}6088$ $.\bar{0}0094$ .02074	$.\bar{0}4872$ $.\bar{0}3059$ $.\bar{0}2117$ .01660 .00847	$.\bar{0}5830$ .00318 .02337
.0227	.0447	.0005	$.\bar{0}672$	$.\bar{0}754$	$.\bar{0}318$	.0227	.0449	.0005	$.\bar{0}672$	$.\bar{0}752$	$.\bar{0}318$
$.\bar{0}0277$ $.\bar{0}0078$ .00693	$.\bar{0}0209$ $.\bar{0}0003$ .00522 .00364 .00008	$.\bar{0}0247$ .00369 .00724 .00005 $.\bar{0}1106$	$.\bar{0}0003$ .00414 .00008 $.\bar{0}0653$ $.\bar{0}1216$	.00375 .00292 $.\bar{0}1100$ $.\bar{0}0611$ $.\bar{0}0731$	.00466 $.\bar{0}1164$ .00110	$.\bar{0}0078$ .00693 $.\bar{0}0277$	.00522 .00364 .00008 $.\bar{0}0209$ $.\bar{0}0003$	.00724 .00005 $.\bar{0}1106$ $.\bar{0}0247$ .00369	.00008 $.\bar{0}0653$ $.\bar{0}1216$ $.\bar{0}0003$ .00414	$.\bar{0}1100$ $.\bar{0}0611$ $.\bar{0}0731$ .00375 .00292	$.\bar{0}1164$ .00110 .00466
.0034	.0068	$.\bar{0}026$	$.\bar{0}145$	$.\bar{0}178$	.0059	.0034	.0068	$.\bar{0}026$	$.\bar{0}145$	$.\bar{0}178$	$.\bar{0}059$

TABLE 4-1 (Continued)

.0̄0042 .0̄0012 .00105	.0̄0031 .00014 .00078 .00055 .0̄0042	.0̄0037 .00080 .00110 .0̄0025 .0̄0239	.00014 .00098 .0̄0043 .0̄0141 .0̄0287	.00081 .00054 .0̄0237 .0̄0144 .0̄0136	.00110 .0̄0275 .00020	.0̄0012 .00105 .0̄0042	.00078 .00055 .0̄0043 .0̄0031 .00014	.00110 .0̄0025 .0̄0239 .0̄0037 .00080	.0̄0043 .0̄0141 .0̄0287 .00014 .00098	.0̄0237 .0̄0144 .0̄0136 .00081 .00054	.0̄0275 .00020 .00110
.0005	.0007	.0̄011	.0̄036	.0̄038	.0̄015	.0005	.0007	.0̄011	.0̄036	.0̄038	.0̄015
.0̄0004 .0̄0002 .00011	.0̄0005 .00006 .00011 .00006 .0̄0018	.0̄0004 .00020 .00011 .0̄0011 .0̄0059	.00006 .00021 .0̄0018 .0̄0035 .0̄0061	.00020 .00014 .0̄0059 .0̄0031 .0̄0034	.00023 .0̄0059 .00005	.0̄0002 .00011 .0̄0004	.00011 .00006 .0̄0018 .0̄0005 .00006	.00011 .0̄0011 .0̄0059 .0̄0004 .00020	.0̄0018 .0̄0035 .0̄0061 .00006 .00021	.0̄0060 .0̄0031 .0̄0034 .00020 .00014	.0̄0059 .00005 .00023
.0001	0	.0̄004	.0̄009	.0̄009	.0̄003	.0001	0	.0̄004	.0̄009	.0̄009	.0̄003
	.0̄0001 .00002 .00002 .0̄0007	.00005 .0̄0004 .0̄0015	.00002 .00005 .0̄0007 .0̄0009 .0̄0015	.00005 .00003 .0̄0015 .0̄0007 .0̄0007	.00006 .0̄0014 .00001		.00002 .0̄0007 .0̄0001 .00002	.0̄0004 .0̄0015 .00005	.0̄0007 .0̄0009 .0̄0015 .00002 .00005	.0̄0015 .0̄0007 .0̄0007 .00005 .00003	.0̄0014 .00001 .00006
0	0	.0̄001	.0̄002	.0̄002	.0̄001	0	0	.0̄001	.0̄002	.0̄002	.0̄001
	.00001 .0̄0002	.00001 .0̄0001 .0̄0003	.00001 .00001 .0̄0002 .0̄0002 .0̄0003	.00001 .00001 .0̄0003 .0̄0002 .0̄0002	.00001 .0̄0003		.0̄0002 .00001	.0̄0001 .0̄0003 .00001	.0̄0002 .0̄0002 .0̄0003 .00001 .00001	.0̄0003 .0̄0002 .0̄0002 .00001 .00001	.0̄0003 .00001
0	0	0	.0̄001	.0̄001	0	0	0	0	.0̄001	.0̄001	0
1.2363	2.2388	.0̄276	2̄.0235	1̄.7357	.8̄460	1.2363	2.2388	.0̄276	2̄.0235	1̄.7357	.8̄460

8) Numberical Check

$$JM_j = r_{kj}\ JM_k + m_j + r_{ij}\ JM_i + r_{pj}\ JM_p + r_{nj}\ JM_n + r_{mj}\ JM_m \qquad (3\text{-}84)$$

$$JM_A = (-.061772)(2.2388) + (-.03449)(1.2363) + (.15443)(2.2388) + 1.0714 = +1.2362$$

$$JM_B = (-.091954)(1.2363) + (-.055755)(-.0276) + (.22989)(1.2363) + (.08103)(2.2388) + (.16367)(-.0276) + 1.8899 = 2.2389$$

$$JM_C = (-.054937)(2.2388) + (.054856)(-2.0235) + (.16127)(2.2388) + (.09712)(-.0276) + (.16457)(-2.0235) - .0410 = -.0276$$

$$JM_D = (-.054856)(-.0276) + (-.054937)(-1.7357) + (.16457)(-.0276) + (.09712)(-2.0235) + (.16127)(-1.7357) - 1.6393 = -2.0234$$

$$JM_E = (-.055755)(-2.0235) + (-.091954)(-.8460) + (.16367)(-2.0235) + (.08103)(-1.7357) + (.22989)(-.8460) - 1.2600 = -1.7357$$

$$JM_F = (-.061772)(-1.7357) + (.15443)(-1.7357)$$

$$+ (-.03449)(-.8460) - .7143$$

$$= - .8459$$

9) Final Moments

$$M_{jk} = D^*_{jk}\ JM_j + C^*_{kj}\ D^*_{kj}\ JM_k + C^*_{nj}{}''\ D^*_{nj}{}''\ JM_n$$

$$+ C^*_{pj}\ D^*_{pj}\ JM_p + FM^*_{jk} \qquad (3\text{-}96)$$

$$M_{jn} = D^*_{jn}\ JM_j + C^*_{nj}\ JM_n + FM^*_{jn} \qquad (3\text{-}101)$$

$$M_{ji} = D^*_{ji}\ JM_j + C^*_{ij}\ D^*_{ij}\ JM_i + C^*_{nj}{}'\ D^*_{nj}{}'\ JM_n$$

$$+ C^*_{mj}\ D^*_{mj}\ JM_m + FM^*_{ji} \qquad (3\text{-}104)$$

$M_{AB} = -1.194$	$M_{GH} = -1.194$
$M_{AG} = +1.194$	$M_{GA} = +1.194$
$M_{BA} = -0.613$	$M_{HG} = -0.613$
$M_{BH} = +1.452$	$M_{HB} = +1.452$
$M_{BC} = -0.839$	$M_{HJ} = -0.839$
$M_{CB} = -1.201$	$M_{JH} = -1.201$
$M_{CJ} = -0.018$	$M_{JC} = -0.018$
$M_{CD} = +1.219$	$M_{JL} = +1.219$
$M_{DC} = +0.781$	$M_{LJ} = +0.781$
$M_{DL} = -1.332$	$M_{LD} = -1.332$
$M_{DE} = +0.551$	$M_{LM} = +0.551$
$M_{ED} = +0.759$	$M_{ML} = +0.759$

$M_{EM} = -1.126$ $M_{ME} = -1.126$

$M_{EF} = +0.367$ $M_{MN} = +0.367$

$M_{FE} = +0.817$ $M_{NM} = +0.817$

$M_{FN} = -0.817$ $M_{NF} = -0.817$

Example No. 2

The structure shown in Figure 4-6 is one truss in a dam. Spacing of the trusses is such that each resists a hydrostatic load of indicated magnitude. The moments of interis for all members are shown beside each respective member. The equations used in determining the constants used in this example are referenced to the corresponding equation in the derivation. The results of this example can be compared to Example No. 2 in J. J. Tuma and J T. Oden, "String Polygon Analysis of Frames with Straight Members," Proceedings, ASCE, Vol. 87, No. ST7, October, 1961, pp. 63-96

1) Elastic Constants

a) Stiffness Factors

$$K_{jk} = \frac{4EI_{jk}}{L_{jk}}$$

$K_{12} = K_{21} = .16000$ EI

$K_{23} = K_{32} = .14667$ EI

$K_{34} = K_{43} = .13333$ EI

$K_{27} = K_{72} = .13333$ EI

$K_{36} = K_{63} = .20000 \ EI$

$K_{45} = K_{54} = .40000 \ EI$

$K_{56} = K_{65} = .12650 \ EI$

$K_{67} = K_{76} = .13915 \ EI$

$K_{78} = K_{87} = .13311 \ EI$

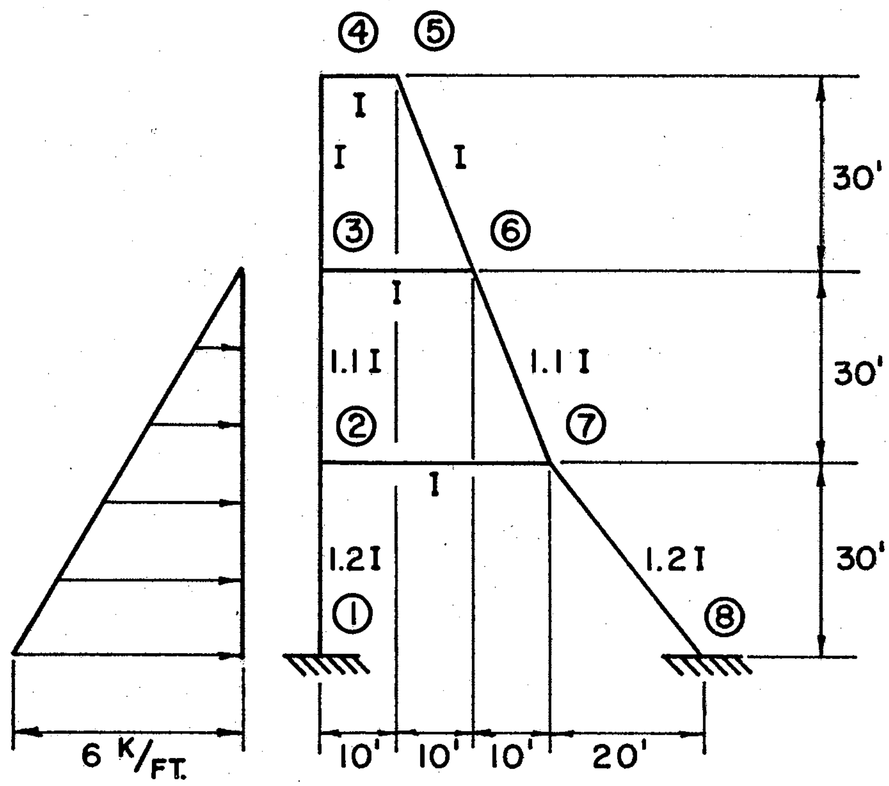


Figure 4-6. One Vierendeel Truss (Typical for Dam).

b) Carry-over Values

All carry-over values are 0.5 because all members are prismatic.

c) Sidesway Stiffness Factors

$$S_{jk} = K_{jk}\ (1 + C_{jk})$$

$S_{12} = S_{21} = .24000$ EI

$S_{23} = S_{32} = .22000$ EI

$S_{34} = S_{43} = .20000$ EI

$S_{27} = S_{72} = .20000$ EI

$S_{36} = S_{63} = .30000$ EI

$S_{45} = S_{54} = .60000$ EI

$S_{56} = S_{65} = .18975$ EI

$S_{67} = S_{76} = .20873$ EI

$S_{78} = S_{87} = .19967$ EI

2) Equivalents

a) Q Values

$$Q_j = (S_{ji} + S_{nm})\ h_{im} + (S_{ij} + S_{mn})\ h_{jn} + \frac{g_n + g_j}{h_{im}} \left[(CK_{ij} + CK_{mn})\ h_{im} + (K_{ij} + K_{mn})\ h_{jn} \right] \qquad (3\text{-}16)$$

$Q_3 = 18.188$ EI

$Q_2 = 27.153$ EI

$Q_6 = 18.188$ EI

$Q_7 = 27.153$ EI

$Q_1 = 47.875$ EI $\qquad$ $Q_8 = 47.875$ EI

b) T Values

$$T_{jk} = \frac{1}{Q_k}\left[S_{jk} + K_{jk}\frac{g_p + g_k}{h_{jn}}\right] \qquad (3\text{-}30)$$

$$T_{ji} = \frac{1}{Q_j}\left[S_{ji} + CK_{ij}\frac{g_n + g_j}{h_{im}}\right] \qquad (3\text{-}41)$$

$T_{12} = .0061270$ $\qquad$ $T_{21} = .0072411$

$T_{23} = .0094527$ $\qquad$ $T_{32} = .010803$

$T_{34} = .014662$ $\qquad$ $T_{43} = .018327$

$T_{56} = .017388$ $\qquad$ $T_{65} = .013910$

$T_{67} = .010250$ $\qquad$ $T_{76} = .0089684$

$T_{78} = .0060242$ $\qquad$ $T_{87} = .0050974$

3) Modified Stiffness Constants

a) Modified Stiffness Factors

$$K^*_{jk} = K_{jk}\left[1 - T_{jk}\,(h_{kp} + C_{jk}\,h_{jn})\right] \qquad (3\text{-}25)$$

$$K^*_{jn} = K_{jn} \qquad (3\text{-}32)$$

$$K^*_{ji} = K_{ji}\left[1 - T_{ji}\,(h_{im} + C_{ji}\,h_{jn})\right] \qquad (3\text{-}36)$$

$K^*_{21} = .084693$ EI $\qquad$ $K^*_{54} = .40000$ EI

$K^*_{27} = .13333$ EI $\qquad$ $K^*_{56} = .071510$ EI

$K^*_{23} = .098146$ EI $\qquad$ $K^*_{65} = .091308$ EI

$K^*_{32} = .083291$ EI $\qquad$ $K^*_{63} = .20000$ EI

$K^*_{36} = .20000$ EI $\qquad$ $K^*_{67} = .082099$ EI

$K^*_{34} = .094232$ EI $\qquad$ $K^*_{76} = .095472$ EI

K^*_{43} = .072242 EI

K^*_{45} = .40000 EI

K^*_{72} = .13333 EI

K^*_{78} = .080988 EI

b) Summation of K*'s

ΣK^*_2 = .31617 EI

ΣK^*_3 = .37752 EI

ΣK^*_4 = .47224 EI

ΣK^*_5 = .47151 EI

ΣK^*_6 = .37341 EI

ΣK^*_7 = .30979 EI

c) Modified Carry-over Stiffness Factors

$$CK^*_{kj} = K_{kj} \left[C_{kj} - T_{jk} (h_{jn} + C_{kj} h_{kp}) \right] \qquad (3\text{-}26)$$

$$CK^*_{nj} = CK_{nj} \qquad (3\text{-}33)$$

$$CK^*_{ij} = K_{ij} \left[C_{ij} - T_{ji} (h_{jn} + C_{ij} h_{im}) \right] \qquad (3\text{-}37)$$

$$CK^*_{nj}'' = - K_{np} (h_{kp} + C_{np} h_{jn}) T_{jk} \qquad (3\text{-}27)$$

$$CK^*_{pj} = - K_{pn} (h_{jn} + C_{pn} h_{jn}) T_{jk} \qquad (3\text{-}28)$$

$$CK^*_{nj}' = - K_{nm} (h_{im} + C_{nm} h_{jn}) T_{ji} \qquad (3\text{-}38)$$

$$CK^*_{mj} = - K_{mn} (h_{jn} + C_{mn} h_{im}) T_{ji} \qquad (3\text{-}39)$$

CK^*_{21} = .016278 EI

CK^*_{23} = .017876 EI

CK^*_{28} = -.053013 EI

$CK^*_{27''}$ = -.062651 EI

$CK^*_{27'}$ = -.046038 EI

CK^*_{26} = -.052618 EI

CK^*_{27} = .066667 EI

CK^*_{56} = .019260 EI

CK^*_{53} = -.046369 EI

$CK^*_{54''}$ = -.057959 EI

CK^*_{54} = .20000 EI

CK^*_{67} = .019657 EI

CK^*_{65} = .019358 EI

CK^*_{62} = -.052613 EI

CK^*_{32} = .017876 EI	$CK^*_{63''}$ = -.060130 EI
CK^*_{34} = .017796 EI	$CK^*_{63'}$ = -.037095 EI
CK^*_{37} = -.052615 EI	CK^*_{64} = -.046367 EI
$CK^*_{36''}$ = -.060135 EI	CK^*_{63} = .10000 EI
$CK^*_{36'}$ = -.037092 EI	CK^*_{78} = .022452 EI
CK^*_{35} = -.046367 EI	CK^*_{76} = .019655 EI
CK^*_{36} = .10000 EI	CK^*_{71} = -.053012 EI
CK^*_{43} = .017795 EI	$CK^*_{72''}$ = -.062651 EI
CK^*_{46} = -.046366 EI	$CK^*_{72'}$ = -.046037 EI
$CK^*_{45''}$ = -.057959 EI	CK^*_{73} = -.052613 EI
CK^*_{45} = .20000 EI	CK^*_{72} = -.06667 EI

4) Modified Distribution Constants

a) Modified Distribution Factors

$$D^*_{jk} = \frac{K^*_{jk}}{\Sigma K^*_j} \qquad (3\text{-}97)$$

D^*_{21} = .26788	D^*_{54} = .84834
D^*_{27} = .42170	D^*_{56} = .15166
D^*_{23} = .31042	D^*_{65} = .24453
D^*_{32} = .22063	D^*_{63} = .53561
D^*_{36} = .52977	D^*_{67} = .21986
D^*_{34} = .24961	D^*_{76} = .30818
D^*_{43} = .15298	D^*_{72} = .43038
D^*_{45} = .84703	D^*_{78} = .26143

b) Modified Carry-over Distribution Factors

$$C^*_{jk}\ D^*_{jk} = \frac{CK^*_{jk}}{\Sigma K^*_j} \qquad (3\text{-}93)$$

$$C^*_{ji}\ D^*_{ji} = \frac{CK^*_{ji}}{\Sigma K^*_j} \qquad (3\text{-}111)$$

$$C^*_{jp}\ D^*_{jp} = \frac{CK^*_{jn}}{\Sigma K^*_j} \qquad (3\text{-}118)$$

$$C^*_{jn''}\ D^*_{jn''} = \frac{CK^*_{jn''}}{\Sigma K^*_j} \qquad (3\text{-}122)$$

$$C^*_{jn}\ D^*_{jn} = \frac{CK^*_{jn}}{\Sigma K^*_j} \qquad (3\text{-}126)$$

$$C^*_{jn'}\ D^*_{jn'} = \frac{CK^*_{jn'}}{\Sigma K^*_j} \qquad (3\text{-}130)$$

$$C^*_{jm}\ D^*_{jm} = \frac{CK^*_{jm}}{\Sigma K^*_j} \qquad (3\text{-}136)$$

$C^*_{21}\ D^*_{21} = .051485$

$C^*_{28}\ D^*_{28} = -.16767$

$C^*_{27''}\ D^*_{27''} = -.19816$

$C^*_{27}\ D^*_{27} = .21086$

$C^*_{27'}\ D^*_{27'} = -.14561$

$C^*_{26}\ D^*_{26} = -.16642$

$C^*_{23}\ D^*_{23} = .056539$

$C^*_{32}\ D^*_{32} = .047351$

$C^*_{37}\ D^*_{37} = -.13937$

$C^*_{36''}\ D^*_{36''} = -.15929$

$C^*_{36}\ D^*_{36} = .26489$

$C^*_{36'}\ D^*_{36'} = -.098251$

$C^*_{35}\ D^*_{35} = -.12282$

$C^*_{34}\ D^*_{34} = .047139$

$C^*_{43}\ D^*_{43} = .037682$

$C^*_{46}\ D^*_{46} = -.098183$

$C^*_{45''}\ D^*_{45''} = -.12273$

$C^*_{45}\ D^*_{45} = .42351$

$C^*_{56}\ D^*_{56} = .040847$

$C^*_{53}\ D^*_{53} = -.098341$

$C^*_{54''}\ D^*_{54''} = -.12292$

$C^*_{54}\ D^*_{54} = .42417$

$C^*_{67}\ D^*_{67} = .052689$

$C^*_{62}\ D^*_{62} = -.14103$

$C^*_{63''}\ D^*_{63''} = -.16118$

$C^*_{63}\ D^*_{63} = .26805$

$C^*_{63'}\ D^*_{63'} = -.099431$

$C^*_{64}\ D^*_{64} = -.12417$

$C^*_{65}\ D^*_{65} = .051888$

$C^*_{78}\ D^*_{78} = .072474$

$C^*_{71}\ D^*_{71} = -.17121$

$C^*_{72''}\ D^*_{72''} = -.20235$

$C^*_{72}\ D^*_{72} = .21520$

$C^*_{72'}\ D^*_{72'} = -.14861$

$C^*_{73}\ D^*_{73} = -.16983$

$C^*_{76}\ D^*_{76} = .063446$

c) Joint Moment Carry-over Factors

$$r_{ji} = -C^{*}_{ji}\,D^{*}_{ji}$$ (3-85 & 111)

$$r_{jn} = -(C^{*\prime}_{jn}D^{*\prime}_{jn} + C^{*}_{jn}\,D^{*}_{jn} + C^{*\prime\prime}_{jn}D^{*\prime\prime}_{jn})$$ (3-87, 99, 103, & 107)

r_{23} = -.056539

r_{26} = +.16642

r_{27} = +.13291

r_{32} = -.047351

r_{37} = +.13937

r_{36} = -.007459

r_{35} = +.12282

r_{34} = -.047139

r_{43} = -.037682

r_{46} = +.098183

r_{45} = -.30078

r_{56} = -.040847

r_{53} = +.098341

r_{54} = -.30125

r_{67} = -.052689

r_{62} = +.14103

r_{63} = -.007439

r_{64} = +.12417

r_{65} = -.051888

r_{72} = +.13576

r_{73} = +.16983

r_{76} = -.063446

5) Load Constants

a) Fixed End Moments

$$FM_{12} = -\frac{\omega l^2}{12} - \frac{\omega l^2}{20}$$

$$= -360$$

$$FM_{21} = +\frac{\omega l^2}{12} + \frac{\omega l^2}{30}$$

$$= +315$$

$$FM_{23} = -\frac{\omega l^2}{20}$$

$$= -135$$

$$FM_{32} = +\frac{\omega l^2}{30}$$

$$= +90$$

b) Static Loads Moments

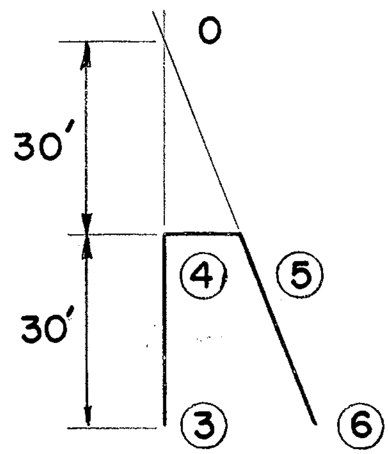


Figure 4-7. Free Body Above $\overline{36}$.

$$SM_{04} = SM_{06} = 0$$

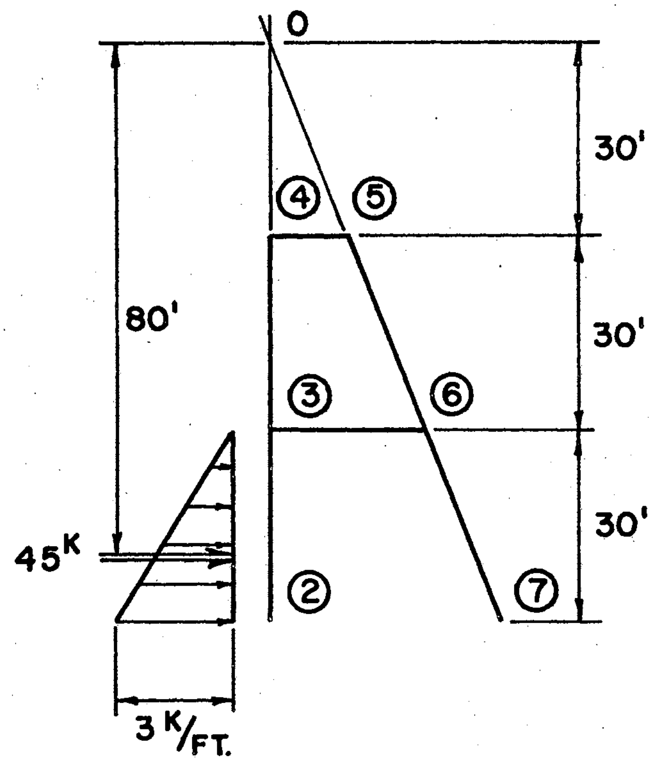


Figure 4-8. Free Body Above $\overline{27}$.

$SM_{03} = SM_{06} = -3600$

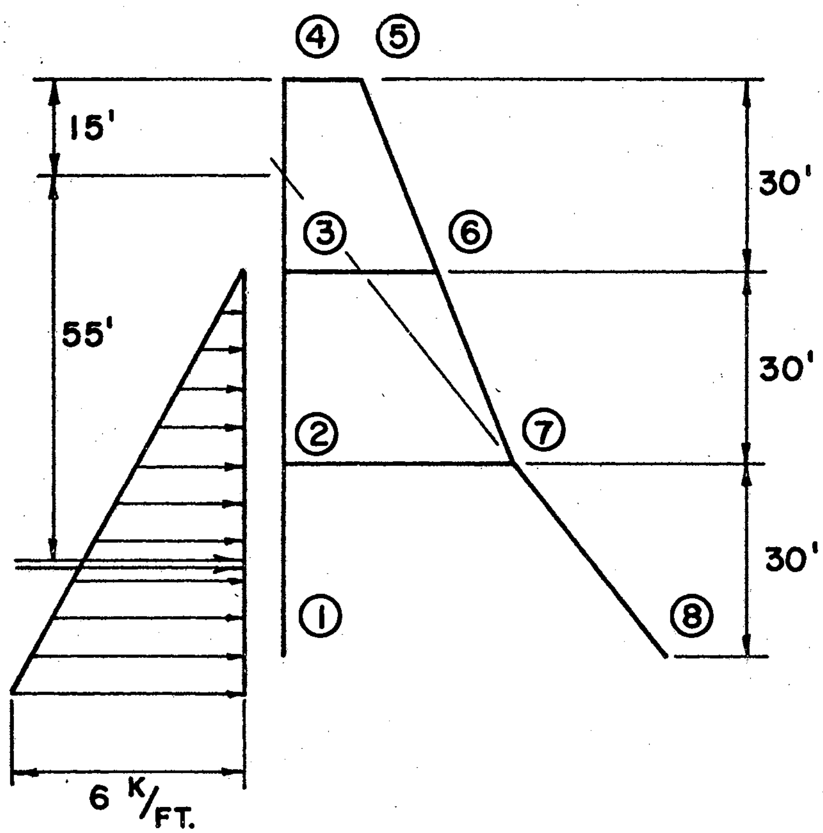


Figure 4-9. Free Body Above $\overline{18}$.

$SM_{02} = SM_{07} = -9900$

c) Fixed End Shears

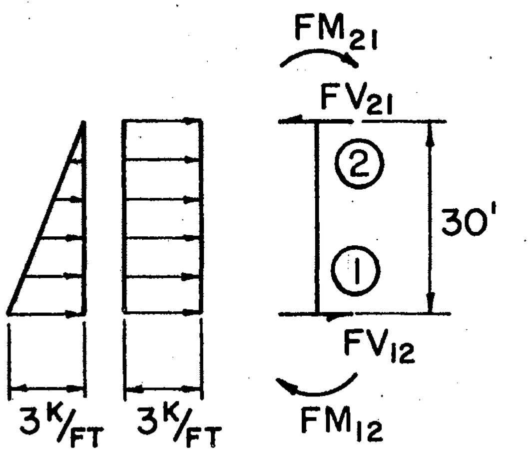


Figure 4-10. Free Body of Member $\overline{12}$.

$FV_{12} = -(1/2)\omega l - (1/3)\omega l$

$= -75$

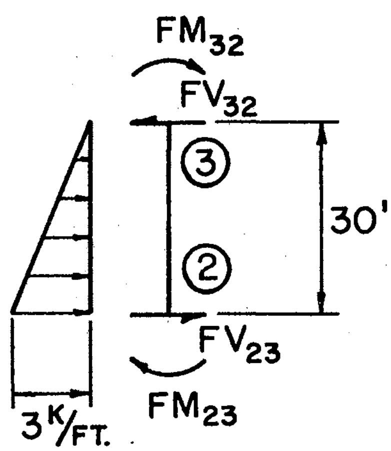


Figure 4-11. Free Body of Member $\overline{23}$.

$FV_{23} = -(1/2)\omega l$

$= -30$

6) Modified Fixed End Moment Constants

a) Modified Fixed End Moments

$$FM^*_{jk} = FM_{jk} - T_{jk}\Big[(FM_{jk} + FM_{np})\,h_{kp} + (FM_{kj} + FM_{pn})\,h_{jn} + (FV_{kj}\,L_k + FV_{pn}\,I_p)\,h_{kp} - SM_{oj}(g_p + g_k)\Big] \tag{3-29}$$

$$FM^*_{jn} = FM_{jn} \tag{3-34}$$

$$FM^*_{ji} = FM_{ji} - T_{ji}\Big[(FM_{ij} + FM_{mn})\,h_{jn} + (FM_{ji} + FM_{nm})\,h_{im} + (FV_{ji}\,I_j + FV_{nm}\,I_n)h_{jn} - SM_{oi}(g_n + g_j)\Big] \tag{3-40}$$

$FM^*_{12} = -914.19$

$FM^*_{21} = -339.96$

$FM^*_{27} = 0$

$FM^*_{23} = -220.07$

$FM^*_{32} = -\ 7.2270$

$FM^*_{34} = 0$

$FM^*_{43} = FM^*_{36} = 0$

$FM^*_{45} = FM^*_{72} = 0$

$FM^*_{54} = 0$

$FM^*_{56} = 0$

$FM^*_{65} = 0$

$FM^*_{63} = 0$

$FM^*_{67} = -\ 92.250$

$FM^*_{76} = -\ 80.715$

$FM^*_{78} = -544.89$

$FM^*_{87} = -461.06$

b) Starting Moments

$$m_j = -\ \Sigma FM^*_j \tag{3-90}$$

$m_2 = +560.03$

$m_3 = +\ 7.2270$

$m_4 = 0$

$m_5 = 0$

$m_6 = +\ 92.250$

$m_7 = +625.61$

TABLE 4-2

7) CARRY-OVER PROCEDURE

2	3	4	5	6	7
$.\bar{0}56539$ → 3 .16642 → 6 .13291 → 7	2 ← $.\bar{0}47351$ $.\bar{0}47139$ → 4 .12282 → 5 $.\bar{0}07459$ → 6 .13937 → 7	3 ← $.\bar{0}37682$ $.\bar{3}0078$ → 5 .098183 → 6	3 ← .098341 4 ← $.\bar{3}0125$ $.\bar{0}40847$ → 6	2 ← .14103 3 ← $.\bar{0}07439$ 4 ← .12417 5 ← .051888 $.\bar{0}52689$ → 7	2 ← .13576 3 ← .16983 6 ← $.\bar{0}63446$
+ 560.03	+ 7.23	0	0	+ 92.25	+ 625.61
	− 31.66			+ 93.20	+ 74.43
	− 24.43				
+ 1.16		+ 1.15	− 3.00	+ .18	− 3.40
		+ 1.15			
	− .04		− .35	+ .11	
			− 3.35		
	− .33	+ 1.01		+ .14	
				+ 185.88	
+ 26.21	− 1.38	+ 23.08	− 9.64		− 9.79
					+ 686.85
+ 93.25	+ 116.65			− 43.58	
+ 120.62					
	− 6.82			+ 20.07	+ 16.03
	+ 108.08				
− 5.12		− 5.09	+ 13.27	− .81	+ 15.06
		+ 19.00			
	− .72		− 5.71	+ 1.87	
			− 2.09		
	− .11	+ .33		+ .04	
				− 22.41	
− 3.16	+ .17	− 2.78	+ 1.16		+ 1.18
					+ 32.27
+ 4.38	+ 5.48			− 2.05	
− 3.90					
	+ .22			− .65	− .52
	+ 5.04				
− .24		− .24	+ .62	− .04	+ .70
		− 2.69			
	+ .10		+ .81	− .27	
			+ 2.59		
	+ .25	− .78		− .11	
				− 3.12	
− .44	+ .02	− .38	+ .16		+ .16
					+ .34
+ .05	+ .06			− .02	
− .63					
	+ .04			− .10	− .08
	+ .47				
− .02		− .02	+ .06	0	+ .07
		− 1.19			
	+ .04		+ .36	− .12	
			+ .58		
	+ .06	− .17		− .02	
				− .26	
− .04	0	− .03	+ .01		+ .01
					0
− .06					
	0			− .01	− .01
	+ .10				
0		0	+ .01	0	+ .01
		− .20			
	+ .01		+ .06	− .02	
			+ .08		
	+ .01	− .02		0	
				− .03	
	0	0	0		0
					0
0					
	+ .02				
0		0	0	0	0
		− .02			
	0		+ .01	0	
			+ .01		
	0	0		0	
				0	
					0
0					
	0				
		0			
+ 676.06	+ 89.28	+ 16.05	− 2.18	− 160.06	+ 719.46

8) Numberical Check

$$JM_j = r_{kj}\ JM_k + m_j + r_{ij}\ JM_i + r_{pj}\ JM_p$$

$$+ r_{nj}\ JM_n + r_{mj}\ JM_m \qquad (3\text{-}84)$$

$$JM_2 = 560.03 + (-.04735)(+89.28) + (+.13576)(+719.46)$$

$$+ (+.14103)(+160.06)$$

$$= + 676.05$$

$$JM_3 = (-.056539)(+676.06) + 7.23 + (-.037682)(+6.05)$$

$$+ (+.16983)(+719.46) + (-.007439)(+160.06)$$

$$+ (+.098341)(-2.18)$$

$$= + 89.18$$

$$JM_4 = (-.047139)(+89.28) + (+.12717)(+160.06)$$

$$+ (-.30125)(-2.18)$$

$$= + 16.32$$

$$JM_5 = (-.051888)(+160.06) + (.12282)(+89.28)$$

$$+ (-.30078)(+16.05)$$

$$= - 2.18$$

$$JM_6 = (-.063446)(719.46) + 92.25 + (-.04084)(-2.18)$$

$$+ (+.16642)(+676.06) + (-.007459)(+89.28)$$

$$+ (+.098183)(+16.05)$$

$$= + 160.11$$

$$JM_7 = 625.61 + (-.052689)(+160.06)$$

$$+ (+.13291)(+676.06) + (+.13937)(89.28)$$

$$= + 719.47$$

9) Final Moments

$$M_{jk} = D^*_{jk}\, JM_j + C^*_{kj}\, D^*_{kj}\, JM_k + C^*_{nj}\, D^*_{nj}\, JM_n + C^*_{pj}\, D^*_{pj}\, JM_p + FM^*_{jk} \qquad (3\text{-}96)$$

$$M_{jn} = D^*_{jn}\, JM_j + C^*_{nj}\, D^*_{nj}\, JM_n + FM^*_{jn} \qquad (3\text{-}101)$$

$$M_{ji} = D^*_{ji}\, JM_j + C^*_{ij}\, D^*_{ij}\, JM_i + C^*_{nj}\, D^*_{nj}\, JM_n + C^*_{mj}\, D^*_{mj}\, JM_m + FM^*_{ji} \qquad (3\text{-}104)$$

$M_{12} = -1002$

$M_{21} = -304$

$M_{27} = +440$

$M_{23} = -136$

$M_{32} = -97$

$M_{54} = +5$

$M_{56} = -5$

$M_{65} = +29$

$M_{63} = +109$

$M_{67} = -138$

$M_{36} = +\ 90$

$M_{34} = +\ 7$

$M_{43} = -\ 13$

$M_{45} = +\ 13$

$M_{76} = +\ 39$

$M_{72} = +\ 452$

$M_{78} = -\ 491$

$M_{87} = -\ 522$

CHAPTER V

SUMMARY AND CONCLUSION

The primary objective of this study was to develope a simplified method of analysis for Vierendeel trusses with inclined chords consisting of members of any cross section. The principles of the carry-over joint moment method were originally presented by Professor Tuma (22, 27, 30, 31, 32).

This method is much shorter than the moment distribution method from which this method is a modification. In a Vierendeel truss there are 2P + 2 joints where the P parameter represents the number of panels, and there are 6P + 2 end moments in all the members of the truss. The time ratio between carry-over joint moment method and moment distribution method is $\frac{2P+2}{6P+2}$. The time ratio varies from .50 for one panel to not reasonably less than .34 for many panels. There is also a further reduction in time for the carry-over method because the distribution and carry-over is completed in one step, whereas the moment distribution method takes two. This lowers the time saved ratio down to .25 through .17 depending on the number of panels used. In a Vierendeel tower the time saved ratio is $\frac{1}{2}\left(\frac{2P}{6P}\right)$ or .167. There are 2P joint in a Vierendeel tower and 6 P end moments in all members. There is an additional time saving due to having one carry-over procedure as compared to many depending upon number of panels for the moment distribution method.

The total amount of time saved by the elimination of the displacement carry-over procedures is very difficult to estimate because additional time is needed to solve for the new constants in the carry-over joint moment method. However, the time needed to calculate the new constants is estimated to be one-half of time required to complete the displacement carry-over procedures.

The total time saving for the carry-over joint method as compared to the moment distribution method is approximately forty-five per cent.

In addition to the time saved there is one more desirable feature of the carry-over joint moment method which is fewer calculations to complete, thereby lowering the chance for error.

SELECTED BIBLIOGRAPHY

1. Maugh, L. C., "Analysis of Vierendeel Trusses by Successive Approximations," Assn Int. des Ponts et Charpentes, Me'moires, Vol. 3, 1935, pp. 333-354.

2. Lightfoot, E., "A Moment Distribution for Vierendeel Bents and Girders with Inclined Chords," Proceedings, Instn. Civ. Engrs. Vol. 10, July, 1958, pp. 321-352.

3. Vierendeel, A., "Calcul des Ponts Vierendeel," Annales des Travaux Public de Belgium, Vol. 32, October, 1931, pp. 729-734.

4. Naylor, N., "Side Sway in Symmetrical Building Frames," Structural Engineer, Vo. 28, April, 1950, pp. 99-102.

5. Matheson, J. L., "Problem of Sway in Complicated Rigid Frames," Journal, Instn. Civ. Engrs., Vol. 24, April, 1945, pp. 99-124.

6. Tsao, C. H., "The Analysis of the Vierendeel Truss and The Wind Stress in Tall Buildings," Ph.D. Thesis, Illinois Institute of Technology, 1952.

7. Vierendeel, A. "Vierendeel Trusses (La Poutre Vierendeel)," Revue Universalle des Mines, Vo. 73, March 1, 1930, pp. 129-133.

8. Wilber, J. B., "Successive Elimination of Unknowns in Slope Deflection Method," Proceedings, ASCE, Vol. 61, December, 1935, pp. 1463-1468.

9. Maugh, L. C., Statically Indeterminate Structures, John Wiley and Sons, Inc., New York, N. Y., 1946.

10. Bateman, E. H., "Open Frame Girder," Journal, Instn. Civ. Engrs., November, 1935, pp. 67-93.

11. Sakai, T., "Practical Calculation Formulae of Parallel Chord Vierendeel Trusses with Constant Stiffness for Full Loads Derived from Differenzengleichung Method," Faculty Enginerring Memoirs, Hokkaido Imperial University, Sapporo, Japan, Vol. 4, November, 1939, pp. 331-354.

12. Muls, F., "Theo'rie Analytique des liqnes d' influence des poutres Vierendeel," Revue Universelle des Mines, Vol. 85, 1942, pp. 29-41.

13. Frocht, M. M., and M. M. Leven., "Photoelastic Analysis of Vierendeel Trusses," Civil Engineering, Vol. 8, October, 1938, pp. 636-688.

14. Bale, L., "Rigid Frames Without Diagonals (Vierendeel Truss)," Proceedings, ASCE, Vol. 67, January, 1941. pp. 93-106, June, 1941, pp. 1157-1165, and September, 1941, pp. 1263-1268.

15. de Medina, F., "La viga Vierendeel con Bridas de igual rigidez," Boletin, Montevideo Facultado de Ingenieria, Vol. 2, July, 1942, pp. 271-292.

16. Decarpentrie, E., "Influence of Curved Haunches in Calculations of Vierendeel Girders," Acier Stahl Steel, Vol. 23, January, 1958, pp. 34-38.

17. Cross, H., "Analysis of Continued Frames by Distributing Fixed End Moments," Transactions, ASCE, Vol. 96, 1932.

18. Young, D., "Analysis of Vierendeel Trusses," Proceedings, ASCE, Vol. 62, August, 1936, pp. 833-860.

19. Wix, H., and H. Dornau, "Beitray zum Momentenausgleichverfahren," Bauingenieur, Vol, 23, September 5, 1942, pp. 267-271.

20. Mijling, F. H., "De rekenmethode Cross gecombineera met de virtueie knoopverplaatsing," Ingenieur, Vol. 62, January 6, 1950, pp. 81-85.

21. Grinter, L. E., and C. H. Tsao, "Joint Translation by Cantilever Moment Distribution," Transactions, ASCE, Vol, 119, 1954, p. 1195.

22. Tuma, J. J., "Analysis of Continuous Beams by Carry-Over Moments," Proceedings, ASCE, Vol. 84, September, 1958, No. 1762.

23. Tuma, J. J., K. S. Havner, and S. E. French., "Analysis of Flat Plates by the Algebraic Carry-Over Method," Vol. 1, School of Civil Engineering Research Publication, Oklahoma State University, Stillwater, No. 1, 1958.

24. Gregory, R. G. "Analysis of Continued Rigid Frames with Joint Translation Prevented by Carry-Over Slopes and Carry-Over Moments," M. S. Thesis, Oklahoma State University, Stillwater, 1959.

25. Sturm, E. R., "Analysis of Multi-Story Rigid Frames with Joint Translation Permitted by Carry-Over Moments," M. S. Thesis, Oklahoma State University, Stillwater, 1959.

26. Riddle, G. M., "Analysis of Two Column Symmetrical Bents with External Ties by Carry-Over Moments," M. S. Thesis, Oklahoma State University, Stillwater, 1959.

27. Tuma, J. J., "Analysis of Continuous Trusses by Carry-Over Moments," Proceedings, ASCE, Vol. 85, December, 1959, No. 2314.

28. Tseng, J., "Analysis of Multistory Towers with Internal Ties by Carry-Over Moments," M. S. Thesis, Oklahoma State University, Stillwater, 1960.

29. Samuel, T., "Analysis of Vierendeel Trusses with Parallel Chords by Joint Moment Carry-Over Procedure," M. S. Thesis, Oklahoma State University, Stillwater, 1961.

30. Lecture by Tuma, J.J., Civil Engineering 620-Seminar in Carry-Over Methods, Oklahoma State University, Stillwater, Spring, 1959.

31. Lecture by Tuma, J. J., Civil Engineering 620-Seminar in Carry-Over Methods, Oklahoma State University, Stillwater, Spring, 1960.

32. Lecture by Tuma, J. J., Civil Engineering 5A4-Theory of Structures III, Oklahoma State University, Stillwater, Fall, 1959-60.

33. Young, D., "Vierendeel Truss," Engineering News-Record, Vol. 107, August 13, 1931, pp. 224-246.

34. Krausche, K., "Zur Berechnung des zweistieligen symmetrischen stockwerkrahmens, etc.," Beton-u Stahlbetonbau, Vol. 46, December, 1951, pp. 275-277.

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